Journal of Financial Economics 121 (2016) 167–194

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/finec

Have we solved the idiosyncratic volatility puzzle?☆

Kewei Hou[a,b,*], Roger K. Loh[c]

[a] The Ohio State University, Fisher College of Business, 2100 Neil Avenue, Columbus, OH 43210, USA
[b] China Academy of Financial Research (CAFR), 211 West Huaihai Road, Shanghai 200030, PR China
[c] Singapore Management University, Lee Kong Chian School of Business, 50 Stamford Road, Singapore 178899, Singapore

ARTICLE INFO

Article history:
Received 18 December 2014
Revised 17 August 2015
Accepted 10 September 2015
Available online 27 February 2016

JEL classification:
G12
G14

Keywords:
Idiosyncratic volatility
Cross-section of stock returns
Lottery preferences
Market frictions

ABSTRACT

We propose a simple methodology to evaluate a large number of potential explanations for the negative relation between idiosyncratic volatility and subsequent stock returns (the idiosyncratic volatility puzzle). Surprisingly, we find that many existing explanations explain less than 10% of the puzzle. On the other hand, explanations based on investors' lottery preferences and market frictions show some promise in explaining the puzzle. Together, all existing explanations account for 29–54% of the puzzle in individual stocks and 78–84% of the puzzle in idiosyncratic volatility-sorted portfolios. Our methodology can be applied to evaluate competing explanations for other asset pricing anomalies.

1. Introduction

Ang, Hodrick, Xing, and Zhang (2006), in a highly influential paper, document a negative relation between idiosyncratic volatility and subsequent stock returns. To the extent that realized idiosyncratic volatility proxies for expected idiosyncratic volatility, this result is very puzzling because traditional asset pricing theories either predict no relation between expected idiosyncratic volatility and expected returns under the assumptions that markets are complete and frictionless and investors are well-diversified, or predict a positive relation under the assumptions that markets are incomplete and investors face sizable frictions and hold poorly diversified portfolios (see, e.g., Merton, 1987; Hirshleifer, 1988). Consequently, many papers have been written trying to explain the puzzle, with each paper proposing a different economic mechanism linking idiosyncratic volatility to subsequent stock returns.[1]

☆ We thank Jack Bao, Geert Bekaert, Hank Bessembinder, Karl Diether, Steve Dimmock, Fangjian Fu, Dong Hong, Chuan-Yang Hwang, Jung-Min Kim, Sehoon Kim, Bob Kimmel, William Leon, David Lesmond, Angie Low, Ralitsa Petkova, René Stulz, Avanidhar Subrahmanyam, Keith Vorkink, Mitch Warachka, Hong Zhang, and seminar participants at Hong Kong Polytechnic University, Nanyang Technological University, National University of Singapore, Ohio State University, Peking University, Seoul National University, Singapore Management University, SUNY Buffalo, University of Central Florida, University of Delaware, University of Exeter, University of Hong Kong, and CICF 2014 (Chengdu) for their comments and suggestions.

* Corresponding author at: The Ohio State University, Fisher College of Business, 2100 Neil Avenue, Columbus, OH 43210, USA. Tel.: +1 614 292 0552; fax: +1 614 292 7062.

E-mail addresses: hou.28@osu.edu (K. Hou), rogerloh@smu.edu.sg (R.K. Loh).

[1] The long list of candidate explanations includes those based on expected idiosyncratic skewness (Boyer, Mitton, and Vorkink, 2010), coskewness (Chabi-Yo and Yang, 2009), maximum daily return (Bali, Cakici, and Whitelaw, 2011), retail trading proportion (Han and Kumar, 2013), one-month return reversal (Fu, 2009; Huang, Liu, Rhee, and Zhang, 2009), illiquidity (Bali and Cakici, 2008; Han and Lesmond, 2011), uncertainty (Johnson, 2004), average variance beta (Chen and Petkova, 2012), and earnings surprises (Jiang, Xu, and Yao, 2009; Wong, 2011). In addition, several papers show that the idiosyncratic volatility puzzle is stronger among stocks with prices of at least five dollars (George and Hwang, 2011), low analyst coverage (Ang, Hodrick, Xing, and Zhang, 2009;

http://dx.doi.org/10.1016/j.jfineco.2016.02.013

However, to date there has been no comprehensive examination about which explanations best explain the puzzle. Further complicating this matter is the fact that existing studies typically differ in terms of empirical methodology and sample construction, thus making direct comparisons of their results difficult.

Motivated by these concerns, this paper provides a simple unified framework to evaluate a large number of candidate explanations of the puzzle. Most studies in this literature typically promote a new explanation of the puzzle while controlling for a limited number of existing explanations. We believe that our paper provides the most comprehensive examination of existing explanations to date. More importantly, our methodology allows us to quantify the fraction of the puzzle that is explained by each candidate explanation, either by itself or after controlling for other competing explanations.

To summarize our methodology, we start from Fama and MacBeth (1973) cross-sectional regressions of month t individual stock returns on month $t-1$ idiosyncratic volatility. We find, as many papers do, that the estimated regression coefficient, which we denote as γ_t, is on average negative and highly statistically significant. Next, we decompose the γ_t coefficient into one or more components, each related to a candidate explanation of the puzzle (e.g., skewness), and a residual component. The ratio of the component related to a particular candidate explanation to the original γ_t coefficient then measures the fraction of the idiosyncratic volatility puzzle that is captured by that explanation, and the ratio of the residual component to γ_t measures the fraction of the puzzle left unexplained by all candidate explanations considered. Our decomposition methodology ensures that the components related to the candidate explanations and the residual component add up to γ_t. This makes for intuitive interpretation and easy comparisons when we pit existing explanations against one another.

To guide our analysis, we break up existing explanations into three groups. The first group of explanations attributes the idiosyncratic volatility puzzle to lottery preferences of investors (they propose different proxies for the lottery feature of a stock, namely, skewness, coskewness, expected idiosyncratic skewness, maximum daily return, and retail trading proportion). The second group of explanations appeals to various forms of market frictions (one-month return reversal, the Amihud illiquidity measure, zero-return proportion, and bid-ask spread) to try to explain the puzzle. Explanations that do not fall naturally into the first two groups (uncertainty, average variance beta, and earnings surprises) are then included in the third group.

Using the sample of Center for Research in Security Prices (CRSP) common stocks from 1963–2012, we find that surprisingly many existing explanations, when evaluated alone, explain less than 10% of the idiosyncratic volatility puzzle. This is true for the explanations based on coskewness, illiquidity, zero-return proportion, uncertainty, and average variance beta. For example, coskewness and analyst dispersion (a proxy for uncertainty) can only explain 1.9% and 5.3%, respectively, of the puzzle. Or consider the Amihud illiquidity measure. Despite being highly correlated with idiosyncratic volatility, it also fails to capture more than 10% of the puzzle.

On the other hand, explanations based on skewness, expected idiosyncratic skewness, maximum daily return, retail trading proportion, one-month return reversal, bid-ask spread, and past earnings surprises show promise in explaining the puzzle. In particular, one-month return reversal alone can explain 33.7% of the puzzle, followed by bid-ask spread at 30.4%, retail trading proportion at 22.3%, expected idiosyncratic skewness at 14.7%, past earnings surprises at 10.9%, and skewness at 10.3%. For the maximum daily return variable proposed by Bali, Cakici, and Whitelaw (2011), it turns out that it can explain the entire puzzle. The problem, however, is that this variable is essentially a range-based measure of volatility and is close to being perfectly collinear with idiosyncratic volatility (correlation of about 0.90). It is therefore not surprising that an alternative proxy for volatility can capture the idiosyncratic volatility puzzle.

Finally, we include all explanations of the puzzle (excluding maximum daily return for reasons mentioned above) in a multivariate framework so that we can evaluate the marginal contribution of each explanation. We are also interested in the total fraction of the puzzle they can collectively explain. We find that after controlling for competing explanations, retail trading proportion explains only 0.2% of the puzzle. Among the other lottery preference-based explanations, expected idiosyncratic skewness explains 4–15%, coskewness explains 3–4%, and skewness explains 2–7% of the puzzle, depending on the specification. Together, the four lottery preference proxies capture a good 10–25% of the puzzle. Among the market friction-based explanations, one-month return reversal explains 1–22%, bid-ask spread explains 8%, the Amihud illiquidity measure explains up to 4%, and zero-return proportion explains less than 2% of the puzzle. Together, the market friction proxies account for 3–24% of the puzzle. Finally, analyst dispersion explains 3–6%, average variance beta explains less than 1%, and past earnings surprises explain 2–5% of the puzzle. Together, this group of explanations accounts for 5–10% of the puzzle in the multivariate analysis. Collectively, all the examined explanations account for 29–54% of the puzzle, with explanations based on lottery preferences and market frictions making the biggest contributions. However, a significant fraction (46–71%) of the puzzle remains unexplained.

In robustness tests, we repeat the multivariate analysis using subsamples of stocks with prices of at least five dollars, low analyst coverage, poor credit ratings, high short-sale constraints, high leverage, low institutional ownership, low book-to-market equity, non-NYSE listings, or for non-January months (which have been shown by previous studies to be associated with a stronger idiosyncratic

George and Hwang, 2011), low credit ratings (Avramov, Chordia, Jostova, and Philipov, 2013), high short-sale constraints (Boehme, Danielsen, Kumar, and Sorescu, 2009; George and Hwang, 2011; Stambaugh, Yu, and Yuan, 2015), high leverage (Johnson, 2004), low institutional ownership (Nagel, 2005), low book-to-market equity (Barinov, 2013), non-NYSE listings (Bali and Cakici, 2008), or for non-January months (George and Hwang, 2011; Doran, Jiang, and Peterson, 2012).

volatility puzzle). We find that existing explanations account for 39–50% of the puzzle on average in these subsamples. In addition, we extend our analysis to idiosyncratic volatility-sorted portfolios to control for measurement errors at the individual stock level, and find that existing explanations capture 78–84% of the puzzle in the portfolio-level analysis.[2] We also examine nonlinear relations in the idiosyncratic volatility puzzle and find that existing explanations account for a similar fraction of the puzzle as in the baseline linear specification. Overall, these robustness results confirm that while lottery preferences and market frictions explain a sizable part of the idiosyncratic volatility puzzle, a significant portion of the puzzle remains unexplained. In the final test, we apply our decomposition methodology to other anomalies to illustrate that our methodology can be used to evaluate candidate explanations for other puzzles in empirical asset pricing.

The rest of the paper is organized as follows. Section 2 describes the data and methodology and gives an overview of the various explanations that have been proposed for the idiosyncratic volatility puzzle. Section 3 evaluates the explanations one at a time, and Section 4 investigates multiple explanations at the same time. Section 5 considers a number of robustness tests, and Section 6 concludes.

2. Data and methodology

2.1. Stock return and idiosyncratic volatility data

We start our sample from the standard CRSP common stock (share codes of 10 or 11) universe from July 1963 to December 2012. Monthly returns are adjusted for delisting following Shumway (1997). To be included in the analysis, we require a firm to have non-missing size and non-negative book-to-market equity (B/M), where size is the most recent June-end market cap and B/M is computed according to Fama and French (2006). We apply a price screen of one dollar to remove penny stocks but also adjust this screen in subsample robustness tests.

We compute idiosyncratic volatility (*IVOL*) following Ang, Hodrick, Xing, and Zhang (2006) as the standard deviation of the residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. We require at least ten daily returns to compute *IVOL*, although our results are unaffected if we require at least 15 daily returns or do not impose any minimum observation restriction. The estimates for *IVOL* start in July 1963, and month $t-1$ estimates of *IVOL* are matched to month t returns from August 1963 to December 2012.

2.2. Candidate variables related to lottery preferences of investors

A battery of candidate variables is constructed as potential explanations of the idiosyncratic volatility puzzle. The first group of explanations concerns the lottery preferences of investors. Barberis and Huang (2008) argue that under cumulative prospect theory, investors overweigh small chances of large gains (hence the lottery preferences). As a result, they prefer positively skewed stocks, causing them to be overpriced, which would then earn low subsequent returns. Several papers attribute the idiosyncratic volatility puzzle to idiosyncratic volatility being correlated with skewness. We measure skewness (denoted *Skew*) using the daily returns in month $t-1$. In addition to the raw skewness measure, we also compute alternative measures of skewness. Chabi-Yo and Yang (2009) develop a model showing that the effect of idiosyncratic volatility on stock returns is related to a stock's coskewness with the market portfolio. We measure coskewness (*Coskew*) as the regression coefficient of squared daily individual stock returns on market returns.[3]

Boyer, Mitton, and Vorkink (2010) use the forecasts from a regression model to proxy for expected idiosyncratic skewness [*E(Idioskew)*] and show that it helps to explain the idiosyncratic volatility puzzle.[4] We obtain the *E(Idioskew)* estimates from the authors for 1988–2005. We then extend their sample period by constructing the measure for 1968–1987 (turnover is dropped from the forecast model for this early period due to lack of turnover data for Nasdaq stocks) and 2006–2012.

We also consider the maximum daily return (*Maxret*) and the retail trading proportion (*RTP*) of a stock, which are proposed by Bali, Cakici, and Whitelaw (2011) and Han and Kumar (2013), respectively, as indicators for stocks that are preferred by lottery-seeking retail investors. *Maxret* is measured using daily returns in month $t-1$. *RTP* is measured as the fraction of the dollar trading volume in month $t-1$ that comes from trades less than or equal to \$5,000, using the Institute for the Study of Security Markets (ISSM) database for 1983–1992 and the Trades and Quotes (TAQ) database for 1993–2000. Following Han and Kumar (2013), we exclude the post-decimalization period due to greater incidence of order-splitting by institutions.

2.3. Candidate variables related to market frictions

The second group of explanations attributes the idiosyncratic volatility puzzle to market frictions. Fu (2009) and Huang, Liu, Rhee, and, Zhang (2009) argue that once we control for the one-month return reversal effect, which

[2] Although the total explained fraction at the portfolio level is significantly higher than that at the individual stock level, our simulation results suggest that it might overstate the true fraction of the puzzle explained by the candidate variables if the candidate variables are already measured precisely at the individual stock level. See Section 5.1 for details.

[3] We have also calculated Harvey and Siddique's (2000) measure of coskewness by regressing daily individual stock returns on squared market returns. The results are similar to those based on Chabi-Yo and Yang's (2009) coskewness measure.

[4] Expected idiosyncratic skewness is estimated by regressing idiosyncratic skewness (measured using the residuals from a regression of past five years of daily returns on the Fama-French factors) on lagged idiosyncratic skewness, idiosyncratic volatility, momentum, turnover, dummy variables for small firms and medium-sized firms, two-digit Standard Industrial Classification (SIC) dummies, and a Nasdaq dummy. Boyer, Mitton, and Vorkink (2010) show that the coefficient on *IVOL* becomes insignificant after controlling for *E(Idioskew)* in Fama-MacBeth regressions using 100 *E(Idioskew)*-sorted portfolios. However, in their individual stock-level Fama-MacBeth regressions, the *IVOL* coefficient remains significant after controlling for *E(Idioskew)*.

is likely driven by microstructure biases, the negative idiosyncratic volatility-return relation is no longer significant. We measure the one-month reversal effect using the month $t-1$ return (*Lagret*).

Illiquidity can also affect the idiosyncratic volatility-return relation. We examine three measures of illiquidity. The Amihud (2002) measure (*Amihud*) is computed as the month $t-1$ average of daily absolute return divided by daily dollar trading volume. We also follow Han and Lesmond (2011) and use the fraction of trading days in month $t-1$ with a zero return (*Zeroret*) as another proxy for illiquidity. The third proxy is the bid-ask spread (*Spread*), which is the average daily percentage bid-ask spread (ask minus bid divided by the average of bid and ask) in month $t-1$. The daily percentage spreads are computed based on the National Best Bid and Offer (NBBO) quotes at every point in time during a trading day (weighted by the average depth of the quotes), using data from ISSM and TAQ for 1984–2012. Han and Lesmond (2011) argue that the bid-ask bounce drives much of the idiosyncratic volatility puzzle.

2.4. Candidate variables related to other explanations

The third group of explanations consists of those that do not fall naturally into the lottery preference or market friction categories. First, idiosyncratic volatility could proxy for the fundamental uncertainty surrounding a stock. Johnson (2004) argues that uncertainty is negatively related to future stock returns because stock is a call option on a levered firm's underlying assets. We measure uncertainty using analyst dispersion (*Dispersion*), which is the standard deviation of analysts' FY1 forecasts scaled by the absolute value of the mean consensus forecast for month $t-1$. Analysts' forecasts are obtained from the Institutional Brokers' Estimate System (I/B/E/S) Summary Estimates unadjusted file.

Chen and Petkova (2012) argue that a stock's exposure to the average variance component of the market variance explains the idiosyncratic volatility puzzle. We replicate their measure of average variance beta (*AvgVar*β) for the sample period of 1968–2012 and include it in the analysis.[5]

We also examine *SUE* (the most recently announced standardized unexpected earnings as of the end of month $t-1$). Jiang, Xu, and Yao (2009) and Wong (2011) show that high idiosyncratic volatility stocks suffer negative earnings surprises, which could explain the subsequent poor return performance of those stocks. *SUE* is measured as the Compustat quarterly earnings before extraordinary items (item IBQ) minus the earnings four quarters ago, divided by the standard deviation of the difference over the last eight quarters. The announcement date of earnings is from item RDQ.

[5] For each month, average variance beta is estimated by regressing a stock's returns over the past 60 months (24-month minimum) on changes in the average variance (AV) of the market portfolio, controlling for changes in the average correlation (AC) of the market portfolio and the Fama and French (1993) factors. AV is the average of the individual stock daily return variances. AC is the average of the pairwise correlations between stocks.

2.5. Decomposition methodology

Our decomposition methodology is based on individual stock-level Fama-MacBeth cross-sectional regressions, which are commonly used in the literature to study the relation between idiosyncratic volatility and returns. For each month t, we regress the cross-section of individual stock characteristic-adjusted returns on their month $t-1$ *IVOL* as follows:

$$R_{it} = \alpha_t + \gamma_t IVOL_{it-1} + \varepsilon_{it}. \tag{1}$$

R_{it} is stock i's characteristic-adjusted return, computed following Daniel, Grinblatt, Titman, and Wermers (1997) (hereafter DGTW).[6] Our results are robust to using raw returns instead of DGTW-adjusted returns. For our baseline sample, the average γ_t coefficient ($\times 100$ and reported in percent) equals –16.955% with a t-statistic of –8.19 (hence the idiosyncratic volatility puzzle).

Next, we regress $IVOL_{it-1}$ on a candidate explanatory variable ($Candidate_{it-1}$):

$$IVOL_{it-1} = a_{t-1} + \delta_{t-1} Candidate_{it-1} + \mu_{it-1}. \tag{2}$$

This regression allows us to assess the relation between idiosyncratic volatility and the candidate variable as any candidate variable that can potentially explain the puzzle must be correlated with idiosyncratic volatility.[7] We then use the regression coefficient estimates to decompose $IVOL_{it-1}$ into two orthogonal components: $\delta_{t-1} Candidate_{it-1}$ is the component of $IVOL_{it-1}$ that is related to the candidate variable and $(a_{t-1} + \mu_{it-1})$ is the residual component that is unrelated to the candidate variable.

The final step is to use the linearity of covariances to decompose the estimated γ_t coefficient from Eq. (1):

$$\begin{aligned}
\gamma_t &= \frac{\mathrm{Cov}[R_{it}, IVOL_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} \\
&= \frac{\mathrm{Cov}[R_{it},\ (\delta_{t-1} Candidate_{it-1} + a_{t-1} + \mu_{it-1})]}{\mathrm{Var}[IVOL_{it-1}]} \\
&= \frac{\mathrm{Cov}[R_{it},\ \delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} \\
&\quad + \frac{\mathrm{Cov}[R_{it},\ (a_{t-1} + \mu_{it-1})]}{\mathrm{Var}[IVOL_{it-1}]} \\
&= \gamma_t^C + \gamma_t^R.
\end{aligned} \tag{3}$$

γ_t^C/γ_t then measures the fraction of the idiosyncratic volatility-return relation (the idiosyncratic volatility puzzle) explained by the candidate variable, and γ_t^R/γ_t measures the fraction of the puzzle left unexplained by the

[6] DGTW-adjusted return is the raw return minus the return on a size-B/M-momentum-matched benchmark portfolio. At the end of June of each year, stocks are first sorted into quintiles based on their market cap using NYSE breakpoints. Then, within each size quintile, stocks are sorted into quintiles according to their B/M ratios from the previous year. In the last step, stocks within each double-sorted size-B/M portfolio are further sorted into quintiles every month based on their returns over the prior 12 months skipping the most recent month. Equal-weighted monthly returns are computed for each characteristic-matched benchmark portfolio.

