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# 1 Direct numerical simulations of turbulent periodic-hill flows with mass-conserving 2 lattice Boltzmann method

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7 Multi-relaxation time lattice Boltzmann method is used to perform direct numeri-  
8 cal simulations of laminar and turbulent pressure-driven flows within a channel with  
9 hill-shape periodic constriction for the first time. The simulations are conducted  
10 on graphics processing unit cluster with two-dimensional domain decomposition to  
11 accelerate the computation. The hill-shape boundary is represented using the inter-  
12 polated bounce back scheme. However, the scheme generates mass leakage across  
13 the boundary, which is more pronounced in the turbulent flow regime, and this may  
14 produce diverging solutions for turbulent flows. The mass leakage due to the local  
15 mass imbalance along the curved boundary is solved by modifying the distribution  
16 functions locally or globally, and both predict similar velocity distributions. Since  
17 the global correction method is more computing time-consuming, the local correc-  
18 tion method is adopted. The present numerical implementation's capability is first  
19 validated by performing direct numerical simulations of turbulent channel flow at  
20  $Re_\tau = 180$ , and the current predicted results agree well with the benchmark solu-  
21 tions. Direct numerical simulations are further conducted for the turbulent flow over  
22 the periodic hill at  $Re_h = 2800$ . Both the mean velocity and turbulent stress com-  
23 pare favorably with the benchmark solutions. The present simulation also correctly  
24 predicts the turbulence splatting effect near the windward hill. Both phenomena are  
25 in good accordance with the benchmark solutions.

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## I. INTRODUCTION

Lattice Boltzmann method (LBM) has been used as an alternative to the Navier-Stokes equation to simulate the dynamic, thermal, and two-phase flows<sup>1-3</sup>. Most applications focus on simple geometric flows at lower Reynolds numbers, because the mesh is in general uniform due to the Lagrangian discretization of the convective term<sup>1</sup>. Despite this limitation, the application of LBM to simulate turbulent flows were also actively pursued. Homogeneous isotropic box turbulence, for example, is frequently examined<sup>4-6</sup>, where in general, the uniform mesh is adopted. LBM was demonstrated to generate compatible results with its Navier-Stokes equation-based counterpart, showing accurate energy spectrum distributions and energy decay.

Further, it would be desirable to perform LBM simulations of wall-bounded turbulent flows. A typical flow examined is the pressure-driven turbulent channel flows and is used as test cases to explore the validity of LBM studies. The predicted results were shown to reproduce the Navier-Stokes DNS data<sup>7</sup>, though the investigated Reynolds number is generally low, such as at  $Re_\tau = 180^{8-11}$ . For higher Reynolds number flows, the van Driest damping function and wall function were adopted to overcome the low near-wall grid resolution of turbulent wall-bounded flows<sup>12,13</sup>. Alternatively, Wu et al.<sup>14</sup> adopted the block-mesh refinement<sup>15</sup> to simulate turbulent channel flows up to  $Re_\tau = 640$ , where a successive grid-refinement resolved the wall layer toward the wall.

On the other hand, turbulent flows along curved surfaces with favorable or adverse pressure gradients are also frequently investigated. To mimic the complex geometry within the Cartesian mesh, several strategies have been proposed<sup>15-18</sup>. For example, Chen et al.<sup>16</sup> extended the extrapolation scheme to curved boundary using the castellated approach. Filippova and Hanel<sup>15</sup> proposed a method using simple linear interpolation between a fictitious equilibrium distribution function and a well-chosen near-boundary distribution function. The weighting factor of the interpolation is determined by the distance between the boundary and the near-boundary lattice. Mei et al.<sup>17</sup> further improved its numerical stability. Bouzidi et al.<sup>19</sup> proposed the interpolated bounce back scheme, where again, the interpolation is based on the wall distance parameter. Lallemand and Luo<sup>18</sup> combined the bounce-back scheme and interpolation scheme to treat a moving curved boundary by the lattice Boltzmann method. This treatment is an extension of that proposed by Bouzidi et al.<sup>19</sup>. An

57 alternate approach was proposed by Lin and his co-workers<sup>2,20</sup>, where a force like corrector  
58 was adopted to enforce the momentum and energy on the boundary nodes.

59 However, Lallenmand and Luo<sup>21</sup> indicated that the interpolation-based schemes destroy  
60 the mass conservation near the boundary. Thus, several mass conserving schemes have been  
61 proposed<sup>22–24</sup>. Further, Sanjeevi et al.<sup>24</sup> performed a systematic study to examine the degree  
62 of mass leakage over the curved boundary. It was found that the currently available schemes  
63 do not conserve mass over the curved boundary. The mass leakage is even more pronounced  
64 when the Reynolds number is high, and different mass conserving strategies were proposed<sup>24</sup>.  
65 However, the Reynolds number explored in previous studies<sup>15–19,22–24</sup> are still in the laminar  
66 flow regime.

67 The aim of the present study is to simulate turbulent wall bounded flows with extensive  
68 separation over curved surface using LBM. Flow separation due to the adverse pressure  
69 gradient has been the focus of studies, which can be caused by the abrupt change of the  
70 geometry<sup>25–27</sup>, or the smoothly expanding boundary<sup>28</sup>. In particular, the recirculation in-  
71 duced by the gradual expanding-geometry along the curved surface is challenging to predict  
72 because the separating point can not be determined in advance, and the related research has  
73 been actively pursued. Thus, turbulent flow over periodic hills has been studied experimen-  
74 tally and numerically<sup>28–31</sup> due to the existence of complex flow patterns such as separation,  
75 recirculation, and reattachment. However, to the knowledge of the authors, LBM based  
76 direct numerical simulations (DNS) of turbulent periodic-hill flows are not available. In the  
77 present study, the focus is to perform direct numerical simulations of turbulent periodic-hill  
78 flow with a multi-relaxation time lattice Boltzmann method. The boundary condition is im-  
79 plemented using the scheme by Bouzidi et al.<sup>19</sup> with mass correction strategies<sup>24</sup>. There are  
80 three issues to be addressed here, i.e., determining the appropriate driving force to achieve  
81 the desired Reynolds number, the extent of the mass leakage of the interpolated bounce  
82 back scheme without mass correction, especially for turbulent flows, and the influence of  
83 the mass correction schemes on the solutions. Finally, turbulent flow predictions are to be  
84 contrasted with the DNS data of Breuer et al.<sup>30</sup> to assess the effectiveness of the present  
85 implementations. The simulation is conducted on message passing interface (MPI)-based  
86 graphics processing unit (GPU) cluster<sup>32,33</sup>. The remainder of this paper is organized as  
87 follows: In Section II, the mathematical formulation of the method is introduced. Finally,  
88 section IV present the conclusion.

