

<sup>1</sup> **Direct numerical simulations of turbulent periodic-hill flows with mass-conserving  
2 lattice Boltzmann method**

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Multi-relaxation time lattice Boltzmann method is used to perform direct numerical simulations of laminar and turbulent pressure-driven flows within a channel with hill-shape periodic constriction for the first time. The simulations are conducted on graphics processing unit cluster with two-dimensional domain decomposition to accelerate the computation. The hill-shape boundary is represented using the interpolated bounce back scheme. However, the scheme generates mass leakage across the boundary, which is more pronounced in the turbulent flow regime, and this may produce diverging solutions for turbulent flows. The mass leakage due to the local mass imbalance along the curved boundary is solved by modifying the distribution functions locally or globally, and both predict similar velocity distributions. Since the global correction method is more computing time-consuming, the local correction method is adopted. The present numerical implementation's capability is first validated by performing direct numerical simulations of turbulent channel flow at  $Re_\tau = 180$ , and the current predicted results agree well with the benchmark solutions. Direct numerical simulations are further conducted for the turbulent flow over the periodic hill at  $Re_h = 2800$ . Both the mean velocity and turbulent stress compare favorably with the benchmark solutions. The present simulation also correctly predicts the turbulence splatting effect near the windward hill. Both phenomena are in good accordance with the benchmark solutions.

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## 26 I. INTRODUCTION

27 Lattice Boltzmann method (LBM) has been used as an alternative to the Navier-Stokes  
28 equation to simulate the dynamic, thermal, and two-phase flows<sup>1–3</sup>. Most applications focus  
29 on simple geometric flows at lower Reynolds numbers, because the mesh is in general uni-  
30 form due to the Lagrangian discretization of the convective term<sup>1</sup>. Despite this limitation,  
31 the application of LBM to simulate turbulent flows were also actively pursued. Homoge-  
32 neous isotropic box turbulence, for example, is frequently examined<sup>4–6</sup>, where in general, the  
33 uniform mesh is adopted. LBM was demonstrated to generate compatible results with its  
34 Navier-Stokes equation-based counterpart, showing accurate energy spectrum distributions  
35 and energy decay.

36 Further, it would be desirable to perform LBM simulations of wall-bounded turbulent  
37 flows. A typical flow examined is the pressure-driven turbulent channel flows and is used  
38 as test cases to explore the validity of LBM studies. The predicted results were shown  
39 to reproduce the Navier-Stokes DNS data<sup>7</sup>, though the investigated Reynolds number is  
40 generally low, such as at  $Re_\tau = 180^{8–11}$ . For higher Reynolds number flows, the van Dri-  
41 est damping function and wall function were adopted to overcome the low near-wall grid  
42 resolution of turbulent wall-bounded flows<sup>12,13</sup>. Alternatively, Wu et al.<sup>14</sup> adopted the block-  
43 mesh refinement<sup>15</sup> to simulate turbulent channel flows up to  $Re_\tau = 640$ , where a successive  
44 grid-refinement resolved the wall layer toward the wall.

45 On the other hand, turbulent flows along curved surfaces with favorable or adverse pres-  
46 sure gradients are also frequently investigated. To mimic the complex geometry within  
47 the Cartesian mesh, several strategies have been proposed<sup>15–18</sup>. For example, Chen et al.<sup>16</sup>  
48 extended the extrapolation scheme to curved boundary using the castellated approach. Fil-  
49 ippova and Hanel<sup>15</sup> proposed a method using simple linear interpolation between a fictitious  
50 equilibrium distribution function and a well-chosen near-boundary distribution function.  
51 The weighting factor of the interpolation is determined by the distance between the bound-  
52 ary and the near-boundary lattice. Mei et al.<sup>17</sup> further improved its numerical stability.  
53 Bouzidi et al.<sup>19</sup> proposed the interpolated bounce back scheme, where again, the interpola-  
54 tion is based on the wall distance parameter. Lallemand and Luo<sup>18</sup> combined the bounce-  
55 back scheme and interpolation scheme to treat a moving curved boundary by the lattice  
56 Boltzmann method. This treatment is an extension of that proposed by Bouzidi et al.<sup>19</sup>. An

57 alternate approach was proposed by Lin and his co-workers<sup>2,20</sup>, where a force like corrector  
58 was adopted to enforce the momentum and energy on the boundary nodes.

59 However, Lallenmand and Luo<sup>21</sup> indicated that the interpolation-based schemes destroy  
60 the mass conservation near the boundary. Thus, several mass conserving schemes have been  
61 proposed<sup>22–24</sup>. Further, Sanjeevi et al.<sup>24</sup> performed a systematic study to examine the degree  
62 of mass leakage over the curved boundary. It was found that the currently available schemes  
63 do not conserve mass over the curved boundary. The mass leakage is even more pronounced  
64 when the Reynolds number is high, and different mass conserving strategies were proposed<sup>24</sup>.  
65 However, the Reynolds number explored in previous studies<sup>15–19,22–24</sup> are still in the laminar  
66 flow regime.

67 The aim of the present study is to simulate turbulent wall bounded flows with extensive  
68 separation over curved surface using LBM. Flow separation due to the adverse pressure  
69 gradient has been the focus of studies, which can be caused by the abrupt change of the  
70 geometry<sup>25–27</sup>, or the smoothly expanding boundary<sup>28</sup>. In particular, the recirculation in-  
71 duced by the gradual expanding-geometry along the curved surface is challenging to predict  
72 because the separating point can not be determined in advance, and the related research has  
73 been actively pursued. Thus, turbulent flow over periodic hills has been studied experimen-  
74 tally and numerically<sup>28–31</sup> due to the existence of complex flow patterns such as separation,  
75 recirculation, and reattachment. However, to the knowledge of the authors, LBM based  
76 direct numerical simulations (DNS) of turbulent periodic-hill flows are not available. In the  
77 present study, the focus is to perform direct numerical simulations of turbulent periodic-hill  
78 flow with a multi-relaxation time lattice Boltzmann method. The boundary condition is im-  
79 plemented using the scheme by Bouzidi et al.<sup>19</sup> with mass correction strategies<sup>24</sup>. There are  
80 three issues to be addressed here, i.e., determining the appropriate driving force to achieve  
81 the desired Reynolds number, the extent of the mass leakage of the interpolated bounce  
82 back scheme without mass correction, especially for turbulent flows, and the influence of  
83 the mass correction schemes on the solutions. Finally, turbulent flow predictions are to be  
84 contrasted with the DNS data of Breuer et al.<sup>30</sup> to assess the effectiveness of the present  
85 implementations. The simulation is conducted on message passing interface (MPI)-based  
86 graphics processing unit (GPU) cluster<sup>32,33</sup>. The remainder of this paper is organized as  
87 follows: In Section II, the mathematical formulation of the method is introduced. Finally,  
88 section IV present the conclusion.

89 **II. MATHEMATICAL FORMULATION**

90 **A. Multi relaxation time lattice Boltzmann model**

91 The D3Q19 multi-relaxation-time (MRT) lattice Boltzmann method<sup>1,34,35</sup> can be ex-  
92 pressed by collision and streaming steps, respectively as in the following:

$$f_i^+(\mathbf{x}, t) = f_i(\mathbf{x}, t) - M_{il}^{-1} S_{lj} [m_j(\mathbf{x}, t) - m_j^{eq}(\mathbf{x}, t)] + G_i(\mathbf{x}, t) \Delta t \quad (1)$$

$$f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i^+(\mathbf{x}, t) \quad (2)$$

93 where  $\mathbf{M}^1$  is a matrix that transforms the distribution function  $f_i$  to the velocity moment,  
94  $m_j = M_{ji} f_i$ .  $\mathbf{S}^1$  is the relaxation time diagonal matrix, and  $G_i$  is the external forcing term.

95 Based on the particle distribution functions, the macroscopic density and velocity can be  
96 obtained as:

$$\sum_i f_i = \rho, \quad \sum_i f_i \mathbf{e}_i = \rho \mathbf{u} \quad (3)$$

97 The equilibrium moments  $m_i^{eq}$  and external force term  $G_i$  are determined as:

$$m_j^{eq} = \underbrace{M_{ji} w_i \rho [1 + \frac{3}{C^2} (\mathbf{e}_i \cdot \mathbf{u}) + \frac{9}{2C^4} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2C^2} (\mathbf{u} \cdot \mathbf{u})]}_{f_i^{eq}}, \quad (4)$$

$$G_i = 3w_i \rho \frac{\mathbf{e}_i \cdot \mathbf{F}}{C^2} \quad (5)$$

99  $C = \Delta x / \Delta t$  is the lattice speed, where  $\Delta x$  and  $\Delta t$  are the lattice width and time step,  
100 respectively. Here,  $\Delta x = \Delta t$ , i.e.  $C = 1$  and  $\mathbf{F}$  is set to be the pressure gradient along the  
101 streamwise direction.

102 The weighting coefficients  $w_i$  are respectively as,  $w_0 = 1/3$ ,  $w_{1-6} = 1/6$ , and  $w_{7-18} =$   
103  $1/36$ . The particle velocity  $\mathbf{e}_i$  is defined as,

$$\mathbf{e}_i = \begin{cases} (0, 0, 0)C, & i = 0 \\ (\pm 1, 0, 0)C, (0, \pm 1, 0)C, (0, 0, \pm 1)C & i = 1 \sim 6 \\ (\pm 1, \pm 1, 0)C, (\pm 1, 0, \pm 1)C, (0, \pm 1, \pm 1)C & i = 7 \sim 18 \end{cases} \quad (6)$$

<sup>104</sup> **B. Boundary conditions**

<sup>105</sup> Along the boundary, due to the inward streaming operations, particle distribution func-  
<sup>106</sup> tion may originate from the undefined nodes external to the flow domain. Therefore measures  
<sup>107</sup> have to be taken to prescribe these unknown particle distribution functions. For example,  
<sup>108</sup> at the near boundary fluid nodes, i.e., solid circular nodes shown in Fig. 1, nodes  $A$ ,  $B$ ,  $C$   
<sup>109</sup> and  $\mathbf{x}_{nw}$  have respectively three, one, two and three unknown distribution functions (vectors  
<sup>110</sup> 1-9). Here  $nw$  denotes the near-wall fluid nodes. The solid square ( $X_w$ ) is the intersection  
<sup>111</sup> of the upstream link (dashed line) with the wall boundary of the respective unknown dis-  
<sup>112</sup> tribution function. For easy identification, the same color is applied for the wall node ( $X_w$ )  
<sup>113</sup> and upstream link (dashed line) for the respective unknown distribution function.

<sup>114</sup> Consider the near wall fluid node at  $X_{nw}$ . If the unknown distribution function is denoted  
<sup>115</sup> as  $f_p(\mathbf{x}_{nw}, t)$ , then  $f_{i \neq p}(\mathbf{x}_{nw}, t + \Delta t) = f_{i \neq p}^+(\mathbf{x}_{nw} - \mathbf{e}_i \Delta t, t)$  (Eq. 2). Also, the unknown  
<sup>116</sup> distribution function is obtained by using the interpolated bounce back scheme proposed by  
<sup>117</sup> Bouzidi et al.<sup>19</sup>, which is based on a wall distance parameter defined as  $q = |\mathbf{x}_{nw} - \mathbf{x}_w| / |\mathbf{e}_p \Delta t|$ ,  
<sup>118</sup> where the respective wall locations are represented by the solid square symbols along the  
<sup>119</sup> solid-fluid boundary shown in Fig. 1. Depending on the value of the parameter  $q$ , Bouzidi  
<sup>120</sup> et al.<sup>19</sup> proposed that,

$$\begin{aligned} q < 0.5, \quad f_p(\mathbf{x}_{nw}, t + \Delta t) &= (1 - 2q)f_{-p}^+(\mathbf{x}_{nw} + \mathbf{e}_p \Delta t, t) + 2qf_{-p}^+(\mathbf{x}_{nw}, t) \\ q \geq 0.5, \quad f_p(\mathbf{x}_{nw}, t + \Delta t) &= (1 - \frac{1}{2q})f_p^+(\mathbf{x}_{nw}, t) + \frac{1}{2q}f_{-p}^+(\mathbf{x}_{nw}, t) \end{aligned} \quad (7)$$

<sup>121</sup> where  $\mathbf{e}_{-p} = -\mathbf{e}_p$  shown in Fig. 1 and open circle ( $\mathbf{x}_{nw} + \mathbf{e}_p$ ) is the second fluid node away  
<sup>122</sup> from the wall.

<sup>123</sup> For  $q = 0.5$ , the scheme recovers the halfway bounce back scheme of Ladd<sup>36</sup>, and the  
<sup>124</sup> mass is conserved. When  $q \neq 0.5$ , the interpolation does not guarantee the local mass con-  
<sup>125</sup> servation, i.e.  $\Delta\rho(\mathbf{x}_{nw}) = \sum f_{-p}^+(\mathbf{x}_{nw}, t) - \sum f_p(\mathbf{x}_{nw}, t + \Delta t) \neq 0$ . As suggested by Sanjeevi  
<sup>126</sup> et al.<sup>24</sup>, there are four possibilities that the imbalance can be added to the distribution  
<sup>127</sup> function, i.e.,

$$f_0(\mathbf{x}_{nw}, t + \Delta t) = f_0^+(\mathbf{x}_{nw}, t) + \Delta\rho(\mathbf{x}_{nw}) \quad (\text{scheme - A}) \quad (8)$$

$$f_i(\mathbf{x}_{nw}, t + \Delta t) = f_i(\mathbf{x}_{nw}, t + \Delta t) + w_i \Delta\rho(\mathbf{x}_{nw}) \quad (\text{scheme - B}) \quad (9)$$

$$f_0(\mathbf{x}_f, t + \Delta t) = f_0^+(\mathbf{x}_f, t) + \frac{\sum \Delta\rho(\mathbf{x}_{nw})}{N_f} \quad (\text{scheme - C}) \quad (10)$$

$$f_i(\mathbf{x}_f, t + \Delta t) = f_i(\mathbf{x}_f, t + \Delta t) + w_i \frac{\sum \Delta\rho(\mathbf{x}_{nw})}{N_f} \quad (\text{scheme - D}) \quad (11)$$

where  $N_f$  is the number of fluid nodes. For Eqs. 8 and 9, the corrections are added to the local nodes and for Eqs. 10 and 11, the corrections are distributed to all the fluid nodes. The influences of adopting Eqs. 8 to 11 on the solutions will be explored in the results section.

### III. RESULTS

#### A. Turbulent channel flow

The numerical procedure is first validated by predicting the turbulent channel flow at  $Re_\tau(u_\tau\delta/\nu)=180$ , where the computational domain is  $12\delta \times 2\delta \times 4.5\delta$  in the streamwise, vertical and spanwise directions, respectively. Two grid densities are adopted, i.e.,  $576 \times 96 \times 288$  and  $1152 \times 192 \times 576$  with corresponding grid densities being  $\Delta^+=3.8$  and 1.9. Since a halfway bounce-back is adopted, therefore, the first grid point is located at  $y^+=1.9$  and 0.95. Fig. 2(a) shows the predicted velocity distributions compared with the results of Moser et al.<sup>7</sup>, where compatible results are predicted, though a slight deviation is observed using the  $576 \times 96 \times 288$  grids. Similar results can be observed in the turbulence intensity predictions, shown in Fig. 2(b).

#### B. Flows over periodic hill

Here, the focus is on the predictions of flow over the periodic hill. Fig. 3 shows the geometry of the periodic hill, and the height of the domain is  $3h$ , instead of the commonly used,  $3.036h^{29,30}$ . For turbulent flow, the width in the spanwise direction is chosen to be  $4.5h^{29,30}$  and for laminar flow the width is  $2.25h$ . The boundary condition is implemented using the scheme by Bouzidi et al.<sup>19</sup> with mass correction<sup>24</sup>, and for the straight boundary,  $q$  is 0.5, i.e., halfway bounce back. The Reynolds number is defined based on the bulk velocity at the hillcrest and the hill height ( $U_{bc}h/\nu$ ), which is half of the Reynolds number based on the height of the flow passage. When simulating flows over the periodic hill, there are three issues to be addressed, i.e., determining the appropriate driving force to achieve the desired Reynolds number, the extent of the mass leakage of the interpolated bounce back scheme without mass correction, especially for turbulent flows, and finally the influence of the mass correction schemes on the solutions. Finally, turbulent flow predictions are to be

156 contrasted with the DNS data of Breuer et al.<sup>30</sup> to assess the effectiveness of the present  
157 implementations.

