Dynamic Spatial Autoregressive Models with Time-varying Spatial Weighting Matrices

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Introduction: A brief literature review

Specification of the weighting matrix W within spatial autoregressive models

- In spatial econometrics [Ord, 1975], the weighting matrix W is a way to connect georeferenced random variables.
- **Exogenous** W. Several definitions: contiguity–based, *k*–neareast neighbors, distance-based criteria, etc.
- An *optimal* W is surely an unrealistic goal [Bavaud, 1998]: no obvious interpretation of the lag in a plane as in time [Whittle, 1954].
- *Are estimates and inferences sensitive to different* W *specifications?* School of thought ...
  1. Smoothing and Semiparametric approaches [McMillen, 2012]
  2. Robustness checks and Economic Theory [Corrado and Fingleton, 2012]
  3. Partial derivatives and summary measures [LeSage and Pace, 2014]
  4. Bayesian approach (prior knowledge)

Introduction: A brief literature review

Model Misspecification problems [Billé and Leorato, 2017]

(1) Estimation methods for potential **Endogenous** W [Qu and Lee [2015], Kelejian and Piras [2014]] (GMM, 2SLS, IV)

(2) Model selection [Seya et al., 2013]

(3) **Estimation of** W. Two main problems:

- W as a parameter $\Rightarrow \rho$ not identified.
- Even with panel data $T << N$, the estimation of $\frac{N\times(N-1)}{2}$ elements of W is intractable.

Current solutions:

- Matrix exponential spatial specification (MESS) [LeSage and Pace, 2007]
- Two-step residual regression estimator (spectral decomposition) [Bhattacharjee and Jensen-Butler, 2013]

Economic systems are essentially *dynamic by nature*, leading to potentially wrong conclusions even with economic W in a static spatial model.

Our motivation: Time–varying W_t matrices

(1) When $T << N$: **Spatial Dynamic Panel Data (SDPD) Models** [Lee and Yu [2012]; Wang and Yu [2015]]
(2) When $T >> N$: **Spatio-temporal model specifications** (exploit more the time information)

- LASSO approach [Otto and Steinert, 2018]
- Parameterizing W_t: $W(\gamma_t, \mathbf{d})$
  1. *Even if there are theoretical reasons indicating that spatial interaction effects are related to distance, it is often not clear from the theory the degree at which the spatial dependence diminishes as distance increases.* [Halleck Vega and Elhorst, 2015]
  2. Although geographical locations/regions are time–invariant, the strength of spatial dependence may also depend on economic variables that are time–varying.

(1) γ_t time–varying distance decay parameter $\Rightarrow$ managing the relative importance of geographical distance

(2) Score Driven (SD) models [[Harvey, 2013]; [Creal et al., 2013]] $\Rightarrow$ exploiting information contained in the score of the conditional distribution of the observables.

Model Specification

A Dynamic SAR model with Time–varying Spatial Weighting matrix (DSWM model)

Let $\mathbf{y}_t$ be a N–dimensional stochastic vector of spatial variables located on a possibly unevenly spaced lattice $Z \subseteq \Re^N$ at time t. We assume that $\mathbf{y}_t$ is generated according to a spatial autoregressive–regressive (SAR) model

$$\mathbf{y}_t = \rho \mathrm{W}\left(\gamma_t, \mathbf{d}\right)\mathbf{y}_t + \mathbf{X}_t\boldsymbol{\beta} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathcal{D}\left(\mathbf{0}, \boldsymbol{\Sigma}_t, \psi\right), \tag{1}$$

where ρ is a static (time–invariant) spatial autocorrelation coefficient, $\mathbf{X}_t = \left(\mathbf{x}_{j,t}; j = 1, \ldots, K\right)$ is a $N \times K$ matrix of exogenous covariates with associated static vector of coefficients $\boldsymbol{\beta} = \left(\beta_j; j = 1, \ldots, K\right)'$, $\boldsymbol{\varepsilon}_t$ is the N–stochastic vector of innovations at each time with diagonal covariance matrix $\Sigma_t = \mathrm{diag}\left(\sigma^2_{i,t}; i = 1, \ldots, N\right)'$, ψ is the shape parameter (Normal and Student's t distributions)

Note: *Cross-sectional heteroskedasticity* if $\sigma^2_i \neq \sigma^2_j$ for at least one $i \neq j$, $i, j = 1, \ldots, N$. *Time-varying shock volatility* through the score driven methodology.

