Idiosyncratic Risk Innovations and the Idiosyncratic Risk-Return Relation

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Stocks with increases in idiosyncratic risk tend to earn low subsequent returns for a few months. However, high idiosyncratic risk stocks eventually earn persistently high returns. These results are consistent with positively priced idiosyncratic risk and temporary underreaction to idiosyncratic risk innovations. Because risk levels and innovations are correlated, the relation between historical idiosyncratic risk and returns may reflect both risk premiums and underreaction and yield misleading inference regarding the price of risk. The results reconcile previous work offering conflicting evidence on the price of idiosyncratic risk and help to discriminate among explanations for the idiosyncratic risk-return relation. (*JEL* G10, G11, G12)

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Introduction

A large number of empirical studies have examined the cross-sectional price of idiosyncratic risk. Recent empirical work has focused on the negative relation between historical idiosyncratic risk and returns documented by Ang et al. (2006, 2009), who find a negative price of idiosyncratic risk. This result is provocative because theory generally suggests that the price of idiosyncratic risk should be zero or positive (see Merton 1987). Some researchers find support for the results of Ang et al. (2006),[1] and others find the results to be fragile and possibly a manifestation of liquidity-related return patterns or skewness rather than reflective of an idiosyncratic risk-return relation.[2]

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[1] See Guo and Savickas (2010) and Peterson and Smedema (2011).

[2] See Bali and Cakici (2008), Huang et al. (2009), Fu (2009), Han and Lesmond (2011), Bali, Cakici, and Whitelaw (2011), and Boyer, Mitton, and Vorkink (2010).

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Overall, the existing literature offers a confusing picture of idiosyncratic risk-related return patterns.

In this paper, we examine the relation between idiosyncratic risk and returns. However, in contrast to prior work, we also consider the relation between idiosyncratic risk innovations and subsequent returns. Such a relation can be caused by frictions and/or investor biases that lead to temporary price underreaction to risk innovations.[3] Empirically, prior studies find evidence of apparent underreaction in a wide variety of settings, suggesting that prices may underreact to risk innovations as well.[4] Also, there is evidence that investors underreact to innovations in volatility when setting option prices (Poteshman 2001). Provided idiosyncratic risk is priced, underreaction to idiosyncratic risk innovations should lead to predictable patterns in returns.

We build a simple model to show that, in the presence of underreaction, risk innovations are negatively related to future returns. This occurs because prices reflect the risk news (which implies a higher discount rate and lower price) with a delay. However, provided that underreaction is temporary and idiosyncratic risk is positively priced, stocks with high idiosyncratic risk will eventually earn higher returns. We calibrate the model to deliver the empirical predictions for the idiosyncratic risk-return relation. We simulate the long-run expected return response to a positive idiosyncratic volatility shock. Our results suggest that expected returns are negative for a few months after the shock, with the most negative return in month one. However, returns become positive over time as the shock gradually decays. This is consistent with a positive long-run price of idiosyncratic risk, but investors temporarily underreact to risk innovations.

We test our model by partitioning historical idiosyncratic volatility into recent (e.g., idiosyncratic volatility over the last six months) and distant (e.g., idiosyncratic volatility lagged six months) components. Because idiosyncratic volatility is persistent, both recent and distant historical idiosyncratic volatility are informative about expected idiosyncratic risk. However, recent information may not be fully assimilated into prices, while distant information is more likely fully priced. Then our model suggests that, controlling for distant idiosyncratic volatility, recent idiosyncratic volatility should be negatively related to subsequent returns. Also, controlling for recent idiosyncratic volatility, distant idiosyncratic volatility should be positively related to subsequent returns.

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3 Such biases and frictions include biased investor beliefs (Barberis, Shleifer, and Vishny 1998; Daniel, Hirshleifer, and Subrahmanyam 1998), slow information diffusion (Hong and Stein 1999; Hong, Torous, and Valkanov 2007), information capacity constraints (Sims 2003), nontrivial transactions costs, and short-sale constraints.

4 Prior research suggests that investors underreact to earnings announcements (Ball and Brown 1968; Bernard and Thomas 1989), prior returns (Jegadeesh and Titman 1993), dividend news (Michaely, Thaler, and Womack 1995), share repurchases (Ikenberry, Lakonishok, and Vermaelen 1995), seasoned equity offerings (Loughran and Ritter 1995), increased R&D expenditures (Eberhart, Maxwell, and Siddique 2004), predictable demographic trends (DellaVigna and Pollet 2007), industry returns (Hong, Torous, and Valkanov 2007) and news about related firms (Cohen and Frazzini 2008; Menzly and Ozbas 2010).

We empirically confirm the predictions of our model using portfolio analysis and stock-level cross-sectional regressions. In the portfolio analysis, we find that after controlling for "recent" idiosyncratic volatility, "distant" idiosyncratic volatility is positively related to future returns. In addition, after controlling for "distant" idiosyncratic volatility, "recent" idiosyncratic volatility is negatively related to future returns. The Fama-MacBeth (1973) regression results echo the portfolio analysis. We also examine the returns of idiosyncratic volatility-sorted portfolios for many months after portfolio formation. Consistent with the model, returns of idiosyncratic volatility-sorted hedge portfolios are negative for about six months after portfolio formation and then are persistently positive. Additionally, the negative returns are concentrated in the month immediately after portfolio formation. This suggests that, in the long-run (i.e., after the effects of temporary underreaction), the idiosyncratic risk-return relation is positive.

Our results are robust to eliminating low-priced or illiquid stocks from the sample. Also, we generally report both value- and equal-weighted results and find that the choice of weighting scheme does not have much of an effect. Therefore, our results are not easily attributed to some subset of small stocks, and appear to be important for the average investor. This is notable because many characteristics known to be associated with returns (market-to-book, size, momentum, and liquidity) are attenuated and sometimes statistically insignificant under value weighting. Finally, we find that well-known liquidity-related return patterns that may explain the relation documented by Ang et al. (2006) cannot explain our results.[5]

Our paper contains three main contributions to the idiosyncratic volatility literature. First, we document a short-lived negative relation between idiosyncratic risk innovations and subsequent returns that explains much of the Ang, Hodrick, Xing, and Zhang (2006) anomaly. Second, we empirically document a long-run positive relation between idiosyncratic risk and returns. Documenting this relation is important because theory generally suggests that the equilibrium price of idiosyncratic risk should be nonnegative.

Third, we develop an underreaction framework to study the idiosyncratic risk-return relation. The framework suggests why different idiosyncratic risk proxies can offer conflicting evidence on the sign of the idiosyncratic risk-return relation. Viewed from our framework, it is not surprising that empirical studies that focus on recent data (e.g., Ang et al. (2006) use a trailing one-month window to calculate historical idiosyncratic volatility) often find a negative idiosyncratic risk-return relation, while studies that focus more on distant data (e.g., Lehmann 1990b uses five years of monthly data) are more likely to find a positive, or insignificant, relation. This occurs because recent data may not be fully assimilated into prices (so that underreaction is relatively important), while distant data is more likely fully priced. Overall, our

[5] See Bali and Cakici (2008), Huang et al. (2009), Fu (2009), and Han and Lesmond (2011).

theoretical and empirical results yield new insights into the nature of the idiosyncratic risk-return relation. In particular, the results suggest a positive price of idiosyncratic risk and underreaction to idiosyncratic risk innovations.

1. A Simple Model of Investor Underreaction to Risk Innovations

In this section, we present a simple model of price underreaction to idiosyncratic risk innovations. The model is intended to highlight how price underreaction can affect inferences from standard empirical procedures. We then deliver the model's empirical implications for the idiosyncratic risk-return relation.

1.1 Motivation for investor underreaction

Should prices underreact to risk innovations? There is apparent evidence of underreaction in a wide variety of settings (see footnote 4). Then it should not be surprising to find underreaction to risk innovations. First, idiosyncratic risk must be estimated from market data and other information sources. The relevant information set could easily be large, diverse, and continuously changing. Second, idiosyncratic risk estimates may be imprecise; this could exacerbate the effects of behavioral biases and investor underreaction (see Zhang 2006). Third, historical idiosyncratic risk is a particularly useful predictor of idiosyncratic risk. Historical idiosyncratic risk is often calculated as the standard deviation of the residuals from a time-series regression of returns on contemporaneous factors (e.g., market returns or the three factors of Fama and French 1996). For much of the historical sample, many investors likely lacked the technical expertise and/or computing power required to calculate this measure for a large number of stocks in real time. In this case, these investors could not use all publicly available information when forming an idiosyncratic risk estimate. Finally, there is evidence that investors underreact to volatility innovations when setting prices for S&P 500 index options (Poteshman 2001). Because nontraded firm-level idiosyncratic volatility estimates almost certainly suffer from more severe underreaction than traded stock index options, this suggests that investors underreact to idiosyncratic volatility innovations.