[7] However, as we will demonstrate later, a high correlation in and of itself does not guarantee that the candidate variable will explain a large fraction of the puzzle.

candidate variable. While the means and variances of the two fractions are unattainable in closed-form, we can use the standard multivariate delta method based on Taylor series expansions to find approximations using the means, variances, and covariances of γ_t^C, γ_t^R, and γ_t (see, e.g., Casella and Berger, 2001):

$$\mathrm{E}\left(\frac{\gamma_t^C}{\gamma_t}\right) \approx \frac{\mathrm{E}(\gamma_t^C)}{\mathrm{E}(\gamma_t)}, \ \mathrm{E}\left(\frac{\gamma_t^R}{\gamma_t}\right) \approx \frac{\mathrm{E}(\gamma_t^R)}{\mathrm{E}(\gamma_t)}, \tag{4}$$

$$\mathrm{Var}\left(\frac{\gamma_t^C}{\gamma_t}\right) \approx \left(\frac{\mathrm{E}(\gamma_t^C)}{\mathrm{E}(\gamma_t)}\right)^2 \times \left(\frac{\mathrm{Var}(\gamma_t^C)}{\left(\mathrm{E}(\gamma_t^C)\right)^2} + \frac{\mathrm{Var}(\gamma_t)}{(\mathrm{E}(\gamma_t))^2} - 2\frac{\mathrm{Cov}(\gamma_t^C, \gamma_t)}{\mathrm{E}(\gamma_t^C)\mathrm{E}(\gamma_t)}\right), \tag{5}$$

and

$$\mathrm{Var}\left(\frac{\gamma_t^R}{\gamma_t}\right) \approx \left(\frac{\mathrm{E}(\gamma_t^R)}{\mathrm{E}(\gamma_t)}\right)^2 \times \left(\frac{\mathrm{Var}(\gamma_t^R)}{\left(\mathrm{E}(\gamma_t^R)\right)^2} + \frac{\mathrm{Var}(\gamma_t)}{(\mathrm{E}(\gamma_t))^2} - 2\frac{\mathrm{Cov}(\gamma_t^R, \gamma_t)}{\mathrm{E}(\gamma_t^R)\mathrm{E}(\gamma_t)}\right). \tag{6}$$

The corresponding estimated means and variances of the fractions are based on the respective time series of γ_t^C, γ_t^R, and γ_t estimates (over T months):

$$\hat{\mathrm{E}}\left(\frac{\gamma_t^C}{\gamma_t}\right) \approx \frac{\overline{\gamma_t^C}}{\overline{\gamma_t}}, \ \hat{\mathrm{E}}\left(\frac{\gamma_t^R}{\gamma_t}\right) \approx \frac{\overline{\gamma_t^R}}{\overline{\gamma_t}}, \tag{7}$$

$$\widehat{\mathrm{Var}}\left(\frac{\overline{\gamma_t^C}}{\overline{\gamma_t}}\right) \approx \frac{1}{T}\left(\frac{\overline{\gamma_t^C}}{\overline{\gamma_t}}\right)^2 \left(\frac{s_{\gamma_t^C}^2}{\overline{\gamma_t^C}^2} + \frac{s_{\gamma_t}^2}{\overline{\gamma_t}^2} - 2\frac{\hat{\rho}_{\gamma_t^C,\gamma_t} s_{\gamma_t^C} s_{\gamma_t}}{\overline{\gamma_t^C}\ \overline{\gamma_t}}\right), \tag{8}$$

and

$$\widehat{\mathrm{Var}}\left(\frac{\overline{\gamma_t^R}}{\overline{\gamma_t}}\right) \approx \frac{1}{T}\left(\frac{\overline{\gamma_t^R}}{\overline{\gamma_t}}\right)^2 \left(\frac{s_{\gamma_t^R}^2}{\overline{\gamma_t^R}^2} + \frac{s_{\gamma_t}^2}{\overline{\gamma_t}^2} - 2\frac{\hat{\rho}_{\gamma_t^R,\gamma_t} s_{\gamma_t^R} s_{\gamma_t}}{\overline{\gamma_t^R}\ \overline{\gamma_t}}\right). \tag{9}$$

Our decomposition methodology is different from the conventional approach to evaluate a candidate variable, which usually involves including the candidate variable as a control in the regression of returns on idiosyncratic volatility:

$$R_{it} = \tilde{\alpha}_t + \tilde{\gamma}_t^R IVOL_{it-1} + \tilde{\gamma}_t^C Candidate_{it-1} + \tilde{\varepsilon}_{it}. \tag{10}$$

In this regression, if $\tilde{\gamma}_t^R$ is zero, researchers typically conclude that the candidate variable explains the idiosyncratic volatility puzzle. If $\tilde{\gamma}_t^R$ is not zero, one might consider using the difference between $\tilde{\gamma}_t^R$ and the original *IVOL* coefficient γ_t from Eq. (1) to measure the fraction of the puzzle that is explained by the candidate variable. This is problematic because the two coefficients are not directly comparable due to the fact that $\tilde{\gamma}_t^R$ is determined by the variation in *IVOL* that is independent of the candidate variable whereas γ_t is determined by the variation in *IVOL* itself. The important advantage of our decomposition methodology is that by requiring both γ_t^C and γ_t^R in Eq. (3) to be determined by the variation in *IVOL*, we ensure that they add up *exactly* to the original γ_t coefficient. This allows us to make a direct statement about the fraction of the idiosyncratic volatility puzzle that is explained by the candidate variable. In addition, unlike the conventional approach, our methodology can easily accommodate multiple candidate variables at the same time so we can objectively quantify the marginal contribution of each candidate variable in a horse race.

It is important to point out that a candidate variable that is highly correlated with idiosyncratic volatility may not necessarily explain a large fraction of the puzzle in our decomposition methodology. This is because the part of idiosyncratic volatility that is related to the candidate variable may not be the part that is responsible for the negative relation between idiosyncratic volatility and returns. In Appendix A, we show that γ_t^C from our decomposition methodology in Eq. (3) is related to the coefficients from the conventional approach in Eq. (10) in the following way: $\gamma_t^C = (\frac{\tilde{\gamma}_t^C}{\delta_{t-1}} + \tilde{\gamma}_t^R) \times \frac{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]}$.[8] This suggests that γ_t^C not only depends on the fraction of the variation of idiosyncratic volatility explained by the candidate variable ($\frac{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]}$), but also on the component of the candidate variable that is uncorrelated with idiosyncratic volatility but correlated with future returns as captured by $\tilde{\gamma}_t^C$ in Eq. (10). Consequently, a candidate variable that is highly positively correlated with idiosyncratic volatility could actually have a small or even negative contribution to the puzzle if the component of the candidate variable that is uncorrelated with idiosyncratic volatility predicts returns positively. Empirically, we show in Section 3 that this is indeed the case for a number of candidate variables we investigate. The bottom line is that our decomposition methodology is not simply picking up candidate variables based solely on their correlations with idiosyncratic volatility. Rather, we attribute a high explanatory power to a variable for capturing a significant fraction of the negative relation between idiosyncratic volatility and returns.

3. Evaluating candidate explanations one at a time

3.1. Sample descriptive statistics

Panel A of Table 1 reports the descriptive statistics of our sample. There are more than two million firm-month observations in our baseline sample. The average raw return is 1.1% per month with a standard deviation of 15.7%. The average DGTW-adjusted return is –0.1% per month with a standard deviation of 14.2%. The average *IVOL* estimated using daily returns is 2.6%. The average market beta (estimated with three years of past monthly returns), size, B/M ratio, and momentum (buy-and-hold return from month $t-12$ to $t-2$) are 1.136, \$1.441 billion, 0.944, and 16.7%, respectively.

[8] We also consider the general case of multiple candidate variables in Appendix A.

Table 1

Sample descriptive statistics.

Sample statistics from 1963 to 2012 are reported. Panel A shows the distribution of firm characteristics and Panel B shows the time-series averages of cross-sectional correlations. The sample consists of all CRSP common stocks with share prices of at least $1 at the end of the previous month. N is the total number of firm-month observations. Return is the raw CRSP monthly return adjusted for delisting according to Shumway (1997). DGTW-adjusted return is the raw return minus the return on a size-B/M-momentum-matched benchmark portfolio. Idiosyncratic volatility (*IVOL*) is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. *Beta* is the regression coefficient of the past three years of monthly returns on market returns. Size and B/M are measured and aligned as in Fama and French (2006), and Momentum is the buy-and-hold month $t-12$ to $t-2$ return. *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *Maxret* is the maximum daily return in month $t-1$. *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings.

Panel A: Distribution of firm characteristics

Variable	Mean	Stdev	N	1st Pctl	10th Pctl	25th Pctl	50th Pctl	75th Pctl	90th Pctl	99th Pctl
Return	0.011	0.157	2123249	−0.360	−0.143	−0.063	0.000	0.071	0.167	0.500
DGTW-adj ret	−0.001	0.142	2123249	−0.335	−0.137	−0.067	−0.008	0.053	0.134	0.441
IVOL	0.026	0.022	2124838	0.003	0.009	0.013	0.021	0.033	0.050	0.106
Size ($m)	1441.3	9373.4	2124838	2.0	9.0	26.1	102.1	483.0	1927.2	24306.8
B/M	0.944	1.521	2124838	0.047	0.202	0.381	0.686	1.153	1.827	4.729
Momentum	0.167	0.709	2124838	−0.747	−0.408	−0.176	0.068	0.347	0.742	2.546
Beta	1.136	0.921	2096757	−0.761	0.202	0.583	1.041	1.566	2.186	4.014
					Lottery preference variables					
Skew	0.258	1.042	2114792	−2.848	−0.801	−0.255	0.216	0.753	1.420	3.335
Coskew	0.006	0.879	2124838	−0.430	−0.061	−0.016	0.001	0.021	0.075	0.504
E(Idioskew)	0.830	0.682	1527963	−0.402	0.119	0.401	0.756	1.169	1.595	2.748
Maxret	0.071	0.077	2124838	0.000	0.020	0.031	0.052	0.086	0.139	0.333
RTP	0.158	0.206	816019	0.000	0.009	0.023	0.070	0.209	0.453	1.000
					Market friction variables					
Lagret	0.016	0.166	2124782	−0.342	−0.139	−0.061	0.000	0.074	0.170	0.537
Amihud	6.352	73.593	1974351	0.000	0.001	0.012	0.155	1.534	8.917	106.32
Zeroret	0.208	0.213	2124838	0.000	0.000	0.048	0.143	0.300	0.500	0.913
Spread	0.032	0.049	1245367	0.000	0.002	0.006	0.015	0.039	0.077	0.226
					Other variables					
Dispersion	0.199	1.237	826678	0.000	0.009	0.019	0.043	0.111	0.306	2.667
AvgVarβ	0.228	7.352	1769692	−19.618	−5.912	−2.282	0.033	2.457	6.575	22.345
SUE	0.169	17.177	1719754	−7.598	−1.595	−0.461	0.186	1.051	2.347	6.344

Panel B: Time-series averages of cross-sectional correlations between firm characteristics

Variable	DGTW-adj ret	IVOL	Beta	Size	BM	Mom	Skew	Coskw	Eiskw	Maxret	RTP	Lagret	Amihud	Zeroret	Spread	Disp	AVβ
IVOL	−0.027	1															
Beta	−0.005	0.179	1														
Size	0.001	−0.138	−0.044	1													
B/M	0.000	0.043	−0.089	−0.065	1												
Momentum	0.006	−0.086	−0.011	0.004	0.027	1											
Skew	−0.012	0.193	0.035	−0.022	0.016	−0.020	1										
Coskew	−0.005	0.052	0.016	−0.003	0.001	−0.012	0.101	1									
E(Idioskew)	−0.010	0.408	0.064	−0.174	0.184	−0.165	0.064	0.013	1								
Maxret	−0.033	0.882	0.154	−0.101	0.031	−0.067	0.476	0.105	0.316	1							
RTP	−0.017	0.472	−0.062	−0.172	0.166	−0.201	0.033	0.010	0.580	0.354	1						
Lagret	−0.045	0.191	−0.007	−0.003	0.025	0.001	0.352	0.080	−0.017	0.389	−0.058	1					
Amihud	0.002	0.308	−0.029	−0.049	0.127	−0.091	0.009	0.003	0.254	0.222	0.402	−0.013	1				
Zeroret	0.004	0.020	−0.146	−0.153	0.164	−0.133	−0.003	−0.007	0.402	0.011	0.461	−0.047	0.213	1			
Spread	−0.011	0.494	−0.101	−0.157	0.182	−0.165	0.034	0.009	0.524	0.358	0.700	−0.014	0.500	0.463	1		
Dispersion	−0.010	0.123	0.064	−0.035	0.061	−0.081	0.015	0.008	0.113	0.097	0.133	−0.021	0.049	0.062	0.104	1	
AvgVarβ	−0.003	0.033	0.087	−0.003	−0.004	0.003	0.006	0.008	0.006	0.028	0.036	0.001	0.008	−0.011	0.016	0.003	1
SUE	0.023	−0.099	−0.015	0.045	−0.062	0.203	0.008	−0.004	−0.107	−0.062	−0.084	0.058	−0.045	−0.083	−0.064	−0.070	0.000

The rest of Panel A reports the descriptive statistics for the three groups (lottery preferences, market frictions, and others) of candidate variables. Among the lottery preference variables, the average *Skew* is 0.258, suggesting that stock returns are on average positively skewed. The average *Maxret* is 7.1%. The average *RTP* is 15.8%, indicating that retail investors typically do not account for a large fraction of the trading volume of a stock. Among the market friction variables, *Lagret* has an average value of 1.6% (higher than the month t average return of 1.1% due to the one-dollar price screen we impose at the end of month $t-1$). The average value of *Zeroret*, an illiquidity proxy, is 20.8%, which indicates that on average about one-fifth of the trading days in a month have a zero return. The average *Spread* is 3.2%. Among the other candidate variables, *Dispersion, AvgVar*β, and *SUE* have average values of 19.9%, 0.228, and 16.9%, respectively.

Panel B of Table 1 reports the time-series averages of cross-sectional correlations. The average correlation between month $t-1$ *IVOL* and month t DGTW-adjusted returns is –0.027, which is consistent with the negative idiosyncratic volatility-return relation documented in the literature. The second column of Panel B shows that *IVOL* is positively correlated with *Skew, Coskew, E(Idioskew), Maxret, RTP, Lagret, Amihud, Zeroret, Spread, Dispersion*, and *AvgVar*β, and negatively correlated with *SUE*. These correlations are generally consistent with the various explanations that have been proposed for the idiosyncratic volatility puzzle. For example, the average correlation between *IVOL* and *Skew* is 0.193, which is consistent with the lottery preference explanation that *IVOL* predicts returns because of its correlation with skewness. Or consider *SUE*. The average correlation between *IVOL* and *SUE* is –0.099. This correlation is in line with the conclusion in Jiang, Xu, and Yao (2009) and Wong (2011) that the poor earnings performance of high idiosyncratic volatility stocks is responsible for their low returns. Among all the candidate variables, the one that has the highest correlation with *IVOL* is *Maxret* (average correlation of 0.882), suggesting that collinearity is a concern for this variable.[9]

3.2. The idiosyncratic volatility puzzle

To set the stage, Table 2 reports the results of monthly Fama-MacBeth cross-sectional regressions of month t individual stock DGTW-adjusted returns on month $t-1$ *IVOL* and different candidate variables.[10] We require at least 50 observations per month so that we have a reasonable sample size for each cross-sectional regression.

Model 1 regresses DGTW-adjusted returns on *IVOL* alone. The sample period is August 1963 to December 2012 with an average of 3,581 stocks per month. The average coefficient on *IVOL* is –16.955% (multiplied by 100 and reported in percent, t = –8.19) and its magnitude and statistical significance are in line with past findings in the literature. Models 2–6 add the lottery preference-based candidate variables one at a time to Model 1. For each model, the number of observations and sample period may differ from those of Model 1 due to data availability of the candidate variable examined. The results from Models 2–6 show that in all but one case, the coefficient on *IVOL* remains negative and statistically significant. Only when *Maxret* is included in the regression does the coefficient on *IVOL* become positive, consistent with the results in Bali, Cakici, and Whitelaw (2011).

Models 7–10 and 11–13 in Table 2 investigate the candidate variables related to market frictions and other explanations, respectively. The results show that the coefficient on *IVOL* is always negative and statistically significant, irrespective of the candidate variable included in the regressions.

The main takeaway from Table 2 is that the negative idiosyncratic volatility-return relation remains significant after controlling for almost all of the candidate explanatory variables (except for *Maxret*). But the question remains: Even if these candidate variables cannot completely explain away the idiosyncratic volatility puzzle, can they at least explain part of it? If so, what fraction of the puzzle can these candidate variables capture? We investigate this next using the decomposition methodology described in Section 2.5.

3.3. Candidate variables related to lottery preferences of investors

We first examine the candidate variables related to lottery preferences of investors. We start off with a detailed account of the decomposition analysis using *Skew* in Panel A of Table 3. Stage 1 regresses month t DGTW-adjusted returns on month $t-1$ *IVOL* and the average coefficient on *IVOL* is –17.401% with a t-statistic of –8.47. Note that this regression excludes firm-month observations with missing *Skew* to ensure that the sample is kept constant when we later add *Skew* to the analysis.

In Stage 2, we add *Skew* to the cross-sectional regressions following the conventional approach in Eq. (10) (this is identical to Model 2 in Table 2). The average coefficient on *Skew* is –0.099% with a t-statistic of –5.53, which is consistent with Barberis and Huang's (2008) prediction that investors overprice positively skewed stocks and as a result the future returns of those stocks are low. Controlling for *Skew*, however, we see that the average coefficient on *IVOL* is still negative and significant (–16.145%, t = –7.67), which suggests that *Skew* cannot fully explain the idiosyncratic volatility puzzle.

We now use our decomposition methodology to assess specifically what fraction of the puzzle is captured by *Skew*. In Stage 3, we regress *IVOL* on *Skew* each month. The average coefficient on *Skew* is 0.367% with a t-statistic of 34.31, suggesting that part of *IVOL* is indeed related to the skewness of a stock (a unit change in *Skew* is associated with a 0.367% change in *IVOL*). However, the adjusted R-squared shows that only 4.3% of the variation in *IVOL* can be explained by *Skew*. The Stage 3 estimated coefficients allow us to separate *IVOL* each month into two components: the first one ($\delta_{t-1}Skew_{it-1}$) is the component of *IVOL* that is related to *Skew* and the second ($a_{t-1}+\mu_{it-1}$) is the residual component that is unrelated to *Skew*.

In Stage 4, we follow Eq. (3) and use the above two components of *IVOL* to decompose the Stage 1 *IVOL* coefficient (γ_t) into a component that is related to *Skew* (γ_t^{Skew}) and a residual component (γ_t^R). The time-series averages

[9] The high correlation between the two variables is not surprising given that price range has been used as a volatility estimator in the literature. See, for example, Alizadeh, Brandt, and Diebold (2002) and Brandt and Diebold (2006). This makes *Maxret* a less satisfactory economic explanation for the puzzle since using *Maxret* as a candidate variable is akin to explaining the idiosyncratic volatility puzzle with another volatility proxy.

[10] Asparouhova, Bessembinder, and Kalcheva (2013) show that microstructure noise introduces an upward bias to stock returns, which could potentially bias the inferences from Fama-MacBeth regressions. In unreported tests, we follow their paper by using (one plus) month $t-1$ return as the weight in the Fama-MacBeth regressions and find that our results are robust to this noise-adjustment procedure.

Table 2
The negative relation between idiosyncratic volatility and returns.

