LatticeKrig: A multi-resolution spatial model for large data.

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National Science Foundation   KAUST, March 2014

Credits

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- Dorit Hammerling, SAMSI/STATMOS/NCAR
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- Finn Lindgren, Bath, Norway

Outline

A flexible spatial estimate for large problems.

- Regional Climate simulation and NARCCAP
- Compact basis functions (b),

  Markov Random fields (H)
- Changes in the seasonality for future climate

Key idea: Introduce sparse basis and precision matrices without compromising the spatial model.

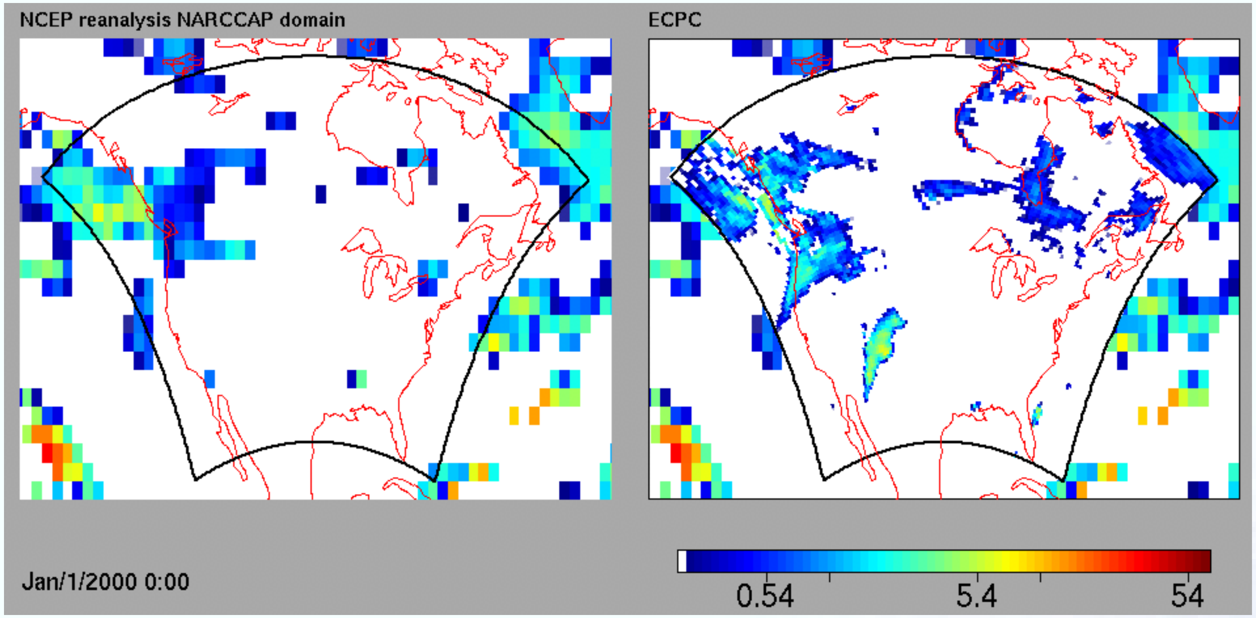
Does not require tapering covariance functions – supports dependence over large distances.

A climate model grid box (?)

An approach to Regional Climate

- Nest a fine-scale weather model in part of a global model's domain.

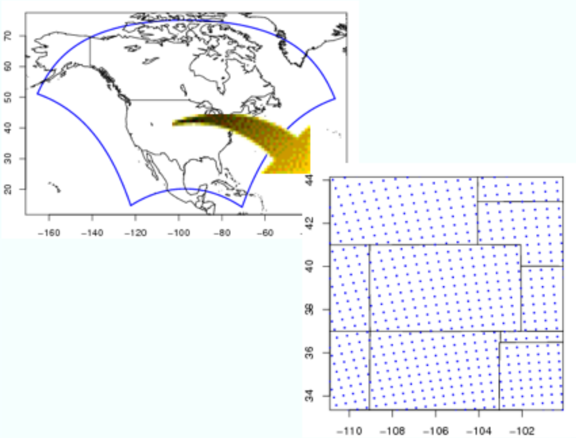
Regional model simulates higher resolution weather based on the global model for boundary values and fluxes.



A snapshot from the 3-dimensional RSM3 model (right) forced by global observations (left)

- Consider different combinations of global and regional models to characterize model uncertainty.

NARCCAP – the design



4GCMS × 6RCMs:

12 runs – balanced half fraction design

Global observations × 6RCMs

X High resolution global atmosphere

GLOBAL MODEL	REGIONAL MODELS					
	MM5I	WRF	HADRM	REGCM	RSM	CRCM
GFDL			●	●	○	
HADCM3	○		●		●	
CCSM	●	■				■
CGCM3		■		●		■
Reanalysis	●	●	●	●	●	●

A designed experiment is amenable to a statistical analysis and can contain more information.

But just 2-d temperatures fields are 72Gb of data.

Climate change

How will the seasonal cycle for temperature change in the future?

Estimating a curve or surface.

An additive statistical model:

Given n pairs of observations (x_i, y_i), $i = 1, \ldots, n$

$$y_i = g(x_i) + \epsilon_i$$

ϵ_i's are random errors and g is an unknown, smooth realization of a Gaussian process.

