# Bayesian Calibration of Numerical Model Output by an R package: ${\rm spCalibration}$

## ${\rm ChenYW}$

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# Contents

1	${f R}$ package and data	2
	1.1 Load R package: spCalibration	2
	1.2 Data layout	2
	1.3 Create grids by CMAQ or INLA	4
	1.4 Mapping matrix: H	5
	1.5 Initialize parameter	5
2	Data truncation	6
3	Model cross validation	7
	3.1 Prediction	8
	3.2 The distributions of parameters	9
4	Complete dataset	9
	4.1 The distributions of parameters	12

```
Setup
```

```
knitr::opts_chunk$set(echo = TRUE, message = FALSE, warning = FALSE)
# , class.source = "BK"
rm(list=ls())
Sys.setlocale("LC_TIME", "English")
```

## [1] "English\_United States.1252"

## 1 R package and data

## 1.1 Load R package: spCalibration

Note: Our all data used in this file was encapsulated in "spCalibration" along with the code.

#### 1.2 Data layout

```
#
                        Fundamental dataset from pollution
   Model_Base_Tab[, c(1, 3:8, 12, 15, 17:19, 24)]
   ##
                   CITY STATION_NAME CMAQ_ID
                                                      LAT DATE_TIME YEAR_MONTH
                                               LON
   ##
         1: Shijiazhuang Shijigongyuan
                                       5724 114.542 38.031 2015-06-01
                                                                       201506
         2: Shijiazhuang Shijigongyuan
                                       5724 114.542 38.031 2015-06-02
                                                                       201506
         3: Shijiazhuang Shijigongyuan
                                       5724 114.542 38.031 2015-06-03
                                                                       201506
         4: Shijiazhuang Shijigongyuan
                                       5724 114.542 38.031 2015-06-04
                                                                       201506
   ##
         5: Shijiazhuang Shijigongyuan
                                       5724 114.542 38.031 2015-06-05
                                                                       201506
   ## 12508:
                 Beijing
                              Shunyi
                                       7932 116.650 40.410 2016-01-27
                                                                       201601
   ## 12509:
                Beijing
                              Shunyi
                                       7932 116.650 40.410 2016-01-28
                                                                       201601
   ## 12510:
                Beijing
                              Shunyi
                                       7932 116.650 40.410 2016-01-29
                                                                       201601
10
   ## 12511:
                Beijing
                              Shunyi
                                       7932 116.650 40.410 2016-01-30
                                                                       201601
   ## 12512:
                Beijing
                              Shunyi
                                       7932 116.650 40.410 2016-01-31
                                                                       201601
12
            REAL_PM25 SITEID
                                LON_X
                                       LAT_Y CMAQ_PM25 NA.Kalman
   ##
             87.20833
                          1 -242926.92 4243177 53.104367
                                                            NA
14
                          1 -242926.92 4243177 47.539544
             44.04167
                                                            NA
15
   ##
             59.45833
                          1 -242926.92 4243177 53.139121
                                                            NA
16
```

```
1 -242926.92 4243177 39.686555
   ##
         4: 75.04167
                                                            NA
                          1 -242926.92 4243177 37.311844
18
   ##
         5: 100.25000
                                                            NA
19
   ## 12508: 110.20833
                           -38955.09 4492669 42.721178
                                                            NΑ
20
   ## 12509: 119.37500
                            -38955.09 4492669 12.222126
                                                            NA
21