## 89 II. MATHEMATICAL FORMULATION

### 90 A. Multi relaxation time lattice Boltzmann model

91 The D3Q19 multi-relaxation-time (MRT) lattice Boltzmann method<sup>1,34,35</sup> can be ex-  
92 pressed by collision and streaming steps, respectively as in the following:

$$f_i^+(\mathbf{x}, t) = f_i(\mathbf{x}, t) - M_{il}^{-1} S_{lj} [m_j(\mathbf{x}, t) - m_j^{eq}(\mathbf{x}, t)] + G_i(\mathbf{x}, t) \Delta t \quad (1)$$

$$f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i^+(\mathbf{x}, t) \quad (2)$$

93 where  $\mathbf{M}^1$  is a matrix that transforms the distribution function  $f_i$  to the velocity moment,  
94  $m_j = M_{ji} f_i$ .  $\mathbf{S}^1$  is the relaxation time diagonal matrix, and  $G_i$  is the external forcing term.

95 Based on the particle distribution functions, the macroscopic density and velocity can be  
96 obtained as:

$$\sum_i f_i = \rho, \quad \sum_i f_i \mathbf{e}_i = \rho \mathbf{u} \quad (3)$$

97 The equilibrium moments  $m_i^{eq}$  and external force term  $G_i$  are determined as:

$$m_j^{eq} = M_{ji} w_i \rho \underbrace{\left[ 1 + \frac{3}{C^2} (\mathbf{e}_i \cdot \mathbf{u}) + \frac{9}{2C^4} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2C^2} (\mathbf{u} \cdot \mathbf{u}) \right]}_{f_i^{eq}}, \quad (4)$$

98

$$G_i = 3w_i \rho \frac{\mathbf{e}_i \cdot \mathbf{F}}{C^2} \quad (5)$$

99  $C = \Delta x / \Delta t$  is the lattice speed, where  $\Delta x$  and  $\Delta t$  are the lattice width and time step,  
100 respectively. Here,  $\Delta x = \Delta t$ , i.e.  $C = 1$  and  $\mathbf{F}$  is set to be the pressure gradient along the  
101 streamwise direction.

102 The weighting coefficients  $w_i$  are respectively as,  $w_0 = 1/3$ ,  $w_{1-6} = 1/6$ , and  $w_{7-18} =$   
103  $1/36$ . The particle velocity  $\mathbf{e}_i$  is defined as,

$$\mathbf{e}_i = \begin{cases} (0, 0, 0)C, & i = 0 \\ (\pm 1, 0, 0)C, (0, \pm 1, 0)C, (0, 0, \pm 1)C & i = 1 \sim 6 \\ (\pm 1, \pm 1, 0)C, (\pm 1, 0, \pm 1)C, (0, \pm 1, \pm 1)C & i = 7 \sim 18 \end{cases} \quad (6)$$

## B. Boundary conditions

Along the boundary, due to the inward streaming operations, particle distribution function may originate from the undefined nodes external to the flow domain. Therefore measures have to be taken to prescribe these unknown particle distribution functions. For example, at the near boundary fluid nodes, i.e., solid circular nodes shown in Fig. 1, nodes  $A$ ,  $B$ ,  $C$  and  $\mathbf{x}_{nw}$  have respectively three, one, two and three unknown distribution functions (vectors 1-9). Here  $nw$  denotes the near-wall fluid nodes. The solid square ( $X_w$ ) is the intersection of the upstream link (dashed line) with the wall boundary of the respective unknown distribution function. For easy identification, the same color is applied for the wall node ( $X_w$ ) and upstream link (dashed line) for the respective unknown distribution function.

Consider the near wall fluid node at  $X_{nw}$ . If the unknown distribution function is denoted as  $f_p(\mathbf{x}_{nw}, t)$ , then  $f_{i \neq p}(\mathbf{x}_{nw}, t + \Delta t) = f_{i \neq p}^+(\mathbf{x}_{nw} - \mathbf{e}_i \Delta t, t)$  (Eq. 2). Also, the unknown distribution function is obtained by using the interpolated bounce back scheme proposed by Bouzidi et al.<sup>19</sup>, which is based on a wall distance parameter defined as  $q = |\mathbf{x}_{nw} - \mathbf{x}_w| / |\mathbf{e}_p \Delta t|$ , where the respective wall locations are represented by the solid square symbols along the solid-fluid boundary shown in Fig. 1. Depending on the value of the parameter  $q$ , Bouzidi et al.<sup>19</sup> proposed that,

$$\begin{aligned} q < 0.5, \quad f_p(\mathbf{x}_{nw}, t + \Delta t) &= (1 - 2q)f_{-p}^+(\mathbf{x}_{nw} + \mathbf{e}_p \Delta t, t) + 2qf_{-p}^+(\mathbf{x}_{nw}, t) \\ q \geq 0.5, \quad f_p(\mathbf{x}_{nw}, t + \Delta t) &= (1 - \frac{1}{2q})f_p^+(\mathbf{x}_{nw}, t) + \frac{1}{2q}f_{-p}^+(\mathbf{x}_{nw}, t) \end{aligned} \quad (7)$$

where  $\mathbf{e}_{-p} = -\mathbf{e}_p$  shown in Fig. 1 and open circle ( $\mathbf{x}_{nw} + \mathbf{e}_p$ ) is the second fluid node away from the wall.

