Equilibrium Underdiversification and the Preference for Skewness

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We develop a one-period model of investor asset holdings where investors have heterogeneous preference for skewness. Introducing heterogeneous preference for skewness allows the model's investors, in equilibrium, to underdiversify. We find support for our model's three key implications using a dataset of 60,000 individual investor accounts. First, we document that the portfolio returns of underdiversified investors are substantially more positively skewed than those of diversified investors. Second, we show that the apparent mean-variance inefficiency of underdiversified investors can be largely explained by the fact that investors sacrifice mean-variance efficiency for higher skewness exposure. Furthermore, we show that idiosyncratic skewness, and not just coskewness, can impact equilibrium prices. Third, the underdiversification of investors does not appear to be coincidentally related to skewness. Stocks most often selected by underdiversified investors have substantially higher average skewness—especially idiosyncratic skewness—than stocks most often selected by diversified investors. (*JEL* G11, G12)

A substantial body of research documents that investors commonly hold portfolios made up of far fewer securities than are necessary to eliminate idiosyncratic risk, [see, e.g., Blume and Friend (1975), Kelly (1995), Odean (1999), Vissing-Jorgensen (1999), Polkovnichenko (2005), and Goetzmann and Kumar (2004)]. In a mean-variance framework, this lack of diversification is a puzzle because standard portfolio theory suggests that underdiversified investors are unnecessarily accepting higher return volatility without any accompanying compensation in expected returns. A number of articles, including Statman (1987), Meulbroek (2005), and Goetzmann and Kumar (2004), show empirically that failure to diversify is costly in terms of the risk-return trade-off achieved by underdiversified investors.

One potential explanation for underdiversification is that investors may consciously choose to remain underdiversified in order to increase the

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likelihood of extreme positive returns, or in other words, to capture higher levels of skewness in their portfolios. Under fairly general conditions, Arditti (1967) and Scott and Horvath (1980) demonstrate that investors prefer positive skewness in return distributions. Simkowitz and Beedles (1978) and Conine and Tamarkin (1981) argue that when the third moment of the return distribution is taken into consideration, investors may optimally choose to remain underdiversified. Diversification then is a two-edged sword: it eliminates *undesired variance* in return distributions, but also eliminates *desired skewness*. Consequently, portfolios that are efficient in a mean-variance-skewness framework only appear to be inefficient when evaluated using a mean-variance framework.

Conine and Tamarkin (1981) construct demand functions of investors who have a preference for skewness and find that these demand functions can lead to optimal portfolios that consist of only a small number of stocks. Yet, as shown by Rubinstein (1973), if all investors have identical cubic utility (preference for skewness), as assumed by Conine and Tamarkin (1981), then investors still hold well-diversified portfolios in equilibrium.

We extend the understanding of the impact of skewness on investor holdings and asset prices both theoretically and empirically. Theoretically, we introduce a model with investors whose demand for mean and variance is the same but preference for skewness is heterogeneous across agents. Our approach extends the results of Conine and Tamarkin (1981) by explicitly allowing for heterogeneity in skewness preference. By relaxing the identical investor assumption, our model allows investors to hold underdiversified portfolios. Models of investor heterogeneity have successfully been used to explain asset prices in other contexts where representative-agent models fall short [see, e.g., Telmer (1993), Heaton and Lucas (1995), and Constantinides and Duffie (1996)], and we find that heterogeneity in skewness preference contributes substantially to the explanation of underdiversification of investors.

Our model demonstrates further that, given the lack of diversification in investor holdings, investors will care about the level of *idiosyncratic* skewness in their portfolio returns. This result stands in contrast to models that incorporate return skewness in the context of fully diversified investors. These models, which include Kraus and Litzenberger (1976) and Harvey and Siddique (2000), predict that only the *coskewness* of a security with the market portfolio will be relevant for investor holdings and asset prices. The prediction of our model that idiosyncratic skewness will be priced is similar to the prediction of Barberis and Huang (2005), although their prediction arises from the assumption of investors having preferences based on cumulative prospect theory, and not from heterogeneous preference for skewness.

Our theoretical investigation leads to three key implications. First, investors with greater demand for skewness will, in equilibrium, hold less-diversified portfolios than investors with less demand for skewness. Second, skewness—and in particular idiosyncratic skewness—will be a priced component of security returns. As a result, investors with skewness preference hold mean-variance-skewness efficient portfolios but may appear to hold inefficient portfolios from a mean-variance perspective. Third, investors with greater preference for skewness will, in equilibrium, select securities (and portfolios) with greater levels of skewness, including idiosyncratic skewness.

Empirically, we test the implications of our model using the portfolio holdings of 60,000 investors at a large discount brokerage house over the period 1991–1996. This dataset is used by Barber and Odean (2000) and others. The empirical results support the three key implications of the theoretical model. First, we document the degree to which underdiversified investors capture skewness in their portfolios. We show that the difference in the portfolio return skewness between underdiversified and diversified investors is large; diversified investor portfolios exhibit very little skewness whereas underdiversified investor portfolios exhibit substantial positive skewness. As a result, the set of the highest-performing portfolios is dominated by underdiversified investors. For example, over a 3-year return horizon, in a ranking of all investor portfolios on returns, among the top one percent of all investors, underdiversified investors outnumber well-diversified investors by a ratio of 26 to 1. Because of this much higher probability of achieving very high returns, investors with a stronger preference for skewness may naturally be attracted to underdiversified portfolios.

Second, we explore the relationship between mean-variance efficiency of portfolios and skewness. Capturing a high level of skewness in underdiversified portfolios may come at the cost of having to accept lower mean returns or higher variance of returns. We find that the skewness coefficient of investor portfolios increases with decreases in the portfolios' Sharpe ratios. In other words, underdiversified investors receive a consistent reward in skewness that at least partially compensates for the lack of mean-variance efficiency in their portfolios. Underdiversified portfolios are much more efficient when skewness of returns is taken into account. Moreover, a regression analysis shows that the trade-off between Sharpe ratios and skewness persists in investor portfolios even after controlling for the level of diversification of the portfolio. We show further that the negative relationship between Sharpe ratios and skewness is shown to be strongest for idiosyncratic skewness rather than for coskewness.[1] The results suggest that underdiversified investors, to some degree, are consciously trading mean-variance efficiency for skewness, and not just

[1] Results could vary for different measures of coskewness. We have investigated other forms of coskewness as detailed in Harvey and Siddique (2000) and found similar results for alternative measures.

obtaining skewness as a by-product of remaining underdiversified for some other reason.

Third, we extend our tests of whether underdiversified investors are consciously seeking higher skewness in their portfolios. We examine the characteristics of stocks within investor portfolios to determine if underdiversified investors are more likely to select stocks with highly skewed returns. We find that highly skewed stocks are relatively popular with underdiversified investors. We document that the degree of skewness in stocks selected by underdiversified investors is surprisingly large.[2] On average, the stocks that are most often selected by the least-diversified investors have skewness coefficients that are roughly *double* the magnitude of the skewness coefficients of stocks most often selected by diversified investors. This result indicates that underdiversified investors have not randomly selected the small number of securities in their portfolios, but rather have intentionally chosen stocks that will be most likely to increase the skewness of their portfolios. Finally, as an interesting sidenote, we show demographically that investors who are younger, male, and less affluent are most likely to demonstrate a stronger preference for skewness.

Our theoretical and empirical findings add to our understanding of why investors fail to diversify. Several other explanations for underdiversification—which are not mutually exclusive of the skewness-preference theory—have been offered in the literature. Barberis and Huang (2005) model underdiversification as resulting from investors' overweighting the probabilities of extreme outcomes and the embedded preference for skewness generated by cumulative prospect theory utility. Shefrin and Statman (2000) argue that underdiversification results because investors construct their portfolio as layers, with bottom layers for downside protection and upper layers for upside potential. In a similar vein, Polkovnichenko (2005) suggests that underdiversification may be explained by investors' desire to "get ahead" as in the prospect theory of Kahneman and Tversky (1979). Both Shefrin and Statman (2000) and Polkovnichenko (2005) have a common link with skewness-preference theory in that a desire for upside potential is a driving factor in investors' decisions. Goetzmann and Kumar (2004) empirically support explanations for underdiversification based on investor overconfidence [Odean (1999)], familiarity bias [Grinblatt and Keloharju (2001)], and lack of wealth or sophistication. Our article contributes to these existing explanations by theoretically connecting skewness preference and equilibrium underdiversification, and by empirically documenting the intentional trade-off investors make between mean-variance efficiency and return skewness.

[2] Goetzmann and Kumar (2004) report a tendency for underdiversified investors to favor skewed stocks but they do not report relative magnitudes of skewness obtained by different clientele.

The article is organized as follows. Section 1 presents the theoretical model. Section 2 describes the data used for our empirical tests. Section 3 explores to what degree skewness is attained through underdiversification. Section 4 studies the relationship between skewness and mean-variance efficiency and the pricing implications of idiosyncratic skewness. Section 5 reports results on the stock selection of underdiversified investors and describes differences in skewness preference across demographic characteristics. Section 6 concludes.

1. A Mean-Variance-Skew Model

In this section we show, using a simple economy, how heterogeneous preference for skewness can lead to equilibrium holdings and asset prices that differ from the standard mean-variance case. Our model builds on the results of Conine and Tamarkin (1981) who, through comparative statics, show how individual investor demand functions may reflect a desire for incomplete diversification in the presence of skewness. Our key extension to Conine and Tamarkin (1981) is to generalize their partial equilibrium comparative statics to equilibrium holdings. Our asset-pricing results extend those of Kraus and Litzenberger (1976) who find that the priced component of skewness is coskewness in a representative-agent model. Using a model with agents whose preferences are described by cumulative prospect theory, Barberis and Huang (2005) generate similar predictions, although their model relies less on skewness in the underlying distribution than our model, given the assumption in their model that agents overweight the probability in the tails of return distributions thereby creating or enhancing the impact of skewness. Our model starts with a more traditional preference structure and relies on the assumption of heterogeneous preference for skewness to generate underdiversified portfolio holdings and the pricing of idiosyncratic skewness.

