CSC165H1: Problem Set1

Pan Chen, Yang Shang

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1. $\forall\, p \in P$, Attacks(p, p) $\Rightarrow \neg$ House(p)
2. $\forall\, p_1 \in P$, ($\exists\, p_2 \in P$, Attacks(p_1, p_2) $\wedge\, p_1 \neq p_2$) $\Rightarrow \neg$ Behaved(p_1)
3. There exists a dog that does not attack any cat.
4. $\forall\, p_1, p_2 \in P$, $(p_1 \neq p_2) \wedge$ Attacks(p_1, p_2) $\wedge$ Attacks(p_2, p_1) $\Rightarrow \neg$ Allows(p_1) $\vee \neg$ Allows(p_2)

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1. HasCSC (s): $\exists$ i $\in \mathbb{N}$, s[i] = C $\wedge$ s[i + 1] = S $\wedge$ s[i + 2] = C
2. Substring(s1, s2): $\exists i \in \mathbb{N}, \forall t \in \mathbb{N}, 0 \leq t < |s_2| \Rightarrow s_1[i+t] = s_2[t]$
3. Palindrome(s): $\forall i \in \mathbb{N}, 0 \leq i < |s| \wedge (s[i] = s[|s| - 1 - i])$
4. False.
   Explain: $|s_1| \leq |s_2|$ doesn't imply Substring(s1, s2).
   Counterexample: s1 = ABC, s2 = WXYZ. $|s_1| \leq |s_2|$ but Substring(s1, s2) = False

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1. $\exists x, y \in \mathbb{R}, x < y \wedge f_1(x) \geq f_1(y)$
2. $(\exists x, y \in \mathbb{R}, x < y \wedge f_2(x) \geq f_2(y)) \wedge (\exists m, n \in \mathbb{R}, m < n \wedge f_2(m) \leq f_2(n))$
3. $\exists x_1, x_2 \in \mathbb{R}, \forall t \in \mathbb{R}, f_3(t) \leq f(x_1) \wedge f_3(t) \leq f_2(x_2) \wedge x_1 \neq x_2$
4. $(\forall x, y \in \mathbb{R}, x < y \Rightarrow f(x) < f(y)) \Rightarrow (\forall m \in \mathbb{R}, \exists n \in \mathbb{R}, f(m) < f(n))$

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1. i. Definition that makes the statement True:
   P(x, y): "x is bigger than y.", where x, y $\in \mathbb{N}$.
   ii. Definition that makes the statement False:
   P(x, y): "x is equal to y.", where x, y $\in \mathbb{N}$.

2. i. **The "True" version:**
   U: {x | x $\in \mathbb{N}$ and x $\geq$ 1000}
   P(x): "x is greater than 100.", where x $\in \mathbb{N}$
   Q(x): "x is greater than 10.", where x $\in \mathbb{N}$
   R(x): "x is greater than 1.", where x $\in \mathbb{N}$
   Explain:

   Since x must be in set U, so x must be a number greater than 100. So, P(x) is always True. Because from mathematics knowledge, we know that if a number is greater than 1000, it has to be greater than 100. Similarly, we know that Q and R are always True.

   So, with P, Q, R being True, $P(x) \Rightarrow Q(x)$ is True, $Q(x) \Rightarrow R(x)$ is True. So, $(P(x) \Rightarrow Q(x)) \Rightarrow R(x)$ is True, $P(x) \Rightarrow (Q(x) \Rightarrow R(x))$ is True. So, $\forall x \in U, ((P(x) \Rightarrow Q(x)) \Rightarrow R(x)) \Leftrightarrow (P(x) \Rightarrow (Q(x) \Rightarrow R(x)))$ is True.

   ii. **The "False" version:**
   U: {x | x $\in \mathbb{N}$ and x $<$ 1000}
   P(x): "x is greater than 10000.", where x $\in \mathbb{N}$
   Q(x): "x is smaller than 10000.", where x $\in \mathbb{N}$
   R(x): "x is equal to 1000.", where x $\in \mathbb{N}$
   Explain:

   Since x must be in set U, so x must be a natural number that is smaller than 1000. So, from basic mathematics knowledge, we know that P is False, because it is impossible that a number smaller than 1000 is greater 10000. Similarly, Q is True, because a number smaller than 1000 must be smaller than 10000 and R is False, because if a number is small than a "value", it can be the "value".

   So, with P, Q, R being False, True, False, respectively, $P(x) \Rightarrow Q(x)$ is True, $Q(x) \Rightarrow R(x)$ is False. So, $(P(x) \Rightarrow Q(x)) \Rightarrow R(x)$ is False, $P(x) \Rightarrow (Q(x) \Rightarrow R(x))$ is True. So, $\forall x \in U, ((P(x) \Rightarrow Q(x)) \Rightarrow R(x)) \Leftrightarrow (P(x) \Rightarrow (Q(x) \Rightarrow R(x)))$ is False.