Research Proposal: Assortment Optimization Considering Behavioral Issues

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Abstract

This short article is the research proposal for my research project: incorporating the theory in behavioral economics into the consumer choice model and then solving the assortment optimization under the new choice model. This is a novel domain, there are few works relating to it, and all of which have been done in recent years. The following of article is organized as follows: in the first section, I will introduce the prospect theory and how the current RM models can be improved; In the second section, I will introduce the context effects and related potential improvements; At the last section, I will introduce further studies and the research priority.

1 Assortment Optimization Considering the Prospect Effects

A quick brief introduction to behavioral economics before entering the main article. There are mainly three streams of literature trying to deal with the behavioral issues of consumers:

1. **Individual choice theories**: such as the prospect theory, the framing effect, etc. These theories are trying to refine the axiom of microeconomics because there are many actual circumstances where the consumer's choice disobeys Von Neuman's utility axiom.

2. **Choice Over Time**: this stream of literature studies how people's utility is discounted with time;

3. **Social Preferences**: study how the choice can be affected by social norms: such as reciprocity and inequality aversion.

- Most research (including this one) relates to the first stream of behavioral literature.

- Most research (including this one) incorporates some individual choice theories to correct consumers' utility and use the corrected utility to calculate the 'new' purchasing probability under the MNL model.

- My intuition is if one wants to incorporate the latter two streams of theories in revenue management, combining them with the Markov choice model may be a good idea. (see (Wang and Wang, 2017) for inspiration). However, this is not the content of this research project.

For a deeper introduction about the behavioral theory, read chapter 20 in Özer and Phillips (2012).

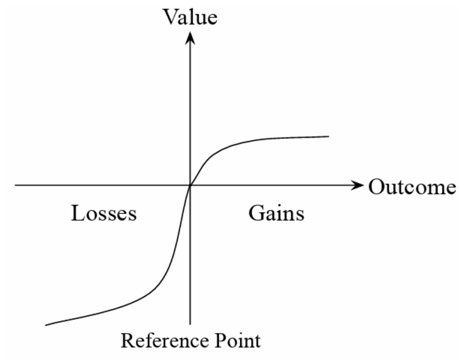
1.1 Historical Research

First, we should go over the **prospect** theory, invented by Daniel Kahneman in 1979 (and he got Nobel Prize for this). The key idea of the prospect theory is that people evaluate an outcome based on the comparison of the outcome with some subjective reference point, rather than based on the absolute outcome itself. Picture 1 is a classic picture showing the utility valuation in the prospect theory, Google it if not understand.

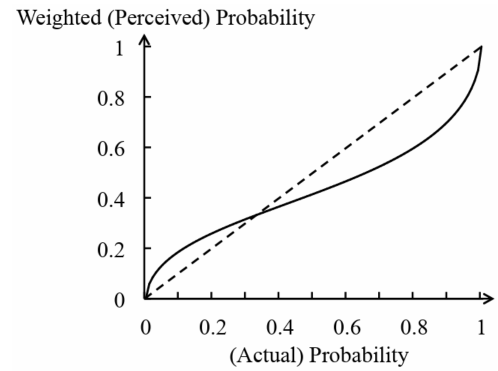
Wang (2018) is the first paper incorporating the prospect effect into revenue management and assortment optimization. The modeling process is quite straightforward. In his article, for any assortment S and its price p (p is a $\|S\|$-dimensional vector), the consumers have a **reference price function** $r(s,p)$, which can be thought as the consumers' expectation of the prices given p and S. For every product i with the baseline utility α_i, the corrected utility is given by:

$$U_i = \alpha_i - \beta p_i + \lambda(r(S,\mathbf{p}) - p_i)^+ - \gamma(p_i - r(S,\mathbf{p}))^+ + \xi_i. \tag{1}$$

Then the author studies the assortment optimization problem given the corrected utility under the MNL. One may wonder what the reference price



(a) Value Function



(b) Probability Weighting Function

Figure 1: Utility Evaluation in Prospect Theory

function looks like. In fact, it's still straightforward. There are five reference price functions studied in the paper:

1. the lowest price: $r_l(S,p) = \min_{k\in S} p_k$;
2. the highest price: $r_h(S,p) = \max_{k\in S} p_k$;
3. the weighted sum: $r_w(S,p) = \sum_{k\in S} w_k p_k$;
4. etc, $\cdots$.