1.2 The model

Because we focus on risk (and discount rates), rather than on cash flows, we adopt a dividend discount model, where expected cash flows are held constant throughout. We assume that discount rate or requried return is determined solely by idiosyncratic risk and the idiosyncratic risk-return relation is positive.[6]

[6] Canonical asset pricing models (e.g., the Sharpe-Lintner CAPM) say that, because investors are free to diversify, idiosyncratic risk is not priced. However, violations of the assumptions underlying these models can lead to a

We assume that the price of the stock follows

$$p_t = \frac{d}{r_t^*} = \frac{d}{\gamma IV_t^*}, \tag{1}$$

where p_t is the price, d is the expected future dividend, r_t^* is the discount rate or required return, and IV_t^* is the "priced" idiosyncratic volatility as perceived by investors. γ is a positive risk-aversion parameter that maps idiosyncratic risk to discount rates.

Under our model, priced or perceived idiosyncratic volatility (IV_t^*) may differ from true idiosyncratic volatility (IV_t). We assume that true idiosyncratic volatility follows an AR(1) process,

$$log(IV_{t+1}) = c + \varphi log(IV_t) + \epsilon_{t+1}, \tag{2}$$

where $\phi \in (0, 1)$ and the error term is white noise $\epsilon \sim N(0, \sigma_\epsilon^2)$. Under this specification, true idiosyncratic volatility is persistent and each stock reverts to its long-run mean idiosyncratic volatility. We assume that the representative investor cannot, or does not, react to ϵ, possibly because ϵ is not observed. Therefore, this information cannot be directly incorporated into prices.

Priced or perceived idiosyncratic volatility (IV^*) evolves according to

$$IV_{t+1}^* = IV_t^* + \Theta(IV_t - IV_t^*). \tag{3}$$

Investors base their idiosyncratic volatility estimates on last period's forecast (IV_t^*) and the forecast error ($IV_t - IV_t^*$). Θ governs the speed with which investors update their forecasts and we consider $\Theta \in (0, 1)$. Under this specification, investors temporarily underreact to risk innovations. However, in the absence of additional shocks (ϵ), priced or perceived idiosyncratic volatility will eventually converge with true idiosyncratic volatility. When $IV_t = IV_t^*$, p_t is determined solely by true idiosyncratic risk (i.e., underreaction does not occur); this can be interpreted as the equilibrium stock price.

Under this model, we derive the expected gross return,

$$\begin{aligned} E_t(R_{t+1}) &= E_t\left(\frac{p_{t+1}+d}{p_t}\right) \\ &= E_t\left(\frac{IV_t^*}{IV_{t+1}^*} + \gamma IV_t^*\right) \\ &= E_t\left[\frac{IV_t^*}{IV_t^* + \Theta(IV_t - IV_t^*)} + \gamma IV_t^*\right]. \end{aligned} \tag{4}$$

nonzero price of idiosyncratic risk. An idiosyncratic risk premium may be caused by constraints or frictions that limit investors' ability to diversify (see Levy 1978; Merton 1987).

It is important to note that in Equation (4), the expected return depends on the representative investor's idiosyncratic volatility forecast error ($IV_t - IV_t^*$), the priced or perceived level of idiosyncratic volatility (IV_t^*), and the speed with which investors update their forecasts (Θ). If Θ is known at time t, then the expected return in the following period becomes known. This type of expected return process is based on the specification of perceived idiosyncratic volatility in Equation (3), and the model's assumptions that the expected cash flows are held constant throughout and discount rates are determined solely by idiosyncratic risk. As a result, our model abstracts away from cash-flow news and news about the other components of discount rates. Under these assumptions, it is straightforward to show that if perceived idiosyncratic volatility is too low (i.e., $IV_t > IV_t^*$), then the next period's expected return will be low (relative to the case in which $IV_t = IV_t^*$). This low expected return corresponds to an expected increase in priced idiosyncratic volatility and the discount rate, which reduces the price of the stock. Also, holding $IV_t - IV_t^*$ constant, higher perceived idiosyncratic volatility will be associated with higher expected returns.

1.3 Empirical implications

The model implies that, controlling for the idiosyncratic volatility forecast error ($IV_t - IV_t^*$), priced or perceived idiosyncratic volatility is positively related to subsequent returns,

$$\frac{\partial E_t(R_{t+1})}{\partial IV_t^*} > 0. \tag{5}$$

Also, controlling for the level of priced idiosyncratic volatility, recent innovations in idiosyncratic volatility are negatively related to subsequent returns. Since innovations in idiosyncratic volatility are positively related to true idiosyncratic volatility, Equation (4) is equivalent to testing,

$$\frac{\partial E_t[R_{t+1}]}{\partial IV_t} < 0, \tag{6}$$

where R_{t+1} is the time $t + 1$ gross return.

Equations (5) and (6) suggest that different idiosyncratic risk proxies can offer conflicting evidence on the sign of the idiosyncratic risk-return relation. Viewed from our framework, it is not surprising that empirical studies that focus on recent data (e.g., Ang et al. 2006 use a trailing one-month window to measure historical or true idiosyncratic volatility) often find a negative idiosyncratic risk-return relation, while studies that focus more on distant data (e.g., Lehmann 1990b uses five years of monthly data) are more likely to find a positive relation. This occurs because recent data may not be fully assimilated into prices (so that underreaction is relatively important), while distant data are more likely to be fully priced. As a result, following Ang et al. (2006), our empirical proxy for true historical idiosyncratic volatility (IV_t) is estimated using the most recent data.[7] And our empirical proxy for perceived idiosyncratic volatility (IV_t^*) is estimated using more distant data, to the extent that distant information is more likely to be fully priced.[8] True idiosyncratic volatility may differ from perceived idiosyncratic volatility since our model assumes that investors cannot react, or cannot slowly react, to idiosyncratic volatility shocks. In Section 3, we describe the empirical proxies in details. We show that historical idiosyncratic volatility estimated from both recent and distant data are informative about future expected idiosyncratic volatility, but they offer opposing pricing implications. We find evidence consistent with investor underreaction to recent idiosyncratic risk innovations, as well as a positive price of idiosyncratic risk.

1.4 Calibration and expected return response

In this section, we calibrate the model to generate empirical predictions for the idiosyncratic risk-return relation under investor underreaction. Figure 1 plots the long-run response of expected return to a one-standard-deviation shock to idiosyncratic volatility, which occurs at month zero, for various Θ values (0.05, 0.2, 0.6, and 0.95). Higher Θ corresponds to less investor underreaction (i.e., they adjust their forecast errors quickly to true level). Using empirical data on idiosyncratic volatility (discussed in the data section) and the calibrated parameters $c = 0.12$, $\phi = 0.85$, $\sigma_\epsilon = 0.2$, and $\gamma = 3$, we show the long-run response of expected returns to a positive idiosyncratic volatility shock in Figure 1.

The return pattern in Figure 1 suggests that with idiosyncratic volatility shocks, expected returns are negative for a few months, with the most negative return in month one. However, returns become positive over time as the shock gradually decays. A reasonable interpretation of this return pattern is that the negative returns in the short-run are likely attributable to a transitory friction (i.e., underreaction to risk innovations), while the long-run (equilibrium) price of idiosyncratic risk is positive.[9] Overall, Figure 1 is consistent with the model's implications of investor underreaction to idiosyncratic risk innovations, resulting in predictable negative returns for a period of time as the shock is gradually incorporated into prices. However, in the long run, as

[7] This can also be seen from Table 1, which suggests that historical idiosyncratic volatility estimated using more recent data (IVR) has higher explanatory power for future true idiosyncratic volatility than that estimated using more distant data (IVD). For example, focusing on the six-month threshold and one-month IVR, the adjusted R^2 is 0.47 using the most recent data versus 0.40 using more distant data. As a result, idiosyncratic volatility estimated from the most recent data (IVR) is a good proxy for true idiosyncratic volatility (IV_t).

[8] In Table 1, when focusing on the one-month threshold and one-month IVR, the adjusted R^2 is 0.37 using the most recent data versus 0.45 using more distant data. This result suggests that when the threshold is short (e.g., one-month), idiosyncratic volatility estimated from more distant data (IVD) captures much information from the recent data (IVR) and generates higher explanatory power for the future true idiosyncratic volatility.

[9] In Figure 1, the long-run mean expected return to which the process is reverting is 0.08%.