158 **1. Determination of driving force**

159 For the periodic-hill geometry considered, there is no analytic solution of the driving  
160 force to reach the expected Reynolds number. Here, a similar measure as in Hsu et al.<sup>37</sup> is  
161 adopted. To achieve the desired bulk velocity the driving force is adjusted via the following  
162 formula, i.e.,

$$F^{IN_t} = F^{(I-1)N_t} + \beta \rho (U_{ref} - U_b^{IN_t-1}) \frac{U_{ref}}{L}, \quad (12)$$
$$I = 1, 2, 3, \dots$$

163 where  $\beta = \max(0.001, \alpha/Re_h)$ .  $N_t$  is the interval of the time step for force adjustment,  
164 which is adopted as 10,000. Thus, the force is updated every 10,000 steps ( $\sim 0.67 L/U_{ref}$ -  
165 flow through time) to avoid generating new disturbance, especially at high Reynolds number  
166 flows.  $U_{ref}$  and  $U_b(t)$  are the target bulk velocity and the predicted bulk velocity at the  
167 hillcrest. Here,  $U_{ref} = U_{bc}$ . The value of  $\alpha$  investigated ranges from 3 to 14. Eq. 12 is  
168 applied to compute laminar and turbulent flows at Reynolds numbers being 100 and 2800,  
169 respectively. The mass conserved scheme adopted is scheme-A of Eq. 8.

170 Fig. 4 shows the time history of the predicted bulk velocity variations ( $U^* = U_b(t)/U_{ref}$ )  
171 with the force adjusted with Eq. 12 for laminar flow at  $Re=100$  using  $\alpha=10$ . The simulation  
172 saturates to the desired bulk velocity monotonically. For turbulent flow, such as at  $Re=2800$ ,  
173 results using  $\alpha = 3$  and 14 produce similar results. Fig. 5 shows the time variations of  
174 the force and bulk velocity for turbulent flow at  $Re=2800$  with  $\alpha = 14$ . The predicted  
175 bulk velocity oscillates around the target velocity, resulting from the compensating force  
176 adjustment and the internal flow instability. After a few transients ( $\sim 200$  flow-through  
177 time), the velocity is time-averaged. As shown in Fig. 5, the time-averaged bulk velocity  
178 approaches the designated bulk velocity as time proceeds. The time-averaged period is  
179 approximately 700 flow-through time ( $U_{ref}/L$ ).

180 **2. Mass leakage**

181 As indicated earlier, the original interpolated bounce back scheme by Bouzidi et al.<sup>19</sup>  
182 may cause mass leakage through the curved boundary<sup>24</sup>. For turbulent flows, this may lead  
183 to diverging solutions. Here, the influences with and without mass correction are explored,  
184 and the mass correction scheme adopted is scheme-A. Simulations are conducted for laminar  
185 flow ( $Re=100$ ) and turbulent flows ( $Re=700$  and  $2800$ ) and the results are shown in Figs. 6  
186 and 7. For laminar flow at  $Re=100$ , as shown in Fig. 6, coarse grid generates higher degree  
187 of mass leakage. However, at 40 flow-through times, the mass leakages are around 2%, 0.4%  
188 and 0.00001% respectively for  $144 \times 48$ ,  $288 \times 96$  grid and  $576 \times 192$  grids. For turbulent  
189 flows, the mass correction is more influential, as shown in Fig. 7. The mass leakages at  
190 40 flow-through times are 0.25% and 2.5%, respectively, for  $Re=700$  and  $2800$ . For flow  
191 at  $Re=2800$ , simulation with the original interpolated bounce back scheme is unstable and  
192 prone to diverge as time progresses.

193 **3. Influences of mass correction schemes**

194 Here, to save computational time, the influence of the correction schemes, i.e., Eqs. 8  
195 to 11 on the solutions are explored first for laminar flow at  $Re=100$  using  $576 \times 192$  grid.  
196 Figs. 8(a) to 8(c) show the predicted velocity and pressure distributions at eight selected  
197 locations.. The deceleration and acceleration of the flow due to the hill's presence is clearly  
198 observed and its associated decrease or rise of the predicted pressure levels. No perceivable  
199 difference is observed using the four schemes at such a fine grid, even for the vertical velocity,  
200 which is relatively smaller than the streamwise velocity.

201 Zoomed views of the pressure contours at the windward hill are shown in Fig. 9, where  
202 results using scheme-A and scheme-C are presented. At two to three grid spacing above  
203 the hill boundary, the predicted contours are similar. However, using scheme-A, there is a  
204 local increase in the pressure level near the boundary, whereas scheme-C generates smoother  
205 results. These are consistent with those observed in Sanjeevi et al.<sup>24</sup>. It should be further  
206 noted here that schemes (A) and (B) show a similar rise in the local pressure distributions at  
207 the windward hill. The pressure distributions using schemes (C) and (D) are identical and  
208 do not show such a local rise of pressure at the corresponding region. Despite this deficiency,

predicted velocity distributions are exactly the same using the four schemes. For the lines representing different schemes collapse and the difference can not be observed, as shown in the zoomed view of the velocity contours shown in Fig. 10.

Figs. 11 and 12 show the predicted mean and turbulent quantities at  $Re=2800$  using scheme-A and C. Again, no perceivable differences are observed among the results. Since, Eqs. 10 and 11 requires global operations, and are thus time consuming compared to Eqs. 8 and 9, therefore, in subsequent simulations, Scheme-A is adopted for simplicity.

#### 4. *Turbulent periodic-hill flow at $Re=2800$*

Here, the focus is to validate the turbulent periodic-hill flow predictions using the direct numerical simulation data of Breuer et al.<sup>30</sup>, where the numerical procedure is based on the Navier-Stokes equation using curvilinear grid discretized with the second-order accurate scheme. The adopted Reynolds number based on the bulk velocity  $U_b$  and hill height  $h$  is 2800. The computational domain is shown in Fig. 3. Here, the mass conservation scheme-A is adopted for simplicity. The grids adopted in the streamwise, vertical and spanwise directions are  $576 \times 192 \times 288$  and  $864 \times 288 \times 432$ . Since two grids generate compatible results (not shown here),  $864 \times 288 \times 432$  grid is used. Also,  $H=3.036h$  is also adopted in the simulations using  $864 \times 292 \times 432$  grid to examine the influence of the height on the solution.

Fig. 13 shows the predicted mean velocity distributions at ten selected locations, and the results are contrasted with the DNS data of Breuer et al.<sup>30</sup>. The usage of  $H=3h$  as height causes a slight departure from the benchmark solution of the mean streamwise velocity distributions shown in Fig. 13(a) in the region near the top boundary, and the simulated results using  $3.036h$  agree quite well with the benchmark solutions. Despite the slight difference of the adopted height, the predicted results agree quite well with Breuer et al.'s DNS data for both the streamwise and vertical velocity components. The shear layer and recirculation zone are predicted well by the present scheme. The top boundary's height has a marginal impact on the bottom wall-flow, and this was also observed by Fröhlich et al.<sup>29</sup>.

Figs. 14(a)-14(c) and 14(d) show respectively the predicted turbulence intensities and shear stresses. Apart from the slight deviation of the predicted turbulence intensity near the top boundary due to the different height adopted, in general, the agreements are good.

<sup>239</sup> The rise of the turbulence level near the shear layer at the height of the hillcrest is also in  
<sup>240</sup> accordance with the DNS data of Breuer et al.<sup>30</sup>. The rise of the bottom wall spanwise tur-  
<sup>241</sup> bulence intensity at  $x/h=8$ , which is greater than the corresponding streamwise turbulence  
<sup>242</sup> intensity, is due to the pressure strain-induced splatting effect<sup>29</sup> resulting from the presence  
<sup>243</sup> of the windward hill.

<sup>244</sup> **IV. CONCLUSION**

<sup>245</sup> Turbulent pressure-driven flows within a channel with hill-shape periodic constriction  
<sup>246</sup> are simulated with multi-relaxation time lattice Boltzmann method on GPU cluster. A  
<sup>247</sup> methodology is proposed to determine the appropriate driving force to achieve the desired  
<sup>248</sup> Reynolds number. The hill-shape boundary is mimicked by the interpolated bounce back  
<sup>249</sup> scheme by Bouzidi et al.<sup>19</sup>. However, the scheme generates mass leakage across the boundary,  
<sup>250</sup> which is more pronounced in the turbulent flow regime, and this may produce diverging  
<sup>251</sup> solutions for turbulent flows. The mass leakage due to the local mass imbalance along the  
<sup>252</sup> curved boundary is solved by modifying the distribution function locally or globally, as  
<sup>253</sup> suggested by Sanjeevi et al.<sup>24</sup>. The locally modified method produces a slight increase in  
<sup>254</sup> pressure locally in contrast to the global correction methods. On the other hand, the mass  
<sup>255</sup> correction strategies marginally influence the predicted velocity distributions. Since the  
<sup>256</sup> global correction method is more computing time-consuming, the local correction method  
<sup>257</sup> is adopted. The capability of the present numerical implementation is first validated by  
<sup>258</sup> performing direct numerical simulations of turbulent channel flow at  $Re_\tau = 180$ , and the  
<sup>259</sup> current predicted results agree well with the benchmark solution by Moser et al.<sup>7</sup>. Direct  
<sup>260</sup> numerical simulations of turbulent flow over the periodic hill are conducted at  $Re_h = 2800$ ,  
<sup>261</sup> and the predicted results are contrasted with the DNS data of Breuer et al<sup>30</sup>. Both the mean  
<sup>262</sup> velocity and turbulent stress compare favorably with the benchmark solutions. The present  
<sup>263</sup> simulation also correctly predicts the turbulence splatting effect, i.e., spanwise turbulence  
<sup>264</sup> intensity being higher than the corresponding streamwise turbulence intensity, resulting from  
<sup>265</sup> the windward hill's presence. Both phenomena are in good accordance with the DNS data  
<sup>266</sup> by Breuer et al.<sup>30</sup>.

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**271 DATA AVAILABILITY STATEMENT**

272 The data that support the findings of this study are available from the corresponding  
273 author upon reasonable request.

**274 REFERENCES**

- 275 <sup>1</sup>D. d'Humierès, I. Ginzburg, M. Krafczyk, P. Lallemand, and L.-S. Luo, "Multiple-  
276 relaxation-time lattice boltzmann models in three dimensions," *Phil. Trans. R. Soc. A.*  
277 **360**, 437 – 451 (2002).
- 278 <sup>2</sup>K.-H. Lin, C.-C. Liao, S.-Y. Lien, and C.-A. Lin, "Thermal lattice boltzmann simulations  
279 of natural convection with complex geometry," *Computers and Fluids* **69**, 35 – 44 (2012).
- 280 <sup>3</sup>A. Fakhari, M. Geier, and T. Lee, "A weighted multiple-relaxation-time lattice boltzmann  
281 method for multiphase flows and its application to partial coalescence cascades," *Journal  
282 of Computational Physics* **341**, 22 – 43 (2017).
- 283 <sup>4</sup>N. Satofuka and T. Nishioka, "Parallelization of lattice boltzmann method for incompress-  
284 ible flow computations," *Computational Mechanics* **23**, 164–171 (1999).
- 285 <sup>5</sup>H. Yu, S. S. Girimaji, and L.-S. Luo, "Lattice boltzmann simulations of decaying homo-  
286 geneous isotropic turbulence," *Phys. Rev. E* **71**, 016708 (2005).
- 287 <sup>6</sup>W. A. Kareem, S. Izawa, A.-K. Xiong, and Y. Fukunishi, "Lattice boltzmann simulations  
288 of homogeneous isotropic turbulence," *Computers and Mathematics with Applications* **58**,  
289 1055 – 1061 (2009).
- 290 <sup>7</sup>R. D. Moser, J. Kim, and N. N. Mansour, "Direct numerical simulation of turbulent  
291 channel flow up to  $\text{ret}=590$ ," *Physics of Fluids* **11**, 943–945 (1999).
- 292 <sup>8</sup>P. Lammers, K. Beronov, R. Volkert, G. Brenner, and F. Durst, "Lattice bgk direct  
293 numerical simulation of fully developed turbulence in incompressible plane channel flow,"  
294 *Computers and Fluids* **35**, 1137 – 1153 (2006).

- <sup>295</sup> <sup>9</sup>K. Suga, Y. Kuwata, K. Takashima, and R. Chikasue, “A d3q27 multiple-relaxation-time lattice boltzmann method for turbulent flows,” Computers and Mathematics with Applications **69**, 518 – 529 (2015).
- <sup>298</sup> <sup>10</sup>Y. Koda and F.-S. Lien, “The lattice boltzmann method implemented on the gpu to simulate the turbulent flow over a square cylinder confined in a channel,” Flow, Turbulence and Combustion **94**, 495–512 (2015).
- <sup>301</sup> <sup>11</sup>Y.-H. Lee, L.-M. Huang, Y.-S. Zou, S.-C. Huang, and C.-A. Lin, “Simulations of turbulent duct flow with lattice boltzmann method on gpu cluster,” Computers and Fluids **168**, 14 – 20 (2018).
- <sup>304</sup> <sup>12</sup>K. N. Premnath, M. J. Pattison, and S. Banerjee, “Generalized lattice boltzmann equation with forcing term for computation of wall-bounded turbulent flows,” Phys. Rev. E **79**, 026703 (2009).
- <sup>307</sup> <sup>13</sup>S. Wilhem, J. Jacob, and P. Sagaut, “An explicit power-law-based wall model for lattice boltzmann method-reynolds averaged numerical simulations of the flow around airfoils,” Physics of Fluids **30**, 065111 (2018).
- <sup>310</sup> <sup>14</sup>C.-M. Wu, Y.-S. Zhou, and C.-A. Lin, “Direct numerical simulations of turbulent channel flows with mesh-refinement lattice boltzmann methods on gpu cluster,” Computers and Fluids **210**, 104647 (2020).
- <sup>313</sup> <sup>15</sup>O. Filippova and D. Hanel, “Grid refinement for lattice-bgk models,” Journal of Computational Physics **147**, 219–228 (1998).
- <sup>315</sup> <sup>16</sup>S. Chen, D. Martinez, and R. Mei, “On boundary conditions in lattice boltzmann methods,” Physics of Fluids **8**, 2527–2536 (1996).
- <sup>317</sup> <sup>17</sup>R. Mei, L.-S. Luo, and W. Shyy, “An accurate curved boundary treatment in the lattice boltzmann method,” Journal of Computational Physics **155**, 307–330 (1999).
- <sup>319</sup> <sup>18</sup>P. Lallemand and L.-S. Luo, “Lattice boltzmann method for moving boundaries,” Journal of Computational Physics **184**, 406–421 (2003).
- <sup>321</sup> <sup>19</sup>M. Bouzidi, M. Firdaouss, and P. Lallemand, “Momentum transfer of a boltzmann-lattice fluid with boundaries,” Physics of Fluids **13**, 3452 – 3459 (2001).
- <sup>323</sup> <sup>20</sup>C. Chang, C.-H. Liu, and C.-A. Lin, “Boundary conditions for lattice boltzmann simulations with complex geometry flows,” Computers and Mathematics with Applications **58**, 940 – 949 (2009).