For  $q = 0.5$ , the scheme recovers the halfway bounce back scheme of Ladd<sup>36</sup>, and the mass is conserved. When  $q \neq 0.5$ , the interpolation does not guarantee the local mass conservation, i.e.  $\Delta \rho(\mathbf{x}_{nw}) = \sum f_{-p}^+(\mathbf{x}_{nw}, t) - \sum f_p(\mathbf{x}_{nw}, t + \Delta t) \neq 0$ . As suggested by Sanjeevi et al.<sup>24</sup>, there are four possibilities that the imbalance can be added to the distribution function, i.e.,

$$f_0(\mathbf{x}_{nw}, t + \Delta t) = f_0^+(\mathbf{x}_{nw}, t) + \Delta \rho(\mathbf{x}_{nw}) \quad (\text{scheme} - A) \quad (8)$$

$$f_i(\mathbf{x}_{nw}, t + \Delta t) = f_i(\mathbf{x}_{nw}, t + \Delta t) + w_i \Delta \rho(\mathbf{x}_{nw}) \quad (\text{scheme} - B) \quad (9)$$

$$f_0(\mathbf{x}_f, t + \Delta t) = f_0^+(\mathbf{x}_f, t) + \frac{\sum \Delta \rho(\mathbf{x}_{nw})}{N_f} \quad (\text{scheme} - C) \quad (10)$$

$$f_i(\mathbf{x}_f, t + \Delta t) = f_i(\mathbf{x}_f, t + \Delta t) + w_i \frac{\sum \Delta \rho(\mathbf{x}_{nw})}{N_f} \quad (\text{scheme} - D) \quad (11)$$

where  $N_f$  is the number of fluid nodes. For Eqs. 8 and 9, the corrections are added to the local nodes and for Eqs. 10 and 11, the corrections are distributed to all the fluid nodes. The influences of adopting Eqs. 8 to 11 on the solutions will be explored in the results section.

### III. RESULTS

#### A. Turbulent channel flow

The numerical procedure is first validated by predicting the turbulent channel flow at  $Re_\tau(u_\tau\delta/\nu)=180$ , where the computational domain is  $12\delta \times 2\delta \times 4.5\delta$  in the streamwise, vertical and spanwise directions, respectively. Two grid densities are adopted, i.e.,  $576 \times 96 \times 288$  and  $1152 \times 192 \times 576$  with corresponding grid densities being  $\Delta^+=3.8$  and  $1.9$ . Since a halfway bounce-back is adopted, therefore, the first grid point is located at  $y^+=1.9$  and  $0.95$ . Fig. 2(a) shows the predicted velocity distributions compared with the results of Moser et al.<sup>7</sup>, where compatible results are predicted, though a slight deviation is observed using the  $576 \times 96 \times 288$  grids. Similar results can be observed in the turbulence intensity predictions, shown in Fig. 2(b).

#### B. Flows over periodic hill

Here, the focus is on the predictions of flow over the periodic hill. Fig. 3 shows the geometry of the periodic hill, and the height of the domain is  $3h$ , instead of the commonly used,  $3.036h$ <sup>29,30</sup>. For turbulent flow, the width in the spanwise direction is chosen to be  $4.5h$ <sup>29,30</sup> and for laminar flow the width is  $2.25h$ . The boundary condition is implemented using the scheme by Bouzidi et al.<sup>19</sup> with mass correction<sup>24</sup>, and for the straight boundary,  $q$  is  $0.5$ , i.e., halfway bounce back. The Reynolds number is defined based on the bulk velocity at the hillcrest and the hill height  $(U_{bc}h/\nu)$ , which is half of the Reynolds number based on the height of the flow passage. When simulating flows over the periodic hill, there are three issues to be addressed, i.e., determining the appropriate driving force to achieve the desired Reynolds number, the extent of the mass leakage of the interpolated bounce back scheme without mass correction, especially for turbulent flows, and finally the influence of the mass correction schemes on the solutions. Finally, turbulent flow predictions are to be

contrasted with the DNS data of Breuer et al.<sup>30</sup> to assess the effectiveness of the present implementations.

### 1. Determination of driving force

For the periodic-hill geometry considered, there is no analytic solution of the driving force to reach the expected Reynolds number. Here, a similar measure as in Hsu et al.<sup>37</sup> is adopted. To achieve the desired bulk velocity the driving force is adjusted via the following formula, i.e.,

$$F^{IN_t} = F^{(I-1)N_t} + \beta \rho (U_{ref} - U_b^{IN_t-1}) \frac{U_{ref}}{L}, \quad (12)$$

$$I = 1, 2, 3, \dots$$

where  $\beta = \max(0.001, \alpha/Re_h)$ .  $N_t$  is the interval of the time step for force adjustment, which is adopted as 10,000. Thus, the force is updated every 10,000 steps ( $\sim 0.67 L/U_{ref}$ -flow through time) to avoid generating new disturbance, especially at high Reynolds number flows.  $U_{ref}$  and  $U_b(t)$  are the target bulk velocity and the predicted bulk velocity at the hillcrest. Here,  $U_{ref} = U_{bc}$ . The value of  $\alpha$  investigated ranges from 3 to 14. Eq. 12 is applied to compute laminar and turbulent flows at Reynolds numbers being 100 and 2800, respectively. The mass conserved scheme adopted is scheme-A of Eq. 8.

Fig. 4 shows the time history of the predicted bulk velocity variations ( $U^* = U_b(t)/U_{ref}$ ) with the force adjusted with Eq. 12 for laminar flow at  $Re=100$  using  $\alpha=10$ . The simulation saturates to the desired bulk velocity monotonically. For turbulent flow, such as at  $Re=2800$ , results using  $\alpha = 3$  and 14 produce similar results. Fig. 5 shows the time variations of the force and bulk velocity for turbulent flow at  $Re=2800$  with  $\alpha = 14$ . The predicted bulk velocity oscillates around the target velocity, resulting from the compensating force adjustment and the internal flow instability. After a few transients ( $\sim 200$  flow-through time), the velocity is time-averaged. As shown in Fig. 5, the time-averaged bulk velocity approaches the designated bulk velocity as time proceeds. The time-averaged period is approximately 700 flow-through time ( $U_{ref}/L$ ).



## 180 2. *Mass leakage*

181 As indicated earlier, the original interpolated bounce back scheme by Bouzidi et al.<sup>19</sup>  
182 may cause mass leakage through the curved boundary<sup>24</sup>. For turbulent flows, this may lead  
183 to diverging solutions. Here, the influences with and without mass correction are explored,  
184 and the mass correction scheme adopted is scheme-A. Simulations are conducted for laminar  
185 flow (Re=100) and turbulent flows (Re=700 and 2800) and the results are shown in Figs. 6  
186 and 7. For laminar flow at Re=100, as shown in Fig. 6, coarse grid generates higher degree  
187 of mass leakage. However, at 40 flow-through times, the mass leakages are around 2%, 0.4%  
188 and 0.00001% respectively for  $144 \times 48$ ,  $288 \times 96$  grid and  $576 \times 192$  grids. For turbulent  
189 flows, the mass correction is more influential, as shown in Fig. 7. The mass leakages at  
190 40 flow-through times are 0.25% and 2.5%, respectively, for Re=700 and 2800. For flow  
191 at Re=2800, simulation with the original interpolated bounce back scheme is unstable and  
192 prone to diverge as time progresses.