Firm-level Fama-MacBeth cross-sectional regressions are estimated each month from August 1963 to December 2012. The dependent variable is DGTW-adjusted returns. Stocks with prices less than $1 at the end of the previous month are excluded. Time-series averages of the coefficients (×100) and the associated time-series *t*-statistics (in parentheses) are reported. Idiosyncratic volatility (*IVOL*) is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *Maxret* is the maximum daily return in month $t-1$. *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12	Model 13
Intercept	0.337***	0.355***	0.339***	0.456***	0.270***	0.433***	0.244***	0.396***	0.345***	0.379***	0.249***	0.349***	0.285***
	(6.11)	(6.47)	(6.06)	(7.79)	(4.74)	(3.88)	(4.27)	(7.20)	(6.66)	(4.98)	(2.95)	(6.07)	(4.75)
IVOL	−16.955***	−16.145***	−17.349***	−20.882***	10.740***	−23.129***	−10.831***	−21.220***	−17.024***	−21.679***	−13.803***	−17.063***	−15.669***
	(−8.19)	(−7.67)	(−8.28)	(−9.56)	(2.85)	(−6.67)	(−4.88)	(−9.38)	(−8.30)	(−7.23)	(−3.70)	(−8.29)	(−7.01)
Skew		−0.099***											
		(−5.53)											
Coskew			−0.380**										
			(−2.49)										
E(Idioskew)				0.022									
				(0.45)									
Maxret					−9.352***								
					(−10.20)								
RTP						−0.021							
						(−0.08)							
Lagret							−4.467***						
							(−13.72)						
Amihud								0.035***					
								(4.63)					
Zeroret									0.168				
									(0.62)				
Spread										2.064			
										(1.26)			
Dispersion											−0.055		
											(−1.08)		
AvgVarβ												−0.006	
												(−1.08)	
SUE													0.108***
													(17.21)
Avg adj R^2	0.004	0.005	0.006	0.005	0.006	0.006	0.011	0.007	0.007	0.007	0.007	0.005	0.005
Avg # firms/mth	3580.5	3563.7	3580.5	2870.8	3580.5	3776.8	3580.4	3327.3	3580.5	3716.6	2258.1	3232.1	3472.3
Startdate	196308	196308	196308	196808	196308	198302	196308	196308	196308	198402	198207	196808	197110
Enddate	201212	201212	201212	201212	201212	200101	201212	201212	201212	201212	201212	201212	201212

Table 3

Decomposing the idiosyncratic volatility puzzle: univariate analysis.

Using firm-level Fama-MacBeth cross-sectional regressions, the negative relation between idiosyncratic volatility (*IVOL*) and DGTW-adjusted returns is decomposed into a component that is related to a candidate variable and a residual component. Stage 1 regresses month t returns on month $t-1$ *IVOL* ($R_{it} = \alpha_t + \gamma_t IVOL_{it-1} + \varepsilon_{it}$). Stage 2 adds a candidate variable ($Candidate_{it-1}$) to the regression. Stage 3 regresses *IVOL* on the candidate variable ($IVOL_{it-1} = a_{t-1} + \delta_{t-1} Candidate_{it-1} + \mu_{it-1}$) to decompose $IVOL_{it-1}$ into two orthogonal components: $\delta_{t-1} Candidate_{it-1}$ and $(a_{t-1} + \mu_{it-1})$. In Stage 4, the γ_t coefficient from Stage 1 is decomposed as: $\gamma_t = \frac{\mathrm{Cov}[R_{it}, IVOL_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} = \frac{\mathrm{Cov}[R_{it},\ \delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} + \frac{\mathrm{Cov}[R_{it},\ (a_{t-1}+\mu_{it-1})]}{\mathrm{Var}[IVOL_{it-1}]} = \gamma_t^C + \gamma_t^R$. The time-series average of γ_t^C divided by the time-series average of γ_t then measures the fraction of the negative idiosyncratic volatility-return relation explained by the candidate variable, and the average γ_t^R divided by the average γ_t measures the fraction of the relation left unexplained by the candidate variable, with the standard errors of the fractions being determined using the multivariate delta method. Stocks with prices less than \$1 at the end of the previous month are excluded from the analysis. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. Panel A examines lottery preference-based candidate variables, where *Skew* is the month $t-1$ skewness of raw daily returns, *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009), *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010), *Maxret* is the maximum daily return in month $t-1$, and *RTP* is the retail trading proportion computed from ISSM and TAQ. Panel B examines market friction-based candidate variables, where *Lagret* is the month $t-1$ return, *Amihud* is the illiquidity measure in Amihud (2002), *Zeroret* is the fraction of trading days in month $t-1$ with a zero return, and *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. Panel B also examines all the other candidate variables, where *Dispersion* is the dispersion in analysts' FY1 earnings forecasts, *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012), and *SUE* is the most recent standardized unexpected earnings. Time-series averages of estimated coefficients (×100) are reported with t-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Lottery preference variables

Stage	Description	Variable	Lottery preference variables									
			Skew		Coskew		E(Idioskew)		Maxret		RTP	
1	DGTW-adj ret	Intercept	0.353***	(6.47)	0.337***	(6.11)	0.411***	(7.34)	0.337***	(6.11)	0.436***	(3.98)
	on IVOL	IVOL	−17.401***	(−8.47)	−16.955***	(−8.19)	−20.138***	(−9.74)	−16.955***	(−8.19)	−23.229***	(−6.58)
2	Add	Intercept	0.355***	(6.47)	0.339***	(6.06)	0.456***	(7.79)	0.270***	(4.74)	0.433***	(3.88)
	candidate	IVOL	−16.145***	(−7.67)	−17.349***	(−8.28)	−20.882***	(−9.56)	10.740***	(2.85)	−23.129***	(−6.67)
	variable	Candidate	−0.099***	(−5.53)	−0.380**	(−2.49)	0.022	(0.45)	−9.352***	(−10.20)	−0.021	(−0.08)
3	IVOL on	Intercept	2.398***	(90.46)	2.474***	(87.71)	1.260***	(61.02)	0.767***	(81.51)	2.096***	(67.02)
	candidate	Candidate	0.367***	(34.31)	0.643***	(9.90)	1.523***	(41.60)	25.850***	(238.67)	5.044***	(45.26)
	variable	Avg adj R^2	4.3%		4.0%		18.4%		77.9%		22.6%	
4	Decompose Stage 1	Candidate	−1.785		−0.321		−2.969		−18.923		−5.189	
	IVOL coefficient		10.3%***	(6.73)	1.9%	(1.08)	14.7%***	(5.80)	112.0%***	(18.72)	22.3%***	(5.92)
		Residual	−15.615		−16.633		−17.168		1.968		−18.040	
			89.7%***	(58.88)	98.1%***	(56.09)	85.3%***	(33.52)	−11.6%*	(−1.95)	77.7%***	(20.58)
		Total	−17.401***	(−8.47)	−16.955***	(−8.19)	−20.138***	(−9.74)	−16.955***	(−8.19)	−23.229***	(−6.58)
			100%		100%		100%		100%		100%	
Sample period			1963–2012		1963–2012		1968–2012		1963–2012		1983–2001	
Avg # firms/mth			3563.7		3580.5		2870.8		3580.5		3776.8	

(continued on next page)

Table 3
Continued.

Panel B: Market friction and other variables

Stage	Description	Variable	Market friction variables						Other variables							
			Lagret		Amihud		Zeroret		Spread		Dispersion		AvgVarβ		SUE	
1	DGTW-adj ret on IVOL	Intercept	0.337***	(6.12)	0.373***	(6.90)	0.337***	(6.11)	0.389***	(4.89)	0.250***	(2.97)	0.355***	(6.13)	0.367***	(6.14)
		IVOL	−16.964***	(−8.20)	−18.401***	(−8.67)	−16.955***	(−8.19)	−20.032***	(−7.73)	−14.326***	(−3.76)	−17.158***	(−8.29)	−17.431***	(−7.84)
2	Add candidate variable	Intercept	0.244***	(4.27)	0.396***	(7.20)	0.345***	(6.66)	0.379***	(4.98)	0.249***	(2.95)	0.349***	(6.07)	0.285***	(4.75)
		IVOL	−10.831***	(−4.88)	−21.220***	(−9.38)	−17.024***	(−8.30)	−21.679***	(−7.23)	−13.803***	(−3.70)	−17.063***	(−8.29)	−15.669***	(−7.01)
		Candidate	−4.467***	(−13.72)	0.035***	(4.63)	0.168	(0.62)	2.064	(1.26)	−0.055	(−1.08)	−0.006	(−1.08)	0.108***	(17.21)
3	IVOL on candidate variable	Intercept	2.350***	(85.09)	2.376***	(86.39)	2.439***	(89.47)	1.824***	(51.29)	2.185***	(70.67)	2.483***	(91.65)	2.600***	(88.45)
		Candidate	2.324***	(21.51)	0.040***	(19.31)	0.778***	(10.32)	28.474***	(34.63)	0.211***	(16.27)	0.009***	(6.05)	−0.065***	(−34.43)
		Avg adj R^2	8.2%		11.3%		1.0%		28.9%		1.7%		0.5%		1.2%	
4	Decompose Stage 1 IVOL coefficient	Candidate	−5.714		0.441		−0.144		−6.087		−0.764		−0.167		−1.898	
			33.7%***	(6.47)	−2.4%	(−0.69)	0.9%	(0.72)	30.4%***	(5.44)	5.3%*	(1.92)	1.0%*	(1.80)	10.9%***	(7.35)
		Residual	−11.250		−18.842		−16.810		−13.944		−13.562		−16.992		−15.533	
			66.3%***	(12.73)	102%***	(29.64)	99.1%***	(83.49)	69.6%***	(12.46)	94.7%***	(34.12)	99.0%***	(183.00)	89.1%***	(60.18)
		Total	−16.964***	(−8.20)	−18.401***	(−8.67)	−16.955***	(−8.19)	−20.032***	(−7.73)	−14.326***	(−3.76)	−17.158***	(−8.29)	−17.431***	(−7.84)
			100%		100%		100%		100%		100%		100%		100%	
Sample period			1963–2012		1963–2012		1963–2012		1984–2012		1982–2012		1968–2012		1971–2012	
Avg # firms/mth			3580.4		3327.3		3580.5		3716.6		2258.1		3232.1		3472.3	

of γ_t^{Skew} and γ_t^R are –1.785% and –15.615%, respectively. Since by construction the two coefficients sum up to the Stage 1 coefficient of –17.401%, we can readily calculate the fraction of the Stage 1 coefficient attributable to *Skew* as $\frac{-1.785}{-17.401} = 10.3\%$ ($t = 6.73$), and the fraction attributable to the residual component is $\frac{-15.615}{-17.401} = 89.7\%$ ($t = 58.88$). We therefore conclude that *Skew* can explain 10.3% of the idiosyncratic volatility puzzle.

We also examine the other skewness variables in Panel A of Table 3. The results show that *Coskew* explains only 1.9% ($t = 1.08$) of the puzzle, and *E(Idioskew)* explains 14.7% ($t = 5.80$) of the puzzle.

Next, we see that the portion of the puzzle that is explained by *Maxret* is 112.0% ($t = 18.72$).[11] It thus appears that *Maxret* explains the idiosyncratic volatility puzzle entirely. However, considering the near perfect collinearity between *Maxret* and *IVOL* and that *Maxret* is essentially a range-based measure of volatility, this finding might be mechanical.

The last column of Panel A shows that *RTP* explains 22.3% ($t = 5.92$) of the idiosyncratic volatility puzzle. However, we note that this result is obtained over a relatively short sample period (1983–2001) compared to the rest of the candidate variables.

While our methodology directly quantifies the fraction of the idiosyncratic volatility puzzle explained by a lottery preference-based candidate variable, one can also evaluate the validity of the candidate variable as a proxy for lottery preferences by studying its return predictability after controlling for *IVOL*. For example, although *E(Idioskew)* explains more than 10% (14.7%) of the puzzle, it has no independent return predictive power after controlling for *IVOL* according to Stage 2 regressions. If *E(Idioskew)* is a good proxy for the lottery feature of a stock, it seems reasonable to expect both the part of *E(Idioskew)* that is related to *IVOL* and the part that is unrelated to *IVOL* to predict returns negatively. The fact that the latter has no return predictive power (average coefficient $= 0.022\%$ and $t = 0.45$ from Stage 2 regressions) implies that only the part of *E(Idioskew)* that is related to *IVOL* is consistent with the lottery preference-based explanation. One reason for this might be that one of the components of *E(Idioskew)* is *IVOL* itself, thus introducing a potential mechanical relation between the two variables.

Of the other lottery preference-based candidate variables, *RTP* also fails to predict returns after controlling for *IVOL* (average coefficient $= -0.021\%$ and $t = -0.08$). Hence, its ability to measure the lottery feature of a stock must also be caveated since only the part of *RTP* that is correlated with *IVOL* has a negative and significant relation with stock returns. For *Skew, Coskew*, and *Maxret*, their return predictability remains negative and significant after controlling for *IVOL* in Stage 2 regressions, which suggests that these three variables are viable lottery preference proxies (although *Coskew* can only explain a negligible fraction of the *IVOL* puzzle according to our decomposition methodology and *Maxret* is likely to be mechanically related to *IVOL*).

Overall, the results from Table 3 Panel A suggest that most of the lottery preference-based candidate variables show some promise in explaining the idiosyncratic volatility puzzle, with *Maxret, RTP, E(Idioskew)*, and *Skew* each explaining more than 10% of the puzzle. The concern about *Maxret*, however, is its mechanical relation with *IVOL*. And while *RTP* and *E(Idioskew)* capture sizable fractions of the puzzle, their ability to measure the lottery feature of a stock is hindered by the fact that neither of them can predict returns after controlling for *IVOL*.

3.4. Candidate variables related to market frictions

Panel B of Table 3 examines the candidate variables related to market frictions. We see that both *Lagret* and *Spread* explain about one-third of the idiosyncratic volatility puzzle (33.7% for *Lagret* and 30.4% for *Spread* with t-statistics of 6.47 and 5.44, respectively), thus leaving two-thirds of the puzzle explained by the residual component. These results suggest that bid-ask bounce and other microstructure effects (that likely drive the short-term return reversal) contribute significantly to the puzzle.

On the other hand, other illiquidity proxies such as *Amihud* and *Zeroret* fail to explain significant fractions of the puzzle. The explained fraction is 0.9% ($t = 0.72$) for *Zeroret* and actually negative at –2.4% ($t = -0.69$) for *Amihud*. Intuitively, the reason that *Amihud* has a negative contribution to the idiosyncratic volatility puzzle is because it is positively correlated with *IVOL* but its return predictability after controlling for *IVOL* is also positive, which is in the opposite direction of the idiosyncratic volatility puzzle. The low explanatory power of *Amihud* despite its high correlation with *IVOL* shows that our decomposition methodology does not necessarily attribute a large explained fraction to a candidate variable just because it has a high correlation with idiosyncratic volatility.

In sum, some market friction-based candidate variables, namely, *Lagret* and *Spread,* capture sizable portions of the idiosyncratic volatility puzzle, while others (*Amihud* and *Zeroret*) have very little success in explaining the puzzle. We also note that *Lagret* continues to predict returns negatively (and significantly) after controlling for *IVOL*, which is consistent with the evidence in the literature regarding the impact of market frictions on stock returns. On the other hand, the relation between *Spread* and returns becomes insignificant after controlling for *IVOL*.

3.5. Candidate variables related to other explanations

Panel B of Table 3 also examines candidate variables that cannot be grouped into the lottery preference or market friction categories. We first look at *Dispersion* and find that it can only explain a small fraction (5.3%, $t = 1.92$) of the idiosyncratic volatility puzzle.[12]

[11] The reason this fraction is above 100% is because the adding-up constraint in Stage 4 requires the *Maxret* component and the residual component to add up to the Stage 1 coefficient on *IVOL*.

[12] Ang, Hodrick, Xing, and Zhang (2006) show that the Fama-French alpha of an *IVOL* quintile spread portfolio decreases by about two-thirds after controlling for *Dispersion* (using two-way sorts). In unreported results, we find that the reduction in alpha is largely due to requiring stocks to

The next candidate variable we investigate is *AvgVarβ*. This variable contributes very little in explaining the puzzle, with the explained fraction equal to 1.0% (t = 1.80). We also examine whether *SUE* can explain the idiosyncratic volatility puzzle. We find that it captures 10.9% (t = 7.35) of the puzzle, suggesting that the poor earnings performance of high idiosyncratic volatility stocks helps to explain the idiosyncratic volatility puzzle. Finally, we note that while *Dispersion* and *AvgVarβ* have no return predictive power after controlling for *IVOL, SUE* has a positive and significant relation with returns after controlling for *IVOL*, which is consistent with prior studies that show a post-announcement drift following earnings surprises.

3.6. Interaction effects

As mentioned in the introduction, several papers show that there exists variation in the idiosyncratic volatility puzzle across stocks with different price levels, analyst coverage, credit ratings, short interest, leverage, institutional ownership, B/M ratios, and exchange listings. We examine what fractions of the idiosyncratic volatility puzzle come from the interaction effects between idiosyncratic volatility and these conditioning characteristics.[13] To do so, we define *CharRank* as the decile rank of a conditioning characteristic (scaled to be between zero and one and in ascending order of the characteristic except for analyst coverage, credit rating, institutional ownership, and B/M ratio) and then include *CharRank* and *CharRank* × *IVOL* in our decomposition analysis.

Table 4 reports the results for the eight conditioning characteristics. We see from Stage 2 regressions that the coefficients on the interaction term *CharRank* × *IVOL* are mostly negative and statistically significant, which suggests that the idiosyncratic volatility puzzle is stronger among stocks with high prices, low analyst coverage, poor credit ratings, high short interest, high leverage, low institutional ownership, and low B/M ratios. However, we find no evidence that the idiosyncratic volatility puzzle is stronger among stocks with non-NYSE listings. The Stage 4 decomposition results show that the average fraction explained by *CharRank* and *CharRank* × *IVOL* is 83.9% across the eight conditioning characteristics. This large explained fraction is not surprising, however, since *IVOL* itself enters into the interaction term. In unreported results, we break the mechanical relation between *IVOL* and *CharRank* × *IVOL* by replacing *IVOL* in the interaction term with *IVOLRank* (the decile rank of *IVOL*) and obtain a more modest average explained fraction of 46.3% across the conditioning characteristics.

4. Evaluating multiple candidate explanations at the same time

4.1. Multivariate analysis

After investigating each of the candidate variables in isolation, we now turn to multivariate analysis. We want to know the marginal contribution of each variable after controlling for competing candidate variables. In addition, we are interested in the total fraction of the puzzle these candidate variables can collectively explain. The linear adding-up constraint of our decomposition methodology ensures that their combined contributions plus that of the residual component add up to 100% of the puzzle.

Table 5 reports the results of the multivariate analysis. We exclude *Maxret* from the analysis due to its mechanical relation with *IVOL*. The remaining candidate variables are included in Model 1. We see that the 11 candidate variables together explain 29.0% of the idiosyncratic volatility puzzle and the residual component accounts for the remaining 71.0% (t = 5.86) of the puzzle. The largest contributor is *Spread* which captures 7.6% of the puzzle, followed by *Lagret* at 5.7%, although neither of these fractions is statistically different from zero (t-statistics of 0.52 and 1.03, respectively). None of the other candidate variables explains more than 5% of the puzzle though some of the explained fractions are statistically significant, namely, *E(Idioskew)* at 4.2% (t = 2.13), *Dispersion* at 3.4% (t = 2.66), and *SUE* at 2.4% (t = 2.76). For most of the 11 candidate variables, the explained fractions are significantly lower than their univariate contributions. This is likely the result of controlling for other candidate variables as well as the limited sample for Model 1 due to the availability of some of the candidate variables.

In Model 2, we drop both *RTP* and *Spread* to extend the sample period to 1982–2012 (from 1984 to 2001 in Model 1) and increase the average cross-sectional sample size to 1,806 stocks per month (from 1,524 stocks per month in Model 1). The total fraction of the puzzle explained by the remaining nine candidate variables increases only slightly to 29.9%, with *E(Idioskew)* and *Dispersion* contributing 10.7% (t = 1.98) and 5.6% (t = 3.22), respectively, and the other seven candidate variables making up the rest (13.6%). In this model, the residual unexplained fraction is still large and significant at 70.1% (t = 6.56).

In Model 3, we further drop *Dispersion* to extend the sample period to 1971–2012 and increase the cross-sectional sample size to 2,752 stocks per month. The total explained fraction now increases substantially to 54.5% with the residual component capturing the remaining 45.5% (t = 10.06).[14] Interestingly, *Lagret*, which

have non-missing *Dispersion* rather than the pure effect of controlling for *Dispersion* while keeping the sample constant.

[13] Price is the stock price at the end of previous month ($t-1$). Analyst coverage is the number of analysts issuing FY1 earnings forecasts in month $t-1$ (firms with no I/B/E/S coverage are excluded). Credit rating is the Standard & Poor's (S&P) long-term issuer rating reported in Compustat (SPLTICRM) and numerically coded following Avramov, Chordia, Jostova, and Philipov (2013). Short interest is measured in month $t-1$ using the data in Cohen, Diether, and Malloy (2007). Leverage is the Compustat long-term debt (LTDEBT) over total assets (AT) from the previous fiscal year end. Institutional ownership is measured using data reported in the most recent quarterly Thomson 13F filings. Non-NYSE listing is identified using the EXCHCD flag on CRSP.

[14] Unreported results show that if we further drop *SUE* from Model 3, we can extend the sample period back to 1968 and in this case the total fraction of the puzzle explained by the remaining seven candidate variables is 45.8%. In addition, if we add *Maxret* (which is mechanically related to *IVOL*) to Model 1, unsurprisingly, the total explained fraction increases considerably to 85.8%.

Table 4
Decomposing the idiosyncratic volatility puzzle: interaction analysis.