Model Specification

Definition (2.1)

$\mathrm{W}(\gamma_t, \mathbf{d}) = \mathrm{W}_t = \{\omega_{ij,t}\}_{i,j=1}^{N}$.
(a) $\omega_{ij,t} = \omega_{ji,t}, \omega_{ij,t} > 0$, $\omega_{ii,t} = 0 \quad \forall i,j = 1,\ldots,N$ and $\forall t$,
(b) $\mathbf{d} = (d_{ij}; i,j = 1,\ldots,N)$, $d_{ii} = 0, \quad \forall i = 1,\ldots,N$ and $d_{ij} = d_{ji}$
$d_{ij} > 0$ for all $i,j = 1,\ldots,N, \quad i \neq j$,
(c) $\gamma_t > 0$.

Def (2.1)(a): W_t is a sequence of symmetric $N \times N$ matrices of real positive entries with zero on the main diagonal. **Def (2.1)(b)**: $\mathbf{d}$ is a $N \times N$ matrix of strictly exogenous non–stochastic variables such that $\mathbf{d} \in \Re_{+}^{N \times N}$.

Assumption (1)

The rows and the columns of $\mathrm{W}_t, \forall t$ *are uniformly bounded as N goes to infinity.*

Different W_t parameterizations: the function $f(.,.)$

$\mathcal{F}_t = \sigma(\mathbf{y}_{t-s}, \mathbf{X}_{t-s+1}, s \geq 0)$ is the filtration generated by the process of $\mathbf{y}_t$ defined in equation (1).

Definition (2.2)

$\omega_{ij,t} = f(\gamma_t, d_{ij})$, where $f : \Re_+ \to \Re_+$ is an $\mathcal{F}_t$–measurable differentiable distance decay function among the spatial units i and j:
(a) $f(\gamma_t, d_{ij}) > 0$ for $i \neq j$,
(b) $f(\gamma_t, d_{ij}) = 0$ for $i = j$,
(c) $f(\gamma_t, d_{ij}) < f(\gamma_t, d_{hk}) \iff d_{ij} > d_{hk}$,
(d) $\lim_{d_{ij}\to\infty} f(\gamma_t, d_{ij}) = 0$, $\lim_{d_{ij}\to 0} f(\gamma_t, d_{ij}) = 1$,
$\lim_{\gamma_t\to\infty} f(\gamma_t, d_{ij}) = 0$, $\lim_{\gamma_t\to 0} f(\gamma_t, d_{ij}) = 1$.

Any **continuous monotonically decreasing function** $\to$ lower weights as distances increase. In our paper we consider the following $f(.,.)$:

(1) *inverse distance*, $f(\gamma_t, d_{ij}) = \frac{1}{d_{ij}^{\gamma_t}}$

(2) *negative exponential*, $f(\gamma_t, d_{ij}) = \exp(-\gamma_t d_{ij})$

Normalization rules: the function $g(.)$

In order to ensure *stable spatio–temporal processes* ...

Lemma (2.1)

Let $\tau(\mathrm{W}_t)$ denote the **time–varying spectral radius** *of the square N–dimensional W_t matrix at time t:*
$\tau(\mathrm{W}_t) = max\{|e_{1t}|, ..., |e_{Nt}|\}$, where $e_{1t}, ..., e_{Nt}$ are the eigenvalues of W_t at time t. Then, $(\mathbb{I}_N - \rho \mathrm{W}_t)^{-1}$ is non–singular for all the values of ρ in the interval $(-1/\tau(\mathrm{W}_t), 1/\tau(\mathrm{W}_t))$ at each time t.

Assumption (2)

$\rho \in \left(-\frac{1}{\tau(\mathrm{W}_t)}, \frac{1}{\tau(\mathrm{W}_t)}\right)_{\setminus\{0\}}, \forall t.$

Two important issues ...

