Dissecting Anomalies with a Five-Factor Model

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A five-factor model that adds profitability (*RMW*) and investment (*CMA*) factors to the three-factor model of Fama and French (1993) suggests a shared story for several average-return anomalies. Specifically, positive exposures to *RMW* and *CMA* (stock returns that behave like those of profitable firms that invest conservatively) capture the high average returns associated with low market β, share repurchases, and low stock return volatility. Conversely, negative *RMW* and *CMA* slopes (like those of relatively unprofitable firms that invest aggressively) help explain the low average stock returns associated with high β, large share issues, and highly volatile returns. (*JEL* G1, G11, G12)

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Motivated by the dividend discount valuation model, Fama and French (FF; 2015) add profitability and investment factors to the market, *Size*, and value/growth factors of the three-factor model of Fama and French (FF; 1993). In FF (2015) the left-hand-side (LHS) assets used to test the resulting five-factor model are portfolios formed using sorts on *Size* (market capitalization or market cap) and combinations of the book-to-market equity ratio, profitability, and investment. The LHS portfolios are thus just finer sorts on the variables used to construct the factors.

Here we follow the advice of Lewellen, Nagel, and Shanken (2010) and consider anomalies not targeted by the five-factor model and known to cause problems for the FF three-factor model. Accruals (Sloan 1996), net share issues (Ikenberry, Lakonishok, and Vermaelen 1995; Loughran and Ritter 1995), momentum (Jegadeesh and Titman 1993), and volatility (Ang et al. 2006) are prominent examples. There is also long-standing evidence (Black, Jensen, and Scholes 1972; Fama and MacBeth 1973) that the relation between average return and market β is flatter than predicted by the Sharpe (1964)-Lintner

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(1965) CAPM. The goal here is to examine whether the five-factor model and models that use subsets of its factors capture average returns from sorts on these variables and whether portfolios that signal model problems have exposures to the size, profitability, and investment factors typical of stocks that cause problems for the five-factor model in many sorts in FF (2015).

Given the large number of anomalies researchers have discovered in stock returns, one might ask why the additions to the FF (1993) three-factor model are profitability and investment factors. FF (2015) argue that these are natural choices, suggested by the dividend discount model. Miller and Modigliani (1961) show that in the dividend discount model, M_t, the time t market cap of a firm's stock, is,

$$M_t = \sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}. \tag{1}$$

In this equation, $Y_{t+\tau}$ is equity earnings for period $t+\tau$, $dB_{t+\tau} \equiv B_{t+\tau} - B_{t+\tau-1}$ is the change in book equity, and r, the internal rate of return on expected cash flows to shareholders, is approximately the long-term expected stock return. Dividing by time t book equity gives

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{2}$$

Equation (2) implies that if we hold constant everything except the current value of the stock, M_t, and the expected stock return, r, a lower value of M_t, or equivalently a higher book-to-market equity ratio, B_t/M_t, implies a higher expected return. Similarly, if we hold M_t, B_t, and the stream of future investments ($dB_{t+\tau}$) fixed, higher expected profitability implies higher expected cash flows to shareholders and a higher expected stock return. Finally, given M_t, B_t, and the stream of future earnings, higher expected investment implies lower expected cash flows and a lower expected return. In short, (2) implies B_t/M_t is a noisy proxy for expected return because the market cap M_t also responds to forecasts of earnings and investment.

Most of our tests use variants of the five-factor time-series regression

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}. \tag{3}$$

In this equation R_{it} is the month t return on one of the portfolios from sorts of stocks on *Size* and an anomaly variable, R_{Ft} is the risk-free rate (the one-month U.S. Treasury-bill rate observed at the beginning of month t), R_{Mt} is the return on the value-weight (VW) portfolio of NYSE-AMEX-NASDAQ stocks, SMB_t (small minus big) and HML_t (high minus low B/M) are the *Size* and value factors of the FF three-factor model, RMW_t (robust minus weak) is a profitability factor, and CMA_t (conservative minus aggressive) is an investment factor.

The bottom line from our tests is that the list of anomalies shrinks when we use the five-factor model, in part because anomaly returns become less anomalous and in part because the returns for different anomalies have similar five-factor exposures (regression slopes in (3)) that suggest they are related phenomena. With two exceptions, accruals and momentum, the five-factor model shrinks anomaly average returns left unexplained by the FF three-factor model. Moreover, the successes and failures of the model are linked to patterns in the slopes for RMW_t and CMA_t that are common to the sorts on β, net share issues, and volatility. The high average returns associated with low β, share repurchases, and low volatility that are left unexplained by the three-factor model are absorbed by positive five-factor exposures to RMW_t and CMA_t, typical of profitable firms that invest conservatively. At the other extreme, the low average returns associated with high β, large share issues, and high return volatility that are left unexplained by the three-factor model are substantially captured by negative five-factor exposures to RMW_t and CMA_t, typical of less profitable firms that invest aggressively.

In the sorts on net share issues and volatility, the portfolios that cause the most serious problems for the five-factor model are in the smaller *Size* quintiles and the highest quintiles of share issues and volatility. These portfolios have negative exposures to RMW_t and CMA_t that lower estimates of their expected returns, but not enough to explain their low average returns. Most interesting, the common patterns in the five-factor slopes for these portfolios suggest they share the lethal traits—small stocks whose returns behave like those of relatively unprofitable firms that invest aggressively—that plague the five-factor model in FF (2015).

Accruals pose special problems. For other anomalies, the five-factor model improves the description of average returns of the FF three-factor model. For accruals the five-factor model does worse. The problem is that in the sorts on accruals, portfolios in the smallest *Size* quintile (microcaps) have negative RMW_t slopes but they do not have the predicted low average returns. Hou, Xue, and Zhang (2015) also find that sorts on accruals produce average returns that escape explanation by a model similar to ours.

For the anomalies discussed above, adding a momentum factor to the five-factor model has little effect on performance, simply because the sorts do not produce portfolios with large momentum tilts. For portfolios formed on momentum, however, the five-factor model does poorly, with regression intercepts about as disperse as average returns on the portfolios. Adding a momentum factor improves model performance, but leaves nontrivial unexplained momentum returns among small stocks.

1. The Factors

Dropping the time subscript on the variables, the tests of the five-factor model use the $R_M - R_F$, *SMB*, and *HML* factors of the three-factor model of FF (1993),

Table 1
Averages, standard deviations, and *t*-statistics for monthly factor returns, July 1963–December 2014 (618 months)

	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	*MOM*
Mean	0.51	0.27	0.36	0.25	0.32	0.69
SD	4.46	3.07	2.86	2.14	1.99	4.22
***t*-statistic**	2.83	2.20	3.15	2.88	4.04	4.05

$R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate. At the end of each June, NYSE, AMEX, and NASDAQ stocks are allocated to two *Size* groups (small and big) using the NYSE median market-cap breakpoints. Stocks are allocated independently to three B/M groups (low to high), using NYSE 30th and 70th percentile breakpoints. The intersections of the two sorts produce six value-weight *Size*-B/M portfolios. In the sort for June of year t, B is book equity at the end of the fiscal year ending in year $t-1$ and M is market cap at the end of December of year $t-1$, adjusted for changes in shares outstanding between the measurement of B and the end of December. We use Compustat data to compute book equity, defined as (1) stockholders equity (or the par value of preferred plus total common equity or assets minus liabilities, in that order) minus (2) the redemption, liquidation, or par value of preferred (in that order) plus (3) balance sheet deferred taxes, if available, minus (4) postretirement benefits, if available. We fill in missing book equity data for NYSE stocks as in Davis, Fama, and French (2000). *HML* is the average of the returns on the two high B/M portfolios from the 2×3 sorts minus the average of the returns on the two low B/M portfolios. The profitability and investment factors, *RMW* (robust minus weak) and *CMA* (conservative minus aggressive), are formed in the same way as *HML*, except the second sort variable is operating profitability or investment. Operating profitability, *OP*, in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenue minus the cost of goods sold, minus selling, general, and administrative expenses, minus interest expense, all divided by book equity. Investment, *Inv*, is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets. In the separate *Size*-B/M, *Size-OP*, and *Size-Inv* sorts, there are three versions of *SMB*, one for each 2×3 sort, and *SMB* is the average of the three, or equivalently, it is the average of the returns on the nine small stock portfolios from the three sorts minus the average of the nine big stock portfolios. The momentum factor, *MOM*, is defined in the same way as *HML*, except the factor is updated monthly rather than annually. To form the six *Size-Prior* $2-12$ portfolios at the end of month $t-1$, *Size* is the market cap of a stock at the end of $t-1$ and *Prior* $2-12$ is its cumulative return for the 11 months from $t-12$ to $t-2$. The table shows average monthly returns (mean), the standard deviations of monthly returns (SD), and the t-statistics for the average returns.

augmented with similar profitability and investment factors. The *SMB* and *HML* factors of the original model use independent sorts of stocks into two *Size* groups and three book-to-market equity (B/M) groups (independent 2×3 sorts). The *Size* breakpoint is the NYSE median market cap, and the B/M breakpoints are the 30th and 70th percentiles of B/M for NYSE stocks. The intersections of the sorts produce six VW portfolios. The *Size* factor SMB_{BM} is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns. The value factor *HML* is the average of the two high B/M portfolio returns minus the average of the two low B/M portfolio returns. The profitability and investment factors, *RMW* and *CMA*, are constructed in the same way as *HML*, except the second sort is on operating profitability (*OP*) or investment (*Inv*). Definitions of the sort variables and details of factor construction are in Table 1.

The 2×3 sorts used to construct *RMW* and *CMA* produce two additional *Size* factors: SMB_{OP} and SMB_{Inv}. The *Size* factor *SMB* used in the tests is the average of the returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the returns on the nine big stock portfolios.

No combination of the factors in (3) explains average returns on portfolios formed on momentum. Thus, in the tests in which the LHS asset returns to be explained are for momentum portfolios, we include a momentum factor, *MOM*, among the right-hand-side (RHS) explanatory returns. *MOM* is defined like *HML*, except that it is updated monthly rather than annually, and the sort for portfolios formed at the end of month $t-1$ is based on the cumulative average returns from t–12 to t–2, called *Prior 2–12*. Note that *MOM* is reconstituted monthly using the most recently available data, whereas *SMB*, *HML*, *RMW*, and *CMA* are updated annually using data that, except for *Size*, are at least six months old. (See Table 1.)

Our sample is the 618 months from July 1963 to December 2014 (henceforth 1963–2014). The average monthly returns on the factors for this period are all more than two standard errors above zero (Table 1). The average equity premium ($R_M - R_F$) for 1963–2014 is large, 0.51% per month, but the monthly standard deviation is also substantial, 4.46%, and the t-statistic for the average premium is 2.83. The average *HML* return has a larger t-statistic, 3.15, the result of combining a smaller average premium, 0.36% per month, with a smaller standard deviation, 2.86%. The profitability factor, *RMW*, has the lowest average premium, 0.25% per month, but because of its low standard deviation, 2.14%, its t-statistic is 2.88. The investment and momentum factors, *CMA* and *MOM*, have the largest t-statistics, 4.04 and 4.05. The large t-statistic for *CMA* combines a moderate factor premium of 0.32% per month with the lowest factor standard deviation, 1.99%. In contrast, the large t-statistic for *MOM* combines the second highest standard deviation, 4.22%, with the highest average return, 0.69% per month. FF (2015) provide more detail on factor construction and the behavior of factor returns.

The momentum factor plays a critical role when the LHS returns in asset pricing regressions are for momentum portfolios. But including *MOM* produces small changes in model performance when the LHS portfolios (here and in FF 2015) are not formed on momentum. Thus, we put the momentum factor aside except when we address the momentum anomaly. We have also tried models that add the liquidity factor of Pástor and Stambaugh (2003) to different versions of regression (3). Skipping the details, the portfolios examined here (and in FF 2015) have trivial loadings on the traded and nontraded versions of the liquidity factor, and including the traded version produces only tiny changes in regression intercepts.

2. Summary Asset Pricing Tests

We turn now to our central task: examining how well variants of the five-factor model capture average returns on portfolios formed on *Size* and each of the anomaly variables. Two-way sorts on *Size* and an anomaly variable allow us to see how anomaly returns, and explanations of them provided by different models, vary across *Size* groups. This section presents summary tests. Later sections examine regression intercepts and pertinent slopes for each anomaly variable. We begin by introducing the left-hand-side portfolios in the tests.

2.1 The LHS anomaly portfolios

2.1.1 Market β. Many studies, from Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) to Frazzini and Pedersen (2014) find that the relation between univariate market β and average stock return is flatter than predicted by the CAPM of Sharpe (1964) and Lintner (1965). We construct 25 VW portfolios at the end of June each year from independent sorts of stocks into quintiles of *Size* at the end of June and β estimated using the preceding five years (two years minimum) of past monthly returns. As in all our sorts, the quintile breakpoints use NYSE stocks, but the sample is NYSE, AMEX, and NASDAQ stocks on both CRSP and Compustat with data for the variables in the sort and share codes 10 or 11. For a bit of color, stocks in the bottom and top *Size* quintiles are often called microcaps and megacaps.

2.1.2 Net share issues. Share repurchases tend to be followed by large average returns (Ikenberry, Lakonishok, and Vermaelen 1995), and average returns after share issues tend to be low (Loughran and Ritter 1995). We form 35 portfolios from independent sorts of stocks into *Size* quintiles and seven net share issues (*NI*) groups. Portfolio formation follows the same rules as the *Size*-β sorts, except the second sort is on *NI* and there are seven groups—negative *NI* (net repurchases), zero *NI*, and quintiles of positive *NI* (net issues). The choice of one repurchase group is a bit arbitrary but in line with the fact that net repurchases are less frequent than net issues, and for big stocks a finer breakdown of repurchases would produce undiversified portfolios. For portfolios formed in June of year t, *NI* is the change in the natural log of split-adjusted shares outstanding from the fiscal year-end in $t-2$ to the fiscal year-end in $t-1$.

