Here are one simulation example and two real data examples.

For the simulation example, I used the leaky integrate-and-fire model Liam specified earlier:

$$\mathrm{d}V = -g \cdot (V(t) - E_{\mathrm{leak}}) + k \cdot I(t), \qquad \text{initial condition } V_0 = V_{\mathrm{reset}}. \tag{1}$$

where $g = 0.1$, $k = 1$, $E_{\mathrm{leak}} = -70$ mV, $V_{\mathrm{reset}} = -75$ mV, $V_{\mathrm{thres}} = -55$ mV.

In Fig. 1, the panels on the left are six true rasters by injecting a square pulse from 100ms to 400 ms with a range of magnitude between 1.43 nA and 1.63 nA

This simulated data is then fitted with a GLM model

$$\log \lambda_t = b_0 + \sum_{j}^{\mathrm{lag}} k_j \cdot x_{t-j} + \sum_{j}^{\mathrm{lag}} h_j \cdot n_{t-j}, \tag{2}$$

where $\exp(b_0)$ is the baseline firing rate, and $x_{t-j}$ and $n_{t-j}$ are stimulus and spiking history up to 40 ms lag. The panels on the right are the predicted rasters.

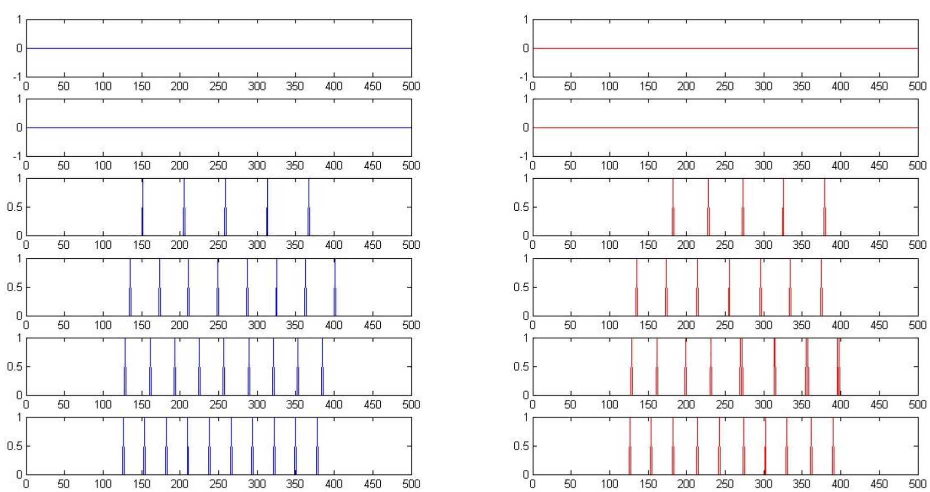

Fig. 1: Simulation example with square current I(t) between 100 ms and 400 ms. *Left* True rasters. *Right* Predicted rasters.

The same GLM model, with a 6 ms lag, is fitted to real spiking data from cell 1 at two different locations. True rasters from the 15 trials at each location (5 trials each at 100 mV, 50 mV, and 25 mV) are plotted in the left panels of Fig. 2 and 3 (decreasing magnitudes on the y-axis). Predicted rasters are plotted in the right panels of Fig. 2 and 3 (10 trials each at the three different magnitudes).