The impact of investor heterogeneity has been investigated extensively in asset pricing. Dynamic consumption models with investor heterogeneity (or incomplete markets) explain features of asset prices that the traditional complete-markets representative-agent model cannot. For example, Constantinides and Duffie (1996) find investor heterogeneity over labor income shocks helps to resolve the equity premium puzzle and note more generally that "(t)he joint hypothesis of incomplete consumption insurance and consumer heterogeneity offers the prospect of enriching the pricing implications of the representative-consumer model."[3] In incomplete markets/heterogeneous investor models, agents

[3] While our choice of using a one-period Capital Asset Pricing Model (CAPM) type model precludes the analysis of incomplete markets and consumption insurance as referenced in the quote from Constantinides and Duffie (1996), our approach allows us to simply illustrate idiosyncratic skewness' potential impact on holdings and prices. More general models of equilibrium preference for skewness is the subject of future work.

are unable to share risk perfectly, a unique state-price vector does not exist, and consequently, agents may hold underdiversified portfolios and idiosyncratic risks may be priced [see also Telmer (1993) and Heaton and Lucas (1995)].

In our one-period model we assume that the investing universe consists of three risky assets and a riskless bond that pays an interest rate r. The return vector of the three securities is denoted as $\mathbf{R} = [R_1, R_2, R_3]$ and the covariance structure of the risky securities is denoted as $\mathbf{V}$ with standard elements. In addition, we allow for the distribution of asset returns to be skewed.

Two types of agents exist in our economy. The first type, which we refer to as "Traditional Investor", has a standard quadratic utility function over wealth

$$U(W) = E(W) - \frac{1}{2\tau} Var(W), \tag{1}$$

where W represents the future wealth and τ is the coefficient of risk aversion, $\tau > 0$. The second type, "Lotto Investor", has identical preferences as traditional investor over mean and variance, but also has preference for skewness:

$$U(W) = E(W) - \frac{1}{2\tau} Var(W) + \frac{1}{3\phi} Skew(W), \tag{2}$$

where ϕ is the coefficient governing preference for skewness, $\phi > 0$. Positive values of ϕ indicate a preference for positive skewness. Our choice of utility functions merits three comments. First, although the shortcomings of quadratic utility (e.g., increasing absolute risk aversion, satiation) are well documented, quadratic utility has been shown to be a reasonable approximation of more standard expected utility functions [e.g., Levy and Markowitz (1979) and Hlawitschka (1994)]. For example, Equation (1) has been shown to be equivalent to negative exponential utility in the presence of normally distributed returns. Second, unlike other common expected utility functions (power utility, CRRA utility), Equations (1) and (2) isolate the impact of heterogeneity in skewness preference while maintaining homogeneity across preference for mean and variance. Third, our choice of utility functions allows our model to nest some traditional equilibria. Specifically, Equations (1) and (2) can lead to equilibrium portfolio separation (i.e., standard mean-variance holdings) under certain restrictions, as shown in Cass and Stiglitz (1970). On one hand, as $\phi \to \infty$, Lotto Investor approaches quadratic utility and the model defaults to the seminal Markowitz (1959) mean-variance model for both holdings and asset prices. On the other hand, if only Lotto Investors exist, our economy would consist of a homogeneous population of skewness preferring investors, and equilibrium holdings would be the well-diversified market portfolio and prices would be described by the Kraus and Litzenberger (1976) coskewness results. Consequently, using utility specifications (1) and (2) allows our model to not only investigate heterogeneous skewness preference equilibria, but also to default to diversified-holdings equilibria in special cases.

Let $\mathbf{X}_j = [x_{j,1}, x_{j,2}, x_{j,3}]$ be an $n \times 1$ vector that denotes investor j's dollar amount invested in each of the three risky assets generating the following representation for end-of-period wealth, $W_j = W_{0,j}(1 + r) + \mathbf{X}'_j(\mathbf{R} - r\mathbf{1})$, where $W_{0,j}$ represents investor j's original endowment.

To solve a particular investor's maximization problem, Equation (1) or Equation (2) is maximized subject to the end-of-period wealth constraint. Traditional Investor's demand function is straightforward:

$$\mathbf{X}_{\mathrm{T}} = \tau \mathbf{V}^{-1}(\mathbf{R} - r), \tag{3}$$

where subscript T signifies Traditional Investor, and is recognized as the traditional mean-variance demand function that leads to complete diversification holdings and CAPM pricing relationships. For Lotto Investor, the solution is not as straightforward, as shown in the following first-order condition:

$$(\mathbf{R} - r\mathbf{1}) - \frac{1}{\tau}\mathbf{V}\mathbf{X}_{\mathrm{L}} + \frac{1}{\phi}[\big(x_{\mathrm{L},1}\mathbf{M}_1 + x_{\mathrm{L},2}\mathbf{M}_2 + x_{\mathrm{L},3}\mathbf{M}_3\big)\mathbf{X}_{\mathrm{L}}] = 0, \tag{4}$$

where subscript L signifies Lotto Investor and $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ are matrices containing skewness elements generically denoted as $M_{ijk} = E[(R_i - \overline{R}_i)(R_j - \overline{R}_j)(R_k - \overline{R}_k)]$ and $\overline{R}_i = E[R_i]$. Three general types of skewness elements exist in the 'M' skewness matrices: M_{iii}, which represents the idiosyncratic skewness of asset 'i', M_{iij}, which represents the curvilinear interaction of the assets 'i' and 'j', and M_{ijk}, which represents the triplicate product moment of the assets 'i', 'j', and 'k'. More discussion of matrix representation of portfolio skewness is provided in Appendix 1 [see also Conine and Tamarkin (1981)]. In general, analytic solutions for the demand functions of Lotto Investor ($\mathbf{X}_{\mathrm{L}}$) are not available given the nonlinear portion of the first-order condition, $\phi[(x_{\mathrm{L},1}\mathbf{M}_1 + x_{\mathrm{L},2}\mathbf{M}_2 + x_{\mathrm{L},3}\mathbf{M}_3)\mathbf{X}_{\mathrm{L}}]$. However, if Lotto Investor were to purge his preference for skewness ($\phi \to \infty$), then the first-order condition would default to the traditional case and Equation (3) would describe his demand for securities.

To illustrate the holding and pricing implications of heterogeneous skewness preference, we simplify the model to a case where only one asset exhibits skewness. This simplification also allows us to obtain solutions, albeit numerical, to Lotto Investor's demand function and, more importantly, to investigate equilibrium holdings and prices. Table 1 provides values for parameters used in the numerical solutions including the risk-aversion and skewness preference parameters (τ, ϕ) and the

Table 1
Model parameter values

Parameter	Variable	Value
Risk-aversion coefficient	τ	2.50
Skewness-preference coefficient	Φ	2.50
Variance of asset 1 returns	σ_1^2	0.20*
Variance of asset 2 returns	σ_2^2	0.35*
Variance of asset 3 returns	σ_3^2	0.25*
Correlation coefficient, assets 1 and 2	$\rho_{1,2}$	0.08
Correlation coefficient, assets 1 and 3	$\rho_{1,3}$	0.15
Correlation coefficient, assets 2 and 3	$\rho_{2,3}$	0.10

This table presents values of parameters used in the numerical solutions of the model. Covariance parameters are chosen to match post-war U.S. stock market data as in Campbell, Lettau, Malkiel, and Xu (2001). Values with an asterisk (*) correspond to annualized numbers.

covariance matrix (**V**) elements. Our choice of setting $\tau = 2.5$ is consistent with general perceptions governing the risk-aversion coefficient ranges.

The covariance parameters are chosen to be generally consistent with historical levels of individual stock variances and correlations following Campbell et al. (2001). Specifically, assets one and three are given the lowest variances and highest correlations to mimic typical stock characteristics relative to asset two. Intuitively, asset one might represent a large-cap stock with low variance and high average correlations to other stocks, whereas asset three might represent a small-cap stock with a higher variance and slightly lower correlations than asset one. We assume that the return distributions of assets one and three are completely characterized by the first two moments (i.e., no skewness). In contrast, the return distribution of asset two is assumed to be skewed. For this reason, asset two is given the highest variance of the risky assets thereby maintaining a positive relationship between the second and third moments in stock returns. We assume that asset two's skewness is idiosyncratic which, given our representation of skewness, is equivalent to setting $M_{ijk} = 0$ for all i, j, k except for the case $i = j = k = 2$.[4] Similar to Barberis and Huang (2005), this restriction on the assets' skewness allows us to isolate the potential impact that idiosyncratic skewness may have on equilibrium holdings and prices in our economy.

We obtain the demand functions and associated equilibrium holdings for the two investors using the parameter values in Table 1 and a -0.35 to 0.35

[4] This restriction precludes the possibility of coskewness between assets and may give the impression that our agents irrationally care only about asset specific skewness and not portfolio skewness. While Equation (2) illustrates that Lotto investor does care about portfolio skewness, our restriction on the form of skewness implies that portfolio skewness will only be a function of asset two's skewness. We chose this restriction to illustrate a setting where idiosyncratic skewness impacts holdings and prices. In unreported results, we investigate other cases of portfolio skewness including coskewness and find results similar to other articles such as Kraus and Litzenberger (1976) and Harvey and Siddique (2000).

range of skewness values for asset two. For comparison purposes, we begin with the case where $\phi \to \infty$; a reduction of our model to the standard mean-variance case of identical agents. In this setting, the equilibrium holdings for each investor are simply to hold one-half of the supply of each asset. Given the existence of three risky assets, each investor will place one-third of their wealth into each of the three assets. For comparison's sake we call this equally weighted portfolio the *diversified* portfolio. In fact, investors would still hold this equally weighted portfolio if all investors were to exhibit *identical* preference for skewness ($\phi > 0$) as noted earlier following Cass and Stiglitz (1970). In this case, as in most identical-agent models, agents must hold identical risky portfolios for markets to clear. A notable exception to this rule is Barberis and Huang (2005), where identical agents hold either the market portfolio or an underdiversified portfolio given the nonlinearities of the agents' preference functions. In our model, only when we allow the investors to have heterogeneous skewness preferences do equilibrium holdings differ.