                            -38955.09 4492669 37.876560
   ## 12510: 86.25000
                         76
                                                            NΑ
22
   ## 12511: 64.91667
                            -38955.09 4492669 15.412478
                                                            NΑ
   ## 12512: 15.62500
                         76 -38955.09 4492669 4.373301
                                                            NΑ
   #
                     the coordinates of monitoring station
2
   head(Site)
        SITEID
                      CITY STATION_NAME
                                          LON
                                                 LAT
                                                        LON_X
                                                               LAT_Y
   ## 1:
             1 Shijiazhuang Shijigongyuan 114.542 38.031 -242926.9 4243177
   ## 2:
             2 Shijiazhuang Xinangaojiao 114.467 38.012 -249721.6 4241671
   ## 3:
             4 Shijiazhuang Zhigongyiyuan 114.455 38.051 -250376.2 4246107
             5 Shijiazhuang Renminhuitang 114.521 38.052 -244560.1 4245683
   ## 5:
             6 Shijiazhuang Xibeishuiyuan 114.502 38.140 -245328.9 4255626
   ## 6:
             7 Shijiazhuang
                               Gaoxinqu 114.605 38.040 -237292.7 4243672
   dataset of CMAQ
2
   CMAQ_PM25
   ##
             CMAQ_ID DATE_TIME CMAQ_PM25
                                            LON
                                                    LAT
                                                           LON_X
                                                                  LAT Y
   ##
          1:
                4619 2015-06-01 9.824296 113.5025 36.71028 -349353.9 4105037
          2:
                4619 2015-06-02 10.246346 113.5025 36.71028 -349353.9 4105037
   ##
                4619 2015-06-03 17.661194 113.5025 36.71028 -349353.9 4105037
          4:
               4619 2015-06-04 4.019376 113.5025 36.71028 -349353.9 4105037
   ##
               4619 2015-06-05 3.203634 113.5025 36.71028 -349353.9 4105037
   ##
          5:
   ## 459812:
               11438 2016-01-27 86.914974 119.7762 39.98793 224742.0 4431396
      459813:
               11438 2016-01-28 20.297575 119.7762 39.98793 224742.0 4431396
               11438 2016-01-29 20.376099 119.7762 39.98793 224742.0 4431396
      459814:
10
   ## 459815:
               11438 2016-01-30 34.814252 119.7762 39.98793 224742.0 4431396
11
               11438 2016-01-31 41.928738 119.7762 39.98793 224742.0 4431396
   ## 459816:
12
             YEAR_MONTH
   ##
13
                 201506
   ##
          1:
14
          2:
                 201506
15
                 201506
16
   ##
          3:
                201506
   ##
          4:
17
                 201506
18
   ##
          5:
19
                 201601
   ## 459812:
20
   ## 459813:
                 201601
21
   ## 459814:
                 201601
22
                 201601
   ## 459815:
   ## 459816:
                 201601
24
```

```
the coordinates of CMAQ lattice
   cmaq_site
   ##
          CMAQ_ID
                     LON
                             LAT
                                    LON_X LAT_Y
                                                      CITY
                                                            ID
   ##
             4619 113.5025 36.71028 -349353.9 4105037
                                                             1
        1:
   ##
        2:
             4638 113.5744 38.30076 -325010.6 4281459 Shijiazhuang
             4639 113.5783 38.38442 -323712.1 4290736 Shijiazhuang
        3:
             4640 113.5822 38.46806 -322412.8 4300010 Shijiazhuang
   ##
        4:
             4641 113.5860 38.55172 -321112.5 4309285 Shijiazhuang
   ## 2495:
            11199 119.5909 40.25370 210047.3 4461493
                                                 Qinhuangdao 2495
   ## 2496:
            11317 119.6677 39.99614 215508.6 4432649
                                                 Qinhuangdao 2496
            11318 119.6783 40.07928 216765.3 4441846
                                                 Qinhuangdao 2497
   ## 2497:
   ## 2498:
            11319 119.6890 40.16242 218020.3 4451043
                                                 Qinhuangdao 2498
11
            11438 119.7762 39.98793 224742.0 4431396
   ## 2499:
                                                Qinhuangdao 2499
12
```