2.1.3 Volatility. Ang et al. (2006) find that stocks with highly volatile returns tend to have low average returns whether volatility is measured as the variance of daily returns or as the variance of the residuals from the FF three-factor model. We construct VW portfolios using monthly sorts on *Size* and *Var* or *Size* and *RVar*, where *Var* is the variance of daily returns, and *RVar* is the variance of daily residuals from the FF three-factor model, both estimated using 60 (with a minimum 20) days of lagged returns. We examine quintiles of *Size* and *Var* or *RVar* but in contrast to other sorts, the NYSE breakpoints for *Var* and *RVar* are set separately for each *Size* quintile. This reflects the fact that with unconditional NYSE breakpoints, the highest *Var* and *RVar* quintiles are mostly microcaps, and the megacap portfolios in the highest volatility quintiles are thin and sometimes empty.

2.1.4 Accruals. Sloan (1996) is the seminal paper in the literature on the low returns associated with high accruals. Accruals arise because accounting decisions cause book earnings to differ from cash earnings. Our tests of the accruals anomaly use 25 VW portfolios formed from the intersection of independent sorts of stocks into *Size* and accrual (*AC*) quintiles. The portfolios are formed at the end of June of each year t. *Size* is market cap at the end of June and accruals are the change in operating working capital per split-adjusted share from the fiscal year-end in $t-2$ to $t-1$ divided by book equity per share in $t-1$.

2.1.5 Momentum. Jegadeesh and Titman (1993) document momentum in stock returns. For example, the relative performance of stocks in months $t-12$ to $t-2$ tends to persist in month t. We form 25 VW momentum portfolios every month. The portfolios for month t, formed at the end of month $t-1$, are the intersection of quintiles from independent sorts on *Size* (market cap at the end of $t-1$), and *Prior 2–12* (the sum of a stock's monthly returns from $t-12$ to $t-2$).

The troublesome portfolios in our asset pricing tests are typically in the smaller *Size* quintiles. For perspective, with NYSE *Size* breaks, the microcap quintile on average contains 57% of NYSE-AMEX-NASDAQ stocks but only 2.9% of aggregate market cap. The next *Size* quintile on average includes 14.7% of stocks but only 3.7% of aggregate market cap. In contrast, the megacap *Size* quintile on average accounts for 8.6% of NYSE-AMEX-NASDAQ stocks, but they are 74.8% of aggregate market cap, and the two largest *Size* quintiles together average 87.3% of aggregate market cap.

2.2 Summary tests

If an asset pricing model captures expected returns, the intercept is indistinguishable from zero in the time-series regression of any asset's excess return (its return in excess of the risk-free rate) on the model's factor returns. Table 2 shows Gibbons, Ross, and Shanken's (1989) *GRS* statistic, which tests this hypothesis for variants of regression (3). The variants of (3) we examine include the three-factor model of FF (1993), in which the explanatory returns are $R_M - R_F$, *SMB*, and *HML*. Also shown are results for three four-factor models that combine $R_M - R_F$ and *SMB* with pairs of *HML*, *RMW*, and *CMA*, and the five-factor model, which is the full version of (3).

We estimate all regression slopes as constants, so time variation in the slopes is a potential problem. Like most of the asset-pricing literature, our models and tests also assume there are no market frictions, for example, transactions costs and taxes.

The *GRS* results are easily summarized. The test rejects the models we consider and, except for one anomaly, the *GRS* p-values for the rejections round to zero to at least three decimal places. Thus, all the models are incomplete descriptions of expected returns. Asset pricing models, however, are simplified propositions about expected returns that are rejected in tests with power. We are less interested in whether competing models are rejected than in their relative performance, which we judge using *GRS* and other statistics. We want to identify the model that is the best (but imperfect) story for average returns.

Table 2
Summary statistics for tests of three-, four-, and five-factor models, July 1963–December 2014 (618 months)

Model factors	*GRS*	$p(GRS)$	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{Aa_i^2}{A\bar{r}_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	$A(R^2)$
25 *Size-β* portfolios							
Mkt	2.26	0.000	0.232	0.98	0.99	0.18	0.75
Mkt SMB HML	1.61	0.032	0.106	0.45	0.19	0.38	0.89
Mkt SMB HML RMW	1.73	0.016	0.083	0.35	0.13	0.56	0.89
Mkt SMB HML CMA	1.51	0.055	0.095	0.40	0.17	0.44	0.89
Mkt SMB RMW CMA	1.68	0.021	0.069	0.29	0.10	0.81	0.89
Mkt SMB HML RMW CMA	1.68	0.021	0.072	0.31	0.10	0.76	0.89
35 *Size-NI* portfolios							
Mkt SMB HML	4.30	0.000	0.136	0.51	0.39	0.18	0.87
Mkt SMB HML RMW	3.74	0.000	0.111	0.41	0.24	0.29	0.88
Mkt SMB HML CMA	4.03	0.000	0.130	0.49	0.35	0.21	0.87
Mkt SMB RMW CMA	3.15	0.000	0.100	0.37	0.19	0.39	0.88
Mkt SMB HML RMW CMA	3.16	0.000	0.098	0.37	0.18	0.38	0.88
25 *Size-Var* portfolios							
Mkt SMB HML	6.02	0.000	0.217	0.72	0.84	0.06	0.87
Mkt SMB HML RMW	5.28	0.000	0.147	0.49	0.49	0.10	0.89
Mkt SMB HML CMA	6.08	0.000	0.213	0.71	0.81	0.07	0.87
Mkt SMB RMW CMA	5.05	0.000	0.130	0.43	0.36	0.14	0.88
Mkt SMB HML RMW CMA	5.03	0.000	0.131	0.44	0.36	0.14	0.89
25 *Size-RVar* portfolios							
Mkt SMB HML	7.15	0.000	0.222	0.71	0.80	0.06	0.88
Mkt SMB HML RMW	6.33	0.000	0.144	0.46	0.44	0.09	0.90
Mkt SMB HML CMA	7.22	0.000	0.222	0.70	0.77	0.06	0.88
Mkt SMB RMW CMA	5.95	0.000	0.118	0.37	0.32	0.14	0.89
Mkt SMB HML RMW CMA	5.94	0.000	0.120	0.38	0.32	0.13	0.90
25 *Size-AC* portfolios							
Mkt SMB HML	3.68	0.000	0.113	0.48	0.26	0.27	0.91
Mkt SMB HML RMW	4.57	0.000	0.143	0.61	0.40	0.17	0.91
Mkt SMB HML CMA	3.29	0.000	0.096	0.41	0.21	0.34	0.91
Mkt SMB RMW CMA	3.70	0.000	0.127	0.54	0.32	0.22	0.91
Mkt SMB HML RMW CMA	3.77	0.000	0.126	0.54	0.31	0.23	0.91
25 *Size-Prior 2–12* portfolios							
Mkt SMB HML	5.06	0.000	0.319	0.97	1.11	0.06	0.85
Mkt SMB HML RMW	4.69	0.000	0.305	0.93	0.96	0.07	0.85
Mkt SMB HML CMA	4.74	0.000	0.297	0.90	0.97	0.07	0.85
Mkt SMB RMW CMA	4.25	0.000	0.280	0.85	0.78	0.09	0.84
Mkt SMB HML RMW CMA	4.24	0.000	0.272	0.83	0.74	0.09	0.85
Mkt SMB HML MOM	3.87	0.000	0.133	0.40	0.17	0.20	0.91
Mkt SMB HML RMW MOM	3.72	0.000	0.118	0.36	0.14	0.23	0.92
Mkt SMB HML CMA MOM	3.73	0.000	0.135	0.41	0.17	0.20	0.91
Mkt SMB RMW CMA MOM	3.56	0.000	0.119	0.36	0.15	0.24	0.92
Mkt SMB HML RMW CMA MOM	3.55	0.000	0.117	0.36	0.14	0.23	0.92

This table tests how well three-, four-, and five-factor models explain monthly excess returns on the 25 *Size-β* (beta) portfolios, the 35 *Size-NI* (net share issues) portfolios, the 25 *Size-Var* (total variance) portfolios, the 25 *Size-RVar* (residual variance) portfolios, the 25 *Size-AC* (accruals) portfolios, and the 25 *Size-Prior 2–12* (momentum) portfolios. The table shows (1) the factors in each regression model, (2) the *GRS* statistic testing whether the expected values of all 25 or 35 intercept estimates are zero, (3) $p(GRS)$, the p-value for the *GRS* statistic, (4) the average absolute value of the intercepts, $A|a_i|$, (5) $A|a_i|/A|\bar{r}_i|$, the average absolute value of the intercepts over the average absolute value of $\bar{r}_i$, which is the average excess return on portfolio i minus the average VW market portfolio excess return, (6) $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$, (7) $As^2(a_i)/Aa_i^2$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over $A\bar{r}_i^2$, and (8) AR^2, the average value of the regression R^2 corrected for degrees of freedom. Each sort uses all stocks with data for the two sort variables at the portfolio formation date. *Mkt* is the excess return on the VW market portfolio, $R_M - R_F$. The other factors are defined in Table 1.

Using A to indicate an average value, the other statistics we use to evaluate competing models include the average absolute intercept, $A|a_i|$, and two ratios that measure the dispersion of the intercepts (unexplained LHS average excess returns) produced by a model relative to the dispersion of LHS average excess returns. We require baselines or reference points to measure dispersion. Since the asset-pricing hypothesis is that the true intercepts are zero, the appropriate reference point for the intercepts is zero. What is the best reference point for the dispersion of the LHS average excess returns? Our current answer is different from that in FF (2015).

In FF (2015), the dispersion of the LHS average excess returns is measured relative to the simple average of all LHS average excess returns. From an asset pricing perspective, however, the average VW market excess return is a better reference point for three reasons: (1) We take Merton's (1973) ICAPM to be a central motivation for multifactor models. The VW market portfolio is the centerpiece of the ICAPM: in the language of Fama (1996), the VW market portfolio is multifactor efficient in all versions of the ICAPM. (2) More simply, the VW market portfolio is an attractive reference point because it is the aggregate of the portfolios chosen by investors. (3) Any model that includes $R_M - R_F$ as a factor perfectly (and trivially) explains the VW market portfolio excess return. In contrast, the EW average of the LHS portfolio excess returns has no special role in asset pricing. For example, as a LHS portfolio, its excess return almost surely produces a nonzero intercept in an asset pricing regression, and the intercept is different for different asset pricing models and different sets of LHS portfolios.

Define $\bar{r}_i$ as the difference between the time-series average excess return on LHS portfolio i and the average excess return on the VW market. The first measure of the relative dispersion of the intercepts is $A|a_i|/A|\bar{r}_i|$, the average absolute intercept divided by the average absolute value of $\bar{r}_i$. The second is $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over $A\bar{r}_i^2$, the average squared value of $\bar{r}_i$.

In the end, the denominators of $A|a_i|/A|\bar{r}_i|$ and $Aa_i^2/A\bar{r}_i^2$ are just scaling variables: for a given set of LHS portfolios, the denominators are the same for all asset pricing models. They just give perspective on the dispersion of the intercepts in the numerators of the ratios. Switching the reference point from the average VW market return used here to the EW average of the LHS average returns used in FF (2015) produces the same ordering of intercept dispersion for different models.

Finally, in Fama and French (2015), we show results for a variant of $Aa_i^2/A\bar{r}_i^2$ that adjusts numerator and denominator for measurement error. In those tests, adjusted ratios tend to be a bit smaller than unadjusted ratios. In ongoing tests on international data, the double adjustment sometimes produces extreme ratios, positive and negative. The sample period in the international tests is much shorter, and we suspect the problem is measurement error in estimates of measurement error, which is especially troublesome in the denominator of the adjusted ratio since it can lead to explosive ratios of either sign.

We do not show double-adjusted ratios here. Instead we show estimates of the proportion of the dispersion of the intercept estimates attributable to sampling error. Thus, the intercept estimate a_i is the true intercept, α_i, plus an estimation error, ε_i,

$$a_i = \alpha_i + \varepsilon_i \tag{4}$$

Since α_{i} is a constant, the expected value of a_i^2 is

$$E(a_i^2) = \alpha_i^2 + \mathrm{E}(\varepsilon_i^2). \tag{5}$$

Averaging over the LHS assets, we have

$$A\mathrm{E}(a_i^2) = A\alpha_i^2 + A\mathrm{E}(\varepsilon_i^2). \tag{6}$$

The expected value of ε_i is zero, so $\mathrm{E}(\varepsilon_i^2)$ is the variance of a_i due to estimation error, which we estimate with the squared sample standard error of a_i, $s^2(a_i)$. The sample estimate of $A\mathrm{E}(a_i^2)$ is Aa_i^2. The ratio $As^2(a_i)/Aa_i^2$ is then our estimate of the proportion of the dispersion (second moment) of the intercept estimates due to estimation error.

Note that low values of $A|a_i|/A|\bar{r}_i|$ and $Aa_i^2/A\bar{r}_i^2$ are good news for an asset pricing model: they say that intercept dispersion (the dispersion of LHS average returns left unexplained by the model) is low relative to the dispersion of the LHS average returns. In contrast, high values of $As^2(a_i)/Aa_i^2$ are good news: they say that much of the dispersion of the intercept estimates is due to sampling error rather than to dispersion of the true intercepts.

2.2.1 Market $\boldsymbol{\beta}$. The *GRS* rejections of our asset pricing models are weakest for the *Size*-β portfolios. Since the β anomaly is a purported violation of the CAPM, we include the CAPM among the models tested. The CAPM is rejected with a *GRS* p-value that is zero to three decimal places. The ratios $A|a_i|/A|\bar{r}_i|$ and $Aa_i^2/A\bar{r}_i^2$ are 0.98 and 0.99, so the dispersion of CAPM intercepts almost matches the dispersion of average LHS portfolio returns. And $As^2(a_i)/Aa_i^2$ estimates that only about 18% of the dispersion of the intercepts is due to sampling error. Similar negative results are observed in tests of the CAPM on portfolios from the other anomaly sorts, and to save space we show no CAPM results for other anomalies. We see later that the CAPM is rejected in the β sorts because the model predicts that the slope in the relation between average excess return and β is the average excess market return, but the actual relation is essentially flat.