Random Effects/Linear model for g

$\{b_j\}$: m basis functions

$$g(x) = \sum_j b_j(\boldsymbol{x})\boldsymbol{\beta}_j$$

A linear model:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

Random effects:

$$\boldsymbol{\beta} \sim MN(0, \rho\boldsymbol{Q}^{-1}) \text{ and } \boldsymbol{\epsilon} \sim MN(0, \sigma^2\boldsymbol{I})$$

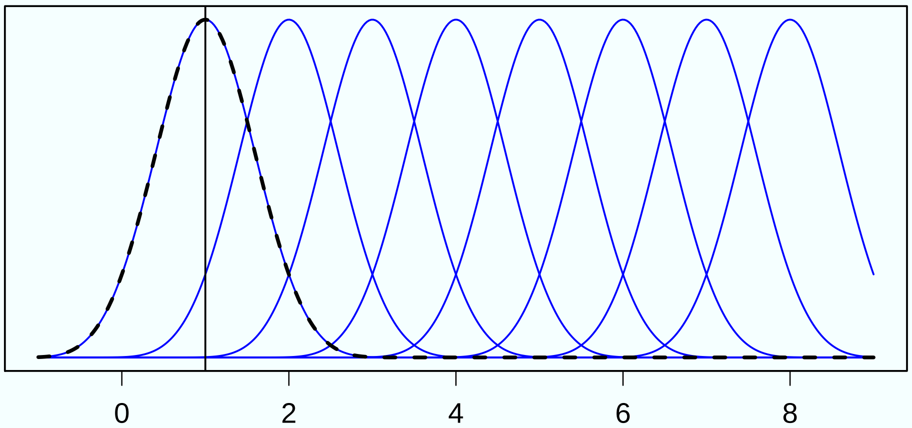
Implied Covariance:

$$E[g(\boldsymbol{x})g(\boldsymbol{x}')] = \sum_{j,k} b_j(\boldsymbol{x})\rho\boldsymbol{Q}^{-1}_{j,k}b_k(\boldsymbol{x}')$$

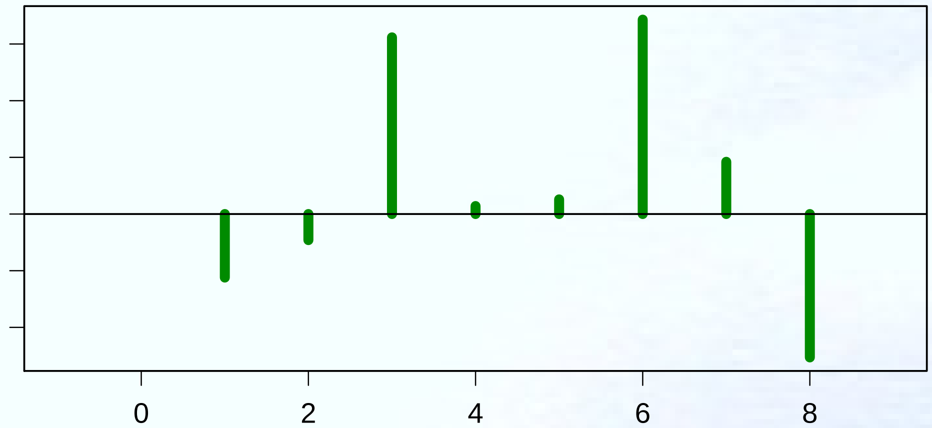
$\lambda = \sigma^2/\rho$ plays an important role as a parameter.

An example of a 1-d basis ...