#### 1.3 Create grids by CMAQ or INLA

```
A. Create directly grids by CMAQ lattices
   # temp <- cmaq_site</pre>
   \# coordinates(temp) = \sim LON X + LAT Y
   # coords <- temp@coords
   # Neighbor_order <- dnearneigh(coords, 0, 1e4)
   # # View(Neighbor_order)
   # adjacent.matrix <- matrix(0, nrow(coords), nrow(coords))</pre>
   # for(i in 1:nrow(coords))
11
      for( j in 1: length(Neighbor_order[[i]]))
12
13
       Ad.num <- Neighbor_order[[i]][j]</pre>
14
       adjacent.matrix[i, Ad.num] = -1
     7
16
   # }
17
   # grid <- list(grid.coords = cmaq_site,
18
               adjacent.matrix = adjacent.matrix)
19
   20
                  B. Create grids by INLA
   22
23
   grid <- ProduceGrid(Site,</pre>
                          # Monitoring stations data
24
                  max.edge = c(0.3, 0.7), # max.edge, offset and cutoff: see INLA
25
                  offset = c(0.4, 0.6),
26
                  cutoff = 0.1,
27
                  col = "black",
28
                  size = 1)
29
```

grid\$plot.grid



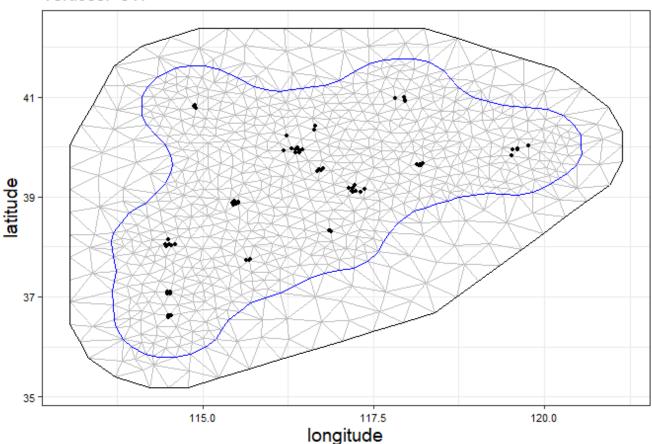


Figure 1.1: The irregular division of spatial domain by INLA

### 1.4 Mapping matrix: H

## 1.5 Initialize parameter

### 1.5.1 Prior

2. DATA TRUNCATION CONTENTS

#### 1.5.2 Importance sampling

#### 1.5.3 Initialization

```
initial.para <- list(
    beta = list(E_beta = c(2, 0.5), Sigma2 = diag(2))
    , theta1 = list(E_theta1 = 0.005, Sigma2 = 2)
    , k = list(E_k = 5, a = 2, b = 1)
    , k0 = list(E_k0 = 5, a = 2, b = 1)
    , theta2 = list(mu = c(1))
    , tau2 = list(E_tau2 = 1, a = 2, b = 1)
    )
}</pre>
```

#### 2 Data truncation

#### 3 Model cross validation

```
leave one city out
city_num = 1
GSD = Test_Train_Fun(Site, Data_Str, Yts_Xts, city_num)
##
##
## Test city: Baoding ...
Train = GSD$Train
Test = GSD$Test
Train$Y_ts = sqrt(Train$Y_ts)
Train$X_ts = sqrt(Train$X_ts)
Test$X_ts = sqrt(Test$X_ts)
#
                   fit model
# CV <- spVBEnKs(data = Train,
#
           prior = prior,
           IS = IS,
#
#
           para = initial.para,
           true.para = NULL,
           parallel = TRUE,
           verbose = TRUE,
#
#
           verbose. VB = TRUE,
           Ensemble.size = 50,
#
#
           cs = 0.4,
           ct = 1,
#
           Remove. CPU. Count = 2,
           N.Chunk = 1,
#
#
           itMax = 1e2,
           tol.vb = 1e-3,
#
           tol.real = 1e-3)
load( "./data/Generate_Data/Test/CV.Rdata")
output
print(CV)
##
## Call: spVBEnKs(data = Train, prior = prior, IS = IS, para = initial.para,
##
     true.para = NULL, parallel = TRUE, verbose = TRUE, verbose.VB = TRUE,
     Ensemble.size = 50, cs = 0.4, ct = 1, Remove.CPU.Count = 2,
##
     N.Chunk = 1, itMax = 100, tol.vb = 0.001, tol.real = 0.001)
##
##
## Iterations: 12
##
```

```
## Log-likelihood: 171854.001
##
  Parameter estimation:
         beta0 beta1 tau_2 theta1 theta2
                                                    k0 iter
  True:
            NA
                        NΑ
                                NA
                                       NΑ
                                             NA
                                                    NA
                                                          0
  Init: 2.000 0.500 1.000
                                                          0
                             0.005
                                    1.000 5.000 5.000
  Esti: 1.281 0.672 0.995
                            0.008
                                    0.601 9.639 4.357
                                                         12
##
## (...output suppressed due to large dimension!)
```