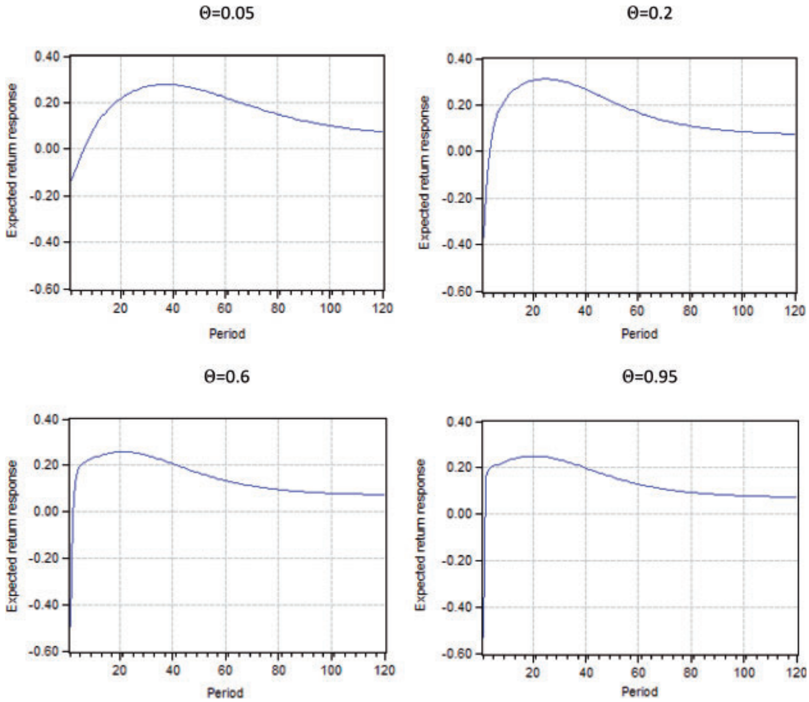


Figure 1
Expected return response to a positive idiosyncratic volatility shock
This figure presents the long-run response of expected return to a one-standard-deviation shock to idiosyncratic volatility for different Θ values 0.05, 0.2, 0.6, and 0.95. The parameters are calibrated with $c = 0.12$, $\phi = 0.85$, $\sigma_\epsilon = 0.2$, and $\gamma = 3$.

the price adjusts to incorporate the shock, the expected return will reflect the underlying risk. We empirically confirm these predictions.

Price underreaction can have an important effect on empirical research focused on the price of idiosyncratic risk. In the presence of investor underreaction, Figure 1 suggests that expected returns are negative for a period of time after a positive shock to true idiosyncratic volatility. If underreaction persists for several months, an empirical study that forms portfolios monthly may infer that the idiosyncratic risk-return relation is negative. However, Figure 1 was generated under an assumption of a positive price of idiosyncratic risk. In the following sections, we examine the empirical results from the data, and we obtain similar results consistent with this interpretation.

2. Data

2.1 Stock sample filters

This section describes the methods and data used in our empirical examination of the idiosyncratic risk-return relation. We obtain data from CRSP and Compustat. Stocks with a lagged price less than one dollar are removed from the sample. In the absence of such a filter, Ang et al. (2006, 2009) find a negative cross-sectional relation between stock returns and idiosyncratic risk. However, Bali and Cakici (2008) show that this result is driven by small, illiquid stocks. Because our paper's primary objective is to understand economically important return patterns associated with idiosyncratic risk, we impose a filter to remove small and illiquid stocks from the sample.

2.2 "Recent" and "distant" idiosyncratic volatility

Following Ang et al. (2006, 2009) and others, idiosyncratic volatility is calculated as the standard deviation of the residuals from a time-series regression of individual stock returns on the contemporaneous factors of Fama and French (1996),

$$r_{i,t} = \alpha + \beta_{i,MKT} MKT_t + \beta_{i,HML} HML_t + \beta_{i,SMB} SMB_t + \epsilon_{i,t}. \tag{7}$$

We show later in the paper that our results are robust to using the market model (i.e., omitting the HML and SMB factors). Daily data are used in the factor regressions.

We distinguish between "recent" historical idiosyncratic volatility (IVR; calucated using data from day $-t$ to day –7) and "distant" historical idiosyncratic volatility (IVD; calculated using data from day $-t-365$ to $-t$). t is the threshold that partitions the historical data. Although the threshold t is based on calendar days, we require at least $t/2-5$ trading day observations to calculate IVR and 125 trading day observations to calculate IVD. Although we often focus on a six-month (183-day) IVR-IVD threshold, thresholds of one and twelve months (38 and 365 days, respectively) are also examined.[10,11] When calculating IVR, we exclude data from the most recent seven calendar days to alleviate the effects of short-term reversals.

2.3 Proxying for expected idiosyncratic volatility

Idiosyncratic volatility calculated from a relatively long historical time series (six months or greater) of daily data is a good predictor, in the cross-section, of subsequent realized idiosyncratic volatility. In particular, this measure of historical idiosyncratic volatility is a better predictor than historical idiosyncratic volatility calculated using a shorter (e.g., one month) time series of daily data, historical idiosyncratic volatility calculated using monthly data, and predicted idiosyncratic volatility from a monthly EGARCH model (as in

[10] The one-month threshold is set to 38 days because a week is skipped before measuring returns. Given this skipped week, a 31-day threshold corresponds to three weeks of data.

[11] Our choice of IVR-IVD thresholds is guided by previous research related to price underreaction, which we explore as an explanation of our results later in the paper. Bernard and Thomas (1989) and Jegadeesh and Titman (1993) (respectively) show that earnings announcement underreaction and momentum persist for about a year.

Table 1
Predictive cross-sectional idiosyncratic volatility regressions, R^2

	One-year IV			One-month IV		
	IVR-IVD threshold			IVR-IVD threshold		
Explanatory variables	1	6	12	1	6	12
IVR	0.55	0.78	0.75	0.37	0.47	0.47
IVD	0.72	0.59	0.52	0.45	0.40	0.35
IVM	0.45	0.45	0.45	0.30	0.30	0.30
PV(1,1)	0.28	0.28	0.28	0.18	0.18	0.18
PV(*i,j*)	0.12	0.12	0.12	0.08	0.08	0.08
IVR, IVD, IVM, PV(1,1), PV(*i,j*)	0.77	0.80	0.76	0.50	0.50	0.48

This table reports time-series averages of R^2 from monthly cross-sectional regressions of realized idiosyncratic volatility on an intercept, IVR, IVD, 60-month trailing idiosyncratic volatility calculated using monthly data (IVM), out-of-sample conditional idiosyncratic volatility derived from an EGARCH (1, 1) model (PV(1,1)), and out-of-sample predicted idiosyncratic volatility derived from the best-fitting EGARCH (*i,j*) model, where $i, j \leq 3$ (PV(*i,j*)). Results are reported for one-year (365-day) realized idiosyncratic volatility (Columns 2-4) and one-month (31-day) realized idiosyncratic volatility (Columns 5-7). Data span 1966-2012.

Fu 2009). This can be seen in Table 1, where we report the time-series average R^2 of cross-sectional regressions of realized idiosyncratic volatility (calculated using daily data) on IVR, IVD, and other measures of historical and predicted idiosyncratic volatility. Individually, IVR and IVD offer the greatest explanatory power.[12] Adding other measures to a model that includes IVR and IVD yields essentially no increase in explanatory power. Also, one-month IVR is, individually, a suboptimal predictor of subsequent idiosyncratic volatility. Use of a longer historical sample (e.g., six-month IVR) or including IVD yields greater explanatory power.

We conclude that historical idiosyncratic volatility calculated from a long time series of daily data is an appealing instrument, in the cross-section, for expected idiosyncratic volatility. This is important because, as stressed by Fu (2009), theory relates expected risk and expected returns, not historical risk and expected returns.[13] For this reason, we use IVR and IVD (with various thresholds) to investigate the idiosyncratic risk-return relation.

2.4 The cross-sectional distribution of idiosyncratic volatility

Table 2 reports descriptive statistics of single-sorted and sequentially sorted stock portfolios, using a six-month IVR-IVD threshold. We refer to IVR- and IVD-sorted portfolios as IVR1-IVR5 and IVD1-IVD5, respectively. We add

[12] These results are similar to those of Guo, Kassa, and Ferguson (2014), who find that one-month historical idiosyncratic volatility is a better predictor of subsequent idiosyncratic volatility than predicted volatility from a monthly EGARCH model.

[13] As noted by Fu (2009), historical idiosyncratic volatility need not be a good point estimate of expected idiosyncratic volatility. However, historical idiosyncratic volatility may still be useful when forming portfolios with dispersion in expected idiosyncratic volatility, or when generating a point estimate of expected idiosyncratic volatility (e.g., through an AR(1) model).

Table 2
Idiosyncratic volatility-sorted portfolio descriptive statistics

Sort	IVR	IVD	IVC	ME	MB	PRET6	PRET1	MAXRET	ILLIQ	Cap. share	ΔEarn
All	2.81	2.82	−0.01	11.79	0.39	6.75	1.41	0.94	0.07		2.07
All SD	1.73	1.65	1.14	1.80	0.91	28.37	13.33	1.08	0.06		19.54
IVR1	1.15	1.32	−0.17	13.05	0.43	5.05	0.92	0.43	0.03	0.56	1.08
IVR2	1.76	1.91	−0.15	12.63	0.39	6.17	1.16	0.62	0.04	0.26	1.18
IVR3	2.40	2.55	−0.15	11.91	0.39	6.73	1.27	0.83	0.06	0.11	1.04
IVR4	3.27	3.38	−0.11	11.18	0.40	6.35	1.21	1.10	0.08	0.05	1.54
IVR5	5.44	4.93	0.52	10.19	0.36	9.48	2.50	1.72	0.13	0.02	5.80
IVD1	1.29	1.19	0.10	13.08	0.42	5.04	1.03	0.46	0.03	0.58	1.13
IVD2	1.88	1.81	0.07	12.63	0.38	5.84	1.18	0.65	0.04	0.25	1.19
IVD3	2.51	2.45	0.06	11.89	0.37	6.13	1.23	0.86	0.06	0.11	1.31
IVD4	3.34	3.30	0.04	11.16	0.38	6.13	1.29	1.12	0.08	0.05	1.85
IVD5	5.01	5.34	−0.34	10.20	0.43	10.64	2.32	1.61	0.12	0.02	4.93
IVRS1	1.82	2.55	−0.73	12.12	0.42	6.10	0.94	0.67	0.04	0.30	1.24
IVRS2	2.28	2.67	−0.39	12.01	0.42	6.55	1.12	0.79	0.05	0.22	1.35
IVRS3	2.63	2.78	−0.15	11.88	0.41	6.58	1.24	0.89	0.06	0.20	1.72
IVRS4	3.07	2.92	0.15	11.69	0.39	6.83	1.42	1.01	0.07	0.17	2.30
IVRS5	4.23	3.17	1.06	11.26	0.32	6.71	2.32	1.34	0.10	0.12	3.84
IVDS1	2.55	1.83	0.72	12.20	0.33	4.74	1.27	0.84	0.06	0.33	1.94
IVDS2	2.65	2.33	0.31	12.00	0.37	5.61	1.29	0.88	0.06	0.22	1.90
IVDS3	2.75	2.69	0.06	11.85	0.40	6.71	1.36	0.92	0.06	0.19	2.05
IVDS4	2.89	3.11	−0.22	11.66	0.42	7.63	1.49	0.98	0.07	0.16	2.14
IVDS5	3.18	3.18	−0.94	11.25	0.43	9.08	1.64	1.08	0.07	0.10	2.38