(1) The **parameter space of** ρ in model (1), with no further assumptions, is not fixed, i.e. $\rho \in \left(-\frac{1}{\tau(\mathrm{W}_t)}, \frac{1}{\tau(\mathrm{W}_t)}\right)_{\setminus\{0\}}$

(2) The spectral–normalization rule [Kelejian and Prucha, 2010] is not sufficient in our case since the autoregressive coefficient of the equivalent normalized model will be $\rho^* = \rho \times \tau(\mathrm{W}_t)$

Normalization rules: the function $g(.)$

Definition (2.3)

$g(\mathrm{W}_t) = \mathrm{W}_t^*$, where $g(\cdot)$ is a $\mathcal{F}_t$–measurable differentiable normalizing function such that $\tau(\mathrm{W}_t^*) = 1, \ \forall t$.

Then, we finally consider the following model

$$\mathbf{y}_t = \rho^* \mathrm{W}_t^* \mathbf{y}_t + \mathbf{X}_t \beta + \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} \mathcal{D}(\mathbf{0}, \mathbf{\Sigma}_t, \psi), \tag{2}$$

where $\mathrm{W}_t^* = \{\omega_{ij,t}^*\}_{i,j=1}^N$ and $\omega_{ij,t}^* = g_{ij}(\gamma_t, \mathrm{W}_t)$, with $\tau(\mathrm{W}_t^*) = 1$. The inverse matrix $(\mathbb{I} - \rho^* \mathrm{W}_t^*)^{-1}$ exists for all values of ρ^* in the interval $(-1, 1)_{\setminus\{0\}}$. Then, the reduced form of model (2) is

$$\mathbf{y}_t = \mathbf{A}_t^{-1} \mathbf{X}_t \beta + \mathbf{A}_t^{-1} \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} \mathcal{D}(\mathbf{0}, \mathbf{\Sigma}_t, \psi) \tag{3}$$

and the conditional distributions of $\mathbf{y}_t$ and ε_t are

$$\mathbf{y}_t | \mathcal{F}_{t-1} \sim \mathcal{D}\left(\mathbf{y}_t; \mathbf{A}_t^{-1} \mathbf{X}_t \beta, \mathbf{A}_t^{-1} \mathbf{\Sigma}_t \mathbf{A}_t^{-1\prime}, \psi\right) \tag{4}$$

and

$$\varepsilon_t | \mathcal{F}_{t-1} \sim \mathcal{D}(\mathbf{0}; \mathbf{\Sigma}_t, \psi), \tag{5}$$

respectively.

Filtering procedure for time–varying parameters

γ_t is updated through a filter based on the *scaled score* of the conditional density in (4). Let $\gamma_t = \exp(\tilde{\gamma}_t)$. The filter is

$$\tilde{\gamma}_{t+1} = (1 - \xi)\,\kappa + \alpha \tilde{s}_t + \xi \tilde{\gamma}_t, \tag{6}$$

where the condition $|\xi| < 1$ is imposed to ensure stationarity of the process and

$$\tilde{s}_t = \tilde{\mathcal{I}}_t\left(\tilde{\gamma}_t, \boldsymbol{\eta}, \mathbf{X}_t\right)^{-1/2} \tilde{\nabla}_t\left(\mathbf{y}_t, \tilde{\gamma}_t, \boldsymbol{\eta}, \mathbf{X}_t\right), \tag{7}$$

where $\boldsymbol{\eta} = \left(\rho^*, \mathrm{diag}\left(\boldsymbol{\Sigma}\right)', \boldsymbol{\beta}'\right)'$. *Time–varying shock volatilities*: given the conditional density in (5) we introduce time–variation in the $\sigma^2_{i,t}$ by letting $\sigma^2_{i,t} = \exp(\tilde{\sigma}^2_{i,t})$ and

$$\tilde{\sigma}^2_{i,t+1} = (1 - \xi_\sigma)\kappa_{\sigma,i} + \alpha_\sigma s_{i,t} + \xi_\sigma \tilde{\sigma}^2_{i,t}, \tag{8}$$

where $s_{i,t}$ is the scaled score of the conditional distribution $\mathbf{y}_t|\mathcal{F}_{t-1}$ evaluated with respect to $\tilde{\sigma}^2_{i,t}$.