- <sup>326</sup> <sup>21</sup>P. Lallemand and L.-S. Luo, "Lattice boltzmann method for moving boundaries," Journal  
<sup>327</sup> of Computational Physics **184**, 406 – 421 (2003).
- <sup>328</sup> <sup>22</sup>P.-H. Kao and R.-J. Yang, "An investigation into curved and moving boundary treatments  
<sup>329</sup> in the lattice boltzmann method," Journal of Computational Physics **227**, 5671 – 5690  
<sup>330</sup> (2008).
- <sup>331</sup> <sup>23</sup>J. Bao, P. Yuan, and L. Schaefer, "A mass conserving boundary condition for the lattice  
<sup>332</sup> boltzmann equation method," Journal of Computational Physics **227**, 8472 – 8487 (2008).
- <sup>333</sup> <sup>24</sup>S. K. P. Sanjeevi, A. Zarghami, and J. T. Padding, "Choice of no-slip curved boundary  
<sup>334</sup> condition for lattice boltzmann simulations of high-reynolds-number flows," Phys. Rev. E  
<sup>335</sup> **97**, 043305 (2018).
- <sup>336</sup> <sup>25</sup>B. F. Armaly, F. Durst, J. C. F. Pereira, and B. Schönung, "Experimental and theoretical  
<sup>337</sup> investigation of backward-facing step flow," Journal of Fluid Mechanics **127**, 473–496  
<sup>338</sup> (1983).
- <sup>339</sup> <sup>26</sup>K. Basnet and G. Constantinescu, "The structure of turbulent flow around vertical plates  
<sup>340</sup> containing holes and attached to a channel bed," Physics of Fluids **29**, 115101 (2017).
- <sup>341</sup> <sup>27</sup>K. Basnet and G. Constantinescu, "Effect of a bottom gap on the mean flow and turbulence  
<sup>342</sup> structure past vertical solid and porous plates situated in the vicinity of a horizontal  
<sup>343</sup> channel bed," Physical Review Fluids **4**, 044604 (2019).
- <sup>344</sup> <sup>28</sup>G. Almeida, D. Durao, and M. Heitor, "Wake flows behind two-dimensional model hills,"  
<sup>345</sup> Experimental Thermal and Fluid Science **7**, 87–101 (1993).
- <sup>346</sup> <sup>29</sup>J. Fröhlich, C. P. Mellen, W. Rodi, L. Temmerman, and M. A. Leschziner, "Highly resolved  
<sup>347</sup> large-eddy simulation of separated flow in channel with streamwise periodic constriction,"  
<sup>348</sup> Journal of Fluid Mechanics **526**, 19–66 (2005).
- <sup>349</sup> <sup>30</sup>M. Breuer, N. Peller, C. Rapp, and M. Manhart, "Flow over periodic hills - numerical  
<sup>350</sup> and experimental study in a wide range of reynolds numbers," Computers and Fluids **38**,  
<sup>351</sup> 433–457 (2009).
- <sup>352</sup> <sup>31</sup>P.-H. Chang, C.-C. Liao, H.-W. Hsu, S.-H. Liu, and C.-A. Lin, "Simulations of laminar  
<sup>353</sup> and turbulent flows over periodic hills with immersed boundary method," Computers and  
<sup>354</sup> Fluids **92**, 233–243 (2014).
- <sup>355</sup> <sup>32</sup>P.-Y. Hong, L.-M. Huang, L.-S. Lin, and C.-A. Lin, "Scalable multi-relaxation-time lattice  
<sup>356</sup> boltzmann simulations on multi-gpu cluster," Computers and Fluids **110**, 1 – 8 (2015).

- <sup>357</sup> <sup>33</sup>X. Shi, T. Agrawal, C.-A. Lin, F.-N. Hwang, and T.-H. Chiu, "A parallel nonlinear  
<sup>358</sup> multigrid solver for unsteady incompressible flow simulation on multi-gpu cluster," Journal  
<sup>359</sup> of Computational Physics **414**, 109447 (2020).
- <sup>360</sup> <sup>34</sup>D. d'Humières, "Generalized lattice-boltzmann equationsd," Prog. Aeronaut. Astronaut.  
<sup>361</sup> **450-458**, 6546–6562 (1992).
- <sup>362</sup> <sup>35</sup>P. Lallemand and L.-S. Luo, "Theory of the lattice boltzmann method: Dispersion, dissipa-  
<sup>363</sup> tion, isotropy, galilean invariance, and stability," Phys. Rev. E **61**, 6546–6562 (2000).
- <sup>364</sup> <sup>36</sup>A. J. C. Ladd, "Numerical simulations of particulate suspensions via a discretized boltz-  
<sup>365</sup> mann equation. part 1. theoretical foundation," Journal of Fluid Mechanics **271**, 285 –  
<sup>366</sup> 309 (1994).
- <sup>367</sup> <sup>37</sup>H.-W. Hsu, J.-B. Hsu, W. Lo, and C.-A. Lin, "Large eddy simulation of turbulent couette-  
<sup>368</sup> poiseuille and couette flows inside a square duct," Journal of Fluid Mechanics **702**, 89 –  
<sup>369</sup> 101 (2012).

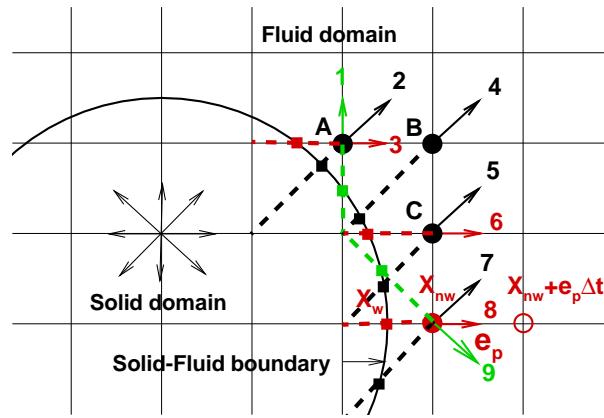


FIG. 1. Solid-fluid boundary treatment (solid square: wall boundary nodes associated with respective arrow; solid circle: near wall fluid node (A, B, C,  $X_{nw}$ ); open circle: the second fluid node away from the wall; arrows (1-9): unknown distribution functions).

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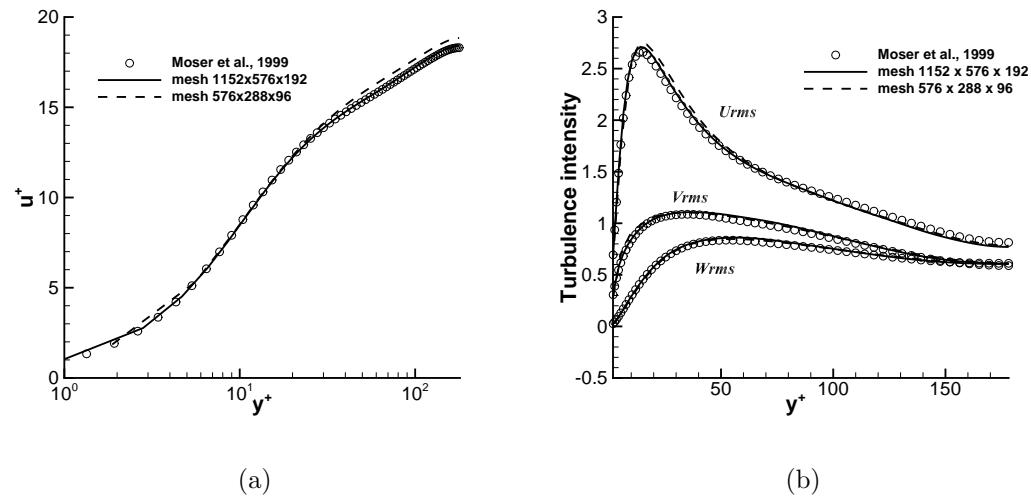


FIG. 2. Predicted streamwise velocity and the turbulent intensities of channel flow at  $Re_\tau = 180$ .  
(a) Streamwise velocity, (b) Turbulence intensities.

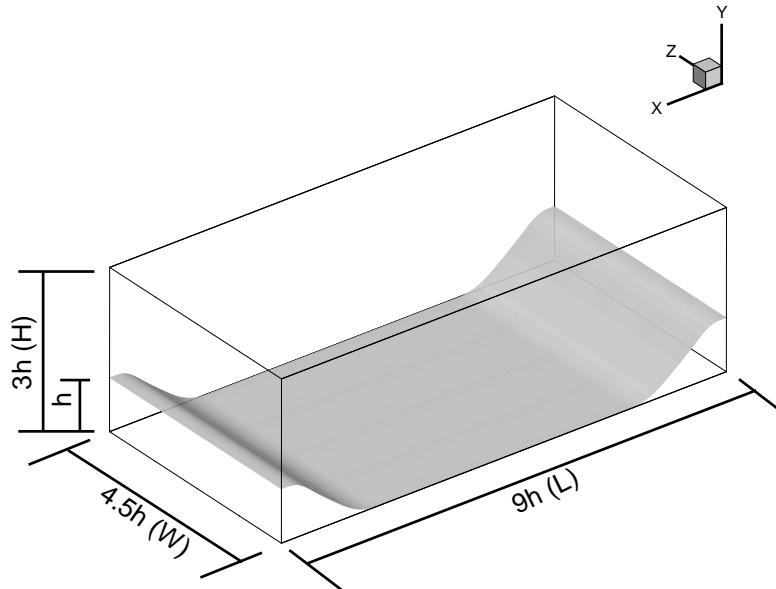


FIG. 3. The geometry of periodic hill.

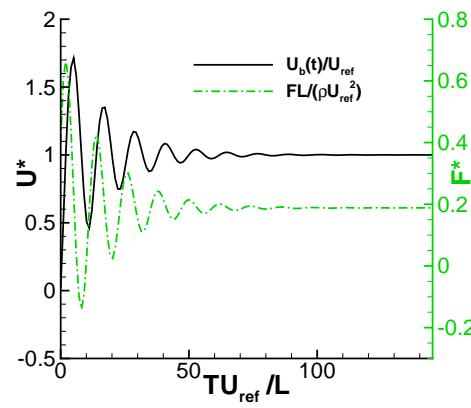


FIG. 4. Predicted bulk velocity and force variations (Re=100, 576 × 192 × 144 grid.)

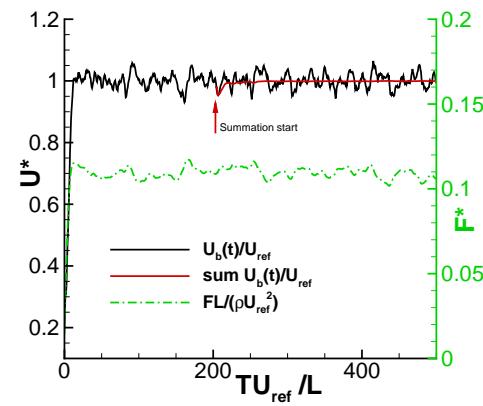


FIG. 5. Predicted bulk velocity and force variations (Re=2800, 864 × 288 × 432 grid.)

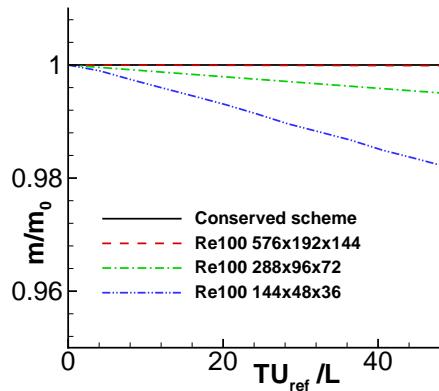


FIG. 6. Mass variations within the computational domain with and without mass correction (Re=100)

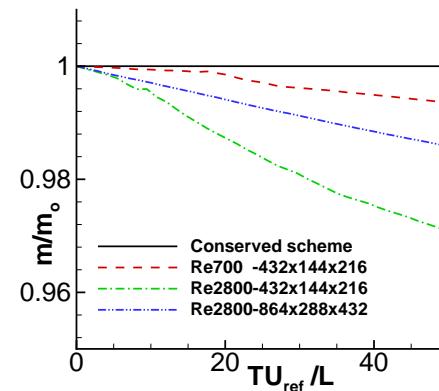


FIG. 7. Mass variations within the computational domain with and without mass correction (Re=700 and 2800)

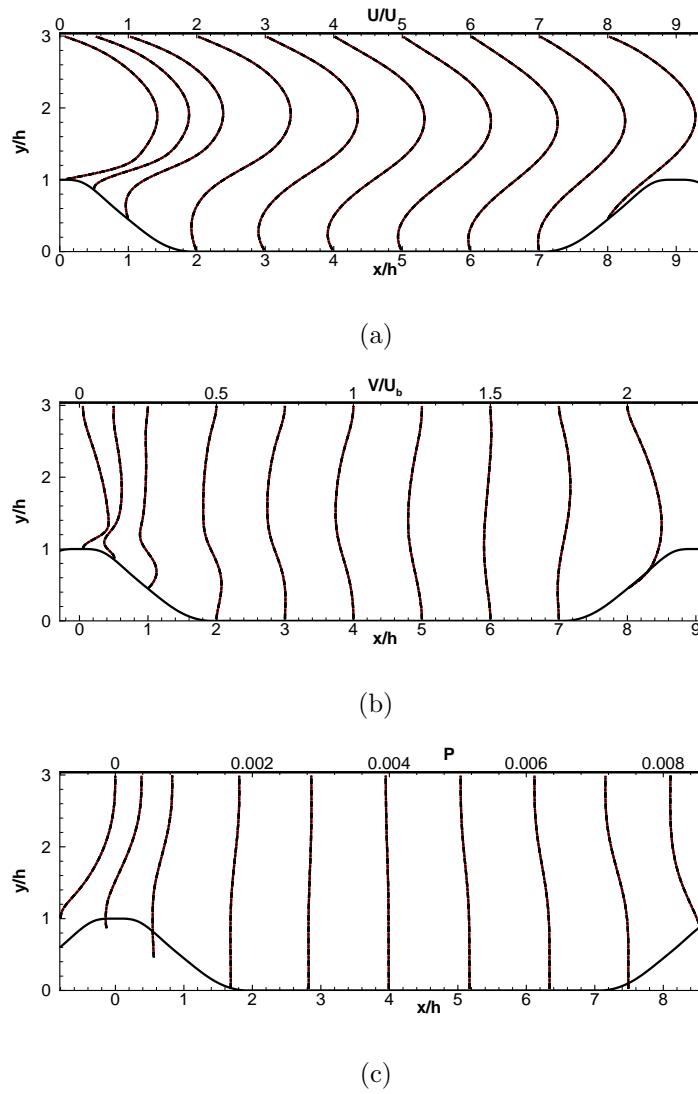


FIG. 8. Predicted velocity and pressure distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=100$ , —: Scheme-A; - - -: Scheme-B; - · - : scheme-C; - · · - : Scheme-D,  $576 \times 192$  grid). (a) U-velocity, (b) V-velocity, (c) Pressure.

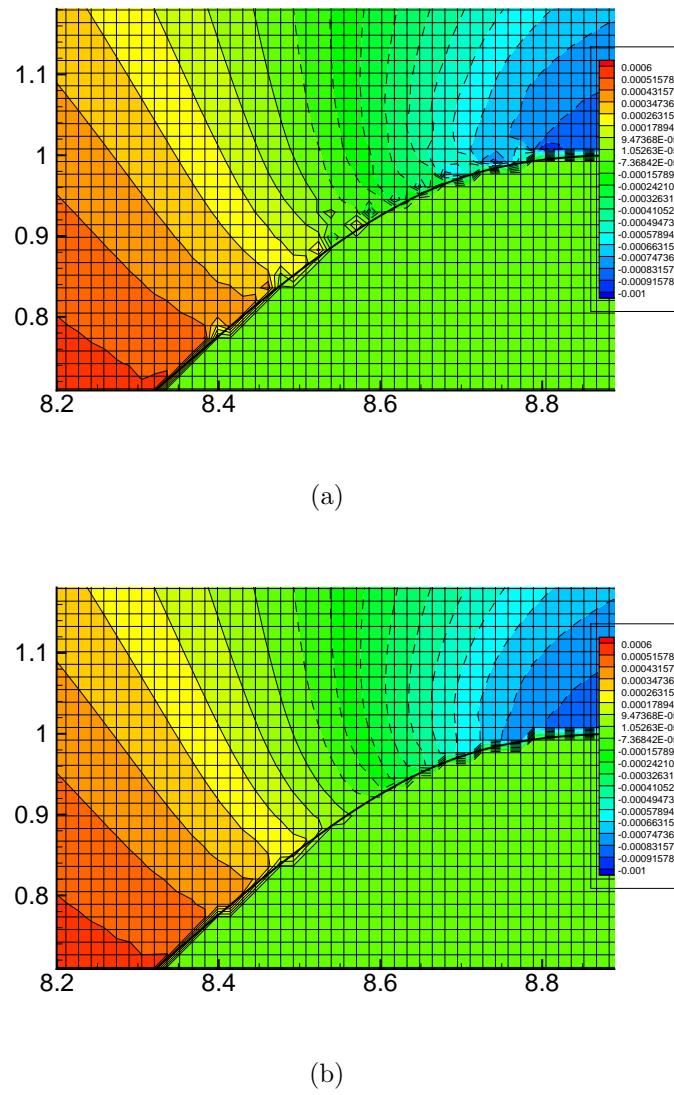


FIG. 9. Distributions of zoom-view pressure contours-Scheme-A and scheme-C ( $Re=100$ ,  $576 \times 192$  grid). (a) Scheme-A, (b) Scheme-C.