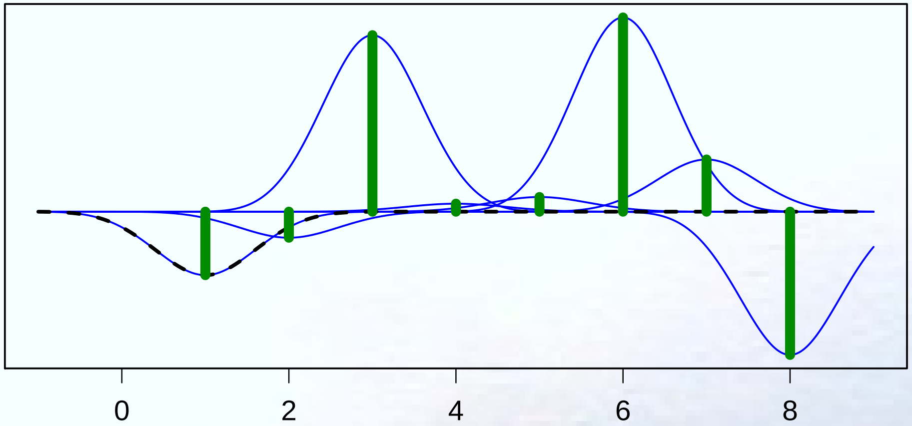
8 basis functions



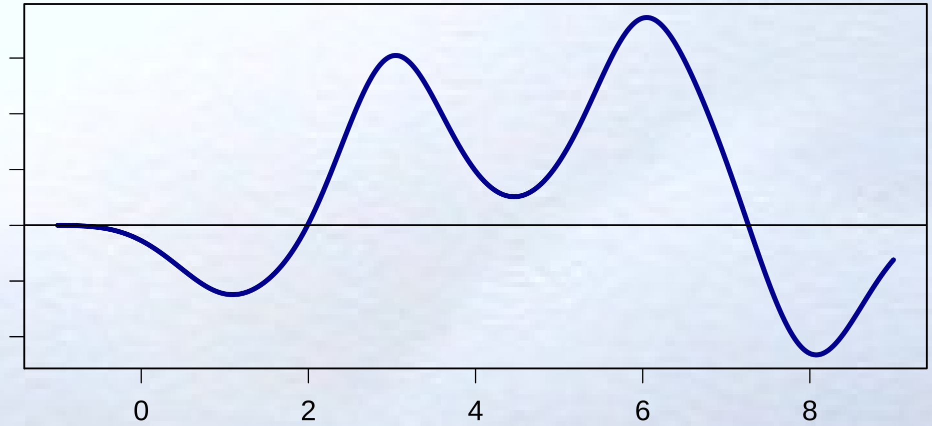
8 (random) weights



weighted basis

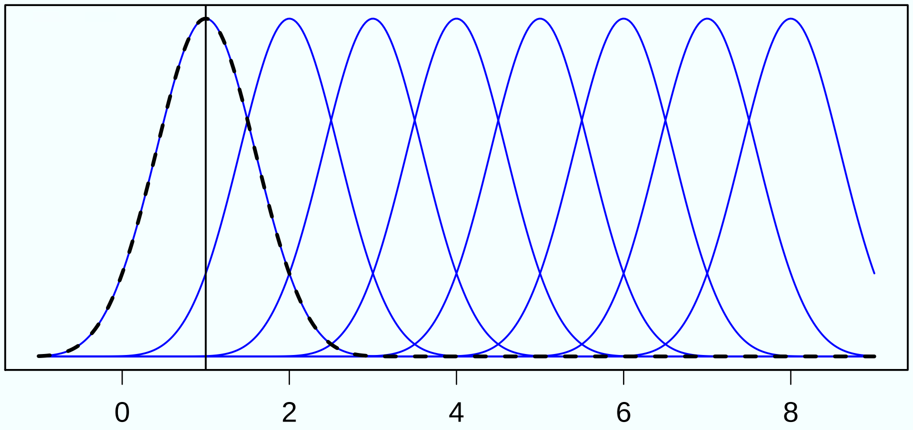


Random curve

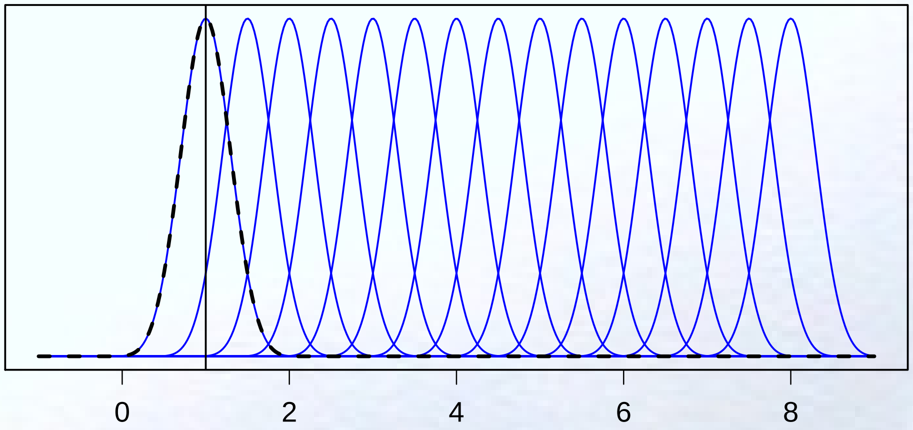


A Multiresolution

8 basis functions

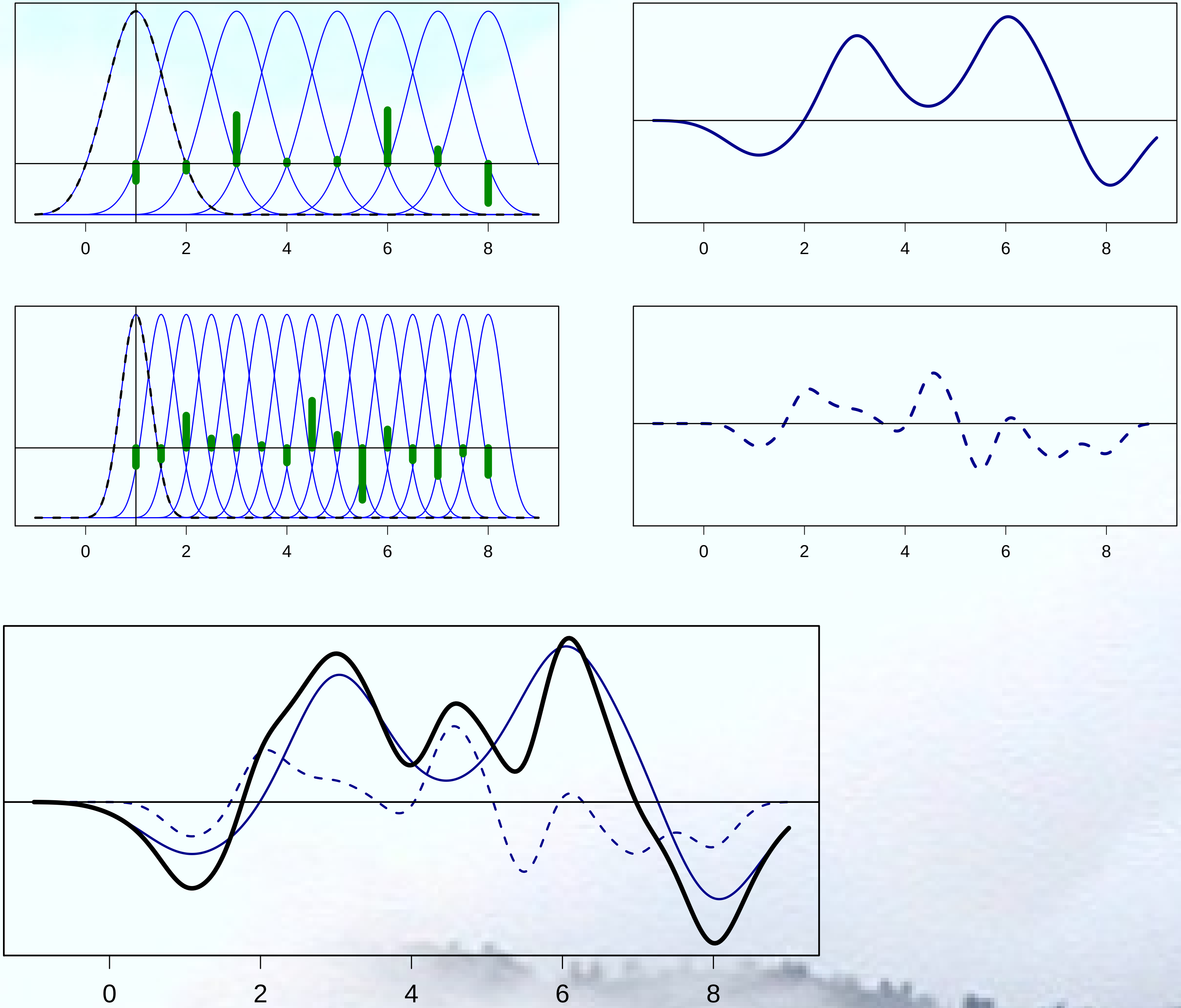


16 basis functions



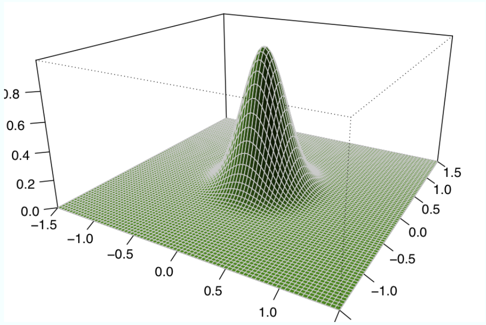
⋮

Adding them up



A recipe for 2-d RBFs

Basis function$_j(\boldsymbol{x}) = \varphi(||\boldsymbol{x} - \boldsymbol{u}_j||/\theta)$



2-d Wendland

- φ is a positive definite, compactly supported function – a nice bump.
- $\{\boldsymbol{u}_j\}$ basis centers on a regular grid
- θ scale set to provide some overlap