In earlier drafts of this paper, the FF (1993) three-factor model easily passes the *GRS* test on the 25 *Size*-β portfolios (GRS = 1.07; p-value = 0.371) and, like Novy-Marx (2014), we conclude that returns on β-sorted portfolios do not identify problems for the three-factor model. Adding 2014 to the 1963–2013 sample of earlier drafts changes that inference; the three-factor *GRS* statistic increases to 1.61 (Table 2) and the *GRS* p-value shrinks to 0.032. The *GRS* test on the *Size-*β portfolios also rejects our other models at conventional levels, but the rejections are weaker than for other anomalies.

Adding 2014 to the 1963–2013 sample has little effect on other measures of performance. Judged on anything but the *GRS* test, the best performers (in a dead heat) in the tests on the *Size-*β portfolios are the five-factor model and the four-factor model that drops *HML*. The average absolute intercepts from the two models are 0.072% and 0.069%, versus 0.106% for the FF three-factor model. The $A|a_i|/A|\bar{r}_i|$ ratios are 0.31 and 0.29 for the five- and four-factor models, so measured in units of return, the dispersion of unexplained average returns is about 30% as large as the dispersion of average returns. In units of return squared ($Aa_i^2/A\bar{r}_i^2$) the dispersion of unexplained average returns is about 10% as large as the dispersion of average returns. The ratios $As^2(a_i)/Aa_i^2$ are 0.76 and 0.81, which suggests that more than three-quarters of the second moments of the intercept estimates for the two models is due to sampling error and only about one-fourth is due to dispersion in the true intercepts. All this is consistent with relatively weak rejections on the *GRS* test.

2.2.2 Net share issues. All the asset pricing metrics in Table 2 agree that the five-factor model and the four-factor model that drops *HML* provide the best descriptions of average *Size-NI* portfolio returns. Thus, adding profitability and investment factors enhances estimates of expected returns for portfolios formed on *Size* and net issues. The average absolute intercepts produced by the two models are 0.098% and 0.100% per month. The ratio $A|a_i|/A|\bar{r}_i|$ is 0.37 for both models, so in units of return, the dispersion of the estimated intercepts is 37% as large as the dispersion of average *Size-NI* portfolio returns. In units of return squared, $Aa_i^2/A\bar{r}_i^2$ is 0.18 and 0.19 for the two models, and the ratios $As^2(a_i)/Aa_i^2$ estimate that about 40% of Aa_i^2 is due to sampling error in the intercept estimates. These results suggest that, despite rejection on the *GRS* test, the two models perform well in the tests on the *Size-NI* portfolios.

When we later examine the intercepts produced by the five-factor model (Section 4), we see that its problems are largely in portfolios of small stocks in the highest *NI* quintile, which have negative exposures to *RMW* and *CMA* like those of small firms that invest a lot despite low profitability. This is the lethal combination that plagues the five-factor model in FF (2015).

2.2.3 Volatility. We also see later (Section V) that the same lethal combination plays a big role in the rejection of the five-factor model in tests on the *Size-Var* and *Size-RVar* (total and residual variance) portfolios. Again, the *GRS* test and other summary statistics in Table 2 imply that the five-factor model and the four-factor model that drops *HML* provide the best descriptions of average *Size-Var* and *Size-RVar* portfolio returns. On all metrics, however, the volatility portfolios pose stronger challenges to the two models than the *Size-NI* portfolios. The average absolute intercepts and seven of the eight ratios comparing the dispersion of the intercepts to the dispersion of the LHS average returns are larger for the volatility portfolios than for the net issues portfolios, and less of the dispersion of the intercepts for the volatility portfolios can be attributed to sampling error.

2.2.4 Accruals. In the tests on the six different sets of LHS portfolios in FF (2015), and in the tests on other LHS portfolios examined here, the five-factor model typically performs better than the FF three-factor model. This is not true for the 25 *Size-AC* portfolios. The culprit is the profitability factor *RMW*. The three models that include *RMW* produce larger *GRS* statistics and are weaker on other metrics than the two models that do not include *RMW*. In contrast, models that include the investment factor *CMA* perform relatively well, except when they include *RMW*. For the *Size-AC* portfolios, the four-factor model that drops *RMW* delivers the best performance on all metrics. The performance of this model in the tests for accruals is similar to that of the best models in the tests for net issues and volatility. We see later that the poor performance of the five-factor model in the accruals tests owes a lot to microcaps.

2.2.5 Momentum. Models that do not include *MOM* fail badly as descriptions of average returns on the 25 *Size-Prior 2–12* portfolios. For example, the estimates of intercept dispersion relative to the dispersion of average excess returns, $\mathrm{A}|a_i|/\mathrm{A}|\bar{r}_i|$, range from 0.97 for the FF three-factor model to 0.83 for the five-factor model. All are far above the values of this ratio in the sorts for other anomaly variables.

When we include the momentum factor, *MOM*, all models are still rejected on the *GRS* test, but explanatory power improves. The best models on the *GRS* test are the six-factor model that adds *MOM* to the five-factor model and the five-factor model that drops *HML*. The performance of these two models is essentially identical on all metrics. The average absolute intercept is 0.117% per month with *HML* and 0.119% without. In units of return, the dispersion of the intercepts relative to the dispersion of *Size-Prior 2–12* average returns, $A|a_i|/A|\bar{r}_i|$, is 0.36 for both models, and in units of return squared, $Aa_i^2/A\bar{r}_i^2$, it is 0.14 for one and 0.15 for the other. These are strong numbers, but they are achieved by adding a momentum factor constructed with a coarser version of the sorts for the 25 *Size-Prior 2–12* portfolios, a luxury not allowed in the tests for other anomalies. Moreover, when *MOM* is among the factors, other models, including Carhart's (1997) model, which adds *MOM* to the FF three-factor model, perform almost as well as the six-factor model.

2.3 The five-factor model versus the FF three-factor model: A simple test

For almost all sorts examined here and in FF (2015), the five-factor model performs better than the FF three-factor model. Are the differences statistically reliable? If we assume expected returns are governed by a linear factor model and some stocks have nonzero exposures to *RMW* and *CMA*, we can use the

GRS test to show formally that the profitability and investment factors add information about expected returns to the three-factor model.

The *GRS* test on the intercepts from FF three-factor regressions to explain *RMW* and *CMA* tells us whether adding *RMW* and *CMA* improves the mean-variance efficient set produced by combining the risk-free rate, $R_M - R_F$, *SMB*, and *HML*. The regression estimates (t-statistics in parentheses) are

$$\underset{(4.08)}{RMW_t = 0.33} - \underset{(-2.64)}{0.05}(R_{Mt} - R_{Ft}) \underset{(-8.43)}{-0.23}SMB_t \underset{(0.33)}{+0.01}HML_t + e_t, \quad R^2 = 0.14 \tag{7}$$

$$\underset{(3.62)}{CMA_t = 0.20} - \underset{(-6.70)}{0.09}(R_{Mt} - R_{Ft}) \underset{(0.52)}{+0.01}SMB_t \underset{(22.12)}{+0.45}HML_t + e_t. \quad R^2 = 0.53 \tag{8}$$

The intercepts in these regressions—0.33 (t=4.08) for *RMW* and 0.20 (t= 3.62) for *CMA*—are large, even by the standards suggested by Harvey, Liu, and Zhu (2015). The *GRS* statistic (21.09; p-value zero to at least five decimal places) confirms that *RMW* and *CMA* jointly add to the information about expected returns in $R_M - R_F$, *SMB*, and *HML*.

Factor redundancy tests like regressions (7) and (8) are definitive. If a factor's average return is captured by its exposures to the other factors in a model, that factor adds nothing to the model's explanation of average returns, and no set of LHS portfolios can overturn this conclusion (Fama 1998; Barillas and Shanken 2015). Conversely, if a factor's average return is not captured by its exposures to the other factors in a model, that factor has a role in explaining average returns in the model. This does not mean this factor is important for explaining average returns for all sets of LHS portfolios. For example, exposures to an important factor may be negligible in a particular LHS sort, in which case that factor may not help explain average returns in that sort.

2.4 An equivalent five-factor model

In the summary tests of Table 2, the five-factor model and the four-factor model that drops *HML* perform almost identically on all metrics. This result is in line with the evidence in FF (2015) that *HML* is redundant for describing U.S. average returns, at least for 1963–2014. Specifically, the large average *HML* return (0.36% per month; t=3.15 in Table 1) is absorbed by the exposures of *HML* to other factors, especially the profitability and investment factors, *RMW* and *CMA*. For the time period and version of the factors used here, the regression to explain *HML* (t-statistics in parentheses) is

$$HML_t = \underset{(-0.47)}{-0.04} + \underset{(0.31)}{0.01}(R_{Mt} - R_{Ft}) \underset{(0.88)}{+0.03}SMB_t \underset{(5.37)}{+0.22}RMW_t \underset{(23.24)}{+1.04}CMA_t + e_t. \quad R^2 = 0.51 \tag{9}$$

In contrast, $R_M - R_F$, *SMB*, *RMW*, and *CMA* have substantial marginal information about average returns. Skipping the details, the intercepts in the regressions to explain each of these factor returns with the other four are 0.81 (t=5.00) for $R_M - R_F$, 0.36 (t=3.09) for *SMB*, 0.42 (t=5.33) for *RMW*, and 0.27 (t=4.98) for *CMA*.

The trivial intercept in (9) implies that nothing is lost in the explanation of average returns if we drop *HML* from the five-factor model. Exposures to *HML* are, however, important for understanding the portfolio types that cause asset-pricing problems. We want to keep *HML*, but we also want other factors to have slopes that reflect the fact that, at least in U.S. data for 1963–2014, the four-factor model that drops *HML* captures average stock returns as well as the five-factor model. A twist on the five-factor model meets these goals. Define *HMLO* (orthogonal *HML*) as the sum of the intercept and residual from (9). Substituting *HMLO* for *HML* in (3) produces an alternative five-factor regression:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HMLO_t + r_i RMW_t + c_i CMA_t + e_{it}. \tag{10}$$

The intercept and residual in (10) are the same as in the five-factor regression (3), so the two regressions are equivalent for judging model performance. For example, the *GRS* test and other results for the five-factor model in Table 2 do not change if we use (10) rather than (3). The *HMLO* slope in (10) is also the same as the *HML* slope in (3), so (10) produces the same estimate of the value tilt of the LHS portfolio. But the estimated mean of *HMLO* (the intercept in the *HML* regression (9)) is near zero (−0.04; t = −0.47), so its slope adds little to the estimate of the expected LHS return from (10). The slopes on other factors in (10) are the same as in the four-factor model that drops *HML*, so other factors have slopes that reflect the fact that they capture the information in *HML* about average returns.

For more insight into model performance, we next examine the asset pricing regressions for the anomaly portfolios in more detail. For each anomaly, we first document the patterns in average returns we seek to explain. We then examine intercepts and pertinent slopes from asset pricing regressions.

3. Market β

Panel A of Table 3 shows average monthly excess returns (returns in excess of the one-month U.S. Treasury-bill rate) on the 25 VW *Size*-β portfolios. The results confirm previous evidence that there is no clear relation between β and average return. For example, the portfolio in the highest β quintile of a *Size* quintile tends to have a slightly higher average return than the portfolio in the lowest β quintile. But the portfolio in the highest β quintile also has a lower average return than portfolios in the middle three β quintiles of a S*ize* quintile, which have similar average returns. There is, however, a size effect in every β quintile: given β, average return is highest for microcaps and lowest for megacaps.

Table 3
Average excess returns and characteristics of stocks in the 25 *Size*-β portfolios, July 1963–December 2014 (618 months)

	Low β	2	3	4	High β	Low β	2	3	4	High β
Panel A: Average excess returns and SD										
			Mean					**SD**		
Small	0.73	0.90	0.92	0.98	0.79	4.41	5.10	5.95	6.51	8.23
2	0.72	0.86	0.95	0.89	0.72	4.30	4.77	5.48	6.21	7.92
3	0.69	0.86	0.84	0.80	0.76	3.85	4.65	5.22	5.96	7.69
4	0.67	0.76	0.74	0.58	0.75	3.89	4.63	5.17	5.84	7.57
Big	0.49	0.52	0.49	0.50	0.41	3.63	4.25	4.88	5.69	7.15
Panel B: Average *B/M*, *OP*, *Inv*, and prior β characteristics										
			B/M					***OP***		
Small	0.99	1.04	1.04	1.04	0.94	0.15	0.14	0.16	0.37	0.02
2	0.88	0.87	0.84	0.82	0.76	0.26	0.25	0.47	0.27	0.23
3	0.85	0.79	0.75	0.72	0.68	0.26	0.30	0.31	0.31	0.25
4	0.80	0.72	0.69	0.69	0.66	0.29	0.31	0.30	0.29	0.31
Big	0.65	0.51	0.57	0.56	0.63	0.35	0.40	0.34	0.37	0.36
			Inv					**Prior β**		
Small	0.10	0.09	0.10	0.11	0.14	0.31	0.84	1.15	1.51	2.49
2	0.12	0.12	0.13	0.15	0.20	0.38	0.84	1.16	1.51	2.40
3	0.12	0.12	0.13	0.15	0.23	0.38	0.84	1.15	1.50	2.32
4	0.10	0.12	0.13	0.14	0.21	0.40	0.84	1.15	1.50	2.24
Big	0.10	0.12	0.14	0.15	0.21	0.42	0.83	1.15	1.48	2.10

At the end of June from 1963 to 2014, we form value-weight portfolios using independent sorts of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and quintiles of β (market beta), with NYSE breakpoints for both variables. The intersections of the two sorts produce 25 *Size*-β portfolios. For portfolios formed in June of year t, *Size* is market capitalization at the end of June and β is market beta estimated by regressing a stock's monthly return on the current market return, estimated using the 60 (with a minimum 24) months of returns preceding June of t. Panel A shows means and standard deviations of monthly excess returns on the 25 portfolios. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation. For portfolios formed at the end of June of year t, B/M, *OP*, and *Inv* in panel B are value-weight averages (market-cap weights) of the variables for the firms in a portfolio. Specifically, B/M is the value-weight average ratio of book equity at the fiscal year-end in calendar year $t-1$ and market cap at the end of December of $t-1$; *OP* is the value-weight average ratio of operating profits and book equity for the fiscal year ending in $t-1$; and *Inv* is the value-weight average rate of growth of total assets for the fiscal year ending in $t-1$. Panel B also shows the β estimates used to form portfolios in June of each year t, first averaged across the stocks in a portfolio and then averaged across years.

