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Calculation and comparison of delta-neutral and multiple-Greek dynamic hedge returns inclusive of market frictions

Thomas O. Meyer*

Department of Commerce, Massey University at Albany Campus, QB 2.51, Private Bag 102-904, NSMC, Auckland, New Zealand

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Abstract

Evidence provided by traders in derivative-asset markets suggests that use of "delta-neutral" (DN) and "multiple-Greek" (MG) hedging strategies are a common and effective approach in achieving desired hedged investment goals. The principal objective of this research is to develop a model that calculates position returns for both DN and MG hedging effectiveness and incorporates Standard Portfolio Analysis of Risk (SPAN) margin requirements (MRs) as well as transaction costs (TCs). The results of this analysis show that a DN hedging approach activated by an increase in the implied volatility of the option produces a more effective hedge on a risk–return trade-off basis than the other hedging approaches examined.

JEL classification: G12; G13
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1. Introduction

Traders in derivative-asset markets employ a variety of dynamic strategies combining offsetting positions in options and/or futures to achieve desired "hedged" investment

* Tel.: +64-9-443-9799x9460; fax: +64-9-441-8177.
E-mail address: T.O.Meyer@Massey.ac.nz (T.O. Meyer).

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objectives. Evidence provided by market practitioners suggests that use of so-called delta-neutral (DN) and multiple-Greek (MG) hedging strategies is a common and effective approach. In fact, use of these general hedging approaches are so commonplace that discussion and examples of their application appear in the publication of London International Financial Futures and Options Exchange (LIFFE), which provides them to any interested party.[1]

DN strategies are derived from the well-known option-pricing model of Black and Scholes (1973) or specifically, the Black (1976) model for valuing interest rate options on futures contracts. "Delta" is the term used to refer to the partial derivative ($\partial C/\partial F$ or $\partial P/\partial F$) for the change in the option (call or put) price with respect to a change in the underlying asset (futures) price from the Black option-pricing model. The general goal of a DN hedging approach is to make a combined option/futures portfolio immune to changes in the underlying asset. However, a DN-hedged portfolio is only immune to small changes in the underlying asset over the next short time period. To maintain delta neutrality the portfolio position must frequently be adjusted or *rebalanced*. The counterbalance to the potential effectiveness of frequent portfolio rebalancing is the possibly large amount of transaction costs (TCs) that the position incurs as it is dynamically adjusted.

In practice, traders are also concerned with changes in the hedged portfolio's value in response to changes in other variables that affect option prices. Some of these other important partial derivatives relate to changes in delta itself (gamma), changes in time-to-expiration (theta), and changes in the volatility of the underlying asset (vega). This research also examines an approach that combines all of these partial derivatives into an MG hedge ratio based on an approach suggested in a LIFFE (1995) publication.

The objectives of this research are as follows. First, a model is developed for use by practitioners that allows the calculation of dynamically hedged-option position returns that specifically accounts for the effects of margin requirement (MR) costs/returns as well as TCs within the framework suggested in a 1995 report prepared by LIFFE and Price Waterhouse (1995). This model is then validated using short-term interest rate derivatives market-price data to ensure that the model produces results, which intuition suggests should be expected. Second, tests are conducted to determine if the inclusion of margin and TCs produces nontrivial differences in mean returns or return volatility. Third, the effectiveness of DN hedges is compared to MG hedges. Finally, two risk-activated strategies drawn from practitioner comments are developed and tested against several automatic rebalancing approaches and a passive approach for mean-variance hedging effectiveness.

First, the empirical findings show that there is a significant difference between mean returns (107 of 162 comparisons) and return variances (75 of 162 cases) for naked options and DN-hedged option positions when market frictions are included vs. the comparative cases where these frictions are ignored. Second, DN hedges are surprisingly found to produce both significantly higher means and lower return variances in a large majority (76%) of cases compared to the more theoretically justified MG hedges. Finally, comparison of risk-

[1] *Short-Term Interest Rates: Futures and Options—An Introduction and Strategy Examples* (LIFFE, 1995).

activated hedges to automatically rebalanced hedges and a passive hedging strategy shows that a DN hedging approach based on an increase in the implied volatility of the option produces a more effective hedge on a risk–return trade-off basis than the other hedging approaches examined.

2. Review of the literature

An important assumption of the Black and Scholes (1973) option-pricing model is that capital markets are perfect, i.e., that there are no TCs. Another assumption is that asset trading takes place continuously so that the replicating portfolio used to determine the option's price is also continuously rebalanced. Boyle and Emanuel (1980) consider the distribution of hedged portfolio returns when rebalancing takes place on a discrete basis and find it to be particularly skewed and leptokurtic. They examine whether weekly rebalancing is optimal. Boyle and Emanuel conclude that hedging errors will be reasonably small if rebalancing is relatively frequent and can be ignored if the errors are uncorrelated with market return. Gilster and Lee (1984) modify the Black and Scholes pricing model to include the effects of TCs (as well as different borrowing and lending rates). Their empirical tests show that TCs of daily rebalancing were reasonably small and that the discrete rebalancing frequencies of the continuous-time option-pricing model do not seem to be a problem. Leland (1985) argues that the Black and Scholes arbitrage-based option-pricing model is invalidated by the inclusion of TCs. He develops an alternative replicating strategy that depends on the level of TCs and the rebalancing frequency. His approach is essentially an adjustment to the volatility used in the Black and Scholes formula. Boyle and Vorst (1992) rework Leland's analysis in a binomial framework. Their adjustment to the variance used in the model differs from Leland's model because although the binomial assumption provides the correct variance, it changes the expected absolute price change in any subinterval. Benet and Luft (1995) examine DN hedging of SPX stock index options and S&P 500 index futures in the presence of MRs and TCs. They examine 1-, 2-, and 4-week rebalancing intervals. They also use substantial changes in delta as a rebalancing trigger. Using the Howard and D'Antonio (1984) risk–return measure, they find that with inclusion of option premiums, TCs, and initial MRs, futures hedges are more effective than option hedges. Clewlow and Hodges (1997) build upon an earlier work by Hodges and Neuberger (1989) that examines writing and hedging a European call option in the presence of proportional TCs. Clewlow and Hodges employ a stochastic optimal control approach for DN hedging portfolios in the presence of TCs. The strategies developed focus on a band within which delta must be maintained. A fixed TC component leads to rebalancing to the inner band when an outer control limit is reached. Gallus (1999) notes that in a complete market model based on geometric Brownian motion a delta-hedging strategy may be used to price many types of exotic options. However, he shows that for specific contingent-claims digital options, if the underlying asset does not follow the assumed price process then DN hedging may actually increase the risk of the option writer.

A number of these studies analyze the Black and Scholes (1973) option-pricing model in the presence of TCs. However, none of these studies explicitly focus on incorporating the daily changes in MRs arising from the Standard Portfolio Analysis of Risk (SPAN) margining system using market prices for short-term interest rate options and futures. Further, none of these studies develop a model that allows practitioners to determine position returns in a manner that reflects the accounting recommendations developed by LIFFE in conjunction with Price Waterhouse.

3. The model and "market imperfections" considered

3.1. Two risk-activated hedging strategies

To analyze DN and MG hedging effectiveness, the model developed here is based explicitly on the example cited on pages 67–69 in *Short-Term Interest Rates: Futures and Options—An Introduction and Strategy Examples* (LIFFE, 1995). In this example, the position is rebalanced on a daily basis using settlement prices. Through informal interviews, several traders have offered anecdotal evidence that suggests two important considerations when they trade. First, one trader who admitted that he had conducted analysis similar to this research concluded that 3-day (2-day) rebalancing is optimal for at-the-money calls (puts) when comparing automatic, daily rebalancing schemes. His comments suggest that daily rebalancing strategies may be employed by some traders. Second, the traders suggested that position risk is an important aspect of why they use dynamic, DN, or MG hedging techniques. They agreed with the author's suggestion that hedge rebalancing in response to increased risk might be a practical and potentially useful approach. Therefore, two dynamic, nonautomatically rebalanced trading strategies are devised that focus upon the risk characteristics of the overall hedged position.[2]

There are two objective measures of market volatility that a hedger in the LIFFE markets might utilize daily to determine if the hedge should be rebalanced. The first measure is whether the overall position's daily MR has increased according to the SPAN margining system because of its increased risk. As is noted in the proceeding discussion, the position's MR is recalculated every day based on the position's maximum loss (ML) under 16 different SPAN risk scenarios. If the maximum (potential) loss rises, then traders are required to post increased margin. Thus, an increase in the ML is an obvious risk indicator upon which to base a positional rebalancing. The first risk-activated strategy is then to rebalance the hedged position whenever the maximum loss (termed the *maximum loss* strategy) indicated by the SPAN system increases. The second measure is based on changes in the implied volatility of the option for which the futures contract is the underlying asset. Changes in the option's implied volatility may be taken as indication of increased risk, and therefore, also act as a rebalancing trigger. The second risk-activated hedging strategy, termed the *increased volatility* (IV) hedge, is then to rebalance the hedged position whenever the implied volatility of the option increases.

[2] These approaches are conceptually similar to that analyzed in Benet and Luft (1995).

An additional point that must be noted is that the hedging strategies analyzed here are based on using end-of-day (settlement) price data. For example, Black and Scholes (1972) assume that the hedged portfolio in their empirical tests is rebalanced on a daily basis. Other researchers including Boyle and Emanuel (1980) and more recently, Clewlow and Hodges (1997) focus on (minimal) daily hedge rebalancing. In fact, the minimum, automatic rebalancing frequency considered by Benet and Luft (1995) and Leland (1985) is 1 week.

