

# Bayesian Calibration of Numerical Model Output by an R package: spCalibration

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Setup

```
knitr::opts_chunk$set(echo = TRUE, message = FALSE, warning = FALSE)
# , class.source = "BK"
rm(list=ls())
suppressMessages(Sys.setlocale("LC_TIME", "English"))

## [1] "English_United States.1252"
```

## 1 R package and data

### 1.1 Load R package: spCalibration

```
1 #####
2 #               load   created R package: spCalibration
3 #####
4 suppressMessages(library(spCalibration))

1 ## Number of platforms: 1
2 ## - platform: NVIDIA Corporation: OpenCL 1.2 CUDA 10.1.0
3 ##   - context device index: 0
4 ##     - GeForce RTX 2080 Ti
5 ## checked all devices
6 ## completed initialization

1 gcQuiet(FALSE, verbose=TRUE)

1 ##          used   (Mb) gc trigger   (Mb) max used   (Mb)
2 ## Ncells 3476141 185.7    6631861 354.2  4988532 266.5
3 ## Vcells 5336461  40.8    10146329 77.5   7379939  56.4
```

**Note:** Our all data used in this file was encapsulated in “spCalibration” along with the code.

### 1.2 Data layout

```
1 #####
2 #               Fundamental dataset from pollution
3 #####
4 Model_Base_Tab[, c(1, 3:8, 12, 15, 17:19, 24)]

1 ##          CITY STATION_NAME CMAQ_ID    LON    LAT  DATE_TIME YEAR_MONTH
2 ##      1: Shijiazhuang Shijigongyuan   5724 114.542 38.031 2015-06-01    201506
3 ##      2: Shijiazhuang Shijigongyuan   5724 114.542 38.031 2015-06-02    201506
4 ##      3: Shijiazhuang Shijigongyuan   5724 114.542 38.031 2015-06-03    201506
5 ##      4: Shijiazhuang Shijigongyuan   5724 114.542 38.031 2015-06-04    201506
6 ##      5: Shijiazhuang Shijigongyuan   5724 114.542 38.031 2015-06-05    201506
7 ##      ---
8 ## 12508:    Beijing      Shunyi      7932 116.650 40.410 2016-01-27    201601
9 ## 12509:    Beijing      Shunyi      7932 116.650 40.410 2016-01-28    201601
10 ## 12510:    Beijing      Shunyi      7932 116.650 40.410 2016-01-29    201601
11 ## 12511:    Beijing      Shunyi      7932 116.650 40.410 2016-01-30    201601
```

```

12 ## 12512:      Beijing      Shunyi      7932 116.650 40.410 2016-01-31      201601
13 ##      REAL_PM25 SITEID      LON_X      LAT_Y CMAQ_PM25 NA.Kalman
14 ##      1: 87.20833      1 -242926.92 4243177 53.104367      NA
15 ##      2: 44.04167      1 -242926.92 4243177 47.539544      NA
16 ##      3: 59.45833      1 -242926.92 4243177 53.139121      NA
17 ##      4: 75.04167      1 -242926.92 4243177 39.686555      NA
18 ##      5: 100.25000      1 -242926.92 4243177 37.311844      NA
19 ##      ---
20 ## 12508: 110.20833      76 -38955.09 4492669 42.721178      NA
21 ## 12509: 119.37500      76 -38955.09 4492669 12.222126      NA
22 ## 12510: 86.25000      76 -38955.09 4492669 37.876560      NA
23 ## 12511: 64.91667      76 -38955.09 4492669 15.412478      NA
24 ## 12512: 15.62500      76 -38955.09 4492669 4.373301      NA

```

```

1 #####
2 #               the coordinates of monitoring station
3 #####
4 head(Site)

```

```

1 ##      SITEID      CITY STATION_NAME      LON      LAT      LON_X      LAT_Y
2 ## 1:      1 Shijiazhuang Shijigongyuan 114.542 38.031 -242926.9 4243177
3 ## 2:      2 Shijiazhuang Xinangaojiao 114.467 38.012 -249721.6 4241671
4 ## 3:      4 Shijiazhuang Zhigongyiyuan 114.455 38.051 -250376.2 4246107
5 ## 4:      5 Shijiazhuang Renminhuitang 114.521 38.052 -244560.1 4245683
6 ## 5:      6 Shijiazhuang Xibeishuiyuan 114.502 38.140 -245328.9 4255626
7 ## 6:      7 Shijiazhuang      Gaoxinqu 114.605 38.040 -237292.7 4243672