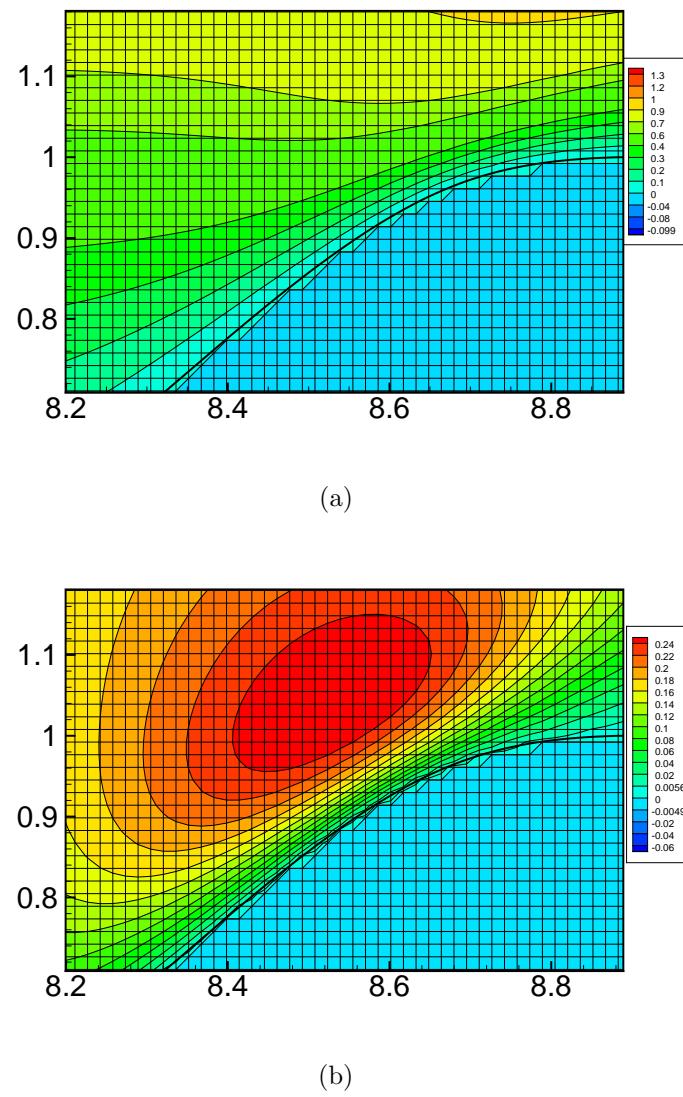


FIG. 10. Distributions of zoom-view streamwise and vertical velocity contours ( $Re=100$ , —: Scheme-A; - - -: Scheme-B; - · - : scheme-C; - · · - : Scheme-D,  $576 \times 192$  grid). (a) U-velocity, (b) V-velocity.

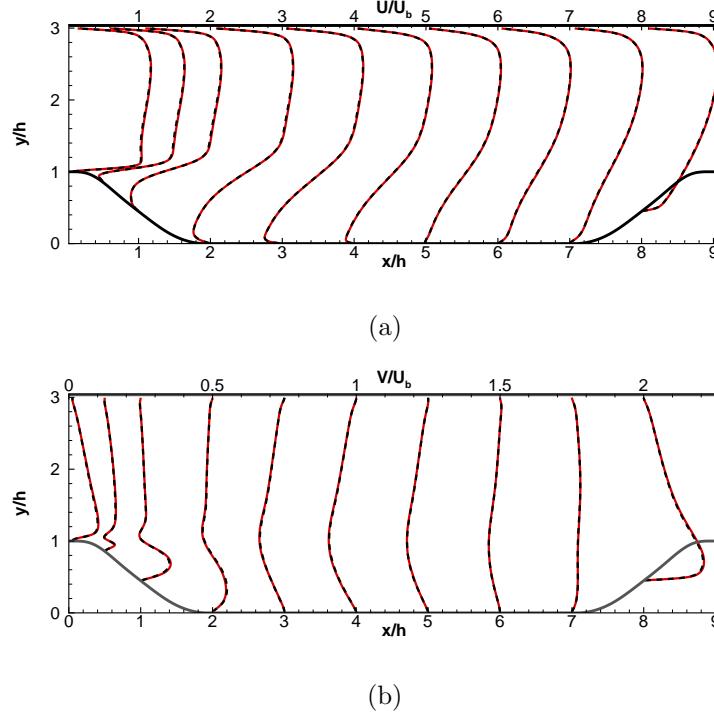


FIG. 11. Predicted mean velocity distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=2800$ , —: Scheme-A; - - -: Scheme-C,  $864 \times 288 \times 462$  grid). (a) U-velocity, (b) V-velocity.

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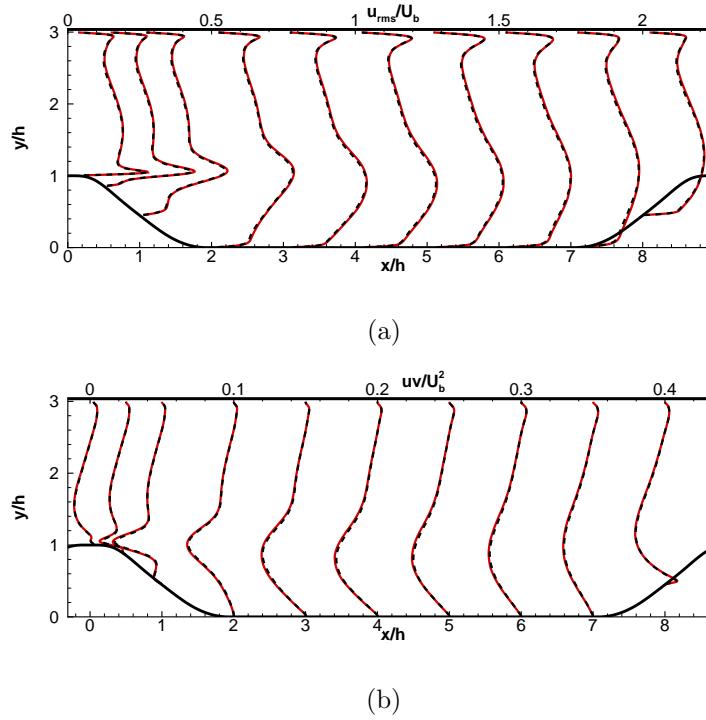


FIG. 12. Predicted turbulence distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ . ( $Re=100$ , —: Scheme-A; - - -: Scheme-C,  $864 \times 288 \times 462$  grid). (a) Streamwise turbulence intensity, (b) Shear stress.

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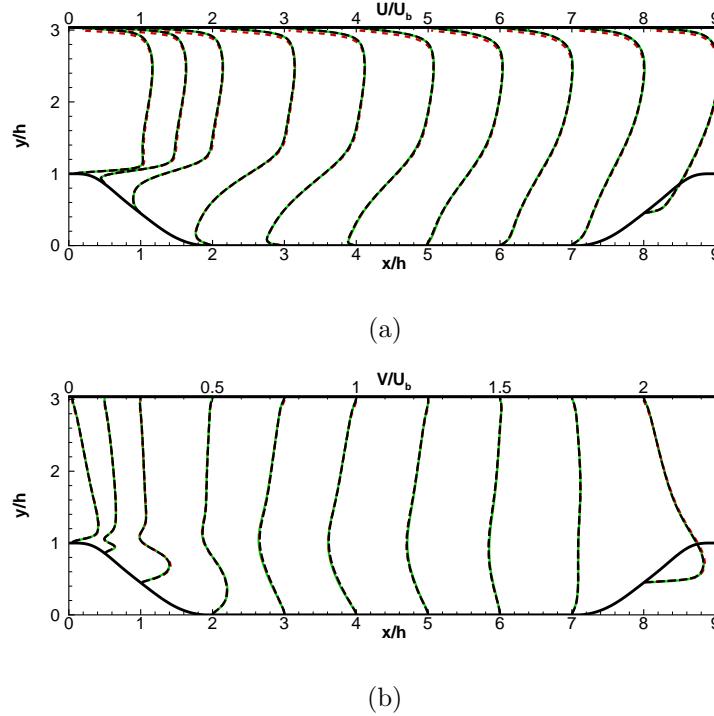


FIG. 13. Predicted mean velocity distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ - $Re=2800$ .  
 (—: Breuer et al. (2009); - - -:  $H=3h$ ,  $864 \times 288 \times 462$  grid; - - - -:  $H=3.036h$ ,  $864 \times 292 \times 462$  grid). (a) Mean streamwise velocity, (b) Mean vertical velocity.