Clearly, large institutional investors could be expected to base their position rebalancing on intraday price changes and adjust them accordingly. However, potential intraday rebalancing would necessarily increase the TCs of any strategy analyzed and quite probably to a significant extent. Evidence from practitioners suggests that some DN traders do in fact use end-of-day position adjustment.[3] This fact should not be taken as indicating that intraday price changes are unimportant. Rather, it may suggest that typical practice for some hedgers is to focus on end-of-day rebalancing. In any event, in this study, no strategy will enjoy a comparative advantage because they are all based on end-of-day data.

To develop the final model utilized here, the following sections describe, respectively, the hypotheses analyzed, the SPAN system for determining initial MRs, the TCs, and data sets employed. This discussion is then followed by development of the equations and overall return model incorporating all MRs/TCs.

3.2. Hypotheses analyzed

In this analysis, several issues regarding hedging effectiveness are considered. To more clearly focus on each of these issues, the hypotheses examined here are enumerated specifically as follows. In a practical application of DN or MG hedging, a trader will incur TCs and cash inflows/outflows related to initial and variation margins, which will be dependent on the frequency of portfolio rebalancing. These costs and cash flows may seriously affect the returns earned on the hedged portfolio. Thus,

Hypothesis 1: A comparison of hedged returns that include these transaction and margin costs/returns to the returns earned on the portfolio without considering these costs will show that mean returns and return variances differ significantly.

A DN hedging strategy rebalances the portfolio based on the option's sensitivity to changes in the underlying futures contract. Theoretically, an MG hedging approach should be superior as it also takes into account changes due to decreasing time-to-expiration, implied volatility, and changes in delta itself. Hypothesis 2 then follows:

Hypothesis 2: The MG hedging strategy is expected to produce significantly higher mean returns and/or significantly lower return variances in comparison to a DN hedging strategy.

[3] Additionally, London SPAN "risk arrays are calculated centrally each day using the closing market prices to illustrate how much the portfolio would gain or lose using the closing market prices and initial margin parameters" (LIFFE, 1996, p. 38).

There is a trade-off between return-variability reduction benefits through increased frequency of rebalancing and the higher TCs of this frequent rebalancing. These higher costs will necessarily lower the portfolio's return. The determination of a superior hedging approach should clearly consider the risk–return trade-off. The hedging benefit per unit of risk (termed *HBS*) developed by Howard and D'Antonio (1987) is utilized here for this purpose. As the HBS measure includes both risk and return in its calculation, it is an empirical question whether increased returns or decreased volatility will dominate the generation of superior HBS measures. Automatic rebalancing schemes may tend to increase TCs unnecessarily in comparison to risk-triggered hedging approaches and both will certainly generate greater TCs than a passive hedging strategy. However, increased rebalancing frequency is expected to reduce position variance. Hypothesis 3 may be stated as:

Hypothesis 3: Hedges based on increased-risk rebalancing are expected to provide a superior risk–return trade-off compared to automatic hedge rebalancing strategies.

Where appropriate, findings in the Results section will be referenced to the particular hypothesis that is being tested.

3.3. Determining initial MRs via the SPAN system and variation margin

3.3.1. The London Clearing House

The primary purpose of the London Clearing House (LCH) is to act, in relation to its members, as central counterparty for contracts traded on London's futures and options exchanges. To limit and cover the potential loss, LCH collects margin on all open positions and recalculates members' margin liabilities on a daily basis. The two major types of margin are initial and variation margin.

3.3.1.1. Initial margin requirements.

Span parameters and scanning range. LCH uses London SPAN to calculate initial MRs for LIFFE.[4] London SPAN builds on and adapts the SPAN framework developed by the Chicago Mercantile Exchange (Chicago Board Options Exchange, 1995). In conjunction with the exchange, LCH sets initial margin parameters for each contract. The two main parameters are a futures price move, known either as the *initial margin rate* or the *futures scanning range*, and an *implied volatility shift*. These are set with reference to historical data on prices and volatilities and other factors such as known price-sensitive events. The parameters are kept under continuous review by LCH but do not change on a daily basis. London SPAN parameters as of 25 February 1997 are used in the analysis throughout for consistent calculation of returns.

London SPAN divides contracts into groups of futures and futures options relating to a single underlying asset (e.g., Short Sterling futures and options on Short Sterling futures). These groups are referred to as "portfolios." At the first stage of calculation, London SPAN simulates how the value of a portfolio would react to the changing market conditions defined

[4] This section draws upon *Understanding London SPAN* published by the London Clearing House (1994).

Scenario	Futures Price Changes	Implied Volatility Changes
1	Futures price down 3/3 range	Volatility up
2	Futures price down 3/3 range	Volatility down
3	Futures price down 2/3 range	Volatility up
4	Futures price down 2/3 range	Volatility down
5	Futures price down 1/3 range	Volatility up
6	Futures price down 1/3 range	Volatility down
7	Futures price unchanged	Volatility up
8	Futures price unchanged	Volatility down
9	Futures price up 1/3 range	Volatility up
10	Futures price up 1/3 range	Volatility down
11	Futures price up 2/3 range	Volatility up
12	Futures price up 2/3 range	Volatility down
13	Futures price up 3/3 range	Volatility up
14	Futures price up 3/3 range	Volatility down
15	Futures up extreme move	Volatility unchanged
16	Futures down extreme move	Volatility unchanged

Fig. 1.

in the initial margin parameters. This is done by forming a series of market scenarios and evaluating the portfolio under each set of conditions.

London SPAN uses 16 market scenarios in conjunction with the scanning range and volatility shift parameter to determine the potential profits/losses for each contract (futures month or option series) by comparing the current (market) price with the calculated contract price under each scenario. Futures prices are determined directly through the various scenarios. Option prices are calculated using the Black (1976) model based on the various futures prices and volatilities in the 16 scenarios. The 16 profits/losses for each contract then form a *risk array*. Fig. 1 above details the 16 market scenarios used in the calculation of London SPAN to form the risk array.

Risk arrays and scanning risk. Risk arrays are calculated each day using the closing futures and options prices. By valuing each net position (future or option) with the appropriate array and then combining arrays, London SPAN determines which scenario generates the ML for the portfolio, which may consist of either naked or combined positions. This ML is then referred to as the *scanning risk*. Scanning risk is the principal input into the calculation of the initial MR. Under SPAN, the initial MR changes each day as rates move and as the relative values of the portfolio components change. Changes in the initial MR need to be funded as long as positions remain open.[5]

3.3.1.2. Variation margins. Each day, open futures and options contracts are “marked-to-market” and daily profits or losses are paid through variation margin. For LIFFE financial options, payment of premium on initiation is not mandatory. If an option gradually becomes

[5] *Futures and Options—Accounting and Administration* (LIFFE & Price Waterhouse, 1995, p. 7).

Short Sterling	£ 3.00
Euromark	DM 8.00
Euroswiss	*Sf* 6.50

Fig. 2.

worthless, then the premium is effectively paid over time via the variation margin.[6] Having determined the profit or loss on a marked-to-market basis, the whole of the profit or loss should be recognized immediately. This recognizes the fact that each day a trader effectively decides either to keep a position open or to close it.[7] Changes in variation margin, therefore, need to be explicitly accounted for in the daily portfolio-return calculations.

3.4. Transaction costs

A sample of brokerage firms in Ireland and the United Kingdom was contacted in an effort to determine typical institutional TCs. Several firms responded and the round-trip costs used in this analysis are essentially an average of those provided. The derivative contracts examined here are denominated in their own domestic currency, so the TCs in the three relevant currencies are given above in Fig. 2.[8]

3.5. Data sets analyzed

The daily closing (settle) prices, as well as other data, e.g., implied volatilities for the futures options and contracts examined here are provided by LIFFE. The short-term interest rate markets analyzed are the 3-month Short Sterling, Euromark, and Euroswiss contracts. In each market, the period considered essentially dates from the inception of futures option trading. For Short Sterling, analysis begins with the March 1989 contract. Euromark and Euroswiss analysis begin with the March 1991 and September 1993 contracts, respectively. Analysis for all contracts ends with the March 1998 contract inclusive.

For each contract maturity, three calls and three puts are chosen for analysis. The contracts chosen are those with the three strike prices closest to the average futures (underlying asset) price over the period of analysis.[9] For each contract, a period of 130 trading days is analyzed, which typically commences about 280 (calendar) days from the option's expiration. This

[6] Ibid.

[7] Ibid. (p. 21).

[8] These transaction costs are somewhat lower than those employed by Benet and Luft (1995) who report that interviewed market participants put round-trip transaction costs at \$8–10.

[9] Options that are close to being "at-the-money" are utilized in this analysis to avoid the possibility that it might be optimal to exercise any of the puts early. Hull (1997, pp. 162–66) shows that it will never be optimal to exercise a call (on a non-dividend-paying stock) early and only optimal for a similar put if it is sufficiently deeply in-the-money.

normally corresponds to the advent of trading in the contract and this approach insures that final portfolio reversal occurs well before the expiration month. The objective of using this investment period is to minimize the compression in option deltas (to their value at expiration) as expiration approaches. In addition, to minimize any potential beginning- or end-of-the-week effects, all positions are initiated on a Wednesday. Once the returns for a given contract have been calculated, they are aggregated into a composite file for each option. Return analysis described in the Results section is then based on these composite return files. This approach conforms with the LIFFE guidelines stated as: "A suitable report to management evaluating hedge performance should contain details of:. . .the hedge efficiency being achieved, the trend over time being a more significant measure of performance than the result of any individual open hedging transaction."[10]

3.6. Portfolio return model

3.6.1. Theoretical option-pricing model

The Black (1976) pricing model is the most widely used and recognized option-pricing model for LIFFE options. The models for pricing short-term interest rate options[11] are as follows:

$$C = [(100 - X) \times N(-d_2)] - [(100 - F) \times N(-d_1)], \text{ and} \tag{1}$$

$$P = [(100 - F) \times N(d_1)] - [(100 - X) \times N(d_2)], \tag{2}$$

where

$$d_1 = \left[\frac{\ln\left(\frac{R_f}{R_x}\right) + \frac{1}{2}S^2T}{S\sqrt{T}}\right],$$

$$d_2 = d_1 - (S\sqrt{T}),$$

$$N(-d_1) = 1 - N(d_1) \text{ and}$$

$$N(-d_2) = 1 - N(d_2),$$

with C as the call premium, P as the put premium, F as the futures price, X as the strike price, R_f as the rate implied by futures price (i.e., 100 – futures), R_x as the rate implied by strike price (i.e., 100 – strike), S as the volatility of 3-month rates measured by annual standard deviation, T as the time to expiration in years, and $N(d)$ as the cumulative probability distribution function for a standardized normal variable.