Using firm-level Fama-MacBeth cross-sectional regressions, the negative relation between idiosyncratic volatility (*IVOL*) and DGTW-adjusted returns is decomposed into components that are related to a conditioning characteristic, its interaction with *IVOL*, and a residual component. Stocks with prices less than $1 at the end of the previous month are excluded. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. The conditioning characteristics are price, analyst coverage, credit rating, short interest, leverage, institutional ownership, B/M ratio, and a non-NYSE dummy. *CharRank* is the decile rank (scaled between zero and one and in ascending order of the characteristic except for analyst coverage, credit rating, institutional ownership, and B/M ratio) of a conditioning characteristic. For non-NYSE listing, *CharRank* is equal to one for non-NYSE stocks and zero otherwise. The standard errors of the fractions of the puzzle explained are determined using the multivariate delta method. Time-series averages of estimated coefficients (×100) are reported with *t*-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Stage	Description	Variable	Decile rank of a conditioning characteristic							
			Price		Analyst coverage		Credit rating		Short interest	
1	DGTW-adj ret on IVOL	Intercept	0.337***	(6.11)	0.293***	(3.54)	0.350***	(3.91)	0.374***	(2.72)
		IVOL	−16.955***	(−8.19)	−15.815***	(−4.51)	−23.596***	(−4.95)	−13.404***	(−3.99)
2	Add candidate variables	Intercept	0.783***	(9.66)	0.156	(1.45)	−0.116	(−0.77)	0.191	(0.85)
		IVOL	−20.782***	(−7.93)	−12.949**	(−2.10)	5.367	(0.61)	−2.526	(−0.67)
		CharRank	−0.439***	(−4.04)	0.286**	(2.09)	0.687***	(2.95)	0.528**	(2.00)
		CharRank × IVOL	−13.835**	(−2.57)	−6.765	(−0.97)	−39.717***	(−3.70)	−27.280***	(−5.31)
3	IVOL on candidate variables	Intercept	3.493***	(79.31)	1.891***	(64.48)	1.328***	(56.25)	3.860***	(73.25)
		CharRank	−4.867***	(−80.55)	−2.335***	(−56.94)	−1.440***	(−48.41)	−5.765***	(−68.93)
		CharRank × IVOL	150.153***	(257.86)	122.170***	(331.44)	110.809***	(432.41)	150.476***	
		Avg adj R^2	52.5%		83.7%		90.8%		64.4%	
4	Decompose Stage 1 IVOL coefficient	CharRank	0.912		1.120		3.642		4.469	
			−5.4%	(−0.62)	−7.1%	(−1.28)	−15.4%***	(−3.18)	−33.3%**	(−2.27)
		CharRank × IVOL	−8.303		−15.256		−27.807		−17.108	
			49.0%***	(5.66)	96.5%***	(10.24)	118%***	(19.81)	128%***	(7.32)
		Residual	−9.564		−1.680		0.569		−0.765	
			56.4%***	(16.59)	10.6%*	(1.95)	−2.4%	(−0.61)	5.7%	(0.69)
		Total	−16.955***	(−8.19)	−15.815***	(−4.51)	−23.596***	(−4.95)	−13.404***	(−3.99)
			100%		100%		100%		100%	
Sample period			1963–2012		1982–2012		1986–2012		1988–2005	
Avg # firms/mth			3580.5		2685.1		960.0		2917.7	
			Leverage		Inst. own.		B/M		Non-NYSE	
1	DGTW-adj ret on IVOL	Intercept	0.337***	(6.12)	0.393***	(5.62)	0.337***	(6.11)	0.337***	(6.11)
		IVOL	−16.978***	(−8.21)	−19.215***	(−7.65)	−16.955***	(−8.19)	−16.955***	(−8.19)
2	Add candidate variables	Intercept	0.328***	(4.78)	0.232***	(2.84)	0.173**	(2.52)	0.330***	(5.68)
		IVOL	−12.287***	(−4.78)	−13.232***	(−2.94)	−11.571***	(−4.31)	−19.962***	(−5.98)
		CharRank	0.041	(0.52)	0.258**	(2.19)	0.357***	(4.90)	0.071	(1.10)
		CharRank × IVOL	−10.284***	(−3.14)	−9.513*	(−1.73)	−11.815***	(−3.96)	2.196	(0.65)
3	IVOL on candidate variables	Intercept	2.635***	(78.77)	2.090***	(75.50)	2.631***	(83.25)	1.785***	(103.28)
		CharRank	−3.740***	(−77.11)	−2.597***	(−70.58)	−3.747***	(−78.81)	−1.785***	(−103.28)
		CharRank × IVOL	142.964***	(387.60)	122.161***	(320.12)	138.984***	(438.62)	100.000***	(100000)
		Avg adj R^2	68.3%		85.4%		66.7%		83.2%	
4	Decompose Stage 1 IVOL coefficient	CharRank	2.376		3.382		0.392		1.305	
			−14.0%***	(−2.84)	−17.6%***	(−3.58)	−2.3%	(−1.58)	−7.7%**	(−2.16)
		CharRank × IVOL	−15.493		−20.707		−13.688		−14.854	
			91.3%***	(13.44)	108%***	(15.33)	80.7%***	(20.64)	87.6%***	(14.98)
		Residual	−3.862		−1.890		−3.658		−3.406	
			22.7%***	(7.28)	9.8%***	(3.01)	21.6%***	(6.60)	20.1%***	(5.99)
		Total	−16.978***	(−8.21)	−19.215***	(−7.65)	−16.955***	(−8.19)	−16.955***	(−8.19)
			100%		100%		100%		100%	
Sample period			1963–2012		1979–2012		1963–2012		1963–2012	
Avg # firms/mth			3572.4		3964.5		3580.5		3580.5	

explained a small and statistically insignificant fraction of the puzzle in Model 2, is the biggest contributor in Model 3 at 21.5% ($t = 5.74$), followed by *E(Idioskew)* at 15.1% ($t = 6.24$), *Skew* at 6.5% ($t = 6.35$), and *SUE* at 5.1% ($t = 7.58$). The rest of the candidate variables (*Coskew, Amihud, Zeroret*, and *AvgVarβ*) together only explain 6.2% of the puzzle.

The results from Table 5 are summarized in Panel A of Fig. 1 where we plot the marginal contributions of the three groups of candidate variables (lottery preferences,

Table 5
Decomposing the idiosyncratic volatility puzzle: multivariate analysis.

Using firm-level Fama-MacBeth cross-sectional regressions, the negative relation between month $t-1$ idiosyncratic volatility (*IVOL*) and month t DGTW-adjusted returns is decomposed into a number of components each related to a candidate variable and a residual component. Stocks with prices less than \$1 at the end of the previous month are excluded. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 earnings forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings. The standard errors of the fractions of the puzzle explained are determined using the multivariate delta method. Time-series averages of estimated coefficients ($\times$100) are reported with t-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Stg.	Description	Variable	Model 1			Model 2			Model 3		
			Coeff.	Fraction	t-stat	Coeff.	Fraction	t-stat	Coeff.	Fraction	t-stat
1	DGTW-adj ret on IVOL	Intercept	0.263**		(2.04)	0.254***		(2.90)	0.412***		(7.17)
		IVOL	−18.560***		(−3.17)	−14.231***		(−3.49)	−19.028***		(−8.89)
2	Add candidate variables	Intercept	0.373***		(2.63)	0.287***		(3.01)	0.364***		(5.53)
		IVOL	−25.582***		(−4.27)	−13.203***		(−3.63)	−12.629***		(−5.14)
		Skew	0.158***		(5.07)	0.120***		(4.12)	0.092***		(4.73)
		Coskew	0.348		(0.63)	0.225		(0.45)	0.058		(0.30)
		E(IdioSkew)	−0.151		(−1.07)	−0.065		(−0.75)	−0.087		(−1.47)
		RTP	−2.338***		(−3.01)						
		Lagret	−4.391***		(−7.13)	−3.829***		(−8.46)	−5.253***		(−13.78)
		Amihud	−0.073		(−0.49)	0.014		(0.27)	0.008***		(3.25)
		Zeroret	−0.298		(−0.98)	0.157		(0.40)	−0.135		(−0.49)
		Spread	24.488***		(3.14)						
		Dispersion	−0.132***		(−2.86)	−0.104*		(−1.95)			
		AvgVarβ	0.003		(0.21)	0.006		(0.53)	−0.004		(−0.67)
		SUE	0.061***		(6.22)	0.051***		(7.07)	0.117***		(15.88)
3	IVOL on candidate variables	Intercept	1.361***		(46.27)	1.446***		(65.63)	1.342***		(70.82)
		Skew	0.093***		(13.71)	0.088***		(11.53)	0.199***		(29.93)
		Coskew	0.267*		(1.90)	0.665***		(5.24)	0.284***		(4.53)
		E(IdioSkew)	0.198***		(7.47)	0.798***		(31.57)	1.381***		(49.82)
		RTP	1.543***		(10.45)						
		Lagret	0.326***		(2.72)	0.196*		(1.71)	1.565***		(16.02)
		Amihud	−0.127***		(−9.87)	0.116***		(11.32)	0.024***		(28.15)
		Zeroret	−3.039***		(−28.27)	−0.562***		(−5.96)	−1.395***		(−22.30)
		Spread	56.846***		(43.70)						
		Dispersion	0.078***		(14.72)	0.129***		(22.01)			
		AvgVarβ	0.010***		(7.61)	0.007***		(4.59)	0.009***		(8.36)
		SUE	−0.010***		(−11.94)	−0.019***		(−19.05)	−0.031***		(−27.09)
		Avg adj R^2	47.5%		(66.25)	26.2%		(51.67)	37.1%		(76.29)
4	Decompose Stage 1 IVOL coefficient	Skew	−0.450	2.4%	(1.51)	−0.432	3.0%	(1.56)	−1.246	6.5%***	(6.35)
		Coskew	−0.520	2.8%	(0.99)	−0.505	3.5%	(0.73)	−0.593	3.1%***	(2.95)
		E(IdioSkew)	−0.772	4.2%**	(2.13)	−1.516	10.7%**	(1.98)	−2.874	15.1%***	(6.24)
		RTP	−0.043	0.2%	(0.08)						
		Lagret	−1.050	5.7%	(1.03)	−0.072	0.5%	(0.07)	−4.085	21.5%***	(5.74)
		Amihud	0.351	−1.9%	(−0.69)	−0.531	3.7%	(0.69)	−0.726	3.8%	(1.60)
		Zeroret	−0.248	1.3%	(0.28)	0.136	−1.0%	(−0.47)	0.186	−1.0%	(−1.02)
		Spread	−1.412	7.6%	(0.52)						
		Dispersion	−0.640	3.4%***	(2.66)	−0.793	5.6%***	(3.22)			
		AvgVarβ	−0.150	0.8%	(0.81)	0.032	−0.2%	(−0.12)	−0.060	0.3%	(0.67)
		SUE	−0.448	2.4%***	(2.76)	−0.579	4.1%***	(3.12)	−0.973	5.1%***	(7.58)
		Residual	−13.178	71.0%***	(5.86)	−9.972	70.1%***	(6.56)	−8.657	45.5%***	(10.06)
		Total	−18.560***	100%	(−3.17)	−14.231***	100%	(−3.49)	−19.028***	100%	(−8.89)
		Sample period	1984–2001			1982–2012			1971–2012		
		Avg # firms/mth	1524.4			1806.0			2752.4		

market frictions, and others) and the residual unexplained fractions using bar charts. We see that variables related to lottery preferences of investors contribute the most in explaining the idiosyncratic volatility puzzle, accounting for 10–25% of the puzzle. Market friction-based candidate variables explain 3–24% of the puzzle and variables related to other explanations account for 5–10% of the puzzle. On the other hand, the unexplained fraction is still very large at 46–71%. Therefore, while the lottery preferences of investors and market frictions prove to be important economic drivers of the idiosyncratic volatility puzzle, a significant portion of the

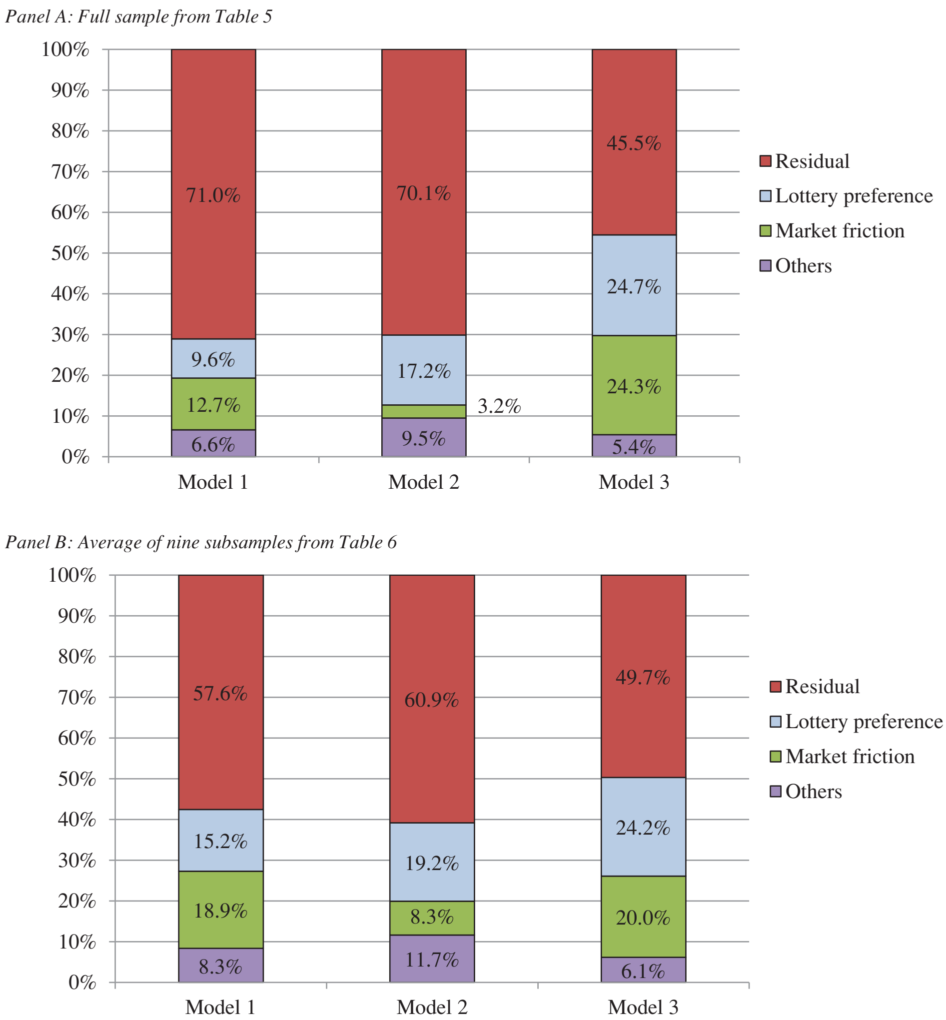

Fig. 1. Average fractions of the idiosyncratic volatility puzzle explained by various groups of candidate variables. The fractions of the idiosyncratic volatility puzzle explained by various groups of candidate variables from Table 5 are plotted in Panel A. Panel B plots the average fractions across the nine subsamples from Table 6. Lottery preference-based candidate variables consist of *Skew*, *Coskew*, *E(Idioskew)*, and *RTP*. Market friction-based candidate variables consist of *Lagret*, *Amihud*, *Zeroret*, and *Spread*. Candidate variables related to other explanations consist of *Dispersion*, *AvgVarβ*, and *SUE*. Model 1 includes all the candidate variables in the multivariate decomposition analysis. Model 2 excludes *RTP* and *Spread*, and Model 3 further excludes *Dispersion*. Residual represents the fraction of the puzzle that cannot be explained.

puzzle cannot be accounted for by the explanations we examine.

4.2. Subsample analysis

In this subsection, we repeat the multivariate decomposition analysis in Table 5 for subsamples of stocks that have been shown to exhibit a stronger idiosyncratic volatility puzzle. This allows us to investigate whether a candidate variable works as well in those subsamples as it does in the full sample. The subsamples we examine include stocks that have prices of at least five dollars, low analyst coverage (three or fewer analysts), poor credit ratings (the lowest three credit rating deciles), high short interest (the highest three short-interest deciles), high leverage (the highest three leverage deciles), low institutional ownership (the lowest three institutional ownership deciles), low B/M ratios (the lowest three B/M deciles), or non-NYSE listings. In addition, we also examine the idiosyncratic volatility puzzle in non-January months (Doran, Jiang, and Peterson, 2012).

Table 6 reports the results of the multivariate decomposition analysis for the nine subsamples. To conserve space, we only report the Stage 4 results. The table shows that a bigger fraction of the idiosyncratic volatility puzzle is typically explained by the candidate variables in these subsamples than in the full sample. Specifically, the total explained fraction is 26–54% (across the three models) for stocks with prices of at least five dollars, 37–50% for low analyst coverage stocks, 47–63% for poor credit rating stocks, 34–52% for high short interest stocks, 41–49% for high leverage stocks, 54–66% for low institutional

Table 6

Decomposing the idiosyncratic volatility puzzle: subsample analysis.

Using firm-level Fama-MacBeth cross-sectional regressions, the negative relation between month $t-1$ idiosyncratic volatility (IVOL) and month t DGTW-adjusted returns is decomposed into a number of components each related to a candidate variable and a residual component for subsamples of stocks with prices of at least \$5 (Panel A), low analyst coverage (1–3 analysts, Panel B), poor credit ratings (lowest three deciles, Panel C), high short interest (highest three deciles, Panel D), high leverage (highest three deciles, Panel E), low institutional ownership (lowest three deciles, Panel F), low B/M ratios (lowest three deciles, Panel G), non-NYSE listings (Panel H), and non-January months (Panel I). IVOL is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. Skew is the month $t-1$ skewness of raw daily returns. Coskew is the coskewness measure in Chabi-Yo and Yang (2009). E(Idioskew) is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). RTP is the retail trading proportion computed from ISSM and TAQ. Lagret is the month $t-1$ return. Amihud is the illiquidity measure in Amihud (2002). Zeroret is the fraction of trading days in month $t-1$ with a zero return. Spread is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. Dispersion is the dispersion in analysts' FY1 earnings forecasts. AvgVarβ is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). SUE is the most recent standardized unexpected earnings. The standard errors of the fractions of the puzzle explained are determined using the multivariate delta method. Time-series averages of estimated coefficients ($\times$100) are reported with t-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	t-stat	Coeff.	Fraction	t-stat	Coeff.	Fraction	t-stat
Panel A: Prices ≥\$5									
Skew	−0.345	1.6%	(1.08)	−0.390	2.3%	(1.45)	−1.227	5.4%***	(5.76)
Coskew	−0.581	2.7%	(0.93)	−1.028	6.1%	(1.47)	−0.737	3.2%**	(2.40)
E(IdioSkew)	−0.655	3.1%	(1.46)	−2.190	13.0%***	(3.25)	−2.008	8.8%***	(4.16)
RTP	−0.734	3.5%*	(1.85)						
Lagret	−1.495	7.1%	(1.40)	−1.269	7.5%	(1.33)	−5.352	23.4%***	(6.75)
Amihud	0.981	−4.6%*	(−1.67)	−1.255	7.4%***	(2.64)	−1.783	7.8%***	(4.71)
Zeroret	1.096	−5.2%	(−0.91)	0.216	−1.3%	(−0.57)	0.192	−0.8%	(−0.49)
Spread	−2.508	11.9%	(1.29)						
Dispersion	−0.502	2.4%*	(1.84)	−0.657	3.9%**	(2.25)			
AvgVarβ	−0.170	0.8%	(0.69)	−0.177	1.0%	(0.44)	−0.195	0.9%*	(1.88)
SUE	−0.495	2.3%***	(2.69)	−0.609	3.6%***	(3.38)	−1.333	5.8%***	(6.49)
Residual	−15.740	74.4%***	(7.40)	−9.549	56.5%***	(6.87)	−10.451	45.6%***	(9.56)
Total	−21.147***	100%	(−3.46)	−16.907***	100%	(−4.22)	−22.896***	100%	(−8.91)
Sample period	1984–2001			1982–2012			1971–2012		
Avg # firms/mth	1448.1			1687.8			2267.9		
Panel B: Low analyst coverage									
Skew	−1.704	7.0%***	(2.66)	−0.637	3.6%*	(1.77)	−0.992	4.9%**	(2.35)
Coskew	−1.380	5.7%	(1.06)	0.081	−0.5%	(−0.10)	−1.369	6.7%*	(1.82)
E(IdioSkew)	−1.158	4.8%**	(2.33)	−1.946	11.0%**	(2.16)	−2.594	12.7%***	(3.35)
RTP	−0.297	1.2%	(0.29)						
Lagret	−3.036	12.5%**	(2.32)	−0.029	0.2%	(0.03)	−1.023	5.0%	(1.03)
Amihud	2.075	−8.5%	(−1.52)	−1.439	8.1%	(1.49)	−0.936	4.6%	(0.93)
Zeroret	0.601	−2.5%	(−0.32)	1.151	−6.5%*	(−1.74)	0.600	−2.9%	(−1.22)
Spread	−5.497	22.6%	(1.18)						
Dispersion	−0.348	1.4%	(0.58)	−2.358	13.3%***	(3.34)			
AvgVarβ	−0.269	1.1%	(0.79)	−0.228	1.3%	(1.02)	−0.167	0.8%	(0.89)
SUE	−1.038	4.3%***	(2.75)	−1.153	6.5%***	(3.27)	−1.442	7.1%***	(4.45)
Residual	−12.303	50.5%***	(4.21)	−11.174	63.0%***	(6.72)	−12.439	61.1%***	(8.30)
Total	−24.352***	100%	(−3.73)	−17.732***	100%	(−4.01)	−20.363***	100%	(−4.70)
Sample period	1984–2001			1983–2012			1982–2012		
Avg # firms/mth	367.7			440.7			763.3		
Panel C: Poor credit ratings									
Skew	−0.397	1.8%	(0.30)	−0.789	5.0%	(0.86)	−0.427	1.8%	(0.74)
Coskew	−1.007	4.6%	(0.42)	−2.168	13.7%	(1.29)	−0.656	2.7%	(0.48)
E(IdioSkew)	−1.029	4.7%	(0.78)	−0.628	4.0%	(0.55)	−2.730	11.3%***	(2.66)
RTP	−1.023	4.7%	(0.49)						
Lagret	−0.859	3.9%	(0.58)	−1.836	11.6%	(1.29)	−2.054	8.5%*	(1.70)
Amihud	1.307	−5.9%	(−0.57)	−1.082	6.9%	(0.87)	−4.346	18.0%***	(3.36)
Zeroret	−4.386	20.0%	(1.13)	−0.835	5.3%	(1.25)	0.063	−0.3%	(−0.15)
Spread	−6.524	29.7%	(0.89)						
Dispersion	0.538	−2.4%	(−0.77)	0.389	−2.5%	(−0.57)			
AvgVarβ	0.148	−0.7%	(−0.22)	0.326	−2.1%	(−0.60)	−0.017	0.1%	(0.05)
SUE	−0.498	2.3%	(0.80)	−0.813	5.2%	(1.36)	−1.253	5.2%***	(2.81)
Residual	−8.246	37.5%*	(1.82)	−8.337	52.9%***	(2.98)	−12.791	52.8%***	(5.80)
Total	−21.975**	100%	(−2.08)	−15.774**	100%	(−2.33)	−24.210***	100%	(−4.42)
Sample period	1987–2001			1986–2012			1986–2012		
Avg # firms/mth	142.4			176.7			234.9		
Panel D: High short interest									
Skew	−0.322	1.8%	(0.62)	−0.030	0.1%	(0.06)	−0.357	1.1%	(0.94)
Coskew	0.690	−3.8%	(−0.43)	−0.880	3.6%	(0.68)	−1.170	3.5%*	(1.92)