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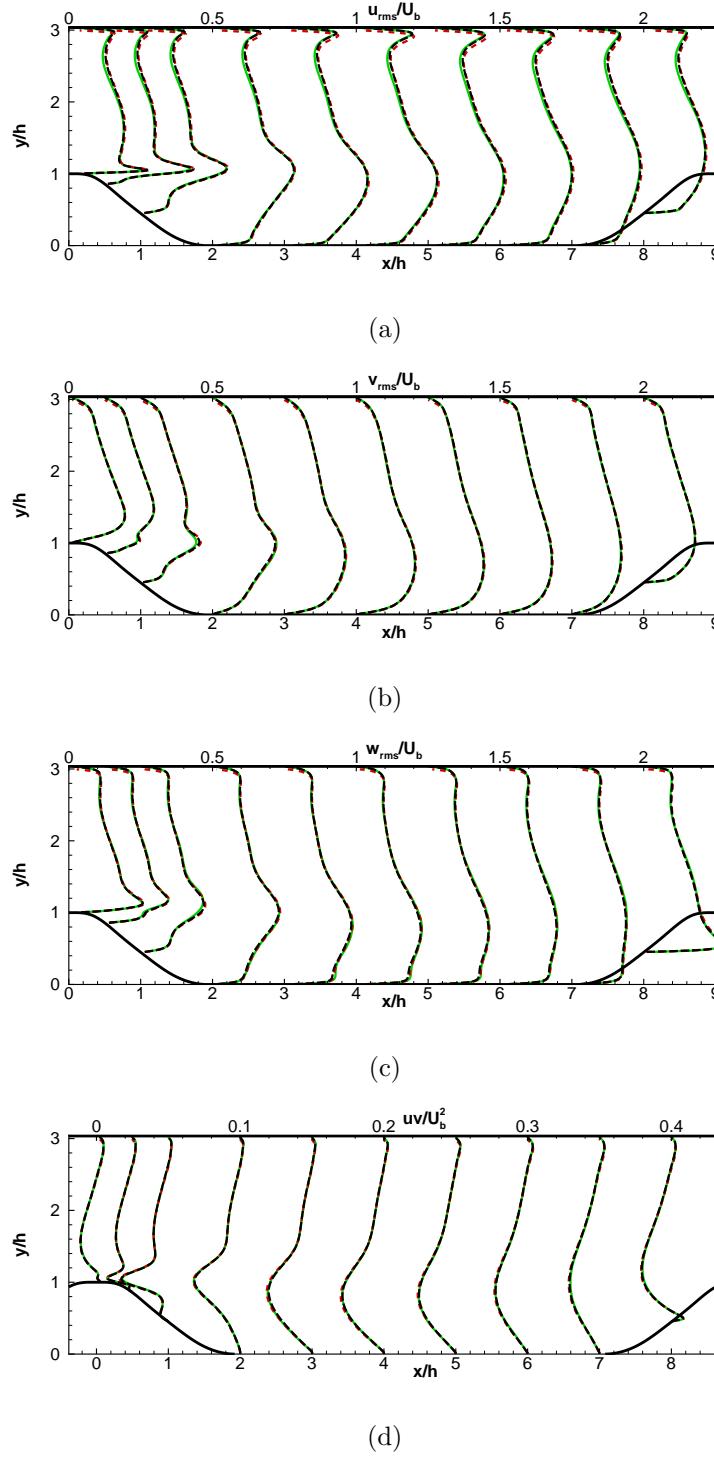
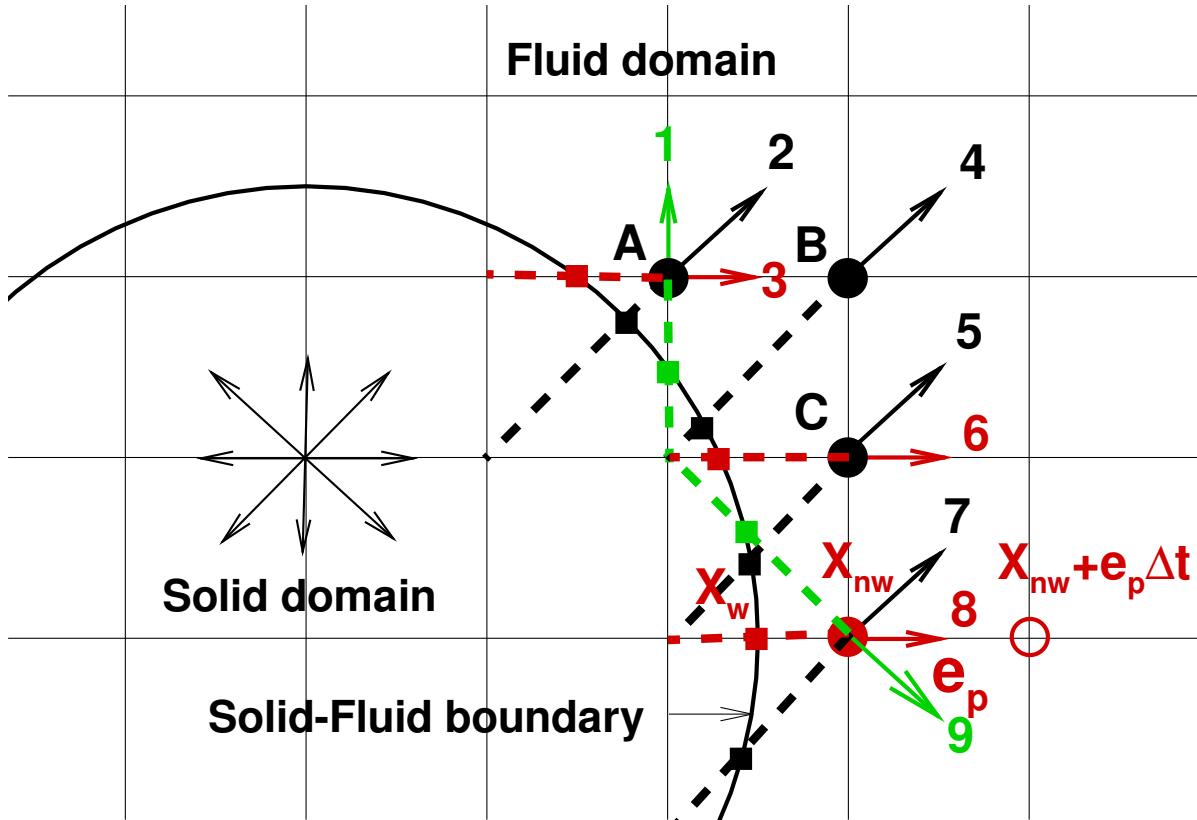
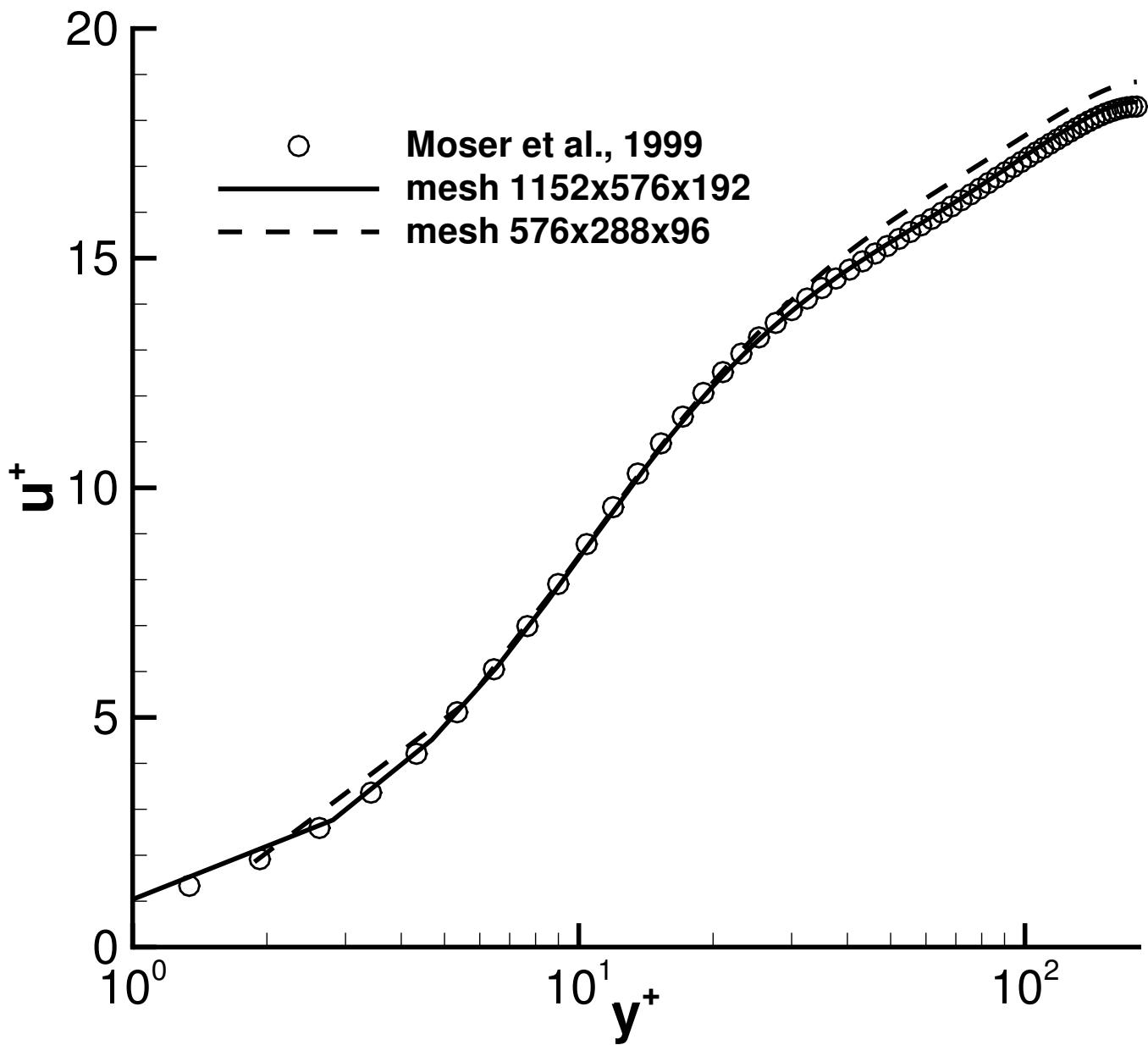
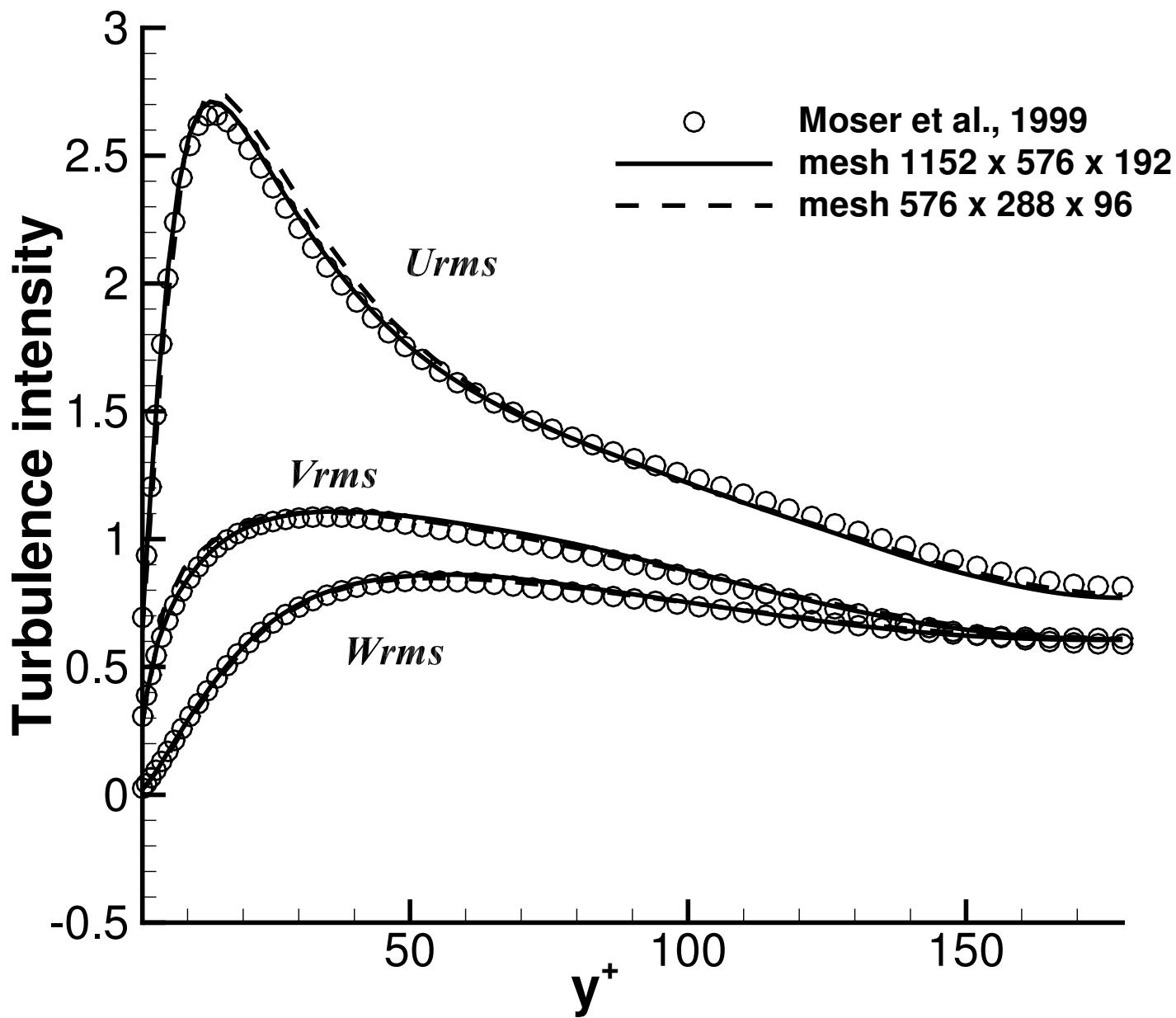
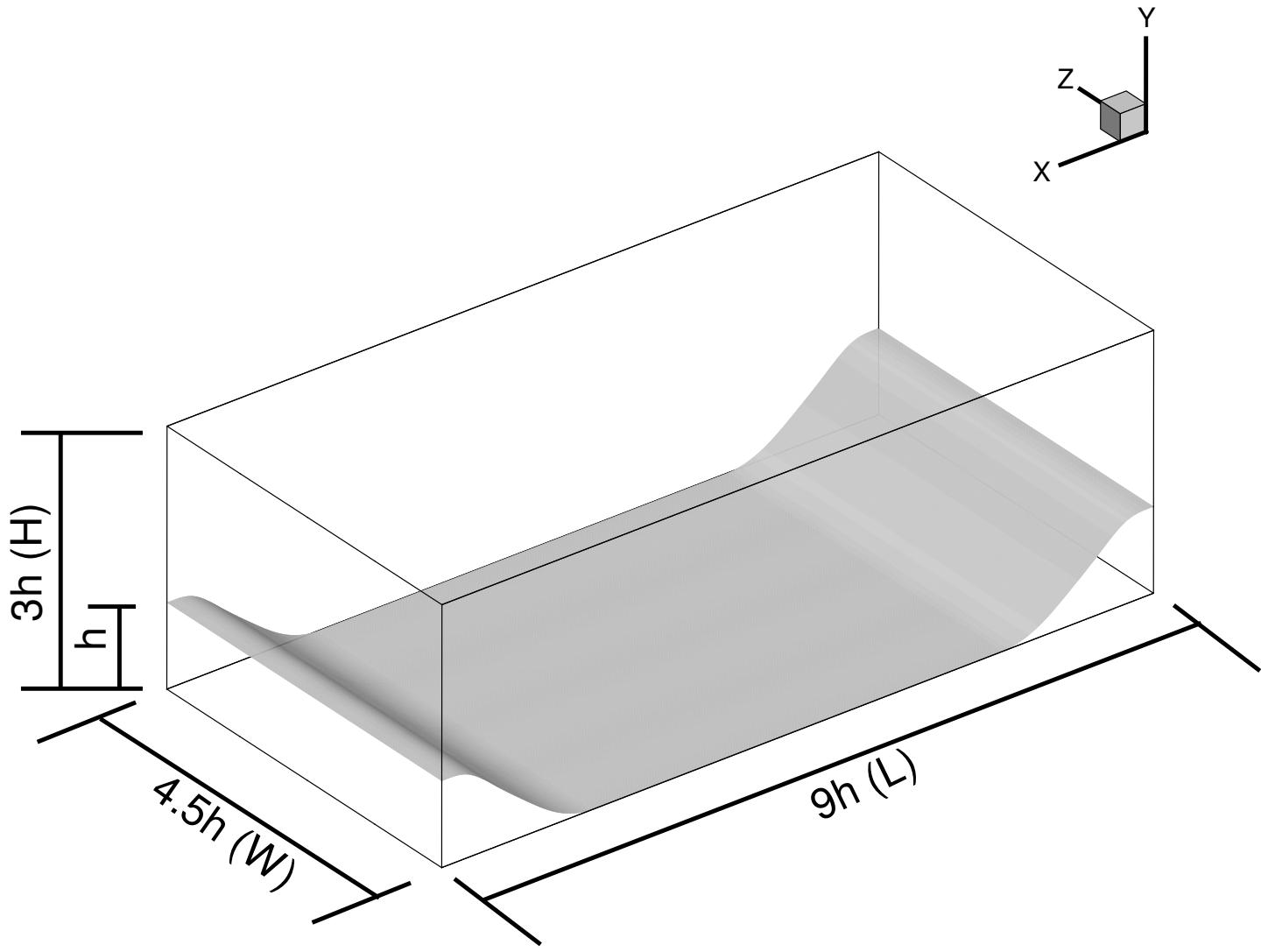


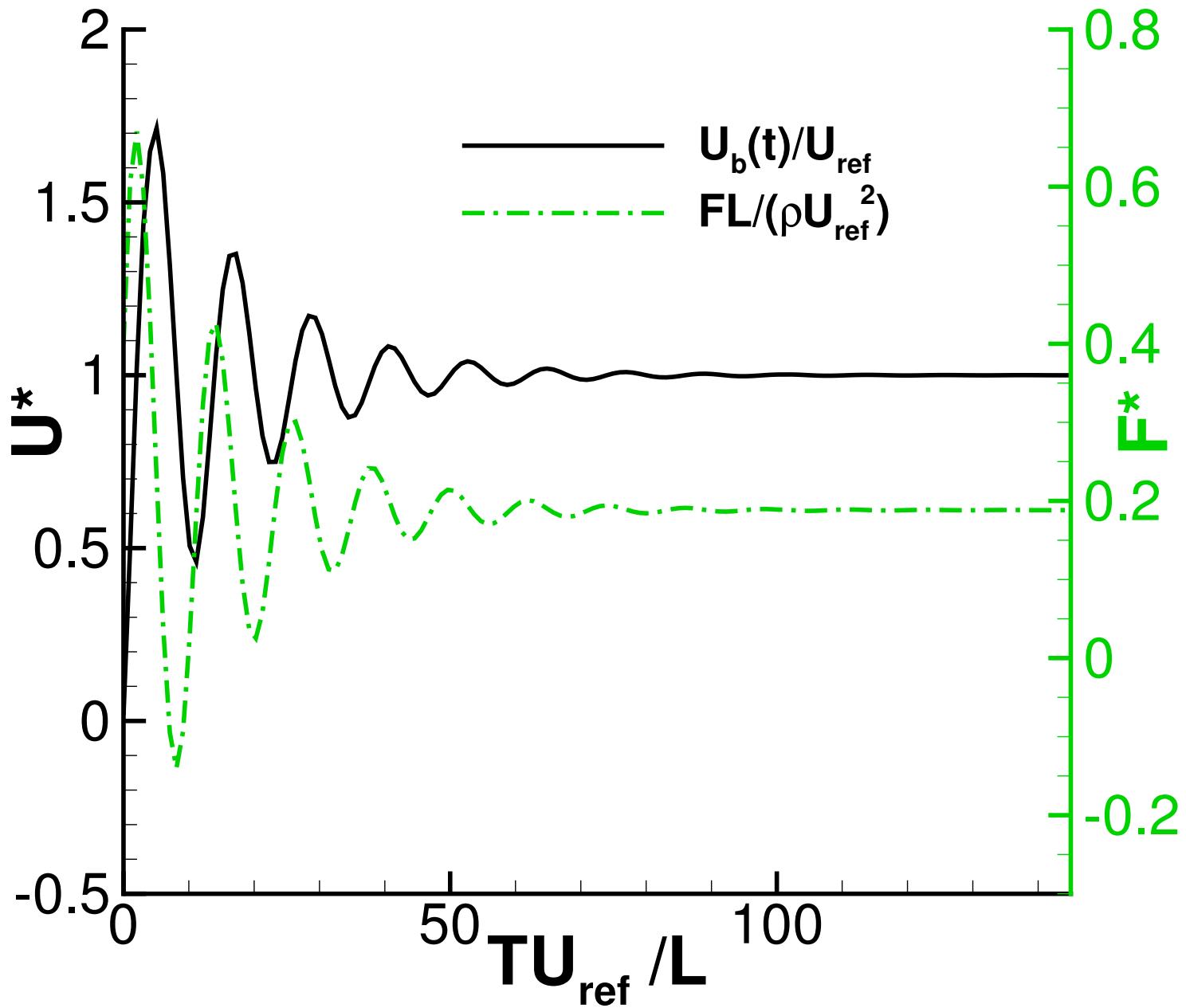
FIG. 14. Predicted turbulence intensity and shear stress distributions at  $x/h=0.05, 0.5, 1, 2, 3, 4, 5, 6, 7$  and  $8$ - $Re=2800$ . (—: Breuer et al. (2009); -·-·-:  $H=3h$ ,  $864 \times 288 \times 462$  grid; -··-:  $H=3.036h$ ,  $864 \times 292 \times 462$  grid). (a) Streamwise turbulence intensity, (b) Vertical turbulence intensity, (c) Spanwise turbulence intensity, (d) Shear stress.