## 193 3. *Influences of mass correction schemes*

194 Here, to save computational time, the influence of the correction schemes, i.e., Eqs. 8  
195 to 11 on the solutions are explored first for laminar flow at Re=100 using  $576 \times 192$  grid.  
196 Figs. 8(a) to 8(c) show the predicted velocity and pressure distributions at eight selected  
197 locations.. The deceleration and acceleration of the flow due to the hill's presence is clearly  
198 observed and its associated decrease or rise of the predicted pressure levels. No perceivable  
199 difference is observed using the four schemes at such a fine grid, even for the vertical velocity,  
200 which is relatively smaller than the streamwise velocity.

201 Zoomed views of the pressure contours at the windward hill are shown in Fig. 9, where  
202 results using scheme-A and scheme-C are presented. At two to three grid spacing above  
203 the hill boundary, the predicted contours are similar. However, using scheme-A, there is a  
204 local increase in the pressure level near the boundary, whereas scheme-C generates smoother  
205 results. These are consistent with those observed in Sanjeevi et al.<sup>24</sup>. It should be further  
206 noted here that schemes (A) and (B) show a similar rise in the local pressure distributions at  
207 the windward hill. The pressure distributions using schemes (C) and (D) are identical and  
208 do not show such a local rise of pressure at the corresponding region. Despite this deficiency,



209 predicted velocity distributions are exactly the same using the four schemes. For the lines  
210 representing different schemes collapse and the difference can not be observed, as shown in  
211 the zoomed view of the velocity contours shown in Fig. 10.

212 Figs. 11 and 12 show the predicted mean and turbulent quantities at  $Re=2800$  using  
213 scheme-A and C. Again, no perceivable differences are observed among the results. Since,  
214 Eqs. 10 and 11 requires global operations, and are thus time consuming compared to Eqs.  
215 8 and 9, therefore, in subsequent simulations, Scheme-A is adopted for simplicity.

#### 216 4. *Turbulent periodic-hill flow at $Re=2800$*

217 Here, the focus is to validate the turbulent periodic-hill flow predictions using the direct  
218 numerical simulation data of Breuer et al.<sup>30</sup>, where the numerical procedure is based on  
219 the Navier-Stokes equation using curvilinear grid discretized with the second-order accurate  
220 scheme. The adopted Reynolds number based on the bulk velocity  $U_b$  and hill height  $h$  is  
221 2800. The computational domain is shown in Fig. 3. Here, the mass conservation scheme-  
222 A is adopted for simplicity. The grids adopted in the streamwise, vertical and spanwise  
223 directions are  $576 \times 192 \times 288$  and  $864 \times 288 \times 432$ . Since two grids generate compatible  
224 results (not shown here),  $864 \times 288 \times 432$  grid is used. Also,  $H=3.036h$  is also adopted in  
225 the simulations using  $864 \times 292 \times 432$  grid to examine the influence of the height on the  
226 solution.

227 Fig. 13 shows the predicted mean velocity distributions at ten selected locations, and the  
228 results are contrasted with the DNS data of Breuer et al.<sup>30</sup>. The usage of  $H=3h$  as height  
229 causes a slight departure from the benchmark solution of the mean streamwise velocity  
230 distributions shown in Fig. 13(a) in the region near the top boundary, and the simulated  
231 results using  $3.036h$  agree quite well with the benchmark solutions. Despite the slight  
232 difference of the adopted height, the predicted results agree quite well with Breuer et al. 's  
233 DNS data for both the streamwise and vertical velocity components. The shear layer and  
234 recirculation zone are predicted well by the present scheme. The top boundary's height has  
235 a marginal impact on the bottom wall-flow, and this was also observed by Fröhlich et al.<sup>29</sup>.

236 Figs. 14(a)-14(c) and 14(d) show respectively the predicted turbulence intensities and  
237 shear stresses. Apart from the slight deviation of the predicted turbulence intensity near  
238 the top boundary due to the different height adopted, in general, the agreements are good.

239 The rise of the turbulence level near the shear layer at the height of the hillcrest is also in  
 240 accordance with the DNS data of Breuer et al.<sup>30</sup>. The rise of the bottom wall spanwise tur-  
 241 bulence intensity at  $x/h=8$ , which is greater than the corresponding streamwise turbulence  
 242 intensity, is due to the pressure strain-induced splatting effect<sup>29</sup> resulting from the presence  
 243 of the windward hill.

#### 244 IV. CONCLUSION

245 Turbulent pressure-driven flows within a channel with hill-shape periodic constriction  
 246 are simulated with multi-relaxation time lattice Boltzmann method on GPU cluster. A  
 247 methodology is proposed to determine the appropriate driving force to achieve the desired  
 248 Reynolds number. The hill-shape boundary is mimicked by the interpolated bounce back  
 249 scheme by Bouzidi et al.<sup>19</sup>. However, the scheme generates mass leakage across the boundary,  
 250 which is more pronounced in the turbulent flow regime, and this may produce diverging  
 251 solutions for turbulent flows. The mass leakage due to the local mass imbalance along the  
 252 curved boundary is solved by modifying the distribution function locally or globally, as  
 253 suggested by Sanjeevi et al.<sup>24</sup>. The locally modified method produces a slight increase in  
 254 pressure locally in contrast to the global correction methods. On the other hand, the mass  
 255 correction strategies marginally influence the predicted velocity distributions. Since the  
 256 global correction method is more computing time-consuming, the local correction method  
 257 is adopted. The capability of the present numerical implementation is first validated by  
 258 performing direct numerical simulations of turbulent channel flow at  $Re_\tau = 180$ , and the  
 259 current predicted results agree well with the benchmark solution by Moser et al.<sup>7</sup>. Direct  
 260 numerical simulations of turbulent flow over the periodic hill are conducted at  $Re_h = 2800$ ,  
 261 and the predicted results are contrasted with the DNS data of Breuer et al.<sup>30</sup>. Both the mean  
 262 velocity and turbulent stress compare favorably with the benchmark solutions. The present  
 263 simulation also correctly predicts the turbulence splatting effect, i.e., spanwise turbulence  
 264 intensity being higher than the corresponding streamwise turbulence intensity, resulting from  
 265 the windward hill's presence. Both phenomena are in good accordance with the DNS data  
 266 by Breuer et al.<sup>30</sup>.