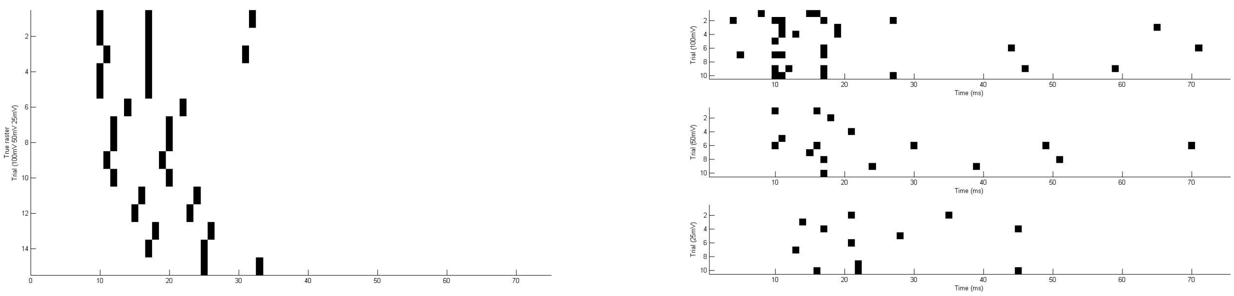

Fig. 2: Cell 1 location 51. *Left* True rasters from the 15 trials at each location (5 trials each at 100 mV, 50 mV, and 25 mV from top to bottom on the y-axis). *Right* Predicted rasters (top to bottom input current: 100 mV; 50 mV; 25 mV)
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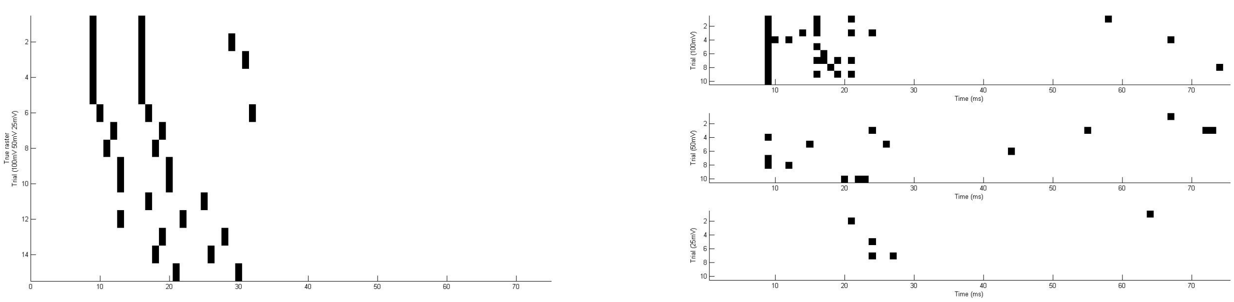

Fig. 3: Cell 1 location 61. *Left* True rasters from the 15 trials at each location (5 trials each at 100 mV, 50 mV, and 25 mV from top to bottom on the y-axis). *Right* Predicted rasters (top to bottom input current: 100 mV; 50 mV; 25 mV)
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For now, the parameter vector $\vec{k}$ only modulates stimulus history in time, not in space. We could augment the GLM model so that $\vec{k}$ is a spatiotemporal kernel. Or, we could let $\vec{k}$ be location-dependent by incorporating known structure (see cascading firing across locations in Fig. 4) in the data, i.e., let location $a$ be a state, filter $\vec{k}$ across the grid with an empirical state-transition matrix $p(a_t|a_{t-1})$ (see Fig. 5).

Let me know whether this all makes sense.

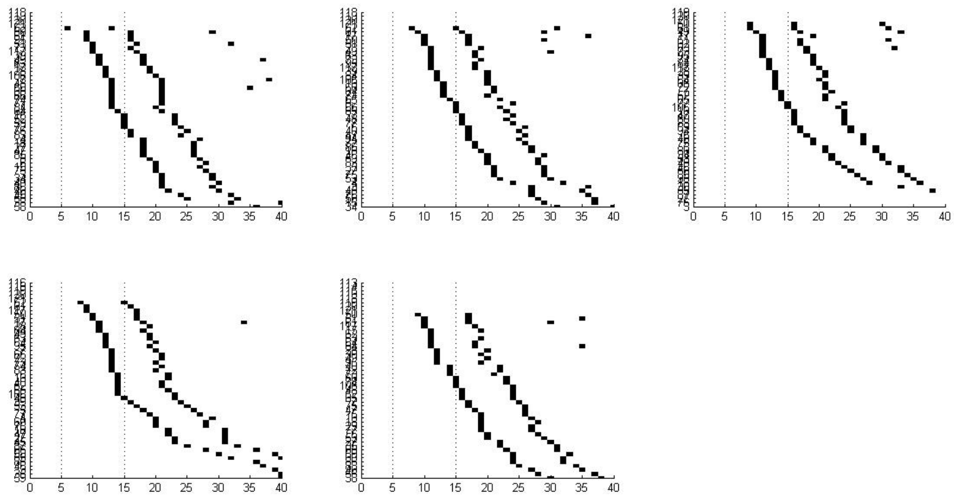

Fig. 4: Rasters for cell 1 of all five 100 mV trials. Y-axis: location, sorted by the time of first spike.