[10] *Futures and Options—Accounting and Administration* (LIFFE & Price Waterhouse, 1995, p. 41).

[11] *Short-Term Interest Rates: Futures and Options* (LIFFE, 1995, p. 64). See also Stoll and Whaley (1993, p. 372) for similar valuation formulas.

Actual LIFFE market prices are used in all portfolio-return calculations. However, as noted above, the Black (1976) model prices are used by the SPAN system to determine initial MRs. In this model, the underlying futures prices are assumed log-normally distributed. The partial derivatives[12] of this version of the Black model with respect to F(Delta) are as follows:

$$\delta_C(\text{Delta}) = \partial C/\partial F = N(-d_1). \tag{3}$$

$$\delta_P(\text{Delta}) = \partial P/\partial F = -N(d_1). \tag{4}$$

Delta is the expected change in the option's value for a small change in the futures price and serves as the hedge ratio in the DN hedging strategy. Delta sensitivity for a long call (long put) position is positive (negative), meaning that the call position benefits from an increase in the futures price whereas the put position loses.

3.6.2. Calculation of the MG hedge ratio

Option values are also sensitive to changes in other variables in the model. Theta is the expected change in the option's value as the time-to-expiration decreases. The sensitivity of both calls and puts to a decrease in time-to-expiration is negative. The calculation of theta is given in Eq. (5) and is the same for both calls and puts (as is also true of vega and gamma that follow).

$$\text{Theta}_C(\partial C/\partial T) = \text{Theta}_P(\partial P/\partial T) = [R_X \times S \times Z(d_1)]/(2\sqrt{T}), \tag{5}$$

where

$$Z(d_1) = \partial N(d_1)/\partial d_1 = \left[\frac{\mathrm{e}^{-(d_1)^2/2}}{\sqrt{2\pi}}\right].$$

Vega (sometimes called *kappa*) represents the expected change in the option's value for a 1% change in the option's volatility. There is a direct relationship between option volatility and option value. Eq. (6) shows the calculation of vega utilized in this research.

$$\text{Vega}_C(\partial C/\partial S) = \text{Vega}_P(\partial P/\partial S) = R_F \times \sqrt{T} \times Z(d_1). \tag{6}$$

Gamma is defined as the expected change in the option delta for an incremental change in the value of the underlying asset. A long call or put position will have positive gamma sensitivity, meaning that delta increases if the underlying futures price increases.

$$\text{Gamma}_C(\partial \Delta_C/\partial F) = \text{Gamma}_P(\partial \Delta_P/\partial F) = Z(d_1)/(R_F \times S \times \sqrt{T}). \tag{7}$$

The approach suggested by LIFFE for aggregating these four sensitivities into a combined hedge ratio is surprisingly simple. "The overall sensitivity of a portfolio can be obtained by

[12] Ibid. LIFFE also provides calculated deltas as part of the daily settlement price information it supplies to interested parties. Note that the deltas in Eqs. (3) and (4) differ from traditional "stock option" deltas, but are consistent with the short-term interest rate option valuation formulas given in Eqs. (1) and (2) above, as well as those in Stoll and Whaley (1993, p. 372).

adding up the delta, gamma, theta, and vega of each individual option position on the same underlying (asset)" (LIFFE, 1995, p. 66). Thus, the multiple-Greek hedge ratios (MGHR) for hedging calls and puts are calculated as in Eqs. (8) and (9) below.

$$\mathrm{MGHR}_C = \mathrm{Delta}_C + \mathrm{Gamma}_C - \mathrm{Theta}_C + \mathrm{Vega}_C. \tag{8}$$

$$\mathrm{MGHR}_P = \mathrm{Delta}_P + \mathrm{Gamma}_P - \mathrm{Theta}_P + \mathrm{Vega}_P. \tag{9}$$

In the models developed below, the discussion is couched in terms of DN hedges, but it applies equally to MG hedges. The only difference in the model (and the analysis that is conducted) is that the MG hedge ratio is substituted for the DN hedge ratio.

3.6.3. Initial and variation margins

As discussed above, SPAN initial margins are calculated on a daily basis for naked or combination positions. In the analysis here, return comparisons are made for naked option positions vs. futures-hedged portfolio positions. The formulas presented below are therefore developed for option-only positions and for hedged positions. Assume that time i represents any day between initiation at time 0 and final position closure after n days. Time k is some date greater than or equal to day 2, and is less than or equal to day n. To represent cumulative returns of day 0 through day i, k is used, where the overall investment is n days. Reversal i refers to the daily reversal from a position opened on day $i-t$ and reversed on day i, whereas t represents the number of days elapsing since the most recently preceding trading day. It should be noted that the detailed formulas below generally relate to a long call position hedged using short futures and a daily DN rebalancing frequency. The initial SPAN MR for a call-only position (SPAN call-only margin, or SCOM) is given in Eq. (10) and the SPAN initial hedge margin (SIHM) is given in Eq. (11).

$$\mathrm{SCOM}_i = \mathrm{Max\ Loss}_i(\mathrm{SPAN\ Call\ Loss}_i). \tag{10}$$

$$\begin{aligned}\mathrm{SIHM}(x)_i &= \mathrm{Max\ Loss}_i(\mathrm{SPAN\ Call\ Loss}_i + \mathrm{SPAN\ DN\ Future\ Loss}_x)\\ &= \mathrm{Max\ Loss}_i(\mathrm{SCL}_i + \mathrm{SFL}_i),\end{aligned} \tag{11}$$

where $\mathrm{SIHM}(x)_i$ is the SPAN initial hedge margin (recalculated daily) for reversal i and rebalancing frequency x.

Note that the scenario generating the ML SPAN call-only margin in Eq. (10) will not necessarily be the same scenario as that generating span call loss in Eq. (11). The SPAN scenario where the ML occurs could easily differ where the call-only position is held compared to where the call position is combined with offsetting futures into a portfolio. Six different automatic rebalancing frequencies are examined. DN H1A refers to daily-adjusted hedges where the returns include all TCs and margin returns/costs. DN H2A, H3A, H4A, and H5A refer to DN hedges with a 2-day, 3-day, weekly, and biweekly rebalancing frequency, respectively. DN H6A refers to a DN strategy, which when initiated is DN but may be termed passive as the futures position is not subsequently adjusted. The ML strategy is then labeled DN H7A, and DN H8A refers to the IV approach.

The call-only daily variation margin (CDVM) is given as follows:

$$\mathrm{CDVM}_i(\text{Long Calls}) = (-C_{i-t} + C_i)\mathrm{CP} \times \mathrm{TV} \times 100, \tag{12}$$

where C_i is the settle call futures option price on day i, CP is the call (or put) position (assumed to equal 100 contracts), and TV is the tick value.

Multiplication of the daily positional profit/loss by 100 is used to convert tick value to an actual (£Stg./DM/Sf) value. Cumulative variation margin then represents net profit/loss to date. Call-only cumulative variation margin (CCVM) from initiation at $i=0$, to the reversal at k, is then given as:

$$\mathrm{CCVM}_{i,k} = \sum_{i=1}^{k} \mathrm{CDVM}_i = \gamma_{i,k}. \tag{13}$$

The next necessary distinction regards the initial (base or non-delta-adjusted) futures position vs. the delta-adjusted futures component. The initial futures daily variation margin (IFDVM) is given in Eq. (14). Then the initial futures cumulative variation margin (IFCVM) at reversal k is given in Eq. (15). Since the call or put is assumed to be held long, the futures hedge for a call (put) is assumed to be a short (long) position and this is reflected in the IFP term.

$$\mathrm{IFDVM}_i = (-F_{i-t} + F_i)\mathrm{IFP} \times \mathrm{TV} \times 100, \tag{14}$$

$$\mathrm{IFCVM}_{i,k} = \sum_{i=1}^{k} \mathrm{IFDVM}_i = \phi_{i,k}, \tag{15}$$

where F_i is the settle short-term interest rate futures on day i and IFP is the initial futures position (initial delta($-$CP)).