Filtering procedure for time–varying parameters

Given the conditional distributions in equations (4) and (5), the log–likelihood contribution (Normal case) of $\mathbf{y}_t$ is proportional to

$$\log p\left(\mathbf{y}_t|\gamma_t,\boldsymbol{\eta},\mathbf{X}_t\right) \propto -\frac{1}{2}\log|\mathbf{\Sigma}_t| + \log|\mathbf{A}_t| - (1/2)\,\boldsymbol{\nu}_t'\boldsymbol{\nu}_t, \qquad (9)$$

where

$$\boldsymbol{\nu}_t'\boldsymbol{\nu}_t = \left(\mathbf{A}_t\mathbf{y}_t - \mathbf{X}_t\beta\right)'\mathbf{\Sigma}_t^{-1}\left(\mathbf{A}_t\mathbf{y}_t - \mathbf{X}_t\beta\right) \qquad (10)$$

Given a spatial sample of time dimension T, model parameters can be easily estimated through Maximum Likelihood (ML), i.e.

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}\in\mathbf{\Theta}} \sum_{t=1}^{T} \log p\left(\mathbf{y}_t|\boldsymbol{\theta},\mathbf{X}_t\right), \qquad (11)$$

where $\boldsymbol{\theta} = \left(\boldsymbol{\eta}',\kappa,\alpha,\xi,\kappa_{\sigma,i},\alpha_\sigma,\xi_\sigma\right)'$ are time–invariant parameters to be estimated, and $\mathbf{\Theta}$ is a compact set where model parameters take values [[Harvey, 2013]; [Blasques et al., 2014]].

The role of γ_t

The goal: consistently estimate γ_t

- $\gamma_t \in \Re_+$ is the *time–varying distance decay parameter*, measuring the relative importance of higher–order neighbourhoods in space and over time t.
- Interactions may continue even over a first or second–order neighbourhood, suggesting that a *sparse* matrix might be non appropriate most of the times.
- Useful to selecting appropriate weighting schemes and knowing the optimal level at which the correlation rapidly decreases over time, without any a priori knowledge of the spatial process $\Rightarrow$ avoinding **model misspecification** due to prespecified W_t.
- Interpretation: $\gamma_t \rightarrow 0$, the higher the role played by the spatial units at greater distances, i.e. *higher–order neighbourhoods* $\Rightarrow$ *dense* matrices are maybe more appropriate.

Dynamic indicators of spatial association ϖ_t

Problem: the domain of γ_t is unbounded

Dynamic indicator

$$\varpi_t = 1 - \frac{\lambda(R_t(\infty))}{\lambda(R_t(\gamma_t))} \tag{12}$$

such that $\varpi_t \in (0, 1)$, and where $R_t(\infty)$ (no spatial association) and $\lambda(.)$ (highest eigenvalue). When γ_t diverges, the spatial units become uncorrelated and $\lambda(R_t(\infty)) = 1$.

- $\gamma_t \to 0 \Rightarrow \varpi_t \to 1$, stronger connections among the spatial units (dense W_t more appropriate),
- $\gamma_t \to \infty \Rightarrow \varpi_t \to 0$, viceversa.

Monte Carlo simulations: FSP of MLE

(1) **Finite sample properties of MLE**

- Number of series $B = 1000$ of pseudo observations from the true model (eq. (4) and (6))
- No time–varying shock volatilities
- $\rho^* = \{-0.7, -0.3, 0.3, 0.7\}$, $N = \{50, 100\}$, $\sigma = 1.0$ (cross-sectional homoskedasticity)
- $\alpha = 0.02$, $\xi = 0.97$, $\kappa = \log 2 \approx 0.6931$ for the γ_t recursion

Note: The true value of the $\kappa = 0.6931$ parameter implies an unconditional level of the γ_t dynamic of 2, i.e. $\mathbb{E}[\gamma_t] = 2$ for all the considered specifications.