For more technical details, see the original paper.

1.2 Potential Improvement

One can see the reference price function used in the paper is pretty *naive*. Consider an assortment with two products A, B, both of which contain the same unit product but are different in volume and price. In this scenario, the reference price function can not depict the prospect theory well because *these functions only take the price of other products into consideration, but the perceived utility can affect the reference price.*

My thought is: the reference price function can be replaced by $r(S,p,\alpha)$, where α_i is the baseline utility for item i. A simple and intuitive structure of $r(S,p,\alpha)$ can be:

$$r_i = \frac{\sum_j p_j}{\sum_j \alpha_j} \cdot \alpha_i, j \in S \tag{2}$$

We can also use $r(S, p, c)$, where c is the cost vector (maybe these two improvements are equivalent, I'm not sure). Using c instead of α can bring to more convenience in the next section's improvements. Under $r(S, p, c)$, the assortment optimization can be formulated by:

$$\max_{S \subseteq \mathcal{N}} \frac{\sum_{i \in S}(p_i - c_i) \cdot \exp(U_i)}{1 + \sum_{i \in S} \exp(U_i)}, \tag{3}$$

where $r_i(S, p, c) = \frac{\sum_j p_j}{\sum_j c_j} \cdot c_i, j \in S$ and U_i follows a similar pattern with equation 1.

2 Assortment Optimization Considering the Context Effects

The last section introduces the first improvement, and I will extend it further in this section. But first, we should review the context effects and how they are used in past studies.

2.1 Historical Studies

Context effects are found in empirical studies, the widely-accepted three context effects:

1. attraction effect: the product C can lead an attraction effect for B;
2. compromise effect: the product D can lead a compromise effect for B;
3. similarity effect: consider three products A, B, C, where B and C share more similarity. For assortment A, B, the inclusion of new product C could decrease the purchasing probability of B more than C because they are more similar.

One can try to understand the three effects based on the figure 2, otherwise google it.
The first paper incorporating the context effects in the choice model is Rooderkerk et al. (2011). This is a paper in marketing and doesn't consider the assortment optimization (no one would because the model is a little bit trivial). In their paper, the utility for item i contains three parts:

$$\mathrm{z}_{\mathrm{hi}}^{\mathrm{S}} = \underbrace{V_i}_{\text{baseline utility}} + \underbrace{VC_i^S}_{\text{context-dependent utility}} + \varepsilon_{\mathrm{i}}, \tag{4}$$