This table reports time-series averages of idiosyncratic volatility sorted stock portfolio characteristics. For all stocks, characteristic means (All) and standard deviations (All SD) are reported. For sorted stock portfolios, only means are reported. Characteristics are distant idiosyncratic volatility (IVD), recent idiosyncratic volatility (IVR), the change in idiosyncratic volatility (IVC, defined as IVR-IVD), log of the market value of equity (ME), log of the market-to-book ratio (MB), 1- and 6-month prior returns (PRET1 and PRET6), log of the trailing one-year average of the absolute value of the daily return divided by dollar volume (ILLIQ; see Amihud 2002), maximum daily return over the last omnth (MAXRET; see Bali et al. 2011), market capitalization share, and earnings change contemporaneous with the IVR measurement period (defined as one-year earnings ending in the IVR measurement period less lagged one-year earnings, with the difference scaled by lagged market capitalization). Volatility and returns are reported as a percent. IVD and IVR quintile portfolios are formed by sorting stocks by IVD and IVR (respectively). IVRS portfolios are formed by first sorting stocks on IVD into quintiles, then within each IVD quintile, stocks are further sorted into subquintiles by IVR. IVRS1 is the portfolio of stocks with the lowest IVR within each IVD portfolio, and IVRS5 is the portfolio of stocks with the highest IVR within each IVD portfolio. IVDS portfolios are formed similarly. Data span 1966-2012.

an "S" to indicate a sequentially sorted portfolio. For example, IVRS portfolios are formed by first sorting stocks on IVD into quintiles, and then within each IVD quintile, stocks are further sorted into subquintiles by IVR. IVRS1 is the portfolio of stocks with the lowest IVR within each IVD portfolio, and IVRS5 is the portfolio of stocks with the highest IVR within each IVD portfolio.[14] The columns labeled IVR and IVD report the idiosyncratic volatility associated with each portfolio (e.g., the IVR5 portfolio contains stocks with an average daily idiosyncratic volatility of 5.44% over the recent historical period (IVR) and 4.93% over the distant historical period (IVD)). Table 2 demonstrates that a single sort on IVD is very similar to a sort on IVR; both IVD and IVR increase in a similar way from IVR1 to IVR5 and from IVD1 to

[14] Similarly, IVDS portfolios are formed by first sorting stocks on IVR into quintiles, and then within each IVR quintile, stocks are further sorted into subquintiles by IVD. IVDS1 is the portfolio of stocks with the lowest IVD within each IVR portfolio, and IVDS5 is the portfolio of stocks with the highest IVD within each IVR portfolio.

IVD5. This occurs because idiosyncratic volatility, not surprisingly, exhibits positive autocorrelation.[15] However, sequential sorts break the tight link between IVD and IVR. The IVDS portfolios exhibit substantial variation in IVD and little variation in IVR. Similarly, the IVRS portfolios exhibit substantial variation in IVR but little variation in IVD.

Table 2 also reports the mean change in idiosyncratic volatility (IVC, defined as the difference between IVR and IVD) for the portfolios. Both the IVRS- and IVDS-sorted portfolios exhibit substantial variation and are monotonic in IVC. This occurs because varying IVR while holding IVD constant is equivalent to varying IVC while holding IVD constant. Then the IVRS and IVDS portfolios can be used to examine the relation between idiosyncratic risk innovations and subsequent returns.

2.5 Idiosyncratic volatility and firm size

Table 2 contains additional portfolio descriptive statistics. On average, high idiosyncratic volatility stocks are small and illiquid. A difficulty encountered when interpreting the returns of hedge portfolios formed by single sorts on idiosyncratic volatility (e.g., IVR5 less IVR1) is that such sorts are similar to a sort on size or liquidity (i.e., high idiosyncratic volatility portfolios tend to contain many small, illiquid stocks and the reverse). Then, any difference in mean returns across the portfolios could be driven by a subset of small, illiquid firms. This could be true even if portfolio returns are value weighted; because the average firm in the IVR5 and IVD5 portfolio is small, even small stocks could receive a large portfolio weight. For this reason, return patterns associated with small stocks (e.g., bid-ask bounce, reversals, short-selling constraints) are a plausible explanation for anomalous returns associated with the IVR5 or IVD5 portfolios or any portfolios formed from these portfolios (e.g., the IVR hedge portfolio). Also, nonzero mean returns of IVD or IVR hedge portfolios may have little economic importance because the returns may be driven by a small subset of small stocks.

Such concerns are alleviated when examining the returns of the IVRS and IVDS hedge portfolios because, by construction, the IVRS5 hedge portfolio must contain stocks from the IVD1 portfolio (which consists of many large stocks). Similarly, the IVDS5 portfolio must contain stocks from the IVR1 portfolio. Also, the IVRS portfolios can be interpreted as a sorting procedure that induces variation in IVR, while controlling for IVD; this should alleviate microstructure concerns because idiosyncratic risk innovations are less obviously related to liquidity and size than idiosyncratic risk levels. Consistent with this, the IVRS and IVDS portfolios exhibit much more size balance than the IVR and IVD portfolios. For example, the IVR5 portfolio consists of, on

[15] Consistent with this, the cross-sectional IVR-IVD correlation is quite high. The time-series average of the IVR-IVD cross-sectional Pearson correlation is 0.78. The time-series average of the Spearman correlation is 0.87.

average, 2% of total market capitalization. The IVRS5 portfolio consists of, on average, 12% of total market capitalization.

3. The Cross-Sectional Price of Idiosyncratic Volatility

In this section, we explore the relation between idiosyncratic volatility and the cross-section of returns. Our model suggests that priced or perceived idiosyncratic volatility is positively related to subsequent returns,

$$\frac{\partial E_t(R_{t+1})}{\partial IV_t^*} > 0. \tag{8}$$

Also, our model suggests that true idiosyncratic volatility is negatively related to subsequent returns,

$$\frac{\partial E_t(R_{t+1})}{\partial IV_t} < 0. \tag{9}$$

Following Ang et al. (2006), our empirical proxy for true historical idiosyncratic volatility (IV_t) is estimated using the most recent data. And our empirical proxy for perceived idiosyncratic volatility (IV_t^*) is estimated using more distant data, to the extent that distant information is more likely to be fully assimilated into prices. True idiosyncratic volatility may differ from perceived idiosyncratic volatility since our model assumes that investors cannot react, or they slowly react, to idiosyncratic volatility shocks. We empirically confirm the predictions of our model using portfolio analysis and stock-level cross-sectional regressions.

3.1 Portfolio analysis

In this section, we examine the idiosyncratic risk-return relation using portfolio level analysis. Panel A of Table 3 reports the mean returns of the hedge (high-minus-low idiosyncratic volatility) portfolio for the IVD, IVR, IVDS, and IVRS sorting procedures. We report raw portfolio returns and a four-factor alpha (using the three factors of Fama and French 1996 and a momentum factor). Equal- and value-weighted returns are reported for IVR-IVD thresholds of one, six, and twelve months.