The daily delta adjustment (DA_i) for reversal i and then the cumulative delta adjustment (CDA) as of reversal k are given in turn.

$$\mathrm{DA}_i = (\delta_{i-t} - \delta_i)(-\mathrm{CP}). \tag{16}$$

$$\mathrm{CDA}_{i,k} = \sum_{i=1}^{k} [(\delta_{i-t} - \delta_i)(-\mathrm{CP})] = \varphi_{i,k}. \tag{17}$$

The call position term appears in Eqs. (16) and (17) to convert the decimal values of the daily delta adjustment to a contract basis. The ith delta-adjusted daily variation margin (DADVM) as of reversal k is then shown in Eq. (18).

$$\mathrm{DADVM}_{i,k} = \varphi_{i,k}[(-F_{i-t} + F_i)\mathrm{TV} \times 100]. \tag{18}$$

The delta-adjusted cumulative variation margin (DACVM) as of reversal k then follows in Eq. (19).

$$\mathrm{DACVM}_{i,k} = \sum_{i=1}^{k} \mathrm{DADVM}_i = \eta_{i,k}. \tag{19}$$

3.6.4. Return on initial and variation margins

The returns/costs on initial and variation margins are the next input to overall calculation of the positions' returns in the model. The approach used here to account for these returns or costs is based on guidelines suggested by LIFFE. In general, the initial margin is treated as a use of cash and thereby generates an opportunity cost of funds. Capital gains/losses from futures or options positions as represented by changes in the variation margins are assumed to be allocated to the cash account.[13] The question arising then in determining the corresponding costs/returns for these cash flows is "What cash interest rate to use?" The LIFFE suggestion is that "a notional rate must be used...that should be the rate earned on the rest of the cash part of the portfolio."[14] The notional margin rate (r_{m}) adopted here from the LIFFE Recommendations (1992, p. 32) is an annual rate of 3%.

The equations used for calculating margin costs/returns are explicitly formulated to account for the considerations enumerated above, namely daily revision of the initial SPAN MR and changes in the variation margin. Thus, the return for each daily reversal is calculated and then summed up to reflect the return calculation as of reversal k. For the naked option position, the cumulative option SPAN initial margin cost (COIMC) is shown in Eq. (20).

$$\mathrm{COIMC}_{i,k} = \sum_{i=1}^{k} [\mathrm{SCOM}_i (r_{\mathrm{m}}(i-(i-t))/365)] = \kappa_{i,k}. \tag{20}$$

Similarly, the cumulative hedge SPAN initial margin cost (CHIMC) for rebalancing frequency x as of reversal k is given as:

$$\mathrm{CHIMC}_{i,k} = \sum_{i=1}^{k} [\mathrm{SIHM}(x)_i (r_{\mathrm{m}}(i-(i-t))/365)] = \lambda_{k,n}. \tag{21}$$

Given the above considerations, the same calculation approach leads to the formulas for the cumulative option variation margin return (COVMR) in Eq. (22) and the cumulative hedge variation margin return (CHVMR) in Eq. (23).

$$\mathrm{COVMR}_{i,k} = \sum_{i=1}^{k} [(\gamma_i)(r_{\mathrm{m}}(i-(i-t))/365)] = \theta_{i,k}. \tag{22}$$

$$\mathrm{CHVMR}_{i,k} = \sum_{i=1}^{k} [(\gamma_i + \phi_i + \eta_i)(r_{\mathrm{m}}(i-(i-t))/365)] = v_{i,k}. \tag{23}$$

3.6.5. Transaction costs

The estimated TCs employed in the model are given above on a round-trip basis. This cost approach simplifies the calculation of the returns, which are analyzed on the basis of daily position reversal. To deal with the different positions analyzed, TCs are subdivided into three

[13] *The Reporting and Performance Measurement of Financial Futures and Options in Investment Portfolios* (LIFFE Recommendations, 1992, pp. 29, 36–41).

[14] Ibid. (p. 34).

categories. These components are the following: initial option position TCs (OTC), base futures position TCs (BFTC), and delta-adjusted futures TCs (DAFTC). Thus, total hedge transaction costs (THTC) as of reversal k are given as:

$$\mathrm{THTC}_{i,k} = \mathrm{OTC}_{i,k} + \mathrm{BFTC}_{i,k} + \mathrm{DAFTC}_{i,k}. \tag{24}$$

3.6.6. Return calculation model

In the finance literature, it is well recognized that return calculations involving futures and options are somewhat difficult due to the fractional margin required to support the larger "market-value" of derivative positions. LIFFE Recommendations (1992, p. 31) stipulate that "it is not possible to measure performance on a 'margin payment' basis, i.e., by using the capital gain on the futures position against either that of the equity or the cash components of the portfolio. This would lead ultimately to nonsensical figures for the return on parts of the portfolio.…Therefore, an **adjustment must be made to an associated economic exposure basis, equivalent to that made in the reporting process**" (emphasis in the original). To account for this, the market-value economic exposure is directly reflected here for naked options or on a "net-basis" in the hedged portfolio returns.

Tests of Hypothesis 1 for the significance of including TCs and margin returns, require comparison of the relevant returns with, and without, inclusion of those costs and/or returns. The efficacy of DN and MG hedging strategies is tested by calculating returns for naked-option positions as well as for the corresponding futures-hedged portfolio positions. The four return models below are developed to deal with these considerations. The long call and put returns without TCs/MRs as of reversal k are given in Eqs. (25) and (26), respectively.

$$R(C)_{\mathrm{NTC},k} = \left[\frac{(-C_0 + C_k)}{C_0}\right]\left[\frac{365}{k}\right]. \tag{25}$$

$$R(P)_{\mathrm{NTC},k} = \left[\frac{(-P_0 + P_k)}{P_0}\right]\left[\frac{365}{k}\right], \tag{26}$$

where P_k is the settle put futures option price on day k.

The return calculation for the call-only position inclusive of all TCs and MRs is given in Eq. (27).

$$R(C)_{\mathrm{ATC},k} = \left[\frac{(\gamma_{i,k} + \theta_{i,k} - \kappa_{i,k}) + (-\mathrm{SCOM}_0 + \mathrm{SCOM}_k) - \mathrm{OTC}_k}{((C_0 \times \mathrm{CP} \times \mathrm{TV} \times 100) + \mathrm{SCOM}_k) + \gamma_{i,k}}\right]\left[\frac{365}{k}\right]. \tag{27}$$

Two points of clarification should be mentioned concerning the formulation given in Eq. (27). First, the difference between the SPAN MR at initiation and at reversal is included in the numerator to account for the daily recalculation as described earlier. Second, the SPAN initial

MR as of reversal at day k is included in the denominator because it represents the amount of funds tied up in initial margin as of that date. Further, SCOM_k is considered part of the investment since the cost of funds tied up in initial margin as represented in the $\kappa_{i,k}$ term are in the numerator.

The return on a delta-adjusted hedge where TCs/margin returns are ignored (NTC) is given in Eq. (28). The absolute value of the long call position offset by the short futures is used in the denominator to reflect the net economic exposure basis of the offsetting positions of the hedged portfolio.

$$R(H)_{\mathrm{NTC},k} = \left[\frac{\eta_{i,k}}{(|(C_0 \times \mathrm{CP}) + (F_0 \times \mathrm{IFP})|\mathrm{TV} \times 100)}\right]\left[\frac{365}{k}\right]. \tag{28}$$

Finally, the return on the delta-adjusted hedge considering all TCs and margin returns (ATC) is given in Eq. (29) as follows.

$$R(H)_{\mathrm{ATC},k} = \left[\frac{(\gamma_{i,k} + \phi_{i,k} + \eta_{i,k} + v_{i,k} - \lambda_{i,k}) + (-\mathrm{SIHM}_0 + \mathrm{SIHM}_k) - \mathrm{THTC}_k}{((|(C_0 \times \mathrm{CP}) + (F_0 \times \mathrm{IFP})|\mathrm{TV} \times 100) + \mathrm{SIHM}_k) + \eta_{i,k}}\right]\left[\frac{365}{k}\right]. \tag{29}$$

It may be noted that the preceding formulations all utilize the initial option and/or futures prices as the base for the return calculations. Clearly, this approach yields return calculations that are effectively decreased in magnitude by the annualization factor as the reversal date gets farther from the initiation date. The calculation approach is derived from the method suggested in LIFFE Recommendations (1992, p. 49). The approach obviously compresses the returns that could be calculated alternatively on the basis of using the daily return calculated as $[(P_{i-t} - P_i)/P_{i-t}]$ where P_i equals the price on day i. However, since all returns here are calculated on a similar basis, they are comparable and do not reflect any particular bias arising from the calculation approach regarding the hypotheses examined.

4. Results

Discussion of the analysis of returns on short-term interest rate contract hedging generally proceeds in order of the hypotheses proposed earlier. Hypothesis 1 states that it is expected that there will be a significant difference between position returns and return variability where a comparison is made of returns that include TCs/MRs (termed *inclusive returns*) to returns excluding (termed *exclusive returns*) them. Evidence regarding this hypothesis is given for the at-the-money calls and puts in Table 1. This table provides annualized returns and standard deviations for naked option positions as well as the DN- and MG-hedged portfolios for each of the eight strategies. An indication of whether the

Table 1
Risk–return measures for unhedged options vs. DN and MG futures-hedged portfolios