Table 4 shows intercepts and slopes for the 25 *Size*-β portfolios produced by the CAPM and the five-factor model (10). The CAPM regressions (panel A) show that sorts on prior β estimates produce large spreads in β estimated using post-sort returns. Since average returns do not increase systematically with β, it is not surprising that the CAPM intercepts are strongly positive for low β portfolios. Megacaps aside, most of the CAPM intercepts in the four lower quintiles of β are reliably positive and those in the highest β quintile are near zero. Megacaps produce the smallest intercept in each β quintile, and the only reliably negative CAPM intercept, -0.31% ($t=-2.29$), is for megacaps in the highest β quintile.

The five-factor model cures the systematic problems of the CAPM in the tests on the 25 *Size*-β portfolios. The strong positive CAPM intercepts in the four lower *Size* and β quintiles disappear in the five-factor results (panel B of Table 4). The negative CAPM intercept for the megacap portfolio in the highest

Table 4
Regressions for the 25 *Size*-β portfolios, July 1963–December 2014 (618 months)

	Low β	2	3	4	High β	Low β	2	3	4	High β
Panel A: CAPM: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + e_{it}$										
			a					$t(a)$		
Small	0.34	0.44	0.39	0.36	0.04	2.96	3.45	2.60	2.45	0.22
2	0.33	0.40	0.41	0.28	−0.07	3.11	3.82	3.68	2.23	−0.42
3	0.32	0.39	0.31	0.19	−0.02	3.70	4.66	3.33	1.80	−0.13
4	0.30	0.28	0.19	−0.04	−0.03	3.39	3.74	2.50	−0.41	−0.19
Big	0.15	0.07	−0.03	−0.11	−0.31	1.77	1.03	−0.50	−1.42	−2.29
			b					$t(b)$		
Small	0.75	0.90	1.04	1.21	1.48	29.13	31.51	31.18	36.27	33.20
2	0.77	0.90	1.07	1.21	1.55	32.61	38.58	43.25	43.53	44.32
3	0.71	0.93	1.05	1.20	1.54	36.51	49.90	50.34	50.08	49.26
4	0.72	0.95	1.08	1.22	1.53	36.46	57.55	63.68	61.60	51.71
Big	0.67	0.89	1.04	1.20	1.42	35.21	61.82	72.92	68.40	46.97
Panel B: Five-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHMLO_t + r_iRMW_t + c_iCMA_t + e_{it}$										
			a					$t(a)$		
Small	0.07	0.06	0.04	0.08	−0.03	1.07	0.98	0.63	1.27	−0.31
2	0.04	0.02	0.05	−0.06	−0.15	0.57	0.31	0.90	−0.99	−2.03
3	0.09	0.10	0.00	−0.11	−0.02	1.16	1.70	0.07	−1.55	−0.25
4	0.07	−0.02	−0.09	−0.24	0.08	0.84	−0.24	−1.37	−2.98	0.72
Big	−0.07	−0.07	−0.09	−0.09	−0.06	−1.08	−1.38	−1.52	−1.14	−0.45
			b					$t(b)$		
Small	0.66	0.81	0.89	1.02	1.14	40.86	51.46	56.20	69.41	56.64
2	0.73	0.86	0.99	1.10	1.29	40.23	63.70	69.69	75.42	70.14
3	0.75	0.92	1.01	1.13	1.31	40.85	63.62	66.61	65.78	59.72
4	0.81	1.01	1.10	1.19	1.35	40.68	65.09	69.59	62.12	50.15
Big	0.83	0.98	1.08	1.18	1.30	50.97	75.20	74.85	59.75	39.24
			s					$t(s)$		
Small	0.77	0.93	1.12	1.14	1.40	34.05	42.18	50.18	54.95	49.30
2	0.60	0.74	0.81	0.95	1.11	23.81	39.13	40.50	46.68	42.85
3	0.27	0.50	0.61	0.72	0.87	10.56	24.94	28.51	29.87	28.29
4	0.04	0.24	0.32	0.38	0.56	1.31	10.88	14.37	14.13	14.88
Big	−0.28	−0.17	−0.11	0.06	0.06	−12.27	−9.32	−5.35	2.06	1.25
			h					$t(h)$		
Small	0.30	0.26	0.22	0.22	−0.09	9.35	8.32	7.18	7.72	−2.38
2	0.33	0.30	0.24	0.17	−0.06	9.37	11.42	8.76	5.99	−1.67
3	0.38	0.19	0.21	0.21	−0.12	10.71	6.80	6.90	6.30	−2.71
4	0.33	0.21	0.13	0.20	−0.09	8.38	6.76	4.15	5.38	−1.63
Big	0.10	0.04	0.06	0.09	−0.10	3.03	1.71	2.07	2.41	−1.57
			r					$t(r)$		
Small	0.05	0.18	0.03	0.01	−0.47	1.49	5.73	0.83	0.44	−11.67
2	0.13	0.32	0.30	0.21	−0.22	3.59	11.71	10.33	7.12	−5.80
3	0.13	0.25	0.29	0.28	−0.17	3.60	8.51	9.57	8.21	−3.95
4	0.17	0.33	0.35	0.23	−0.33	4.26	10.59	10.89	6.05	−6.21
Big	0.31	0.31	0.26	0.11	−0.26	9.41	11.81	8.98	2.77	−3.96
			c					$t(c)$		
Small	0.33	0.42	0.38	0.23	−0.08	9.32	12.39	11.05	7.28	−1.90
2	0.38	0.42	0.33	0.29	−0.10	9.60	14.25	10.65	9.12	−2.52
3	0.40	0.33	0.30	0.25	−0.24	10.20	10.66	9.08	6.66	−4.98
4	0.45	0.39	0.30	0.17	−0.26	10.52	11.70	8.84	4.16	−4.40
Big	0.44	0.19	0.01	−0.14	−0.44	12.60	6.77	0.17	−3.33	−6.16

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size*-β portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the orthogonal value factor, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*. The table shows CAPM (panel A) and five-factor (panel B) intercepts and regressions slopes.

β quintile also becomes inconsequential (−0.06; $t=-0.45$). The only blemish on the five-factor model is the intercept for the intersection of the fourth *Size* and fourth β quintiles, −0.24 ($t=-2.98$).

The improvements in the description of average returns on the *Size*-β portfolios provided by the five-factor model trace to patterns in the five-factor regression slopes that absorb the patterns in average returns. Panel B of Table 4 shows that portfolios in the four lower β quintiles tilt toward value (positive *HMLO* slopes) and portfolios in the highest β quintile have a growth tilt (negative *HMLO* slopes), but since the average *HMLO* return is close to zero these tilts have little impact on five-factor intercepts. The heavy lifting is done by the *RMW* and *CMA* slopes. Microcaps aside, the five-factor *RMW* slopes are strongly positive in the four lower β quintiles. The *RMW* slopes become strongly negative in the highest β quintile, especially for microcaps. In all *Size* quintiles the *CMA* slopes are positive in the lower β quintiles but turn strongly negative in the highest β quintile. In short, the returns on low β stocks behave like those of profitable firms that invest conservatively, whereas the returns on high β stocks track those of less profitable firms that invest a lot.

The *RMW* and *CMA* slopes of the five-factor model increase the predicted returns on the low β portfolios of the *Size*-β sorts and reduce the predicted returns on the high β portfolios. But the low and high β portfolios have similar average returns, so five-factor intercepts close to zero imply that the slopes for the market and/or *SMB* lean against the *RMW* and *CMA* slopes. Panel B of Table 4 shows that the spreads in the five-factor market slopes for the lowest and highest β portfolios of given *Size* quintiles are large, from 0.47 to 0.56. The average market premium for 1963–2014 is 0.51% per month, so the spreads in average returns predicted by the spreads in R_M–R_F slopes are also large, 0.24% to 0.29% per month. Surprisingly, in every *Size* quintile *SMB* slopes increase monotonically from low β to high β quintiles, and the spreads are again large, from 0.34 for megacaps to 0.63 for microcaps. The average *SMB* return for 1963–2014 is 0.27% per month, so the spreads in average returns predicted by the spreads in the *SMB* slopes range from 0.09% (megacaps) to 0.17% per month (microcaps). In short, the five-factor market and *SMB* slopes, and the associated premiums, offset *RMW* and *CMA* slopes and premiums to capture average returns that show little tendency to increase with CAPM β.

The results suggest that average returns vary with multivariate β (market slope b) in the way predicted by the five-factor model, even though there is little relation between CAPM β and average returns. For perspective, we estimate a four-factor model that drops the market factor from the five-factor model (3) and uses returns on the 25 *Size*-β portfolios measured in excess of the market return as LHS variables. In other words, we set all multivariate βs equal to 1.0. Skipping the details, the intercepts from this model are negative for the five portfolios in the lowest β quintile, and three of five are more than two standard errors below zero. The intercepts for the five portfolios in the highest β quintile are positive, and three of five are more than two standard errors above zero. Thus, setting multivariate βs equal to 1.0 produces forecasts of average returns that are too high for low β portfolios and too low for high β portfolios. We conclude that there is a positive relation between multivariate β and average returns, and the average premium for multivariate β conforms well to the five-factor model.

Since lots of what is common in the story for average returns for different sets of LHS anomaly portfolios centers on the slopes for *RMW* and *CMA*, an interesting question is whether the slopes line up with profitability (*OP*) and investment (*Inv*) characteristics. Like the *CMA* slopes of the *Size*-β portfolios, average investment increases from lower to higher β quintiles (Table 3). But contradicting the *RMW* slopes, profitability (*OP*) is not systematically lower for high β portfolios, except perhaps for microcaps. This is not surprising. Multivariate regression slopes estimate marginal effects, holding constant other explanatory variables, so the slopes need not line up with univariate characteristics.

Since characteristics do not always line up with regression slopes, we are careful when describing the slopes. For example, we say that strong negative *RMW* and *CMA* slopes for the portfolios in the highest β quintile imply that returns on these stocks "behave like" those of unprofitable firms that invest aggressively. Table 3 shows these firms have grown rapidly, but except for microcaps, they have not been a lot less profitable than lower β portfolios of the same *Size* quintile.

4. Net Share Issues

Panel A of Table 5 shows average excess returns for the 35 VW portfolios from independent sorts of stocks into *Size* quintiles and seven net share issues (*NI*) groups. Repurchases (negative *NI*) are associated with higher average returns. In all *Size* groups average returns are similar for the lowest three quintiles of positive *NI*, but average returns are lower in the fourth quintile. The striking result, and the result that will be difficult to explain fully, is the extreme low average returns of the five portfolios in the highest *NI* quintile (largest net issues). Though not our main interest, there is a *Size* effect in every *NI* group: microcaps have higher average returns than megacaps.

The summary tests in Table 2 say that the five-factor model improves the description of average returns on the *Size*-*NI* portfolios provided by the FF three-factor model. Table 6 shows the three-factor and five-factor intercepts and the five-factor *HMLO*, *RMW*, and *CMA* slopes. We do not show $R_M - R_F$ and *SMB* slopes since they are similar for different models and so cannot explain the intercept improvements produced by the five-factor model.

In previous research, repurchases are associated with positive unexplained average returns. The three-factor intercepts for the repurchase portfolios are positive, 0.11% to 0.24% per month, and 1.96 to 3.62 standard errors from zero. The intercepts are smaller in the five-factor model, and the largest, 0.10%

Table 5
Average excess returns and characteristics of stocks in the 35 *Size-NI* portfolios, July 1963–December 2014 (618 months)

NI →	Neg	Zero	Low	2	3	4	High	Neg	Zero	Low	2	3	4	High
Panel A: Average excess returns and SD														
				Mean							**SD**			
Small	1.05	0.78	0.89	0.94	1.02	0.71	0.24	5.67	5.72	6.36	6.42	6.85	7.12	7.73
2	0.92	0.82	0.92	0.90	0.87	0.80	0.32	5.31	5.66	5.84	6.13	6.31	6.52	7.09
3	0.97	0.76	0.84	0.93	0.82	0.72	0.29	5.09	5.46	5.26	5.64	5.84	6.00	6.53
4	0.94	0.57	0.65	0.70	0.81	0.61	0.35	4.91	4.90	5.07	5.32	5.45	5.79	6.17
Big	0.62	0.61	0.49	0.47	0.59	0.43	0.18	4.18	4.78	4.25	4.54	4.85	5.41	5.12
Panel B: Average *B/M*, *OP*, *Inv*, and *NI* characteristics														
				B/M							***OP***			
Small	1.05	1.17	1.03	0.93	0.86	0.75	0.68	0.31	0.21	0.18	0.22	0.23	0.06	−0.10
2	0.86	1.00	0.86	0.79	0.73	0.67	0.63	0.27	0.31	0.28	0.30	0.54	0.23	0.18
3	0.78	0.92	0.84	0.73	0.68	0.65	0.64	0.31	0.30	0.27	0.26	0.30	0.23	0.26
4	0.69	0.90	0.79	0.68	0.67	0.65	0.68	0.34	0.27	0.28	0.28	0.32	0.31	0.24
Big	0.59	0.81	0.59	0.52	0.53	0.61	0.71	0.37	0.32	0.36	0.35	0.46	0.32	0.30
				Inv							***NI***			
Small	0.06	0.06	0.07	0.08	0.10	0.14	0.59	−4.99	0.00	0.14	0.52	1.28	3.85	36.60
2	0.08	0.11	0.10	0.11	0.13	0.18	0.58	−4.58	0.00	0.14	0.52	1.28	3.89	34.34
3	0.08	0.11	0.09	0.11	0.14	0.19	0.53	−3.85	0.00	0.14	0.52	1.29	3.94	28.96
4	0.07	0.07	0.09	0.11	0.13	0.18	0.45	−3.34	0.00	0.14	0.52	1.28	3.96	24.82
Big	0.08	0.10	0.09	0.12	0.13	0.18	0.41	−2.00	0.00	0.13	0.51	1.28	3.84	22.65

At the end of June each year from 1963 to 2014, we form value-weight (VW) portfolios using independent sorts of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and into seven *NI* (net share issues) groups, including stocks with negative *NI* (repurchases), zero *NI*, and quintiles of positive *NI* (net issues), using NYSE breakpoints for both variables. The intersections of the two sorts produce 35 *Size-NI* portfolios. For portfolios formed in June of year t, *Size* is market capitalization at the end of June and *NI* is the change in the natural log of split-adjusted shares outstanding from the fiscal year-end in $t-2$ to the fiscal year-end in $t-1$. Panel A shows means and standard deviations of monthly excess returns on the 35 portfolios. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation, as defined in Table 3. Panel B also shows the time-series average values of *NI* used to form portfolios each year.

per month, is only 1.73 standard errors from zero. The intercept improvements produced by the five-factor model center on the *RMW* and *CMA* slopes. The repurchase portfolios have strong positive exposures to *CMA*, *RMW*, and, megacaps aside, *HMLO*. In other words, their returns covary positively with the returns of value stocks and stocks of profitable, low investment firms. Positive exposures to *RMW* and *CMA* increase five-factor estimates of expected returns and lead to negligible intercepts. In short, the repurchase anomaly disappears in the five-factor model.