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## 271 DATA AVAILABILITY STATEMENT

272 The data that support the findings of this study are available from the corresponding  
273 author upon reasonable request.

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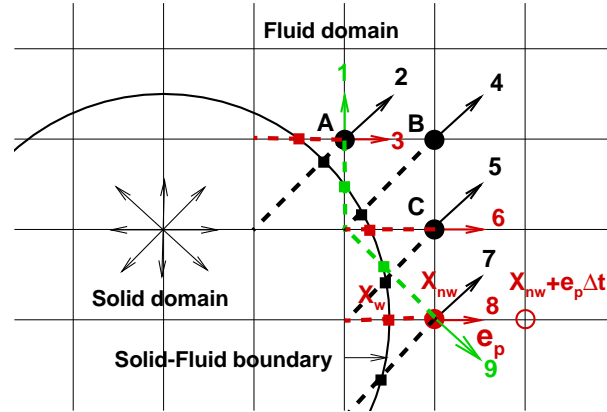


FIG. 1. Solid-fluid boundary treatment (solid square: wall boundary nodes associated with respective arrow; solid circle: near wall fluid node (A, B, C,  $X_{nw}$ ); open circle: the second fluid node away from the wall; arrows (1-9): unknown distribution functions).



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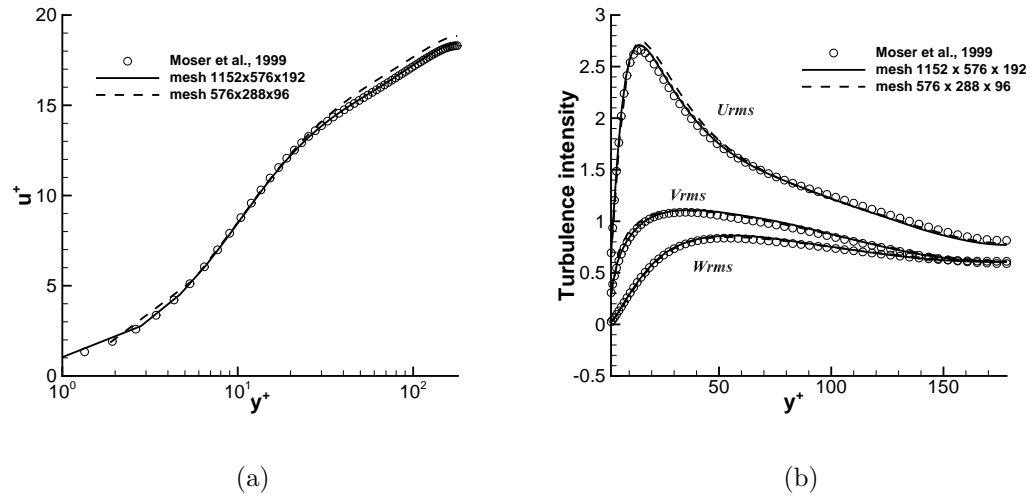


FIG. 2. Predicted streamwise velocity and the turbulent intensities of channel flow at  $Re_\tau = 180$ .

(a) Streamwise velocity, (b) Turbulence intensities.

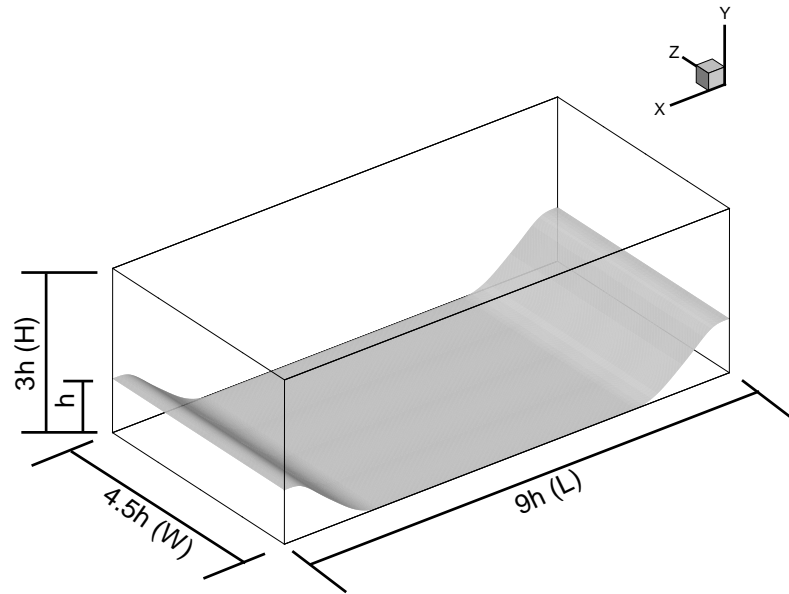


FIG. 3. The geometry of periodic hill.

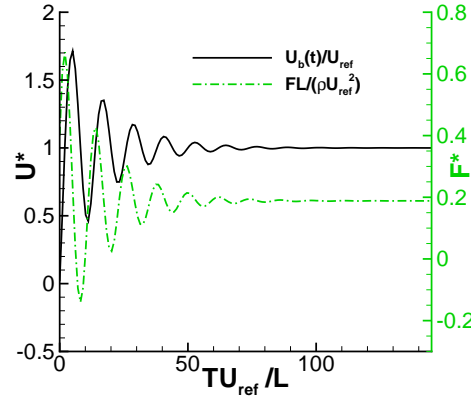


FIG. 4. Predicted bulk velocity and force variations (Re=100,  $576 \times 192 \times 144$  grid.)