Option/hedge	DN hedges		MG hedges		Option/hedge	DN hedges		MG hedges	
	Rtn (%)	S.D. (%)	Rtn (%)	S.D. (%)		Rtn (%)	S.D. (%)	Rtn (%)	S.D. (%)
Market: Short Sterling									
Call 2A	−38.856**	1268.535**	−38.856**	1268.535**	Put 2A	69.890	1760.818**	69.890	1760.818**
Call 2B	114.901	1160.517	114.901	1160.517	Put 2B	69.997	736.201	69.997	736.201
H1A	−0.435**	2.744	−0.606	3.445	H1A	−0.204*	3.381	7.608**	47.291**
H1B	−0.287	2.775	−0.508	3.418	H1B	−0.050	3.451	16.142	81.642
H2A	−0.348**	2.932	−0.537	3.423	H2A	−0.244*	3.327	7.496**	47.372**
H2B	−0.205	2.983	−0.445	3.400	H2B	−0.098	3.393	16.019	81.637
H3A	−0.305*	2.899	−0.513	3.373	H3A	−0.261*	3.323	7.119**	46.591**
H3B	−0.168	2.938	−0.425	3.351	H3B	−0.118	3.387	15.340	79.431
H4A	−0.317*	3.172*	−0.525	3.392	H4A	−0.239*	3.307	7.733**	47.636**
H4B	−0.177	3.263	−0.439	3.370	H4B	−0.101	3.369	16.449	82.038
H5A	−0.224	3.418**	−0.462	3.348	H5A	−0.240	3.201	7.342**	46.903**
H5B	−0.089	3.549	−0.387	3.333	H5B	−0.117	3.267	15.958	81.113
H6A	−0.033	4.530**	−0.334	3.250	H6A	−0.314*	2.273	6.206**	44.357**
H6B	0.134	4.933	−0.264	3.240	H6B	−0.214	2.291	13.276	67.918
H7A	−0.388*	2.829	−0.575	3.489	H7A	−0.194*	3.404	7.201**	48.194**
H7B	−0.254	2.860	−0.486	3.465	H7B	−0.052	3.475	16.058	83.006
H8A	−0.363*	2.914	−0.543	3.396	H8A	−0.216	3.336	7.837**	47.486**
H8B	−0.243	2.953	−0.467	3.375	H8B	−0.091	3.398	16.259	81.201
Market: Euromark									
Call 2A	−46.620**	952.406**	−46.620**	952.406**	Put 2A	−8.744**	587.822**	−8.744**	587.822**
Call 2B	209.327	1168.265	209.327	1168.327	Put 2B	−76.190	511.195	−76.190	511.195
H1A	−0.430**	2.051	−0.917*	2.964	H1A	−0.524**	1.722	−3.923	43.20**
H1B	−0.196	2.023	−0.777	2.893	H1B	−0.374	1.678	−3.721	46.88
H2A	−0.367**	2.167	−0.873*	2.851	H2A	−0.517**	1.706	−3.534	42.06**
H2B	−0.147	2.155	−0.744	2.792	H2B	−0.376	1.663	−3.337	46.04
H3A	−0.450**	2.003	−0.932	3.019	H3A	−0.511**	1.708	−3.144	41.30**
H3B	−0.240	1.957	−0.807	2.956	H3B	−0.374	1.663	−2.954	45.52
H4A	−0.345**	2.191	−0.851	2.809	H4A	−0.534**	1.686	−3.861	42.61**
H4B	−0.139	2.176	−0.731	2.759	H4B	−0.400	1.642	−3.558	46.38
H5A	−0.323**	2.237	−0.830	2.776	H5A	−0.530**	1.662	−3.385	41.38**

H5B	−0.130	2.215	−0.724	2.740	H5B	−0.406	1.619	−3.111	45.64
H6A	0.196*	4.524**	−0.430*	2.106	H6A	−0.454**	1.504*	−2.997	40.04**
H6B	0.447	4.971	−0.338	2.101	H6B	−0.342	1.453	−2.304	44.64
H7A	−0.531**	1.959*	−0.981	3.078	H7A	−0.500**	1.665	−3.761	42.80**
H7B	−0.327	1.894	−0.859	3.013	H7B	−0.366	1.620	−3.338	46.61
H8A	−0.371**	2.067	−0.859	2.907	H8A	−0.501**	1.708	−4.033	42.91**
H8B	−0.190	2.007	−0.755	2.864	H8B	−0.381	1.665	−3.804	46.86
Market: Euroswiss									
Call 2A	72.634**	3128.722**	72.634**	3128.722**	Put 2A	−123.201**	900.875**	−123.201**	900.875**
Call 2B	631.667	1520.845	631.667	1520.845	Put 2B	−178.718	723.383	−178.718	723.383
H1A	0.042*	3.586	−1.622	3.052	H1A	−0.110	2.880	−5.415	56.313**
H1B	0.254	3.681	−1.495	2.952	H1B	−0.001	2.793	−5.359	61.403
H2A	0.167*	3.655	−1.558	2.992	H2A	−0.123	2.842	−4.956	56.082**
H2B	0.364	3.746	−1.442	2.891	H2B	−0.022	2.759	−4.914	61.242
H3A	0.164	3.616	−1.548	2.943	H3A	−0.118	2.836	−5.747	57.714**
H3B	0.354	3.707	−1.438	2.843	H3B	−0.020	2.753	−5.690	62.676
H4A	0.194	3.678	−1.599	2.887	H4A	−0.105	2.824	−6.637	57.781**
H4B	0.378	3.768	−1.425	2.786	H4B	−0.010	2.741	−6.592	62.495
H5A	0.350	3.877	−1.411	2.786	H5A	−0.126	2.797	−6.164	57.517**
H5B	0.521	3.972	−1.325	2.692	H5B	−0.039	2.714	−6.057	62.409
H6A	1.088	4.738	−0.815	2.465**	H6A	−0.178	2.644	−5.681	56.573**
H6B	1.283	4.870	−0.742	2.352	H6B	−0.100	2.556	−5.136	62.064
H7A	0.013	3.630	−1.642	3.004	H7A	−0.127	2.794	−5.998	57.519**
H7B	0.201	3.727	−1.532	2.906	H7B	−0.031	2.709	−5.747	62.350
H8A	0.184	3.730	−1.507	2.910	H8A	−0.076	2.833	−5.457	56.271**
H8B	0.344	3.818	−1.419	2.814	H8B	0.009	2.749	−5.407	61.469

Return A includes TCs and margin returns whereas Return B excludes them. Call or Put 2 refers to the at-the-money option. H(x) refers to either the DN or MG hedge with rebalancing frequency x. H1 is the daily adjusted hedge, H2, H3, H4, and H5 are the second-day, third-day, weekly, and biweekly adjusted hedges, respectively. H6 is the passive hedge strategy. H7 (H8) is the ML (IV) hedging strategy. Rtn is the annualized option or hedged position return. S.D. is the standard deviation of returns. t indicates whether the paired t test for a significant difference in mean return A vs. B is significant. F indicates whether the folded F statistic testing for a significant difference in variances is significant.

* Significance at the 5% level.

** Significance at the 1% level.

Market/Option	Delta-Neutral Hedges Significant t-Tests	Delta-Neutral Hedges Significant F-Tests	Multiple-Greek Hedges Significant t-Tests	Multiple-Greek Hedges Significant F-Tests
Short Sterling Calls	23/27	15/27	3/27	5/27
Short Sterling Puts	19/27	11/27	15/27	27/27
Euromark Calls	27/27	20/27	17/27	8/27
Euromark Puts	27/27	16/27	5/27	27/27
Euroswiss Calls	8/27	10/27	3/27	4/27
Euroswiss Puts	3/27	3/27	3/27	19/27
Total	**107/162**	**75/162**	**46/162**	**90/162**

Fig. 3. Summary comparison of naked options, DN, and MG hedge returns inclusive of margin and TCs vs. exclusive returns.

parametric statistics for significant differences in means (paired t tests) and variability (F tests) is also shown in Table 1. These statistics have been estimated using the SAS PROC TTEST (SAS Institute, 1985b).[15] The forms of the paired t and the "folded" F tests are given in Eqs. (30) and (31) below.

$$\text{Paired } t \text{ statistic} = \left[\frac{(\mu_1 - \mu_2)}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}}\right], \text{ and} \tag{30}$$

$$\text{Folded } F \text{ test} = \left[\frac{\text{Larger of } (\sigma_1^2, \sigma_2^2)}{\text{Smaller of } (\sigma_1^2, \sigma_2^2)}\right], \tag{31}$$

where μ_i is the sample mean of return series i and σ_i^2 is the sample variance of return series i.

The in-the-money and out-of-the-money results are not provided in the table[16] in the interest of brevity. However, in the interest of completeness, those results are included in the summary statistics that are detailed above in Fig. 3. The number of returns included for each of the six options in a given market are as follows: Short Sterling (4773), Euromark (3741), and Euroswiss (2451). As is shown in Table 1, and indeed is true for all 18 options, there is a significant difference between inclusive and exclusive returns for all naked options in terms of mean returns and variances (standard deviations reported) as evidenced by paired t and F tests. Interestingly, comparison of mean inclusive to exclusive returns shows that the sign of

[15] Additionally, SAS PROC UNIVARIATE (SAS Institute, 1985a) is used to test the (difference between compared) returns for normality. The Kolmogorov D statistic testing for normality rejects the hypothesis that any of the return distributions are normal. Given this, a signed rank test is used to test the hypothesis that the population mean is zero. Results of this nonparametric test for the various comparisons described in Hypotheses 1–3 show that the t tests reported here understate the level of significant differences in mean returns.

[16] These and other additional results are available from the author.

the naked option return changes from negative (for inclusive returns) to positive (for exclusive returns) in 6 of 18 cases.

Table 1 shows that there are consistent, significant differences in both *t* and *F* tests for MG hedges in Short Sterling puts. Consistent differences in mean returns are significant for DN hedges in both Euromark calls and puts. MG comparisons for *F* values are consistently significant for both Euromark and Euroswiss puts. Summary results for all 18 options and hedges combined as well as based on each market are provided in Fig. 3. In this figure, the results are considered significant if the *k* value equals 5% or less. Viewing the overall results based on the combined figures it is clear that there is a significant difference in means between inclusive and exclusive returns in a large majority (66%) of cases for DN hedges although there is none for MG hedges. Conversely, a majority of MG hedges (55.6%) yield significantly lower variances in the presence of margin and TCs while only 46.3% of these comparisons are significantly different for DN hedges. When the inclusive vs. exclusive comparison is made by market, it is clear that the highest number of significant differences is evidenced by the Short Sterling and Euromark markets for DN hedges (and to a lesser extent the MG hedges). The Euroswiss market evidences the fewest number of significant differences, which is perhaps largely due to the relatively lower number of returns in its analysis. In sum, these overall results suggest that the effect of including TCs and MRs in the return calculations produce nontrivial differences as compared to returns where these costs/returns are ignored. Thus, in all results evaluated henceforth, the analysis cites inclusive returns rather than exclusive returns.

