



What can we expect from consensus decision-making?

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#### **Outline**

- we introduce the definitions of argumentation framework and semantics
- brief look at a result from graph aggregation
- we define several semantic properties
- we study the interaction of semantic properties, aggregation rules and its properties

#### First Taste

Who should be responsible for blocking negotiation in their region.:

- I My government cannot negotiate with your government because your government doesn't even recognize my government
- A Your government doesn't recognize my government either
  - I But your government is a terrorist government



## **Background: Abstract Argumentation Frameworks**

An abstract argumentation framework (AF) is a pair AF =  $\langle Arg, \rightarrow \rangle$ , where,

- Arg is a finite set of arguments
- → is an irreflexive binary attack-relation on Arg



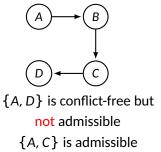
A is **not** attacked by any argument, B is *attacked* by A, C, D *attack* each other.

P.M. Dung. On the Acceptability of Arguments and its Fundamental Role in NMR, LP and *n*-Person Games. *Artificial Intelligence*, 77(2):321–357, 1995.

## Background: Conflict-Freeness and Admissibility

#### Given an AF, we say that $\Delta \subseteq Arg$ is:

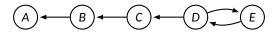
- conflict-free if there exist no arguments  $A, B \in \Delta$  such that  $A \rightarrow B$
- admissible if it is conflict-free and defends every single one of its members



# Background: Conflict-Freeness and Admissibility

- A Diesel cars should be banned from entering the city centre in order to decrease pollution.
- B There are few alternatives: there are not enough charging stations around.
- C Setting up more charging stations.

- D In times of financial crisis, the city should not commit to spending additional money.
- E Health and climate change issues are important, so the city has to spend what is needed to tackle pollution.

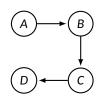


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\{E\}, \{D\}, \{D,E\}, \{B,D\} are admissible sets of AF
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## **Background: Preferred Extension**

Given an AF, we say that  $\Delta \subseteq Arg$  is:

 a grounded extension iff Δ is a maximal admissible set w.r.t. set inclusion (accept as more argument as possible)

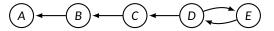


{A} is an admissible set butnot a preferred extension{A, C} is a preferred extension

## **Background: Preferred Extension**

- A Diesel cars should be banned from in the city centre in order to decrease pollution.
- B There are few alternatives: there are not enough charging stations around.
- C Set up more charging stations.

- D In times of financial crisis, the city should not commit to spending additional money.
- E Health and climate change issues are important, so the city has to spend what is needed to tackle pollution.



{A, C, E}, {B, D} are the preferred extensios of AF

## **Background: Grounded Semantics**

The characteristic function of AF is the function  $f_{AF}: 2^{Arg} \rightarrow 2^{Arg}$  with  $f_{AF}: \Delta \mapsto \{A \in Arg \mid \Delta \text{ defends } A\}$ .

The grounded extension of AF is the least fixed point of its characteristic function  $f_{AF}$ .



$$f_{AF}^1(\emptyset) = \{A\}, f_{AF}^2(\emptyset) = \{A, C\}, f_{AF}^3(\emptyset) = \{A, C\}, f_{AF}^2(\emptyset) = f_{AF}^3(\emptyset)$$
  
so the grounded extension of AF is  $\{A, C\}$ .

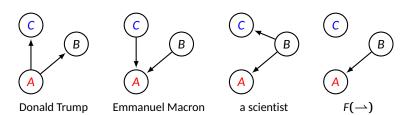
## **Collective Argumentation**

Fix a set of arguments. Given n agents and a profile of attack relations  $\rightarrow = (\rightarrow_1, \dots, \rightarrow_n)$ . How should we aggregate this information?

### An example

Let F be the majority rule. Three figures each provide an AF on the same set of three issues:

- A Increasing consumption of fossil fuels.
- B Preventting global warming
- C Lowering or eliminating electric car tax



# **Graph Aggregation**

**Theorem 1** For  $|Arg| \ge 3$ , any unanimous, grounded, neutral, and independent agggegation rule F that preserves some property P that is *implicative* and *disjunctive* must be a dictatorship.

