





# Variable Spatiotemporal Framework for Multivariate Time Series Prediction

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## **Outline**



- > Introduction & Motivation
- > VSTGN Framework
- Experimental Evaluation
- > Conclusion



#### **Introduction & Motivation**

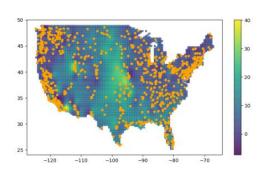


# Multivariate Time Series Forecasting:

- Predicting future values of multiple interdependent variables by jointly modeling their temporal dependencies and cross-variable interactions.
- Ubiquitous applications: Transportation, Energy, Climate.

# Spatiotemporal Graph Neural Network Framework (STGNN):

- Spatial Dependency Modeling: GCN-based  $\,\mathcal{S}_{ heta_s}$
- Temporal Dependency Modeling: RNN-based  $\mathcal{T}_{\theta_t}$
- Joint Spatiotemporal Learning:  $X_{t+1} = \mathcal{T}_{\theta_t}(\mathcal{S}_{\theta_s}(\mathcal{R}X_t))$





#### **Introduction & Motivation**



# Modeling Spatial Dependency in Spatiotemporal Data:

the general form of a GCN-based model:

$$\mathbf{H}^{(l+1)} = ext{AGGREGATE}\left(\mathbf{H}^{(l)}, \mathbf{A}; \Theta^{(l)}
ight)$$

- ✓ H: Input or updated feature vectors.
- ✓ A: a correlation matrix.
- θ: a learnable mapping function that projects input features into latent spaces suitable for downstream tasks.

# "Static-dynamic" misalignment limitations:

- Static correlation matrix.(Static adjacency matrix or spatially adaptive correlation matrix)
- Static projection function. (Shared parameters across time)



#### **Introduction & Motivation**



# Proposed Solution and Key Contributions:

- We propose VSTGN, a novel spatiotemporal GNN framework that adaptively constructs real-time correlation matrices from observational data, capturing dynamic relational changes.
- VSTGN introduces a spatiotemporally varying projection function that jointly accounts for temporal periodicity and spatial heterogeneity.
- Experiments on real-world datasets demonstrate that our model outperforms state-of-the-art baselines.

Achieveing "dynamic-dynamic" alignment with real-world scenarios.



#### VSTGN Framework



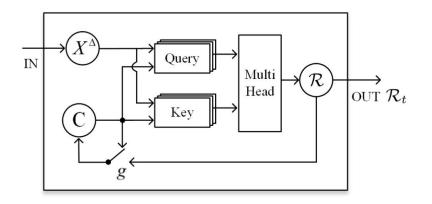
# Adaptive Dynamic Correlation Modeling--Gated Attention Unit (GAU)

Node correlation construction based on local data and self-attention mechanism.

$$egin{aligned} oldsymbol{Q}_t &= \left[ \left[ oldsymbol{X}_t^\Delta | oldsymbol{C}_{t-1} 
ight]^T oldsymbol{W}_q, \quad oldsymbol{K}_t &= \left[ \left[ oldsymbol{X}_t^\Delta | oldsymbol{C}_{t-1} 
ight]^T oldsymbol{W}_k, \ & ext{head}_{t,i} &= ext{softmax}(rac{oldsymbol{Q}_t oldsymbol{K}_t^T}{\sqrt{d_{model}}}), \ oldsymbol{\mathcal{R}}_t &= ext{sigmoid}(\llbracket ext{head}_{t,0}, ..., ext{head}_{t,i} 
black 
black oldsymbol{W}_r), \end{aligned}$$

Real-time correlation update based on Gated mechanism

$$egin{aligned} oldsymbol{g}_t &= \operatorname{sigmoid}(\left[\left[oldsymbol{X}_t^{\Delta} | oldsymbol{C}_{t-1}
ight]\right]^T oldsymbol{W}_u), \ oldsymbol{C}_t &= (\mathbf{1} - oldsymbol{g}_t) \otimes oldsymbol{C}_{t-1} + oldsymbol{g}_t \otimes oldsymbol{\mathcal{R}}_t, \end{aligned}$$





#### VSTGN Framework



Spatiotemporal varying Projection Function Learning--Spatiotemporal Embedding Layer (SEL)

Spatial variability

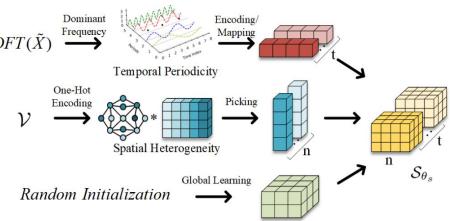
$$S_i = \sigma(\text{One\_Hot}(v_i)W_s) \in \mathbb{R}^{1 \times d_s},$$

- Temporal variability
  - ✓ Dominant frequency identification

$$\boldsymbol{w} = \text{Top\_p}(\text{DFT}(\tilde{\boldsymbol{X}})) \in \mathbb{R}^{1 \times p},$$

- ✓ Periodic encoding
- $\Gamma_t = [\Psi(\frac{2\pi}{w_1}I_t + \varphi_1), ..., \Psi(\frac{2\pi}{w_p}I_t + \varphi_p)],$
- ✓ Temporal feature learning

$$oldsymbol{T}_t = ext{MLP}(oldsymbol{W}_c oldsymbol{\Gamma}_t) \in \mathbb{R}^{d_s imes d_t}$$



✓ Spatiotemporal varying mapping function:

$$oldsymbol{P}_{i,t} = oldsymbol{S}_i oldsymbol{T}_t oldsymbol{W}_g \in \mathbb{R}^{1 imes f}$$



