

# Aggregation of Support-Relations of Bipolar Argumentation Frameworks\*

Extended Abstract

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## ABSTRACT

In many real-life situations, individuals may have different opinions on support-relations between arguments. When confronted with such situations, we may wish to aggregate individuals' argumentation views on support-relations into a collective view, which is acceptable to the group. In this paper, we assume that under bipolar argumentation theory, individuals are equipped with a set of arguments and a set of attacks between arguments, but with possibly different support-relations. Using the methodology in social choice theory, we analyze what semantic properties of bipolar argumentation frameworks can be preserved by desirable aggregation rules during aggregation of support-relations.

## KEYWORDS

bipolar argumentation framework; social choice theory; support relation; graph aggregation

## 1 INTRODUCTION

The attack relation has played a significant role in formal argumentation [1, 9, 12]. However, recent years have seen a revived interest in the support relation between arguments in argumentation systems [2–5]. In these systems, an argument can not only attack another argument, but it can also support another one. For example, an argument can support another argument by confirming its premise or undermine at least one of its attackers. The support relation between arguments is vital in modeling debates in real life.

Due to the incompleteness of information, or different positions, agents may have different opinions regarding the support relation between arguments. The bipolar argumentation framework [3–5] is a formalism of Dung's abstract argumentation framework which adds the capability of modeling the support relation between arguments, and will be considered in this paper.

Agents many have different opinions on support-relations, which form argumentative stances. When a group of agents are engaged in a debate, we may wish to aggregate stances possessed by agents to obtain a collective decision agree on by the group. Given that there is a broad discussion of aggregation of argumentation systems

with the attack relation [6, 7, 10, 13], it is far from being clear what consensus can be achieved when the support relation is involved in this process.

The goal of this paper is to investigate the aggregation of views of a group of agents in the context of bipolar argumentation. Given a set of arguments and a set of attack-relations between these arguments, agents might conflict with one another upon support-relations between arguments, and we may wish to aggregate such support-relations. Following the model introduced by Chen and Endriss [6] which originally developed by Endriss and Grandi [11], we consider the preservation of properties of bipolar argumentation frameworks, i.e., whether properties satisfied by individuals are preserved by aggregation rules.

We obtain both positive and negative results. For positive results, we show which semantic properties can be preserved by desirable aggregation rules. For negative results, we introduce two meta-properties for bipolar argumentation frameworks. These meta-properties have connections with the choice-theoretic axioms during aggregation. We then show that if a property  $P$  belongs to the family of the instances of the meta-properties, then any aggregation rule that satisfies certain basic axioms and that is supposed to preserve  $P$  must be a dictatorship. We then show that a semantic property of BAF satisfy these two meta-properties, which implies that the preservation of such property during aggregation of bipolar argumentation frameworks guarantees to impossibility results.

## 2 THE MODEL

An abstract bipolar argumentation framework (BAF) is an extension of Dung's abstract argumentation framework [9] in which a general support relation between arguments is added. Formally, a BAF is a triple  $\langle Arg, \rightarrow, \rightsquigarrow \rangle$ , where  $Arg$  is a set of arguments,  $\rightarrow$  is a binary relation on  $Arg$ , which is called the attack relation,  $\rightsquigarrow$  is a binary relation on  $Arg$ , which is called the support relation. Given two arguments  $A, B \in Arg$ , if  $A \rightarrow B$  holds, then we say that  $A$  attacks  $B$ , if  $A \rightsquigarrow B$ , then we say that  $A$  supports  $B$ .

The attack relation and the support relation must verify the following constraint:  $\rightarrow \cap \rightsquigarrow = \emptyset$ , we call it *essential constraint*.

**Definition 2.1.** Given two arguments  $A, B \in Arg$ , a *supported attack* for  $B$  by  $A$  is a sequence  $(A_1, \dots, A_n)$  of arguments of  $Arg$  such that  $A_1 \rightsquigarrow, \dots, \rightsquigarrow A_{n-1}, A_{n-1} \rightarrow A_n, A = A_1, A_n = B$ , and  $n \geq 2$ . A *secondary attack* for  $B$  by  $A$  is a sequence  $(A_1, \dots, A_n)$  of arguments of  $Arg$  such that  $A_1 \rightarrow A_2, A_2 \rightsquigarrow, \dots, \rightsquigarrow A_n, A = A_1, A_n = B$ , and  $n \geq 2$ .

**Definition 2.2.** Let  $\Delta \subseteq Arg$ , let  $A \in Arg$ .  $\Delta$  *set-attacks*  $A$  iff there exists a supported attack or a secondary attack for  $A$  from an

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element of  $\Delta$ . Let  $\Delta \subseteq \text{Arg}$  be a set of arguments,  $\Delta$  is conflict-free iff  $\nexists A, B \in \Delta$  such that  $\{A\}$  set-attacks  $B$ .