(continued on next page)

Table 6 (*continued*)

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
E(IdioSkew)	−1.637	9.1%	(0.94)	−5.761	23.3%***	(3.30)	−12.122	36.1%***	(7.22)
RTP	−1.862	10.3%	(1.09)						
Lagret	2.473	−13.7%	(−0.99)	1.361	−5.5%	(−0.92)	−0.586	1.7%	(0.57)
Amihud	0.361	−2.0%	(−0.60)	−1.127	4.6%	(0.96)	−3.228	9.6%***	(3.38)
Zeroret	1.606	−8.9%	(−0.72)	0.244	−1.0%	(−0.33)	0.646	−1.9%	(−1.17)
Spread	−8.038	44.5%*	(1.78)						
Dispersion	−0.591	3.3%	(1.46)	−1.139	4.6%**	(2.32)			
AvgVarβ	−0.265	1.5%	(0.48)	−0.364	1.5%	(0.90)	−0.017	0.1%	(0.08)
SUE	−0.112	0.6%	(0.32)	−0.750	3.0%*	(1.66)	−0.761	2.3%***	(3.07)
Residual	−10.349	57.4%***	(2.90)	−16.306	65.9%***	(6.51)	−15.992	47.6%***	(8.31)
Total	−18.045*	100%	(−1.95)	−24.752***	100%	(−3.68)	−33.587***	100%	(−6.93)
Sample period	1990–2001			1988–2005			1988–2005		
Avg # firms/mth	428.0			440.1			601.6		
Panel E: High leverage									
Skew	−0.214	0.9%	(0.52)	−0.168	1.0%	(0.54)	−0.757	3.4%***	(3.76)
Coskew	−0.773	3.2%	(0.69)	−0.886	5.3%	(0.91)	−1.670	7.4%***	(3.12)
E(IdioSkew)	−0.198	0.8%	(0.18)	−0.506	3.0%	(0.35)	−1.618	7.2%**	(2.02)
RTP	−1.142	4.8%	(1.27)						
Lagret	−0.847	3.5%	(0.78)	0.291	−1.7%	(−0.24)	−3.335	14.8%***	(4.63)
Amihud	1.475	−6.1%	(−1.41)	−3.326	19.8%***	(2.99)	−2.000	8.9%***	(3.24)
Zeroret	0.333	−1.4%	(−0.28)	0.243	−1.4%	(−0.72)	0.206	−0.9%	(−0.93)
Spread	−8.818	36.7%***	(2.73)						
Dispersion	−0.722	3.0%*	(1.83)	−1.409	8.4%***	(2.89)			
AvgVarβ	−0.444	1.8%	(1.29)	−0.354	2.1%	(1.37)	−0.115	0.5%	(0.86)
SUE	−0.422	1.8%**	(1.98)	−0.753	4.5%**	(2.56)	−1.343	6.0%***	(6.78)
Residual	−12.252	51.0%***	(4.08)	−9.957	59.2%***	(5.78)	−11.938	52.9%***	(10.80)
Total	−24.024***	100%	(−2.94)	−16.826***	100%	(−3.26)	−22.570***	100%	(−8.27)
Sample period	1984–2001			1983–2012			1971–2012		
Avg # firms/mth	562.6			638.7			877.8		
Panel F: Low institutional ownership									
Skew	−1.880	11.2%	(1.46)	−1.813	8.1%**	(2.13)	−1.883	8.1%***	(4.99)
Coskew	−2.973	17.8%	(1.19)	−1.606	7.1%	(1.08)	−1.539	6.6%**	(2.25)
E(IdioSkew)	−0.248	1.5%	(0.21)	−3.253	14.5%**	(2.04)	−3.502	15.0%***	(4.79)
RTP	1.321	−7.9%	(−0.45)						
Lagret	−3.092	18.5%	(1.30)	−0.996	4.4%	(0.56)	−4.277	18.4%***	(4.63)
Amihud	−4.045	24.2%	(0.71)	−2.933	13.1%**	(1.98)	−0.921	4.0%	(1.53)
Zeroret	−0.832	5.0%	(0.41)	0.082	−0.4%	(−0.11)	0.355	−1.5%	(−0.84)
Spread	4.285	−25.6%	(−0.41)						
Dispersion	−0.901	5.4%	(0.99)	0.543	−2.4%	(−0.53)			
AvgVarβ	−0.182	1.1%	(0.35)	0.051	−0.2%	(−0.10)	−0.247	1.1%	(1.51)
SUE	−2.421	14.5%	(1.46)	−2.243	10.0%***	(2.74)	−1.627	7.0%***	(5.79)
Residual	−5.771	34.5%	(1.62)	−10.299	45.8%***	(3.91)	−9.660	41.5%***	(5.68)
Total	−16.740	100%	(−1.48)	−22.466***	100%	(−3.18)	−23.300***	100%	(−6.61)
Sample period	1990–2001			1983–2012			1979–2012		
Avg # firms/mth	131.5			145.1			743.3		
Panel G: Low B/M ratios									
Skew	−0.391	2.0%	(0.63)	−0.574	4.3%	(1.29)	−1.005	4.3%***	(3.48)
Coskew	−1.700	8.7%	(1.13)	−0.035	0.3%	(0.04)	−0.472	2.0%	(0.88)
E(IdioSkew)	−1.498	7.7%*	(1.75)	−1.571	11.9%*	(1.76)	−3.424	14.6%***	(4.58)
RTP	0.311	−1.6%	(−0.32)						
Lagret	−0.756	3.9%	(0.53)	0.371	−2.8%	(−0.30)	−3.242	13.8%***	(3.51)
Amihud	−0.349	1.8%	(0.41)	0.257	−1.9%	(−0.34)	−0.238	1.0%	(0.28)
Zeroret	−3.171	16.2%	(1.59)	−0.025	0.2%	(0.06)	−0.166	0.7%	(0.79)
Spread	8.651	−44.2%	(−1.25)						
Dispersion	−0.482	2.5%	(0.40)	−0.897	6.8%	(1.34)			
AvgVarβ	−0.276	1.4%	(0.53)	−0.499	3.8%	(1.22)	−0.278	1.2%*	(1.66)
SUE	−0.769	3.9%**	(2.18)	−0.683	5.2%**	(2.00)	−1.175	5.0%***	(5.21)
Residual	−19.154	97.8%***	(3.64)	−9.556	72.3%***	(5.49)	−13.490	57.4%***	(10.50)
Total	−19.583**	100%	(−2.59)	−13.212***	100%	(−2.89)	−23.490***	100%	(−7.59)
Sample period	1984–2001			1982–2012			1971–2012		
Avg # firms/mth	452.3			594.0			728.3		
Panel H: Non-NYSE listings									
Skew	−2.146	6.4%**	(2.47)	−0.636	3.1%**	(2.12)	−2.117	10.2%***	(5.98)
Coskew	−1.696	5.0%	(0.71)	−0.750	3.6%	(1.08)	−0.440	2.1%	(1.42)

(*continued on next page*)

Table 6 (*continued*)

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
E(IdioSkew)	−1.292	3.8%*	(1.66)	−2.468	11.9%***	(3.14)	−3.010	14.5%***	(5.73)
RTP	−0.131	0.4%	(0.11)						
Lagret	0.787	−2.3%	(−0.58)	0.310	−1.5%	(−0.31)	−5.279	25.5%***	(5.78)
Amihud	1.331	−4.0%	(−0.56)	−2.036	9.8%**	(2.40)	−0.341	1.6%	(0.47)
Zeroret	2.324	−6.9%	(−1.13)	0.741	−3.6%	(−1.10)	0.536	−2.6%	(−1.48)
Spread	−13.107	39.0%**	(2.47)						
Dispersion	−3.457	10.3%***	(3.01)	−2.089	10.1%***	(4.44)			
AvgVarβ	0.155	−0.5%	(−0.19)	−0.310	1.5%	(1.38)	−0.145	0.7%	(1.25)
SUE	−2.033	6.1%**	(2.43)	−0.834	4.0%***	(3.78)	−1.299	6.3%***	(6.79)
Residual	−14.340	42.7%***	(4.34)	−12.697	61.1%***	(8.76)	−8.619	41.6%***	(7.68)
Total	−33.606***	100%	(−4.04)	−20.768***	100%	(−4.99)	−20.714***	100%	(−8.49)
Sample period	1984–2001			1983–2012			1971–2012		
Avg # firms/mth	596.5			874.7			1571.4		
Panel I: Non-January months									
Skew	−0.347	2.0%	(1.13)	−0.196	1.6%	(0.81)	−1.105	5.2%***	(6.19)
Coskew	−0.569	3.2%	(1.04)	−0.364	2.9%	(0.49)	−0.544	2.5%***	(2.63)
E(IdioSkew)	−0.594	3.3%	(1.54)	−1.143	9.1%	(1.46)	−3.534	16.5%***	(7.44)
RTP	−0.167	0.9%	(0.28)						
Lagret	−1.074	6.0%	(0.99)	−0.037	0.3%	(0.04)	−3.773	17.6%***	(5.47)
Amihud	−0.004	0.0%	(0.01)	−0.530	4.2%	(0.65)	−1.458	6.8%***	(3.34)
Zeroret	−0.505	2.8%	(0.53)	0.128	−1.0%	(−0.41)	0.216	−1.0%	(−1.13)
Spread	−0.445	2.5%	(0.15)						
Dispersion	−0.628	3.5%**	(2.47)	−0.791	6.3%***	(2.85)			
AvgVarβ	−0.208	1.2%	(1.05)	−0.192	1.5%	(0.84)	−0.106	0.5%	(1.16)
SUE	−0.388	2.2%**	(2.57)	−0.502	4.0%**	(2.58)	−1.015	4.7%***	(7.97)
Residual	−12.858	72.3%***	(5.39)	−8.879	71.0%***	(5.47)	−10.091	47.1%***	(11.49)
Total	−17.788***	100%	(−2.91)	−12.506***	100%	(−2.89)	−21.408***	100%	(−9.88)
Sample period	1984–2000			1982–2012			1971–2012		
Avg # firms/mth	1523.2			1805.7			2754.7		

ownership stocks, 2–43% for low B/M stocks, 39–58% for non-NYSE stocks, and 28–53% for non-January months, compared with 29–54% for the full sample reported in Table 5.

We plot in Panel B of Fig. 1 the average fraction explained by each group of candidate variables and the average unexplained fraction across the nine subsamples. The first bar chart shows that the candidate variables in Model 1 collectively explain an average of 42.4% of the idiosyncratic volatility puzzle while the residual component captures the remaining 57.6%. Comparing across the three groups of candidate variables, we see that market friction-based variables combine to explain an average of 18.9% of the puzzle across the nine subsamples, followed by lottery preference-based variables which explain 15.2%, and variables related to other explanations which explain 8.3% of the puzzle.

The second and third bar charts in Panel B show that the total explained fractions are 39.1% and 50.3% for Models 2 and 3, respectively, thus leaving 60.9% and 49.7% of the puzzle unexplained. Lottery preference-based candidate variables now dominate the other candidate variables as they combine to explain 19.2% and 24.2% of the puzzle in Models 2 and 3, respectively. The contribution of market friction-based candidate variables is 8.3% in Model 2 and 20.0% in Model 3. Finally, candidate variables related to other explanations together explain 11.7% and 6.1% of the puzzle in Models 2 and 3, respectively.

Overall, the results in Table 6 show that the lottery preference-based and market friction-based candidate variables also perform relatively well among the subsample of stocks where the idiosyncratic volatility puzzle is stronger. However, at least half of the puzzle still remains unexplained in these subsamples.

5. Additional robustness tests

In this section, we show that our decomposition methodology is robust to using portfolios to control for measurement errors at the individual stock level and to allowing for nonlinear relations in the idiosyncratic volatility puzzle. We also show that the methodology can be used to evaluate explanations for other asset pricing anomalies.

5.1. Portfolio-level analysis

Thus far, our decomposition analysis has been based on cross-sectional regressions estimated at the individual stock level. The advantage of using individual stocks, as opposed to portfolios, is that it is robust to data mining and loss of information concerns. However, this does raise the question about measurement errors at the individual stock level as both idiosyncratic volatility and many of the candidate variables we investigate are generated regressors.[15]

[15] We note that measurement errors will also affect the conventional approach of including the candidate variables as controls in the regression of returns on idiosyncratic volatility. Therefore, this issue is not unique to our decomposition methodology.

To see the effect of measurement errors, let us assume that both idiosyncratic volatility and the candidate variable are measured with error, e.g., $\widetilde{IVOL} = IVOL + u$, and $\widetilde{Candidate} = Candidate + v$, where the true variables (*IVOL* and *Candidate*) are unobservable and u and v are mean-zero measurement errors. We show in Appendix B that the measurement error in idiosyncratic volatility has no effect on the mean or standard error of the fraction of the idiosyncratic volatility puzzle explained by the candidate variable (i.e., γ_t^C/γ_t). However, the measurement error in the candidate variable does lead to a downward bias in the mean as well as the standard error of the fraction by a factor of $\frac{\mathrm{Var}(Candidate)}{\mathrm{Var}(Candidate)+\mathrm{Var}(v)}$, although the t-statistic for the fraction is still unbiased.

To address this concern about measurement errors, we follow the literature and perform robustness checks by using idiosyncratic volatility-sorted portfolios instead of individual stocks in the decomposition analysis. The motivation for using portfolios is that if the errors in an estimated variable are not perfectly correlated across stocks, we can improve the precision of the estimates by grouping stocks into portfolios because the errors will tend to offset each other. The disadvantage of aggregating stocks into portfolios, as pointed out by Ang, Liu, and Schwarz (2010), is that it loses information by reducing the cross-sectional variation in the estimated variable. This can lead to a mechanical increase in the correlation between a candidate variable and idiosyncratic volatility, which causes the fraction of the idiosyncratic volatility puzzle explained by the candidate variable to be overstated.

At the beginning of each month t, we sort individual stocks into 200 portfolios based on their month $t-1$ *IVOL*.[16] We compute the portfolio-level variables by taking the value-weighted averages of the individual stock-level variables. Month $t-1$ portfolio-level *IVOL* and candidate variables are then matched with value-weighted month t DGTW-adjusted returns of each portfolio for the decomposition analysis.

Panel A of Table 7 reports the univariate decomposition results and Panel B reports the multivariate results. We first see that the idiosyncratic volatility puzzle remains highly significant among the idiosyncratic volatility-sorted portfolios. The average Fama-MacBeth regression coefficient of portfolio returns on *IVOL* ranges from −21.080% (t = −7.88) to −28.318% (t = −9.07), depending on the candidate variable examined. The results in Panel A also show that the candidate variables that are promising in the individual stock-level analysis tend to perform well in the portfolio-level analysis. Specifically, among the seven candidate variables that capture at least 10% of the puzzle in the individual stock-level univariate analysis (*Skew, E(Idioskew), Maxret, RTP, Lagret, Spread*, and *SUE*), all but *SUE* continue to explain more than 10% (in some cases substantially so) of the puzzle in the portfolio-level univariate analysis whereas the explained fraction for *SUE* is only slightly below 10%. On the other hand, *Coskew, Amihud, Zeroret*, and *Dispersion*, which fail to capture more than 10% of the puzzle in the individual stock-level analysis, now each account for more than 10% of the puzzle in the portfolio-level analysis.[17]

In Panel B, we put all the candidate variables (except *Maxret*) through our multivariate decomposition framework. The results in Panel B show that *RTP* and *E(Idioskew)* are the biggest contributors among the lottery preference-based candidate variables, whereas *Spread, Amihud*, and *Lagret* dominate in the market friction category. Variables related to other explanations account for very little of the puzzle in the portfolio-level multivariate analysis. Collectively, all the candidate variables explain 78–84% of puzzle (depending on the model) and the residual component accounts for the remaining 16–22% of the puzzle. Thus, although the total explained fraction in the portfolio-level multivariate analysis is larger than that from the individual stock-level analysis, there is still a nontrivial portion of the idiosyncratic volatility puzzle left unexplained.

As argued above, the overall increase in the explanatory power by the candidate variables in the portfolio-level analysis can come from two sources—a reduction of measurement errors in the candidate variables and/or a mechanical increase in the correlation with idiosyncratic volatility due to portfolio averaging. In the analysis described below, we use simulations to try to disentangle these two effects.

Specifically, we simulate a negative idiosyncratic volatility-return relation at the individual stock level and introduce a noisy candidate variable, which contains a true component (explaining r percent of the simulated idiosyncratic volatility puzzle) and an independent noise component. We vary the true explained fraction (r) from 1% to 100% at 1% intervals and the amount of signal in the candidate variable measured by the candidate informativeness ratio $k = \frac{\mathrm{Var}(Candidate)}{\mathrm{Var}(Candidate)+\mathrm{Var}(Noise)}$ from 10% to 100% also at 1% intervals (we require the candidate variable to contain a minimum of 10% of signal). We then apply our decomposition methodology to the simulated individual stock sample to estimate the fraction of the idiosyncratic volatility puzzle that is explained by the noisy candidate variable for each of the $100 \times 91 = 9,100$ (r, k) combinations. The estimation results show that the asymptotic relation between the measurement error and estimated fraction in Appendix B also holds in finite sample simulations. For example, for a candidate variable with a true fraction (r) of 50% and a candidate informativeness ratio (k) of 50%, the average estimated fraction from 20

[16] We have also used three-digit SIC industry portfolios and found that on average the candidate variables explain a smaller fraction of the puzzle in these industry portfolios than in the idiosyncratic volatility-sorted portfolios. The relation between idiosyncratic volatility and returns is insignificant for broader industry portfolios such as two-digit SIC or Fama-French 49-industry portfolios.