Four level multi-resolution starting with 11×11 *grid has 8804 basis functions.*

A recipe for Q

Recall: $g(\boldsymbol{x}) = \sum_j b_j(\boldsymbol{x})\boldsymbol{\beta}_j$

β at each resolution level is a Markov random field:

$$\alpha\boldsymbol{\beta}_j - \sum_{\text{neighbors}} \boldsymbol{\beta}_l = e_j \quad \text{i.e. } H\boldsymbol{\beta} = \boldsymbol{e}$$

- $\{e_j\}$ are uncorrelated N(0,1)
- H has only 5 nonzero elements in each row.

Weights in lattice format:

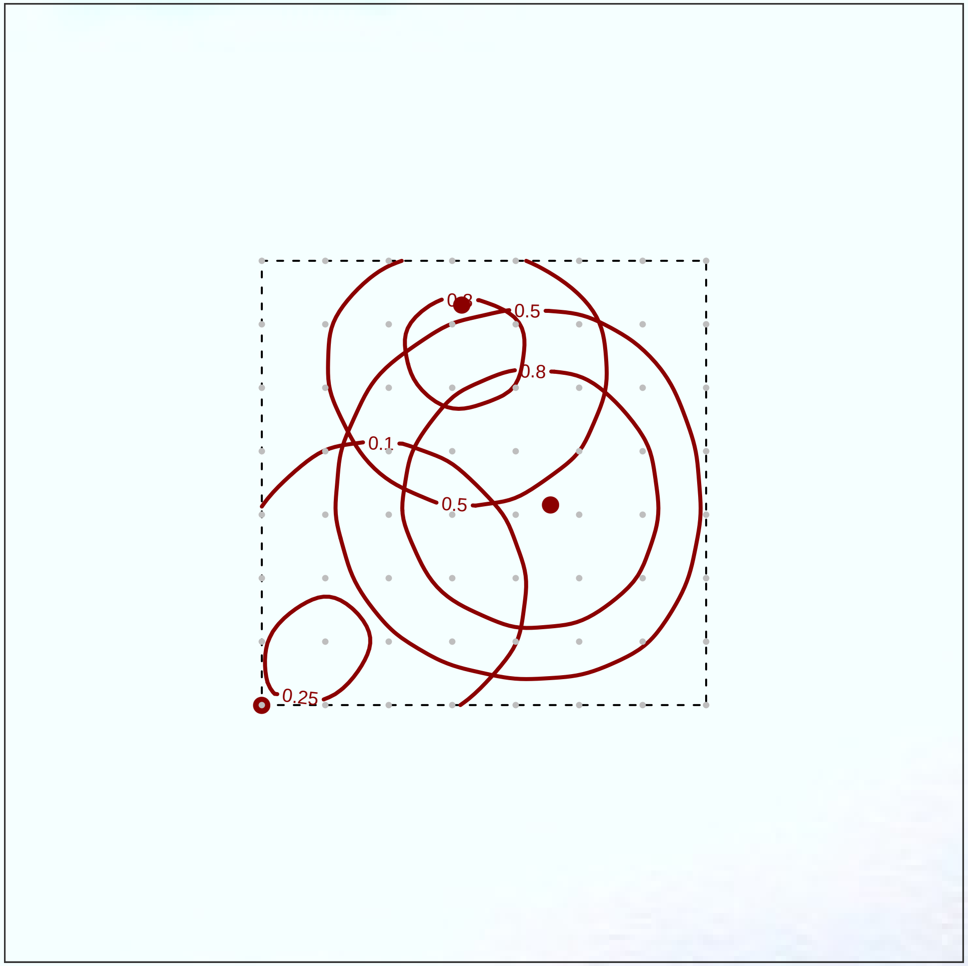
$$\begin{array}{ccccc} . & . & . & . & . \\ . & . & -1 & . & . \\ . & -1 & \alpha & -1 & . \\ . & . & -1 & . & . \\ . & . & . & . & . \end{array}$$