Monte Carlo simulations: FSP of MLE

$N = 100$	T	DGP1					DGP2				
		ρ =-0.7	α =0.02	β =0.97	κ =0.6931	σ =1	ρ =-0.3	α =0.02	β =0.97	κ =0.6931	σ =1
Median	500	-0.6998	0.0201	0.9671	0.7024	1.0000	-0.2998	0.0193	0.9567	0.7008	0.9999
	1000	-0.7000	0.0200	0.9672	0.7113	1.0000	-0.3003	0.0203	0.9652	0.6913	1.0000
	2000	-0.6999	0.0200	0.9686	0.7013	0.9999	-0.2997	0.0200	0.9676	0.6948	1.0000
SD	500	0.0028	0.0011	0.0129	0.1241	0.0049	0.0059	0.0095	0.0809	0.0905	0.0040
	1000	0.0019	0.0008	0.0090	0.1040	0.0033	0.0039	0.0058	0.0496	0.0649	0.0031
	2000	0.0014	0.0005	0.0058	0.0825	0.0021	0.0034	0.0047	0.0368	0.0772	0.0023
MSE	500	8.0E-06	1.2E-06	0.0001	0.0154	2.4E-05	3.4E-05	9.0E-05	0.0076	0.0082	1.6E-05
	1000	3.8E-06	6.4E-07	9.8E-05	0.0110	1.1E-05	1.5E-05	3.4E-05	0.0028	0.0042	9.7E-06
	2000	2.1E-06	3.0E-07	3.7E-05	0.0068	4.4E-06	1.1E-05	2.2E-05	0.0014	0.0060	5.5E-06
		DGP3					DGP4				
		ρ =0.3	α =0.02	β =0.97	κ =0.6931	σ =1	ρ =0.7	α =0.02	β =0.97	κ =0.6931	σ =1
Median	500	0.3004	0.0194	0.9561	0.6879	1.0000	0.6999	0.0199	0.9667	0.7011	0.9999
	1000	0.3002	0.0202	0.9638	0.7000	0.9999	0.6999	0.0199	0.9680	0.7016	1.0000
	2000	0.3000	0.0199	0.9663	0.6960	1.0000	0.6999	0.0200	0.9689	0.6952	1.0000
SD	500	0.0056	0.0081	0.0953	0.0863	0.0044	0.0026	0.0011	0.0136	0.1200	0.0043
	1000	0.0040	0.0065	0.0455	0.0745	0.0032	0.0020	0.0008	0.0089	0.1021	0.0030
	2000	0.0029	0.0044	0.0456	0.0551	0.0021	0.0014	0.0005	0.0062	0.0826	0.0019
MSE	500	3.2E-05	6.6E-05	0.0105	0.0074	2.0E-05	7.2E-06	1.3E-06	0.0002	0.0144	1.8E-05
	1000	1.6E-05	4.2E-05	0.0024	0.0056	1.0E-05	4.3E-06	6.5E-07	9.0E-05	0.0104	9.5E-06
	2000	8.8E-06	2.0E-05	0.0022	0.0030	4.5E-06	2.1E-06	3.2E-07	4.4E-05	0.0068	3.7E-06

Monte Carlo simulations: FSP of MLE

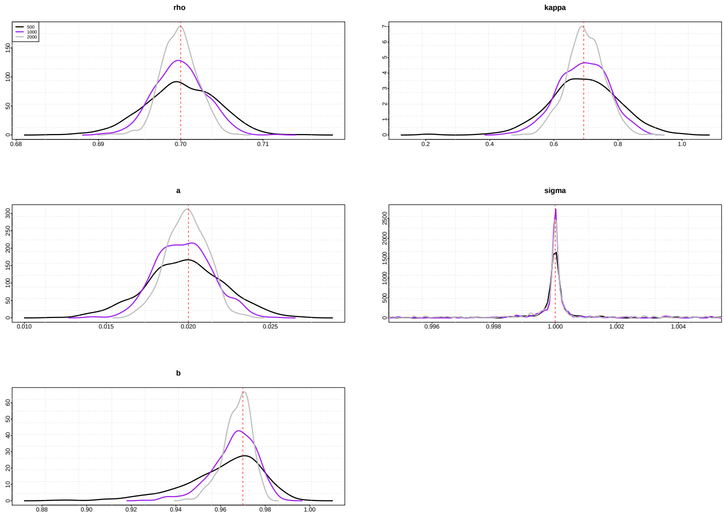


Figure: Finite sample properties with a spatial dimension N=50, $T = \{500, 1000, 2000\}$ and a positive autocorrelation coefficient ρ^*=0.7.