[17] This is especially true for *Amihud*, which sees its contribution increase considerably from −2.4% in the individual stock-level analysis to 51.8% in the portfolio-level analysis. This large increase is partly due to a mechanical increase in the correlation between *Amihud* and *IVOL* as a result of portfolio averaging (the correlation almost doubles from 0.308 at the individual stock level to 0.610 at the portfolio level). In unreported results, when we re-compute portfolio-level *Amihud* and *IVOL* using value-weighted daily portfolio returns instead of taking averages of the individual stock-level *Amihud* and *IVOL* (to avoid the mechanical increase in the correlation), we obtain a more modest increase in the explained fraction to 26.9%.

Table 7
Decomposing the idiosyncratic volatility puzzle: portfolio-level analysis.
Using portfolio-level Fama-MacBeth cross-sectional regressions, the negative relation between month $t-1$ idiosyncratic volatility (*IVOL*) and month t DGTW-adjusted returns is decomposed into a number of components each related to a candidate variable and a residual component. Panels A and B report the results of univariate and multivariate analyses, respectively, using idiosyncratic volatility-sorted portfolios. At the beginning of each month t, we sort individual stocks into 200 portfolios based on their month $t-1$ *IVOL*. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. The portfolio-level *IVOL*, candidate variables, and returns are computed as value-weighted averages of the firm-level variables. Firm-level *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *Maxret* is the maximum daily return in month $t-1$. *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings. Stocks with prices less than \$1 at the end of the previous month are excluded from the analysis. The standard errors of the fractions of the puzzle explained are determined using the multivariate delta method. Time-series averages of estimated coefficients (×100) are reported with *t*-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Univariate analysis

Candidate	IVOL coeff.		Candidate component			Residual component		
	Coeff.	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
Skew	−25.884***	(−11.95)	−3.448	13.3%***	(6.56)	−22.436	86.7%***	(42.71)
Coskew	−25.884***	(−11.95)	−3.087	11.9%***	(5.09)	−22.797	88.1%***	(37.56)
E(Idioskew)	−27.984***	(−13.22)	−11.971	42.8%***	(15.22)	−16.012	57.2%***	(20.36)
Maxret	−25.884***	(−11.95)	−22.753	87.9%***	(50.08)	−3.131	12.1%***	(6.89)
RTP	−28.318***	(−9.07)	−18.820	66.5%***	(19.54)	−9.497	33.5%***	(9.86)
Lagret	−25.884***	(−11.95)	−6.621	25.6%***	(6.73)	−19.263	74.4%***	(19.59)
Amihud	−25.988***	(−11.99)	−13.474	51.8%***	(15.76)	−12.514	48.2%***	(14.64)
Zeroret	−25.884***	(−11.95)	−3.739	14.4%***	(9.04)	−22.145	85.6%***	(53.51)
Spread	−21.080***	(−7.88)	−12.441	59.0%***	(13.93)	−8.638	41.0%***	(9.67)
Dispersion	−22.618***	(−8.53)	−2.609	11.5%***	(5.50)	−20.010	88.5%***	(42.19)
AvgVarβ	−27.633***	(−13.05)	−0.755	2.7%***	(3.01)	−26.879	97.3%***	(107.14)
SUE	−26.873***	(−12.44)	−2.635	9.8%***	(9.62)	−24.239	90.2%***	(88.50)
Average				33.1%			66.9%	

Panel B: Multivariate analysis

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
Skew	−0.426	1.6%*	(1.81)	−0.682	2.9%***	(2.63)	−1.155	4.2%***	(5.29)
Coskew	−1.174	4.4%***	(3.00)	−0.625	2.7%	(1.49)	−0.768	2.8%**	(2.27)
E(IdioSkew)	−1.883	7.1%***	(4.79)	−4.729	20.3%***	(7.59)	−5.375	19.7%***	(10.67)
RTP	−5.763	21.8%***	(7.34)						
Lagret	−1.697	6.4%*	(1.86)	−2.681	11.5%***	(3.49)	−3.585	13.1%***	(5.70)
Amihud	−2.998	11.3%***	(6.03)	−7.361	31.6%***	(11.00)	−8.689	31.8%***	(15.13)
Zeroret	0.340	−1.3%	(−1.39)	−1.035	4.4%***	(5.09)	−0.572	2.1%***	(3.27)
Spread	−7.553	28.5%***	(8.28)						
Dispersion	−0.758	2.9%***	(4.08)	−0.892	3.8%***	(4.41)			
AvgVarβ	−0.081	0.3%	(0.59)	−0.226	1.0%	(1.33)	−0.165	0.6%	(1.43)
SUE	−0.363	1.4%***	(3.60)	−0.498	2.1%***	(4.27)	−0.866	3.2%***	(7.98)
Residual	−4.100	15.5%***	(5.27)	−4.554	19.6%***	(5.55)	−6.114	22.4%***	(8.63)
Total	−26.457***	100%	(−7.62)	−23.282***	100%	(−8.78)	−27.288***	100%	(−12.64)
Sample period	1984–2001			1982–2012			1971–2012		

rounds of simulations is 24.9999%, almost identical to 25% as predicted by the asymptotic analysis in Appendix B.[18]

Next, to study the effect of portfolio averaging on measurement errors, we group individual stocks into 200 portfolios based on their simulated idiosyncratic volatility. Portfolio-level variables are computed as value-weighted averages of simulated individual stock-level variables using simulated market cap as weights. We then use the decomposition methodology to estimate what fraction of the portfolio-level idiosyncratic volatility puzzle can be explained by the portfolio-level candidate variable. This allows us to study the tradeoff between the reduction of measurement errors through portfolio averaging (which will reduce the downward bias in the estimated fraction) and loss of information and a mechanical increase in the correlation between the candidate variable and idiosyncratic volatility (which will overstate the fraction explained by the candidate variable).

Fig. 2 plots the portfolio-level simulation results for (r, k) values at 10% intervals (even though the actual simulations are done at 1% intervals). The figure shows that

[18] The reason we obtain such accurate results is that we perform 20 rounds of simulations for the entire sample of firm-month observations. When we reduce the sample size of the simulations, the results become noisier.

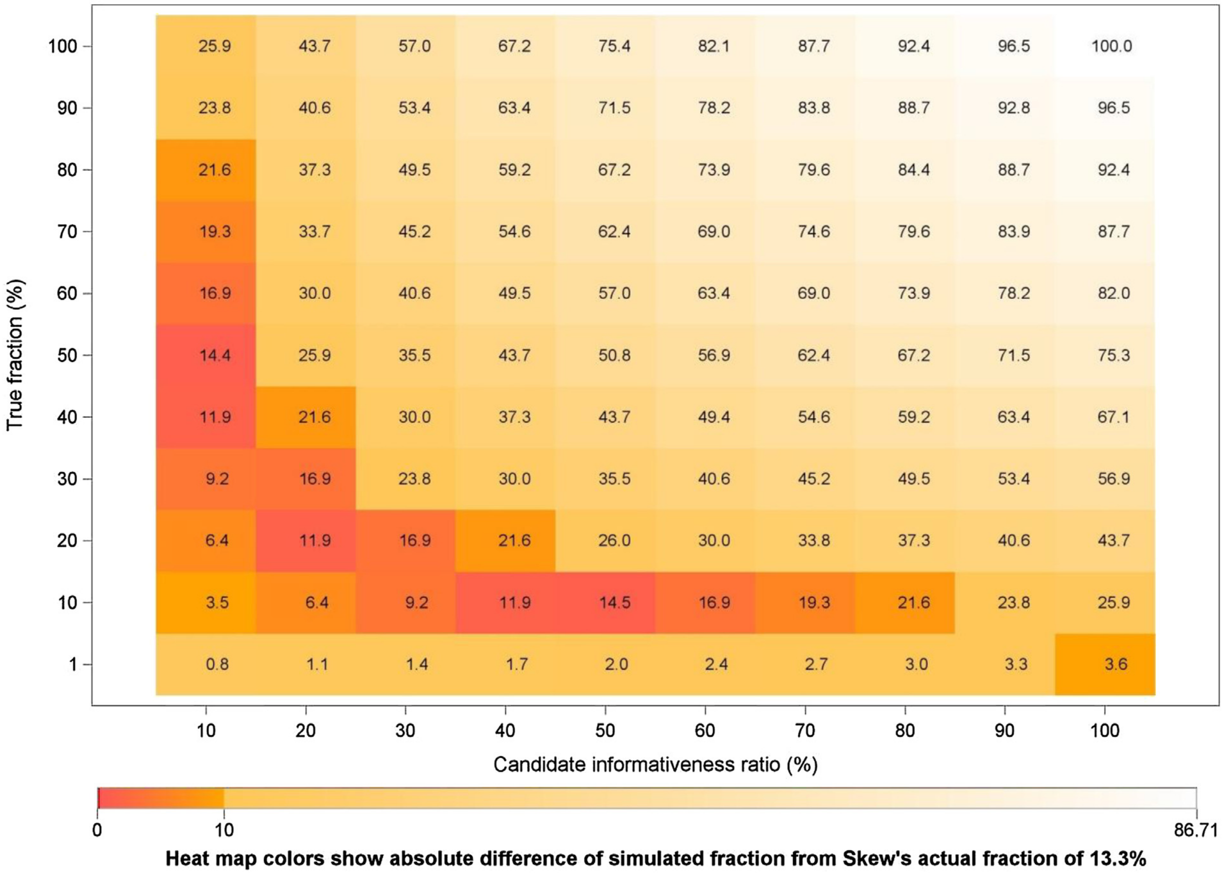

Fig. 2. Simulated fraction for 200 portfolios. The fraction of the portfolio-level simulated idiosyncratic volatility puzzle explained by a simulated noisy candidate variable is plotted. Specifically, we simulate a negative idiosyncratic volatility-return relation at the individual stock level and introduce a noisy candidate variable, which contains a true component (explaining r percent of the simulated idiosyncratic volatility puzzle) and an independent noise component. We vary the true explained fraction (r) from 1% to 100% at 1% intervals and the amount of signal in the candidate variable measured by the candidate informativeness ratio $k = \frac{\mathrm{Var}(Candidate)}{\mathrm{Var}(Candidate)+\mathrm{Var}(Noise)}$ from 10% to 100% also at 1% intervals. Individual stocks are then grouped into 200 portfolios based on their simulated idiosyncratic volatility, and portfolio-level variables are computed as value-weighted averages of simulated firm-level variables using simulated market cap as weights. For each of the $100 \times 91 = 9,100$ (r, k) combinations, we apply our decomposition methodology to estimate the fraction of the portfolio-level idiosyncratic volatility puzzle that is explained by the portfolio-level candidate variable. The figure plots the simulated fractions for (r, k) values at 10% intervals due to space constraints. The heat map colors in the figure describe the absolute differences between the simulated fractions and the actual fraction of the idiosyncratic volatility puzzle explained by *Skew* (13.3%) reported in Panel A of Table 7, where redder colors indicate smaller absolute differences.

for a given (r, k) combination, grouping stocks into 200 portfolios increases the estimated fraction significantly. For example, for a candidate variable with a true fraction of 50% and a candidate informativeness ratio of 50% at the individual stock level, the portfolio-level estimated fraction is 50.8%, which is very close to the true fraction of 50% and doubles the individual stock-level estimated fraction of 25%. This example suggests that grouping stocks into 200 portfolios can significantly reduce (or even eliminate) the error-induced downward bias in the estimated fractions at the individual stock level. On the other hand, for a candidate variable that is already measured precisely at the individual stock level, Fig. 2 shows that portfolio grouping can actually overstate the true fraction explained by the candidate variable. As a case in point, consider a candidate variable with a true fraction of 50% and a candidate informativeness ratio of 70% at the individual stock level, the portfolio-level estimated fraction is 62.4%, more than 10% higher than the true fraction of 50%.

To gain further insights on the measurement error issue, we relate the above portfolio-level simulation results to the actual fractions explained by the candidate variables examined in our paper. Take *Skew* as an example. *Skew* explains an actual fraction of 13.3% of the idiosyncratic volatility puzzle in the portfolio-level analysis in Table 7. Looking at Fig. 2, there are many possible (r, k) combinations that produce simulated fractions close to 13.3% at the portfolio level. For example, the true fraction explained by *Skew* could be 10% and the candidate informativeness ratio could be 50% at the individual stock level which would give a portfolio-level simulated fraction of 14.5%. This is fairly close to the actual fraction of 13.3%. Another possibility is that the true fraction is 50% and the candidate informativeness ratio is 10%—this would give a portfolio-level simulated fraction of 14.4%. These two possibilities are not the only close matches. The heat map colors in Fig. 2 describe the absolute difference between the simulated fractions and *Skew*'s actual fraction of 13.3%, where redder colors denote smaller absolute differences.

For a given candidate informativeness ratio between 10–100%, we compute a precision-weighted true fraction explained by *Skew*, where precision is defined as the reciprocal of the squared difference between the actual fraction and the simulated fraction. Table 8 reports that the average precision-weighted true fraction across all candidate informativeness ratios between 10–100%, assuming that we have a diffuse prior about the amount of noise in *Skew* at the individual stock level, is 12.1%. This is very close to the actual explained fraction of 13.3%. On the other hand, if we assume that *Skew* is measured precisely (imprecisely) with a candidate informativeness ratio between 50–100% (10–49%), the average precision-weighted true fraction is considerably lower (higher) at 6.7% (18.9%), which suggests that the actual fraction of 13.3% is overstating (understating) the true fraction explained.

The results for the other candidate variables, also reported in Table 8, paint a similar picture. The average precision-weighted true fraction for a diffuse-prior candidate variable (10–100% candidate informativeness ratio) is typically very close to the actual explained fraction (31.5% versus 33.1%, averaged across all 12 candidate variables). On the other hand, the average precision-weighted true fraction for a clean candidate variable (50–100% candidate informativeness ratio) is significantly lower than the actual fraction (24.3% versus 33.1%, averaged across all 12 candidate variables), and that for a noisy candidate variable (10–49% candidate informativeness ratio) is significantly higher than the actual fraction (40.6% vs. 33.1%, averaged across all 12 candidate variables).[19] Overall, these results suggest that when one is agnostic about the amount of noise in a candidate variable, grouping stocks into 200 portfolios comes very close to uncovering the true fraction explained by the candidate variable. On the other hand, if one is confident that the candidate variable is measured precisely (imprecisely) at the individual stock level, then grouping stocks into 200 portfolios is likely to overstate (understate) the true fraction explained by the candidate variable.

We also perform the simulation analysis by grouping stocks into smaller numbers of portfolios (100, 50, and 25 portfolios) and report their results as well as those from the individual stock-level analysis in Table 8. We see that while the actual fraction explained by a candidate variable goes up when we group stocks into fewer portfolios (from an average of 33.1% across all 12 candidate variables for 200 portfolios to 54.5% for 25 portfolios), the precision-weighted true fraction remains very stable when we reduce the number of portfolios. It varies within a tight range from 24 to 29% for the clean scenario, 32–36% for the diffuse-prior scenario, and 41–45% for the noisy scenario. These results suggest that grouping stocks into a smaller number of portfolios may not necessarily lead to a further reduction in the measurement errors. Instead, it increases the likelihood that the estimated fraction will overstate the true fraction explained by a candidate variable. We therefore conclude that grouping individual stocks into 200 portfolios achieves the appropriate balance between concerns of measurement errors in the candidate variables versus loss of information and overstating the explained fractions by the candidate variables (especially when one is agnostic about the amount of noise in the candidate variables).

In sum, the analysis in this subsection confirms that our main findings are robust to using idiosyncratic volatility-sorted portfolios to mitigate the measurement errors at the individual stock level. It also shows that our decomposition methodology can be applied to characteristic-sorted portfolios in addition to individual stocks.

5.2. Nonlinear relations

To be consistent with existing literature, we adopt simple linear specifications in Eqs. (1) and (2) to study to what extent different candidate variables can explain the idiosyncratic volatility puzzle. However, our decomposition methodology can easily accommodate nonlinear relations in the idiosyncratic volatility puzzle.

One possible source of nonlinearity is in the relation between idiosyncratic volatility and returns. Previous studies such as Ang, Hodrick, Xing, and Zhang (2006) argue that much of the idiosyncratic volatility puzzle is driven by high idiosyncratic volatility stocks earning low returns. To investigate this possibility, we define a dummy variable, *HIGHIVOL*, which equals one when *IVOL* belongs to the top decile in month $t-1$ and zero otherwise. Panel A of Table 9 confirms that there is indeed a negative and significant relation between *HIGHIVOL* and subsequent stock returns with the average Fama-MacBeth regression coefficient on *HIGHIVOL* ranging from −0.713% ($t=-3.71$) to −1.155% ($t=-5.40$). We then use our decomposition methodology to study the *HIGHIVOL*-return relation. We find that most of the candidate variables that prove useful in explaining the linear *IVOL*-return relation also capture sizable fractions of the relation between *HIGHIVOL* and returns in the univariate analysis. The multivariate analysis in Panel B of Table 9 shows that all the candidate variables (excluding *Maxret*) combine to explain 25–48% of the negative relation between *HIGHIVOL* and returns, thus leaving more than 50% of the relation unexplained. This finding is again in line with the results from our multivariate decomposition of the linear *IVOL*-return relation.

Another source of nonlinearity comes from potential nonlinear relations between idiosyncratic volatility and the candidate variables. For example, perhaps only the extreme values of the candidate variables are useful in explaining the *HIGHIVOL*-return relation. To investigate this, we replace a candidate variable with a dummy that equals one when the candidate variable belongs to the extreme decile and zero otherwise. Panel C of Table 9 shows that this treatment leaves the univariate contribution largely unaffected for most of the candidate variables, except for *Coskew* whose explained fraction improves to 23.3% from 2.5% in Panel A. Panel D of Table 9 shows that together, these binary candidate variables (again excluding *Maxret*)

[19] The precision-weighted fractions for the clean and noisy scenarios can, in essence, be viewed as providing a confidence interval for the true fraction explained by a candidate variable.

Table 8
Precision-weighted true fractions from simulation analysis.

A precision-weighted true explained fraction is computed for each candidate variable based on the simulation analysis. Specifically, we simulate a negative idiosyncratic volatility-return relation at the individual stock level and introduce a noisy candidate variable, which contains a true component (explaining r percent of the simulated idiosyncratic volatility puzzle) and an independent noise component. We vary the true explained fraction (r) from 1% to 100% at 1% intervals and the amount of signal in the candidate variable measured by the candidate informativeness ratio $k = \frac{\mathrm{Var}(Candidate)}{\mathrm{Var}(Candidate)+\mathrm{Var}(Noise)}$ from 10% to 100% also at 1% intervals. For each of the $100 \times 91 = 9,100$ (r, k) combinations, we then apply our decomposition methodology to estimate the fraction of the simulated idiosyncratic volatility puzzle that is explained by the noisy candidate variables for the simulated individual stock sample as well as for 200, 100, 50, and 25 portfolios sorted on simulated idiosyncratic volatility. The actual true fraction explained by a candidate variable at each level of portfolio aggregation is then matched to the corresponding simulated explained fraction to compute the precision-weighted true fraction explained by the candidate variables, where precision is defined as the reciprocal of the squared difference between the actual fraction and the simulated fraction. Three versions of the precision-weighted true fraction are reported: a clean version (50–100% candidate informativeness ratio), a diffuse-prior version (10–100% candidate informativeness ratio), and a noisy version (10–49% candidate informativeness ratio). Firm-level *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *Maxret* is the maximum daily return in month $t-1$. *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings. Portfolio-level variables are computed as value-weighted averages of the firm-level variables.