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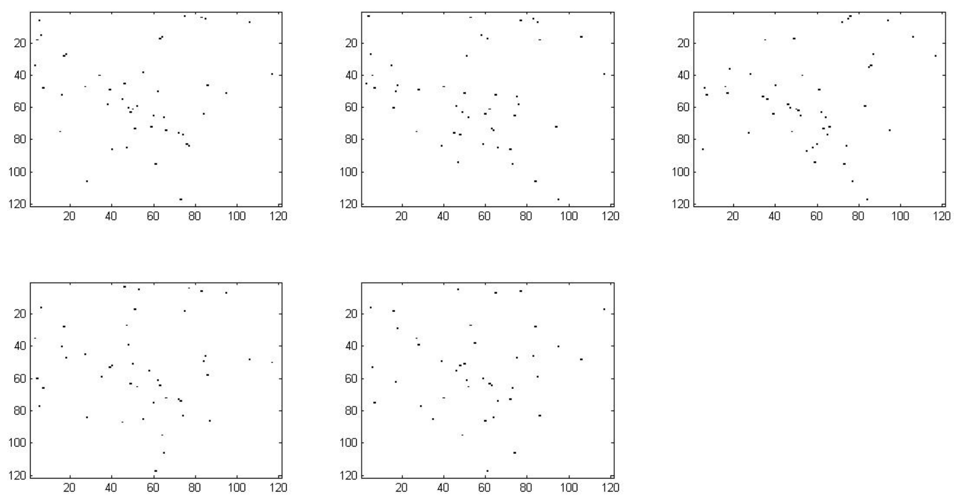

Fig. 5: Empirical transition (the relation in time between 2 locations) matrix $p(a_t|a_{t-1})$ for cell 1 of all five 100 mV trials. X-axis: location $a_{t-1}$; Y-axis: location $a_t$. The transition matrix is fairly consistent across trials

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A point process with conditional intensity given by

$$\lambda(t) = f(V(t)), \tag{3}$$

where the sub-threshold voltage $V(t)$ is the solution to the following differential equation

$$\mathrm{d}V = -g(V(t) - E_{\text{leak}}) + k \cdot I(t), \tag{4}$$

with initial value $V(t_{i-1}) = V_{\text{reset}}$, where $t_{i-1}$ is the time of the most recent event. Once $V(t) > V_{\text{thres}}$, an event is triggered, and $V(t)$ resets to $V_{\text{reset}}$.

Namely,

$$V(t) = E_{\text{leak}} + (V_{\text{reset}} - E_{\text{leak}}) \exp(-g(t - t_{i-1})) + k \int_{t_{i-1}}^{t} I(s) \exp(-g(t-s)) \mathrm{d}s. \tag{5}$$

Therefore,

$$\lambda(t) = f(V(t)) = \max(0, V(t) - V_{\text{thres}}). \tag{6}$$

This linear rectifier function can be approximated by

$$\max(0, V(t) - V_{\text{thres}}) = \log(1 + \exp(V(t) - V_{\text{thres}})). \tag{7}$$

Now, we rewrite $\lambda(t)$ in the following way

$$h(\lambda(t)) = (E_{\text{leak}} - V_{\text{thres}}) + (V_{\text{reset}} - E_{\text{leak}})a(t) + kb(t), \tag{8}$$

where $\{E_{\text{leak}} - V_{\text{thres}}, V_{\text{reset}} - E_{\text{leak}}, k\}$ is the set of parameters.

Combining Eq. 5 and 6, we get

$$a(t) = \exp(-g(t - t_{i-1})); \tag{9}$$

and

$$b(t) = \int_{t_{i-1}}^{t} I(s) \exp(-g(t-s)) \mathrm{d}s. \tag{10}$$

and the link function

$$h(\lambda(t)) = f^{-1}(\lambda(t)) = \log(\exp(\lambda(t)) - 1) \tag{11}$$