Lab4 GCN

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一、实验目标

- 1. 熟悉图卷积神经网络的基本原理
- 2. 了解网络层数对图卷积神经网络性能的影响
- 3. 了解不同激活函数, self loop, DropEdge, PairNorm等技术对图卷积神经网络性能的 影响。

二、数据集介绍

Dataset	Nodes	Edges	Classes	Features
Citeseer	3327	4732	6	3703
Cora	2708	5429	7	1433

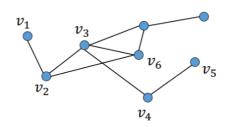
PPI数据集共24张图,平均每张图有2371个节点,共56944个节点818716条边,每个节点 特征长度为50,基因本体基作为label(总共121个),label不是one-hot编码。

三、实验原理

1.图的基本概念

图的矩阵A表示:

- 邻接矩阵 $A_{ij}=1$, 如果 v_i 和 v_j 相邻;
- 度矩阵 $D=diag(d(v_1),\ldots,d(v_N))$, $d(v_i)=\sum_{v_j\in N_{v_i}}A_{ij}$.



度矩阵 D A

拉普拉斯矩阵的性质L = D - A:

- LI = DI AI = d d = 0
- IL = 0
- L是半正定的,最小特征值为0,其特征向量为1

归一化后的拉普拉斯矩阵:

• 对称归一化: $L^{sym} = D^{-\frac{1}{2}}LD^{-\frac{1}{2}}$

随机游走归一化: L^{rw} = D⁻¹L

2.图傅里叶变换

 $L = U\Lambda U^T$ 为其特征值分解,U的列向量类比于傅里叶变换中的基。

对图上信号z的图傅里叶变换:

3.图傅里叶逆变换

 $L=U\Lambda U^T$ 为其特征值分解,U的列向量类比于傅里叶变换中的基。

对图上信号 2 的图傅里叶逆变换:

$$f(i) = \sum_{l=1}^{n} \hat{f}(\lambda_{l}) u_{l}(i) = \mathbf{u}(i)^{T} \hat{f}$$

$$\begin{pmatrix} f(1) \\ f(2) \\ \vdots \\ f(N) \end{pmatrix} = \begin{pmatrix} u_{1}(1) & u_{2}(1) & \dots & u_{N}(1) \\ u_{1}(2) & u_{1}(2) & \dots & u_{N}(2) \\ \vdots & \vdots & \ddots & \vdots \\ u_{1}(N) & u_{2}(N) & \dots & u_{N}(N) \end{pmatrix} \begin{pmatrix} \hat{f}(\lambda_{1}) \\ \hat{f}(\lambda_{2}) \\ \vdots \\ \hat{f}(\lambda_{N}) \end{pmatrix} \qquad \qquad \qquad \qquad f = U \hat{f}$$

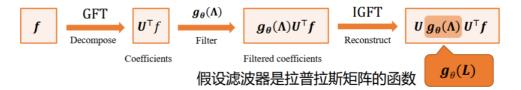
$$\longleftarrow f = U \hat{f}$$

$$\vdash f = U \hat{f}$$

$$\vdash$$

4.多项式卷积核

局部化: L对图信号f的操作Lf相当于在图上传播一步



多项式卷积核:

$$oldsymbol{g}_{ heta}(oldsymbol{\Lambda}) = \sum_{k=0}^K heta_k oldsymbol{\Lambda}^k$$

$$\hat{g}(\Lambda) = \begin{bmatrix} \sum\limits_{k=0}^K \theta_k \lambda_1^k & & & \\ & \sum\limits_{k=0}^K \theta_k \lambda_2^k & & & \\ & & \cdots & & \\ & & \sum\limits_{k=0}^K \theta_k \lambda_N^k & \end{bmatrix}$$

切比雪夫多项式,通过如下递归定义:

•
$$T_0(x) = 1$$
; $T_1(x) = x$
• $T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x)$