Candidate	Actual fraction explained (%)					Precision-weighted true fraction (%): If informativeness ratio is 50–100% (clean candidate)					If informativeness ratio is 10–100% (diffuse-prior candidate)					If informativeness ratio is 10–49% (noisy candidate)				
	Firm	200port	100port	50port	25port	Firm	200port	100port	50port	25port	Firm	200port	100port	50port	25port	Firm	200port	100port	50port	25port
Skew	10.3	13.3	19.7	26.7	39.0	14.4	6.7	7.9	8.5	10.4	26.6	12.1	14.2	15.3	18.6	42.2	18.9	22.3	23.9	29.0
Coskew	1.9	11.9	20.4	30.3	38.3	2.8	5.9	8.2	10.0	10.1	5.0	10.6	14.8	18.1	18.0	7.9	16.7	23.3	28.4	28.2
E(Idioskew)	14.7	42.8	48.6	52.0	59.1	20.5	27.1	26.0	22.5	21.4	35.6	43.6	42.3	38.2	36.8	54.8	64.5	63.2	58.3	56.5
Maxret	112.0	87.9	91.7	94.9	97.8	69.4	84.3	85.4	86.7	87.6	63.4	76.4	77.7	79.0	80.0	55.7	66.4	67.8	69.2	70.3
RTP	22.3	66.5	69.4	80.2	89.3	31.0	54.6	49.5	58.5	71.4	47.1	64.6	62.1	67.5	73.5	67.7	77.3	78.1	78.9	76.3
Lagret	33.7	25.6	27.2	35.3	49.0	46.8	13.9	11.6	12.3	15.0	59.0	25.7	21.2	22.5	27.2	74.6	40.7	33.4	35.4	42.8
Amihud	−2.4	51.8	59.8	73.6	84.5	9.9	36.0	37.0	46.3	56.9	12.9	52.0	53.1	60.4	67.1	16.8	72.5	73.6	78.3	80.2
Zeroret	0.9	14.4	19.5	25.3	34.2	1.4	7.2	7.8	8.1	8.6	2.5	13.2	14.0	14.3	15.2	3.8	20.7	22.0	22.2	23.6
Spread	30.4	59.0	65.8	75.4	82.0	42.2	44.3	44.4	49.3	50.9	56.1	58.4	58.7	62.3	63.7	73.8	76.4	77.1	78.9	80.1
Dispersion	5.3	11.5	15.8	29.6	39.3	7.5	5.7	6.1	9.7	10.5	13.8	10.2	11.0	17.5	18.8	21.9	16.1	17.1	27.5	29.4
AvgVarβ	1.0	2.7	5.0	9.0	14.4	1.6	1.4	1.9	3.0	3.5	2.7	2.2	3.0	4.3	4.9	4.2	3.3	4.4	5.9	6.7
SUE	10.9	9.8	13.8	18.7	27.1	15.2	4.9	5.4	5.6	6.3	28.0	8.6	9.4	9.7	10.8	44.3	13.4	14.6	15.0	16.6
Average	20.1	33.1	38.1	45.9	54.5	21.9	24.3	24.3	26.7	29.4	29.4	31.5	31.8	34.1	36.2	39.0	40.6	41.4	43.5	45.0

explain 42–56% of the *HIGHIVOL*-return relation, compared with 25–48% of the relation explained in Panel B. Still, a sizable 44–58% of the *HIGHIVOL*-return relation remains unexplained.

Overall, the analysis above shows that our results are robust to modifying the decomposition methodology to accommodate nonlinearity in the idiosyncratic volatility puzzle.[20]

5.3. Decomposing other anomalies

In this paper, we treat the negative idiosyncratic volatility-return relation as a puzzle and use the candidate variables proposed in the literature (e.g., maximum daily return) to try to explain the puzzle. In this subsection, to show the flexibility of our methodology, we turn the tables and use idiosyncratic volatility as a candidate variable to

[20] We have also experimented with other nonlinear specifications, which include using log *IVOL*, winsorized *IVOL*, decile ranks of *IVOL*, and adding squared terms of the candidate variables, and found very similar results. To conserve space, they are not reported.

Table 9

Decomposing the idiosyncratic volatility puzzle: nonlinear relations.

Using firm-level Fama-MacBeth cross-sectional regressions, the negative relation between a dummy variable for having high idiosyncratic volatility in month $t-1$ (*HIGHIVOL*) and month t DGTW-adjusted returns is decomposed into a number of components each related to a candidate variable and a residual component. *HIGHIVOL* equals one when *IVOL* belongs to the highest decile in month $t-1$ and zero otherwise. Panels A and B report the results of univariate and multivariate analyses, respectively, using *HIGHIVOL* and the original candidate variables. Panels C and D report the results of univariate and multivariate analyses, respectively, replacing each candidate variable with a dummy variable which equals one when the candidate variable belongs to the highest decile (lowest decile for *SUE*) and zero otherwise. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. *Skew* is the month $t-1$ skewness of raw daily returns. *Coskew* is the coskewness measure in Chabi-Yo and Yang (2009). *E(Idioskew)* is the expected idiosyncratic skewness measure in Boyer, Mitton, and Vorkink (2010). *Maxret* is the maximum daily return in month $t-1$. *RTP* is the retail trading proportion computed from ISSM and TAQ. *Lagret* is the month $t-1$ return. *Amihud* is the illiquidity measure in Amihud (2002). *Zeroret* is the fraction of trading days in month $t-1$ with a zero return. *Spread* is the average daily bid-ask spread in month $t-1$ from ISSM and TAQ. *Dispersion* is the dispersion in analysts' FY1 forecasts. *AvgVarβ* is a stock's exposure to the average variance component of the market variance as in Chen and Petkova (2012). *SUE* is the most recent standardized unexpected earnings. Stocks with prices less than \$1 at the end of the previous month are excluded from the analysis. The standard errors of the fractions of the puzzle explained are determined using the multivariate delta method. Time-series averages of estimated coefficients (×100) are reported with *t*-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Univariate analysis using HIGHIVOL

Candidate	HIGHIVOL coeff.		Candidate component			Residual component		
	Coeff.	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
Skew	−0.909***	(−9.51)	−0.069	7.6%***	(6.26)	−0.840	92.4%***	(76.19)
Coskew	−0.904***	(−9.44)	−0.023	2.5%***	(3.00)	−0.881	97.5%***	(117.03)
E(Idioskew)	−1.092***	(−10.71)	−0.178	16.3%***	(7.44)	−0.914	83.7%***	(38.11)
Maxret	−0.904***	(−9.44)	−0.785	86.9%***	(17.98)	−0.119	13.1%***	(2.72)
RTP	−1.155***	(−5.40)	−0.259	22.4%***	(4.82)	−0.896	77.6%***	(16.68)
Lagret	−0.904***	(−9.44)	−0.242	26.8%***	(6.79)	−0.662	73.2%***	(18.54)
Amihud	−0.929***	(−9.42)	0.005	−0.5%	(−0.18)	−0.933	100%***	(36.47)
Zeroret	−0.904***	(−9.44)	0.001	−0.2%	(−0.20)	−0.905	100%***	(126.29)
Spread	−1.057***	(−6.37)	−0.327	30.9%***	(5.00)	−0.730	69.1%***	(11.17)
Dispersion	−0.713***	(−3.71)	−0.038	5.3%**	(2.05)	−0.675	94.7%***	(36.39)
AvgVarβ	−0.909***	(−8.73)	−0.010	1.1%**	(2.08)	−0.899	98.9%***	(191.37)
SUE	−0.912***	(−7.98)	−0.076	8.3%***	(7.79)	−0.836	91.7%***	(85.77)
Average				17.3%			82.7%	

Panel B: Multivariate analysis using HIGHIVOL

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
Skew	−0.035	2.2%*	(1.69)	−0.006	0.6%	(0.53)	−0.051	4.9%***	(6.75)
Coskew	−0.149	9.4%*	(1.86)	−0.069	7.3%*	(1.94)	−0.026	2.5%***	(3.35)
E(IdioSkew)	0.012	−0.8%	(−0.36)	−0.066	7.0%*	(1.87)	−0.158	15.0%***	(7.55)
RTP	0.031	−2.0%	(−0.52)						
Lagret	−0.032	2.0%	(0.39)	0.009	−1.0%	(−0.14)	−0.210	20.0%***	(5.87)
Amihud	−0.057	3.6%	(0.84)	−0.034	3.5%	(0.51)	−0.049	4.7%*	(1.94)
Zeroret	0.002	−0.1%	(−0.05)	0.006	−0.7%	(−0.82)	0.025	−2.3%***	(−3.16)
Spread	−0.118	7.5%	(0.80)						
Dispersion	−0.020	1.3%	(1.01)	−0.025	2.6%	(1.50)			
AvgVarβ	−0.017	1.1%	(0.95)	−0.030	3.1%*	(1.68)	−0.007	0.7%	(1.63)
SUE	−0.008	0.5%	(0.79)	−0.021	2.2%***	(3.24)	−0.032	3.1%***	(8.06)
Residual	−1.188	75.3%***	(7.52)	−0.713	75.3%***	(6.83)	−0.541	51.6%***	(13.07)
Total	−1.578	100%	(−3.71)	−0.947	100%	(−4.07)	−1.050	100%	(−9.61)
Sample period	1984–2001			1982–2012			1971–2012		
Avg # firms/mth	1524.4			1806.0			2752.4		

(continued on next page)

Table 9
Continued.

Panel C: Univariate analysis using HIGHIVOL and dummy candidate variables

Candidate	HIGHIVOL coeff.		Candidate component			Residual component		
	Coeff.	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
High Skew	−0.909***	(−9.51)	−0.071	7.8%***	(6.61)	−0.838	92.2%***	(78.57)
High Coskew	−0.904***	(−9.44)	−0.210	23.3%***	(11.49)	−0.693	76.7%***	(37.87)
High E(Idioskew)	−1.092***	(−10.71)	−0.188	17.2%***	(8.32)	−0.904	82.8%***	(39.99)
High Maxret	−0.904***	(−9.44)	−0.799	88.4%***	(19.09)	−0.105	11.6%**	(2.51)
High RTP	−1.155***	(−5.40)	−0.128	11.1%***	(2.91)	−1.027	88.9%***	(23.32)
High Lagret	−0.904***	(−9.44)	−0.278	30.8%***	(8.88)	−0.626	69.2%***	(19.97)
High Amihud	−0.929***	(−9.42)	−0.045	4.8%	(1.58)	−0.884	95.2%***	(31.29)
High Zeroret	−0.904***	(−9.44)	0.011	−1.2%*	(−1.73)	−0.915	101%***	(142.66)
High Spread	−1.057***	(−6.37)	−0.252	23.8%***	(4.99)	−0.805	76.2%***	(15.96)
High Dispersion	−0.745***	(−3.93)	−0.082	11.0%***	(3.10)	−0.663	89.0%***	(24.98)
High AvgVarβ	−0.909***	(−8.73)	−0.014	1.6%*	(1.71)	−0.894	98.4%***	(107.11)
Low SUE	−0.912***	(−7.98)	−0.045	4.9%***	(7.51)	−0.867	95.1%***	(145.55)
Average				18.6%			81.4%	

Panel D: Multivariate analysis using HIGHIVOL and dummy candidate variables

Candidate	Model 1			Model 2			Model 3		
	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat	Coeff.	Fraction	*t*-stat
High Skew	−0.032	2.0%*	(1.89)	−0.007	0.7%	(0.69)	−0.040	3.8%***	(6.61)
High Coskew	−0.198	12.2%***	(2.72)	−0.119	12.2%***	(3.18)	−0.146	13.9%***	(9.24)
High E(Idioskew)	−0.091	5.6%**	(2.50)	−0.139	14.3%***	(2.94)	−0.113	10.8%***	(7.95)
High RTP	−0.031	1.9%	(0.68)						
High Lagret	−0.106	6.5%*	(1.65)	−0.099	10.1%**	(2.47)	−0.187	17.8%***	(7.71)
High Amihud	−0.048	2.9%	(0.91)	0.011	−1.1%	(−0.18)	−0.091	8.7%***	(3.47)
High Zeroret	−0.084	5.2%	(0.98)	−0.002	0.2%	(0.34)	0.012	−1.1%**	(−2.45)
High Spread	−0.023	1.4%	(0.16)						
High Dispersion	−0.068	4.2%**	(2.45)	−0.069	7.1%***	(3.07)			
High AvgVarβ	0.001	−0.1%	(−0.06)	−0.013	1.3%	(1.00)	−0.005	0.4%	(1.08)
Low SUE	−0.012	0.8%	(0.95)	−0.020	2.0%***	(2.68)	−0.020	1.9%***	(7.50)
Residual	−0.935	57.5%***	(5.35)	−0.519	53.1%***	(6.00)	−0.459	43.8%***	(12.08)
Total	−1.627	100%	(−3.94)	−0.977	100%	(−4.26)	−1.050	100%	(−9.60)
Sample period	1984–2001			1982–2012			1971–2012		
Avg # firms/mth	1530.6			1813.0			2752.4		

explain the relations between returns and variables such as maximum daily return and past earnings surprise because the return predictability of these variables has also been cited in the literature as being anomalous relative to traditional asset pricing theories.

For brevity, we focus our analysis on three anomaly variables—*Maxret*, *Lagret*, and *SUE*—and use *IVOL* as the only candidate variable, although the analysis can be easily extended to other anomaly variables or to include other candidate explanatory variables.

Table 10 shows that *IVOL*, when considered alone, explains 67.5% of the negative relation between *Maxret* and subsequent returns. This number, though smaller than the fraction of the *IVOL* puzzle explained by *Maxret* (112.0%), is still impressive and identifies idiosyncratic volatility as a major contributor to the maximum daily return puzzle. In contrast, *IVOL* can only explain 5.9% of the one-month return reversal effect based on *Lagret* and 7.3% of the post-earnings announcement drift based on *SUE*, compared to 33.7% and 10.9% of the *IVOL* puzzle explained by *Lagret* and *SUE*, respectively. In short, the above results show that our decomposition methodology can be used to evaluate explanations for other asset pricing anomalies.

6. Conclusion

In this paper, we propose a simple methodology to examine a large number of explanations that have been proposed in the literature for the negative relation between idiosyncratic volatility and subsequent stock returns (the idiosyncratic volatility puzzle). The main advantage of our approach is that it allows us to quantify the contribution of each explanation either by itself or when evaluated against competing explanations.

We find that, surprisingly, many existing explanations explain less than 10% of the idiosyncratic volatility puzzle. On the other hand, explanations based on investors' lottery preferences and market frictions show some promise in explaining the puzzle. Taken together, however, all existing explanations still leave a sizable portion of the puzzle unexplained. Our main findings are robust to subsample analysis, using portfolios instead of individual stocks, and potential nonlinearity in the idiosyncratic volatility puzzle. Finally, our decomposition methodology can also be applied to evaluate competing explanations for other asset pricing anomalies.

Table 10
Decomposing other anomalies.
Using firm-level Fama-MacBeth cross-sectional regressions, the relations between three anomaly variables (*Maxret*, *Lagret*, and *SUE*) and DGTW-adjusted returns are decomposed into a component that is related to *IVOL* and a residual component. Stage 1 regresses month t returns on an anomaly variable. Stage 2 adds *IVOL* as the candidate variable to the regression. Stage 3 regresses the anomaly variable on *IVOL* to decompose the anomaly variable into two orthogonal components. In Stage 4, the coefficient on the anomaly variable from Stage 1 is decomposed into a component that is related to *IVOL* and a residual component. The time-series average of the *IVOL* component divided by the time-series average of the Stage 1 coefficient on the anomaly variable then measures the fraction of the anomaly explained by *IVOL*, and the average residual component divided by the average Stage 1 coefficient measures the fraction of the anomaly left unexplained by *IVOL*. *IVOL* is the standard deviation of residuals from a regression of daily stock returns in month $t-1$ on the Fama and French (1993) factors. *Maxret* is the maximum daily return in month $t-1$, *Lagret* is the month $t-1$ return, and *SUE* is the most recent standardized unexpected earnings. Stocks with prices less than \$1 at the end of the previous month are excluded from the analysis. The standard errors of the fractions of the anomaly explained are determined using the multivariate delta method. Time-series averages of estimated coefficients (×100) are reported with t-statistics in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Stage	Description	Variable	Anomaly variable					
			Maxret		Lagret		SUE	
1	DGTW-adj ret on anomaly	Intercept	0.347***	(8.87)	−0.033	(−1.15)	−0.100***	(−4.79)
		Anomaly	−6.421***	(−12.02)	−4.324***	(−13.64)	0.115***	(18.06)
2	Add IVOL	Intercept	0.270***	(4.74)	0.244***	(4.27)	0.285***	(4.75)
		Anomaly	−9.352***	(−10.20)	−4.467***	(−13.72)	0.108***	(17.21)
		IVOL	10.740***	(2.85)	−10.831***	(−4.88)	−15.669***	(−7.01)
3	Anomaly variable on IVOL	Intercept	−0.910***	(−25.80)	−2.460***	(−15.11)	61.341***	(22.99)
		IVOL	304.335***	(202.66)	162.213***	(20.49)	−1559.1***	(−34.87)
		Avg adj R^2	77.9%		8.2%		1.2%	
4	Decompose Stage 1 anomaly coefficient	IVOL	−4.337		−0.253		0.008	
			67.5%***	(18.05)	5.9%***	(3.26)	7.3%***	(4.94)
		Residual	−2.084		−4.071		0.106	
			32.5%***	(8.67)	94.1%***	(52.31)	92.7%***	(62.34)
		Total	−6.421***	(−12.02)	−4.324***	(−13.64)	0.115***	(18.06)
			100%		100%		100%	
Sample period			1963–2012		1963–2012		1971–2012	
Avg # firms/mth			3580.5		3580.4		3472.3	

Appendix A. The relation to the conventional approach

In this appendix, we demonstrate the relation between our decomposition methodology in Eq. (3) and the conventional approach of regressing returns on idiosyncratic volatility and a candidate variable in Eq. (10). Specifically, for each month t, we can substitute Eq. (2) into Eq. (10) and obtain:

$$\begin{aligned} R_{it} &= \tilde{\alpha}_t + \tilde{\gamma}_t^R(a_{t-1} + \mu_{it-1} + \ \delta_{t-1} Candidate_{it-1}) \\ &\quad + \tilde{\gamma}_t^C Candidate_{it-1} + \tilde{\varepsilon}_{it} \\ &= \tilde{\alpha}_t + \tilde{\gamma}_t^R(a_{t-1} + \mu_{it-1}) + (\tilde{\gamma}_t^C \\ &\quad + \delta_{t-1}\tilde{\gamma}_t^R) Candidate_{it-1} + \tilde{\varepsilon}_{it} \\ &= \tilde{\alpha}_t + \tilde{\gamma}_t^R(a_{t-1} + \mu_{it-1}) + \bar{\gamma}_t^C Candidate_{it-1} + \tilde{\varepsilon}_{it}, \end{aligned} \tag{11}$$

where $\bar{\gamma}_t^C$, which equals $\tilde{\gamma}_t^C + \delta_{t-1}\tilde{\gamma}_t^R$, is identical to the coefficient of regressing returns on the candidate variable alone because $(a_{t-1} + \mu_{it-1})$ and $Candidate_{it-1}$ are uncorrelated by construction. We can then rewrite γ_t^C from Eq. (3) as follows:

$$\begin{aligned} \gamma_t^C &= \frac{\mathrm{Cov}[R_{it}, \delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} \\ &= \frac{\mathrm{Cov}[R_{it}, \delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]} \times \frac{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} \\ &= \frac{\bar{\gamma}_t^C}{\delta_{t-1}} \times \frac{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]} \\ &= \left(\frac{\tilde{\gamma}_t^C}{\delta_{t-1}} + \tilde{\gamma}_t^R\right) \times \frac{\mathrm{Var}[\delta_{t-1} Candidate_{it-1}]}{\mathrm{Var}[IVOL_{it-1}]}. \end{aligned} \tag{12}$$

To show the relation more generally for k candidate variables, we simplify the notation by denoting $IVOL_{it-1}$ as $\mathbf{V}$ and R_{it} as $\mathbf{R}$, both are $n \times 1$ vectors where n is the number of firms in the month t cross-sectional regression. We also denote an $n \times 1$ vector of ones by $\boldsymbol{\iota}$ and we can now rewrite Eq. (1) as:

$$\mathbf{R} = \boldsymbol{\iota}\alpha + \mathbf{V}\gamma + \boldsymbol{\varepsilon}. \tag{13}$$

Next, we regress $\mathbf{V}$ on $\boldsymbol{\iota}$ and the $n \times k$ matrix of k candidate variables (measured contemporaneously with $\mathbf{V}$ in month $t-1$) denoted by $\mathbf{C} = (\mathbf{C}_1 \quad \cdots \quad \mathbf{C}_\mathbf{k})$, where $\mathbf{C}_\mathbf{j}$ is an $n \times 1$ vector:

$$\mathbf{V} = \boldsymbol{\iota}a + \ \mathbf{C}\boldsymbol{\delta}^\mathbf{C} + \boldsymbol{\mu}, \tag{14}$$

where $\boldsymbol{\delta}^\mathbf{C}$ is a $k \times 1$ vector of coefficients. In the last step, we decompose the idiosyncratic volatility-return relation γ into k components each related to a candidate variable and a residual component:

$$\begin{aligned} \gamma &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\mathbf{V}'\mathbf{r} \\ &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\left(\mathbf{C}\boldsymbol{\delta}^\mathbf{C} + \boldsymbol{\iota}a + \boldsymbol{\mu}\right)'\mathbf{r} \\ &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\left(\mathbf{C}\boldsymbol{\delta}^\mathbf{C}\right)'\mathbf{r} + \left(\mathbf{v}'\mathbf{v}\right)^{-1}(\boldsymbol{\iota}a + \boldsymbol{\mu})'\mathbf{r}, \end{aligned} \tag{15}$$

where $\mathbf{v}$ and $\mathbf{r}$ (both $n \times 1$ vectors) are demeaned versions of $\mathbf{V}$ and $\mathbf{R}$, respectively. The first term in the last line of Eq. (15) represents the combined contribution of all k candidate variables and the second term represents the unexplained component of the idiosyncratic volatility puzzle. The contribution of the jth candidate variable is then $\gamma_j^C = (\mathbf{v}'\mathbf{v})^{-1}(\mathbf{C_j}\delta_j^C)'\mathbf{r}$.