```

```

1 #####
2 #               dataset of CMAQ
3 #####
4 CMAQ_PM25

```

```

1 ##      CMAQ_ID DATE_TIME CMAQ_PM25      LON      LAT      LON_X      LAT_Y
2 ##      1:      4619 2015-06-01 9.824296 113.5025 36.71028 -349353.9 4105037
3 ##      2:      4619 2015-06-02 10.246346 113.5025 36.71028 -349353.9 4105037
4 ##      3:      4619 2015-06-03 17.661194 113.5025 36.71028 -349353.9 4105037
5 ##      4:      4619 2015-06-04 4.019376 113.5025 36.71028 -349353.9 4105037
6 ##      5:      4619 2015-06-05 3.203634 113.5025 36.71028 -349353.9 4105037
7 ##      ---
8 ## 459812: 11438 2016-01-27 86.914974 119.7762 39.98793 224742.0 4431396
9 ## 459813: 11438 2016-01-28 20.297575 119.7762 39.98793 224742.0 4431396
10 ## 459814: 11438 2016-01-29 20.376099 119.7762 39.98793 224742.0 4431396
11 ## 459815: 11438 2016-01-30 34.814252 119.7762 39.98793 224742.0 4431396
12 ## 459816: 11438 2016-01-31 41.928738 119.7762 39.98793 224742.0 4431396
13 ##      YEAR_MONTH
14 ##      1:      201506
15 ##      2:      201506
16 ##      3:      201506
17 ##      4:      201506
18 ##      5:      201506
19 ##      ---

```

```

20 ## 459812:      201601
21 ## 459813:      201601
22 ## 459814:      201601
23 ## 459815:      201601
24 ## 459816:      201601

1 #####
2 #               the coordinates of CMAQ lattice
3 #####
4 cmaq_site

1 ##          CMAQ_ID      LON      LAT      LON_X      LAT_Y      CITY      ID
2 ##      1:      4619 113.5025 36.71028 -349353.9 4105037      Handan      1
3 ##      2:      4638 113.5744 38.30076 -325010.6 4281459 Shijiazhuang      2
4 ##      3:      4639 113.5783 38.38442 -323712.1 4290736 Shijiazhuang      3
5 ##      4:      4640 113.5822 38.46806 -322412.8 4300010 Shijiazhuang      4
6 ##      5:      4641 113.5860 38.55172 -321112.5 4309285 Shijiazhuang      5
7 ##      ---
8 ## 2495:      11199 119.5909 40.25370  210047.3 4461493  Qinhuangdao 2495
9 ## 2496:      11317 119.6677 39.99614  215508.6 4432649  Qinhuangdao 2496
10 ## 2497:      11318 119.6783 40.07928  216765.3 4441846  Qinhuangdao 2497
11 ## 2498:      11319 119.6890 40.16242  218020.3 4451043  Qinhuangdao 2498
12 ## 2499:      11438 119.7762 39.98793  224742.0 4431396  Qinhuangdao 2499

```

### 1.3 Create grids by CMAQ or INLA

```

1 #####
2 #               A. Create directly grids by CMAQ lattices
3 #####
4 # temp <- cmaq_site
5 # coordinates(temp) = ~ LON_X + LAT_Y
6 # coords <- temp@coords
7 # Neighbor_order <- dnearneigh(coords, 0, 1e4)
8 # # View(Neighbor_order)
9 # adjacent.matrix <- matrix(0, nrow(coords), nrow(coords))
10 # for(i in 1:nrow(coords))
11 # {
12 #   for( j in 1: length(Neighbor_order[[i]]))
13 #   {
14 #     Ad.num <- Neighbor_order[[i]][j]
15 #     adjacent.matrix[i, Ad.num] = -1
16 #   }
17 # }
18 # grid <- list(grid.coords = cmaq_site,
19 #              adjacent.matrix = adjacent.matrix)
20 #####
21 #               B. Create grids by INLA
22 #####
23