Validation of the model proposed to account for all option and futures returns, as well as the margin and TCs, is an additional aspect of this research. It is intuitively obvious that a DN (MG) hedging approach is expected to be successful at reducing portfolio variability as compared to naked option positions. When hedging effectiveness is measured in terms of variance reduction both the DN and MG approaches are indeed effective based on the *F* test. For all six options in each of the three markets, all eight of the DN hedging frequencies (144 comparisons) yield highly significant *F* tests. Similarly, uniform variance reduction for the MG hedges is also observed for all comparisons. Given the uniformity of the *F* statistics, these results are not reproduced here although it is of interest to report the extent to which option-only variability is eliminated through hedging on average. Accordingly, the average S.D. of returns for all naked options is calculated to be 1360%, whereas the average S.D. for all DN hedges is 3%. Based on these averages, the average reduction in return S.D. achieved through DN hedging is found to be 99.78%. Thus, it might be said that, on average, 99.78% of naked option return variability is eliminated through DN hedging. Similar risk-reduction results are found for CG hedges. A potentially more interesting question is whether apart from risk reduction the DN or CG hedges actually improve returns. Table 2 details the results of the *t* tests for significant differences in naked-option means vs. hedged returns. The daily-adjusted returns are provided as these *t* statistics are largely representative of returns for all hedge strategies. Focusing on daily-adjusted hedges may provide a conservative portrayal of hedge returns as this strategy would tend to have the highest rebalancing TCs. As the top panel of Table 2 shows, 8 of the 18 DN-hedged means are significantly higher (or less negative) than the unhedged-option means. Conversely, in three cases, the unhedged-option

Table 2
Comparison of unhedged-option return to daily-adjusted DN and combined Greek hedge return using paired t test for difference in means

Market	Option	μ_O vs. μ_H	t statistic	Option	μ_O vs. μ_H	t statistic
DN hedges						
Short Sterling	Call 1	(−)<(−)	−0.0175	Put 1	(+)>(−)	0.9058
	Call 2	(−)<(−)	−2.0905*	Put 2	(+)>(−)	2.7470**
	Call 3	(−)<(−)	−2.5523**	Put 3	(+)>(−)	4.3515**
Euromark	Call 1	(+)>(−)	0.0731	Put 1	(−)<(+)	−4.2661**
	Call 2	(−)<(−)	−2.9635**	Put 2	(−)<(−)	−0.8551
	Call 3	(−)<(−)	−3.7674**	Put 3	(+)>(−)	1.3069
Euroswiss	Call 1	(+)>(−)	2.0657*	Put 1	(−)<(+)	−8.6825**
	Call 2	(+)>(+)	1.1456	Put 2	(−)<(−)	−6.7575**
	Call 3	(+)>(+)	0.6574	Put 3	(−)<(+)	−3.4974**
MG hedges						
Short Sterling	Call 1	(−)<(−)	−0.0085	Put 1	(+)>(+)	1.4772
	Call 2	(−)<(−)	−2.0812*	Put 2	(+)>(+)	2.4467**
	Call 3	(−)<(−)	−2.3845**	Put 3	(+)>(+)	4.3189**
Euromark	Call 1	(+)>(−)	0.0939	Put 1	(−)<(+)	−4.5253**
	Call 2	(−)<(−)	−2.9322**	Put 2	(−)<(−)	−0.5002
	Call 3	(−)<(−)	−3.7383**	Put 3	(+)>(+)	1.1638
Euroswiss	Call 1	(+)>(−)	2.0956*	Put 1	(−)<(−)	−8.6025**
	Call 2	(+)>(−)	1.1719	Put 2	(−)<(−)	−5.5937**
	Call 3	(+)>(−)	0.7121	Put 3	(−)<(−)	−3.4486**

Call 2 and Put 2 are the at-the-money options. Call 1 and Put 3 are the in-the-money options. Call 3 and Put 1 are the out-of-the-money options. μ_O vs. μ_H shows a comparison of the mean of the unhedged option to the mean of the (daily-adjusted) DN or MG hedge return. The (+) or (−) signs indicate whether the mean is greater than or less than zero. The inequality sign shows which of the two means is greater (or less negative). t Statistic shows the parametric test statistic testing for a significant difference in sample means.

* Significance at the 5% level.

** Significance at the 1% level.

mean is positive, the hedge mean is negative, and the t statistics are significant. The remaining seven comparisons do not generate significant t statistics. The bottom panel of Table 2 depicts results for the MG hedges that essentially mirror the DN results. Reference to the hedged means in Table 1 generally shows these means are quite close to zero (although this is not true for MG-hedged puts). So, the overall conclusion that may be reached regarding hedging returns is that in 44% of the cases examined here, the DN hedges transformed significant option losses into portfolio returns near zero, even after accounting for all of the costs of undertaking the hedges. In the seven cases where the mean returns compared are not significantly different, at a minimum, the hedges minimized portfolio losses.

Hypothesis 2 asserts that MG hedges should provide greater risk reduction, higher hedged means, or both, in comparison to DN hedges. The results of the parametric analysis of this hypothesis are shown in Table 3 for the at-the-money options.

Table 3
Comparison of DN to MG hedge returns

Comparison	At-the-money call			At-the-money put		
	t statistic	F statistic	Lower S.D.	t statistic	F statistic	Lower S.D.
Short Sterling						
DN H1A vs. MG H1A	2.683**	1.58**	DN	−11.384**	195.66**	DN
DN H2A vs. MG H2A	2.900**	1.36**	DN	−11.260**	202.69**	DN
DN H3A vs. MG H3A	3.228**	1.35**	DN	−10.914**	196.63**	DN
DN H4A vs. MG H4A	3.097**	1.14**	DN	−11.534**	207.52**	DN
DN H5A vs. MG H5A	3.425**	1.04	MG	−11.142**	214.66**	DN
DN H6A vs. MG H6A	3.724**	1.94**	MG	−10.141**	380.73**	DN
DN H7A vs. MG H7A	2.871**	1.52**	DN	−10.574**	200.40**	DN
DN H8A vs. MG H8A	2.774**	1.36**	DN	−11.688**	202.59**	DN
Euromark						
DN H1A vs. MG H1A	8.276**	2.09**	DN	4.807**	629.4**	DN
DN H2A vs. MG H2A	8.639**	1.73**	DN	4.384**	607.5**	DN
DN H3A vs. MG H3A	8.148**	2.27**	DN	3.896**	584.7**	DN
DN H4A vs. MG H4A	8.694**	1.64**	DN	4.772**	638.5**	DN
DN H5A vs. MG H5A	8.693**	1.54**	DN	4.218**	620.4**	DN
DN H6A vs. MG H6A	7.669**	4.62**	MG	3.882**	709.0**	DN
DN H7A vs. MG H7A	7.550**	2.47**	DN	4.657**	661.2**	DN
DN H8A vs. MG H8A	8.374**	1.98**	DN	5.031**	630.9**	DN
Euroswiss						
DN H1A vs. MG H1A	17.497**	1.38**	MG	4.658**	382.4**	DN
DN H2A vs. MG H2A	18.080**	1.49**	MG	4.260**	389.5**	DN
DN H3A vs. MG H3A	18.178**	1.51**	MG	4.822**	414.1**	DN
DN H4A vs. MG H4A	18.258**	1.62**	MG	5.590**	418.6**	DN
DN H5A vs. MG H5A	18.259**	1.94**	MG	5.191**	423.0**	DN
DN H6A vs. MG H6A	17.640**	3.70**	MG	4.811**	457.8**	DN
DN H7A vs. MG H7A	17.393**	1.46**	MG	5.048**	423.7**	DN
DN H8A vs. MG H8A	17.706**	1.64**	MG	4.728**	394.4**	DN

DN (MG) H1A is the daily-adjusted delta-neutral (multiple-Greek) hedge including all TCs/MRs. H2A, H3A, H4A, and H5A are the second-day, third-day, weekly, and biweekly adjusted hedges. H6A is the passive hedge strategy. H7A is the ML hedging strategy and H8A is the IV hedging strategy. t Statistic compares the DN hedge mean return to the corresponding MG hedge mean. Similarly, F statistic tests for a significant difference in return variances. Lower S.D. indicates whether the DN or MG hedge produces the lower standard deviation.

** Significance at the 1% level.

The form of the numerator in the t statistic is the DN mean minus the MG mean. Thus, a positive t statistic means that the DN mean is higher (or less negative) than the MG mean. All of the 48 t statistics shown in Table 3 are highly significant, although out of 144 comparisons (including the 12 options not shown), only 119 are significant. The results in Table 3 indicate that DN hedges have higher means than their MG counterparts except for Short Sterling puts. Summing all results, DN-hedged returns exceed the MG returns in 110 cases, of which 101 exhibit significant t statistics.

All of the F test statistics shown in Table 3 are highly significant except one case. Indeed, 136 comparisons of the total 144 turn out to be significant. The MG hedges consistently produce lower variances for only Euroswiss calls. The DN hedges lead to uniformly lower variances for all the puts shown in Table 3 and for most of the Short Sterling and Euromark calls. Overall, DN-hedged variances are lower in 109 cases and of these 108 are significant (including all 72 put comparisons). Based on the analysis for these three markets, the DN hedges prove to yield higher and less variable returns in a large majority of cases. Given the overall dominance of the DN hedges the remaining analysis and discussion focuses on those results.