Monte Carlo simulations: Filtering Properties

(2) **Flexibility of the score updating mechanism assumed for the γ_t recursion**

We assume the following artificial deterministic and stochastic patterns

- Constant: $\gamma_t = 2$
- Sine: $\gamma_t = 3 + 2\sin(2\pi t/200)$
- Fast Sine: $\gamma_t = 3 + 2\sin(2\pi t/20)$
- Step: $\gamma_t = 4 - 3\,(t > 500)$
- Ramp: $\gamma_t = \mod(t/400)/100 + 1$
- AR: $\gamma_t = \exp(\tilde{\gamma}_t)$, $\tilde{\gamma}_t = 0.015 + 0.98\tilde{\gamma}_{t-1} + 0.1\eta_t, \eta_t \sim \mathcal{N}(0,1)$.

We simulate $B = 1000$ series of length $T = 1000$ and $N = 25$, whereas $\sigma = 1$ and $\rho^* = 0.7$ are fixed and are not estimated.

Monte Carlo simulations: Filtering Properties

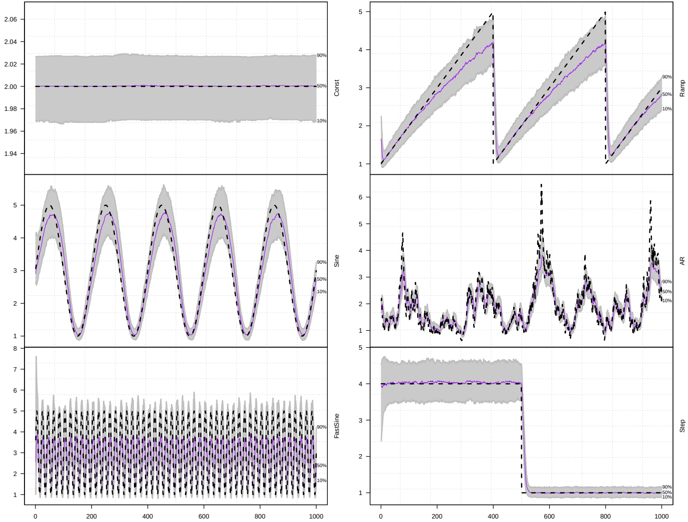


Figure: Filtering properties of the model.

(2) House price dynamics in the UK

Recent literature on house prices [Shi and Lee [2017]; Billé et al. [2017]; Bailey et al. [2016]; Holly et al. [2010]]

(1) Quality adjusted (to price variations) regional house price series from Nationwide Building Society website [Holly et al., 2011].

(2) Period from Q1 1974 to Q2 2018 (178 quarterly observations).

(3) Starting from NUTS data from the UK Office of National Statistics website, Nationwide regions are obtained by aggregating different NUTS regions.

(4) Explanatory Variables: population level, unemployment index, etc. [Brady, 2011].

(5) **d**: geographical distances based on Euclidean distances between centroids of the regions.

Aim: *identifying periods of time in which the evaluated time–series are more inter–connected, suggesting a common behaviour among them*

(2) House price dynamics in the UK

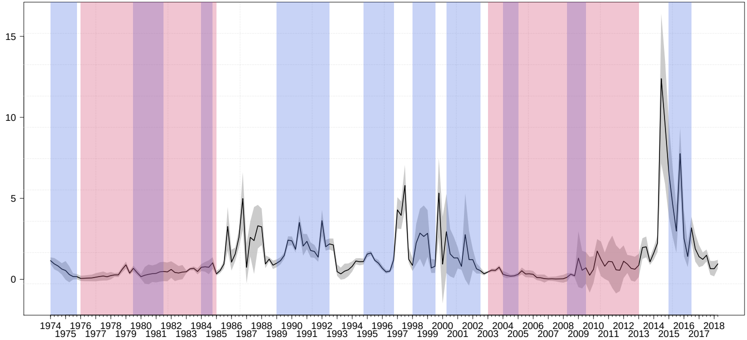


Figure: Filtered γ_t for the **Student's t model** for UK house price dynamics and $(10\% - 90\%)$ in–sample simulation–based confidence bands (gray bands). The blue shaded bars indicate periods of UK recession according to the OECD based Recession Indicators for the United Kingdom from the Peak through the Trough (GBRRECDM), see Federal Reserve Bank of St. Louis [2018].