For our base case of $\phi = 2.5$, we find that both investors, and particularly Lotto Investor, hold underdiversified portfolios when asset two's return distribution is skewed. Panel A of Figure 1 provides numerical solutions to Traditional Investor's equilibrium holdings of the three assets over a range of values for skewness levels of asset two. Panel B of Figure 1 provides the analogous results for Lotto Investor. Panel B shows that Lotto Investor's demand for the skewed security (asset two) depends on the sign and magnitude of the skewness, consistent with a preference for positive skewness. In the extreme case, when asset two has positive skewness of 0.35, asset two makes up approximately 50% of Lotto Investor's portfolio. Lotto Investor holds this underdiversified portfolio because his preference structure is *different* from Traditional Investor's in a manner that cannot be mitigated by the existence of a risk-free asset or mutual fund theorem.[5]

Figure 1 illustrates that Traditional Investor's level of underdiversification is also related to the skewness of asset two. Although Traditional Investor holds an underdiversified portfolio, he is more diversified than Lotto Investor, because he holds greater amounts of both asset one and asset three, unlike Lotto Investor who only increases holdings of asset two. For example, in the case where asset two has positive skewness of 0.35, Traditional Investor's equilibrium portfolio variance increases by less than 2% relative to the *diversified* portfolio, while Lotto Investor's portfolio variance is more than 20% greater than the *diversified* portfolio's variance. Given that Traditional Investor's holdings are more heavily loaded into assets one and three, both of which are less risky than asset two, this result is not surprising. The relatively high-variance choice for asset two drives this

[5] In unreported solutions to the model, we find that greater preference for skewness (lower values of ϕ) will lead to more extreme underdiversification by Lotto Investor.

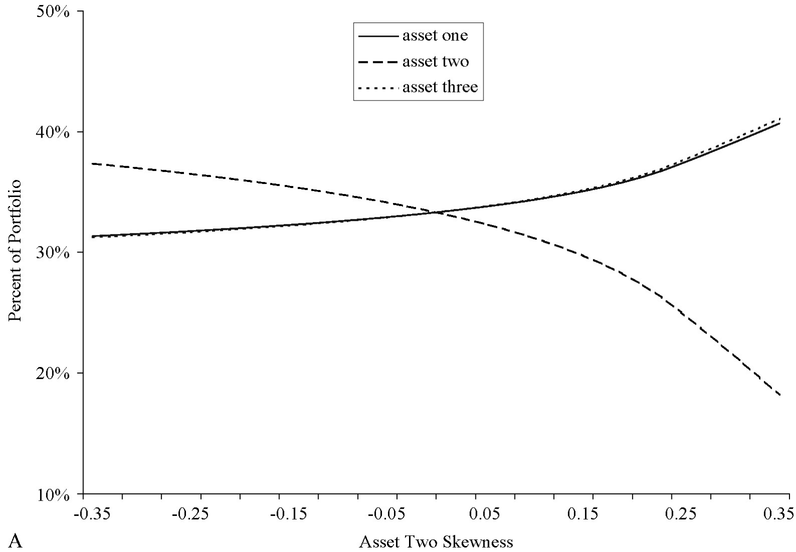

Figure 1
Composition of risky asset portfolios for Traditional Investor (no skewness preference) in Panel A and Lotto Investor (positive skewness preference) in Panel B, as a function of asset two skewness levels.

result, but should not be interpreted as artificial given the well-known positive relationship between skewness and variance—stocks with high return skewness will also have high return variance. So, while both investors hold underdiversified portfolios in our model, the impact on portfolio volatility of underdiversification is much greater for the investor with preference for skewness and, consequently, Lotto Investor is more likely to appear to make irrational holding decisions from a mean-variance perspective. The holding predictions are similar to Malkiel and Xu (2002), who investigate a model in which investors hold underdiversified portfolios; however, exogenous constraints on demand holdings drive the underdiversification in their model. In our model, the investors choose to hold underdiversified portfolios driven by different preferences for skewness.

Lotto Investor's preference for skewness not only impacts security holdings but also causes asset prices to deviate from the mean-variance case. Figure 2 plots the expected excess returns for each of the three securities in our model as a function of asset two's skewness level. The most striking result of Figure 2 is that asset two's expected excess return varies substantially relative to asset two's idiosyncratic skewness level. This relationship generalizes extant asset-pricing intuition, including Kraus and Litzenberger (1976), where coskewness is the exclusive priced component of total skewness. The qualitative relationship between idiosyncratic skewness and expected return of asset two in Figure 2 coincides with the cumulative prospect theory model of Barberis and Huang (2005).

Intuitively, the decline (increase) in asset two's expected return as idiosyncratic skewness increases (decreases) follows from Lotto Investor's willingness to trade mean-variance efficiency for positive skewness or upside potential in his portfolio. Consequently, when asset two's skewness is 0.35, Lotto Investor is willing to hold a portfolio that has a Sharpe ratio that is more than 20% lower than that of his investing counterpart

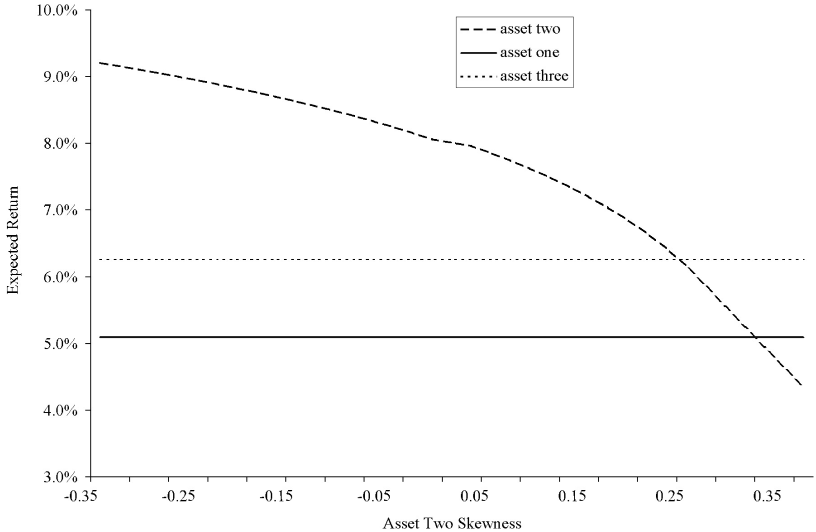

Figure 2
Expected excess returns of three risky assets as a function of asset two skewness levels.

Traditional Investor (0.14 vs. 0.17). In contrast to asset two's expected return dependence on skewness, the expected returns of the two non-skewed securities (one and three) are not affected by the skewness level of asset two given the similar perspective of both Lotto Investor and Traditional Investor regarding the possibilities of these two securities; consequently assets one and three can be priced using standard mean-variance methods. With nonzero idiosyncratic skewness, asset two's expected returns would deviate from the CAPM; lower (higher) returns than predicted when skewness is positive (negative). Given that the security variances are held constant in our model investigation, a plot of Sharpe ratios for the three securities would be scaled versions of the plots in Figure 2. We use the predicted negative relationship between the skewness and the Sharpe ratio of asset two to help formulate our empirical investigations of mean-variance efficiency in subsequent sections of the article and can also help to reconcile the wide variations in the mean-variance efficiency of investor portfolios found by Goetzmann and Kumar (2004).

The three key implications we take from this model to the data are as follows: first, investors with greater demand for skewness will, in equilibrium, hold less-diversified portfolios than investors with less or no demand for skewness; second, mean-variance-skewness efficiency of asset prices could include the case where idiosyncratic skewness is a priced component of returns in addition to coskewness, and consequently underdiversified investors will hold mean-variance-skewness efficient portfolios but may appear to hold inefficient portfolios in a mean-variance perspective (i.e. Sharpe ratios); and third, investors with greater preference for skewness will, in equilibrium, hold securities (and consequently portfolios) with greater levels of, potentially idiosyncratic, skewness.

2. Data and Descriptive Statistics

Our primary data source is a record of monthly investor portfolio holdings at a major U.S. discount brokerage house.[6] The database consists of the portfolios of 78,000 households and contains portfolio positions for the period of January 1991 through November 1996. Many of the households have multiple accounts listed, and we aggregate all holdings to the household level. We exclude from our analysis household portfolios that have no investment in individual equities at any point during the sample period, reducing our sample to 65,562 household portfolios.

[6] This database is used and described in other studies including Barber and Odean (2000), Barber and Odean (2001), Barber et al. (2003), Barber and Odean (2003), Graham and Kumar (2004), Goetzmann and Kumar (2004), and Barber et al. (2004).

Table 2
Descriptive statistics of portfolios by level of diversification

Portfolio size (No. of) stocks)	Number of observations	Portfolio value ($) Mean	25th percentile	Median	75th percentile	Standard deviation	Equity fund holdings Average value ($)	% of portfolios holding funds
			Panel A: Portfolios as of January 1991					
1	14,696	11,083	1706	4529	10,100	36,267	1360	9.5
2	9585	13,884	3625	6875	13,537	36,780	1391	10.1
3	6240	18,267	5131	9498	18,135	38,714	1495	10.3
4	4265	22,598	6763	12,108	22,450	53,739	1573	10.6
5	2898	28,745	8350	15,393	31,175	60,669	1768	10.6
6–9	5518	41,072	11,686	22,112	43,625	79,348	2250	12.1
10+	3809	129,294	25,637	56,939	123,550	505,494	2764	13.3
All	47,011	27,839	4100	9376	22,377	154,301	1646	10.5
			Panel B: Portfolios as of January 1993					
1	12,460	14,586	2,211	5850	13,171	48,433	3969	14.8
2	8919	20,267	4875	9660	19,275	85,007	3977	15.1
3	6030	24,223	7250	13,156	24,429	55,957	3583	15.8
4	4555	31,127	9750	17,198	30,175	64,418	4190	16.5
5	3324	38,061	11,615	20,625	37,531	80,738	3265	15.9
6–9	6987	59,535	17,593	30,728	58,087	329,706	4642	17.8
10+	5585	171,512	39,768	76,888	167,111	366,229	8790	22.7
All	47,860	44,938	6313	15,034	36,252	192,472	4555	16.6
			Panel C: Portfolios as of January 1996					
1	5737	15,889	1375	5523	14,000	54,049	5737	22.0
2	3957	23,768	4829	10,863	22,650	78,075	10,951	23.7
3	2685	34,133	8400	16,131	31,994	144,177	12,377	25.8
4	2138	39,781	12,033	21,903	41,432	80,631	10,837	28.2
5	1614	49,372	15,546	28,295	50,926	82,812	16,270	28.6
6–9	3582	85,821	21,670	39,298	72,031	562,537	12,896	30.6
10+	3592	257,487	50,983	107,821	246,310	674,404	23,357	35.9
All	23,305	71,826	6930	20,123	51,438	361,565	13,187	27.2

This table presents descriptive statistics of the household portfolios in the dataset. The dataset consists of the portfolio holdings of 60,000 investors at a large brokerage house in the U.S. The three panels correspond to three representative points in time over the sample period (January 1991, January 1993, and January 1996). The household portfolios are sorted based on number of stocks held in the portfolio.