There are no serious problems in the three-factor and five-factor intercepts for portfolios with zero *NI* and in the two lowest quintiles of positive *NI*. Three portfolios in the third quintile of *NI* have three-factor and five-factor intercepts near or more than 2.0 standard errors above zero. These unexplained average returns are positive, but the net issues anomaly is about the low average stock returns of firms that issue stock. Chance is a possible explanation.

The net issues anomaly is strong in the three-factor regressions for portfolios in the highest *NI* quintile, with negative intercepts from −0.28% per month ($t=-3.03$) for the portfolio in the fourth *Size* quintile to −0.57% ($t=-6.20$)

Table 6
Regressions for the 35 *Size-NI* portfolios, July 1963 to December 2014 (618 months)

$NI \rightarrow$	Neg	Zero	Low	2	3	4	High	Neg	Zero	Low	2	3	4	High
Panel A: Three-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + e_{it}$														
				a							**t(a)**			
Small	0.21	−0.02	−0.00	0.06	0.18	−0.14	−0.57	3.47	−0.29	−0.02	0.88	2.52	−1.84	−6.20
2	0.11	0.04	0.09	0.07	0.07	0.04	−0.43	1.96	0.36	1.24	1.00	1.04	0.57	−5.70
3	0.24	0.03	0.05	0.17	0.12	0.01	−0.37	3.62	0.23	0.72	2.46	1.63	0.15	−4.57
4	0.23	−0.01	−0.06	0.02	0.20	0.01	−0.28	3.41	−0.04	−0.86	0.33	2.77	0.14	−3.03
Big	0.15	0.15	0.07	0.05	0.14	0.01	−0.36	2.71	1.09	0.99	0.78	1.98	0.17	−4.24
Panel B: Five-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HMLO_t + r_i RMW_t + c_i CMA_t + e_{it}$														
				a							**t(*a*)**			
Small	0.10	−0.05	−0.03	0.02	0.22	−0.04	−0.36	1.73	−0.66	−0.44	0.33	3.11	−0.53	−4.36
2	−0.03	−0.06	0.01	−0.01	0.05	0.12	−0.24	−0.64	−0.53	0.09	−0.19	0.69	1.79	−3.34
3	0.08	−0.01	−0.02	0.12	0.11	0.09	−0.11	1.29	−0.10	−0.29	1.69	1.43	1.25	−1.56
4	0.06	−0.18	−0.19	−0.07	0.23	0.14	0.01	1.00	−1.41	−2.52	−0.90	3.12	1.71	0.13
Big	0.01	0.00	−0.07	0.03	0.16	0.20	−0.18	0.15	0.02	−0.99	0.49	2.22	2.40	−2.23
				h							**t(*h*)**			
Small	0.26	0.23	0.21	0.12	−0.06	−0.03	−0.22	9.18	5.70	5.85	3.55	−1.83	−0.93	−5.45
2	0.25	0.35	0.25	0.07	0.02	−0.03	−0.12	9.88	6.43	7.65	2.16	0.77	−1.01	−3.46
3	0.22	0.30	0.38	0.14	−0.00	0.06	−0.03	7.49	5.07	10.87	4.09	−0.11	1.57	−0.97
4	0.22	0.07	0.23	0.12	−0.03	−0.11	0.06	7.16	1.17	6.48	3.27	−0.78	−3.03	1.55
Big	0.00	0.12	0.00	−0.03	−0.03	−0.12	0.32	0.09	1.75	0.07	−0.85	−0.76	−3.03	8.14
				r							**t(*r*)**			
Small	0.27	0.09	0.07	0.04	−0.22	−0.25	−0.61	9.23	2.31	1.89	1.14	−6.18	−6.91	−15.04
2	0.37	0.38	0.29	0.14	0.06	−0.17	−0.41	14.15	6.87	8.37	4.12	1.72	−5.33	−11.76
3	0.44	0.10	0.29	0.14	0.03	−0.10	−0.45	14.89	1.67	8.24	3.95	0.88	−2.66	−12.57
4	0.39	0.26	0.28	0.21	−0.07	−0.35	−0.57	12.51	4.17	7.77	5.70	−1.93	−9.07	−13.90
Big	0.22	0.21	0.29	0.09	0.01	−0.40	−0.13	8.60	3.11	8.81	2.83	0.28	−9.76	−3.25
				c							**t(*c*)**			
Small	0.44	0.32	0.33	0.29	0.06	−0.11	−0.32	14.03	7.21	8.43	7.74	1.63	−2.84	−7.29
2	0.46	0.36	0.29	0.28	0.05	−0.16	−0.44	16.56	5.96	7.77	7.31	1.27	−4.62	−11.68
3	0.37	0.45	0.41	0.22	−0.01	−0.17	−0.56	11.63	6.91	10.64	5.74	−0.13	−4.21	−14.57
4	0.49	0.54	0.47	0.28	−0.08	−0.20	−0.43	14.33	7.88	11.90	6.91	−2.14	−4.68	−9.79
Big	0.33	0.54	0.19	−0.10	−0.16	−0.43	−0.20	12.12	7.22	5.40	−2.82	−4.17	−9.76	−4.62

The LHS variables are the monthly excess returns on the 35 *Size-NI* (net share issues) portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML*, or its orthogonal counterpart, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*. Panel A shows the regression intercepts from the FF three-factor model and panel B shows regression intercepts and *HMLO*, *RMW*, and *CMA* slopes from the five-factor model (10).

for microcaps. The five-factor intercepts for these portfolios are less extreme due to negative *RMW* and *CMA* slopes that lower five-factor estimates of expected returns. But the net issues anomaly survives in the five-factor model: the intercepts for four of the five portfolios in the highest *NI* quintile are negative and three are more than 2.2 standard errors below zero.

The unexplained average returns associated with large net issues have lots in common with the five-factor asset pricing problems in the sorts on *Size*, B/M, *OP*, and *Inv* in FF (2015). The portfolios in the highest *NI* quintile have negative *RMW* and *CMA* slopes, so their returns behave like those of the stocks of firms with low profitability and high investment. Small stocks with this combination of *RMW* and *CMA* exposures are the major problem for the five-factor model in many LHS sorts in FF (2015). But the highest *NI* megacap portfolio also has a negative five-factor intercept, −0.18% ($t = -2.23$), and high investment despite low profitability is not a problem among large stocks in FF (2015).

The *RMW* and *CMA* slopes for the *Size-NI* portfolios in Table 6 line up with their average profitability and investment characteristics, *OP* and *Inv*, in Table 5. Firms that repurchase are on average more profitable than firms that make large share issues (Table 5), but the decline in *RMW* slopes from repurchasers to extreme issuers is sharper (Table 6). There is stronger correspondence between *CMA* slopes and *Inv*. Firms that repurchase on average have the lowest rates of investment, which is in line with strong positive *CMA* slopes, and large share issues signal high rates of investment matched by strong negative *CMA* slopes.

The jumps in *NI* and *Inv* from the fourth to the fifth quintile of *NI* are impressive. Net issues average less than 4% of stock outstanding in the fourth *NI* quintile, rising to 22.65% (megacaps) to 36.60% (microcaps) in the fifth quintile. Investment is 14% to 19% of assets in the fourth *NI* quintile, rising to 41% (megacaps) to 59% (microcaps) in the fifth. The extreme rates of investment and net issues in the fifth *NI* quintile suggest that lots of these firms do mergers financed with stock, a combination known to be associated with low stock returns (Loughran and Vijh 1997). The overlap among new issues, mergers financed with stock, and the combination of low profitability and high investment that is a five-factor asset-pricing problem here and in FF (2015) is an interesting topic for future research.

5. Volatility

Table 7 shows summary statistics for the 25 VW *Size-RVar* (residual variance) portfolios. Table 8 shows intercepts and slopes for the portfolios from the five-factor regression (10), along with the intercepts from the FF three-factor model. The corresponding results for the 25 *Size-Var* (total variance) portfolios are similar and are in Tables A1 and A2 of the Appendix.

For megacaps there is no relation between average return and *RVar* (Table 7). For microcaps, the portfolios in the two highest *RVar* quintiles have lower average returns and the average excess return of the portfolio in the highest

Table 7
Average excess returns and characteristics of stocks in the 25 *Size-RVar* portfolios, July 1963–December 2014 (618 months)

RVar →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Means and SD of portfolio excess returns										
			Mean					**SD**		
Small	1.01	1.17	1.08	0.81	−0.20	4.18	5.64	6.48	7.52	8.99
2	0.92	1.03	1.07	0.97	0.18	4.12	5.30	5.96	6.82	8.45
3	0.75	0.91	0.93	0.93	0.36	3.79	4.86	5.42	6.20	7.79
4	0.72	0.75	0.75	0.78	0.44	3.84	4.53	5.17	5.72	7.41
Big	0.47	0.52	0.51	0.48	0.45	3.65	4.15	4.55	5.06	6.40
Panel B: Average *B/M*, *OP*, *Inv*, and *RVar* characteristics										
			B/M					***OP***		
Small	1.01	0.96	0.93	0.90	0.94	0.24	0.28	0.33	0.28	−0.09
2	0.89	0.83	0.80	0.77	0.73	0.34	0.31	0.30	0.28	0.18
3	0.85	0.77	0.73	0.71	0.68	0.28	0.30	0.32	0.31	0.27
4	0.82	0.73	0.69	0.68	0.64	0.29	0.32	0.35	0.31	0.30
Big	0.60	0.58	0.58	0.59	0.55	0.35	0.36	0.36	0.35	0.40
			Inv					***RVar***		
Small	0.11	0.14	0.17	0.21	0.22	2.05	4.64	7.54	12.94	41.16
2	0.10	0.13	0.16	0.21	0.30	1.41	2.90	4.40	6.66	16.81
3	0.10	0.12	0.14	0.18	0.29	1.14	2.23	3.35	5.03	12.50
4	0.09	0.11	0.13	0.15	0.25	1.03	1.84	2.68	3.95	9.70
Big	0.10	0.11	0.12	0.15	0.21	0.83	1.40	1.94	2.76	5.85

Panel A shows means and standard deviations of monthly excess returns on value-weight portfolios formed monthly using a first pass sort of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and second-pass sorts into quintiles of *RVar* (residual variance), using NYSE breakpoints for both variables. The *RVar* sorts are conditional on *Size* quintile. The intersections of the two sorts produce 25 *Size-RVar* portfolios. For portfolios formed at the beginning of month t, *Size* is the market cap of a stock at the beginning of t and *RVar* is the variance of its daily residuals from the FF three-factor model estimated using 60 (with a minimum 20) days of lagged returns. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation, as defined in Table 3. Panel B also shows the time-series average values of *RVar* used to form portfolios each month.

RVar quintile is −0.20% per month. For the middle three *Size* quintiles, there is no clear relation between average return and volatility in the lowest four volatility quintiles, but the portfolios in the highest volatility quintile have much lower average returns.

The summary tests in Table 2 say that the five-factor model provides a better description of average returns on the *Size-RVar* portfolios than the FF three-factor model. Table 8 shows a clear pattern in the three-factor intercepts. In every *Size* quintile, the portfolios in the lowest three *RVar* quintiles have positive three-factor intercepts and the portfolios in the highest *RVar* quintile have negative intercepts. The pattern is weak for megacaps but progressively stronger for smaller *Size* quintiles. For microcaps, the three-factor intercept for the portfolio in the lowest *RVar* quintile is 0.34% per month ($t = 5.20$), and the intercept for the portfolio in the highest *RVar* quintile is −1.23% ($t = -7.66$).