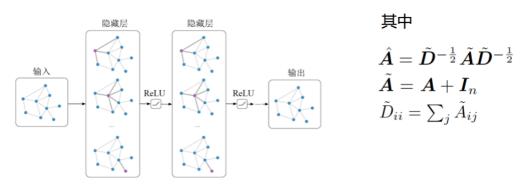
$$\widehat{g}(\mathbf{\Lambda}) = \sum_{k=0}^{K} \theta_k T_k(\widetilde{\mathbf{\Lambda}}) \qquad \text{ } \sharp \mathbf{P} \ \widetilde{\mathbf{\Lambda}} = \frac{2\mathbf{\Lambda}}{\lambda_{max}} - \mathbf{I}$$

$$\boldsymbol{U}\hat{g}(\boldsymbol{\Lambda})\boldsymbol{U}^{\mathsf{T}}\boldsymbol{f} = \sum_{k=0}^{K} \theta_{k} T_{k}(\tilde{\boldsymbol{L}})\boldsymbol{f}$$
, 其中 $\tilde{\boldsymbol{L}} = \frac{2L}{\lambda_{max}} - \boldsymbol{I}$

5.GCN

叠加多层GCN得到最终的模型(常取两层):

$$\hat{\boldsymbol{Y}} = f(\boldsymbol{X}, \boldsymbol{A}) = \operatorname{Softmax} \left(\hat{\boldsymbol{A}} \operatorname{ReLU} (\hat{\boldsymbol{A}} \boldsymbol{X} \boldsymbol{W}^0) \boldsymbol{W}^1 \right)$$



四、核心代码

网络用多层GCN模块堆叠而成:

```
class GCNBlock(nn.Module):
        def __init__(self, in_feats, out_feats, activation=None,
    pair_norm=False, add_self_loops=False, drop_edge=0):
 3
            super(GCNBlock, self).__init__()
             self.conv = SAGEConv(in_feats, out_feats, 'mean')
 4
            self.conv = GraphConv(in_feats, out_feats)
 5
 6
            self.dropedge = DropEdge(p=drop_edge)
            self.self_loops = AddSelfLoop()
 8
            self.norm = PairNorm()
            self.activation = activation
 9
            self.pair_norm = pair_norm
10
            self.add_self_loops = add_self_loops
11
            self.drop_edge = drop_edge
12
13
        def forward(self, g, in_feat):
14
            if self.drop_edge:
15
                g = self.dropedge(g)
16
17
                self.add_self_loops = True
            if self.add_self_loops:
18
                g = self.self_loops(g)
19
            h = self.conv(q, in_feat)
20
            if self.pair_norm:
21
                h = self.norm(h)
22
            if self.activation:
23
                h = self.activation(h)
24
25
            return h
26
    class GCN(nn.Module):
27
        def __init__(self, in_feats, n_hidden, n_classes, n_layers,
28
    activation, dropout=0.,
29
                     pair_norm=False, add_self_loops=False,
    drop_edge=0):
            super(GCN, self).__init__()
30
31
            if dropout:
                self.dropout = nn.Dropout(p=dropout)
32
            else:
33
                self.dropout = 0.
34
35
            self.layers = nn.ModuleList()
36
            # input layer
            self.layer1 = GCNBlock(in_feats, n_hidden, activation,
37
    pair_norm=pair_norm,add_self_loops=add_self_loops,
    drop_edge=drop_edge)
            self.layers = nn.ModuleList()
38
39
            # hidden layers
40
            for i in range(n_layers - 1):
```

```
41
                self.layers.append(GCNBlock(n_hidden, n_hidden,
    activation, pair_norm=pair_norm,add_self_loops=add_self_loops,
    drop_edge=drop_edge))
42
            # output layer
43
            self.layers.append(GCNBlock(n_hidden, n_classes,
    activation=None,
    pair_norm=pair_norm,add_self_loops=add_self_loops,drop_edge=drop_e
    dge))
44
        def forward(self, g, features):
45
            h = self.layer1(g, features)
46
            for idx, layer in enumerate(self.layers):
47
48
                if idx > 0 and self.dropout:
                    h = self.dropout(h)
49
                h = layer(g, h)
50
            if self.dropout:
51
                h = self.dropout(h)
52
53
            return h
```