```

```

24 grid <- ProduceGrid(Site,      # Monitoring stations data
25                     max.edge = c(0.3, 0.7), # max.edge, offset and cutoff: see INLA
26                     offset = c(0.4, 0.6),
27                     cutoff = 0.1,
28                     col = "black",
29                     size = 1)
30 grid$plot.grid

```

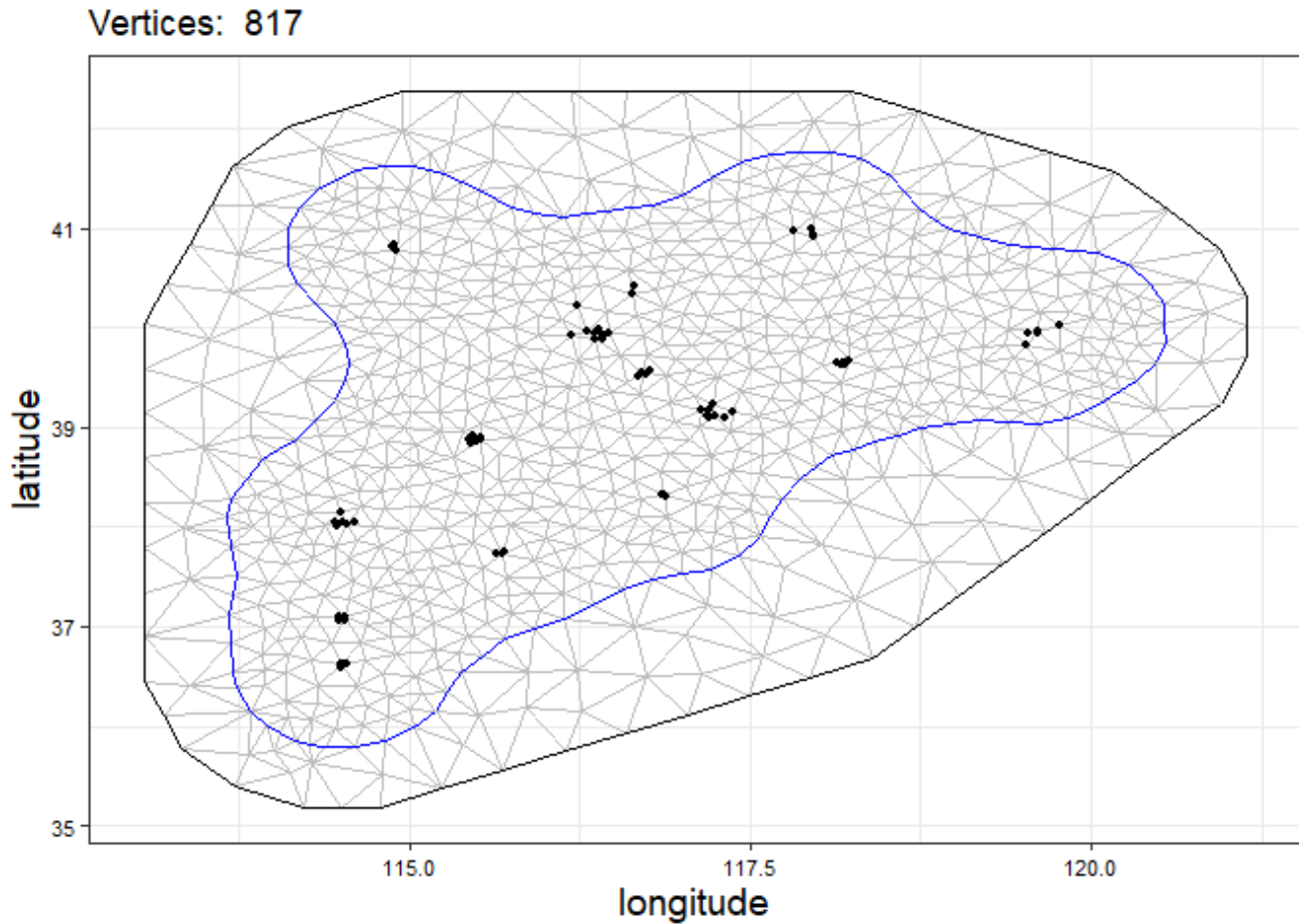


Figure 1.1: The irregular division of spatial domain by INLA

## 1.4 Mapping matrix: H

```

Data_Str <- ProduceHmatrix(grid, Site, threshold = 1e5)
# save(grid, file = "./data/BaseTable/Data_Str.Rda")
# load("./data/BaseTable/Data_Str.Rda")
#####

```

## 1.5 Initialize parameter

### 1.5.1 Prior

```
#####
#                               initialize parameters
#####
{
  #####
  # true.para <- list(beta = rep(NA, 2), nugget.tau2 = NA,
  #                   theta = c(NA, NA), k0 = NA, k = NA)
  #####
  #####
  #                               prior distribution
  #####
  prior <- list(
    beta = list(mu = c(0, 1), Sigma2 = 1e5*diag(2))
    , nugget.tau2 = list(a = 2, b = 1)
    , theta1 = list(mu = 0.005, Sigma2 = 1e5)
  )
}
```

### 1.5.2 Importance sampling

```
IS.size <- 50
# rinvgamma(IS.size, shape = 0.5, rate = 1)
IS <- list(theta2_random_sample = runif(IS.size, 1e-5, 5)
  , K_random_sample = runif(IS.size, 5, 15)
  , K0_random_sample = runif(IS.size, 5, 15)
  , G_theta2_weight = rep(1/IS.size, IS.size)
  , G_k_weight = rep(1/IS.size, IS.size)
  , G_k0_weight = rep(1/IS.size, IS.size)
  , IS.size = IS.size
  , Thresh = c(1, 1, 1)
)
```

### 1.5.3 Initialization

```
initial.para <- list(
  beta = list(E_beta = c(2, 0.5), Sigma2 = diag(2))
  , theta1 = list(E_theta1 = 0.005, Sigma2 = 2)
  , k = list(E_k = 5, a = 2, b = 1)
  , k0 = list(E_k0 = 5, a = 2, b = 1)
  , theta2 = list(mu = c(1))
  , tau2 = list(E_tau2 = 1, a = 2, b = 1)
)
```

## 2 Data truncation

```