Problems remain, but the five-factor model shrinks the troublesome intercepts of the three-factor model. Four of the five three-factor intercepts for portfolios in the lowest *RVar* quintile are more than two standard errors

Table 8
Regressions for the 25 *Size-RVar* portfolios, July 1963 to December 2014 (618 months)

RVar →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor: $R_{it}–R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_i + e_{it}$										
			a					***t*(*a*)**		
Small	0.34	0.31	0.13	−0.19	−1.23	5.20	4.33	1.75	−1.86	−7.66
2	0.26	0.20	0.20	0.04	−0.72	4.16	3.04	2.77	0.56	−7.16
3	0.15	0.17	0.13	0.08	−0.43	2.32	2.53	1.82	1.06	−4.58
4	0.15	0.10	0.03	0.05	−0.29	2.02	1.45	0.45	0.65	−2.87
Big	0.08	0.11	0.01	−0.06	−0.08	1.33	2.03	0.18	−1.02	−0.86
Panel B: Five-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHMLO_t + r_iRMW_t + c_iCMA_t + e_{it}$										
			a					***t*(*a*)**		
Small	0.22	0.17	0.09	−0.08	−0.85	3.41	2.52	1.10	−0.80	−5.63
2	0.12	0.02	0.05	−0.05	−0.46	1.97	0.34	0.69	−0.66	−4.85
3	0.01	0.03	−0.03	−0.03	−0.20	0.21	0.43	−0.47	−0.42	−2.27
4	0.02	−0.06	−0.12	−0.04	−0.04	0.25	−0.86	−1.82	−0.50	−0.42
Big	−0.00	−0.02	−0.10	−0.05	0.15	−0.08	−0.38	−1.91	−0.85	1.68
			b					***t*(*b*)**		
Small	0.72	0.99	1.10	1.15	1.12	46.57	59.76	58.42	46.41	30.55
2	0.78	1.01	1.11	1.24	1.26	53.64	69.67	69.28	68.73	54.47
3	0.78	1.00	1.10	1.20	1.25	50.88	66.65	70.02	66.09	57.16
4	0.82	1.00	1.13	1.20	1.26	46.39	62.05	70.24	64.42	54.72
Big	0.83	0.96	1.05	1.11	1.18	59.85	77.16	80.24	75.87	55.31
			s					***t*(*s*)**		
Small	0.69	0.94	1.04	1.19	1.36	31.92	40.56	39.37	34.50	26.57
2	0.56	0.77	0.85	0.94	1.10	27.27	37.81	37.55	37.12	34.05
3	0.31	0.48	0.58	0.69	0.80	14.52	23.10	26.27	27.13	26.18
4	0.10	0.19	0.25	0.32	0.50	4.10	8.41	11.13	12.21	15.49
Big	−0.27	−0.25	−0.16	−0.13	0.02	−14.06	−14.08	−8.82	−6.20	0.64
			h					***t*(*h*)**		
Small	0.37	0.37	0.37	0.30	0.21	12.41	11.63	10.08	6.19	2.90
2	0.32	0.34	0.30	0.24	−0.08	11.27	11.99	9.56	6.71	−1.78
3	0.34	0.38	0.33	0.23	−0.16	11.19	13.13	10.59	6.52	−3.74
4	0.36	0.31	0.25	0.17	−0.16	10.50	9.76	7.99	4.56	−3.47
Big	0.14	−0.04	0.04	0.04	−0.18	5.09	−1.48	1.68	1.49	−4.30
			r					***t*(*r*)**		
Small	0.39	0.45	0.25	−0.09	−0.81	12.48	13.52	6.69	−1.76	−11.06
2	0.41	0.52	0.48	0.37	−0.56	13.96	17.92	14.90	10.34	−12.02
3	0.38	0.50	0.51	0.41	−0.50	12.29	16.48	16.25	11.31	−11.30
4	0.37	0.47	0.44	0.29	−0.58	10.28	14.56	13.65	7.73	−12.54
Big	0.22	0.25	0.27	0.01	−0.48	7.99	9.87	10.24	0.24	−11.35
			c					***t*(*c*)**		
Small	0.50	0.47	0.34	0.05	−0.24	15.09	13.26	8.36	0.99	−3.05
2	0.50	0.53	0.39	0.17	−0.51	15.79	16.86	11.07	4.38	−10.09
3	0.53	0.44	0.41	0.21	−0.53	15.99	13.63	11.91	5.40	−11.24
4	0.56	0.45	0.38	0.19	−0.50	14.44	12.84	10.82	4.71	−10.01
Big	0.23	0.17	0.17	0.01	−0.57	7.47	6.43	6.08	0.25	−12.26

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-RVar* (residual variance) portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML*, or its orthogonal counterpart, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*. Panel A shows intercepts from the FF three-factor model, and panel B shows five-factor intercepts and slopes from (10).

above zero. In the five-factor model only the intercept for microcaps (0.22% per month; $t = 3.41$) is more than two standard errors from zero. Four of the five three-factor intercepts for portfolios in the highest *RVar* quintile are more than 2.8 standard errors below zero. The five-factor model pulls all four of these intercepts toward zero, but two are still extreme, -0.85% per month, $t = -5.63$, for microcaps and -0.46%, $t = -4.85$, for the second *Size* quintile.

Panel B of Table 8 shows the five-factor regression slopes for the 25 *Size*-*RVar* portfolios. Market slopes increase strongly from the low *RVar* to the high *RVar* portfolios. Within *Size* quintiles, there is a strong positive relation between *SMB* slope and *RVar*; stocks with higher residual return volatility behave like smaller stocks. The positive correlations of *RVar* with market and *SMB* slopes help explain why *Size*-*Var* and *Size*-*RVar* portfolios produce much the same results in our tests. Megacaps aside, *HMLO* slopes are strongly positive in the bottom four quintiles of *RVar*, but microcaps aside, they turn negative in the highest *RVar* quintile. In words, low residual volatility tends to be associated with value and high residual volatility tends to be associated with growth. Keep in mind, however, that the average *HMLO* return is close to zero, so *HMLO* slopes add almost nothing to the description of average returns.

Higher five-factor market and *SMB* slopes for stocks with more volatile residual returns go in the wrong direction to explain the pattern in the *Size*-*RVar* average portfolio returns. The improvements in the description of average return provided by the five-factor model come from its *RMW* (profitability) and *CMA* (investment) slopes. Major lifting is done by the *RMW* slopes, which are strongly positive in the lower three quintiles of *RVar* and strongly negative in the highest *RVar* quintile. The *CMA* slopes have a similar though less pronounced pattern. Thus, the improvements in the explanation of average returns provided by the five-factor model trace to the fact that the returns of low volatility stocks behave like those of firms that are profitable but conservative in terms of investment, whereas the returns of high volatility stocks behave like those of firms that are relatively unprofitable but nevertheless invest aggressively. Novy-Marx (2014) also finds that profitability exposures are important in capturing the low average returns of high volatility stocks. (His model does not include an investment factor.)

Average values of *Inv* that increase with *RVar* (Table 7) confirm the suggestion from the *CMA* slopes that higher residual volatility is associated with more investment. For stocks in the bottom two *Size* quintiles, lower profitability (*OP*) in the highest *RVar* quintile is roughly consistent with lower *RMW* slopes. In the highest three *Size* quintiles, however, average *OP* shows no relation to *RVar* – another example of multivariate regression slopes that do not line up with a univariate characteristic.

The five-factor model does not completely capture average returns on the 25 *Size*-*RVar* portfolios, but its major problems are familiar. Specifically, strong negative exposures to *RMW* and *CMA* capture the low average returns of big stocks that have high *RVar*. But strong negative exposures to *RMW* and *CMA* miss a large part of the lower average returns of high *RVar* small stocks. Small stocks with strong negative exposures to *RMW* and *CMA* are the lethal combination that escapes explanation in many of the sorts here and in FF (2015).

Table 9
Average excess returns and characteristics of stocks in the 25 *Size-AC* portfolios, July 1963–December 2014 (618 months)

AC →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Means and SD of portfolio excess returns										
			Mean					**SD**		
Small	0.89	0.91	0.83	0.96	0.60	7.14	6.46	6.15	6.51	7.22
2	0.84	0.86	0.80	0.78	0.64	6.58	5.77	5.76	6.20	6.85
3	0.85	0.81	0.85	0.79	0.59	6.25	5.42	5.21	5.64	6.62
4	0.75	0.69	0.65	0.74	0.71	5.73	5.09	4.92	5.15	6.26
Big	0.67	0.48	0.52	0.50	0.26	5.20	4.14	4.05	4.57	5.20
Panel B: Average *B/M*, *OP*, *Inv*, and *AC* characteristics										
			B/M					***OP***		
Small	0.91	0.99	0.99	0.92	0.76	−0.06	0.15	0.16	0.18	0.24
2	0.75	0.85	0.79	0.74	0.62	0.19	0.22	0.24	0.25	0.44
3	0.71	0.77	0.78	0.68	0.56	0.31	0.25	0.25	0.27	0.30
4	0.65	0.75	0.74	0.63	0.51	0.33	0.27	0.27	0.30	0.41
Big	0.52	0.59	0.57	0.47	0.41	0.43	0.33	0.33	0.36	0.67
			Inv					***AC***		
Small	0.09	0.15	0.16	0.18	0.31	−57.78	−2.24	1.66	6.03	50.26
2	0.14	0.16	0.18	0.18	0.32	−39.77	−2.18	1.66	5.97	47.45
3	0.15	0.16	0.15	0.18	0.31	−66.10	−2.15	1.67	5.90	38.96
4	0.13	0.12	0.14	0.16	0.25	−22.48	−2.12	1.60	5.84	22.28
Big	0.15	0.12	0.12	0.13	0.20	−16.71	−2.06	1.58	5.64	27.88

Panel A shows means and standard deviations of monthly excess returns on 25 value-weight (VW) portfolios formed yearly at the end of June using independent sorts of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and quintiles of *AC* (accruals), with NYSE breakpoints for both variables. For portfolios formed in June of year t, *Size* is market capitalization at the end of June and *AC* is the change in operating working capital per split-adjusted share from the fiscal year-end in $t-2$ to $t-1$ divided by book equity per split-adjusted share in $t-1$. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation, as defined in Table 3. Panel B also shows the time-series average values of *AC* used to form portfolios each year.

6. Accruals

Panel A of Table 9 shows average excess returns on the 25 VW *Size-AC* portfolios. Average returns are similar for the lower four *AC* quintiles of each of the four smallest *Size* quintiles. For megacaps the lowest *AC* quintile has a higher average return than the portfolios in the middle three quintiles. In four of the five *Size* quintiles, average returns are much lower for the highest *AC* quintile. The megacap portfolio in the highest *AC* quintile has by far the lowest average excess return in the matrix, 0.26% per month. There is also a *Size* effect in every *AC* quintile: The microcap portfolio in each *AC* quintile has a higher average return than the megacap portfolio.

Panel A of Table 10 shows regression intercepts for the 25 *Size-AC* portfolios from the FF three-factor model, the four-factor model that adds *CMA* (the best-performing model in the summary tests on the *Size-AC* portfolios in Table 2), and the five-factor model that also adds *RMW*. The FF three-factor model overestimates average returns on four of five portfolios in the highest *AC* quintile, producing intercepts from 1.90 to 4.03 standard errors below zero. The exception is the portfolio in the second largest *Size* quintile, which, unlike the other portfolios in the highest *AC* quintile, does not have a low average return (Table 9). The FF three-factor model underestimates average returns on 19 of the 20 portfolios in the lower four *AC* quintiles. The most extreme positive intercept, 0.28% per month ($t = 3.32$), is for the megacap portfolio in the lowest *AC* quintile.

The four-factor model that adds *CMA* to the FF three-factor model moves all the troublesome negative intercepts in the highest *AC* quintile toward zero, and only those for the two smallest *Size* quintiles are more than two standard errors from zero. The four-factor intercept for the megacap portfolio in the lowest *AC* quintile is a bit larger than the three-factor intercept (0.30, t = 3.47, versus 0.28,

Table 10
Regressions for the 25 *Size-AC* portfolios, July 1963 to December 2014 (618 months)

	a					***t*(*a*)**				
AC →	**Low**	**2**	**3**	**4**	**High**	**Low**	**2**	**3**	**4**	**High**
Panel A: Regression intercepts										
Three-factor: *Mkt, SMB,* and *HML*										
Small	0.02	0.12	0.06	0.16	−0.28	0.26	1.61	0.95	2.48	−4.03
2	0.02	0.12	0.06	−0.00	−0.18	0.28	1.87	0.95	−0.04	−2.86
3	0.13	0.14	0.18	0.08	−0.18	1.68	2.10	2.74	1.25	−2.21
4	0.10	0.06	0.01	0.15	0.03	1.23	0.92	0.09	2.23	0.35
Big	0.28	0.10	0.12	0.07	−0.17	3.32	1.93	2.21	1.15	−1.90
Four-factor: *Mkt, SMB, HML,* and *CMA*										
Small	−0.04	0.05	0.02	0.13	−0.27	−0.47	0.74	0.26	2.01	−3.96
2	0.00	0.10	0.03	−0.00	−0.15	0.07	1.53	0.42	−0.03	−2.38
3	0.13	0.12	0.16	0.10	−0.11	1.70	1.73	2.37	1.57	−1.31
4	0.08	0.02	0.02	0.13	0.07	0.95	0.32	0.24	1.97	0.87
Big	0.30	0.08	0.09	0.10	−0.11	3.47	1.52	1.70	1.67	−1.21
Five-factor: *Mkt, SMB, HMLO, RMW,* and *CMA*										
Small	0.12	0.20	0.13	0.22	−0.19	1.63	3.09	2.03	3.44	−2.70
2	0.06	0.10	0.04	−0.01	−0.17	0.93	1.52	0.60	−0.10	−2.62
3	0.20	0.16	0.17	0.08	−0.19	2.45	2.31	2.53	1.18	−2.30
4	0.09	0.09	−0.02	0.12	0.07	1.06	1.29	−0.27	1.79	0.82
Big	0.35	0.10	0.04	0.05	−0.20	4.00	1.94	0.64	0.83	−2.21
Panel B: Regression slopes										
Three-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + e_{it}$										
			h					***t*(*h*)**		
Small	−0.02	0.00	0.02	0.00	−0.04	−0.87	0.04	0.87	0.04	−1.47
2	−0.00	0.07	0.05	0.03	−0.09	−0.14	3.12	2.32	1.18	−4.20
3	−0.07	0.02	0.08	0.06	−0.05	−2.64	0.97	3.45	2.62	−1.76
4	−0.05	0.10	0.18	−0.02	−0.13	−1.74	3.94	7.88	−0.81	−4.33
Big	−0.27	−0.03	0.02	−0.10	−0.19	−8.84	−1.88	1.02	−4.34	−5.91
Four-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + c_i CMA_t + e_{it}$										
			h					***t*(*h*)**		
Small	−0.15	−0.14	−0.08	−0.06	−0.03	−4.04	−4.13	−2.54	−2.09	−1.01
2	−0.03	0.03	−0.02	0.03	−0.03	−1.04	0.83	−0.70	0.96	−1.01
3	−0.07	−0.03	0.03	0.11	0.12	−1.77	−0.95	0.98	3.49	3.01
4	−0.10	0.01	0.21	−0.05	−0.03	−2.55	0.16	6.55	−1.69	−0.86
Big	−0.23	−0.08	−0.04	−0.03	−0.05	−5.76	−3.22	−1.56	−0.87	−1.23
			c					***t*(*c*)**		
Small	0.28	0.31	0.22	0.14	−0.01	5.07	6.25	4.81	3.18	−0.13
2	0.06	0.10	0.16	−0.01	−0.14	1.40	2.26	3.67	−0.12	−3.23
3	−0.02	0.12	0.12	−0.10	−0.38	−0.30	2.52	2.41	−2.29	−6.56
4	0.11	0.21	−0.05	0.08	−0.21	1.88	4.24	−1.01	1.63	−3.60
Big	−0.08	0.10	0.14	−0.16	−0.31	−1.26	2.72	3.50	−3.61	−4.89