Precision matrix for β is sparse: $\boldsymbol{Q} = (\boldsymbol{H}^T\boldsymbol{H})$

Assumed covariance model

$8\times$ 8 grid with no grid points of buffer

Covariance function for a single level $\alpha = 4.2$

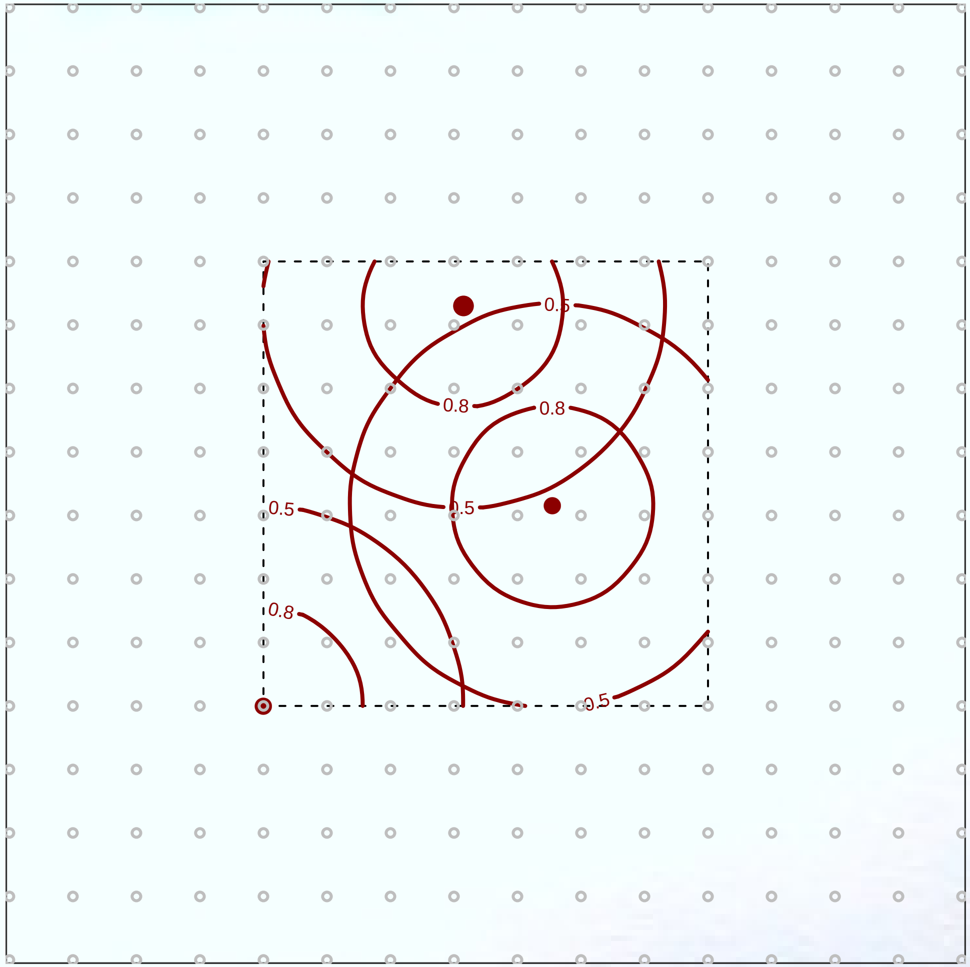


Stationary? This is really poor!