Now, take the conventional approach of regressing $\mathbf{R}$ on $\mathbf{V}$ and $\mathbf{C}$:

$$\mathbf{R} = \boldsymbol{\iota}\tilde{\alpha} + \mathbf{V}\tilde{\gamma}^R + \mathbf{C}\tilde{\boldsymbol{\gamma}}^{\mathbf{C}} + \tilde{\boldsymbol{\varepsilon}}. \tag{16}$$

We can rewrite Eq. (16) by substituting in Eq. (14) as follows:

$$\begin{aligned}\mathbf{R} &= \boldsymbol{\iota}\tilde{\alpha} + (\boldsymbol{\iota}a + \mathbf{C}\boldsymbol{\delta}^{\mathbf{C}} + \boldsymbol{\mu})\tilde{\gamma}^R + \mathbf{C}\tilde{\boldsymbol{\gamma}}^{\mathbf{C}} + \tilde{\boldsymbol{\varepsilon}} \\ &= \boldsymbol{\iota}\tilde{\alpha} + (\boldsymbol{\iota}a + \boldsymbol{\mu})\tilde{\gamma}^R + \mathbf{C}(\tilde{\boldsymbol{\gamma}}^{\mathbf{C}} + \boldsymbol{\delta}^{\mathbf{C}}\tilde{\gamma}^R) + \tilde{\boldsymbol{\varepsilon}}.\end{aligned} \tag{17}$$

Because $\mathbf{C}$ and $(\boldsymbol{\iota}a + \boldsymbol{\mu})$ are uncorrelated by construction, the coefficient on the jth candidate variable $(\tilde{\gamma}_j^C + \delta_j^C\tilde{\gamma}^R)$ should be identical to the slope coefficient when $\mathbf{R}$ is regressed on the regression residual of $\mathbf{C_j}$ on the other k–1 candidate variables. Specifically, we define an $n \times (k+1)$ matrix $\mathbb{C} = (\boldsymbol{\iota} \quad \mathbf{C}_1 \quad \cdots \quad \mathbf{C_k})$, an $(k + 1) \times (k + 1)$ matrix $\mathbf{J}$ which is an identity matrix except that the $(j + 1)$th diagonal term is set to zero, and $\boldsymbol{\theta}$ which is the $(k + 1) \times 1$ vector of coefficients from regressing $\mathbf{C_j}$ on $\mathbb{C}\mathbf{J}$. Then we have:

$$\begin{aligned}\left(\tilde{\gamma}_j^C + \delta_j^C\tilde{\gamma}^R\right) &= \left[\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)\right]^{-1}\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\mathbf{r} \\ \left(\tilde{\gamma}_j^C + \delta_j^C\tilde{\gamma}^R\right) &= \left[\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)\right]^{-1}\left[\mathbf{C_j}'\mathbf{r} - \left(\mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\mathbf{r}\right] \\ \mathbf{C_j}'\mathbf{r} &= \left[\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)\right]\left(\tilde{\gamma}_j^C + \delta_j^C\tilde{\gamma}^R\right) \\ &\quad + \left(\mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\mathbf{r}.\end{aligned} \tag{18}$$

We can then rewrite Eq. (15) to give us the relation between γ_j^C (the contribution of the jth candidate variable to the idiosyncratic volatility puzzle) and $\tilde{\gamma}_j^C$ [the coefficient on the jth candidate variable in Eq. (16)]:

$$\begin{aligned}\gamma_j^C &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\left(\mathbf{C_j}\delta_j^C\right)'\mathbf{r} \\ &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\delta_j^C\mathbf{C_j}'\mathbf{r} \\ &= \left(\mathbf{v}'\mathbf{v}\right)^{-1}\delta_j^C\left\{\left[\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\left(\mathbf{C_j} - \mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)\right]\right. \\ &\quad \left.\times\left(\tilde{\gamma}_j^C + \delta_j^C\tilde{\gamma}^R\right) + \left(\mathbb{C}\mathbf{J}\boldsymbol{\theta}\right)'\mathbf{r}\right\}.\end{aligned} \tag{19}$$

When $k = 1$, the above relation collapses to $(\mathbf{v}'\mathbf{v})^{-1}\delta_j^C(\mathbf{C_j}'\mathbf{C_j})(\tilde{\gamma}_{jC} + \delta_j^C\tilde{\gamma}^R)$, which is the matrix form of Eq. (12).

Appendix B. The effect of measurement errors

In this appendix, we analyze the effect of measurement errors on our decomposition methodology. Our decomposition methodology is based on the following two equations:

$$R_{it} = \alpha_t + \gamma_t IVOL_{it-1} + \varepsilon_{it}, \tag{20}$$

$$IVOL_{it-1} = a_{t-1} + \delta_{t-1}C_{it-1} + \mu_{it-1}. \tag{21}$$

For brevity, we denote $Candidate_{it-1}$ here as C_{it-1}. Let us assume that $IVOL_{it-1}$ and C_{it-1} are not directly observable and can only be measured with error:

$$\widetilde{IVOL}_{it-1} = IVOL_{it-1} + u_{it-1}, \tag{22}$$

$$\tilde{C}_{it-1} = C_{it-1} + v_{it-1}. \tag{23}$$

The following standard assumptions apply to the error terms:

(i) $\mathrm{E}(v_{it-1}) = \mathrm{E}(C_{it-1}v_{it-1}) = \mathrm{E}(IVOL_{it-1}v_{it-1}) = \mathrm{E}(\mu_{it-1}v_{it-1}) = 0$
(ii) $\mathrm{E}(u_{it-1}) = \mathrm{E}(C_{it-1}u_{it-1}) = \mathrm{E}(IVOL_{it-1}u_{it-1}) = \mathrm{E}(\varepsilon_{it}u_{it-1}) = 0$
(iii) $\mathrm{E}(R_{it}u_{it-1}) = \mathrm{E}(R_{it}v_{it-1}) = 0$
(iv) $\mathrm{E}(u_{it-1}v_{it-1}) = 0$
(v) $\mathrm{E}(IVOL_{it-1}\varepsilon_{it}) = \mathrm{E}(C_{it-1}\mu_{it-1}) = 0.$

From Eqs. (21), (22), and (23), the relation between $\widetilde{IVOL}_{it-1}$ and $\tilde{C}_{it-1}$ is:

$$\widetilde{IVOL}_{it-1} = a_{t-1} + \delta_{t-1}\tilde{C}_{it-1} + \omega_{it-1}, \tag{24}$$

where $\omega_{it-1} = \mu_{it-1} - \delta_{t-1}v_{it-1} + u_{it-1}$.

We can then write down the estimator, $\hat{\delta}_{t-1}$, as a function of its true counterpart δ_{t-1}:

$$\begin{aligned}\hat{\delta}_{t-1} &= \frac{\mathrm{Cov}(\widetilde{IVOL}_{it-1}, \tilde{C}_{it-1})}{\mathrm{Var}(\tilde{C}_{it-1})} \\ &= \frac{\mathrm{Cov}(IVOL_{it-1} + u_{it-1}, C_{it-1} + v_{it-1})}{\mathrm{Var}(C_{it-1} + v_{it-1})} \\ &= \frac{\mathrm{Cov}(a_{t-1} + \delta_{t-1}C_{it-1} + \mu_{it-1} + u_{it-1}, C_{it-1} + v_{it-1})}{\mathrm{Var}(C_{it-1}) + \mathrm{Var}(v_{it-1})} \\ &= \frac{\sigma^2_{C,t-1}}{\sigma^2_{C,t-1} + \sigma^2_{v,t-1}} \times \delta_{t-1}.\end{aligned} \tag{25}$$

Next, we decompose $\hat{\gamma}_t$, the estimator of γ_t, using the estimators from Eq. (24):

$$\begin{aligned}\hat{\gamma}_t &= \frac{\mathrm{Cov}(R_{it}, \widetilde{IVOL}_{it-1})}{\mathrm{Var}(\widetilde{IVOL}_{it-1})} \\ &= \frac{\mathrm{Cov}(R_{it}, \hat{\delta}_{t-1}\tilde{C}_{it-1})}{\mathrm{Var}(\widetilde{IVOL}_{it-1})} + \frac{\mathrm{Cov}(R_{it}, \hat{a}_{t-1} + \hat{\omega}_{it-1})}{\mathrm{Var}(\widetilde{IVOL}_{it-1})} \\ &= \hat{\gamma}_t^C + \hat{\gamma}_t^R.\end{aligned} \tag{26}$$

With the above, we can express $\hat{\gamma}_t$ and $\hat{\gamma}_t^C$ as functions of their true counterparts γ_t and γ_t^C:

$$\begin{aligned}\hat{\gamma}_t &= \frac{\mathrm{Cov}(R_{it}, \widetilde{IVOL}_{it-1})}{\mathrm{Var}(\widetilde{IVOL}_{it-1})} \\ &= \frac{\mathrm{Cov}(R_{it}, IVOL_{it-1} + u_{it-1})}{\mathrm{Var}(IVOL_{it-1} + u_{it-1})} \\ &= \frac{\mathrm{Cov}(R_{it}, IVOL_{it-1})}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \\ &= \frac{\sigma^2_{IVOL,t-1}}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \times \gamma_t,\end{aligned} \tag{27}$$

and

$$\begin{aligned}\hat{\gamma}_t^C &= \frac{\mathrm{Cov}(R_{it}, \hat{\delta}_{t-1}\tilde{C}_{it-1})}{\mathrm{Var}(\widetilde{IVOL}_{it-1})} = \frac{\mathrm{Cov}[R_{it}, \hat{\delta}_{t-1}(C_{it-1} + v_{it-1})]}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \\ &= \frac{\mathrm{Cov}(R_{it}, \hat{\delta}_{t-1}C_{it-1})}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \\ &= \frac{\sigma^2_{C,t-1}}{\sigma^2_{C,t-1} + \sigma^2_{v,t-1}} \times \frac{\mathrm{Cov}(R_{it}, \delta_{t-1}C_{it-1})}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \\ &= \frac{\sigma^2_{C,t-1}}{\sigma^2_{C,t-1} + \sigma^2_{v,t-1}} \times \frac{\sigma^2_{IVOL,t-1}}{\sigma^2_{IVOL,t-1} + \sigma^2_{u,t-1}} \times \gamma_t^C.\end{aligned} \tag{28}$$

We can further simplify Eq. (28) by denoting $k_{t-1} = \frac{\sigma^2_{C,t-1}}{\sigma^2_{C,t-1}+\sigma^2_{v,t-1}}$, and $\lambda_{t-1} = \frac{\sigma^2_{IVOL,t-1}}{\sigma^2_{IVOL,t-1}+\sigma^2_{u,t-1}}$, where $0 < k_{t-1} < 1$ and $0 < \lambda_{t-1} < 1$.

We see from Eqs. (27) and (28) that both $\hat{\gamma}_t$ and $\hat{\gamma}^C_t$ are biased downwards. The bias in $\hat{\gamma}_t$ is due to the measurement error in *IVOL* while the bias in $\hat{\gamma}^C_t$ is due to the measurement errors in both *IVOL* and the candidate variable. It is then straightforward to use Eq. (4) to show that the mean of the fraction, $\hat{\gamma}^C_t/\hat{\gamma}_t$, is biased downwards with the magnitude of the bias determined by k_{t-1}:

$$\mathrm{E}\left(\frac{\hat{\gamma}^C_t}{\hat{\gamma}_t}\right) \approx \frac{\mathrm{E}\left(\hat{\gamma}^C_t\right)}{\mathrm{E}\left(\hat{\gamma}_t\right)} = \frac{k_{t-1}\lambda_{t-1}\mathrm{E}\left(\gamma^C_t\right)}{\lambda_{t-1}\mathrm{E}(\gamma_t)} = k_{t-1}\frac{\mathrm{E}(\gamma^C_t)}{\mathrm{E}(\gamma_t)}. \tag{29}$$

The variance of the fraction is obtained by rewriting Eq. (5):

$$\mathrm{Var}\left(\frac{\hat{\gamma}^C_t}{\hat{\gamma}_t}\right) \approx \left(\frac{\mathrm{E}\left(\hat{\gamma}^C_t\right)}{\mathrm{E}\left(\hat{\gamma}_t\right)}\right)^2 \times \left(\frac{\mathrm{Var}\left(\hat{\gamma}^C_t\right)}{\left(\mathrm{E}\left(\hat{\gamma}^C_t\right)\right)^2} + \frac{\mathrm{Var}\left(\hat{\gamma}_t\right)}{\left(\mathrm{E}\left(\hat{\gamma}_t\right)\right)^2} - 2\frac{\mathrm{Cov}\left(\hat{\gamma}^C_t, \hat{\gamma}_t\right)}{\mathrm{E}\left(\hat{\gamma}^C_t\right)\mathrm{E}\left(\hat{\gamma}_t\right)}\right). \tag{30}$$

We can verify the effect of measurement error on each of the following terms in (30):

$$\frac{\mathrm{Var}\left(\hat{\gamma}^C_t\right)}{\left(\mathrm{E}\left(\hat{\gamma}^C_t\right)\right)^2} = \frac{k^2_{t-1}\lambda^2_{t-1}\mathrm{Var}\left(\gamma^C_t\right)}{k^2_{t-1}\lambda^2_{t-1}\left(\mathrm{E}\left(\gamma^C_t\right)\right)^2} = \frac{\mathrm{Var}\left(\gamma^C_t\right)}{\left(\mathrm{E}\left(\gamma^C_t\right)\right)^2}, \tag{31}$$

$$\frac{\mathrm{Var}\left(\hat{\gamma}_t\right)}{\left(\mathrm{E}\left(\hat{\gamma}_t\right)\right)^2} = \frac{\lambda^2_{t-1}\mathrm{Var}(\gamma_t)}{\lambda^2_{t-1}(\mathrm{E}(\gamma_t))^2} = \frac{\mathrm{Var}(\gamma_t)}{(\mathrm{E}(\gamma_t))^2}, \tag{32}$$

$$\frac{2\mathrm{Cov}\left(\hat{\gamma}^C_t, \hat{\gamma}_t\right)}{\mathrm{E}\left(\hat{\gamma}^C_t\right)\mathrm{E}\left(\hat{\gamma}_t\right)} = \frac{2k_{t-1}\lambda^2_{t-1}\mathrm{Cov}\left(\gamma^C_t, \gamma_t\right)}{k_{t-1}\lambda^2_{t-1}\mathrm{E}\left(\gamma^C_t\right)\mathrm{E}(\gamma_t)} = \frac{2\mathrm{Cov}\left(\gamma^C_t, \gamma_t\right)}{\mathrm{E}\left(\gamma^C_t\right)\mathrm{E}(\gamma_t)}. \tag{33}$$

As a result, the only source of bias in Eq. (30) is from the first term, $(\frac{\mathrm{E}(\hat{\gamma}^C_t)}{\mathrm{E}(\hat{\gamma}_t)})^2 = k^2_{t-1}(\frac{\mathrm{E}(\gamma^C_t)}{\mathrm{E}(\gamma_t)})^2$. Hence, the standard error of the fraction is biased downwards with the magnitude of the bias determined by k_{t-1}, which is identical to the magnitude of the bias in the mean of the fraction as shown in Eq. (29).

References

Alizadeh, S., Brandt, M.W., Diebold, F.X., 2002. Range-based estimation of stochastic volatility models. Journal of Finance 57, 1047–1092.
Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. Journal of Financial Markets 5, 31–56.
Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2006. The cross-section of volatility and expected returns. Journal of Finance 61, 259–299.
Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2009. High idiosyncratic volatility and low returns: international and further U.S. evidence. Journal of Financial Economics 91, 1–23.
Ang, A., Liu, J., Schwarz, K., 2010. Using stocks or portfolios in tests of factor models. Unpublished working paper. Columbia University.
Asparouhova, E., Bessembinder, H., Kalcheva, I., 2013. Noisy prices and inference regarding returns. Journal of Finance 68, 665–714.
Avramov, D., Chordia, T., Jostova, G., Philipov, A., 2013. Anomalies and financial distress. Journal of Financial Economics 108, 139–159.
Bali, T.G., Cakici, N., 2008. Idiosyncratic volatility and the cross section of expected returns. Journal of Financial and Quantitative Analysis 43, 29–58.
Bali, T.G., Cakici, N., Whitelaw, R.F., 2011. Maxing out: stocks as lotteries and the cross-section of expected returns. Journal of Financial Economics 99, 427–446.
Barberis, N., Huang, M., 2008. Stocks as lotteries: the implications of probability weighting for security prices. American Economic Review 98, 2066–2100.
Barinov, A., 2013. Idiosyncratic volatility, growth options, and the cross-section of returns. Unpublished working paper. University of Georgia.
Boehme, R.D., Danielsen, B.R., Kumar, P., Sorescu, S.M., 2009. Idiosyncratic risk and the cross-section of stock returns: Merton (1987) meets Miller (1977). Journal of Financial Markets 12, 438–468.
Boyer, B., Mitton, T., Vorkink, K., 2010. Expected idiosyncratic skewness. Review of Financial Studies 23, 169–202.
Brandt, M.W., Diebold, F.X., 2006. A no-arbitrage approach to range-based estimation of return covariances and correlations. Journal of Business 79, 61–74.
Casella, G., Berger, R.L., 2001. Statistical Inference, second ed. Duxbury Press, Pacific Grove, California.
Chabi-Yo, F., Yang, J., 2009. Default risk, idiosyncratic coskewness and equity returns. Unpublished working paper. Ohio State University.
Chen, Z., Petkova, R., 2012. Does idiosyncratic volatility proxy for risk exposure? Review of Financial Studies 25, 2745–2787.
Cohen, L., Diether, K.B., Malloy, C., 2007. Supply and demand shifts in the shorting market. Journal of Finance 65, 2061–2096.
Daniel, K., Grinblatt, M., Titman, S., Wermers, R., 1997. Measuring mutual fund performance with characteristic-based benchmarks. Journal of Finance 52, 1035–1058.
Doran, J.S., Jiang, D., Peterson, D.R., 2012. Gambling preference and the New Year effect of assets with lottery features. Review of Finance 16, 685–731.
Fama, E.F., French, K.R., 1993. Common risk factors in the returns of stocks and bonds. Journal of Financial Economics 33, 3–56.
Fama, E.F., French, K.R., 2006. The value premium and the CAPM. Journal of Finance 61, 2163–2185.
Fama, E.F., MacBeth, J.D., 1973. Risk, return and equilibrium: empirical tests. Journal of Political Economy 81, 607–636.
Fu, F., 2009. Idiosyncratic risk and the cross-section of expected stock returns. Journal of Financial Economics 91, 24–37.
George, T.J., Hwang, C.-Y., 2011. Analyst coverage and the cross sectional relation between returns and volatility. Unpublished working paper. Nanyang Technological University.
Han, B., Kumar, A., 2013. Speculative trading and asset prices. Journal of Financial and Quantitative Analysis 48, 377–404.
Han, Y., Lesmond, D., 2011. Liquidity biases and the pricing of cross-sectional idiosyncratic volatility. Review of Financial Studies 24, 1590–1629.
Harvey, C.R., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1295.
Hirshleifer, D., 1988. Residual risk, trading costs, and commodity futures risk premia. Review of Financial Studies 1, 173–193.
Huang, W., Liu, Q., Rhee, S.G., Zhang, L., 2009. Return reversals, idiosyncratic risk, and expected returns. Review of Financial Studies 23, 147–168.
Jiang, G.J., Xu, D., Yao, T., 2009. The information content of idiosyncratic volatility. Journal of Financial and Quantitative Analysis 44, 1–28.
Johnson, T.C., 2004. Forecast dispersion and the cross-section of stock returns. Journal of Finance 59, 1957–1978.
Merton, R.C., 1987. A simple model of capital market equilibrium with incomplete information. Journal of Finance 42, 483–510.
Nagel, S., 2005. Short sales, institutional investors and the cross-section of stock returns. Journal of Financial Economics 77, 277–309.
Shumway, T., 1997. The delisting bias in CRSP data. Journal of Finance 52, 327–340.
Stambaugh, R.F., Yu, J., Yuan, Y., 2015. Arbitrage asymmetry and the idiosyncratic volatility puzzle. Journal of Finance 70, 1903–1948.
Wong, P., 2011. Earnings shocks and the idiosyncratic volatility discount in the cross-section of expected returns. Unpublished working paper. University of South Carolina.