Table 3 confirms the findings of Ang et al. (2006). The one-month IVR hedge portfolio has a negative mean return, especially under value weighting or when using the risk-adjustment model. However, this relation is often not significant when examining raw or equal-weighted returns. IVRS hedge portfolio mean returns are always negative and highly statistically significant. For example, using a six-month IVR-IVD threshold, the equal-weighted IVRS hedge portfolio has a mean return of -51.8 basis points per month, with a heteroscedasticity-robust t-statistic of -4.25. For each threshold and weighting scheme, the statistical evidence for a negative IVRS-return relation is

Table 3
Idiosyncratic volatility-sorted hedge portfolio returns

Panel A: One-month IVR-IVD threshold

	EW raw	EW alpha	VW raw	VW alpha
IVR	−0.363	−0.480***	−0.683***	−0.844***
	(0.278)	(0.178)	(0.284)	(0.174)
IVD	−0.042	−0.259	−0.446	−0.737***
	(0.306)	(0.191)	(0.334)	(0.195)
IVRS	0.549***	−0.504***	−0.683***	−0.668***
	(0.086)	(0.074)	(0.121)	(0.105)
IVDS	0.301*	0.096	−0.028	−0.276**
	(0.161)	(0.104)	(0.186)	(0.130)

Panel B: Six-month IVR-IVD threshold

	EW raw	EW alpha	VW raw	VW alpha
IVR	−0.212	−0.405***	−0.680**	−0.964***
	(0.303)	(0.190)	(0.339)	(0.193)
IVD	0.134	−0.121	−0.237	−0.515***
	(0.294)	(0.183)	(0.313)	(0.176)
IVRS	−0.518***	−0.526***	−0.670***	−0.705***
	(0.122)	(0.097)	(0.154)	(0.128)
IVDS	0.522***	0.367***	0.280**	0.065
	(0.102)	(0.081)	(0.127)	(0.114)

Panel C: Twelve-month IVR-IVD threshold

	EW raw	EW alpha	VW raw	VW alpha
IVR	−0.015	−0.266	−0.490	−0.846***
	(0.305)	(0.187)	(0.333)	(0.193)
IVD	0.230	−0.029	−0.153	−0.410**
	(0.283)	(0.174)	(0.301)	(0.174)
IVRS	−0.372**	−0.443**	−0.573***	−0.646***
	(0.141)	(0.102)	(0.166)	(0.135)
IVDS	0.448***	0.355***	0.338***	0.258**
	(0.084)	(0.076)	(0.107)	(0.112)

Panel D: Monthly measures

	EW raw	EW alpha	VW raw	VW alpha
EGARCH (i, j)	−0.030	−0.123	−0.010	−0.193
	(0.223)	(0.136)	(0.241)	(0.135)
IVM	−0.163	−0.250	−0.328	−0.446***
	(0.290)	(0.164)	(0.296)	(0.156)

This table reports monthly returns (in percent) of idiosyncratic volatility-sorted hedge portfolios. Portfolios are reformed monthly from 1966-2012. Equal- and value-weighted raw returns and four-factor alphas (using the three factors of Fama and French 1996 and a momentum factor) are reported. Panels A, B, and C report mean returns of hedge portfolios formed by sorts on recent historical idiosyncratic volatility (IVR, calculated over the last one, six, or twelve months), distant historical idiosyncratic volatility (IVD, calculated over the year prior to IVR), sequential sorts on IVR then IVD (IVDS), and sequential sorts on IVD then IVR (IVRS). Panel D reports mean returns of hedge portfolios formed by a sort on the last five years of idiosyncratic volatility using monthly data (IVM) and out-of-sample predicted volatility from the best fitting EGARCH(i, j) model, where $i, j \leq 3$ (see Fu 2009; Fink, Fink, and He 2012; Guo, Kassa, and Ferguson 2014). Newey-West standard errors are reported below the returns. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

stronger than the evidence for a negative IVR-return relation. Therefore, controlling for distant idiosyncratic volatility reveals a stronger relation between recent idiosyncratic volatility and returns.

The IVDS hedge portfolio mean return is always positive and significant when using a twelve-month IVR-IVD threshold, always positive and generally statistically significant when using a six-month threshold, and generally insignificant when using a one-month threshold. Therefore, this analysis reveals, under certain conditions, a positive relation between idiosyncratic risk and subsequent returns.

Panel D of Table 3 reports mean returns of hedge portfolios formed using idiosyncratic risk proxies generated from monthly data. The IVM hedge portfolio is formed by sorting stocks on idiosyncratic volatility calculated from monthly data. The IVM hedge portfolios mean raw returns are insignificant, although the alpha is negative under value weighting. We also report the mean returns of a hedge portfolio formed by sorting on predicted volatility from the best-fitting EGARCH (*i,j*) model for each stock, where $i, j \leq 3$ (following Fu 2009). There is little evidence of nonzero returns when examining the EGARCH (*i,j*) hedge portfolio. This is consistent with Guo, Kassa, and Ferguson (2014) and Fink, Fink, and He (2012), who show that use of an unbiased estimator results in an insignificant relation.

Overall, Table 3 suggests that IVD is positively related to future returns, and IVR is negatively related to future returns. These results are consistent with our model of investor underreaction to risk innovations.

3.2 Fama-MacBeth regressions

So far we have tested the significance of IVR and IVD as a determinant of the cross-section of future stock returns at the portfolio level. This portfolio-level analysis has the advantage of being nonparametric in the sense that we do not impose a functional form on the relation between idiosyncratic volatility and future stock returns. The portfolio-level analysis also has two potentially significant disadvantages. First, it throws away a large amount of information in the cross-section via aggregation. Second, it is a difficult setting in which to control for multiple effects or stock characteristics simultaneously. Consequently, we now examine the cross-sectional relation between IVR, IVD, and expected returns at the stock level using Fama and MacBeth (1973) regressions:

$$r_{i,t+1} = \alpha + \beta_{t,IVD} IVD_{i,t} + \beta_{t,IVR} IVR_{i,t} + \gamma_t X_t + \epsilon_{i,t}, \tag{10}$$

where X is a vector of controls. This specification allows us to examine the partial IVR and IVD return relations. Our model implies that $\beta_{t,IVD} > 0$ and $\beta_{t,IVR} < 0$. Our focus on partial IVR and IVD return relations is motivated by our belief that investors may react differently to recent and distant historical idiosyncratic volatility, perhaps due to underreaction to idiosyncratic risk innovations.

Our specification includes IVR and IVD in levels. We also examine a regression of returns on IVR and IVC (equal to IVR–IVD). This regression can be interpreted as a horse race between idiosyncratic risk levels and innovations as explanatory variables. Cross-sectional regressions are run using both OLS and WLS (with weights equal to market capitalizations).[16] The OLS and WLS regressions correspond to equal-weighted and value-weighted approachs (respectively).

Stock characteristics considered in the cross-sectional regressions are market capitalization, market-to-book ratio, prior returns from month -6 to month -2, prior returns over month -1, illiquidity,[17] and the maximum daily return over the prior month (see Bali, Cakici, and Whitelaw 2011). We consider the latter because this characteristic appears to be informative about expected skewness. Results are robust to using other skewness measures, including expected skewness as constructed by Boyer, Mitton, and Vorkink (2010) and historical skewness.

Table 4 reports results using a one-, six-, and twelve-month IVR-IVD threshold. Consistent with the results of Ang et al. (2006), one-month IVR, as the sole explanatory variable, is negatively related to subsequent returns. However, as the sole explanatory variable, IVR is not significant in any of the WLS regressions. This suggests that the negative IVR-return relation is only important for smaller stocks. Also, the negative IVR-return relation is strongest for one-month IVR and weakest (insignificant) for twelve-month IVR, even when using OLS. This is troubling if one interprets the negative IVR-return relation as evidence supporting a negative idiosyncratic risk-return relation because use of a stronger proxy for idiosyncratic risk (twelve-month IVR; see Table 1) yields a weak, and sometimes insignificant, idiosyncratic risk-return relation.

Regressions with both IVR and IVC suggest that there is a more robust relation between idiosyncratic risk innovations and subsequent returns than idiosyncratic risk levels and subsequent returns. In each of these regressions, the t-statistic associated with the IVC parameter is larger in magnitude than the t-statistic associated with the IVR parameter. Of these six regressions, IVR is marginally significant in one case, and insignificant in the others. IVC is always significant, usually at the 1% level. These results suggest that idiosyncratic risk innovations, rather than levels, drive returns, particularly when using better proxies for idiosyncratic risk (six- and twelve-month IVR).

[16] Under WLS, we minimize ?$w_i e_i^2$, where w_i is market capitalization and e_i is the difference between the actual and fitted return. Under OLS, $w_i = 1$.

[17] Following Amihud (2002), illiquidity is calculated as the log of the trailing one-year average of daily $|R_{i,t}|/DVOL_{i,t}$, where $R_{i,t}$ is the return of stock i on day t and $DVOL$ is dollar volume.