Fixing this problem

8× 8 grid with 5 grid points of buffer

Correlation function for a single level $\alpha = 4.2$

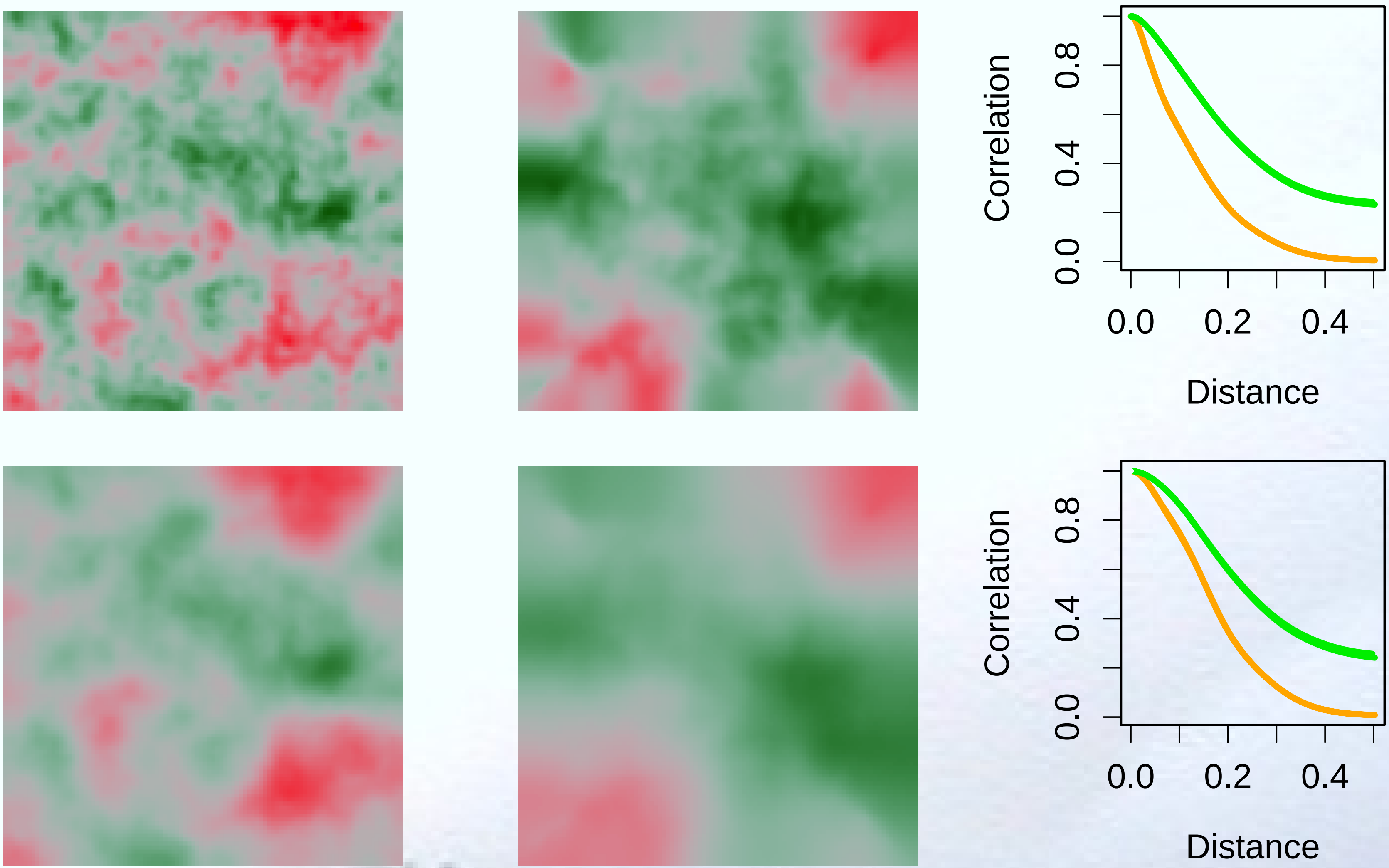


Close to being stationary!

What does the model do?

Combination of 4 levels starting with an 8×8 grid

Uncorrelated weights Correlated weights Correlation functions



Computation

Ridge regression/ conditional expectation/ BLUE/

Posterior conditioned on covariance.

$$\hat{g}(x) = E[g(x)|\boldsymbol{y}, \boldsymbol{P}] = \sum_{k=1,n} \hat{\boldsymbol{\beta}}_k b_k(\boldsymbol{x})$$

$$\hat{\boldsymbol{\beta}} = \left(X^T X + \lambda \boldsymbol{Q}\right)^{-1} X^T \boldsymbol{y}, \quad \lambda = \sigma^2/\rho$$

X^T, $X^T X$, $\boldsymbol{Q}$ are sparse.

Computation continued

Likelihood:

Dominated by finding $log(det(X^T X + \lambda \boldsymbol{Q}))$ take advantage of sparsity.

Conditional simulation for standard errors:
Simulation of fields and the spatial prediction also take advantage of sparsity.

At 20,000 observations:

- Ordinary Kriging about 21 minutes
- LatticeKrig models 5 - 20 seconds.