Table 2 presents descriptive statistics of the portfolios in the sample at three representative points in time: Panel A presents statistics for January 1991, Panel B for January 1993, and Panel C for January 1996.

The average value of the household portfolios in the sample, after averaging each household's portfolio over all months in which it exists, is $60,560. Of this total value, on average 68% is held in individual equities, 9% in equity mutual funds, and 23% in other investments (such as cash and debt instruments). Following Barber and Odean (2000), Barber and Odean (2001), Graham and Kumar (2004), Goetzmann and Kumar (2004), Barber, Odean, and Strahilevitz (2004), and Ivkovic and Weisbenner (2005), we primarily restrict our attention to investments in individual stocks. The average stock-only portfolio in our sample has a value of $40,940, with a median value of $14,323.

Restricting the dataset to equities, of course, eliminates consideration of other investments through which investors may be able to capture skewness. Option investing, in particular, would seem to be a logical way to capture skewness. We find, however, that option investing is relatively uncommon for investors in this database, with less than 1% of investors holding options in early years, and just over 2% of investors holding options in later years. We also find that, on average, investors that are less diversified in equities hold a larger proportion of their overall portfolio in options. Thus, it seems unlikely that including options in our analysis would alter the finding that less diversification is associated with greater skewness.

This investor database is subject to the concern that the portfolios do not necessarily represent the entire investment holdings of the households in the database. Thus, we could be underestimating or overestimating the actual degree of diversification of investors in the database. In addition, the portfolio holdings may reflect investor decision-making only for a portion of investors' wealth, and not their overall attitude toward risk and return. For example, if an investor holds a larger, more-diversified portfolio elsewhere, she may be more inclined to take risks with a smaller portion of her investments. Goetzmann and Kumar (2004) address this concern with the database in detail, and conclude that, by and large, the accounts in this database represent substantial portions of investor wealth. They note that median stock portfolios of investors in the overall U.S. population, according to the Survey of Consumer Finances, were $16,900 in 1992 and $15,300 in 1995. The median portfolio value of over $14,000 in this database is of comparable size, and suggests that the portfolios in the database would represent serious, nontrivial investments for most U.S. households.

In addition to the investor database described above, we obtain monthly return data from the Center for Research in Security Prices (CRSP) to measure the performance of the stocks in the investor database. We merge the two databases by matching CUSIPs in the investor database with CUSIPs in the CRSP database.

3. Skewness in Underdiversified Portfolios

The degree to which underdiversified investors capture skewness in portfolio returns is not well known. In this section, we compare return characteristics of portfolios held by investors in order to assess the magnitude of the differences in portfolio return skewness for different levels of portfolio diversification. To make the comparisons (and throughout the article), we use three different measures of diversification. The first measure, D^1, is defined as

$$D_j^1 = 1/n, \tag{5}$$

where n is the number of securities in household j's portfolio. We use the inverse of the number of securities for the convenience of being directionally consistent with our other two diversification measures, both of which also associate lower levels of diversification with higher values. The D^1 measure of diversification holds the advantage of simplicity (and popular usage), but it does not capture differences in the weighting of securities within portfolios (e.g., a portfolio equally weighted in two securities would be treated the same as a portfolio with 99% weight in one security and 1% weight in another).

Our second diversification measure, D^2, is a Herfindahl index of the weights of each security in the household's portfolio defined as

$$D_j^2 = \sum_{i=1}^{n} w_{ij}^2, \tag{6}$$

where w_{ij} is the weight of security i in household j's portfolio. Higher values of D^2 indicate lower levels of diversification. D^2 has an advantage over D^1 in that it captures differences in weighting within portfolios. However, D^2 still does not capture covariance differences between securities within a household's portfolio (e.g., a portfolio equally weighted in two technology stocks would be treated the same as a portfolio equally weighted in one technology stock and one utility stock).

Our third diversification measure, D^3, accounts for covariances between securities. D^3 is defined as

$$D^3 = \sum_{i=1}^{n} w_{i,j}^2 + \left(1 - \sum_{i=1}^{n} w_{i,j}^2\right) \overline{\rho}_j \tag{7}$$

where $\overline{\rho}_j = \sum_{h=1}^{n} \sum_{i=i}^{n} w_{h,j} w_{i,j} \rho_{hi} / \sum_{h=1}^{n} \sum_{i=i}^{n} w_{h,j} w_{i,j}$. As with D^1 and D^2, higher values of D^3 indicate lower levels of diversification. As can be seen in Equation (7), D^3 demonstrates that diversification can be increased by either reducing the concentration of stocks within the portfolio or by reducing the correlation of stocks within the portfolio. D^3 thus accounts for both the security weights in the portfolio and the portfolio's covariance structure. However, D^3 is also not without weakness. Specifically, although Equation (7) reflects both types of diversification (reducing concentration and reducing correlation), the weight the formula places on each type of diversification is somewhat arbitrary. In addition, D^3 requires an estimate of the correlations in a given portfolio and a fraction of the securities held by investors in the dataset are not found in the CRSP dataset or do not have a return sample sufficiently large enough to estimate covariances.

Consequently, some household portfolios are excluded from our analysis when we report D^3-based results.

Our standard measure of the skewness of portfolio returns is the skewness coefficient,

$$\hat{\mu}^3 = \frac{\frac{1}{60}\sum_{t=1}^{60}(r_t - \mu)^3}{\hat{\sigma}^3}, \tag{8}$$

where the coefficient is calculated from a 60-month window of monthly returns and $\hat{\sigma}^3$ is the cube of the estimated return standard deviation. An important feature of the skewness coefficient is that it is scaled by the variance of returns. Equation (8) adjusts for the fact that variance and skewness are positively correlated, that is, we are essentially measuring the incremental skewness over what would be expected given the level of variance in returns. Equation (8) measures total skewness of returns; in later analysis we will decompose skewness to analyze the elements of coskewness and idiosyncratic skewness.

We sort all household portfolios into deciles according to diversification levels at three representative dates in our sample period (January 1991, January 1993, and January 1996). We calculate returns of these portfolio holdings for 60 subsequent months. We then calculate the average skewness coefficient (as well as the average mean and variance) of the monthly returns for each portfolio. Table 3 presents the average return statistics of each decile. In the table, Panel A presents results for portfolios sorted on D^2 and Panel B for portfolios sorted on D^3. In both panels, results are presented for each of the three time periods. Decile 1 (10) corresponds to the most (least) diversified portfolios. The deciles are of roughly equal size except for Decile 10, which contains all single-stock portfolios. (All three diversification measures, D^1, D^2, and D^3, equal 1 for single-stock portfolios.)

Table 3 shows, first of all, that mean returns tend to be slightly higher for less-diversified investors. The average variance of returns is also considerably higher for less-diversified portfolios. Most important for our purposes, Table 3 demonstrates that skewness coefficients are much higher for less-diversified portfolios. All six cases reported show an increasing trend moving from Decile 1 to Decile 10. In some time periods, the increases in skewness are particularly large. For example, for the 1996 data in Panel A, portfolios in the most-diversified decile have an average skewness coefficient of −0.115, whereas portfolios in the least-diversified decile have an average skewness coefficient of 0.428. The degree of skewness obtained by underdiversified investors is substantial.

Table 3
Monthly return statistics of household portfolios sorted by level of diversification

Panel A: Portfolios sorted on D^2

	January 1991					January 1993					January 1996				
Decile	N	D^2 mean	Mean	Variance	Skewness	N	D^2 mean	Mean	Variance	Skewness	N	D^2 mean	Mean	Variance	Skewness
High div.	3424	0.128	0.015	0.002	0.238	3753	0.106	0.015	0.002	−0.052	1 845	0.095	0.018	0.005	−0.115
2	3424	0.222	0.015	0.004	0.331	3754	0.185	0.015	0.003	0.063	1845	0.170	0.018	0.007	0.096
3	3415	0.296	0.016	0.006	0.355	3754	0.250	0.015	0.004	0.115	1845	0.234	0.018	0.009	0.163
4	3416	0.363	0.016	0.006	0.367	3751	0.317	0.015	0.005	0.173	1843	0.301	0.018	0.011	0.225
5	3398	0.445	0.016	0.009	0.381	3728	0.382	0.015	0.007	0.209	1830	0.373	0.019	0.014	0.309
6	3342	0.511	0.017	0.009	0.402	3694	0.477	0.015	0.008	0.226	1810	0.467	0.019	0.016	0.373
7	3341	0.574	0.017	0.010	0.390	3681	0.537	0.015	0.009	0.250	1794	0.534	0.019	0.020	0.401
8	3298	0.706	0.017	0.013	0.419	3636	0.651	0.016	0.011	0.258	1775	0.652	0.019	0.020	0.426
9	3175	0.918	0.017	0.017	0.381	3534	0.883	0.016	0.013	0.289	1689	0.885	0.019	0.026	0.450
Low div.	13,076	1.000	0.017	0.015	0.437	11,271	1.000	0.016	0.013	0.315	4775	1.000	0.018	0.025	0.428

Panel B: Portfolios sorted on D^3

	January 1991					January 1993					January 1996				
Decile	N	D^3 mean	Mean	Variance	Skewness	N	D^3 mean	Mean	Variance	Skewness	N	D^3 mean	Mean	Variance	Skewness
High div.	3107	0.399	0.016	0.007	0.269	3414	0.295	0.014	0.004	0.185	1703	0.232	0.018	0.007	0.090
2	3102	0.490	0.016	0.006	0.355	3416	0.379	0.015	0.004	0.114	1701	0.306	0.018	0.008	0.086
3	3101	0.545	0.015	0.006	0.342	3408	0.434	0.015	0.005	0.123	1701	0.361	0.018	0.010	0.184
4	3098	0.592	0.015	0.006	0.347	3403	0.485	0.015	0.005	0.129	1698	0.416	0.018	0.011	0.205
5	3090	0.640	0.016	0.007	0.345	3403	0.538	0.015	0.005	0.134	1693	0.474	0.019	0.012	0.239
6	3089	0.689	0.016	0.007	0.351	3393	0.594	0.015	0.006	0.164	1681	0.539	0.019	0.015	0.281
7	3091	0.744	0.016	0.008	0.330	3392	0.660	0.016	0.007	0.161	1688	0.614	0.018	0.014	0.275
8	3068	0.815	0.016	0.011	0.297	3377	0.745	0.016	0.007	0.131	1677	0.710	0.019	0.017	0.332
9	3014	0.930	0.016	0.011	0.341	3323	0.893	0.016	0.010	0.225	1648	0.881	0.019	0.020	0.384
Low div.	13,076	1.000	0.017	0.015	0.427	11,271	1.000	0.016	0.013	0.305	4775	1.000	0.018	0.025	0.428

This table presents return statistics of household portfolios for the 60 months subsequent to portfolio formation. Returns are reported subsequent to three representative months during the sample period (January 1991, January 1993, and January 1996). Portfolios are sorted into deciles corresponding to their level of diversification as measured by D^2 and D^3, which are defined in Equations (6) and (7) respectively. N is the number of portfolios in the decile. The mean return is the average monthly return for portfolios in the decile over the subsequent 60 months. The variance (skewness) of returns is the variance (skewness) of the 60 monthly returns, calculated for each individual portfolio and averaged over all portfolios in the decile.