#####
#                               Data truncation
#####
YearMonth <- c(201511, 201512, 201601) # 201506,201507,201508 # 201511, 201512, 201601
Yts_Xts <- ParYtsXts(Model_Base_Tab, YearMonth = YearMonth)
```

## 3 Model cross validation

```
#####
#                               leave one city out
#####
city_num = 1
GSD = Test_Train_Fun(Site, Data_Str, Yts_Xts, city_num)

##
##
## Test city: Baoding ...

Train = GSD$Train
Test = GSD$Test
Train$Y_ts = sqrt(Train$Y_ts)
Train$X_ts = sqrt(Train$X_ts)
Test$X_ts = sqrt(Test$X_ts)

#####
#                               fit model
#####
# CV <- spVBEKs(data = Train,
#               prior = prior,
#               IS = IS,
#               para = initial.para,
#               true.para = NULL,
#               parallel = TRUE,
#               verbose = TRUE,
#               verbose.VB = TRUE,
#               Ensemble.size = 50,
#               cs = 0.4,
#               ct = 1,
#               Remove.CPU.Count = 2,
#               N.Chunk = 1,
#               itMax = 1e2,
#               tol.vb = 1e-3,
#               tol.real = 1e-3)
load( ". /data/Generate_Data/Test/CV.Rdata")

#####
#                               output
#####
```

```
print(CV)

##
## Call: spVBEKs(data = Train, prior = prior, IS = IS, para = initial.para,
##   true.para = NULL, parallel = TRUE, verbose = TRUE, verbose.VB = TRUE,
##   Ensemble.size = 50, cs = 0.4, ct = 1, Remove.CPU.Count = 2,
##   N.Chunk = 1, itMax = 100, tol.vb = 0.01, tol.real = 0.001)
##
## Iterations: 7
##
## Log-likelihood: 180136.595
##
## Parameter estimation:
##      beta0 beta1 tau_2 theta1 theta2      k  k0 iter
## True:   NA    NA    NA     NA     NA    NA  NA   0
## Init: 2.000 0.500 1.000 0.005    1.0 5.000 5.00   0
## Esti: 1.533 0.623 1.024 0.012    0.9 10.841 5.12   7
##
## (...output suppressed due to large dimension!)
```