五、实验结果

1.Node Classification

节点分类任务中,cora数据集和citeseer数据集是multi-class,用正确率作为评价指标,ppi数据集为multi-label,用F1指标作为评价指标。并且对于ppi数据集,采用20张图片作训练,2张图片作验证,2张图片作测试。

add_self_loop

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
cora	False	1	0.0	False	relu	0.811
cora	True	3	0.0	False	relu	0.817
cora	False	3	0.0	False	relu	0.798
cora	True	5	0.0	False	relu	0.807
cora	False	5	0.0	False	relu	0.791

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
citeseer	True	1	0.0	False	relu	0.703
citeseer	False	1	0.0	False	relu	0.660
citeseer	True	3	0.0	False	relu	0.692
citeseer	False	3	0.0	False	relu	0.674
citeseer	True	5	0.0	False	relu	0.676
citeseer	False	5	0.0	False	relu	0.624

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_f1
ppi	True	1	0.0	False	relu	0.895
ppi	False	1	0.0	False	relu	0.894
ppi	True	3	0.0	False	relu	0.923
ppi	False	3	0.0	False	relu	0.921
ppi	True	5	0.0	False	relu	0.962
ppi	False	5	0.0	False	relu	0.958

对于三个数据集,添加self_loop后的效果都要更好。

网络层数

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
cora	True	2	0.0	False	relu	0.823
cora	True	3	0.0	False	relu	0.817
cora	True	5	0.0	False	relu	0.807
cora	True	10	0.0	False	relu	0.707

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
citeseer	True	1	0.0	False	relu	0.703
citeseer	True	2	0.0	False	relu	0.696
citeseer	True	3	0.0	False	relu	0.688
citeseer	True	5	0.0	False	relu	0.676
citeseer	True	10	0.0	False	relu	0.551

对cora和citeseer数据集来说,浅层的网络效果要略好。

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_f1
ppi	True	1	0.0	False	relu	0.895
ppi	True	2	0.0	False	relu	0.870
ppi	True	3	0.0	False	relu	0.923
ppi	True	5	0.0	False	relu	0.962
ppi	True	10	0.0	False	relu	0.944

对ppi数据集来说,5层的效果要显著更好。

drop edge

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
cora	True	1	0.1	False	relu	0.728
cora	True	1	0.2	False	relu	0.666
cora	True	1	0.5	False	relu	0.619
cora	True	3	0.0	False	relu	0.817
cora	True	3	0.1	False	relu	0.648
cora	True	3	0.2	False	relu	0.640
cora	True	3	0.5	False	relu	0.575

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
citeseer	True	1	0.0	False	relu	0.703
citeseer	True	1	0.1	False	relu	0.652
citeseer	True	1	0.2	False	relu	0.605
citeseer	True	1	0.5	False	relu	0.585
citeseer	True	3	0.0	False	relu	0.688
citeseer	True	3	0.1	False	relu	0.604
citeseer	True	3	0.2	False	relu	0.581
citeseer	True	3	0.5	False	relu	0.562

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_f1
ppi	True	1	0.0	False	relu	0.895
ppi	True	1	0.1	False	relu	0.844
ppi	True	1	0.2	False	relu	0.808
ppi	True	1	0.5	False	relu	0.731
ppi	True	3	0.0	False	relu	0.923
ppi	True	3	0.1	False	relu	0.866
ppi	True	3	0.2	False	relu	0.827
ppi	True	3	0.5	False	relu	0.758

drop_edge在三个数据集上都没有明显的效果。

PairNorm

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
cora	True	1	0.0	True	relu	0.779
cora	True	3	0.0	False	relu	0.817
cora	True	3	0.0	True	relu	0.761
cora	True	5	0.0	False	relu	0.807
cora	True	5	0.0	True	relu	0.772

