Notes on Recursion Theory

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N.J. Cutland, *Computability: An introduction to recursive function theory*, Cambridge University Press, 198

Contents

Prerequisites

basic notations of sets and functions:

- $A \subseteq B$、$A \subset B$ 、$A \cup B$、$A \cap B$、、
- $A \setminus B \coloneqq \{x \mid x \in A, x \notin B\}$
- $\overline{A} \coloneqq \mathbb{N} \setminus A$
- ordered pair: (x, y)
- Cartesian product: $A \times B \coloneqq \{(x, y) \mid x \in A, y \in B\}$
- $A^n \coloneqq A \times \cdots \times A$ (n times)
- function f: $(x, y), (x, z) \in f \Longrightarrow y = z$
- $Dom(f) \coloneqq \{x : f(x)$ is defined $\}$
- $Ran(f) \coloneqq \{f(x) \mid x \in Dom(f)\}$
- $f \colon A \to B$: $Dom(f) = A$
- f is injective: $\forall x, y \in Dom(f), x \neq y \Longrightarrow f(x) \neq f(y)$.
- surjective:
- bijective: injective + surjective
- restriction $f|X$ (or $f\lfloor X$):
- composition $f \circ g$
- inverse image:
- total function:
- partial function:
- Zero function $0 \colon \mathbb{N} \to \mathbb{N}$
- $m \colon \mathbb{N} \to \mathbb{N}$
- equivalence relation, equivalence class
- partial order: irreflexivity(禁自反) + transitivity

1 Computable functions

1.1 Algorithms, effective procedures

1.2 URM: the unlimited register machine (无限寄存器)

1.3 computable functions

Exercise 2.2 (p14) Carry out the computation under the program of example 2.1 with initial configuration $8, 4, 2, 0, 0, \ldots$

Answer: The program of example 2.1 is :

I_1	$J(1,2,6)$	I_4	$J(1,2,6)$
I_2	$S(2)$	I_5	$J(1,1,2)$
I_3	$S(3)$	I_6	$T(3,1)$

	R_1	R_2	R_3	R_4	R_5		Next instruction
Initial configuration	8	4	2	0	0	...	I_1
	8	4	2	0	0	...	I_2 (since $r_1 \neq r_2$)
	8	5	2	0	0	...	I_3
	8	5	3	0	0	...	I_4
	8	5	3	0	0	...	I_5 (since $r_1 \neq r_2$)
	8	5	3	0	0	...	I_2 (since $r_1 = r_1$)
	8	6	3	0	0	...	I_3
	8	6	4	0	0	...	I_4
	8	6	4	0	0	...	I_5 (since $r_1 \neq r_2$)
	8	6	4	0	0	...	I_2 (since $r_1 = r_1$)
	8	7	4	0	0	...	I_3
	8	7	5	0	0	...	I_4
	8	7	5	0	0	...	I_5 (since $r_1 \neq r_2$)
	8	7	5	0	0	...	I_2 (since $r_1 = r_1$)
	8	8	5	0	0	...	I_3
	8	8	6	0	0	...	I_4
	8	8	6	0	0	...	I_6 (since $r_1 = r_2$)
Final configuration	6	8	6	0	0	...	I_7: STOP