One aspect of this analysis that may be driving these comparative results is the simple approach utilized to aggregate the partial derivatives into the MG hedge ratio suggested in the LIFFE publication. Textbook examples as in Hull (1997, Chap. 14) or Stoll and Whaley (1993, Chap. 12) suggest that for each Greek effect that is being hedged, a different option is needed to implement the hedge. Such a strategy employing numerous options would certainly lead to an increase in TCs and reduce hedge returns. Further, it would not work in the LIFFE strategy analyzed here as the base position is considered the option and the hedge asset is the futures contract.

To examine Hypothesis 3, the means and variances of the two risk-triggered hedging strategies are compared to their counterparts from the other six hedging approaches. Table 4 depicts the parametric results for the at-the-money options. The form of the numerator in the t test is the other hedge (OH) mean vs. the ML hedge mean. The t statistics for calls in all three markets are almost uniformly positive and are markedly significant for Euromark calls. Biweekly and passive other hedges have significantly higher means than ML hedges in the Short Sterling and Euroswiss markets for calls. ML hedged-put returns fare better against the other hedges for Short Sterling and the Euromark although the t tests are typically not significant. Out of all 126 comparisons, 27 are significant. ML hedge returns are higher in 40 cases, but only two of these are significant. Table 4 indicates that the ML hedging approach seems to perform better at reducing risk than generating higher returns. ML hedges provide significant variance reduction for Short Sterling and Euromark calls in most cases. In fact, the example where ML hedges offer consistently less variance reduction is for Short Sterling puts. The F statistics are significant in 57 of 126 comparisons. ML hedges yield (significantly) lower hedge variances in (39) 86 cases.

Table 5 provides the results of the parametric tests comparing IV hedges to the other hedges for the at-the-money options. The IV hedges generate mean hedge returns that are not generally significantly different from the other hedges except in comparison to the passive hedges. For calls, the other hedges typically have superior mean returns, whereas the opposite is true for puts. Of all 126 comparisons, only 17 have significant t statistics. The IV-hedged return exceeds its OH counterpart in 82 cases, with five cases being significant. Table 5 also shows that the contest for producing lower variances is a rather back-and-forth affair. The IV hedge for calls appears to be most effective at reducing risk in comparison to biweekly and passive hedges. In contrast, passive hedge returns are significantly lower than IV hedges for puts in all three markets. Overall, 44 (of 126) F statistics are significant. IV hedges generate lower variances in 49 cases and 23 cases are significant.

Table 4
Comparison of DN to ML hedge returns

Comparison	At-the-money call			At-the-money put		
	t statistic	F statistic	Lower S.D.	t statistic	F statistic	Lower S.D.
Short Sterling						
DN H1A vs. DN H7A	– 0.831	1.06 *	OH	– 0.153	1.01	OH
DN H2A vs. DN H7A	0.681	1.07* *	ML	– 0.736	1.05	OH
DN H3A vs. DN H7A	1.412	1.05	ML	– 0.974	1.05	OH
DN H4A vs. DN H7A	1.152	1.26* *	ML	– 0.655	1.06 *	OH
DN H5A vs. DN H7A	2.547* *	1.46* *	ML	– 0.686	1.13* *	OH
DN H6A vs. DN H7A	4.590* *	2.56* *	ML	– 2.026 *	2.24* *	OH
DN H8A vs. DN H7A	0.425	1.06 *	ML	– 0.328	1.04	OH
Euromark						
DN H1A vs. DN H7A	2.184 *	1.10* *	ML	– 0.631	1.07 *	ML
DN H2A vs. DN H7A	3.438* *	1.22* *	ML	– 0.441	1.05	ML
DN H3A vs. DN H7A	1.774	1.05	ML	– 0.284	1.05	ML
DN H4A vs. DN H7A	3.877* *	1.25* *	ML	– 0.890	1.03	ML
DN H5A vs. DN H7A	4.274* *	1.30* *	ML	– 0.780	1.00	OH
DN H6A vs. DN H7A	9.017* *	5.33* *	ML	1.231	1.23* *	OH
DN H8A vs. DN H7A	3.441* *	1.11* *	ML	– 0.031	1.05	ML
Euroswiss						
DN H1A vs. DN H7A	0.284	1.02	OH	0.210	1.06	ML
DN H2A vs. DN H7A	1.473	1.01	ML	0.041	1.03	ML
DN H3A vs. DN H7A	1.456	1.01	OH	0.104	1.03	ML
DN H4A vs. DN H7A	1.734	1.03	ML	0.268	1.02	ML
DN H5A vs. DN H7A	3.135* *	1.14* *	ML	0.006	1.00	ML
DN H6A vs. DN H7A	8.916* *	1.70* *	ML	– 0.659	1.12* *	OH
DN H8A vs. DN H7A	1.629	1.06	ML	0.628	1.03	ML

DN H1A is the daily-adjusted delta-neutral hedge including all TCs/MRs. H2A, H3A, H4A, and H5A are the second-day, third-day, weekly, and biweekly adjusted hedges. H6A is the passive hedge strategy. H7A is the ML hedging strategy and H8A is the IV hedging strategy. t Statistic compares other DN hedge mean returns to the ML hedge mean. Similarly, F statistic tests for a significant difference in return variances. Lower S.D. indicates whether the OH or ML hedge produces the lower standard deviation.

* Significance at the 5% level.

* * Significance at the 1% level.

A risk–return measure is developed by Howard and D'Antonio (1987), which they term the HBS. This measure is used to provide further evidence on Hypothesis 3. The form of their HBS measure is given in Eq. (27) below for the relevant comparison, which is an unhedged option to a DN-hedged position.

$$\mathrm{HBS} = [(r_{\mathrm{m}} + ((\mu_{\mathrm{H}} - r_{\mathrm{m}})/\sigma_{\mathrm{H}})\sigma_{\mathrm{U}} - \mu_{\mathrm{U}})/\sigma_{\mathrm{U}}], \tag{32}$$

where $\mu_{\mathrm{U(H)}}$ is the unhedged (hedged) portfolio mean return and $\sigma_{\mathrm{U(H)}}$ is the unhedged (hedged) portfolio standard deviation.

Table 5
Comparison of DN to IV hedge returns

Comparison	At-the-money call			At-the-money put		
	t statistic	*F* statistic	Lower S.D.	*t* statistic	*F* statistic	Lower S.D.
Short Sterling						
DN H1A vs. DN H8A	− 1.249	1.13**	OH	0.175	1.03	IV
DN H2A vs. DN H8A	0.254	1.01	IV	− 0.412	1.01	OH
DN H3A vs. DN H8A	0.971	1.01	OH	− 0.652	1.01	OH
DN H4A vs. DN H8A	0.736	1.18**	IV	− 0.330	1.02	OH
DN H5A vs. DN H8A	2.131*	1.38**	IV	− 0.355	1.09**	OH
DN H6A vs. DN H8A	4.230**	2.42**	IV	− 1.667	2.15**	OH
DN H7A vs. DN H8A	− 0.425	1.06*	OH	0.328	1.04	IV
Euromark						
DN H1A vs. DN H8A	− 1.238	1.02	OH	− 0.593	1.02	IV
DN H2A vs. DN H8A	0.082	1.10**	IV	− 0.405	1.00	OH
DN H3A vs. DN H8A	− 1.677	1.06	OH	− 0.250	1.00	OH
DN H4A vs. DN H8A	0.530	1.12**	IV	− 0.848	1.03	OH
DN H5A vs. DN H8A	0.956	1.17**	IV	− 0.739	1.06	OH
DN H6A vs. DN H8A	6.968**	4.79**	IV	1.245	1.29**	OH
DN H7A vs. DN H8A	− 3.441**	1.11**	OH	0.031	1.05	OH
Euroswiss						
DN H1A vs. DN H8A	− 1.359	1.08*	OH	− 0.411	1.03	IV
DN H2A vs. DN H8A	− 0.171	1.04	OH	− 0.583	1.01	IV
DN H3A vs. DN H8A	− 0.196	1.06	OH	− 0.521	1.00	IV
DN H4A vs. DN H8A	0.092	1.03	OH	− 0.358	1.01	OH
DN H5A vs. DN H8A	1.518	1.08*	IV	− 0.622	1.03	OH
DN H6A vs. DN H8A	7.418**	1.61**	IV	− 1.299	1.15**	OH
DN H7A vs. DN H8A	− 1.629	1.06	OH	− 0.628	1.03	OH

DN H1A is the daily-adjusted delta-neutral hedge including all TCs/MRs. H2A, H3A, H4A, and H5A are the second-day, third-day, weekly, and biweekly adjusted hedges. H6A is the passive hedge strategy. H7A is the ML hedging strategy and H8A is the IV hedging strategy. *t* Statistic compares the other DN hedge mean returns to the IV hedge mean. Similarly, *F* statistic tests for a significant difference in return variances. Lower S.D. indicates whether the OH or IV hedge produces the lower standard deviation.

* Significance at the 5% level.

** Significance at the 1% level.

As may be readily determined, the higher the excess standardized hedged return in relation to the unhedged return and standard deviation, the higher (more positive) will be the HBS measure.