To understand more intuitively how these high skewness coefficients translate into outcomes that may appeal to underdiversified investors, we look at some examples of outcomes for underdiversified investors compared to diversified investors. We rank all households by their level of diversification, on the basis of D^2, and compare outcomes for the quartile of the most-diversified investors with outcomes for the quartile of the least-diversified investors. We take the portfolio positions of all investors as of January 1993 (other time periods yield similar results) and calculate subsequent returns to those positions over various return horizons from 1 month up to 5 years. We rank all portfolios on the basis of their total-return performance over a given return horizon. In results not tabulated, we find that the least-diversified investors are far more likely to achieve high-ranking returns than are the most-diversified investors. For example, over a 6-month return horizon, among the top 1% of performers in the entire sample, the least-diversified investors outnumber the most-diversified investors 11 to 1. The differences are even more pronounced over other horizons. Over a 3-year return horizon, the least-diversified investors are *26 times* more likely to be among the top 1% of performers than are the most-diversified investors. We present these statistics as examples simply to give intuition behind the possible mindset of underdiversified investors with a preference for skewness. If an investor has a strong desire to achieve very high performance, remaining underdiversified appears to be a reasonable strategy.[7]

In summary, the results in this section show that the return skewness attained by underdiversified investors is substantial. On an intuitive level, investors with a strong preference for skewness may reasonably sacrifice the mean-variance efficiency of diversification to attain higher skewness. In the next section, we more formally examine the trade-off between mean-variance efficiency and skewness.

4. Skewness and Mean-Variance Efficiency

In this section we assess the efficiency of household portfolios when return skewness is taken into account. Our analysis extends Goetzmann and Kumar (2004), who perform a mean-variance analysis of these portfolios and find that diversified portfolios are more likely to be above the capital market line in the mean-variance plane than underdiversified portfolios, indicating that underdiversified portfolios, on average, are less efficient than diversified portfolios. However, if underdiversified portfolios also have more skewness than diversified portfolios, then in a mean-variance-skewness analysis, underdiversified portfolios may not be less efficient.

[7] Of course, the underdiversified investor is also much more likely to appear among the *lowest* 1% of performers. But losses in the lowest percentile are capped at -100%, whereas in the highest percentile in this sample, potential gains run well over 2000% (depending on the time horizon).

To assess the mean-variance-skewness efficiency of the household portfolios, ideally we would construct an efficient frontier in mean-variance-skewness space using the return characteristics of available stocks at times of portfolio formation. However, the large number of calculations required to construct this frontier in three dimensions makes this approach intractable. As an alternative, we study the relationship between the mean-variance efficiency and the skewness of the portfolios. To measure mean-variance efficiency, we construct each household's Sharpe ratio, $(E[r_j] - r_f)/\sigma_j$, using the monthly performance of the securities in their portfolio over the preceding 60 months. In other words, we measure the portfolio *ex ante* performance, which reflects the performance investors would expect at the time of portfolio formation based on prior performance.

Panel A of Figure 3 plots the nonparametric estimated relationship between each household's Sharpe ratio and the portfolio diversification level. We estimate the nonparametric relationship using a kernel regression as follows:

$$sr(D^i) = \frac{\sum_{j=1}^{n} k\left(\frac{D^i_j - D^i}{h}\right) sr_j}{\sum_{j=1}^{n} k\left(\frac{D^i_j - D^i}{h}\right)}, \tag{9}$$

where h is a bandwidth parameter, chosen to minimize mean-squared error following Härdle (1990), $k(\cdot)$ is a kernel function, and n is the number of households.[8] Estimating the relationship between the investors' Sharpe ratios and diversification measures using a kernel regression allows the data to inform the relationship without imposing any parametric structure. Panel A of Figure 3 plots estimates of Equation (9) using D^3 as the diversification measure. Similar results (not shown) are obtained for the other diversification measures. Panel A illustrates the relationship between mean-variance efficiency and diversification for three representative points in time during the sample period. All three plots exhibit a clear downward trend, indicating that Sharpe ratios are lower for less-diversified portfolios. The tight standard error bands indicate these differences are statistically significant.

Moving beyond mean-variance analysis, however, Panel B of Figure 3 demonstrates that underdiversified investors receive a benefit for their lack

[8] Equation (9) is equivalent to the Nadaraya–Watson kernel estimator, which has been shown by Stone (1977) to be a local-linear estimator. Intuitively, this estimator assumes the relationship between households' Sharpe ratios and corresponding diversification measures is a constant within a neighborhood of diversification measures as defined by $(D^i_j - D^i)/h$. The estimated constant is found by minimizing squared errors for all households with diversification measures close to a particular value of D^i.

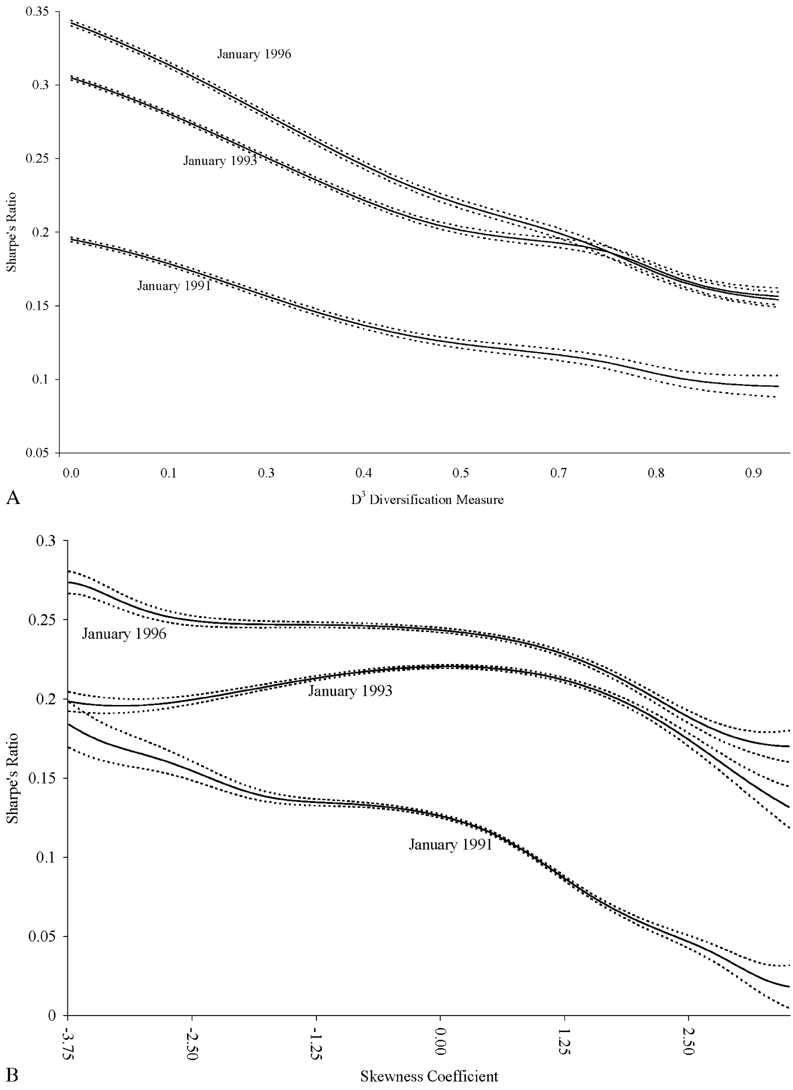

Figure 3
Kernel-estimated relationship between each household's Sharpe ratio and the diversification measure D^3 (Panel A) and household portfolio skewness coefficient (Panel B). Estimates are solid lines, while two-standard error bands are dashed lines.

of mean-variance efficiency. Panel B plots the kernel-estimated relationship between each household's Sharpe ratio and the portfolio skewness (similar to Equation (9) but where each household's diversification measure D^3 is replaced with their portfolio's skewness measure.) Again, the relationship is shown for three representative points in time. Although exceptions over certain ranges exist in each of the three plots, the downward trending relationship between these variables is clear. The differences between the Sharpe ratios of highly skewed portfolios and negatively skewed portfolios are also statistically significant. In other words, household portfolios with higher skewness coefficients have lower mean-variance efficiency. Panel B illustrates a trade-off facing investors: on average, greater skewness comes at the expense of mean-variance efficiency.

Two contrasting interpretations of this negative relationship between skewness and mean-variance efficiency are possible. On one hand, the relationship might demonstrate that investors with greater skewness preference consciously sacrifice mean-variance efficiency in order to obtain greater skewness. On the other, the relationship might demonstrate that underdiversified investors unwittingly obtain greater skewness, whether they have a greater preference for it or not, as a by-product of their failure to diversify. After all, intuition suggests, and the results of Section II document, that skewness increases with lower levels of diversification.

In an effort to determine whether higher skewness in investors' portfolios is consciously chosen or unintended, we turn to regression analysis. Our goal is to determine whether the negative relationship between mean-variance efficiency and skewness, as predicted by our theoretical model, persists even after controlling for the portfolio's level of diversification. A negative relationship between mean-variance efficiency and skewness that persists after controlling for diversification would suggest that something more than just a naïve lack of diversification may be at work. Specifically, finding a negative relationship between mean-variance efficiency and skewness, holding diversification constant, would indicate that investors *consciously* construct their portfolios in a way that will increase skewness at the expense of mean-variance efficiency.