Table 4
Fama-MacBeth cross-sectional regressions with idiosyncratic volatility

Panel A: One-month IVR-IVD threshold

Weight	IVR	IVD	IVC	ME	MB	PRET1	PRET6	ILLIQ	MAXRET
OLS	−0.157***								
	(0.045)								
OLS	−0.118*		−0.124**						
	(0.066)		(0.064)						
OLS	−0.241***	0.124***							
	(0.021)	(0.056)							
OLS	−0.110***	0.005		0.071	−0.215***	−0.051***	0.008***	0.185***	−2.747***
	(0.024)	(0.055)		(0.055)	(0.044)	(0.004)	(0.002)	(0.048)	(0.752)
WLS	−0.079								
	(0.072)								
WLS	−0.068		−0.036						
	(0.186)		(0.091)						
WLS	−0.104**	0.036							
	(0.040)	(0.091)							
WLS	−0.248***	−0.115		−0.092**	−0.088	−0.031***	0.009***	0.027	3.170**
	(0.044)	(0.089)		(0.055)	(0.069)	(0.006)	(0.003)	(0.045)	(1.570)

Panel B: Six-month IVR-IVD threshold

Weight	IVR	IVD	IVC	ME	MB	PRET1	PRET6	ILLIQ	MAXRET
OLS	−0.121*								
	(0.063)								
OLS	−0.066		−0.284***						
	(0.068)		(0.042)						
OLS	−0.350***	0.284***							
	(0.043)	(0.042)							
OLS	−0.323***	0.255***		0.082	−0.234***	−0.052***	0.009***	0.188***	−2.943***
	(0.039)	(0.043)		(0.055)	(0.044)	(0.004)	(0.002)	(0.048)	(0.622)
WLS	−0.089								
	(0.098)								
WLS	−0.024		−0.262***						
	(0.108)		(0.078)						
WLS	−0.286***	0.262***							
	(0.079)	(0.078)							
WLS	−0.406***	0.160***		−0.079	−0.098	−0.030***	0.009***	0.028	0.272
	(0.069)	(0.078)		(0.055)	(0.069)	(0.006)	(0.003)	(0.044)	(1.316)

Panel C: Twelve-month IVR-IVD threshold

Weight	IVR	IVD	IVC	ME	MB	PRET1	PRET6	ILLIQ	MAXRET
OLS	−0.064								
	(0.067)								
OLS	−0.023		−0.242***						
	(0.068)		(0.036)						
OLS	−0.265***	0.242***							
	(0.058)	(0.036)							
OLS	−0.179***	0.202***		0.098*	−0.241***	−0.051***	0.008***	0.182***	−4.409***
	(0.052)	(0.034)		(0.055)	(0.044)	(0.004)	(0.002)	(0.048)	(0.606)
WLS	−0.056								
	(0.104)								
WLS	0.000		−0.269***						
	(0.110)		(0.076)						
WLS	−0.269***	0.269***							
	(0.096)	(0.075)							

(continued)

Table 4
Continued

Panel C: Twelve-month IVR-IVD threshold

Weight	IVR	IVD	IVC	ME	MB	PRET1	PRET6	ILLIQ	MAXRET
WLS	−0.366***	0.181***		−0.063	−0.097	−0.030***	0.008***	0.036	−0.767
	(0.086)	(0.071)		(0.055)	(0.069)	(0.006)	(0.003)	(0.044)	(1.280)

This table reports the average slope coefficients from the Fama and MacBeth (1973) cross-sectional regressions of one-month-ahead stock excess returns on lagged stock characteristics. Characteristics are recent historical idiosyncratic volatility (IVR), distant idiosyncratic volatility (IVD), the log of the market value of equity (ME), the log of the market-to-book ratio (MB), 1- and 6-month prior returns (PRET1 and PRET6), a measure of illiquidity (ILLIQ) based on Amihud (2002), and the maximum daily return over the last month (MAXRET; see Bali et al. 2011). Results are reported using an IVR-IVD threshold of one, six, and twelve months. Results are reported for a standard cross-sectional regression (OLS) and a cross-sectional regression in which each observation is weighted by market capitalization (WLS). Data are monthly and span 1966-2012. Newey-West standard errors are reported below the slope coefficients to determine the statistical significance of the coefficients. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Consistent with innovations, rather than levels, driving returns, the IVR-return relation is far stronger when controlling for IVD (varying IVR while controlling for IVD is equivalent to varying IVC while controlling for IVD). For example, in the twelve-month IVR-IVD threshold OLS regression, including IVD decreases the IVR parameter from -0.064 to -0.265, and the t-statistic from -0.95 to -4.58. Similarly, the IVR parameter is not significant in the univariate WLS regressions, although controlling for IVD reveals a significant IVR parameter.

Table 4 suggests that the estimated IVR and IVD parameters are of a similar magnitude, but have opposing signs (the estimated IVR parameter is always negative and the estimated IVD parameter is always positive). This is also consistent with a negative relation between idiosyncratic risk innovations (IVR–IVD) and subsequent returns. In addition, the IVD and IVR parameters have similar magnitudes in the equal- and value-weighted regressions. This suggests that the IVR and IVD return relations are pervasive, not likely to be explained by return patterns associated with small stocks (e.g., bid-ask bounce or reversals), and likely relevant to the average investor. In contrast, in the WLS regression with only IVR, the slope parameter is never significant. Therefore, the results of this section suggest that the return patterns documented in this paper are of greater economic importance than previously documented results related to idiosyncratic risk.

Overall, the empirical results of Table 4 are consistent with the model of investor underreaction. Controlling for IVD, IVR is negatively related to subsequent returns. Controlling for IVR, IVD is positively related to subsequent returns. In a horse race between IVR and IVC, IVC is generally highly significant, while IVR is not. These results suggest that risk innovations, rather than risk levels, are driving the relation with subsequent returns.

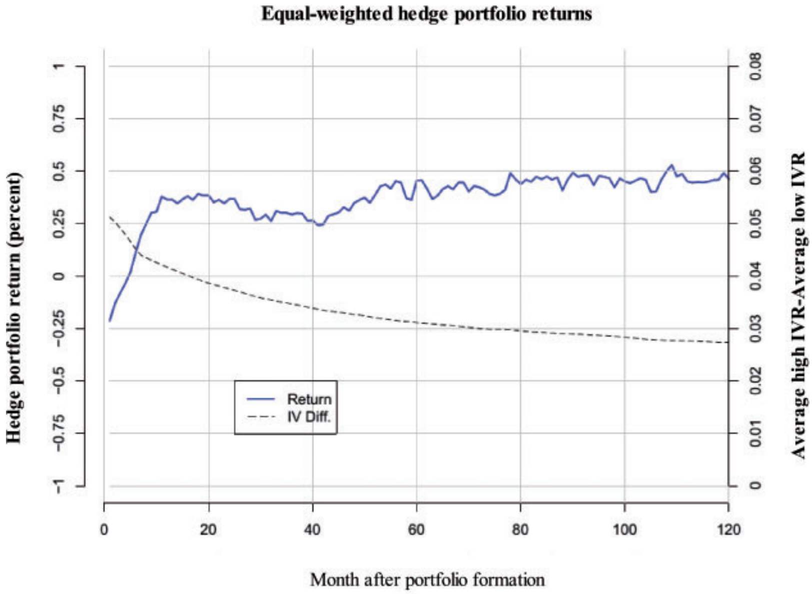
4. Longer Holding Period

So far we have tested that idiosyncratic risk innovations are negatively related to subsequent returns. However, risk innovations and risk levels are correlated (see Table 2). To separate the effects of idiosyncratic risk levels and innovations, we examine the significance of a cross-sectional relation between idiosyncratic volatility and longer horizon future returns. This is motivated by the empirical fact that risk innovations are not very persistent, while risk levels are quite persistent. Therefore, one way to separate the effects of idiosyncratic risk levels and innovations is to examine returns for many months after portfolio formation. Provided the effects of risk levels are more persistent than the effects of risk innovations, sufficiently deferred hedge portfolio returns should primarily reflect the price of idiosyncratic risk.

Importantly, examining deferred returns will only be useful if idiosyncratic volatility is persistent. The time-series average of the cross-sectional correlation between six-month IVR and the six-month IVR in three, five, and ten years after portfolio formation is 0.56, 0.50, and 0.44, respectively (the Spearman correlations are 0.82, 0.77 and 0.68, respectively). This can also be seen in Figure 2, which presents the results of this section graphically. In particular, the dashed plot shows the evolution of the difference in IVR across the high and low idiosyncratic volatility portfolios. Even after ten years, the equal-weighted IVR difference of 0.028 (and value-weighted difference of 0.022) is large compared to the average cross-sectional mean and standard deviation of IVR (0.028 and 0.017; see Table 2). We conclude that IVR can be reasonably used to form portfolios with dispersion in expected idiosyncratic volatility long after portfolio formation.

Table 5 reports twelve-month equal-weighted returns of idiosyncratic volatility-sorted hedge portfolios for up to ten years after portfolio formation. First, we note that the return patterns documented in the prior section are not very persistent. The negative returns of the equal-weighted IVRS hedge portfolio persist for only twelve months after portfolio formation. In fact, the twelve-month equal-weighted IVR hedge portfolio raw return is slightly positive. Although Table 5 does not report value-weighted results, Figure 2 indicates that the negative idiosyncratic risk-return relation is not very persistent even for value-weighted returns. In this figure, equal-weighted IVR hedge portfolio returns are negative for about five months after portfolio formation, while value-weighted returns are negative for about one year. In both cases the negative returns are largest in magnitude in the month immediately after portfolio formation, and then they quickly attenuate. After about eighteen months, the monthly returns of both equal- and value-weighted hedge portfolios are always positive.