(2) House price dynamics in the UK

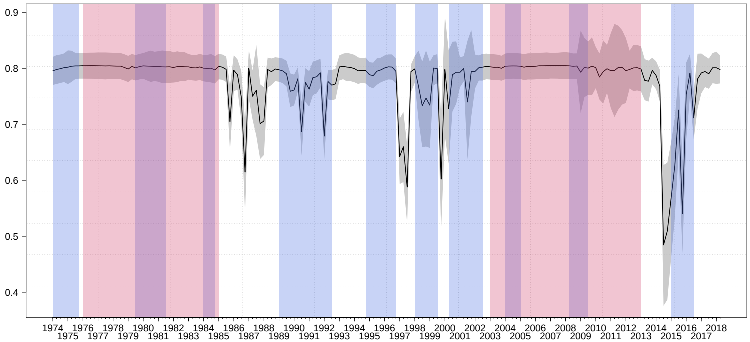
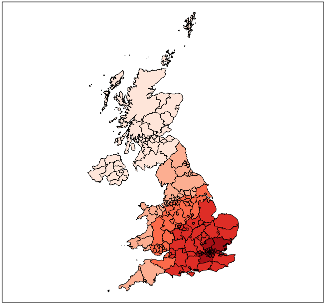
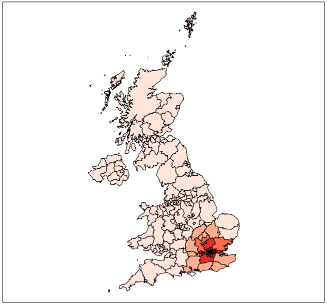


Figure: The evolution of the indicator of spatial association as in Equation (12) for house price dynamics and (10% − 90%) in–sample simulation–based confidence bands (gray bands). The blue shaded bars indicate periods of UK recession according to the OECD based Recession Indicators for the United Kingdom from the Peak through the Trough (GBRRECDM), see Federal Reserve Bank of St. Louis [2018].

(2) House price dynamics in the UK



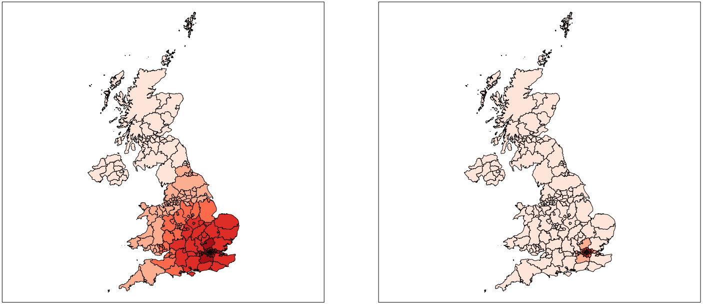
(a) Period 1976-1985



(b) Period 1986-2003

Figure: Higher–order effects of W_t with respect to London for 4 different periods of time. The intra–periods average values of γ_t are: (a) $\gamma_1 = \mathbf{0.415}$, (b) $\gamma_2 = \mathbf{1.734}$.

(2) House price dynamics in the UK



(a) Period 2004-2012 (b) Period 2013-2016

Figure: Higher–order effects of W_t with respect to London for 4 different periods of time. The intra–periods average values of γ_t are: (a) $\gamma_3 = \mathbf{0.492}$, (b) $\gamma_4 = \mathbf{4.054}$.

Conclusions

- A Dynamic Spatial Weighting Matrix (DSWM) model.
- Parametrization of W_t: any continuous monotonically decreasing function (distance decay functions).
- Feasible even with large N.
- Score Driven models, exploiting information from the conditional distributions.
- Finite sample as well as asymptotic properties of the ML estimator. Filtering properties. Filter invertibility.
- UK house prices:
  (1) Periods with substantially different spatial processes (different weighting schemes).
  (2) The magnitude of London's dominance over the other regions as described by Holly et al. [2011] is remarkably different across time.

Future Developments

(1) Including lagged exogenous regressors (i.e. W$\mathbf{X}$), leading to a time–varying spatial Durbin specification of the model, or spatially autocorrelated shocks (i.e. Wε_t), leading to a time–varying spatial autoregressive model with autoregressive (and eventually heteroskedastic) disturbances, can be addressed at the cost of additional calculation starting from our specification.

(2) **Forecasting** spatial-temporal structures

Thank you for your attention!

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