The HBS measures comparing the effectiveness of DN hedges at reducing naked-option risk are provided in Table 6. To compare the various hedging approaches to one another, for a given option, the HBS measure from each approach is ranked against the other HBS measures. These relative rankings provide information as to which hedging approach is most effective at providing the best risk–return trade-off for a particular option in each

Table 6
Comparison of ranked HBS measures for unhedged-option position vs. eight DN hedge returns including all TCs and margin returns

Option	HBS1	R1	HBS2	R2	HBS3	R3	HBS4	R4	HBS5	R5	HBS6	R6	HBS7	R7	HBS8	R8
SSC1	– 1.3762	8	– 1.2969	5	– 1.2924	4	– 1.2238	3	– 1.1318	2	– 0.8540	1	– 1.3278	7	– 1.3054	6
SSC2	– 1.2188	8	– 1.1087	5	– 1.1071	4	– 1.0126	3	– 0.9103	2	– 0.6366	1	– 1.1648	7	– 1.1209	6
SSC3	– 0.9581	8	– 0.8490	4	– 0.8498	5	– 0.7281	3	– 0.6514	2	– 0.4204	1	– 0.8966	7	– 0.8735	6
SSP1	– 0.8565	3	– 0.8815	4	– 0.8872	6	– 0.8869	5	– 0.9132	7	– 1.2785	8	– 0.8471	1	– 0.8532	2
SSP2	– 0.9857	2	– 1.0130	4	– 1.0193	6	– 1.0174	5	– 1.0501	7	– 1.4956	8	– 0.9760	1	– 1.0020	3
SSP3	– 1.0628	2	– 1.0873	4	– 1.0981	6	– 1.0933	5	– 1.1310	7	– 1.6429	8	– 1.0615	1	– 1.0741	3
EMC1	– 1.2283	7	– 1.1826	5	– 1.1943	6	– 1.1696	3	– 1.0802	2	– 0.7328	1	– 1.2374	8	– 1.1718	4
EMC2	– 0.8469	8	– 0.7976	5	– 0.8065	6	– 0.7852	4	– 0.7059	2	– 0.4258	1	– 0.8452	7	– 0.7771	3
EMC3	– 0.4282	8	– 0.3894	5	– 0.3897	6	– 0.3859	4	– 0.3375	2	– 0.1504	1	– 0.4248	7	– 0.3769	3
EMP1	– 0.7735	1	– 0.7935	5	– 0.7912	4	– 0.7903	3	– 0.8100	6	– 0.9369	8	– 0.8219	7	– 0.7803	2
EMP2	– 0.9397	1	– 0.9590	3	– 0.9594	5	– 0.9594	4	– 0.9778	6	– 1.0618	8	– 0.9788	7	– 0.9456	2
EMP3	– 1.3971	1	– 1.4214	4	– 1.4188	3	– 1.4234	5	– 1.4419	6	– 1.5868	8	– 1.4444	7	– 1.4096	2
ESC1	– 1.8856	5	– 1.7656	4	– 1.9762	7	– 1.6507	2	– 1.6532	3	– 0.7205	1	– 2.0272	8	– 1.8859	6
ESC2	– 1.6199	6	– 1.5013	4	– 1.6698	7	– 1.4741	3	– 1.4331	2	– 0.5677	1	– 1.7503	8	– 1.5788	5
ESC3	– 1.1142	5	– 1.0659	2	– 1.2224	8	– 1.1186	6	– 1.0983	4	– 0.4204	1	– 1.1551	7	– 1.0853	3
ESP1	– 1.7158	1	– 1.7307	4	– 1.7278	3	– 1.7674	5	– 1.7931	6	– 1.9646	8	– 1.8287	7	– 1.7210	2
ESP2	– 2.0265	1	– 2.0410	4	– 2.0356	3	– 2.0757	5	– 2.1043	7	– 2.2774	8	– 2.0824	6	– 2.0291	2
ESP3	– 2.4852	1	– 2.4969	3	– 2.4999	4	– 2.5337	5	– 2.5647	7	– 2.7817	8	– 2.5428	6	– 2.4894	2
Mean	– 1.2733	4.22	– 1.2434	4.11	– 1.2748	5.17	– 1.2276	4.06	– 1.2104	4.44	– 1.1086	4.50	– 1.3007	6.06	– 1.2489	3.44
S.D.	0.5111	3.04	0.5115	0.83	0.5289	1.50	0.5231	1.11	0.5457	2.25	0.7206	3.60	0.5459	2.39	0.5228	1.62

HBS(x) is the Howard and D'Antonio (1987) risk–return measure that compares the unhedged option's return and standard deviation to those of the DN hedge with the rebalancing frequency denoted as x. R(x) is the relative ranking of the HBS measure for a given option hedge. SSCz (SSPz), EMCz (EMPz), and ESCz (ESPz) is the abbreviation for Short Sterling calls (puts), Euromark calls (puts), and Euroswiss calls (puts), respectively. When $z=1$ it refers to the in-the-money call and the out-of-the-money put. $z=2$ is the at-the-money call and put. $z=3$ refers to the out-of-the-money call and the in-the-money put. S.D. is the standard deviation.

market. The HBS measures for the hedges are uniformly negative. This measure is based on the excess standardized hedge return so this is not too surprising. The notional risk-free return (r_m) used throughout the analysis is 3%, which is clearly larger than the mean return earned on many of the hedges as is shown in Table 1.

Summary statistics regarding the ranks from Table 6 show that the IV hedge approach ranks as the best HBS-based strategy with an average rank of 3.4. The weekly rebalancing frequency generates an average rank of 4.06. Based on its mean rank, the ML hedging approach comes in last place. Closer examination of Table 6 shows that the passive hedge ranks in first place for all nine calls, but ranks last for all puts. Conversely, daily rebalancing is a poor approach for hedging calls but works very well for puts. The IV hedging strategy is quite effective for puts and is modestly successful for calls.

Wilcoxon signed-rank testing is conducted for both HBS measures and rankings. The results show that IV, 2-day (DN2), and weekly (DN4) rebalanced hedges offer a significantly better risk–return trade-off vs. the 3-day (DN3) and the ML hedge (DN7) strategies based on either ranks or HBS measures. Additionally, the biweekly rebalancing approach (DN5) has significantly lower rankings and less negative HBS measures than DN7.

5. Summary and conclusions

This study examines both DN and MG hedging approaches that are popular with traders in LIFFE short-term interest rate derivative markets. To make the analysis as useful and realistic as possible, "real-world" market imperfections are explicitly incorporated into the hedging model that is developed and then tested empirically. Specifically, the portfolio-return model accounts for the impact of TCs and the costs/returns associated with initial and variation MRs. This study explicitly focuses on incorporating the daily recalculation of MRs arising from the SPAN margining system using market prices for short-term interest rate options and futures. Further, the developed model allows practitioners to determine position returns in a manner that reflects the accounting recommendations developed by LIFFE in conjunction with Price Waterhouse.

There are three principal conclusions derived from the empirical analysis. First, a comparison of returns inclusive of TCs/MRs to where they are excluded evidences statistically significant differences using parametric (and nonparametric) tests in a large number of the comparisons examined. Thus, these market imperfections may be considered nontrivial. The model is validated by analysis showing that hedged portfolio returns (variances) are significantly higher (lower) when portfolio rebalancing occurs less (more) frequently as intuition suggests.

Second, in practice, traders are concerned with position sensitivities other than just delta. An approach described in a LIFFE publication for aggregating delta, vega, theta, and gamma is employed to calculate an MG hedge ratio. All hedging effectiveness analysis is conducted for both DN and MG hedge ratios. In this analysis, DN hedges are surprisingly found to produce both significantly higher means and lower return variances in a large majority of cases compared to the more theoretically justified MG hedges.

Finally, two risk-activated hedge approaches are compared to automatically rebalanced hedges and a passive hedging strategy on the basis of mean-variance hedging effectiveness. The results of this analysis show that a DN hedging approach activated by an increase in the implied volatility of the option produces a more effective hedge on a risk–return trade-off basis than the other hedging approaches examined. Conversely, the risk-activated hedging strategy triggered by an increase in the daily ML calculated by the SPAN margining system does not prove to be an effective hedging approach. Another unexpected result is that 2-day and weekly rebalanced hedges prove significantly better than 3-day rebalanced hedges.

The primary implication of this study for future researchers is that any analysis based on the simplifying assumption of no transaction or margin costs may be seriously flawed or at a minimum may yield misleading results. Several results suggest implications for practitioners. Hedgers who are considering the use of a DN hedging approach should be impressed with the unambiguous and significant risk-reduction characteristics of all the hedging approaches analyzed. Further, the predominance of the passive hedging approach in all three call markets suggests that less frequent position rebalancing may be quite effective for calls and will certainly reduce TCs. By contrast, daily rebalancing is a commendable approach for hedging puts. Hedgers may also wish to consider increases in the implied volatility of the underlying asset as a signal for position rebalancing given its overall effectiveness here for both calls and puts. Finally, hedgers who are concerned about incorporating effects due to gamma, vega, or theta should perhaps look beyond a simple aggregation of these sensitivities into one hedge ratio.

A simplification employed here is the Black and Scholes (1973) assumption of log-normally distributed returns of the underlying asset. The appeal of this assumption is that in the model there is only one unobserved parameter, the variance of returns. Other return distributions like the jump diffusion model exists, which more accurately account for the widely recognized possibility of "fat-tails" and they may be more theoretically desirable. However, the jump diffusion model requires estimation of three unobserved parameters (Simmons, 1997, p. 26), which increases the complexity of its use. Additionally, instead of the Black (1976) European option-pricing model, a variant like the Barone-Adesi and Whaley (1987) model for an efficient analytical approximation of American option values could be used to calculate the hedging deltas and other Greeks.

One final limitation of this research is the fact that only three LIFFE short-term interest rate option and futures markets are analyzed. An interesting extension would be to analyze DN and MG hedging strategies in additional markets and different exchanges.

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