#### VSTGN Framework



# Variable Spatiotemporal Dependence Capture

Variable graph convolution constructed based on GAU and SEL

$$VGNN(\boldsymbol{X}_t) = \sigma(\boldsymbol{\mathcal{R}}_t \boldsymbol{X}_t^T \boldsymbol{P}_t) \in \mathbb{R}^{n \times f},$$

 Combined with a Gated Recurrent Unit to form a complete Variable Spatiotemporal Graph Neural Network (VSTGN):

$$egin{aligned} oldsymbol{r} &= \operatorname{sigmiod}(oldsymbol{\mathcal{R}}_t \left[ \left[ oldsymbol{X}_t | oldsymbol{h}_{t-1}^T 
ight]^T \mathbb{S} oldsymbol{T}_t oldsymbol{W}_r^*), \ oldsymbol{z} &= \operatorname{sigmiod}(oldsymbol{\mathcal{R}}_t \left[ \left[ oldsymbol{X}_t | oldsymbol{h}_{t-1}^T 
ight]^T \mathbb{S} oldsymbol{T}_t oldsymbol{W}_z^*), \ oldsymbol{s} &= \operatorname{tanh}(oldsymbol{\mathcal{R}}_t \left[ \left[ oldsymbol{X}_t | (oldsymbol{h}_{t-1}^T \otimes oldsymbol{r}) 
ight]^T \mathbb{S} oldsymbol{T}_t oldsymbol{W}_s^*), \ oldsymbol{h}_t &= (oldsymbol{1} - oldsymbol{z}) \otimes oldsymbol{s} + oldsymbol{z} \otimes oldsymbol{h}_{t-1}, \end{aligned}$$

 The final predictions are generated by feeding the hidden states into a fully connected neural network.







EXPERIMENTAL RESULTS OF MODELS FOR SINGLE-STEP PREDICTION (	(RMSE/MAE/MAPE).
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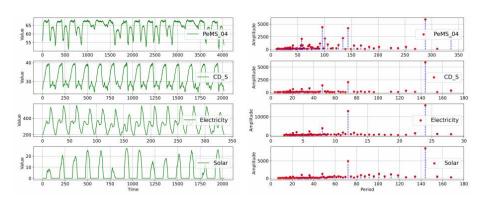
Model	PeMS_04	CD_S	Electricity	Solar
TGCN [6]	28.95/18.61/13.82	4.35/2.79/11.04	0.150/0.083/29.71	0.241/0.172/176.75
ASTGCN [8]	31.81/19.57/13.03	4.03/2.65/10.66	0.126/0.077/30.81	0.235/0.161/274.19
STGCN [27]	27.76/17.59/11.72	3.76/2.31/9.37	0.156/0.091/36.11	0.201/0.169/211.68
DCRNN [28]	28.26/17.97/11.95	3.99/2.43/9.72	0.131/0.086/31.08	0.291/0.196/402.03
AGCRN [10]	28.15/17.91/11.93	3.81/2.37/9.57	0.116/0.074/26.08	0.264/0.123/353.60
DGCN [13]	28.11/18.02/12.03	3.33/2.19/9.12	0.121/0.079/28.11	0.226/0.143/179.96
DGCRN [19]	27.85/17.68/11.79	3.11/2.10/8.77	0.129/0.082/29.21	0.209/0.155/152.32
VSTGN†	27.01/17.11/11.35	3.01/2.02/8.51	0.118/0.073/26.05	0.219/0.169/167.12
VSTGN	26.40/16.86/11.23	2.93/1.96/8.22	0.111/0.070/25.28	0.221/0.163/166.01

Datasets: Traffic, Electricity, and Solar

Our model outperformed baseline methods by an average of 5.36%.

The data demonstrates prominent periodic patterns, mainly at daily, half-daily, and one-third-daily intervals.





(a) Visualization of the four (b) DFT of the four datasets. datasets.

Fig. 2. Data visualization and spectral analysis.



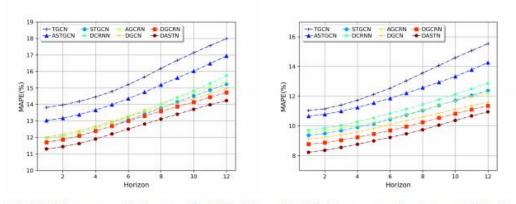


TABLE II MAPE OF THE ABLATION EXPERIMENT.

Model	VSTGN_DC	VSTGN_SE	VSTGN_TE	VSTGN
PeMS_04	11.49	11.67	11.95	11.23
CD_S	8.52	8.83	8.90	8.22
Electricity	26.07	28.92	28.29	25.28
Solar	173.81	174.12	174.01	166.01

TABLE III TRAINING TIME(S) OF MODELS.

Model	DCRNN	AGCRN	DGCN	DGCRN	VSTGN
CD_S	462.81	177.31	870.34	386.92	357.15
PeMS_04	37.67	17.17	76.81	33.91	27.01



(a) Multistep prediction on PeMS\_04. (b) Multistep prediction on CD\_S.

Fig. 3. Multistep prediction performance of VSTGN.

The ablation experiments validate the effectiveness of the GAU and the temporal-spatial embedding component of the SEL module.

VSTGN retains its superiority in multi-step prediction while avoiding substantial computational overhead.



#### **Conclusion**



- This study addresses the misalignment between static modeling approaches and the inherently dynamic nature of multivariate time series systems.
- We propose a novel variable spatiotemporal graph neural network framework that integrates dynamic adaptive node correlations with a spatiotemporal varying projection function.
- Experimental results demonstrate its superior performance in both single-step and multi-step predictions, significantly outperforming state-of-the-art baselines.









# Thank you

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