Climate change

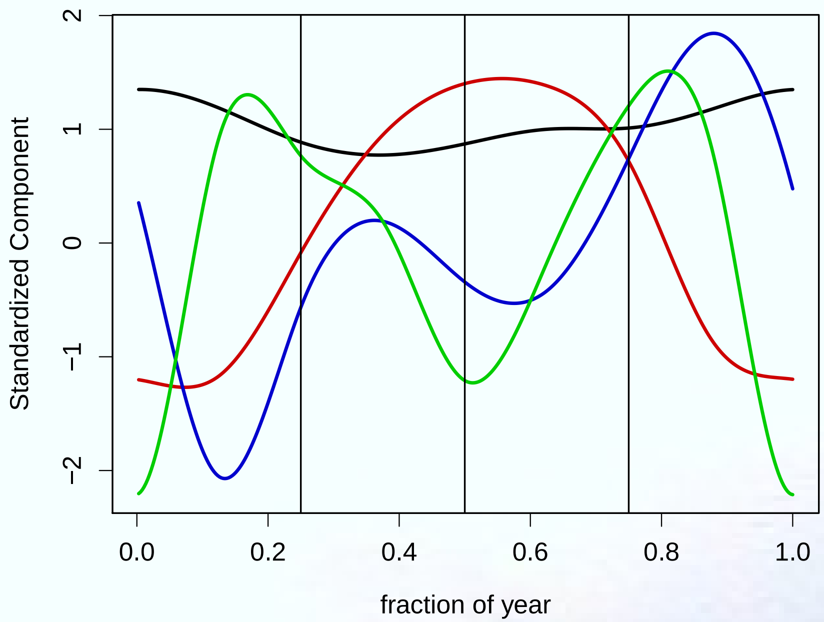
How will the seasonal cycle for temperature change in the future?

Back to NARCCAP

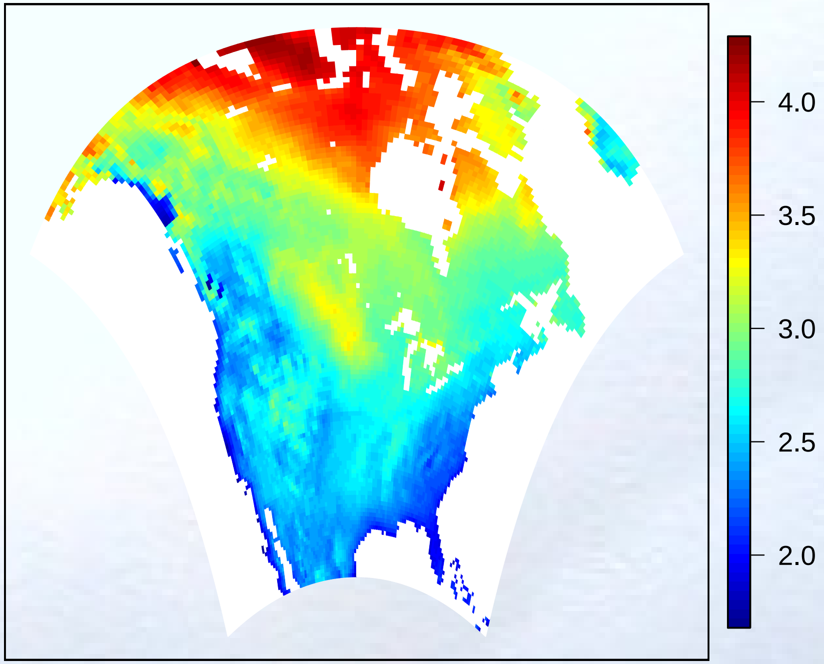
Back to NARCCAP

- A 2×2 subset of NARCCAP (4 global/regional combinations)
- (Future - Present) seasonal cycle expand in 4 principle components ... gives 4 coefficient spatial fields for each model.
- Approximately 8000 spatial locations

Seasonal PCs
(future - present)



First coefficient
CRCM ccsm

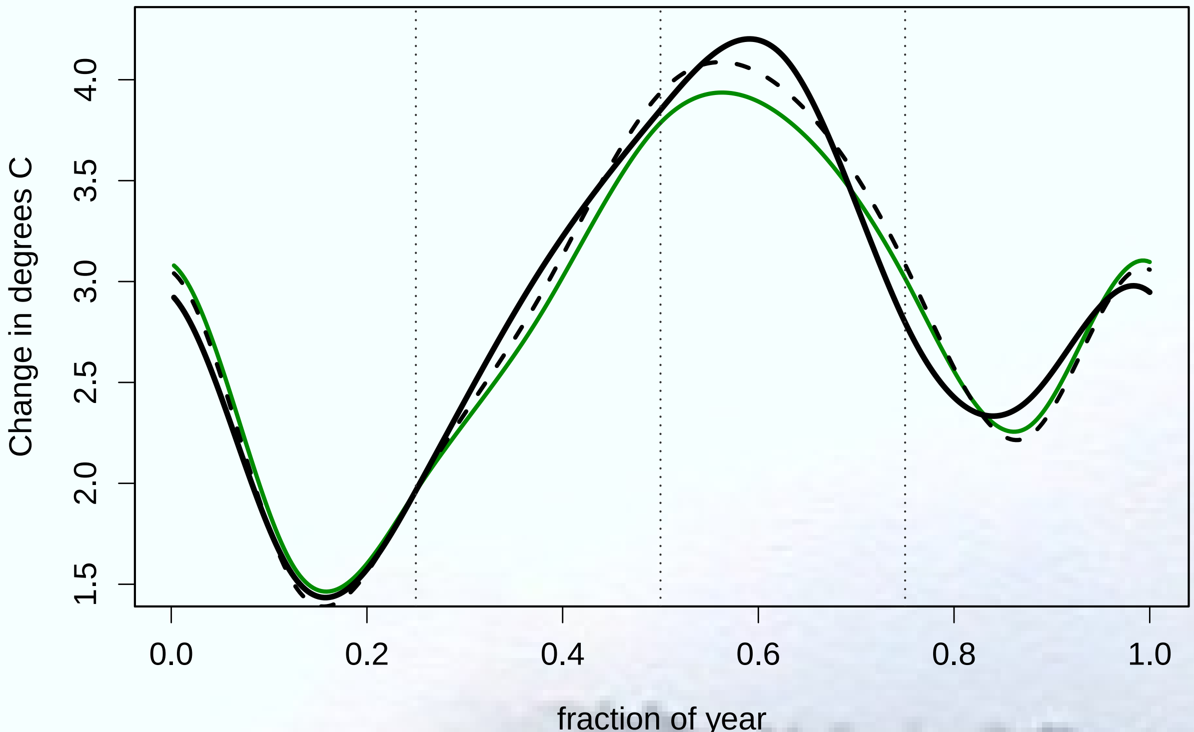


Spatial model

- *Four coefficients of seasonal profile for the four model combinations – and at each grid box ($16 = 4 \times 4$ fields each with 9K locations.)*