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
citeseer	True	1	0.0	False	relu	0.703
citeseer	True	1	0.0	True	relu	0.691
citeseer	True	3	0.0	False	relu	0.688
citeseer	True	3	0.0	True	relu	0.682
citeseer	True	5	0.0	False	relu	0.676
citeseer	True	5	0.0	True	relu	0.678

在cora和citeseer数据集上,pair_norm没有提升模型效果。

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_f1
ppi	True	1	0.0	False	relu	0.895
ppi	True	1	0.0	True	relu	0.732
ppi	True	3	0.0	False	relu	0.923
ppi	True	3	0.0	True	relu	0.919
ppi	True	5	0.0	False	relu	0.962
ppi	True	5	0.0	True	relu	0.966

在ppi数据集上,pair_norm使得模型效果有了一定的提升。

激活函数

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
cora	True	1	0.0	False	tanh	0.823
cora	True	1	0.0	False	sigmoid	0.751
cora	True	3	0.0	False	relu	0.817
cora	True	3	0.0	False	tanh	0.815
cora	True	3	0.0	False	sigmoid	0.319
cora	True	5	0.0	False	relu	0.807
cora	True	5	0.0	False	tanh	0.798
cora	True	5	0.0	False	sigmoid	0.319

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
citeseer	True	1	0.0	False	relu	0.703
citeseer	True	1	0.0	False	tanh	0.690
citeseer	True	1	0.0	False	sigmoid	0.677
citeseer	True	3	0.0	False	relu	0.688
citeseer	True	3	0.0	False	tanh	0.695
citeseer	True	3	0.0	False	sigmoid	0.275
citeseer	True	5	0.0	False	relu	0.676
citeseer	True	5	0.0	False	tanh	0.663
citeseer	True	5	0.0	False	sigmoid	0.253

在cora和citeseer数据集上, relu≈tanh>sigmoid。

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
ppi	True	1	0.0	False	relu	0.895
ppi	True	1	0.0	False	tanh	0.900
ppi	True	1	0.0	False	sigmoid	0.890
ppi	True	3	0.0	False	relu	0.923
ppi	True	3	0.0	False	tanh	0.940
ppi	True	3	0.0	False	sigmoid	0.750
ppi	True	5	0.0	False	relu	0.958
ppi	True	5	0.0	False	tanh	0.984
ppi	True	5	0.0	False	sigmoid	0.424

但在ppi数据集上, tanh>relu>sigmoid。

best result

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_acc
cora	True	1	0.0	False	relu	0.836
citeseer	True	1	0.0	False	relu	0.703

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_f1
ppi	True	5	0.0	False	tanh	0.984

2.Link Prediction

链路预测任务中,采用AUC作为评价指标。由于ppi数据集训练时间过长,仅采用cora和citeseer对比不同技术的效果,并且对于ppi数据集取5张图片来测试效果。

add_self_loop

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	1	0.0	False	relu	0.939
cora	False	1	0.0	False	relu	0.945
cora	True	3	0.0	False	relu	0.943
cora	False	3	0.0	False	relu	0.960
cora	True	5	0.0	False	relu	0.937
cora	False	5	0.0	False	relu	0.953

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
citeseer	True	1	0.0	False	relu	0.962
citeseer	False	1	0.0	False	relu	0.920
citeseer	True	3	0.0	False	relu	0.958
citeseer	False	3	0.0	False	relu	0.924
citeseer	True	5	0.0	False	relu	0.964
citeseer	False	5	0.0	False	relu	0.962