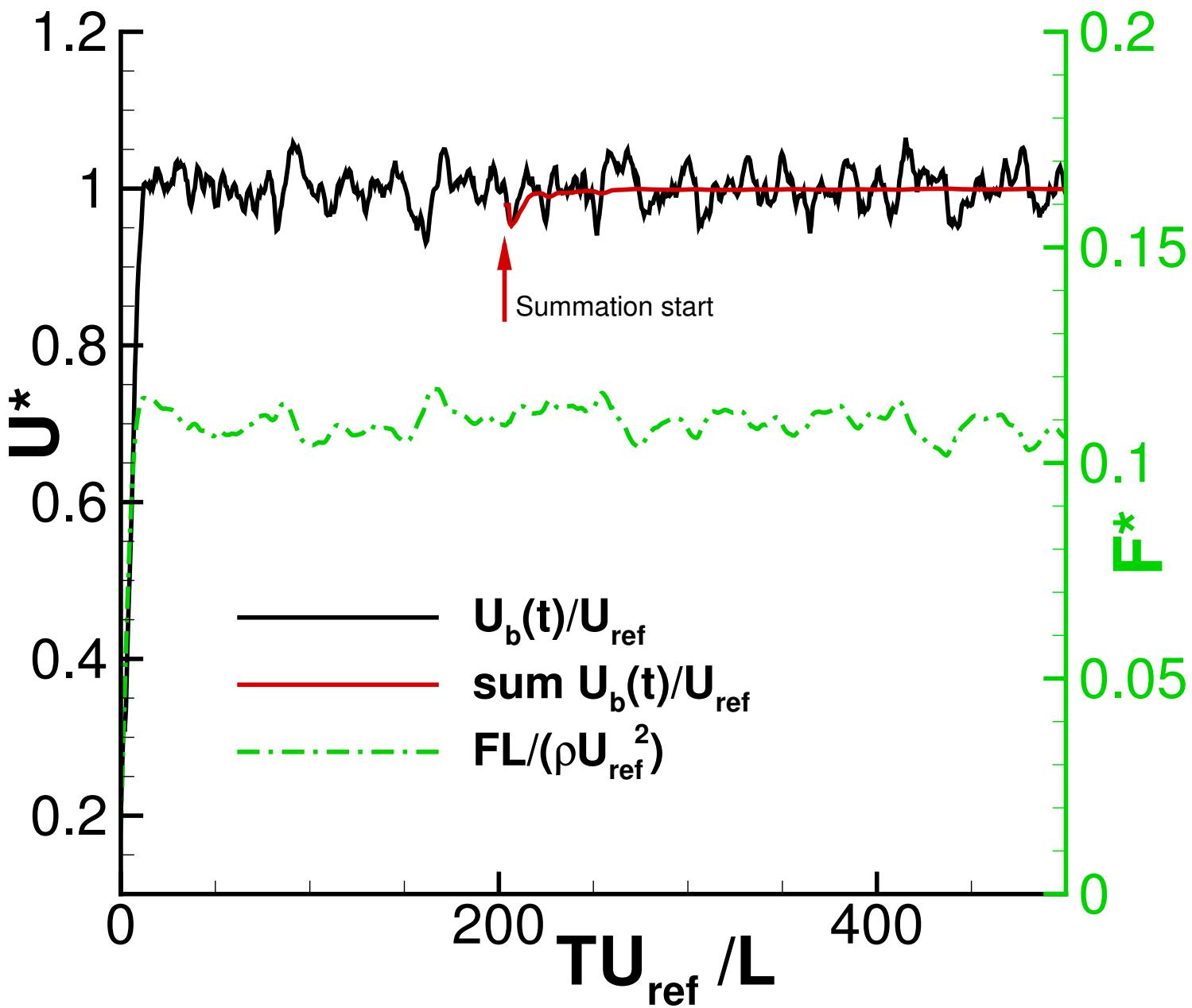


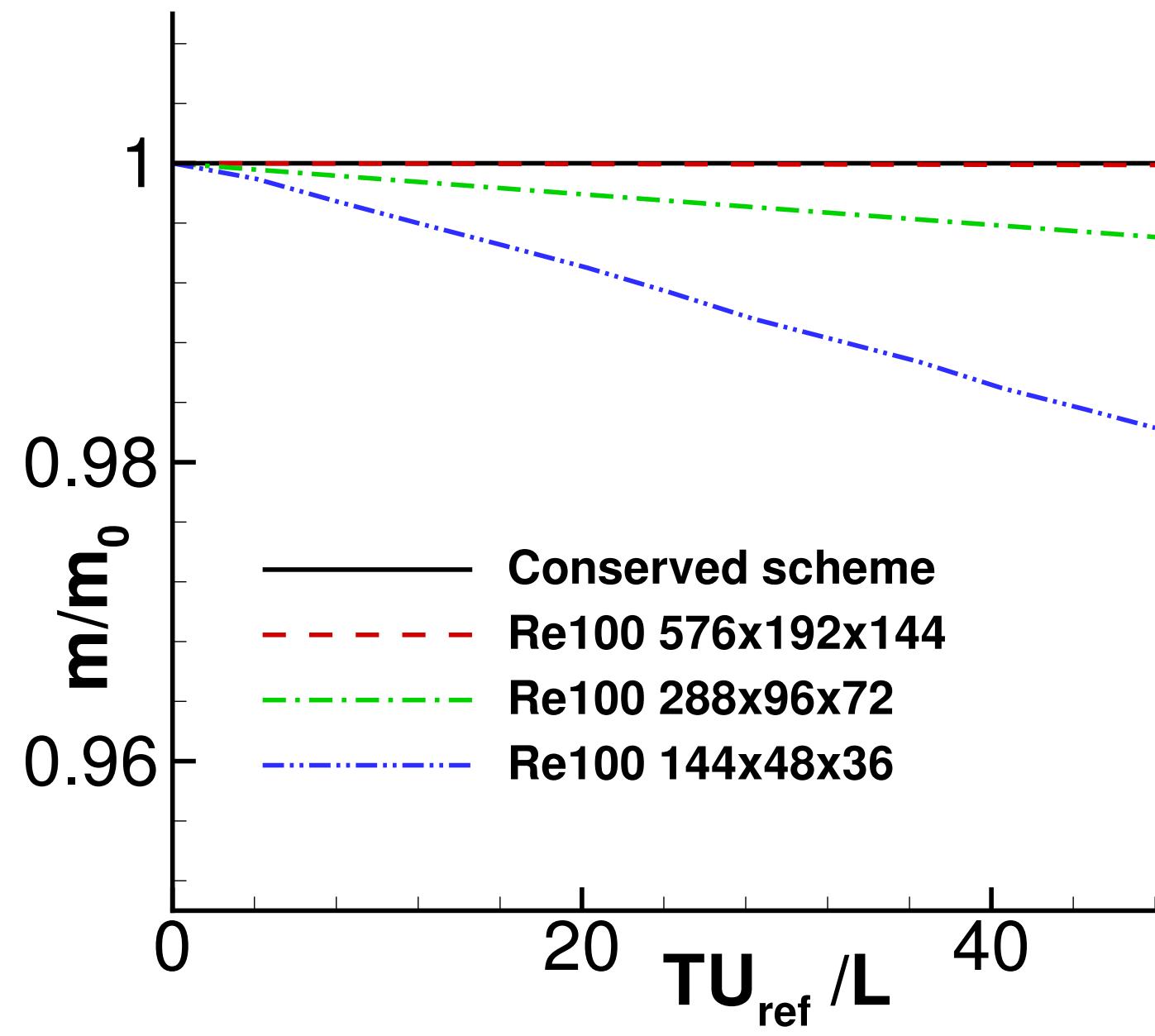


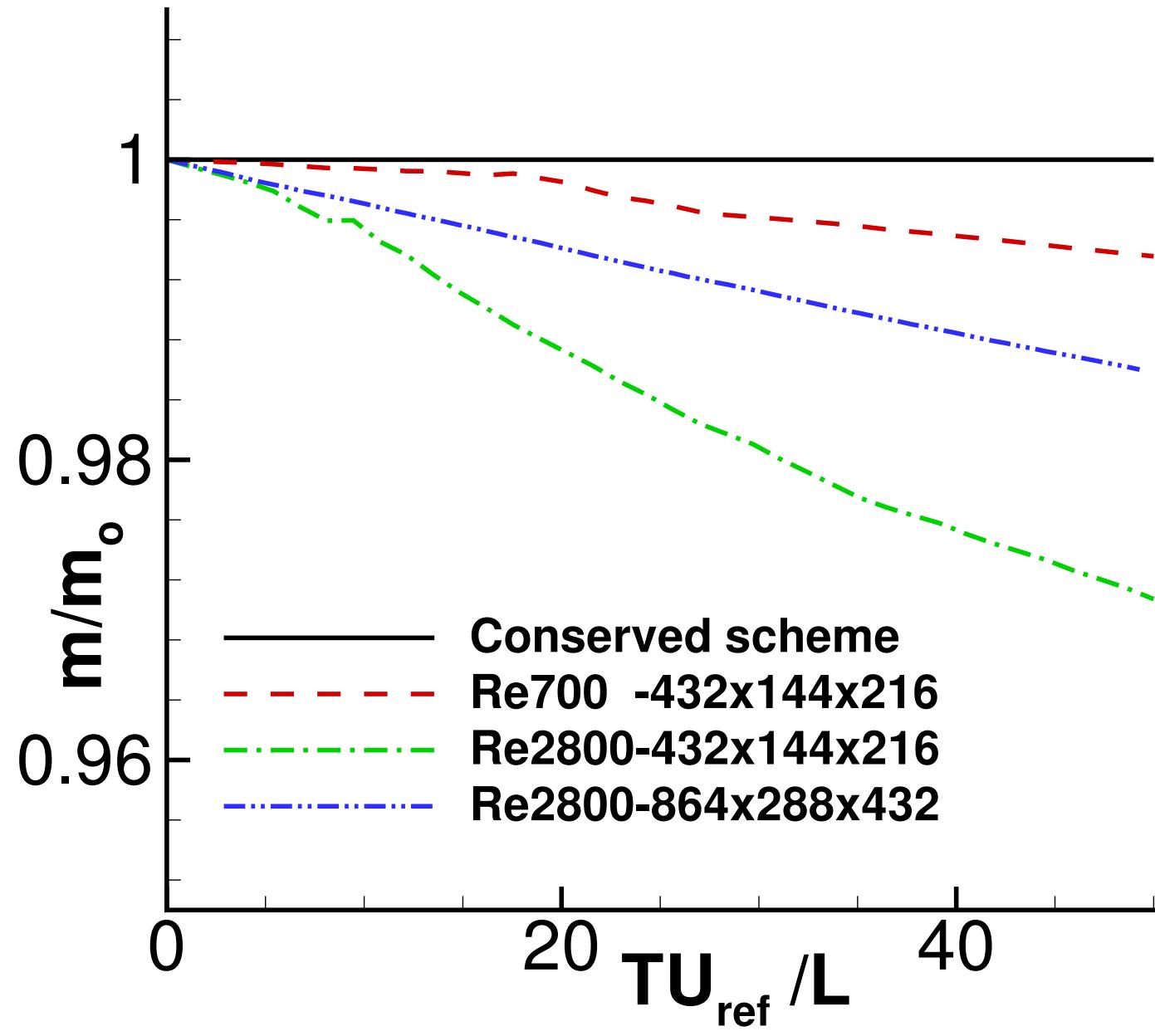


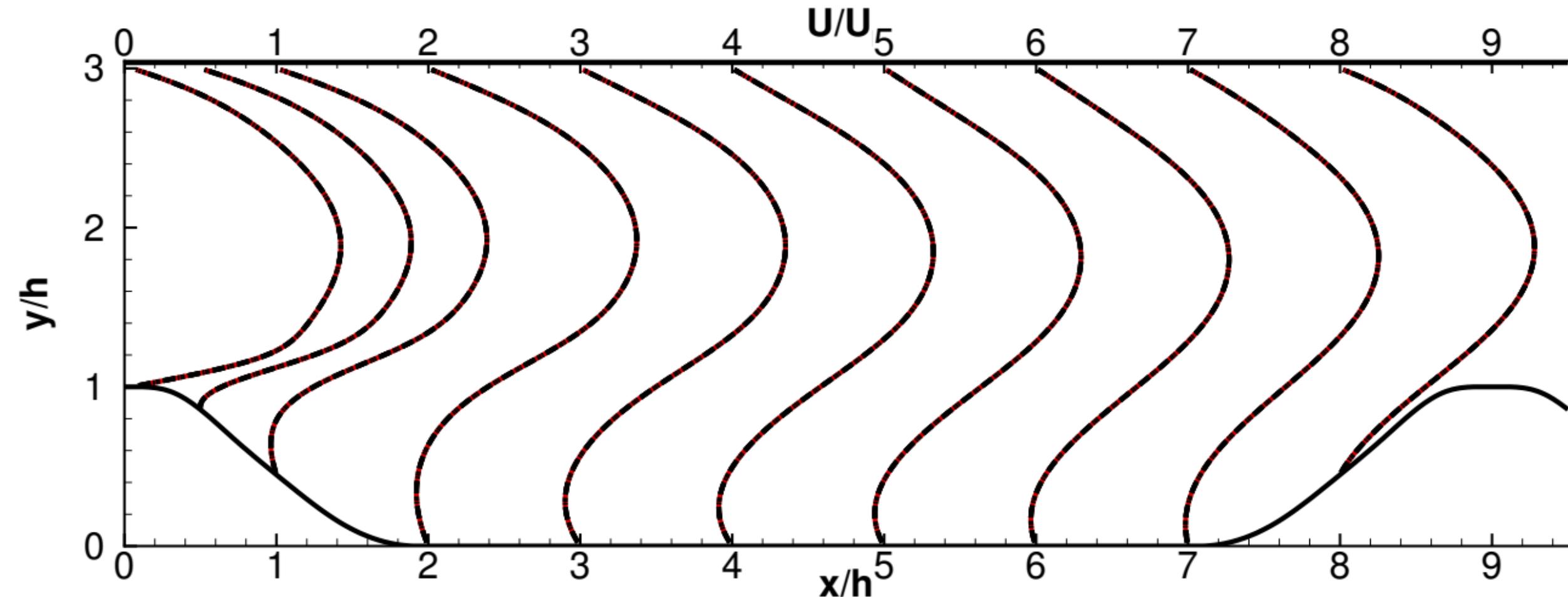


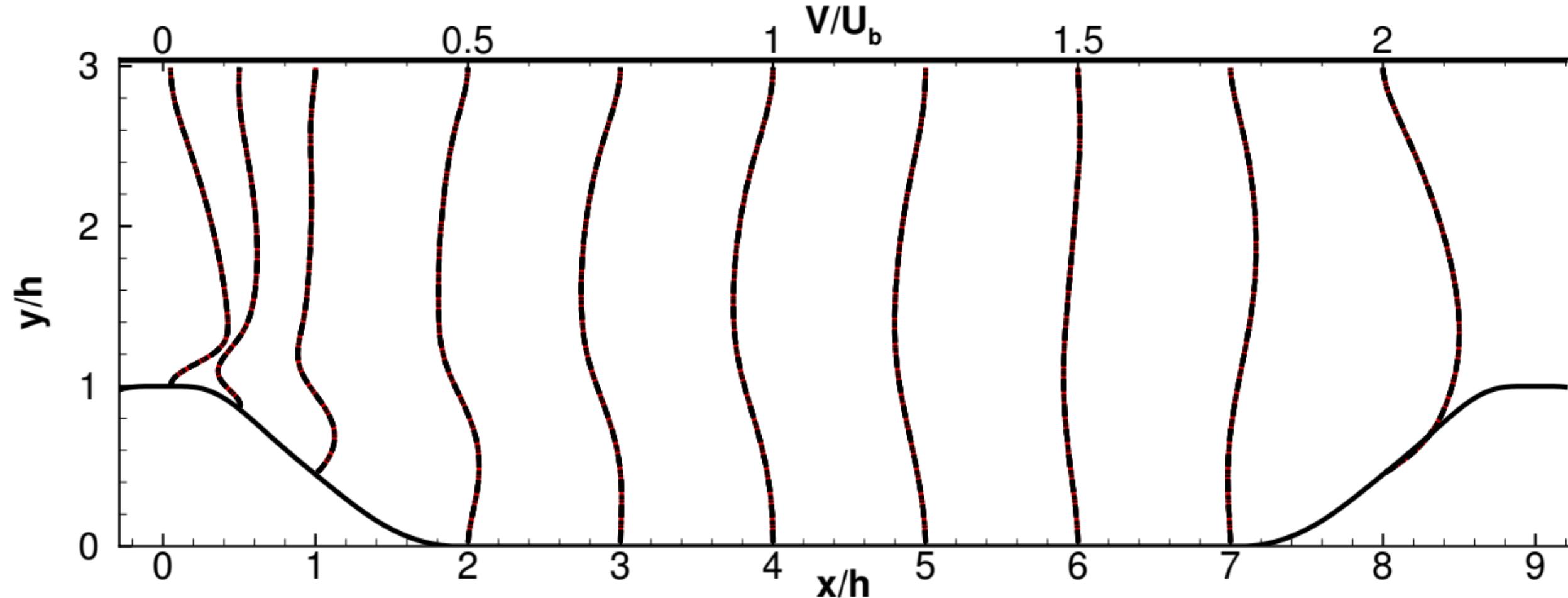


