Aggregation of Abstract Argumentation Frameworks

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Motivation

When a group of agents are engaged in a debate, they may:

- disagree on many details
- but agree on high-level ideas

How should we model such scenarios of *collective argumentation*?

Abstract Argumentation Frameworks

An abstract *argumentation framework (AF)* is a pair $AF = \langle Arg, \rightharpoonup \rangle$, where,

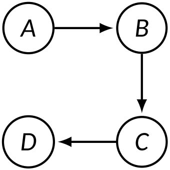
- Arg is a finite set of *arguments*
- $\rightharpoonup$ is an irreflexive binary *attack-relation* on Arg

P.M. Dung. On the Acceptability of Arguments and its Fundamental Role in NMR, LP and n-Person Games. *Artificial Intelligence*, 77(2):321–357, 1995.

Semantics

Given an AF, we say that $\Delta \subseteq Arg$ is:

- *conflict-free* if there exist no arguments $A, B \in \Delta$ such that $A \rightharpoonup B$.
- *admissible* if it is conflict-free and defends every single one of its members.



$\{A, D\}$ is conflict-free but not admissible

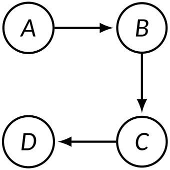
More semantics: stable semantics, preferred semantics, complete semantics, etc.

A set of arguments is call an *extension* if it is acceptable under a given semantics.

Grounded Semantics

The *characteristic function* of AF is the function $f_{AF} : 2^{Arg} \longrightarrow 2^{Arg}$ with $f_{AF} : \Delta \mapsto \{A \in Arg \mid \Delta \text{ defends } A\}$.

The *grounded extension* of AF is the *least fixed point* of its characteristic function f_{AF}.



$$f^1_{AF}(\emptyset) = \{A\}, f^2_{AF}(\emptyset) = \{A, C\}, f^3_{AF}(\emptyset) = \{A, C\}, f^2_{AF}(\emptyset) = f^3_{AF}(\emptyset)$$

so the *grounded extension* of AF is $\{A, C\}$.

Collective Argumentation

Fix a set of *arguments*. Given n *agents* and a *profile* of attack relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$. How should we *aggregate* this information?

Outline of this talk:

- Aggregation rules
- Axioms: properties of aggregation rule
- Preservation of semantic properties of AFs

Aggregation Rules

An *aggregation rule* is a function F mapping any profile of attack-relations n to a single attack-relation.

Examples:

- *Quota Rule*: for $q \in \mathbb{N}$, $F_q(\rightharpoonup) = \left\{ att \in Arg \times Arg \mid \# N^{\rightharpoonup}_{att} \geqslant q \right\}$,
  $N^{\rightharpoonup}_{att}$ denotes the set of supporters of the attack att in profile $\rightharpoonup$.
- *Oligarchic rule*: for $C \subseteq N$, $F_C(\rightharpoonup) = \left\{ att \in Arg \times Arg \mid C \subseteq N^{\rightharpoonup}_{att} \right\}$.

Axioms

Recall that $N^{\rightharpoonup}_{att}$ denotes the set of supporters of the attack *att* in profile $\rightharpoonup$.

Examples for desirable properties of aggregation rules F:

- *anonymous*: $F(\rightarrow_1, \dots, \rightarrow_n) = F(\rightarrow_{\pi(1)}, \dots, \rightarrow_{\pi(n)})$
- *neutral*: $N^{\rightharpoonup}_{att} = N^{\rightharpoonup}_{att'}$ implies $att \in F(\rightharpoonup) \Leftrightarrow att' \in F(\rightharpoonup)$
- *independent*: $N^{\rightharpoonup}_{att} = N^{\rightharpoonup'}_{att}$ implies $att \in F(\rightharpoonup) \Leftrightarrow att \in F(\rightharpoonup')$
- *monotonic*: $N^{\rightharpoonup}_{att} \subseteq N^{\rightharpoonup'}_{att}$ for all profiles $\rightharpoonup, \rightharpoonup'$ and all attacks *att*.
- *unanimous*: $F(\rightarrow_1, \dots, \rightarrow_n) \supseteq (\rightarrow_1) \cap \dots \cap (\rightarrow_n)$
- *grounded*: $F(\rightarrow_1, \dots, \rightarrow_n) \subseteq (\rightarrow_1) \cup \dots \cup (\rightarrow_n)$

Preservation of AF-Properties

What AF-properties are preserved under aggregation?

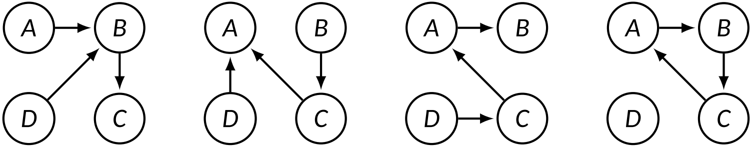
We are interested in *semantic properties* such as:

- *Acyclicity*
- *Nonemptiness* of the grounded extension
- $A \in Arg$ being *acceptable* (under a given semantics)
- $\Delta \subseteq Arg$ *being an extension* (according to a given semantics)

So, in case all agents agree on one of them being satisfied, we would like to see it preserved under aggregation.

Example

Let F be the *majority rule*, consider the following example:



AF_1 AF_2 AF_3 $F(\rightharpoonup)$

Observations:

- The *majority rule* does not preserve *acyclicity*.
- All of AF_1, AF_2, and AF_3 satisfy *nonemptiness* of the grounded extension.

Preservation of Conflict-Freeness

Theorem 1 Every aggregation rule F that is *grounded* preserves *conflict-freeness*.

Idea of the Proof Any grounded aggregation rule would not *invent* an attack between two conflict-free arguments.

Preservation of Grounded Extensions

Theorem 3 For $|Arg| \geqslant 5$, any unanimous, grounded, neutral, and independent aggregation rule F that preserves *grounded extensions* must be a *dictatorship*.

Idea of the Proof

- The proof of this theorem makes use of a technique developed by Endriss and Grandi for graph aggregation
- It is a generalisation of Arrow's seminal result for preference aggregation

U. Endriss and U. Grandi. Graph Aggregation. *Artificial Intelligence*, 245:86–114, 2017.

K.J. Arrow. Social Choice and Individual Values, 2nd ed., John Wiley and Sons, 1963. First edition published in 1951.

Preservation of Acyclicity

Acyclicity is associated with the existence of *single extension*.

Theorem 4 If $|Arg| \geqslant n$, then under any neutral and independent aggregation rule F that preserves *acyclicity* at least one agent must have *veto powers*.

Idea of the Proof

- The proof of this theorem relies on the result for a more general property which is called k-exclusiveness.
- Acyclicity is a k-exclusive property.

Preservation Results

Property	Dictator?	Veto?	Preserve Rule(s)
Argument accetability (This holds for all four semantics)	✓	-	-
Conflict-freeness	-	-	All grounded rules
Admissibility	-	-	Nomination rule
Grounded extension	✓	-	-
Stable extension	-	-	Nomination rule
Coherence	✓	-	-
Nonempty of the GE	-	✓	-
Acyclicity	-	✓	-
Anti-transitivity	-	✓	-

Summary

In this talk, we have:

- defined a model for aggregation of AFs
- defined the desirable properties of AFs
- drawn a picture of the capabilities and limitations of aggregation of AFs

Things could be done in the future:

- study the preservation of preferred and complete extensions
- study further semantic properties of AAF go beyond four classical semantics
- ...