(*continued*)

Table 10
Continued

$AC \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
Five-factor: $R_{it}-R_{Ft}=a_i+b_i(R_{Mt}-R_{Ft})+s_iSMB_t+h_iHMLO_t+r_iRMW_t+c_iCMA_t+e_{it}$										
			h					***t*(*h*)**		
Small	−0.07	−0.07	−0.03	−0.02	0.01	−2.12	−2.09	−0.85	−0.69	0.29
2	−0.00	0.03	−0.02	0.03	−0.04	−0.16	0.84	−0.50	0.86	−1.27
3	−0.04	−0.01	0.04	0.10	0.08	−0.97	−0.32	1.16	3.05	2.00
4	−0.09	0.04	0.19	−0.06	−0.04	−2.35	1.14	5.92	−1.79	−0.87
Big	−0.21	−0.07	−0.07	−0.05	−0.10	−5.07	−2.70	−2.62	−1.68	−2.24
			r					***t*(*r*)**		
Small	−0.38	−0.38	−0.27	−0.22	−0.21	−10.77	−11.64	−8.74	−7.01	−6.23
2	−0.13	0.00	−0.03	0.02	0.03	−4.22	0.04	−1.01	0.54	1.13
3	−0.16	−0.10	−0.03	0.08	0.21	−3.97	−3.01	−0.75	2.39	5.27
4	−0.05	−0.15	0.12	0.01	−0.00	−1.15	−4.49	3.74	0.28	−0.05
Big	−0.17	−0.07	0.12	0.11	0.19	−3.94	−2.77	4.57	3.61	4.40
			c					***t*(*c*)**		
Small	0.04	0.09	0.08	0.03	−0.09	1.08	2.54	2.46	0.95	−2.40
2	0.00	0.13	0.14	0.03	−0.16	0.05	3.69	3.99	0.79	−4.88
3	−0.12	0.07	0.14	0.02	−0.21	−2.80	1.85	3.85	0.55	−4.96
4	−0.00	0.18	0.18	0.03	−0.25	−0.01	4.83	5.20	0.71	−5.48
Big	−0.35	0.00	0.12	−0.16	−0.32	−7.55	0.16	4.25	−4.79	−6.68

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-AC* (accruals) portfolios of Table 2. The RHS variables are the excess market return, $Mkt=R_M-R_F$, the *Size* factor, *SMB*, the value factor, *HML*, or its orthogonal counterpart, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*. Panel A shows three-factor, four-factor, and five-factor regression intercepts. Panel B shows regression slopes for *RMW*, *CMA*, and *HML* or *HMLO* (as relevant).

t = 3.32), but most of the intercepts in the first four quintiles of *AC* move a bit toward zero.

Performance deteriorates in the five-factor model that adds *RMW*, especially for microcaps. Adding *RMW* pushes the intercept for the microcap portfolio in the highest *AC* quintile toward zero (from −0.27, t = −3.96, to −0.19, t = −2.70), but it moves the intercepts for the other four microcap portfolios from values mostly indistinguishable from zero to large positive values that are 1.63 to 3.44 standard errors from zero. The problems of the five-factor model are not limited to microcaps. Adding *RMW* increases the intercepts for all portfolios in the lowest *AC* quintile, and three of the five intercepts for the portfolios in the highest *AC* quintile become more negative.

The regression slopes in panel B of Table 10 help us interpret the intercepts. (We do not show R_M-R_F and *SMB* slopes since they are similar for different models.) For the troublesome portfolios in the three smallest *Size* quintiles and the highest *AC* quintile, the *HML* slopes in the FF three-factor model are close to zero, so *HML* does not help explain the low average returns of these portfolios. The *HML* slope for the megacap portfolio in the highest *AC* quintile is rather strongly negative, which helps explain the extremely low average excess return on this portfolio, 0.26% per month (Table 9), but nevertheless leaves a three-factor intercept, −0.17% per month that is −1.90 standard errors from zero (Table 10). The negative *HML* slope for the megacap portfolio in the lowest *AC* quintile in part explains why the three-factor model does a poor job explaining the high average return of this portfolio.

Microcaps aside, the four-factor model that adds *CMA* to the FF three-factor model produces strong negative *CMA* slopes for the portfolios in the highest *AC* quintile in Table 10. This is consistent with the *Inv* evidence in Table 9 that firms in these portfolios tend to invest aggressively, and it helps explain why this model improves the regression intercepts for these portfolios. Table 9 shows the microcap portfolio in the highest *AC* quintile also invests a lot, but its *CMA* slope is close to zero and its four-factor intercept, −0.27 ($t = -3.96$), is almost unchanged from the three-factor intercept, −0.28. Most of the four-factor *HML* slopes are close to zero and so have little effect on the regression intercepts.

The *RMW* slopes for microcaps in the five-factor model are strongly negative. This is consistent with the evidence in Table 9 that controlling for *AC*, profitability is lowest for microcaps. Driven by a large negative *RMW* slope, the intercept for the microcap portfolio in the highest *AC* quintile shrinks from −0.27 in the four-factor model that does not include *RMW* to −0.19 ($t = -2.70$) in the five-factor model. The *RMW* slopes are, however, also largely responsible for the general deterioration of the intercepts from the four-factor to the five-factor model for the other four microcap portfolios. The reductions predicted by negative *RMW* slopes do not show up in the average returns of these portfolios. Negative *RMW* slopes for portfolios in the lowest *AC* quintile (which do not have low average returns in Table 9) and positive slopes for some of the portfolios in the highest *AC* quintile (which have low average returns) are responsible for the five-factor model's adverse effect on the intercepts for these portfolios.

FF (2015) find that in 5×5 sorts on *Size* and *Inv*, the portfolios in the smaller *Size* quintiles and the highest *Inv* quintile produce intercept problems, even for asset pricing models that include the investment factor *CMA*. This suggests that small firms that invest a lot are a general problem for the asset pricing models we consider. Table 9 shows these firms are prominent in the highest *AC* portfolios of smaller *Size* quintiles, and the intercepts for these portfolios are always among the most extreme in Table 10.

The *Size-AC* portfolios provide a valuable caution. They are the only sorts, here and in FF (2015), in which the five-factor model performs noticeably worse than other models. It is also noteworthy that the problems of the five-factor model in the *Size-AC* sorts trace to the profitability factor since in other sorts *RMW* typically improves the description of average returns, often substantially.

7. Momentum

Table 11 shows average excess returns for monthly independent sorts of stocks into quintiles of *Size* and momentum (*Prior 2–12*). With one exception, there is a *Size* effect in the *Prior 2–12* quintiles; given *Prior 2–12*, average returns are larger for portfolios of small stocks. The exception is the lowest *Prior 2–12* quintile (extreme losers), in which the portfolios in the two smallest *Size*

Table 11
Average excess returns and characteristics of stocks in the 25 *Size-Prior 2–12* portfolios, July 1963–December 2014 (618 months)

Prior 2–12 →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Means and SD of portfolio excess returns										
			Mean					**SD**		
Small	0.03	0.67	0.91	1.05	1.39	8.01	5.88	5.43	5.50	6.78
2	0.14	0.66	0.82	1.01	1.23	7.86	5.88	5.27	5.41	6.74
3	0.27	0.62	0.72	0.78	1.19	7.37	5.53	5.07	5.00	6.31
4	0.20	0.59	0.66	0.79	1.04	7.26	5.52	4.87	4.79	5.89
Big	0.17	0.46	0.39	0.55	0.79	6.79	4.88	4.37	4.31	5.26
Panel B: Average *B/M*, *OP*, *Inv*, and *Prior 2–12* characteristics										
			B/M					***OP***		
Small	0.84	0.95	0.98	1.00	0.99	0.08	0.21	0.28	0.28	0.17
2	0.72	0.80	0.83	0.84	0.82	0.24	0.27	0.28	0.29	0.24
3	0.68	0.75	0.78	0.78	0.73	0.27	0.29	0.29	0.29	0.29
4	0.66	0.71	0.74	0.73	0.71	0.29	0.35	0.33	0.30	0.28
Big	0.56	0.59	0.60	0.60	0.61	0.36	0.34	0.35	0.37	0.39
			Inv					***Prior 2–12***		
Small	0.24	0.14	0.13	0.13	0.14	−31.87	−4.56	9.45	25.08	90.33
2	0.27	0.16	0.15	0.15	0.19	−27.79	−4.33	9.55	25.08	83.61
3	0.24	0.15	0.14	0.14	0.20	−25.91	−4.13	9.56	24.97	77.08
4	0.19	0.14	0.12	0.13	0.18	−23.80	−4.03	9.66	24.90	71.41
Big	0.19	0.13	0.12	0.12	0.16	−20.94	−3.71	9.68	24.82	58.96

Panel A shows means and standard deviations of monthly excess returns on value-weight portfolios formed monthly using independent sorts of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and quintiles of *Prior 2–12* (momentum), with NYSE breakpoints for both variables. The intersections of the two sorts produce 25 *Size-Prior 2–12* portfolios. For portfolios formed at the beginning of month t, *Size* is the market cap of a stock at the beginning of t and *Prior 2–12* is its cumulative return for the 11 months from $t-12$ to $t-2$. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation, as defined in Table 3. Panel B also shows the time-series average values of *Prior 2–12* used to form portfolios each month.

quintiles have extreme low average excess returns, 0.03% and 0.14% per month. There is a strong momentum effect in every *Size* quintile, but it decreases as *Size* increases. The spread in average returns from extreme winners to extreme losers is 1.36% per month for microcaps and 0.62% for megacaps. The average monthly excess return for microcap extreme winners is 1.39%, versus 0.79% for megacap winners.

Table 12 shows intercepts from the five-factor model (3) and the six-factor model that adds *MOM* to (10) and in which *HMLO* is the sum of the intercept (0.04; $t=0.52$) and residual from the regression of *HML* on $R_M - R_F$, *SMB*, *RMW*, *CMA*, and *MOM*. The five-factor model is not much help in capturing the average returns produced by momentum sorts. The five-factor intercepts for extreme losers are strongly negative, the intercepts for extreme winners are strongly positive, and the spreads between the intercepts for extreme winners and losers are similar to the spreads in average returns.

In the dividend discount model (2) that Fama and French (2015) use to motivate the five-factor model, the internal rate of return on expected cash flows to shareholders (r in Equation (2)) is approximately the long-term expected stock return. Momentum is short-term; the relative performance of stocks in

Table 12
Regressions for the 25 *Size-Prior 2–12* portfolios, July 1963–December 2014 (618 months)

Prior 2–12 →	Low	2	3	4	High	Low	2	3	4	High
Panel A: $R_{it}-R_{Ft}=a_i+b_i(R_{Mt}-R_{Ft})+s_iSMB_t+h_iHML_t+r_iRMW_t+c_iCMA_t+e_{it}$										
			a					***t*(*a*)**		
Small	−0.71	−0.20	0.02	0.20	0.62	−4.68	−2.50	0.39	3.06	6.40
2	−0.62	−0.21	−0.05	0.15	0.51	−4.35	−2.58	−0.80	2.79	5.63
3	−0.37	−0.18	−0.11	−0.07	0.52	−2.40	−2.10	−1.75	−1.06	5.34
4	−0.40	−0.21	−0.13	0.02	0.44	−2.41	−2.19	−1.94	0.34	4.09
Big	−0.37	−0.13	−0.17	−0.04	0.33	−2.27	−1.29	−2.72	−0.62	3.08
Panel B: $R_{it}-R_{Ft}=a_i+b_i(R_{Mt}-R_{Ft})+s_iSMB_t+h_iHMLO_t+r_iRMW_t+c_iCMA_t+m_iMOM_t+e_{it}$										
			a					***t*(*a*)**		
Small	−0.23	−0.01	0.09	0.15	0.40	−2.28	−0.09	1.44	2.31	4.85
2	−0.11	0.03	0.01	0.10	0.24	−1.47	0.45	0.09	1.88	3.73
3	0.16	0.06	−0.00	−0.13	0.22	1.79	0.98	−0.02	−2.11	3.30
4	0.16	0.07	−0.02	−0.03	0.11	1.60	1.09	−0.25	−0.45	1.49
Big	0.17	0.19	−0.09	−0.15	−0.02	1.59	2.76	−1.51	−2.61	−0.24
			h					***t*(*h*)**		
Small	0.13	0.29	0.31	0.21	0.05	2.62	9.20	10.27	6.55	1.37
2	0.06	0.20	0.25	0.21	0.01	1.66	7.00	8.92	8.33	0.36
3	0.05	0.21	0.27	0.29	0.00	1.07	6.84	9.51	9.87	0.14
4	0.09	0.14	0.24	0.20	0.04	1.94	4.26	7.90	6.68	1.03
Big	0.02	0.04	0.13	0.08	0.06	0.34	1.20	4.28	2.64	1.73
			r					***t*(*r*)**		
Small	−0.31	0.15	0.26	0.15	−0.17	−6.37	4.75	8.62	4.70	−4.22
2	−0.16	0.27	0.30	0.22	−0.15	−4.50	9.34	11.03	8.68	−4.87
3	−0.18	0.26	0.34	0.37	−0.08	−4.16	8.46	11.84	12.48	−2.44
4	−0.20	0.29	0.37	0.36	−0.09	−4.05	8.74	12.31	11.64	−2.63
Big	0.03	0.24	0.26	0.25	−0.01	0.55	7.43	8.80	8.89	−0.27
			c					***t*(*c*)**		
Small	−0.10	0.33	0.38	0.33	0.02	−1.86	9.81	11.67	9.57	0.38
2	−0.17	0.23	0.32	0.30	−0.13	−4.49	7.44	10.69	10.78	−3.89
3	−0.18	0.22	0.33	0.35	−0.14	−3.84	6.61	10.68	10.91	−3.93
4	−0.09	0.30	0.34	0.30	−0.10	−1.69	8.47	10.42	8.95	−2.46
Big	−0.13	0.19	0.15	0.22	−0.17	−2.34	5.47	4.72	7.25	−4.75
			m					***t*(*m*)**		
Small	−0.69	−0.30	−0.12	0.05	0.30	−31.07	−20.38	−8.90	3.09	16.16
2	−0.72	−0.35	−0.10	0.04	0.37	−43.43	−26.44	−7.75	3.76	25.35
3	−0.74	−0.36	−0.18	0.05	0.41	−36.98	−25.21	−13.92	4.00	27.62
4	−0.79	−0.41	−0.19	0.05	0.45	−35.34	−26.70	−13.35	3.28	27.17
Big	−0.75	−0.45	−0.13	0.15	0.48	−32.18	−29.29	−9.08	11.34	30.72

