Research Proposal

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1 Model Setting

Consider a market consisting of one central platform and n retailers, where each retailer $i \in \mathcal{N}$ has a set of products denoted by S_i, and each product $j \in S_i$ is characterized by a customer value u_j (denote $v_j = e^{u_j}$) and a predetermined price r_j. We assume that all products have zero marginal costs.

Each retailer i offers an assortment $S_i^p \subseteq S_i$ to the central platform. After observing the assortments $\cup_{i \in \mathcal{N}} S_i^p$ offered by all retailers, the platform performs a meta-assortment optimization to determine the subset $S^P \subseteq \cup_{i \in \mathcal{N}} S_i^p$ that will be displayed on the platform. Additionally, the platform has the discretion to set a commission fee rate $\delta \geq 0$. In addition to the central platform, each retailer operates an independent sales channel, free from any commission fees imposed by the platform. Through this channel, each retailer can choose a separate assortment $S_i^r \subseteq S_i \setminus S_i^p$[1].

The demand of the consumer follows an MNL model. We presume that consumer purchasing behavior unfolds in two stages: First, customers evaluate the products available on the central platform. Should they deem the offerings unsatisfactory—if the value of the external option is greater than any product's value within S^p—then they proceed to explore the direct sales channels of the retailers. They will peruse the collective assortment $\cup_{i \in \mathcal{N}} S_i^r$ and make their selection from this pool. If, however, customers cannot find any suitable products even in the direct channels, they will quit purchasing.

[1] It is implicitly assumed that an item featured on the central platform cannot simultaneously be presented on the retailer's own channel. This assumption needs further discussion.

Following similar assumptions made in prior research (Liu et al. (2023)), we posit that if a customer opts not to purchase any product from S^p ($v_j+\epsilon_j < v_0+\epsilon_0$ for all $j \in S^p$), the term ϵ_0 will not be re-sampled when customers subsequently evaluate products in the second stage.

The choice probability for product $j \in S_i^p$ is given as:

$$P_j^p = \mathbf{1}\{j \in S^p\} \frac{v_j}{1+\sum_{k \in S^p} v_k}$$

According to Theorem 2.1 in Gao et al. (2021), the choice probability for product $j \in S_i^r$ is given as:

$$P_j^r = \frac{v_j}{(1+\sum_{k \in S^p} v_k)(1+\sum_{k \in S^p} v_k + \sum_{k \in \cup_{i \in \mathcal{N}} S_i^r} v_k)}$$

The revenue for the platform and firms can be expressed as:

$$\Pi_i = \sum_{j \in S_i^p} P_j^p r_j (1-\delta) + \sum_{j \in S_i^r} P_j^r r_j$$

$$\Pi_p = \sum_{j \in S^p} p_j^p r_j \delta$$

2 Prelimilary Analysis

First, we analyze the case when the commission rate δ is fixed.

Definition 1 *Equilibrium under a fixed δ: an equilibrium is a set of assortments $(S^p, S_i^p, S_i^r \forall i \in \mathcal{N})$, where neither the retailers nor the platform can increase their revenue by offering a different assortment.*

1. Given λ and $S_i^p, \forall i \in \mathcal{N}$, the optimization problem for the platform is deterministic: offer a revenue-ordered assortment. The platform can find a r^* and only offer all products with $r_j \geq r^*$.

2. For any product $j \in S_i^p$ with $r_j < r^*$, the retailer cannot be worse off when it opts not to display it at all. So if there is an equilibrium $(S^{p*}, S_i^{p*}, S_i^{r*})$ where $S_i^{p*} \not\subseteq S^{p*}$ for some i, the actions $(S^{p*}, S_i^{p*} \cap S^{p*}, S_i^{r*})$ is also an equilibrium.

3. For any product $j \in S_i^p$ with $r_j \geq r^*$, the retailer will measure the tradeoff of either sending the product to the platform, which increases the likelihood of it being purchased; or retaining it for their own sales channel to avoid commission fees. It is preferable for the retailer to keep the product in S_i^p if:

$$\sum_{l \in S_i^p} \frac{(1-\delta)v_l r_l}{1 + \sum_{k \in S^p} v_k} + \sum_{l \in S_i^r} \frac{v_l r_l}{(1 + \sum_{k \in S^p} v_k)(1 + \sum_{k \in S^p} v_k + \sum_{k \in \cup_{i \in \mathcal{N}} S_i^r} v_k)} >$$
$$\sum_{l \in S_i^p / j} \frac{(1-\delta)v_l r_l}{1 + \sum_{k \in S^p / j} v_k} + \sum_{l \in S_i^r \cup j} \frac{v_l r_l}{(1 + \sum_{k \in S^p / j} v_k)(1 + \sum_{k \in S^p} v_k + \sum_{k \in \cup_{i \in \mathcal{N}} S_i^r} v_k)}$$

   Rearrange the inequality, we can obtain:

$$\delta < 1 - \frac{r_j(1 + \sum_{k \in S^p} v_k)(1 + \sum_{k \in S^p / j} v_k) + \sum_{l \in S_i^r} v_l r_l}{(r_j \cdot (1 + \sum_{k \in S^p / j} v_k) - \sum_{l \in S_i^p / j} v_l r_l)(1 + \sum_{k \in S^p} v_k + \sum_{k \in \cup_{i \in \mathcal{N}} S_i^r} v_k)} \tag{1}$$

   The right-hand side of the inequality can be close to 1 when n is large, indicating that if δ is not significantly high, it becomes advantageous for all retailers to list their products with $r_j \geq r^*$ on the platform.

4. Then the retailer's decision problem on S_i^r is equivalent to:

$$\max_{S_i^r \subseteq S_i / S_i^p} \sum_{j \in S_i^r} \frac{v_j r_j}{a(b + \sum_{j \in S_i^r} v_j)},$$

   where a and b are some constants. As a result, every S_i^r should be in revenue-ordered.

The intuition for these observations is that when n is large, signifying a highly competitive market, the potential revenue from increased consumer purchases at the initial stage can often surpass the revenue gains from avoiding commission fees in the second stage. Consequently, we have the following lemma:

Lemma 1 *There exists a $\delta^* > 0$, such that for all $\delta \le \delta^*$, there exists an equilibrium with this structure: $S_i^p = \{j \in S_i : r_j \ge r^*\}, S_i^f = \{j \in S_i / S_i^p : r_j \ge r_i^*\}$, and $S^p = \cup_{i \in \mathcal{N}} S_i^p$.*

A drawback of this model: we want to investigate the equilibrium of the game when δ is a decision variable of the platform. However, when n is large, the threshold δ^* can be large as well, in which case the equilibrium remains 'static' if δ is not implausibly high. Few meaningful insight could be extracted from the equilibrium.

3 Related Literature

- Fair assortment optimization (recomendation) of the platform: Lu et al. (2023), Chen et al. (2022), Johnson et al. (2023);
- Assortment optimization considering platform OM: Hense and Hübner (2022), Aouad and Saban (2023), Rios et al. (2023), Chen et al. (2021);
- Xu et al. (2023) studied a model with a similar structure but focusing on pricing and reinforcement learning.

References

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