#### 3.1 Prediction

```
prediction
P <- predict(CV, test = Test, site = Site, method = c("ensemble"),
         transf = "square", ncol.layout = 2)
P$error
  $Diff.Station.RMSE
     34
           35
                36
                      37
                           38
                                 39
  72.812 64.230 105.620
                   66.340 60.315 57.126
##
##
  $Total.Error
   Pearson.before Pearson.after
                        RMSE
                                    NMB
                                         NME
                   0.813 72.898 -26.219 -16.471 32.512
```

P\$ensemble.plot

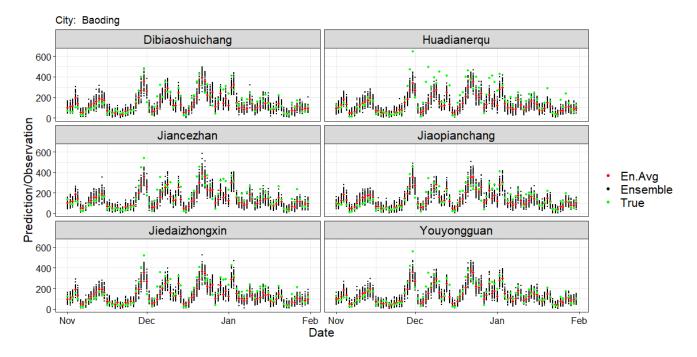


Figure 3.1: Prediction by ensemble

4. COMPLETE DATASET CONTENTS

P\$mean.plot

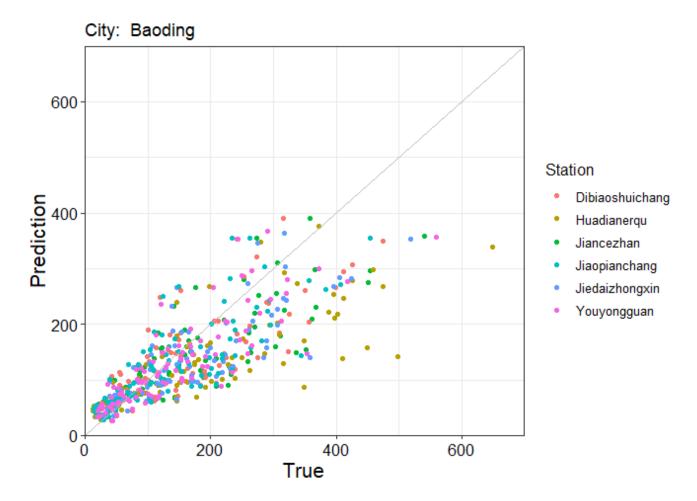


Figure 3.2: Prediction by ensemble mean

### 3.2 The distributions of parameters

## 4 Complete dataset

P\$NoClose.Dist



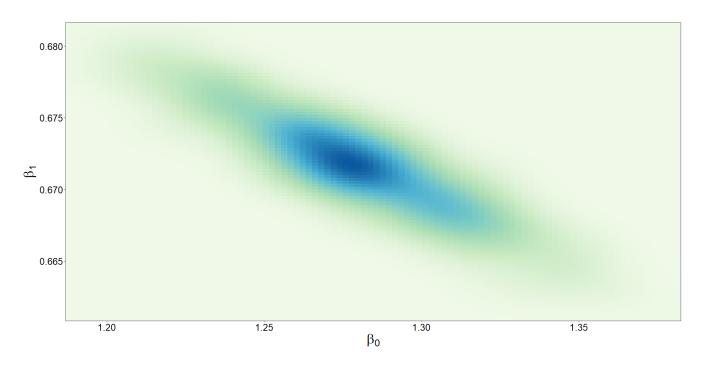


Figure 3.3: The distribution of Beta parameter

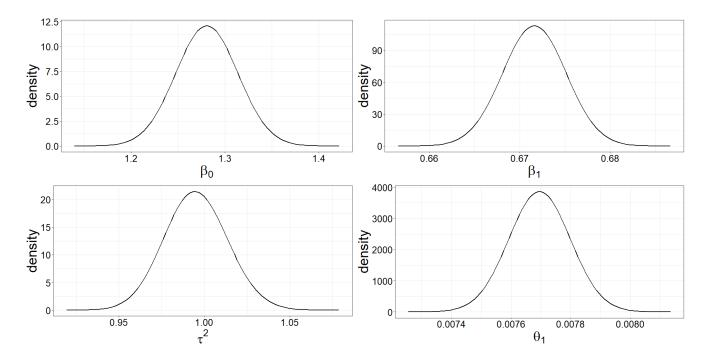