- *"measurement error": the interannual variation of models*

Boulder grid box, CRCM/ccsm

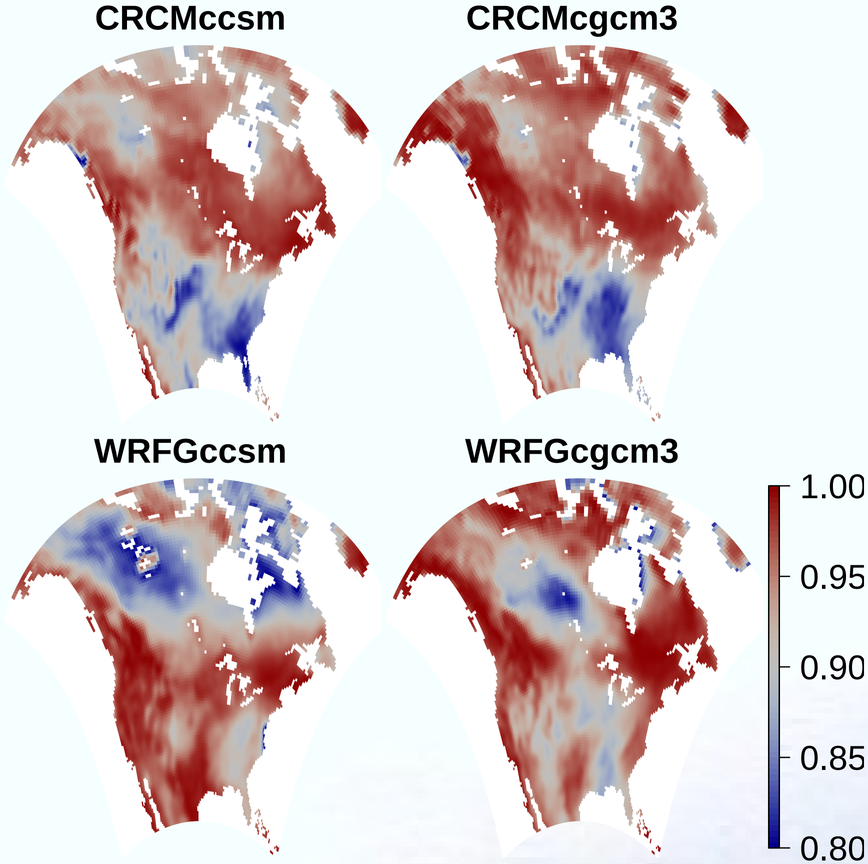


Solid - 8 Bsplines, Dashed - projection from 4 EOFS/PCs, Difference with spatial smoothing

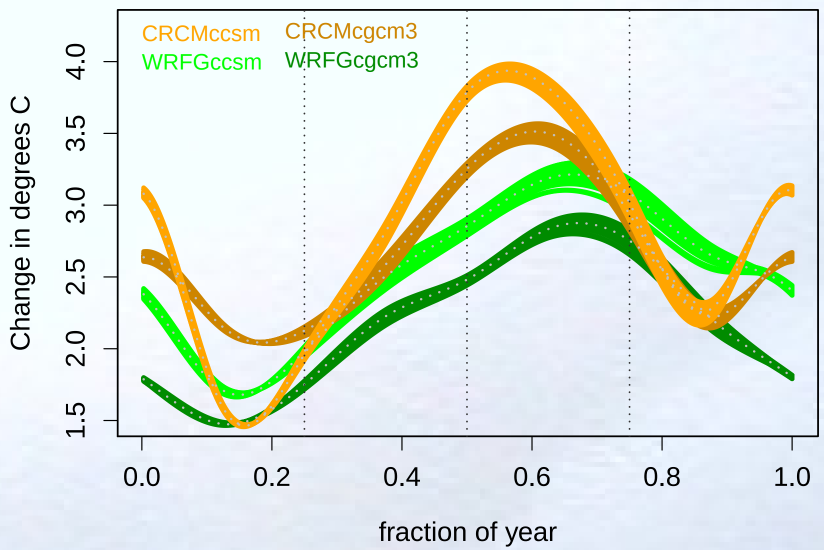
Results

- Thin plate spline-like model (1 level $120 \times 55 \approx 6000$ basis functions)
- λ found by MLE (equivalent to sill and nugget)
- Conditional simulation of fields (facilitates nonlinear statistics)

R^2 for first PC



Inference for Boulder grid box



Summary

- **Computational efficiency gained by compact basis functions and sparse precision matrix.**

- **Can reproduce usual spatial analysis:**
flexible covariances, parameter estimation, spatial prediction, inference.

See `LatticeKrig` *package in R*

Thank you!