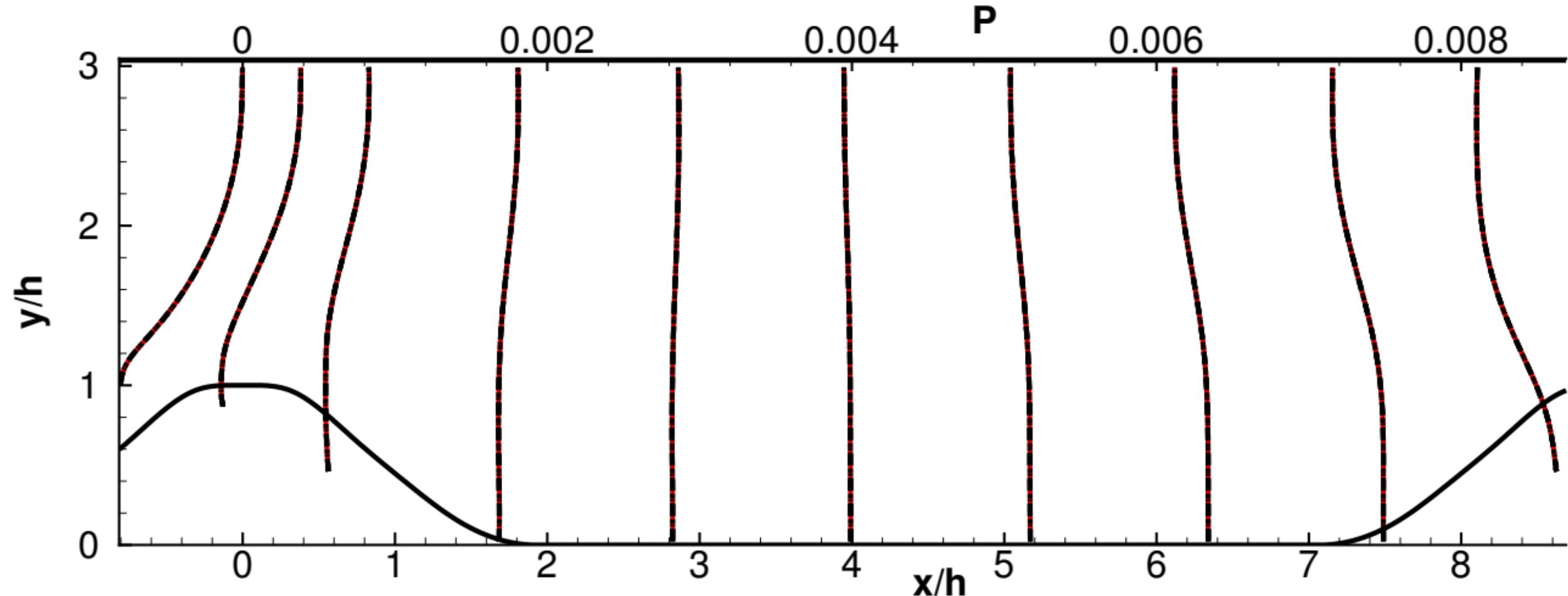


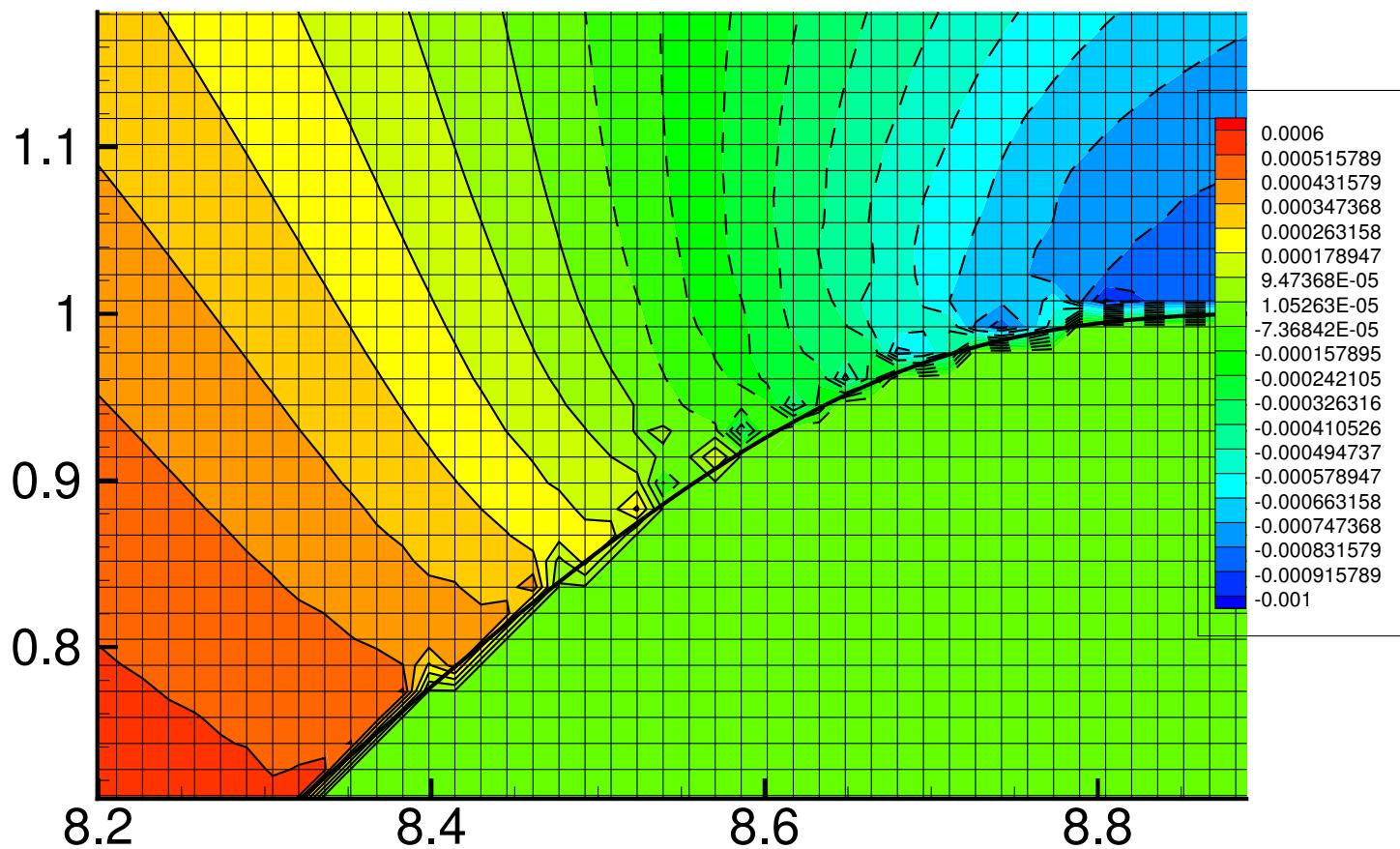


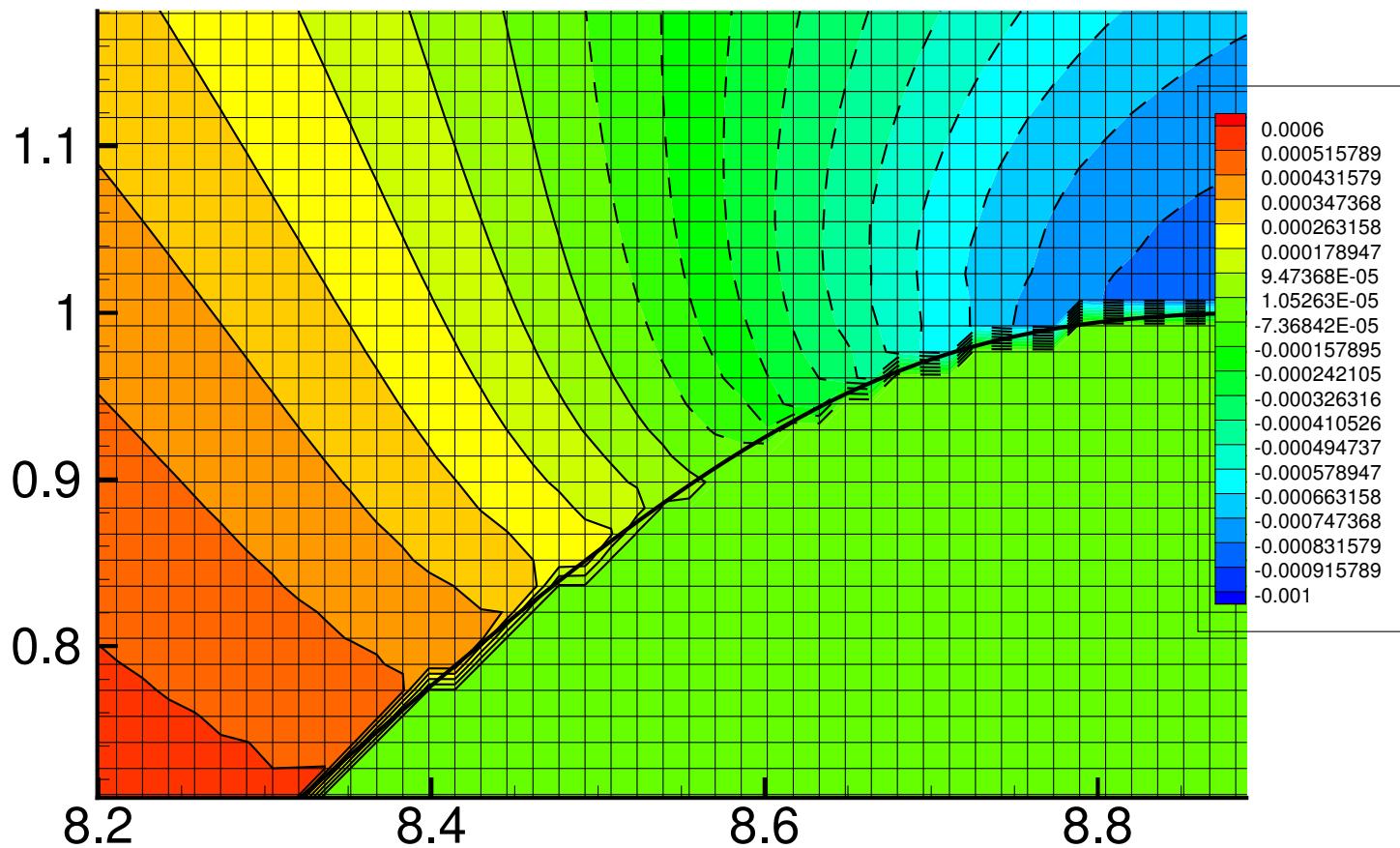


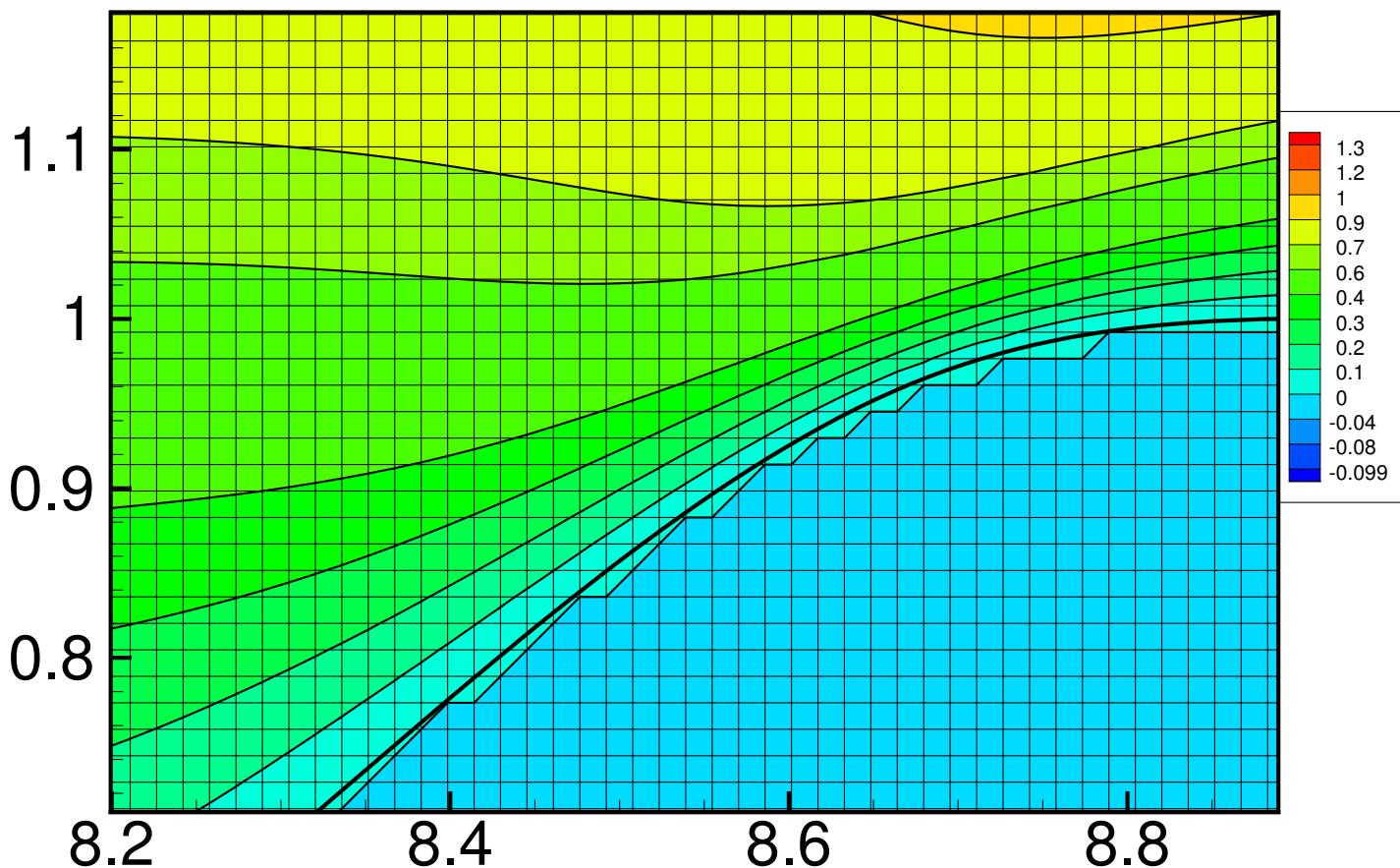


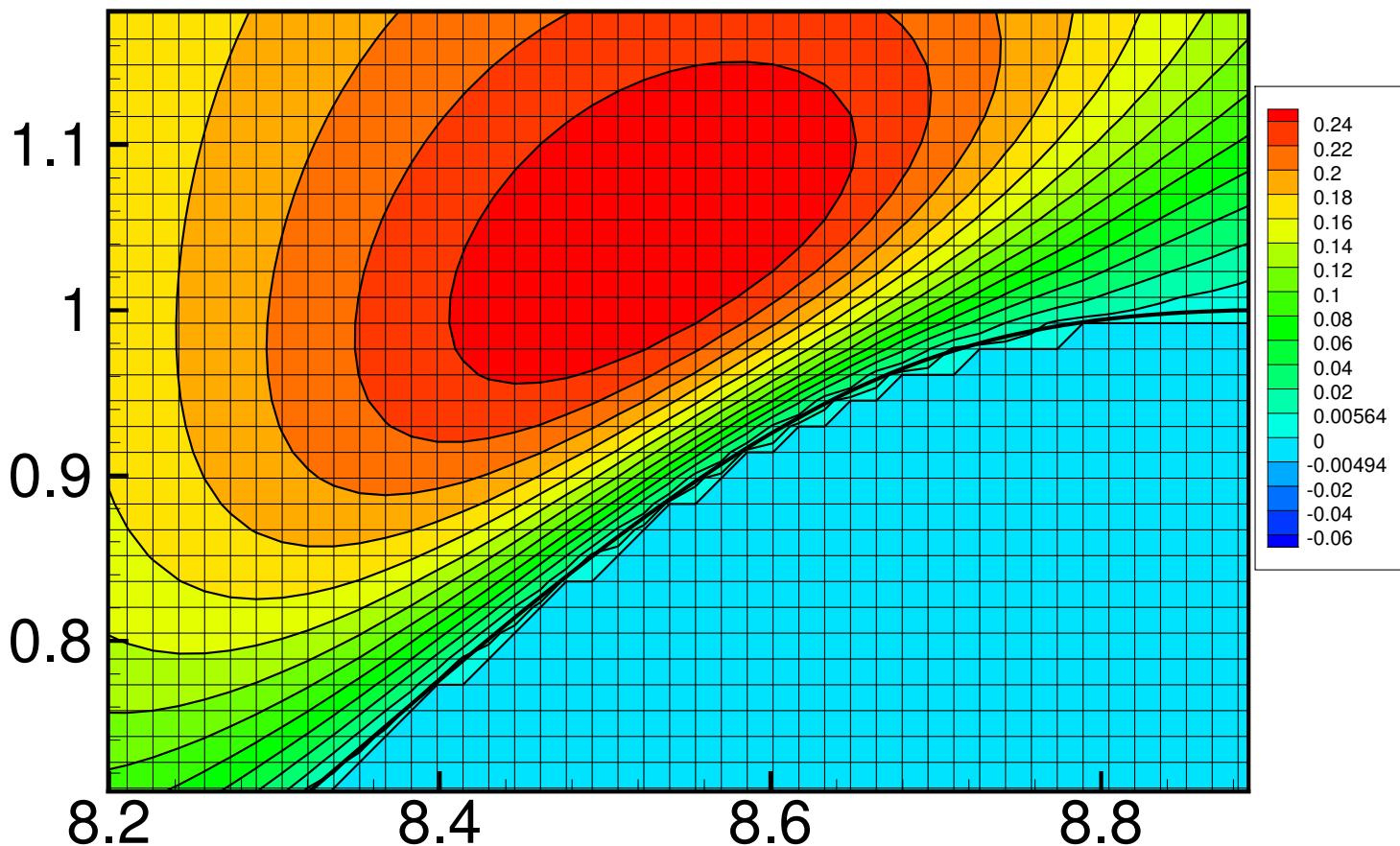