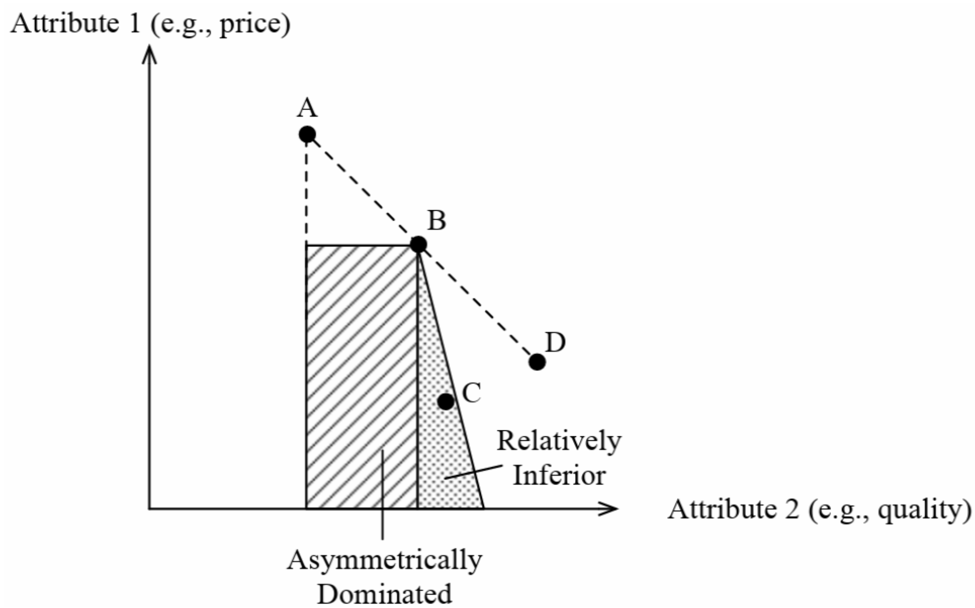


Figure 2: Demonstration of Attraction Effect and Compromise Effect

and the context-dependent utility is calculated using the attribute vector of the products within S based on the Euclidean distance.

It was until 2020 that Yousefi Maragheh et al. (2020) proposed the CMNL (contextual MNL) model that incorporates the context effects into assortment optimization. Their modeling of the context effects is much easier. They define the utility as:

$$U_i = \alpha_i + \sum_j \alpha_{ji}, j \in S, \quad (5)$$

where α_{ji} is the context effects of j on i. The authors use some examples to show that a simple matrix A can capture the three context effects. In fact, it's not (just comparing it with Rooderkerk et al. (2011) and you can see why). **The only way to capture the context effect precisely is by using the attribute data**.

2.2 Potential Improvement

Now, consider the improved model in the last section. We can assume that each product has K attributes that the consumers can perceive. For each attribute k of item i, the seller can decide its level V_{ik}. Because we have the cost variable c, we can assume $c_i = \sum_k V_{ik}$, which means the seller can distribute the resources on different attributes. α_i is the baseline utility, and we can assume its structure as:

1. either predetermined;
2. or $\alpha_i = \sum_k \beta_k V_{ik}$, which is a linear combination on V_{ik}.

We depict the context effects using the attribute, denoted as $T_i = T(S, V)$. The refined utility function is given as:

$$U_i = \alpha_i + r_i(S, p, c) + T_i(S, V) \tag{6}$$

Now, the assortment optimization problem can be formulated by:

$$\begin{aligned} \max_{S \subseteq \mathcal{N}, V} & \frac{\sum_{i \in S}(p_i - c_i) \cdot \exp(U_i)}{1 + \sum_{i \in S} \exp(U_i)}, \\ & \sum_k V_{ik} = c_i, \forall i \in S \end{aligned} \tag{7}$$

One way to make it simpler is to include the price vector in the attribute: $\bar{V}_i \in \mathcal{R}^{K+1} := V_i \cup p_i$, and refine the constraint to $\sum_1^{k-1} V_{ik} = c_i$.
Note that although all the extended models are based on MNL, they are not in the set of RUM (random utility model).

3 The Furthuer Studies

Finishing the problem (the NP-hard proof, optimal assortment structure, heuristics, and even the bound) can cost a lot of time. If we still have time, we can extend it further.

3.1 Incorperating the Focal Effect

In some scenarios, the assortment may have a 'star' product, consumers would over-evaluate its utility, this is called the focal effect. Kovach and Tserenjigmid (2022) design a new choice model (focal luce model: FLM) that considers this phenomenon:

$$v(i, \mathcal{S}) = v_i + \log\left(1 + \delta(\mathcal{S}) \cdot \mathbf{1}\{i \in F(\mathcal{S})\}\right) + \epsilon_i, \tag{8}$$

where $F(\cdot)$ is the *focal function* indicating the star products given the set , and $\delta(\cdot)$ is the *distortion function* indicating the over-evaluation for the star products. Jiang et al. (2023) consider the assortment optimization under the FLM.
Our model can continue to incorporate this effect if the model is tractable.

3.2 Pricing with Assortment Optimization

Another improvement is to consider the pricing and assortment optimization together. Treating p as a decision variable rather than a predetermined variable. This kind of study is becoming more and more common these days.

3.3 Research Output

This framework may not be used (like 99% of other research) because it might be trivial (depending on the structure of r_i and T_i), so the main direction of this paper is to output some **insights** that can guide the process of product design and assortment design.

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