In Table 5, the mean returns of every equal-weighted idiosyncratic volatility-sorted portfolio (IVR, IVD, IVRS, and IVDS) are always positive starting twelve months after portfolio formation (although these returns are not



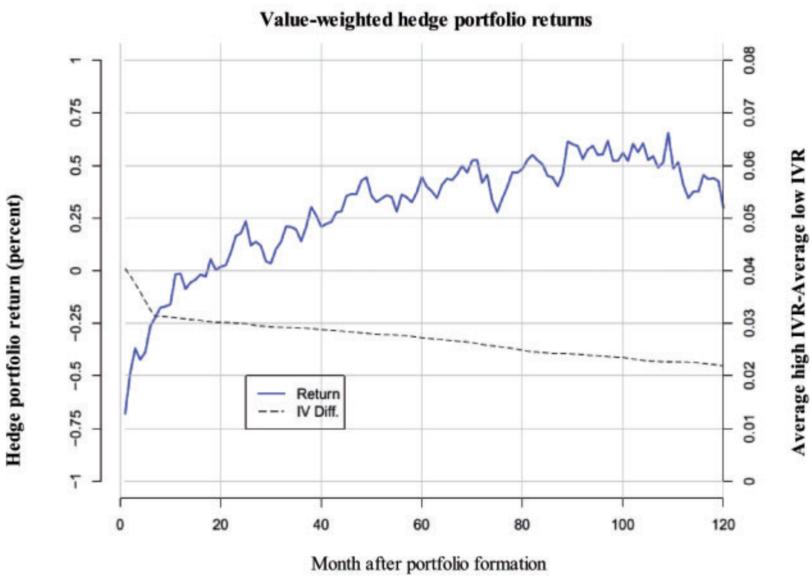


Figure 2
Idiosyncratic volatility hedge portfolio returns
This figure displays mean returns and dispersion in idiosyncratic volatility of high-low idiosyncratic volatility hedge portfolios, by month after portfolio formation. Idiosyncratic volatility is calculated as six-month historical idiosyncratic volatility (IVR). Portfolios are reformed monthly from 1966-2012. Value-weighted returns are calculated using weights at month zero. IV difference is the difference in return measurement period IVR across the extreme quintile portfolios.

Table 5
Twelve-month returns of historical idiosyncratic volatility-sorted hedge portfolios for ten years after portfolio formation

Return (months)	IVR	IVD	IVRS	IVDS	IVR VW
$R_{1,12}$	2.46	3.74	−1.10**	3.61***	−3.24
	(1.78)	(1.76)	(0.52)	(1.06)	(1.94)
$R_{13,24}$	5.40	4.83	2.00	1.34*	0.37
	(3.17)	(3.09)	(1.06)	(0.71)	(2.80)
$R_{25,36}$	4.42	3.78	1.91	0.64	2.01
	(2.63)	(2.56)	(1.53)	(0.48)	(1.59)
$R_{37,48}$	4.13	4.26	1.13	1.32	3.58
	(2.50)	(2.55)	(1.46)	(1.53)	(1.98)
$R_{49,60}$	5.53*	5.47*	1.46	1.59	4.31
	(2.83)	(2.85)	(0.83)	(0.93)	(2.93)
$R_{61,72}$	5.65*	5.94**	1.19	2.32**	5.01*
	(2.68)	(2.61)	(0.86)	(0.87)	(2.59)
$R_{73,84}$	5.78**	5.43*	1.64*	1.70*	5.19**
	(2.54)	(2.56)	(0.80)	(0.91)	(2.49)
$R_{85,96}$	6.13**	5.87**	2.31**	0.87	6.43***
	(2.71)	(2.46)	(0.86)	(0.86)	(2.56)
$R_{97,108}$	5.93***	6.19***	1.46*	1.46	6.33***
	(2.37)	(2.48)	(0.74)	(1.01)	(2.29)
$R_{109,120}$	6.21**	6.02**	1.78*	1.58	5.49**
	(2.66)	(2.67)	(0.99)	(1.09)	(2.74)

This table reports returns (in percent) of equal-weighted idiosyncratic volatility-sorted hedge portfolios for months 1-120 subsequent to portfolio formation, in twelve-month increments. Portfolios are reformed monthly from 1966-2012. Hedge portfolios are formed by single sorts on recent historical idiosyncratic volatility (IVR, calculated over the last six months), distant historical idiosyncratic volatility (IVD, calculated over the year prior to IVR), sequential sorts on IVR then IVD (IVDS), and sequential sorts on IVD then IVR (IVRS). The IVR VW column reports value-weighted results with weights determined at month zero. Newey-West standard errors are reported below each mean return. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

always significant). Starting about five years after portfolio formation, twelve-month returns of the IVR and IVD hedge portfolio are generally around 6% and always at least marginally significant. Combined with previously reported results, this provides evidence that a single measure of idiosyncratic risk (IVR) can be sometimes positively and sometimes negatively related to subsequent returns, depending on the return measurement period. The long-run positive relation is consistent with theory, which predicts a nonnegative price of idiosyncratic risk. Indeed, compensation for risk seems a particularly appealing explanation for the long-run positive returns in Table 5 and Figure 2, as most types of mispricing are likely corrected within five years. In untabulated results, we find that the long-run positive returns of idiosyncratic volatility-sorted portfolios are robust to the use of different return aggregation periods and IVR-IVD thresholds (including the use of one-month IVR), value weighting, and controlling for size and liquidity. Overall, this evidence suggests that the underlying price of idiosyncratic risk is positive, with the negative returns immediately after portfolio formation likely attributable to investor underreaction to risk innovations.

Table 6
Twelve-month returns of historical idiosyncratic volatility-sorted hedge portfolios for ten years after portfolio formation, market model

Return (months)	IVR	IVD	IVRS	IVDS
$R_{1,12}$	6.88**	7.07**	1.95	2.85***
	(3.08)	(2.88)	(1.39)	(1.05)
$R_{13,24}$	8.30***	7.22***	3.79***	1.80*
	(3.00)	(2.72)	(1.50)	(0.96)
$R_{25,36}$	8.36***	8.02***	2.93**	2.34***
	(3.17)	(2.92)	(1.42)	(1.00)
$R_{37,48}$	6.86***	6.89***	1.93**	2.96***
	(2.52)	(2.46)	(0.96)	(0.98)
$R_{49,60}$	6.45***	6.04**	2.28***	2.28***
	(2.38)	(2.27)	(0.97)	(0.88)
$R_{61,72}$	6.57***	6.33***	2.15**	2.36***
	(2.31)	(2.15)	(1.03)	(0.89)
$R_{73,84}$	6.38***	6.01***	2.19*	1.86*
	(2.38)	(2.24)	(1.19)	(0.97)
$R_{85,96}$	6.23***	5.50***	2.84***	1.22
	(2.18)	(1.97)	(1.14)	(0.87)
$R_{97,108}$	6.10***	5.72***	2.61***	1.09
	(1.96)	(1.90)	(0.91)	(0.87)
$R_{109,120}$	6.34***	5.44***	2.93***	0.54
	(1.90)	(1.86)	(0.87)	(0.89)

This table reports returns (in percent) of equal-weighted idiosyncratic volatility-sorted hedge portfolios for months 1-120 subsequent to portfolio formation, in twelve-month increments. Idiosyncratic volatility is computed using the market model. Portfolios are reformed monthly from 1966-2012. Hedge portfolios are formed by single sorts on recent historical idiosyncratic volatility (IVR, calculated over the last six months), distant historical idiosyncratic volatility (IVD, calculated over the year prior to IVR), sequential sorts on IVR then IVD (IVDS), and sequential sorts on IVD then IVR (IVRS). Newey-West standard errors are reported below each mean return. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

5. Robustness Checks and Alternative Explanations

5.1 Market model and alternative samples

The primary sample consists of data from 1966-2012. This facilitates comparison to the work of Ang et al. (2006), who examine a similar sample. In this section, we examine the robustness of our results to two changes. First, we compute idiosyncratic volatility using the market model rather than the three-factor model. Second, we consider an expanded sample (1929-2012).

Under the market model, idiosyncratic volatility is the standard deviation of the residuals from a time-series regression of a stocks returns on the returns of the value-weighted stock index. Idiosyncratic volatility calculated under the market model is highly correlated with idiosyncratic volatility calculated under the three-factor model (pooled sample correlation in excess of 0.99). Therefore, idiosyncratic risk estimates appear to be insensitive to this change in empirical models.

Table 6 reports twelve-month mean returns of hedge portfolios for ten years subsequent to portfolio formation. Results in Table 6 are generally similar to those in Table 5, although we use an expanded sample and the market model. Twelve months after portfolio formation, the mean returns of the IVR, IVD, IVRS, and IVRS portfolios are always positive and generally significant. Therefore, use of the market model and an expanded sample provides additional evidence that the underlying idiosyncratic risk-return relation is positive.

5.2 Alternative explanations

An alternative explanation of our results is that the short- and long-run relations are distinct. Perhaps idiosyncratic risk can be thought of as a composite stochastic process, with a short-run and long-run component, where each component is priced differently. However, simultaneously asserting a positive risk premium for long-run idiosyncratic risk and a negative risk premium for short-run idiosyncratic risk does not seem sensible. It is possible that the short-run idiosyncratic risk-return relation is not driven by idiosyncratic risk, but some characteristic correlated with idiosyncratic risk (e.g., one of the explanations of the negative relation discussed above). However, as discussed above, the negative relation between idiosyncratic risk and returns is fragile and seems to be driven by innovations in idiosyncratic risk. Still, the short-run relation could be caused by some characteristic correlated with idiosyncratic risk innovations. However, we have controlled for many return patterns that may plausibly explain this relation (e.g., short-term reversals, bid-ask bounce, and liquidity). A remaining alternative explanation for the short-run relation is temporary misreaction to cash-flow news. It is difficult to fully eliminate this possibility because cash-flow news is difficult to measure (see Chen and Zhao 2009). However, we control for the portion of prior cash-flow news that is correlated with prior returns, which is likely substantial.