**Definition 2.3.** Let  $\Delta \subseteq \text{Arg}$  be a set of arguments,  $\Delta$  is called *d-admissible* iff  $\Delta$  is conflict-free and defends all its elements.  $\Delta$  is a d-preferred extension if it is maximal (w.r.t. set-inclusion) among all d-admissible sets.

Fix a finite set  $\text{Arg}$  of arguments, a set  $\rightarrow$  of attacks between arguments, and a set  $N = \{1, \dots, n\}$  of  $n$  agents. Each agent  $i \in N$  supplies us with a set of supports  $\rightsquigarrow_i$ , which together with  $\text{Arg}$  and  $\rightarrow$  gives rise to a bipolar argumentation framework  $\langle \text{Arg}, \rightarrow, \rightsquigarrow_i \rangle$ , reflecting her individual views on which supports between arguments are acceptable. A profile of support-relations  $\rightsquigarrow = (\rightsquigarrow_1, \dots, \rightsquigarrow_n)$  is a set of support-relations provided by agents. An aggregation rule  $F : (2^{\text{Arg} \times \text{Arg}})^n \rightarrow 2^{\text{Arg} \times \text{Arg}}$  is a function that maps a given profile of support-relations into a single support-relation. We denote  $N_{\rightsquigarrow}^{\text{sup}}$  by the set of agents who accept  $\text{sup}$  under profile  $\rightsquigarrow$ , i.e.,  $N_{\rightsquigarrow}^{\text{sup}} = \{i \in N \mid \text{sup} \in \rightsquigarrow_i\}$ .

Here we define desirable properties of aggregation rule.

**Definition 2.4.** Given an aggregation rule  $F$ ,  $F$  is *unanimous* if  $(\rightsquigarrow_1) \cap \dots \cap (\rightsquigarrow_n) \subseteq F(\rightsquigarrow)$ ,  $F$  is *grounded* if  $F(\rightsquigarrow) \subseteq (\rightsquigarrow_1) \cup \dots \cup (\rightsquigarrow_n)$ ,  $F$  is *neutral* if for any profile of support-relations  $\rightsquigarrow$ , for any pair of supports  $\text{sup}_1, \text{sup}_2$ ,  $N_{\rightsquigarrow}^{\text{sup}_1} = N_{\rightsquigarrow}^{\text{sup}_2}$  then  $\text{sup}_1 \in F(\rightsquigarrow)$  iff  $\text{sup}_2 \in F(\rightsquigarrow)$ . An aggregation rule  $F$  is *independent* if for any pair of profiles of support-relations  $\rightsquigarrow_1, \rightsquigarrow_2$ , for any support  $\text{sup}$ ,  $N_{\rightsquigarrow_1}^{\text{sup}} = N_{\rightsquigarrow_2}^{\text{sup}}$  then  $\text{sup} \in F(\rightsquigarrow_1)$  iff  $\text{sup} \in F(\rightsquigarrow_2)$ .

**Definition 2.5.** An aggregation rule  $F$  is *dictatorial* if there is an agent  $i$  such that for any profile of support-relations  $\rightsquigarrow$ ,  $F(\rightsquigarrow) = \rightsquigarrow_i$ .

**Definition 2.6.** The unanimous rule is an aggregation rule  $F$  with  $F(\rightsquigarrow) = \{\text{sup} \in \text{Arg} \mid \text{sup} \in (\rightsquigarrow_1) \cap \dots \cap (\rightsquigarrow_n)\}$ .

**Definition 2.7.** Let  $i \in N$  be an agent, the dictatorship rule of individual  $i$  is the aggregation rule with  $F_i = \rightsquigarrow_i$ .

### 3 PROPERTIES OF BAFS

The problem we are considering is the preservation of semantic properties in the context of bipolar argumentation. Given a property  $P \subseteq 2^{\text{Arg} \times \text{Arg}}$  that is a set of supports on  $\text{Arg}$ , and  $P$  is satisfied by all agents, whether the output of the aggregation rule satisfies  $P$ .

**Definition 3.1.** An aggregation rule  $F$  preserves a property  $P$  if whenever for every profile  $\rightsquigarrow$  we have that  $P(\rightsquigarrow_i)$  for all  $i \in N$ , then we have  $P(F(\rightsquigarrow))$ .

The *essential constraint* is an example of a high-level feature that requires that no agent accepts both the attack and the support between a pair of arguments. When we observe that all agents verify such semantic feature in a profile, we would like to see what aggregation rule preserves this basic constraint under aggregation.

We are also interested in the preservation of semantic extensions. Given a set of argument  $\Delta \subseteq \text{Arg}$  that is a d-preferred extension of  $\langle \text{Arg}, \rightarrow, \rightsquigarrow_i \rangle$  for all  $i \in N$ , we are interested under what circumstances  $\Delta$  is a d-preferred extension of  $F(\rightsquigarrow)$  as well.