The LHS variables are the monthly excess returns on the 25 *Size-Prior 2–12* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML*, or its orthogonal counterpart, *HMLO*, the profitability factor, *RMW*, the investment factor, *CMA*, and the momentum factor *MOM*, constructed using independent 2x3 sorts on *Size* and each of B/M, *OP*, *Inv*, and *Prior 2–12*. The table shows intercepts for a five-factor model that does not include *MOM*, and intercepts and slopes for the six-factor model that includes *MOM*. In this table, *HMLO* is the sum of the intercept (0.04, $t=0.51$) and the residual from the regression of *HML* on R_M-R_F, *SMB*, *HML*, *RMW*, *CMA*, and *MOM*.

months $t-12$ to $t-2$ tends to persist for only about nine months starting in t. Because of the long-term return reversals identified by DeBondt and Thaler (1985), *Prior 2–12* is negatively related to longer-term relative returns. Thus, perhaps it is not surprising that the five-factor model, which is targeted at long-run expected returns, fails to capture the positive relation between *Prior 2–12* and current returns.

Adding *MOM* to the five-factor model improves the regression intercepts, but problems remain. Most noticeable is the unexplained momentum among microcaps: the extreme loser portfolio has a rather strong negative intercept, -0.23% per month ($t = -2.28$), the winner portfolio has a strong positive intercept, 0.40% ($t = 4.85$), and the intercepts increase monotonically from losers to winners. There is a weaker momentum pattern in the intercepts of the second *Size* quintile, and there is a weak reverse momentum pattern in the intercepts for the largest (megacap) quintile.

The regression slopes for the six-factor model show why including *MOM* is critical in the tests on the *Size-Prior 2–12* portfolios. The *MOM* slopes increase from strongly negative for losers to strongly positive for winners, which is the pattern in average returns. But *HMLO*, *RMW*, and *CMA* provide little help. Both the *HMLO* premium and the *HMLO* slopes for extreme winners and extreme losers are close to zero. Megacaps aside, *RMW* slopes are negative for extreme losers, and the negative slopes are helpful for explaining low average returns, but *RMW* slopes are also slightly negative for extreme winners. The *CMA* slopes have a similar pattern. Panel B of Table 11 shows there are also no clear patterns in B/M, *OP*, and *Inv* for the 25 *Size-Prior 2–12* portfolios.

8. Conclusions

The list of anomalies shrinks in the five-factor model, in part because anomalous returns become less anomalous and in part because the returns associated with different anomaly variables share factor exposures that suggest they are in large part the same phenomenon.

The flat relation between market β and average return that has long plagued tests of the CAPM is captured in the five-factor model by *RMW* and *CMA* slopes that offset the average return predictions of market and *SMB* slopes. Stocks with higher CAPM market βs have higher five-factor market and *SMB* slopes that raise predictions of their average returns. But low β stocks have positive exposures to the profitability and investment factors of the five-factor model that raise predictions of their average returns, and high β stocks have negative exposures to *RMW* and *CMA* that lower their predicted returns. Thus, low β stock returns behave like those of profitable firms that invest conservatively, whereas high β returns behave like those of less profitable firms that invest aggressively.

The high average returns associated with share repurchases, which are a problem for the FF three-factor model, cease to be an anomaly in the five-factor model. The reason again is that the returns of repurchasers behave like those of profitable firms that invest conservatively. Positive exposures to *RMW* and *CMA* also go a long way toward capturing the average returns of low volatility stocks, whether volatility is measured in terms of total returns or residuals from the FF three-factor model.

Like the returns of relatively unprofitable firms that invest aggressively, the returns of high β stocks, stocks with highly volatile returns, and stocks of firms that make large share issues load negatively on *RMW* and *CMA*. Unlike the average returns of high β portfolios, however, negative five-factor exposures to *RMW* and *CMA* do not fully capture the low average returns associated with large share issues and high volatility. Unexplained average returns are largely concentrated in small stocks, especially microcaps. Small stocks with negative exposures to *RMW* and *CMA* are also a problem for the five-factor model in many of the tests in FF (2015), leading them to dub it the lethal combination.

The five-factor model typically performs better than the FF three-factor model when applied to different sets of LHS portfolios here and in FF (2015). Portfolios formed on *Size* and accruals are an exception. The pricing problems associated with accruals do not seem to have much to do with the lethal combination of slopes that is a common problem in other sorts.

All models that do not include a momentum factor fare poorly in the tests on the 25 *Size-Prior 2–12* portfolios. A six-factor model that includes *MOM* performs well, but by playing a home game; the momentum factor, *MOM*, is just a coarse (2×3 rather than 5×5) version of the sorts used to construct the 25 *Size-Prior 2–12* portfolios. Nevertheless, the six-factor model leaves lots of momentum in microcap returns unexplained.

Table A1
Summary statistics for the 25 *Size-Var* portfolios, July 1963–December 2014 (618 months)

Var →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Means and SD of portfolio excess returns										
			Mean					**SD**		
Small	1.00	1.18	1.09	0.81	−0.18	4.07	5.67	6.50	7.53	9.22
2	0.90	1.03	1.06	0.91	0.27	4.02	5.26	5.90	6.83	8.78
3	0.75	0.85	0.97	0.88	0.44	3.68	4.78	5.39	6.23	8.08
4	0.67	0.74	0.77	0.77	0.49	3.72	4.48	5.10	5.79	7.69
Big	0.43	0.53	0.54	0.46	0.46	3.43	3.99	4.48	5.10	6.74
Panel B: Average *B/M*, *OP*, *Inv*, and *Var* characteristics										
			B/M					***OP***		
Small	1.01	0.96	0.93	0.90	0.94	0.24	0.28	0.33	0.30	−0.10
2	0.89	0.83	0.80	0.78	0.73	0.34	0.31	0.30	0.29	0.18
3	0.84	0.77	0.73	0.72	0.68	0.29	0.30	0.31	0.31	0.27
4	0.81	0.72	0.69	0.68	0.65	0.29	0.33	0.35	0.31	0.30
Big	0.65	0.55	0.56	0.58	0.57	0.34	0.37	0.36	0.36	0.37
			Inv					***Var***		
Small	0.11	0.14	0.17	0.21	0.22	2.53	5.69	9.01	14.88	43.98
2	0.10	0.13	0.16	0.21	0.30	1.89	3.79	5.70	8.52	20.08
3	0.10	0.12	0.14	0.18	0.30	1.54	2.98	4.47	6.69	15.70
4	0.10	0.11	0.12	0.15	0.26	1.41	2.52	3.70	5.43	12.57
Big	0.10	0.11	0.12	0.14	0.21	1.33	2.16	2.98	4.19	8.51

This table shows means and standard deviations of monthly excess returns on value-weight portfolios formed monthly using a first pass sort of NYSE, AMEX, and (beginning in 1973) NASDAQ stocks into *Size* (market capitalization) quintiles and second-pass sorts into quintiles of *Var* (total variance) using NYSE breakpoints for both variables. The *Var* sorts are conditional on *Size* quintile. The intersections of the two sorts produce 25 *Size-Var* portfolios. For portfolios formed at the beginning of month t, *Size* is the market cap of a stock at the beginning of t and *Var* is the variance of its daily returns estimated using 60 (with a minimum 20) days of lagged returns. Panel A shows means and standard deviations of monthly excess returns on the 25 portfolios. Panel B shows time-series means of the portfolio book-to-market equity ratio (B/M), operating profitability (*OP*), and investment (*Inv*) for the fiscal year ending in the calendar year preceding portfolio formation, as defined in Table 3. Panel B also shows the time-series average values of *Var* used to form portfolios each month.

Table A2
Regressions for the 25 *Size-Var* portfolios, July 1963 to December 2014 (618 months)

Var →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + e_{it}$										
			a					***t(a)***		
Small	0.35	0.30	0.13	−0.22	−1.25	5.34	4.09	1.68	−2.33	−7.64
2	0.27	0.21	0.18	−0.03	−0.68	4.24	3.05	2.51	−0.38	−6.08
3	0.18	0.11	0.18	0.01	−0.38	2.68	1.69	2.36	0.08	−3.76
4	0.13	0.10	0.06	0.00	−0.26	1.68	1.31	0.83	0.06	−2.46
Big	0.05	0.11	0.05	−0.06	−0.12	0.73	1.85	0.89	−1.06	−1.21
Panel B: Five-factor: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HMLO_t + r_i RMW_t + c_i CMA_t + e_{it}$										
			a					***t(a)***		
Small	0.23	0.17	0.07	−0.14	−0.87	3.65	2.42	0.95	−1.42	−5.62
2	0.12	0.03	0.04	−0.15	−0.42	2.02	0.48	0.55	−1.97	−3.96
3	0.04	−0.02	0.02	−0.14	−0.16	0.61	−0.34	0.23	−1.87	−1.60
4	0.01	−0.06	−0.11	−0.10	−0.01	0.08	−0.90	−1.67	−1.20	−0.07
Big	−0.04	−0.05	−0.06	−0.11	0.13	−0.57	−0.91	−1.11	−1.69	1.30
			b					***t(b)***		
Small	0.69	0.99	1.10	1.17	1.16	44.19	58.48	57.56	50.14	30.95
2	0.76	1.00	1.10	1.24	1.32	51.99	66.59	68.05	69.30	50.63
3	0.75	0.98	1.09	1.22	1.30	47.76	65.14	65.50	67.60	55.13
4	0.78	0.98	1.12	1.22	1.32	41.83	57.53	67.74	62.51	54.81
Big	0.76	0.93	1.03	1.14	1.24	44.97	70.40	75.24	74.54	52.07
			s					***t(s)***		
Small	0.67	0.94	1.05	1.20	1.39	30.34	39.61	39.38	36.69	26.31
2	0.55	0.76	0.84	0.96	1.13	26.85	35.93	36.90	38.19	30.80
3	0.31	0.48	0.56	0.70	0.83	13.94	22.66	24.23	27.56	25.19
4	0.09	0.20	0.25	0.32	0.51	3.55	8.30	10.71	11.66	15.09
Big	−0.25	−0.22	−0.17	−0.18	0.01	−10.54	−12.08	−8.84	−8.35	0.38
			h					***t(h)***		
Small	0.35	0.42	0.39	0.33	0.24	11.30	12.68	10.54	7.29	3.21
2	0.30	0.36	0.33	0.25	−0.07	10.48	12.43	10.49	7.00	−1.33
3	0.31	0.38	0.34	0.26	−0.17	10.13	13.09	10.57	7.34	−3.62
4	0.36	0.30	0.27	0.21	−0.17	9.78	9.01	8.30	5.62	−3.59
Big	0.16	0.06	0.07	−0.02	−0.09	4.94	2.25	2.51	−0.57	−1.83
			r					***t(r)***		
Small	0.37	0.44	0.27	−0.04	−0.79	11.74	12.97	7.04	−0.86	−10.52
2	0.40	0.52	0.47	0.43	−0.54	13.78	17.39	14.46	11.85	−10.24
3	0.38	0.45	0.54	0.47	−0.47	12.06	14.98	16.05	12.80	−9.96
4	0.36	0.45	0.50	0.33	−0.57	9.60	13.17	15.05	8.42	−11.80
Big	0.19	0.38	0.30	0.09	−0.46	5.66	14.31	11.11	2.85	−9.69
			c					***t(c)***		
Small	0.47	0.50	0.39	0.13	−0.23	13.72	13.60	9.30	2.61	−2.84
2	0.49	0.54	0.40	0.22	−0.47	15.44	16.63	11.41	5.56	−8.32
3	0.50	0.45	0.40	0.32	−0.55	14.79	13.94	11.20	8.08	−10.69
4	0.52	0.47	0.42	0.26	−0.52	12.85	12.71	11.66	6.04	−10.06
Big	0.35	0.24	0.16	0.04	−0.59	9.66	8.51	5.34	1.12	−11.51

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-Var* (total variance) portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML*, or its orthogonal counterpart, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*. Panel A shows intercepts from the FF three-factor model, and panel B shows five-factor intercepts and slopes from (10).

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