Figure 3.4: The distributions of parameters of close form

4. COMPLETE DATASET CONTENTS

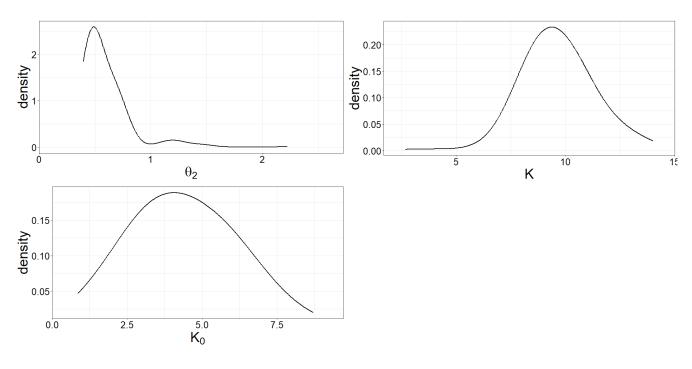


Figure 3.5: The distributions of parameters of no close form

Total\_Data <- list(</pre>

```
n = dim(Yts_Xts_Y_ts)[2]
             , Nt = dim(Yts_Xts\$Y_ts)[1]
             , N.BAUs = Data_Str$N.BAUs
             , Y_ts = Yts_Xts_Y_ts
             , X_ts = Yts_Xts$X_ts
             , BAUs.Dist = Data_Str$BAUs.Dist
             , Adj.Mat = Data_Str$G
             , Hs = Data_Str$Hs
Total_Data$Y_ts = sqrt(Total_Data$Y_ts)
Total_Data$X_ts = sqrt(Total_Data$X_ts)
fit model and prediction
# Total <- spVBEnKs(data = Total_Data,</pre>
#
                prior = prior,
#
                IS = IS,
                para = initial.para,
                true.para = NULL,
                parallel = TRUE,
                verbose = TRUE,
#
                verbose.VB = TRUE,
#
                Ensemble.size = 50,
#
                cs = 0.4,
#
                ct = 1,
                Remove. CPU.Count = 2,
```