#### where

- implicative: there exist a set Att ⊆ Arg × Arg such that
   [att₁, att₂, att₃ ∈ Arg × Arg \Att] → [att₁ ∧ att₂ → att₃]
- disjunctive: there exist a set Att ⊆ Arg × Arg such that
   [att<sub>1</sub>, att<sub>2</sub> ∈ Arg × Arg \ Att] → [att<sub>1</sub> ∨ att<sub>2</sub>]

#### Example:

- Transitivity is implicative
- Completeness is disjunctive
- Connectedness is implicative and disjunctive

### **Semantic Properties**

What AF-properties are preserved under aggregation?

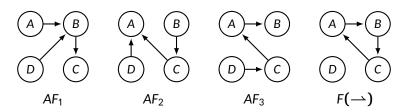
We are interested in *semantic properties* such as:

- acyclicity
- nonemptiness of the grounded extension
- $\Delta \subseteq Arg$  being an extension (according to a given semantics)

So, in case all agents agree on one of them being satisfied, we would like to see it preserved under aggregation.

### Example

Let F be the *majority rule*. Consider the following example:



#### Observations:

- acyclicity is not preserved
- nonemptiness of the grounded extension is preserved

But does the latter result hold in general?

#### **Preservation of Conflict-Freeness**

**Theorem 2** Every aggregation rule *F* that is *grounded* preserves conflict-freeness.

#### **Proof Idea**

 no grounded aggregation rule would invent an attack between two arguments

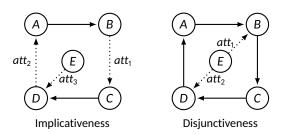
<u>Terminology</u>: an aggregation rule *F* is called *grounded* if  $F(\rightarrow_1, \ldots, \rightarrow_n) \subseteq (\rightarrow_1) \cup \cdots \cup (\rightarrow_n)$  for every profile  $\rightarrow$ .

#### **Preservation of Grounded Extensions**

**Theorem 3** For  $|Arg| \ge 5$ , any unanimous, grounded, neutral, and independent aggregation rule *F* that preserves *grounded extensions* must be a *dictatorship*.

#### **Proof Idea**

Being a grounded extension is a disjunctive and implicative property



- Att =  $\{A \rightarrow B, C \rightarrow D\}$
- $att_1 = B \rightarrow C$ ,  $att_2 = D \rightarrow A$ , and  $att_3 = E \rightarrow D$ .
- the acceptance of att<sub>1</sub> and att<sub>2</sub> imply the acceptance of att<sub>3</sub>

## **Preservation of Acyclicity**

Acyclicity is associated with the existence of a *single extension*.

**Theorem 4** If  $|Arg| \ge n$ , then under any neutral and independent aggregation rule F that preserves *acyclicity* at least one agent must have *veto powers*.

#### **Proof Idea**

- the proof of this theorem relies on a result for a more general property which we call *k*-exclusiveness
- acyclicity is a k-exclusive property

<u>Terminology</u>: Agent  $i \in N$  has veto powers under aggregation rule F, if  $F(\_) \subseteq (\__i)$  for every profile  $\_$ .

#### **Preservation Results**

Property	Rule(s)
Argument acceptability	
(Holds for all four semantics)	dictatorships
Conflict-freeness	all grounded rules
Admissibility	nomination rule
Grounded extension	dictatorships
Stable extension	nomination rule
Coherence	dictatorships
Nonempty of the GE	veto rules
Acyclicity	veto rules

### Summary

#### In this talk, we have:

- defined a model for aggregation of AFs
- defined desirable semantic properties of AFs
- drawn a picture of the capabilities and limitations of aggregation of AFs

#### Things that could be done in the future:

- study the preservation of preferred and complete extensions
- study further semantic properties of AFs, going beyond the four classical semantics

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