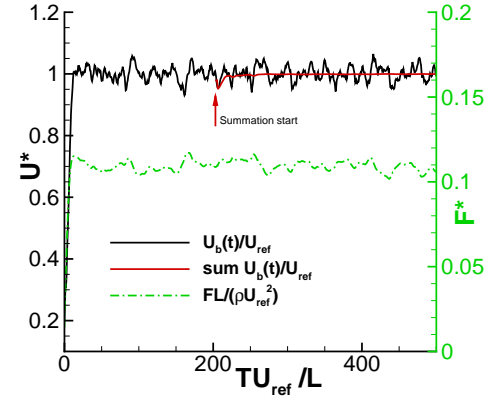


FIG. 5. Predicted bulk velocity and force variations (Re=2800,  $864 \times 288 \times 432$  grid.)

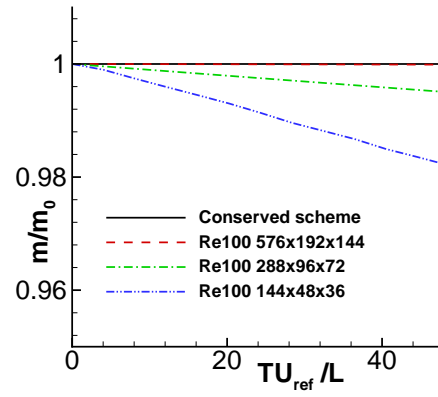


FIG. 6. Mass variations within the computational domain with and without mass correction (Re=100)

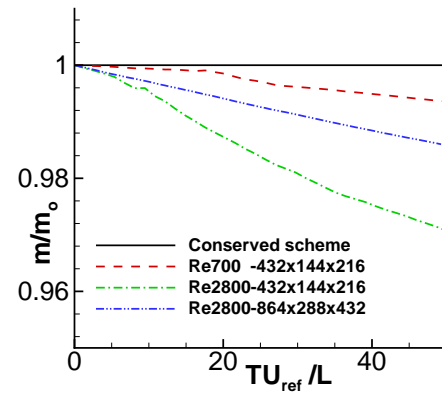


FIG. 7. Mass variations within the computational domain with and without mass correction (Re=700 and 2800)

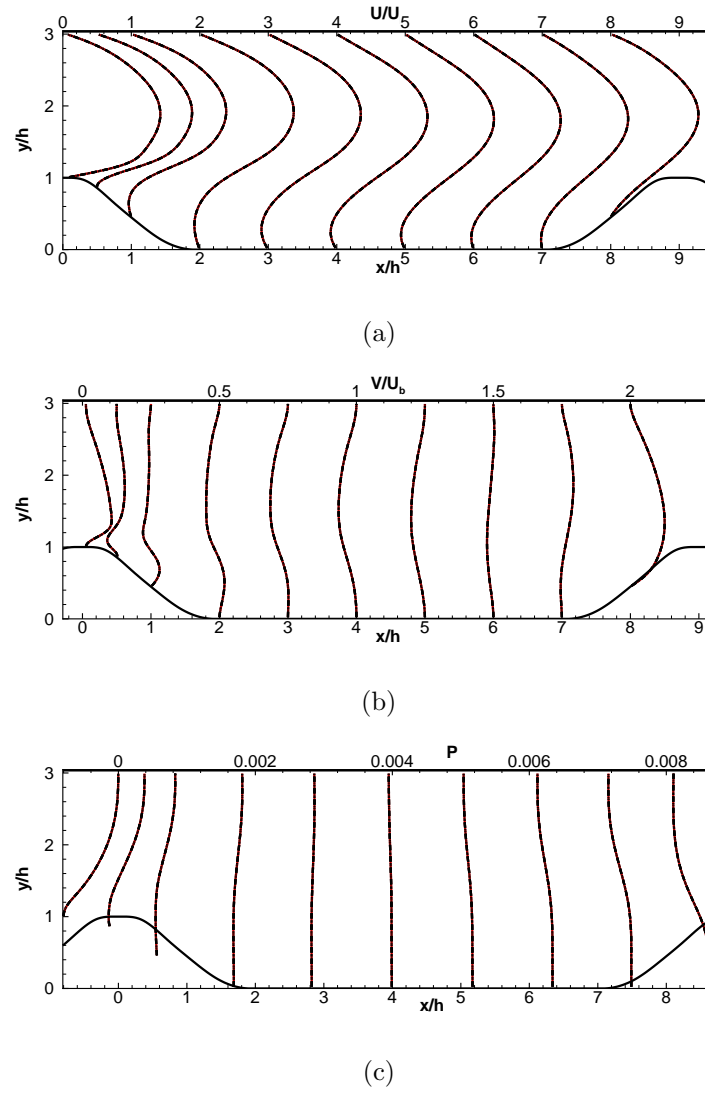
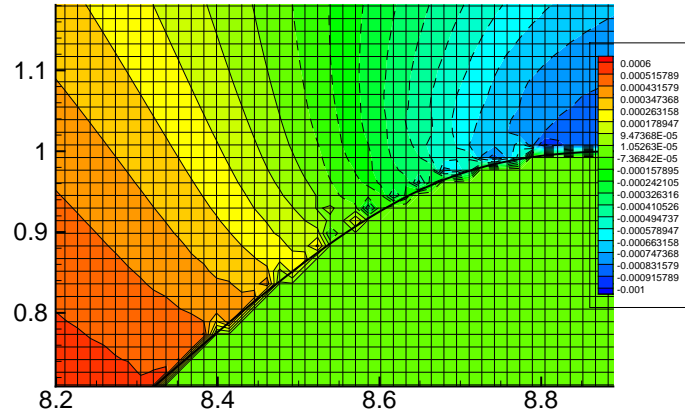


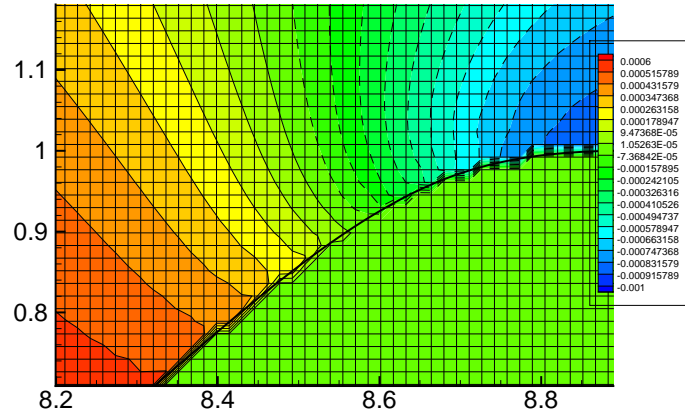
FIG. 8. Predicted velocity and pressure distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=100$ , —: Scheme-A; - - - - -: Scheme-B; ······: scheme-C; - · - · - ·: Scheme-D,  $576 \times 192$  grid). (a) U-velocity, (b) V-velocity, (c) Pressure.