We estimate the following regression model,

$$sr_{i,t} = \alpha_0 + \alpha_1 \hat{\beta}_{i,t} + \alpha_2 \hat{\mu}^3_{i,t} + \alpha_3 D^j_{i,t} + \varepsilon_{i,t}, \tag{10}$$

where $sr_{i,t}$ is the Sharpe ratio of household i's portfolio at month t, $\hat{\beta}_{i,t}$ is the estimated CAPM beta of household i's portfolio at month t, $\hat{\mu}^3_{i,t}$ is the skewness coefficient of household i's portfolio at month t, $D^j_{i,t}$ is diversification measure j ($j = 1, 2,$ or 3) for household i, and $\varepsilon_{i,t}$ is a residual. Given the structure of Equation (10), two estimation and inference complications must be addressed. To see the first complication, we rewrite a simplified version of Equation (10) as $sr_{i,t} = \alpha_0 + \boldsymbol{\alpha}\mathbf{X}_t + \varepsilon_{i,t}$, where $\boldsymbol{\alpha}$ is the vector of slope parameters and $\mathbf{X}_t$ is our vector of regressors. Because the regressors found in $\mathbf{X}_t$ are generated, or estimated, we face an errors-in-variables problem with the estimation of Equation (10), and consequently inference using standard estimation and testing methods may be flawed. To correct for this problem, we adjust the standard errors of our parameter estimates following Pagan (1984) and Shanken (1992) as $\sigma_c^2(\boldsymbol{\alpha}) = \sigma_{OLS}^2(1 + \boldsymbol{\alpha}\boldsymbol{\Sigma}_X^{-1}\boldsymbol{\alpha}) + \boldsymbol{\Sigma}_X/T$,where σ_{OLS}^2 is the standard OLS-estimated parameter variance, $\boldsymbol{\Sigma}_X$ is the covariance matrix of generated regressors, and T is the sample size used to estimate regressors.

The second problem results because both $sr_{i,t}$ and $\hat{\mu}_{i,t}^3$ contain a power of the estimated second moment. Consequently, the regression residual is likely to be correlated with the regressor, $\hat{\mu}_{i,t}^3$, violating standard estimation techniques. Many methods based on instrumental variable techniques exist to correct for this problem. Given the large cross-sectional (up to 40,000 accounts each month) and small time-series (70 months) aspect of our dataset, we correct for this potential correlation by regressing the raw skewness coefficients for each household's portfolio in a given month against the estimated household portfolio standard deviation in that month. We then use the residuals from these regressions as our measures of skewness in Equation (10) instead of the skewness coefficient. This measure of skewness is orthogonal to residual movements based on the standard deviation component of the household Sharpe ratio, satisfying OLS estimation assumptions. Following prior analysis in the article, we estimate Equation (10) using OLS for our three representative points in time over the sample period.

Table 4 reports the coefficients from the estimation of Equation (10). We present three panels that correspond to the three points in time studied. The first row of each panel reports the estimates without a control for diversification included in the model. For all three time periods, the estimated coefficient on skewness is negative, confirming the negative relationship between skewness and mean-variance efficiency, as predicted by our theoretical model and observed empirically in Panel B of Figure 3.. In the second, third, and fourth rows of each panel we control for the level of diversification of the household portfolios, measured by D^1, D^2, and D^3, respectively. A negative relationship between diversification and the Sharpe ratio is also expected on the basis of results already presented. In all nine cases, the coefficient on the diversification measure is negative and significant at the 1% level. Importantly, the coefficient on skewness remains negative while controlling for all three measures of diversification in all three panels. Across all twelve specifications, the t-statistics on α_2 range from -1.31 to -4.11 and, performing a one-tailed test on whether the coefficients are negative, show that all but two of the coefficients are significant at the 5% level for the three time periods reported.[9]

[9] We report results for separate time periods for consistency with the rest of the article and with previous literature. Panel estimation of the coefficients would be complicated by the time-series properties of regression variables and by the large size of the cross-section relative to the time series. To ensure that our three time periods are representative of the entire dataset, we estimate Equation (12) for all months of the dataset and find α_2 to be negative in 66 out of 70 sample months estimating the version of Equation (12) that is reported on the first line of Table 4. T-Statistics constructed using the time-series averages of α_2 are negative and significant at the 5% level for all versions of Equation (12) in Table 4.

Table 4
Tests of Sharpe ratio with skewness and diversification

Date	Constant	β	μ^3	D^1	D^2	D^3	R^2 (%)
January-91	0.1131	0.0112	−0.0760	—	—	—	1.78
	(0.0057)	(0.0096)	(0.0582)				
	0.1542	−0.0056	−0.0639	−0.0778	—	—	3.94
	(0.0077)	(0.0082)	(0.0388)	(0.0048)			
	0.1694	−0.0047	−0.0665	—	−0.0887	—	4.42
	(0.0101)	(0.0084)	(0.0412)		(0.0054)		
	0.1683	0.0039	−0.0716	—	—	−0.0718	1.89
	(0.0201)	(0.0091)	(0.0420)			(0.0090)	
January-93	0.2145	0.0977	−0.0039	—	—	—	0.52
	(0.0013)	(0.0092)	(0.0013)				
	0.2838	0.0524	−0.0045	−0.1414	—	—	9.92
	(0.0017)	(0.0076)	(0.0011)	(0.0024)			
	0.2968	0.0586	−0.0055	—	−0.1473	—	9.43
	(0.0021)	(0.0079)	(0.0013)		(0.0027)		
	0.2968	0.0586	−0.0042	—	—	−0.1436	9.54
	(0.0021)	(0.0079)	(0.0011)			(0.0026)	
January-96	0.2596	0.0729	−0.0495	—	—	—	1.45
	(0.0020)	(0.0060)	(0.0163)				
	0.3487	0.0465	−0.0281	−0.1948	—	—	21.83
	(0.0024)	(0.0048)	(0.0160)	(0.0034)			
	0.3821	0.0482	−0.0273	—	−0.2191	—	24.22
	(0.0030)	(0.0049)	(0.0161)		(0.0036)		
	0.4160	0.0527	−0.0311	—	—	−0.2392	19.18
	(0.0053)	(0.0057)	(0.0165)			(0.0052)	

This table presents coefficient estimates of the following regression model,

$$sr_{i,t} = \alpha_0 + \alpha_1 \beta_{i,t} + \alpha_2 \mu^3_{i,t} + \alpha_3 D^j_{i,t} + \varepsilon_t$$

where $sr_{i,t}$ is the Sharpe ratio of the household i's portfolio on month t, β is the CAPM beta of the household's portfolio, and μ^3 is the residual of a regression of total skewness of the household's portfolio on the household's portfolio standard deviation, all estimated from the prior 60 months returns. D^1, D^2, and D^3 are the diversification measures as defined in Equations (7), (8), and (9) respectively. The models are estimated for three representative points in time in the sample period (January 1991, January 1993, and January 1996). Standard errors, below in parentheses, are constructed following Pagan (1984) for generated regressors.

These regression results show that the negative relationship between the Sharpe ratio and skewness persists even after controlling for the diversification level of household portfolios. While certainly some proportion of underdiversified investors are likely underdiversified for reasons other than skewness preference, the persistence of the negative relationship between mean-variance efficiency and skewness suggests that a material proportion of the underdiversified investors are intentionally selecting securities for their portfolios that will increase skewness at the expense of mean-variance efficiency.

4.1 Decomposition of skewness

We now ask the question of what *type* of skewness drives the results of Table 4. As discussed previously, other models incorporating skewness predict that only coskewness with the market portfolio will be relevant for investor decision-making [Kraus and Litzenberger (1976), Harvey and Siddique (2000)]. In contrast, our model of heterogeneous preference for skewness predicts that idiosyncratic skewness can be priced when a fraction of investors have preference for skewness and hold underdiversified portfolios in equilibrium. To investigate this prediction, we reproduce the mean-variance inefficiency results of Table 4, but this time we decompose the skewness of household portfolio returns to obtain the idiosyncratic component of skewness and the portfolio's coskewness with the market portfolio. Following Harvey and Siddique (2000), we measure the coskewness of the household portfolio, CS_i, as the estimated coefficient on the squared market excess return from the following regression,

$$r_{i,t} - r_f = \alpha + \beta_i(r_{m,t} - r_f) + CS_i(r_{m,t} - r_f)^2 + \varepsilon_{i,t}, \tag{11}$$

where r_m is the market portfolio return. Our coskewness measure represents the contribution of the individual security to the overall skewness of the market portfolio. We measure the idiosyncratic skewness of household portfolio returns, IS_i, as the skewness coefficient defined in Equation (8) constructed using the residuals from Equation (11).[10]

As in Table 4, Table 5 reports the results of regressions of household portfolio Sharpe ratios on a number of explanatory variables including portfolio beta, the level of portfolio diversification (D^3 is used here), and total portfolio skewness. In addition, the regressions in Table 5 include the above measures of coskewness and idiosyncratic skewness. The results show that idiosyncratic skewness, in addition to coskewness, helps to explain Sharpe ratios. The negative signs on the skewness coefficients are consistent with the prediction that investors are willing to sacrifice mean-variance efficiency to obtain greater skewness, and hence portfolios with greater skewness have lower Sharpe ratios. The seventh column of the table shows that idiosyncratic skewness (IS), in particular, is an important determinant of Sharpe ratios. In all specifications and in all time periods in Table 5, the coefficient on idiosyncratic skewness is negative (as predicted by our model) and is always significant at the 1% level. Coskewness (CS) is also shown to be a statistically significant determinant of Sharpe ratios, but the effect of coskewness is less consistent. In the first two time periods the coefficient on coskewness is negative, as expected, but the results for January 1996 show a positive coefficient, opposite of that predicted by Kraus and Litzenberger (1976) and Harvey and Siddique (2000).

The result that idiosyncratic skewness may be relevant for asset pricing, as predicted by our model, runs contrary to most asset-pricing models and prior empirical investigations into pricing of skewness. The behavioral model of Barberis and Huang (2005) is the only other

[10] We also investigated other functional forms of Equation (11), and find the idiosyncratic skewness results in Table 5 to be robust.