4. COMPLETE DATASET CONTENTS

```
#
                    N.Chunk = 1,
#
                    itMax = 1e2,
                    tol.vb = 1e-3,
                    tol.real = 1e-3)
load("./data/Generate_Data/Total.Rdata")
print(Total)
##
## Call: spVBEnKs(data = Total_Data, prior = prior, IS = IS, para = initial.para,
##
       true.para = NULL, parallel = TRUE, verbose = TRUE, verbose.VB = TRUE,
       Ensemble.size = 50, cs = 0.4, ct = 1, Remove.CPU.Count = 2,
##
       N.Chunk = 1, itMax = 100, tol.vb = 0.001, tol.real = 0.001)
##
##
## Iterations: 12
##
## Log-likelihood: 170404.298
##
## Parameter estimation:
##
        beta0 beta1 tau_2 theta1 theta2
                                            k k0 iter
## True: NA
                                           NA
                                                NA
## Init: 2.00 0.500 1.000 0.005 1.000 5.000 5.00
                                                       0
## Esti: 1.22 0.667 1.018 0.008 0.604 9.475 4.33
##
## (...output suppressed due to large dimension!)
```

### 4.1 The distributions of parameters

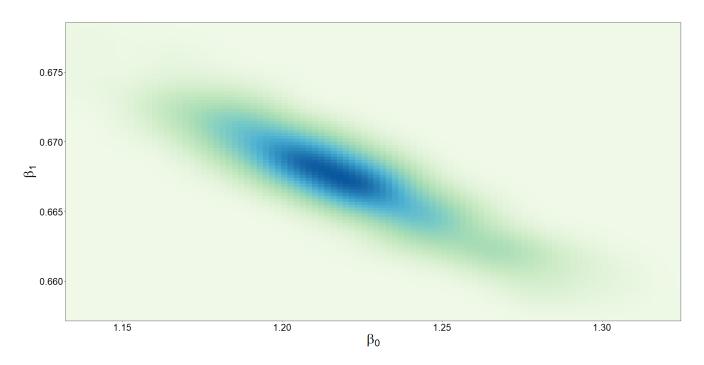


Figure 4.1: The distribution of  $\beta$  parameter

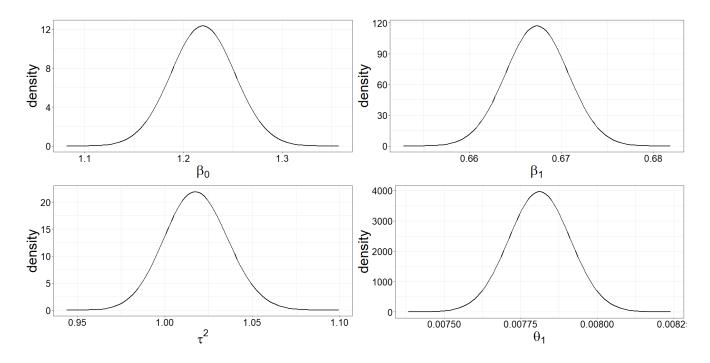


Figure 4.2: The distributions of parameters of close form

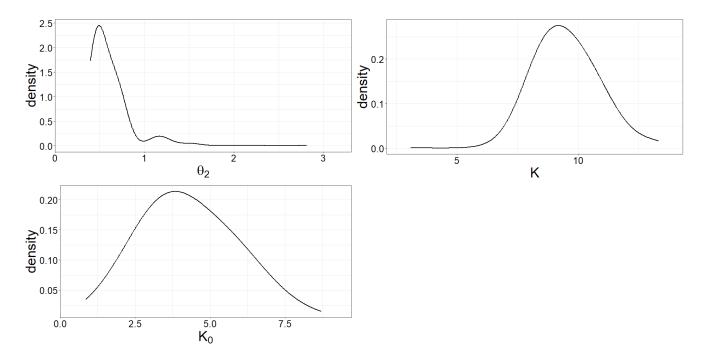


Figure 4.3: The distributions of parameters of no close form