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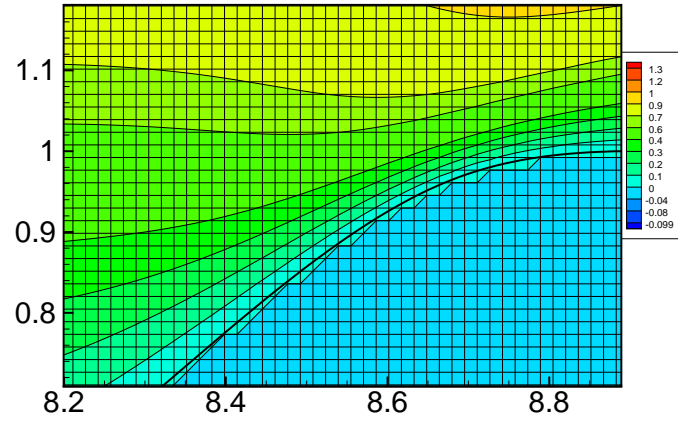


(a)

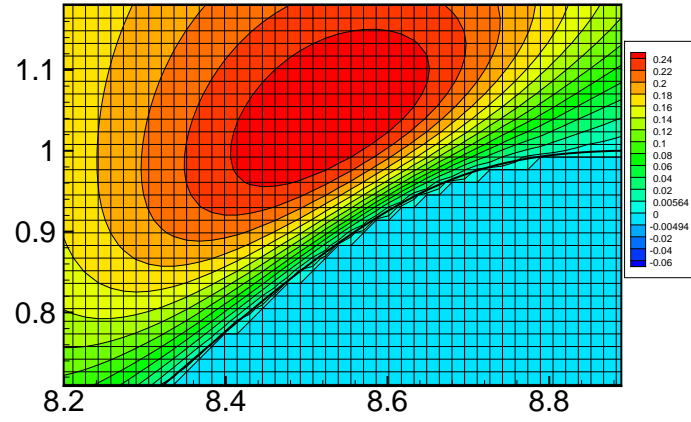


(b)

FIG. 9. Distributions of zoom-view pressure contours-Scheme-A and scheme-C ( $Re=100$ ,  $576 \times 192$  grid). (a) Scheme-A, (b) Scheme-C.



(a)



(b)

FIG. 10. Distributions of zoom-view streamwise and vertical velocity contours ( $Re=100$ , —: Scheme-A; - - - - -: Scheme-B; - · - · - · -: scheme-C; - · - · - · -: Scheme-D,  $576 \times 192$  grid). (a) U-velocity, (b) V-velocity.

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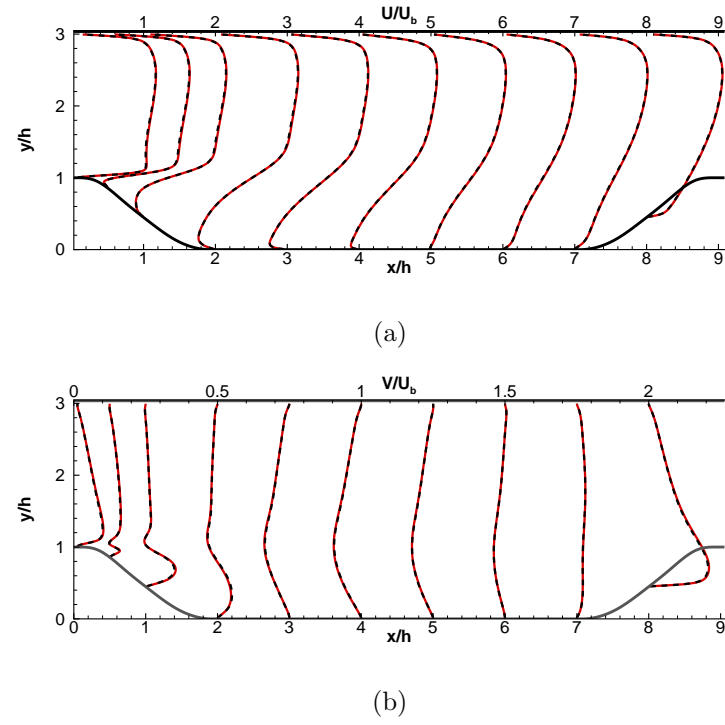


FIG. 11. Predicted mean velocity distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=2800$ , —: Scheme-A; - - - - -: Scheme-C,  $864 \times 288 \times 462$  grid). (a) U-velocity, (b) V-velocity.

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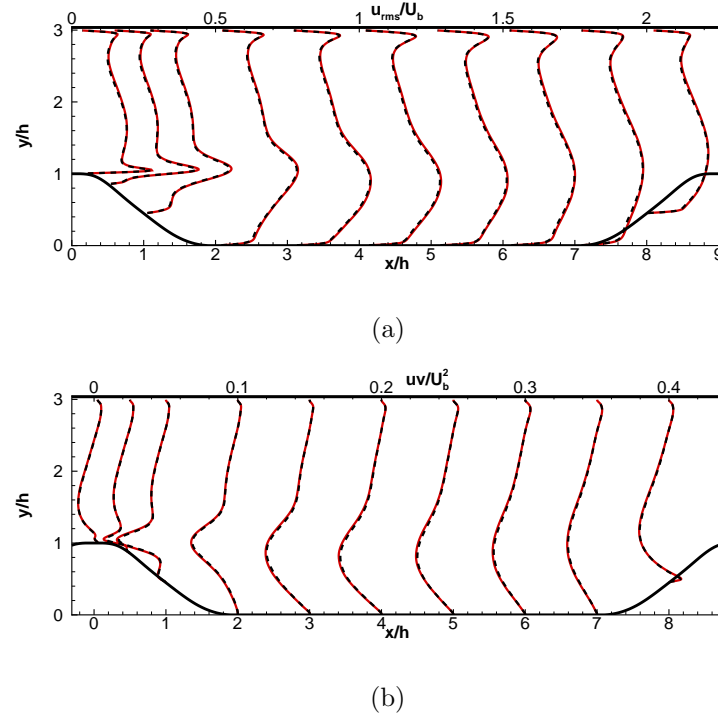


FIG. 12. Predicted turbulence distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=100$ , —: Scheme-A; - - - - -: Scheme-C,  $864 \times 288 \times 462$  grid). (a) Streamwise turbulence intensity, (b) Shear stress.