5.3 Cash flows and the contemporaneous idiosyncratic risk-return relation

Table 2 shows that, on average, high idiosyncratic volatility stocks tend to have high prior returns. The positive contemporaneous relation between returns and idiosyncratic volatility was documented by Duffee (1995), and is consistent with the positive skewness of the average stock's return distribution. However, partial underreaction to idiosyncratic risk innovations and a positive price of idiosyncratic risk suggests that idiosyncratic risk innovations should be negatively related to contemporaneous returns. This apparent contradiction can be explained by changes in expected cash flows. High idiosyncratic risk firms, and firms with increases in idiosyncratic risk, tend to have positive cash-flow news, as measured by the change in earnings contemporaneous with the IVR measurement period. This suggests that, given some innovation in idiosyncratic risk, cash-flow news tend to offset discount rate news (i.e., a positive shock to idiosyncratic risk suggests lower prices due to increased risk, but tends to occur at the same time as good cash flow news, which suggests higher prices). Although it may be of interest to decompose these returns into a cash-flow and discount-rate component, this is a challenging task.

6. Conclusion

In this paper, we document a short-lived negative relation between idiosyncratic risk innovations and subsequent returns and a persistent positive relation between idiosyncratic risk levels and subsequent returns. These relations are consistent with a positive price of idiosyncratic risk and with temporary price underreaction to idiosyncratic risk innovations. Because idiosyncratic risk levels and innovations are correlated, these relations tend to be offset in standard empirical studies that examine the relation between historical idiosyncratic risk and subsequent returns.

We develop a simple model to examine the idiosyncratic risk-return relation in the presence of underreaction. We calibrate the model to deliver the empirical predictions. We simulate the long-run expected return response to a positive idiosyncratic volatility shock. The return pattern suggests that expected returns are negative for a few months after the shock, with the most negative return in month one. However, returns become positive over time as the shock gradually decays. This is consistent with investors' temporary underreaction to risk innovations and a long-run positive price of idiosyncratic risk. We empirically confirm the predictions of our model using portfolio analysis and cross-sectional regressions.

Our framework can simultaneously accommodate theory, which generally suggests a nonnegative price of idiosyncratic risk, and a negative empirical relation between some measures of historical idiosyncratic risk and returns (as documented by Ang et al. 2006). Also, our framework reconciles empirical studies that may estimate different prices of idiosyncratic risk (including differing signs). In the presence of underreaction, studies that examine the relation between recent idiosyncratic risk and immediate subsequent returns should find a negative relation if recent information is not yet fully incorporated into prices. However, studies that use a longer window to calculate historical idiosyncratic risk, or examine deferred returns, are less focused on underreaction and more likely to estimate a positive relation.

We rule out many alternative explanations of our results. We find that return patterns associated with short-term reversals, momentum, and liquidity cannot explain our results. Still, it remains possible that some omitted stock characteristic, correlated with historical idiosyncratic volatility, could explain our findings. However, it is not easy to find an alternative explanation that predicts the return pattern observed in the data (short-lived negative returns related to risk innovations and persistent positive returns related to the level of risk). Overall, we find a positive price of idiosyncratic risk and underreaction to idiosyncratic risk innovations to be a compelling explanation for the return patterns associated with idiosyncratic risk.

References

Amihud, Y. 2002. Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets* 5:31–56.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2006. The cross-section of volatility and expected returns. *Journal of Finance* 61:259–99.

———. 2009. High idiosyncratic volatility and low returns: International and further U.S. evidence. *Journal of Financial Economics* 91:1–23.

Bali, T. G., and N. Cakici. 2008. Idiosyncratic volatility and the cross-section of expected returns. *Journal of Financial and Quantitative Analysis* 43:29–58.

Bali, T. G., N. Cakici, and R. F. Whitelaw. 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics* 99:427–46.

Ball, R., and P. Brown. 1968. An empirical evaluation of accounting income numbers. *Journal of Accounting Research* 6:159–78.

Barberis, N., A. Shleifer, and R. Vishny. 1998. A model of investor sentiment. *Journal of Financial Economics* 49:307–343.

Bernard, V. L., and J. K. Thomas. 1989. Post-earnings-announcement drift: Delayed price response or risk premium? *Journal of Accounting Research* 27:1–36.

———. 1990. Evidence that stock prices do not fully reflect the implications of current earnings for future earnings. *Journal of Accounting and Economics* 13:305–340.

Boyer, B., T. Mitton, and K. Vorkink. 2010. Expected idiosyncratic skewness. *Review of Financial Studies* 23:169–202.

Chen, L., and X. Zhao. 2009. Return decomposition. *Review of Financial Studies* 22:5213–49.

Cohen, L., and A. Frazzini. 2008. Economic links and predictable returns. *Journal of Finance* 63:1977–2011.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor psychology and security market under- and overreactions. *Journal of Finance* 53:1839–85.

DellaVigna, S., and J. M. Pollet. 2009. Investor inattention and friday earnings announcements. *Journal of Finance* 64:709–749.

Duffee, G. R. 1995. Stock returns and volatility: A firm level analysis. *Journal of Financial Economics* 37:399–420.

Eberhart, A. C., W. F. Maxwell, and A. R. Siddique. 2004. An examination of long-term abnormal stock returns and operating performance following R&D increases. *Journal of Finance* 59:623–50.

Fama, E. F., and K. R. French. 1996. Multifactor explanations of asset pricing anomalies. *Journal of Finance* 51:55–84.

Fama, E. F., and J. D. MacBeth. 1973. Risk and return: Some empirical tests. *Journal of Political Economy* 81:607–636.

Fink, J. D., K. E. Fink, and H. He. 2012. Expected idiosyncratic volatility measures and expected returns. *Financial Management* 41:519–53.

Fu, F. 2009. Idiosyncratic risk and the cross-section of expected stock returns. *Journal of Financial Economics* 91:24–37.

Guo, H., H. Kassa, and M. F. Ferguson. 2014. On the relation between EGARCH idiosyncratic volatility and expected stock returns. *Journal of Financial and Quantitative Analysis* 49:271–96.

Guo, H., and R. Savickas. 2010. Relation between time-series and cross-sectional effects of idiosyncratic variance on stock returns. *Journal of Banking and Finance* 34:1637–49.

Han, Y., and D. Lesmond. 2011. Liquidity biases and the pricing of cross-sectional idiosyncratic volatility. *Review of Financial Studies* 24:24–37.

Hong, H., W. Torous, and R. Valkanov. 2007. Do industries lead stock markets? *Journal of Financial Economics* 83:367–396.

Hong, H., and J. C. S. trading. 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance* 54:2143–84.

Huang, W., Q. Liu, S. G. Rhee, and L. Zhang. 2009. Return reversals, idiosyncratic risk, and expected Returns. *Review of Financial Studies* 23:147–68.

Ikenberry, D., J. Lakonishok, and T. Vermaelen. 1995. Market underreaction to open market share repurchases. *Journal of Financial Economics* 39:181–208.

Jegadeesh, N. 1990. Evidence of predictable behavior of security returns. *Journal of Finance* 45:881–98.

Jegadeesh, N., and S. Titman. 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance* 48:65–91.

Lehmann, B. 1990a. Fads, martingales, and market efficiency. *Quarterly Journal of Economics* 105:1–28.

———. 1990b. Residual risk revisited. *Journal of Econometrics* 45:71–97.

Levy, H. 1978. Equilibrium in an imperfect market: A constraint on the number of securities in the portfolio. *American Economic Review* 68:643–58.

Loughran, T., and J. R. Ritter. 1995. The new issues puzzle. *Journal of Finance* 50:23–51.

Menzly, L., and O. Ozbas. 2010. Market segmentation and cross-predictability of Returns. *Journal of Finance* 65:1555–80.

Merton, R. C. 1987. A simple model of capital market equilibrium with incomplete information. *Journal of Finance* 42:483–510.

Michaely, R., R. H. Thaler, and K. L. Womack. 1995. Price reactions to dividend initiations and omissions: overreaction or drift? *Journal of Finance* 50:573–608.

Newey, W. K., and K. D. West. 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55:703–708.

Peterson, D. R., and A. R. Smedema. 2011. The return impact of realized and expected idiosyncratic volatility. *Journal of Banking and Finance* 35:2547–58.

Poteshman, A. M. 2001. Underreaction, Overreaction, and Increasing Misreaction to Information in the Options Market. *Journal of Finance* 56:851–876.

Sims, C. A. 2003. Implications of rational inattention. *Journal of Monetary Economics* 50:665–90.

Zhang, X. F. 2006. Information uncertainty and stock returns. *Journal of Finance* 61:105–137.