Table 5
Tests of Sharpe ratio with a decomposition of skewness

Date	Constant	β	D^3	μ^3	CS	IS	R^2 (%)
January 91	0.1683	0.0039	−0.0718	−0.0716	—	—	1.89
	(0.0201)	(0.0091)	(0.0090)	(0.0420)			
	0.1654	0.0524	−0.0698	—	−0.0289	—	1.35
	(0.0038)	(0.0079)	(0.0039)		(0.0042)		
	0.1665	0.0038	−0.0708	—	—	−0.0489	1.28
	(0.0041)	(0.0042)	(0.0041)			(0.0122)	
	0.1671	0.0375	−0.0870	—	−0.0249	−0.0378	4.21
	(0.0051)	(0.0092)	(0.0046)		(0.0048)	(0.0137)	
January 93	0.2968	0.0586	−0.1436	−0.0042	—	—	9.54
	(0.0021)	(0.0079)	(0.0026)	(0.0011)			
	0.2936	−0.0271	−0.1442		−0.0089	—	9.62
	(0.0019)	(0.0106)	(0.0026)		(0.0013)		
	0.2932	0.0578	−0.1459	—	—	−0.0085	9.34
	(0.0020)	(0.0078)	(0.0027)			(0.0026)	
	0.2936	−0.0272	−0.1440	—	−0.0090	−0.0087	9.76
	(0.0019)	(0.0106)	(0.0026)		(0.0013)	(0.0025)	
January 96	0.4160	0.0527	−0.2392	−0.0311	—	—	19.18
	(0.0053)	(0.0057)	(0.0052)	(0.0165)			
	0.4139	0.0638	−0.2343	—	0.0022	—	19.50
	(0.0045)	(0.0054)	(0.0048)		(0.0010)		
	0.4159	0.0527	−0.2380	—	—	−0.0774	19.37
	(0.0061)	(0.0061)	(0.0055)			(0.0160)	
	0.4160	0.0527	−0.2392	—	0.0018	−0.0650	24.60
	(0.0061)	(0.0063)	(0.0056)		(0.0010)	(0.0160)	

This table presents coefficient estimates of the following regression model,

$$sr_{i,t} = \alpha_0 + \alpha_1\beta_{i,t} + \alpha_2\mu^3_{i,t} + \alpha_3 D^3_{i,t} + \alpha_4 CS_{i,t} + \alpha_5 IS_{i,t} + \varepsilon_t$$

where $sr_{i,t}$ is the Sharpe ratio of household i's portfolio on month t, β is the CAPM beta of the household's portfolio, μ^3 is the residual of a regression of total skewness of the household's portfolio on the household's portfolio standard deviation, CS is the coskewness, and IS is the idiosyncratic skewness of the household's portfolio, all estimated from the prior 60 months returns. Coskewness is calculated as the regression coefficient on a market model including the squared excess market return. Idiosyncratic skewness is calculated using the residuals from the market model regression to construct a skewness coefficient. D^3 is the diversification measure as defined in Equation (7). The models are estimated for three representative points in time in the sample period (January 1991, January 1993, and January 1996). Standard errors, below in parentheses, are constructed following Pagan (1984) for generated regressors.

model, to our knowledge, that predicts underdiversification and the pricing of idiosyncratic skewness. The results of Table 5 suggest that underdiversification and pricing of idiosyncratic skewness may occur even without behavioral preferences or misestimation of tail probabilities. Both Barberis and Huang (2005) and our results suggest that more investigation into the pricing of skewness is warranted.

5. Stock Selection of Underdiversified Investors

In this section, we describe the stock selection behavior of both underdiversified and diversified investors. We test whether there appears to be conscious intent by investors to capture greater skewness in their stock selection. To investigate security selection as it relates to skewness, we construct a security-specific measure that reflects the average level of diversification of all investors that hold the security. This measure, which we refer to as the security's average investor diversification (SAID), is defined as,

$$\overline{SAID_k}^j = \frac{1}{n_k}\sum_{i=1}^{n_k} D_i^j \tag{12}$$

where n_k is equal to the number of investors in our database holding security k at a given point in time and D_i^j corresponds to investor i's diversification measure j ($j = 1, 2, 3$).[11] Equation (12) indicates that $\overline{SAID}^j$ measures the average diversification measure of all investors holding a security, and thus reflects the relative popularity of a stock among diversified and underdiversified clientele. Each month we construct the measure $\overline{SAID}^j$ for all stocks in the database and then sort the stocks into deciles based on their $\overline{SAID}^j$ ranking. To investigate the skewness characteristics of the stocks across the diversification deciles, at each month and for each security we calculate one skewness coefficient using the past 60 months of returns for the security and then calculate another skewness coefficient using the subsequent (to the sorting month) 60 months of returns. We thus calculate both a historical and a forward-looking skewness measure for all stocks and for every month in the dataset.

Table 6 reports the time-series average (as well as minimum and maximum) of the cross-sectional average monthly skewness coefficients for the $\overline{SAID}$-sorted deciles. Panel A presents results for diversification measure D^2, and Panel B presents results for D^3. The results consistently show that underdiversified investors place relatively greater portfolio weight on highly skewed stocks, consistent with Goetzmann and Kumar (2004). We also document that the magnitude of the average skewness of stocks selected by underdiversified investors is large. For example, in the historical returns in Panel A, the average skewness coefficient in the least-diversified decile (D^2) is nearly twice as large as the average skewness coefficient of the most-diversified decile, and is more than twice as large as the average skewness coefficient for deciles two through four.[12] Similar results hold for the forward-looking skewness results and for the D^3 diversification measure. The minimum and maximum columns tell a similar story; upside potential is much higher in securities that are

[11] Goetzmann and Kumar (2004) construct a similar measure.

[12] The results in Table 6 are nearly monotonic in the skewness relationships across diversification deciles apart from decile one which has a larger skewness estimate relative to other well-diversified deciles. We find that many of the stocks that are sorted into decile one are stocks that only a few investors in our dataset (often just one) hold and these households happen to hold well-diversified portfolios. Estimates of $\overline{SAID}^j$ in these cases are extremely noisy and unreliable given the small number of households (n_k) for which we can construct average measures in Equation (12). When we restrict our analysis to only securities that are held by more than three investors, this anomaly in decile one goes away.

Table 6
Skewness of securities ranked by the average diversification levels of their investors

Panel A: Securities' average investor diversification (SAID) determined by D^2

	Average skewness of securities in decile (historical returns)			Average skewness of securities in decile (subsequent returns)		
Decile	Mean	Minimum	Maximum	Mean	Minimum	Maximum
High SAID	0.6154	0.2144	0.8104	0.6187	0.4099	0.7801
2	0.4998	0.1866	0.7244	0.5596	0.4119	0.7521
3	0.4889	0.2159	0.6935	0.5204	0.3444	0.6621
4	0.4867	0.2538	0.6669	0.5384	0.3531	0.7038
5	0.5386	0.2888	0.8304	0.5940	0.4488	0.7599
6	0.6208	0.4207	0.8531	0.6629	0.4636	0.8993
7	0.7589	0.4604	0.9411	0.7409	0.5221	0.9441
8	0.8616	0.6289	1.0329	0.8100	0.6116	0.9458
9	0.9443	0.5289	1.0948	0.8280	0.6385	0.9810
Low SAID	1.0968	0.7737	1.2873	0.9478	0.6950	1.1345

Panel B: Securities' average investor diversification (SAID) determined by D^3

	Average skewness of securities in decile (historical returns)			Average skewness of securities in decile (subsequent returns)		
Decile	Mean	Minimum	Maximum	Mean	Minimum	Maximum
High SAID	0.5092	0.1226	0.6869	0.5547	0.3414	0.7446
2	0.4300	0.1191	0.6316	0.4999	0.3524	0.6813
3	0.4370	0.1628	0.6308	0.4791	0.2945	0.6578
4	0.4936	0.2398	0.7117	0.5401	0.3530	0.7080
5	0.5378	0.3385	0.7789	0.6000	0.4552	0.7694
6	0.6737	0.4622	0.8911	0.7142	0.5122	0.8718
7	0.8103	0.5145	1.0124	0.7807	0.5651	0.9603
8	0.9093	0.7201	1.1060	0.8488	0.6336	1.0216
9	1.0164	0.6222	1.1568	0.8724	0.6546	1.0387
Low SAID	1.1365	0.8000	1.3220	0.9861	0.7220	1.1702

This table presents summary statistics of the return skewness of individual securities. The securities are sorted into deciles based upon the average diversification level of investors holding each security. The average diversification level of investors holding each security is measured by SAID, as in Equation (12). Decile 1 (10) represents the set of securities that are most prevalent in the portfolios of the most (least) diversified investors. The level of diversification is measured by D^2 and D^3, as defined in Equations (6) and (7), respectively. Stocks are sorted into the SAID deciles, and return statistics calculated for each individual security, in each month of the 71-month sample period (January 1991 through November 1996). The first set of statistics are calculated from the 60 months of data *prior to* household portfolio formation, and the second set of statistics are calculated from 60 months of data *after* household portfolio formation. The mean (minimum, maximum) is the mean (minimum, maximum) of the 71 months of average skewness levels calculated for each decile.

popular with underdiversified investors. These results strongly suggest that the skewness that underdiversified investors obtain is not coincidental, but rather is driven at least partly by an intentional process of security selection. In addition, investors who prefer skewness are able to capture substantial upside potential by not only holding an underdiversified portfolio but also by careful security selection; underdiversified investors are successful in choosing stocks with substantially higher skewness than stocks common to diversified investors.[13]

5.1 Heterogeneous preferences for skewness

A key assumption of our theoretical results is that differing skewness preferences can lead to underdiversified equilibrium holdings. We investigate household characteristics that are correlated with skewness in household portfolio returns and report these results in Table 7. We rank households by the skewness of their portfolios, where skewness coefficients are obtained using the prior 60 months of CRSP return data. We do this for our three representative months. We then sort households into deciles on the basis of the skewness of their portfolios for each of the 3 months. Table 7 reports average characteristics of the households for each of the deciles, averaged over the three representative months. In Table 7, decile 1 (10) represents the portfolios with the lowest (highest) skewness.

Table 7 shows that portfolios with higher skewness preference are smaller on average, particularly in the two highest deciles. This higher skewness among smaller portfolios would be expected if investors naturally diversify as they add equity to their accounts. Investors with high skewness preference tend to be slightly younger, especially in the highest decile, where average age is 47 compared to 51 in the lowest deciles. Higher skewness among younger investors is consistent with the oft-repeated view that investors further away from retirement should hold portfolios with more risk. Although most investors in the database are male, Table 7 also shows that males are relatively more likely to hold highly skewed portfolios. In the lowest-skewness decile, investors are 86.6% male and 12.3% female, while in the highest-skewness decile, the makeup is 90.0% male and 8.6% female. That males would hold portfolios with higher skewness seems consistent with Barber and Odean (2001), who argue that higher overconfidence among males affects their trading activity. Affluent traders, as defined internally by the brokerage, are far less likely to hold highly skewed portfolios. The proportion of affluent traders in the lowest decile (20.5%) is almost double the proportion in the highest decile (10.6%). The pattern of less-wealthy individuals disproportionately investing in skewed payoff distributions has been documented in the context of lotteries [see Blalock, Just, and Simon (2004)], and has been discussed in the context of well-known theories such as those in Friedman and Savage (1948) and Kahneman and Tversky (1979). Further multivariate regression analysis (not reported) suggests that age, client type, gender and portfolio size variables have the most robust correlations with portfolio skewness. In summary, younger investors, male investors, and less-affluent investors

[13] In additional tests (not reported), we find that underdiversified investors primarily have greater exposure to idiosyncratic skewness rather than coskewness.