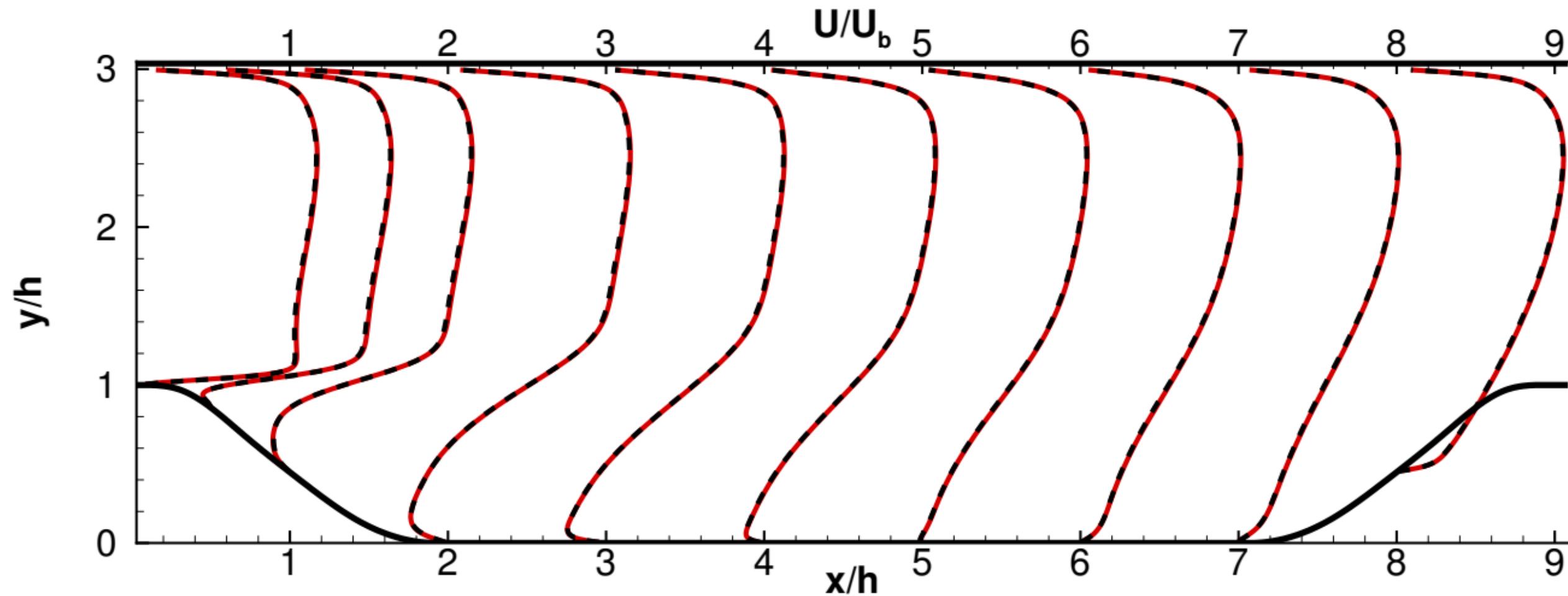


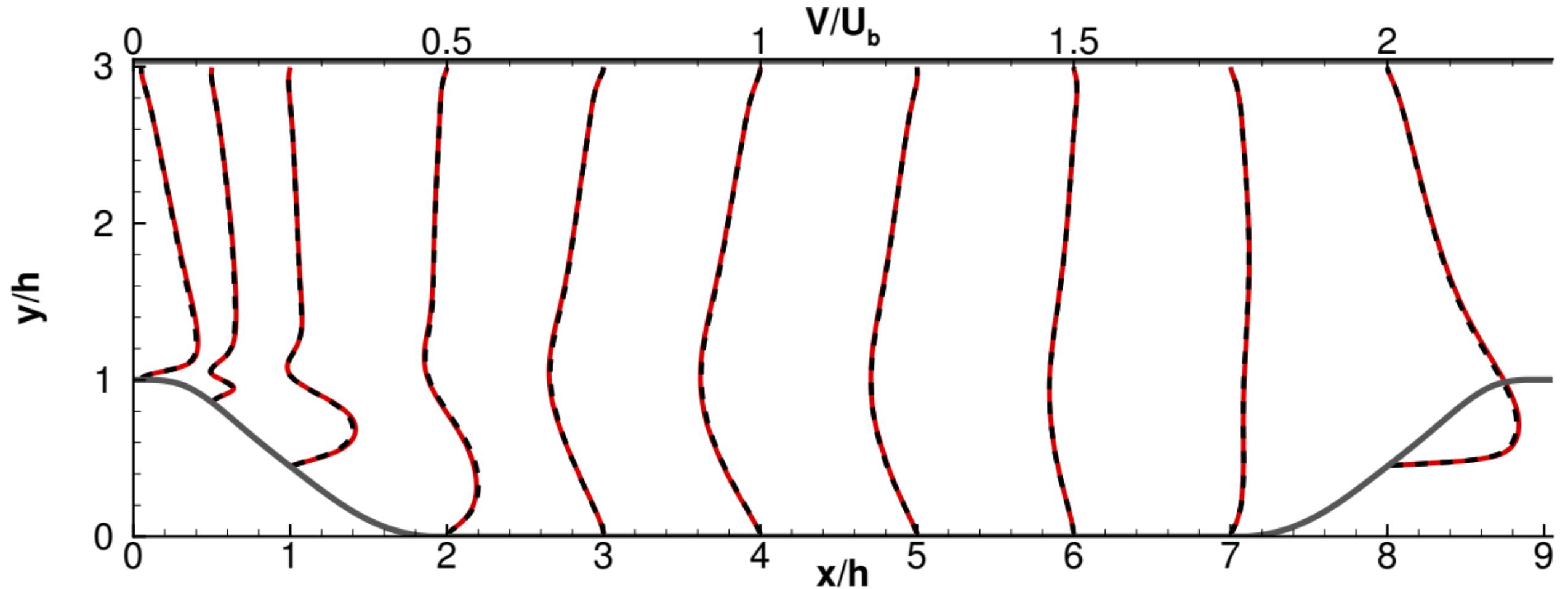


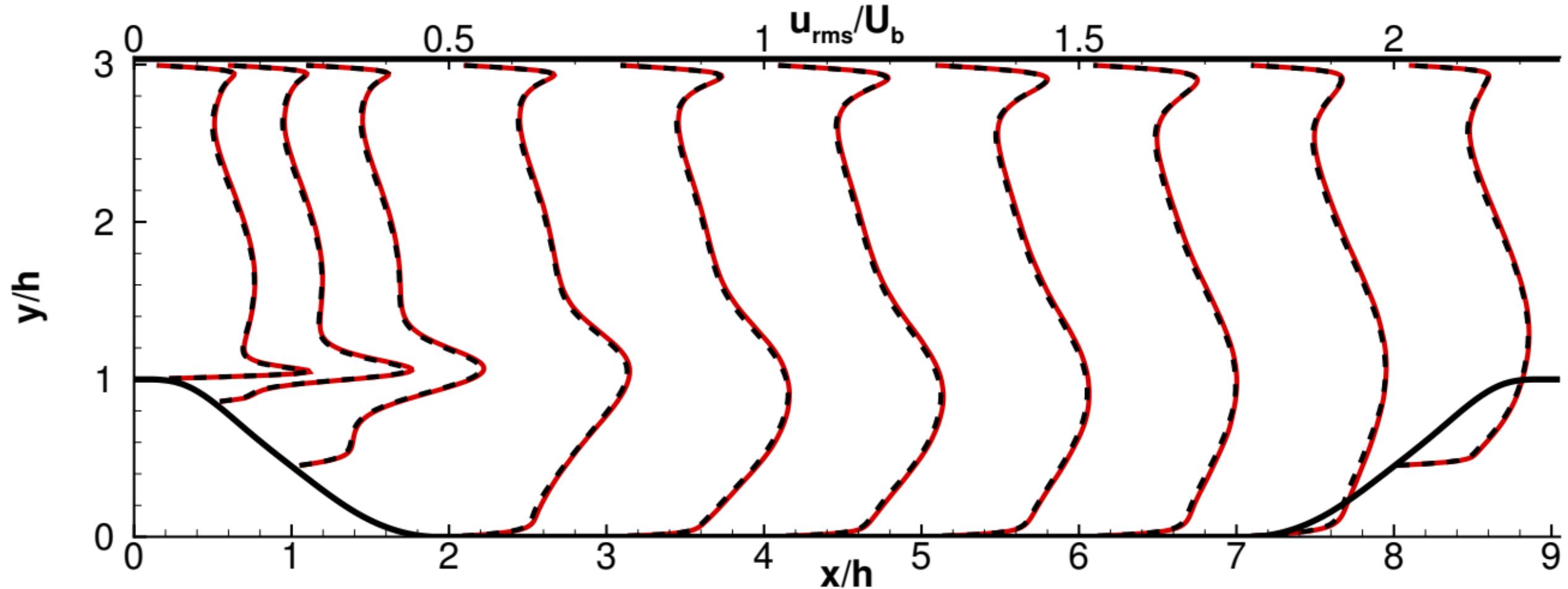


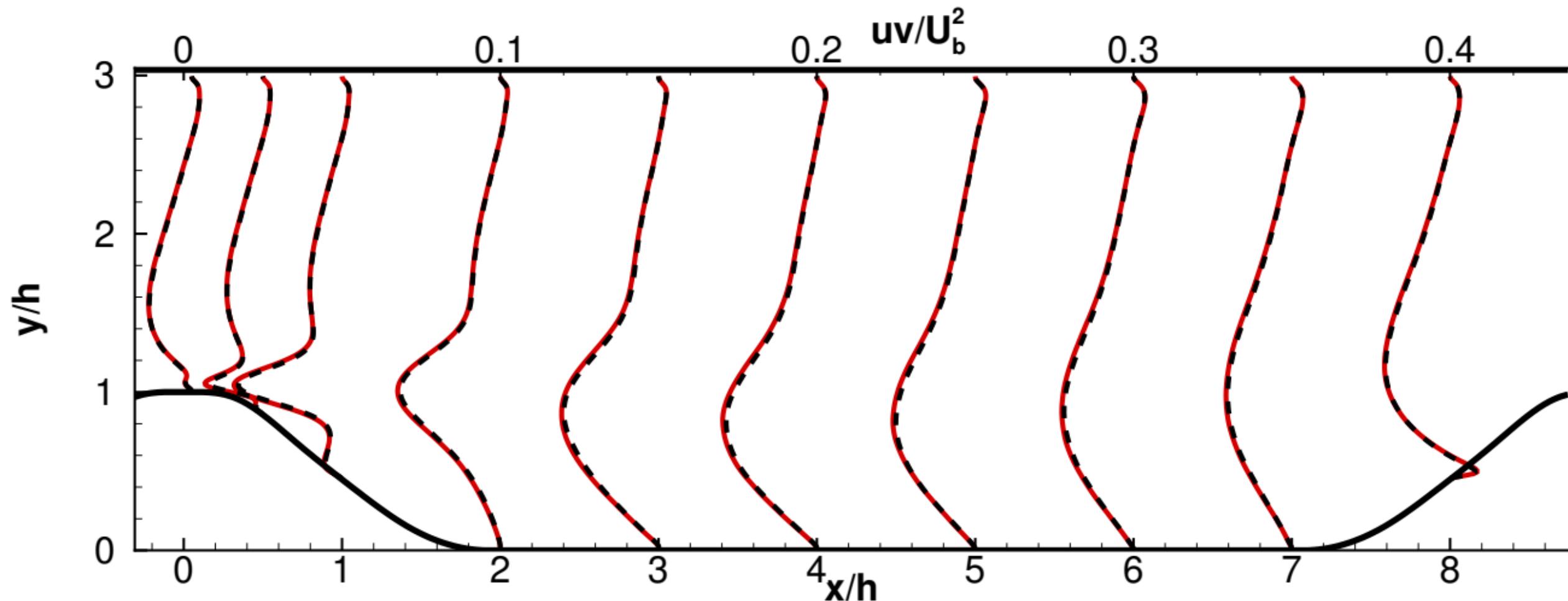


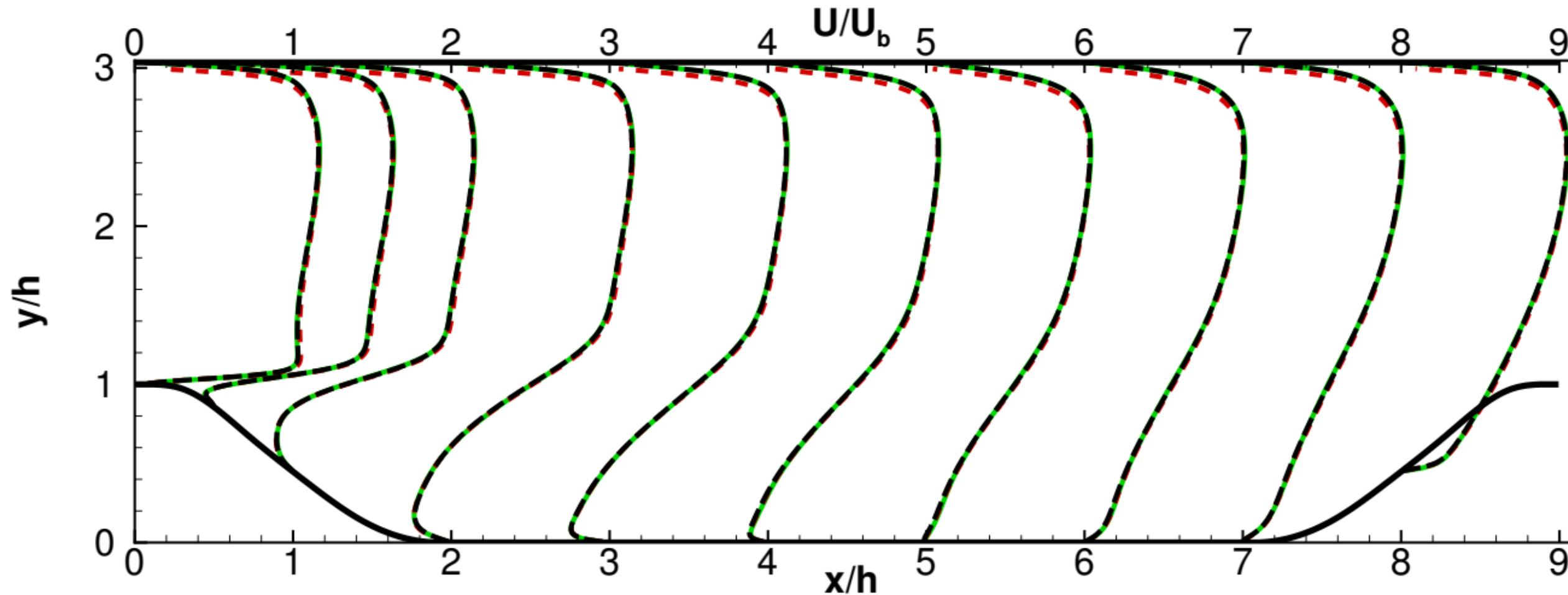












$$U/U_b$$