对于两个数据集,添加self_loop后的效果都要更好。

网络层数

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	1	0.0	False	relu	0.945
cora	True	2	0.0	False	relu	0.945
cora	True	3	0.0	False	relu	0.960
cora	True	5	0.0	False	relu	0.953
cora	True	10	0.0	False	relu	0.896

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
citeseer	True	1	0.0	False	relu	0.962
citeseer	True	2	0.0	False	relu	0.957
citeseer	True	3	0.0	False	relu	0.958
citeseer	True	5	0.0	False	relu	0.964
citeseer	True	10	0.0	False	relu	0.963

对于两个数据集,中浅层的网络效果不错。

drop edge

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	1	0.0	False	relu	0.945
cora	True	1	0.1	False	relu	0.892
cora	True	1	0.2	False	relu	0.900
cora	True	1	0.5	False	relu	0.896
cora	True	3	0.0	False	relu	0.960
cora	True	3	0.1	False	relu	0.909
cora	True	3	0.2	False	relu	0.896
cora	True	3	0.5	False	relu	0.894

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
citeseer	True	1	0.0	False	relu	0.962
citeseer	True	1	0.1	False	relu	0.950
citeseer	True	1	0.2	False	relu	0.932
citeseer	True	1	0.5	False	relu	0.917
citeseer	True	3	0.0	False	relu	0.958
citeseer	True	3	0.1	False	relu	0.919
citeseer	True	3	0.2	False	relu	0.899
citeseer	True	3	0.5	False	relu	0.901

对于两个数据集,drop edge都没有起到很好的效果。

PairNorm

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	1	0.0	False	relu	0.945
cora	True	1	0.0	True	relu	0.953
cora	True	3	0.0	False	relu	0.960
cora	True	3	0.0	True	relu	0.954
cora	True	5	0.0	False	relu	0.953
cora	True	5	0.0	True	relu	0.946

对于cora数据集, pair_norm的效果一般。

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
citeseer	True	1	0.0	False	relu	0.962
citeseer	True	1	0.0	True	relu	0.966
citeseer	True	3	0.0	False	relu	0.958
citeseer	True	3	0.0	True	relu	0.971
citeseer	True	5	0.0	False	relu	0.964
citeseer	True	5	0.0	True	relu	0.958

而对于citeseer数据集,pair_norm的添加使得模型有了明显的提升。

激活函数

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	1	0.0	False	relu	0.945
cora	True	1	0.0	False	tanh	0.952
cora	True	1	0.0	False	sigmoid	0.755
cora	True	3	0.0	False	relu	0.960
cora	True	3	0.0	False	tanh	0.963
cora	True	3	0.0	False	sigmoid	0.603
cora	True	5	0.0	False	relu	0.953
cora	True	5	0.0	False	tanh	0.951
cora	True	5	0.0	False	sigmoid	0.584

在cora数据集上, relu≈tanh>sigmoid。

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
citeseer	True	1	0.0	True	relu	0.966
citeseer	True	1	0.0	True	tanh	0.990
citeseer	True	1	0.0	True	sigmoid	0.988
citeseer	True	3	0.0	True	relu	0.971
citeseer	True	3	0.0	True	tanh	0.981
citeseer	True	3	0.0	True	sigmoid	0.980
citeseer	True	5	0.0	False	relu	0.964
citeseer	True	5	0.0	False	tanh	0.969
citeseer	True	5	0.0	False	sigmoid	0.973

在citeseer数据集上, tanh>relu>sigmoid。

best result

data_name	add_self_loop	n_layers	drop_edge	pair_norm	activations	test_auc
cora	True	3	0.0	False	tanh	0.963
citeseer	True	1	0.0	True	tanh	0.990
ppi	True	3	0.0	False	relu	0.871

六、实验总结

本次实验了解的图神经网络的原理,以及学会了使用DGL,并对比了不同激活函数,self loop, DropEdge, PairNorm等技术对图卷积神经网络性能的影响。