Table 7
Demographic characteristics of investors with different levels of skewness preference

			Gender		Client type			Account type		Marital status		
Decile	Portfolio size ($)	Age	Female	Male	Affluent	Active trader	Other	Retirement	General	Single	Married	Undeclared
Low skewness preference	63,548	51	12.3	86.6	20.5	8.8	70.7	32.4	67.6	22.3	55.5	22.3
2	74,894	51	12.6	86.2	20.9	9.7	69.4	29.9	70.0	21.3	55.8	22.9
3	66,897	51	12.7	86.0	17.8	8.4	73.9	30.5	69.5	22.8	55.2	22.1
4	72,835	51	11.2	87.7	18.5	10.4	71.0	31.8	68.2	22.8	55.6	21.5
5	70,901	50	12.5	86.6	17.7	8.5	73.7	30.9	69.2	23.2	53.9	22.9
6	62,856	51	11.0	88.1	16.7	8.7	74.6	30.8	69.2	22.3	55.6	22.2
7	57,301	50	12.3	86.5	17.2	8.7	74.1	30.5	69.4	23.3	55.1	21.6
8	67,691	50	11.6	87.5	15.4	8.6	76.0	30.8	69.2	22.7	54.4	22.9
9	47,010	50	9.9	88.8	14.4	7.8	77.8	29.9	70.1	23.0	53.4	23.7
High skewness prefrerence	41,005	47	8.6	90.0	10.6	8.6	80.8	30.4	69.6	24.0	54.3	21.8

This table presents average demographic characteristics of investors sorted into deciles based on skewness preference. Decile 1 (10) represents the set of investors with the lowest (highest) preference for skewness based on returns of portfolio holdings over the prior 60 months. Client types of "Affluent" or "Active trader" are as defined internally by the brokerage. Retirement accounts consist of IRA and Keough accounts. Averages are taken over three representative points in time over the sample period (January 1991, January 1993, and January 1996).

are the demographic groups that demonstrate the strongest preference for skewness.

Our database consists of individuals with accounts at a discount brokerage house during the 1990s and it is quite possible that our database consists of a relatively homogeneous population of investing agents. So while we find some heterogeneity across skewness preference in our dataset, more stark differences across preference for skewness are likely to exist outside the scope of our data. In addition, other types of investors such as mutual fund managers may have a preference for skewness that they are unable to fully express owing to constraints on their investing behavior. In contrast to Malkiel and Xu (2002), who model some agents as being constrained to hold an underdiversified portfolio of securities, some investing agents may be constrained in the opposite direction: to hold a diversified portfolio with very small exposure to positive skewness.

6. Conclusion

Investors seem to intuitively understand the trade-off between skewness and diversification. Many have been taught the benefits of diversification, yet recognize that diversification limits upside potential. They may observe, for example, that Bill Gates did not become the richest person in the United States by pursuing a careful diversification strategy. Riepe (2002) captures this mindset by comparing the common advice to "diversify your portfolio" with the common advice to "eat your vegetables". Following these pieces of advice is acknowledged to be good for you, but following them is not perceived to be much fun. Recognizing the trade-off between skewness and diversification, investors with a strong desire for upside potential may eschew prudent diversification in favor of a chance of hitting an investment "home run".

This article shows, theoretically and empirically, that the response of investors to the trade-off between skewness and diversification has real implications for household portfolio formation and asset prices. Theoretically, the model we develop shows how heterogeneity in skewness preference leads to a lack of diversification in equilibrium. Empirically, we show that the trade-off between skewness and diversification is manifest in the stockholdings of individual investors. We show that underdiversified investors obtain higher levels of skewness in their returns, and consequently, have much greater probability of very high payoffs. Moreover, we show that investors appear to intentionally choose return skewness: they trade mean-variance efficiency to obtain skewness, and they select stocks that increase the skewness of their portfolio returns. As further evidence of the link between underdiversification and skewness preference, we show throughout our analysis that idiosyncratic skewness, and not just coskewness with the market portfolio, appears to be relevant for investor holdings and asset prices.

The results presented in this article may lead to other important findings as well. In particular, our theoretical model of heterogeneous skewness preference is likely to lead to asset-pricing relationships, in addition to equity, that differ from the standard mean-variance case. For example, if investors exhibit a strong skewness preference, then options markets, other derivative markets, and fixed-income markets (which exhibit substantial negative skewness in returns) are other arenas in which skewness preference could potentially have a strong influence on prices.

Appendix A:

To represent the skewness structure of an arbitrary portfolio of n securities, we make use of the following matrices $\mathbf{M}_1, \mathbf{M}_2, \ldots \mathbf{M}_n$ defined as:

$$\mathbf{M}_i = \begin{bmatrix} M_{i11} & M_{i12} & \cdots & M_{i1n} \\ M_{i21} & M_{i22} & \cdots & M_{i2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{in1} & M_{in2} & \cdots & M_{inn} \end{bmatrix},$$

with $i = 1, 2, \ldots n$ and where arbitrary elements of $\mathbf{M}_i$, denoted as M_{ijk} where

$$M_{ijk} = E[(R_i - \overline{R}_i)\,(R_j - \overline{R}_j)\,(R_k - \overline{R}_k)],$$

and $\overline{R}_i = E[R_i]$. Three types of elements exists in these skewness matrices: M_{iii}, which represents the idiosyncratic skewness of asset 'i', M_{iij}, which is the curvilinear interaction of the assets 'i' and 'j', and M_{ijk}, which is the triplicate product moment of the assets 'i', 'j', and 'k'. In our simple model, a three security portfolio will require the 3×3 skewness matrices: $\mathbf{M}_1, \mathbf{M}_2$, and $\mathbf{M}_3$. Also, in numerical solutions to our model above, we assume all elements of the skewness matrices are zero except for M_{222} which we allow to vary from -0.35 to 0.35.

To calculate the skewness of a portfolio of n securities we use the following formula:

$$Skew(\mathbf{X}) = \sum_{i=1}^{n} \mathbf{e}_i' \mathbf{X}\mathbf{X}' \mathbf{M}_i \mathbf{X}, \tag{A1}$$

where e_i is a $n \times 1$ vector of zeros with a one in the i^{th} element and $\mathbf{X}$ is the $n \times 1$ vector of portfolio weights. In our model above, the skewness of a three-asset portfolio, given Equation (A1), can be written as

$$[x_1(\mathbf{X}'\mathbf{M}_1) + x_2(\mathbf{X}'\mathbf{M}_2) + x_3(\mathbf{X}'\mathbf{M}_3)]\mathbf{X}$$
$$= \sum_{i=1}^{3} x_i^3 M_{iii} + 3 \sum_{i=1, i\neq j}^{3} \left[\sum_{j=1}^{3} x_i^2 x_j M_{iij} \right] + \sum_{i=1}^{3} \sum_{j=1, i\neq j\neq k}^{3} \sum_{k=1}^{3} x_i x_j x_k M_{ijk}.$$

To obtain the demand function for Lotto Investor we need the first derivative of the portfolio's skewness, $Skew(\mathbf{X})$, with respect to $\mathbf{X}$ shown in the following proposition:

Proposition 1.

$$\frac{\partial Skew(\mathbf{X})}{\partial \mathbf{X}'} = \sum_{i=1}^{n}[\mathbf{X}'\mathbf{M}_i[((\mathbf{X}'\mathbf{e}_i) \otimes I) + \mathbf{X}\mathbf{e}_i'] + \mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i] \tag{A2}$$

Proof. Let $\partial Skew(\mathbf{X})/\partial \mathbf{X}' = \partial \sum_{i=1}^{n} \mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i\mathbf{X}/\partial \mathbf{X}'$ from Equation (A1). Given that the derivative operator can move inside the summation, we show $\partial[\mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i\mathbf{X}]/\partial \mathbf{X}'$ for arbitrary i, which then can easily be extended to obtain the derivative of the sum. Defining $f(\mathbf{X}) = XX'\mathbf{e}_i$ and $g(\mathbf{X}) = X$, we can rewrite $\partial[\mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i\mathbf{X}]/\partial \mathbf{X}'$ as $\partial[f(\mathbf{X})'\mathbf{M}_i g(\mathbf{X})]/\partial \mathbf{X}'$. Using the chain rule for matrix quadratic forms, Mangus and Neudecker (1999, page 177),

$$\frac{\partial[\mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i\mathbf{X}]}{\partial \mathbf{X}'} = g(\mathbf{X})'\mathbf{M}_i\frac{\partial f(\mathbf{X})}{\partial \mathbf{X}'} + f(\mathbf{X})'\mathbf{M}_i\frac{\partial g(\mathbf{X})}{\partial \mathbf{X}'}...$$

While $\partial g(\mathbf{X})/\partial \mathbf{X}'$ is straightforward to obtain, $\partial f(\mathbf{X})/\partial \mathbf{X}'$ is not. Applying the chain rule, as above, to $\partial f(\mathbf{X})/\partial \mathbf{X}'$ from Mangus and Neudecker (1999, page 181) we can show:

$$\frac{\partial(XX'\mathbf{e}_i)}{\partial \mathbf{X}'} = (\mathbf{X}'\mathbf{e}_i) \otimes I + \mathbf{X}\mathbf{e}_i',$$

where $\otimes$ represents the Kronecker product.

Consequently, $\partial[\mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i\mathbf{X}]/\partial \mathbf{X}' = \mathbf{X}'\mathbf{M}_i[\left((\mathbf{X}'\mathbf{e}_i) \otimes I\right) + \mathbf{X}\mathbf{e}_i'] + \mathbf{e}_i'\mathbf{X}\mathbf{X}'\mathbf{M}_i$, which is the term inside the sum of (A2). ∎

We use the result of Proposition 1 to obtain first-order condition for Lotto Investor's utility maximization problem.

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