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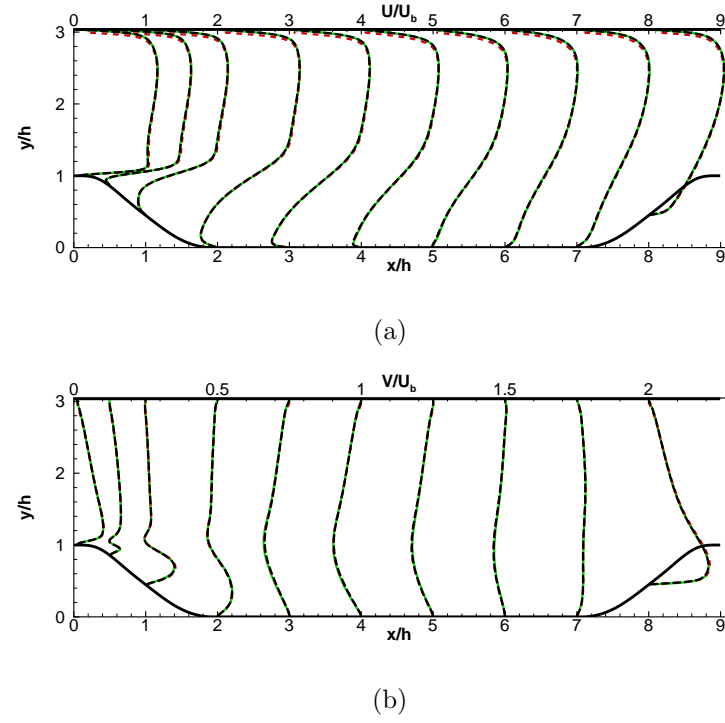


FIG. 13. Predicted mean velocity distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ - $Re=2800$ . (—: Breuer et al. (2009); - - - - -:  $H=3h$ ,  $864 \times 288 \times 462$  grid; - - - - -:  $H=3.036h$ ,  $864 \times 292 \times 462$  grid). (a) Mean streamwise velocity, (b) Mean vertical velocity.

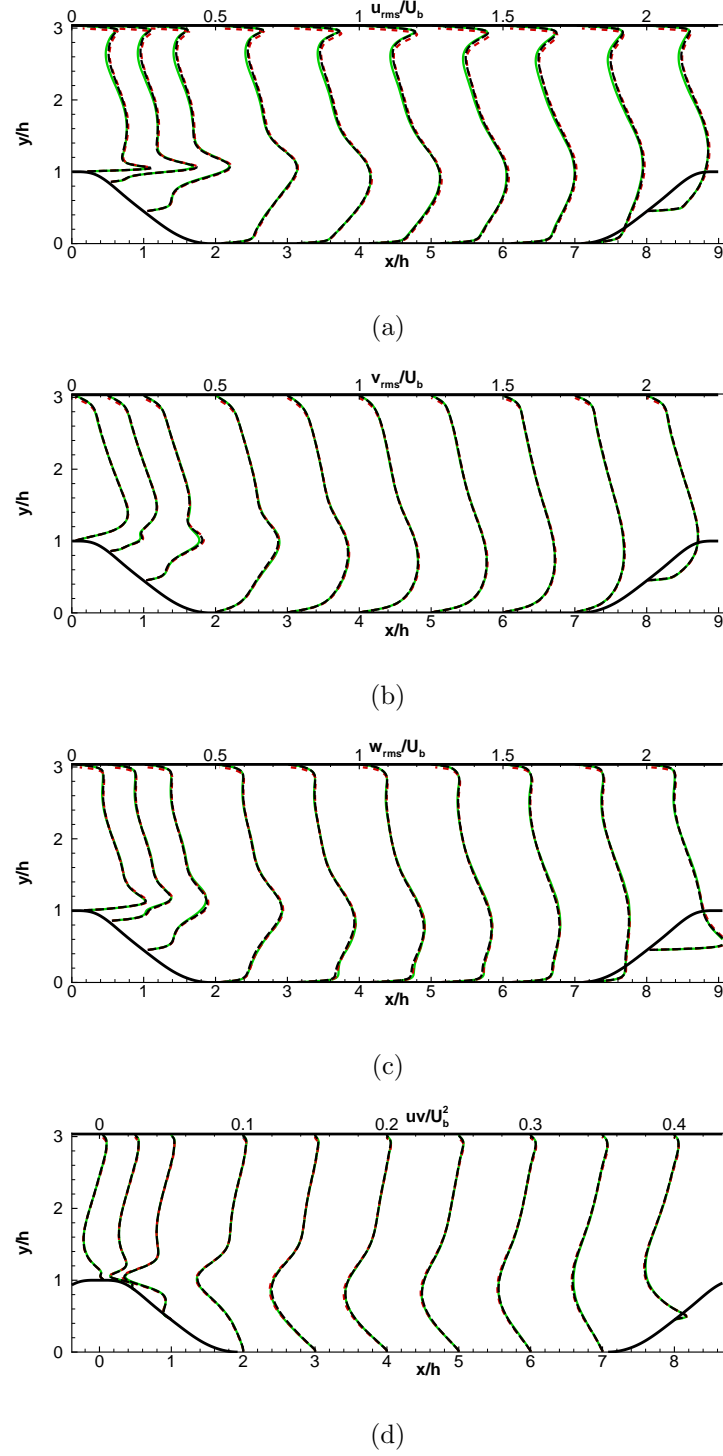


FIG. 14. Predicted turbulence intensity and shear stress distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ - $Re=2800$ . (—: Breuer et al. (2009); - - - :  $H=3h$ ,  $864 \times 288 \times 462$  grid; ····:  $H=3.036h$ ,  $864 \times 292 \times 462$  grid). (a) Streamwise turbulence intensity, (b) Vertical turbulence intensity, (c) Spanwise turbulence intensity, (d) Shear stress.