Besides the properties identified above, we introduce two meta-properties, namely non-simplicity and disjunctiveness.

**Definition 3.2.** A property  $P$  is called **non-simple** if there exist a set  $\text{Sup} \subseteq \text{Arg} \times \text{Arg}$  of supports and three individual supports

$\text{sup}_1, \text{sup}_2, \text{sup}_3 \in \text{Arg} \times \text{Arg} \setminus \text{Sup}$  such that  $\langle \text{Arg}, \rightarrow, \text{Sup} \cup S \rangle$  with  $S \subseteq \{\text{sup}_1, \text{sup}_2, \text{sup}_3\}$  satisfies  $P$  if and only if  $S \neq \{\text{sup}_1, \text{sup}_2, \text{sup}_3\}$ .

**Definition 3.3.** A property  $P$  is called **disjunctive** if there exist a set  $\text{Sup} \subseteq \text{Arg} \times \text{Arg}$  of supports and two individual supports  $\text{sup}_1, \text{sup}_2 \in \text{Arg} \times \text{Arg} \setminus \text{Sup}$  such that  $\langle \text{Arg}, \rightarrow, \text{Sup} \cup S \rangle$  with  $S \subseteq \{\text{sup}_1, \text{sup}_2\}$  satisfies  $P$  if and only if  $S \neq \emptyset$ .

### 4 PRESERVATION RESULTS

Recall that a grounded rule is an aggregation rule for which only supports with at least one supporter will be collectively accepted. A bipolar AF satisfies essential constraint if it does not contain two arguments for which the first one simultaneously attacks and supports the same other one.

**PROPOSITION 4.1.** Every aggregation rule  $F$  that is grounded preserves the essential constraint.

For the preservation of conflict-freeness, we obtain a positive but relatively less broad result.

**PROPOSITION 4.2.** The unanimous rule preserves conflict-freeness.

Let  $\text{sup} \in \rightsquigarrow$  be a support, let  $N = \{1, \dots, n\}$  be a finite set of individuals (or agents, we assume that there are two or more agents), and let  $\rightsquigarrow$  be a profile of support-relations. Recall that  $N_{\rightsquigarrow}^{\text{sup}}$  is the set of agents who accept  $\text{sup}$  under profile  $\rightsquigarrow$ . A *winning coalition*  $\mathcal{W} \subseteq N$  is a set of agents who can decide whether to accept or reject a given support  $\text{sup}$ . Given an aggregation rule  $F$ , if  $F$  is neutral and independent, then  $F$  can be fully determined by a single set  $\mathcal{W}$  of winning coalitions, i.e., for every profile  $\rightsquigarrow$  and every support  $\text{sup}$  it is the case that  $\text{sup} \in F(\rightsquigarrow) \Leftrightarrow N_{\rightsquigarrow}^{\text{sup}} \in \mathcal{W}$ . In our proofs, we will rely on the concept of *ultrafilters* familiar from model theory [8]. An *ultrafilter* is a collection of subsets of  $N$  satisfying *closure under intersection*, *maximality*, and  $\emptyset \notin \mathcal{W}$ . Here is a more formal definition.

**Definition 4.3.** An *ultrafilter*  $\mathcal{W}$  on a set  $N$  is a collection of subsets of  $N$  satisfying the following conditions:

- (1)  $\emptyset \notin \mathcal{W}$
- (2) for any pair of sets  $C_1, C_2 \subseteq N$ ,  $C_1 \cap C_2 \in \mathcal{W}$  (closure under intersection)
- (3) for any set  $C$ , one of  $C$  and  $N \setminus C$  is in  $\mathcal{W}$  (maximality)

We restate the simple result, which interprets a well-known fact of ultrafilter in our context.

*Let  $F$  be an independent and neutral aggregation rule and let  $\mathcal{W}$  be the corresponding set of winning coalitions for supports, i.e.,  $\text{sup} \in F(\rightsquigarrow) \Leftrightarrow N_{\rightsquigarrow}^{\text{sup}} \in \mathcal{W}$  for all  $\text{sup} \in \rightsquigarrow$ . Then  $F$  is dictatorial if and only if  $\mathcal{W}$  is an ultrafilter.*

**LEMMA 4.4.** Let  $P$  be a property that is non-simple, and disjunctive. Then, for  $|\text{Arg}| \geq 3$ , any unanimous, grounded, neutral, and independent aggregation rule  $F$  that preserves  $P$  must be a dictatorship.

If properties we are interested in are non-simple, and disjunctive, then we can apply Lemma 4.4 to achieve axiomatic results for them.

**THEOREM 4.5.** For  $|\text{Arg}| \geq 5$ , any unanimous, grounded, neutral, and independent aggregation rule  $F$  that preserves d-preferred extensions must be a dictatorship.

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