### 3.1 Prediction

```
#####
#                               prediction
#####
P <- predict(CV, test = Test, site = Site, method = c("ensemble"),
             transf = "square", ncol.layout = 2)

P$error

## $Diff.Station.RMSE
##      34      35      36      37      38      39
## 70.200 63.093 100.160 65.252 61.534 53.623
##
## $Total.Error
##   Pearson.before Pearson.after  RMSE      MB      NMB      NME
## 1           0.628           0.816 70.546 -21.943 -13.785 31.907

P$ensemble.plot

P$mean.plot
```

### 3.2 The distributions of parameters

```
#####
#                               the distributions of parameters
#####
P <- plot(CV, n = 500, title.size = 30, text.size = 20, line.size = 1)
P$Beta.plot
```



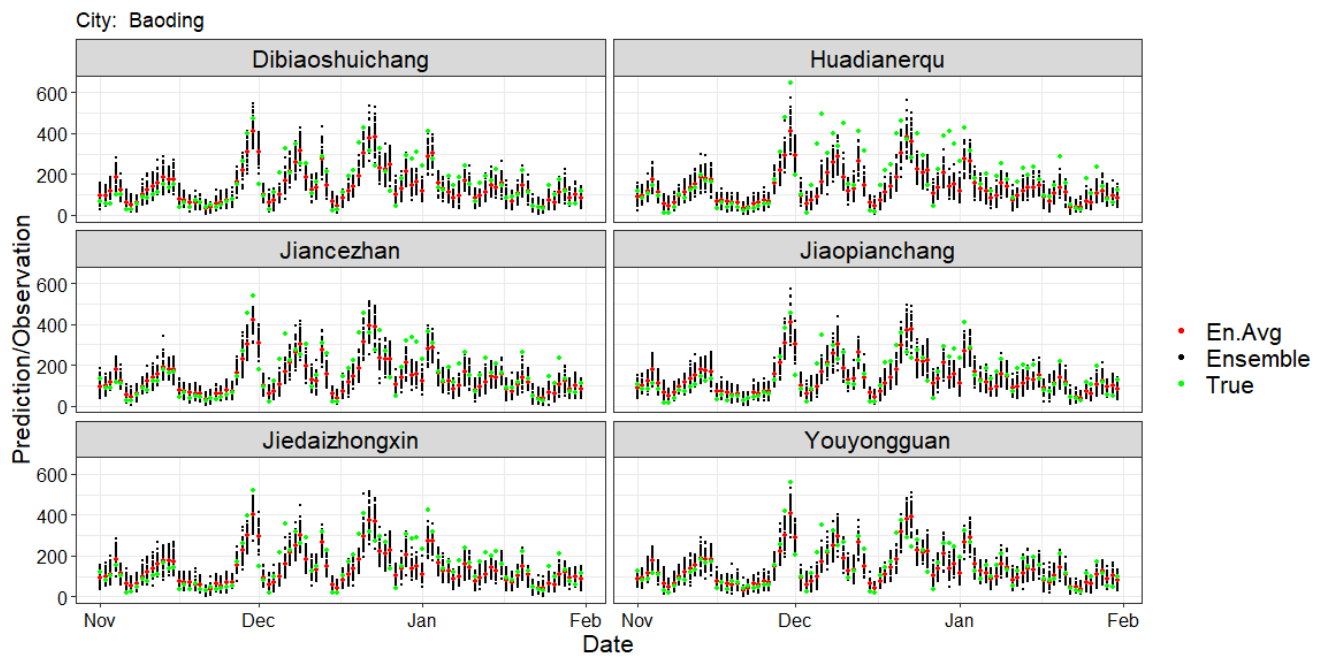


Figure 3.1: Prediction by ensemble

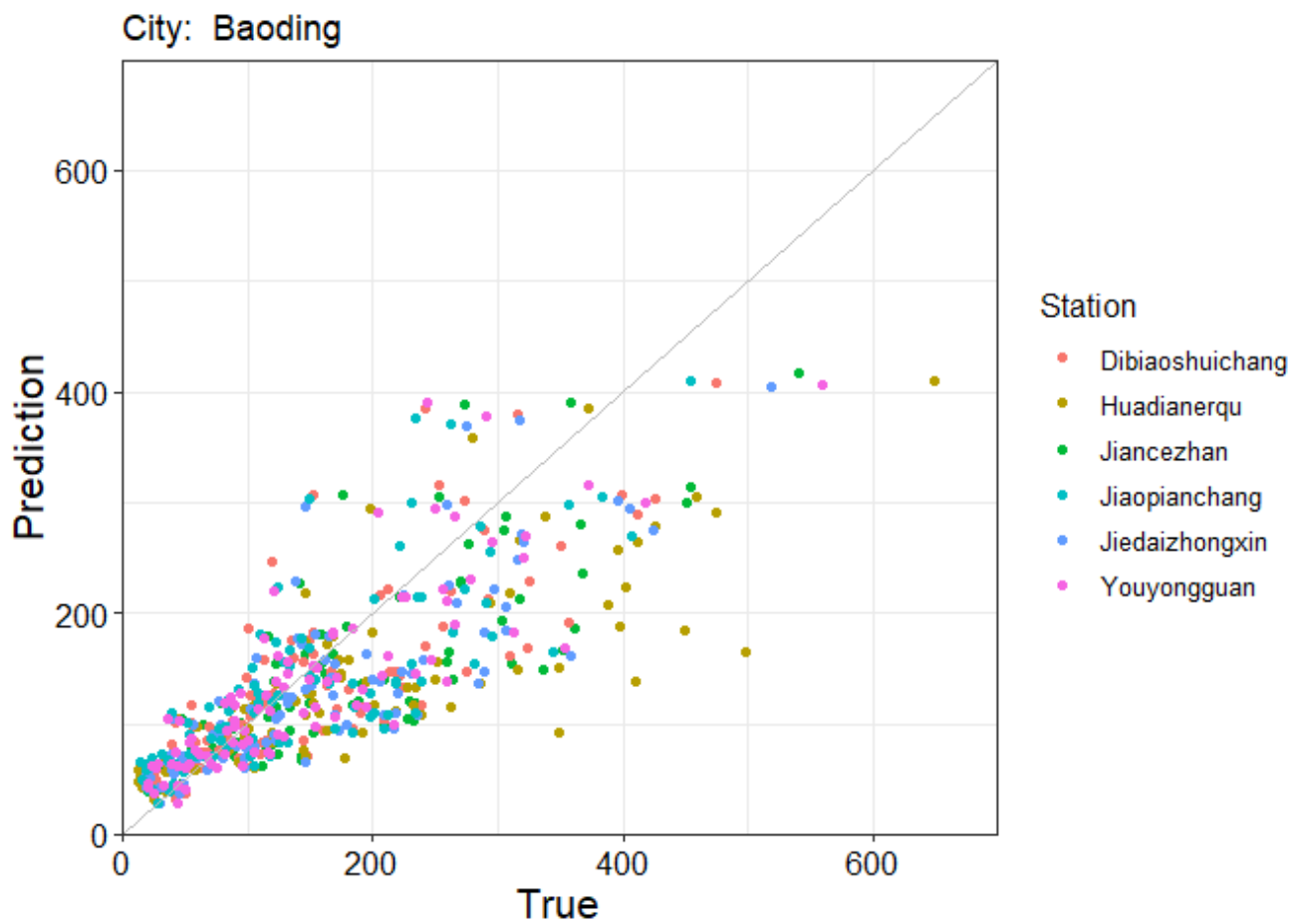


Figure 3.2: Prediction by ensemble mean

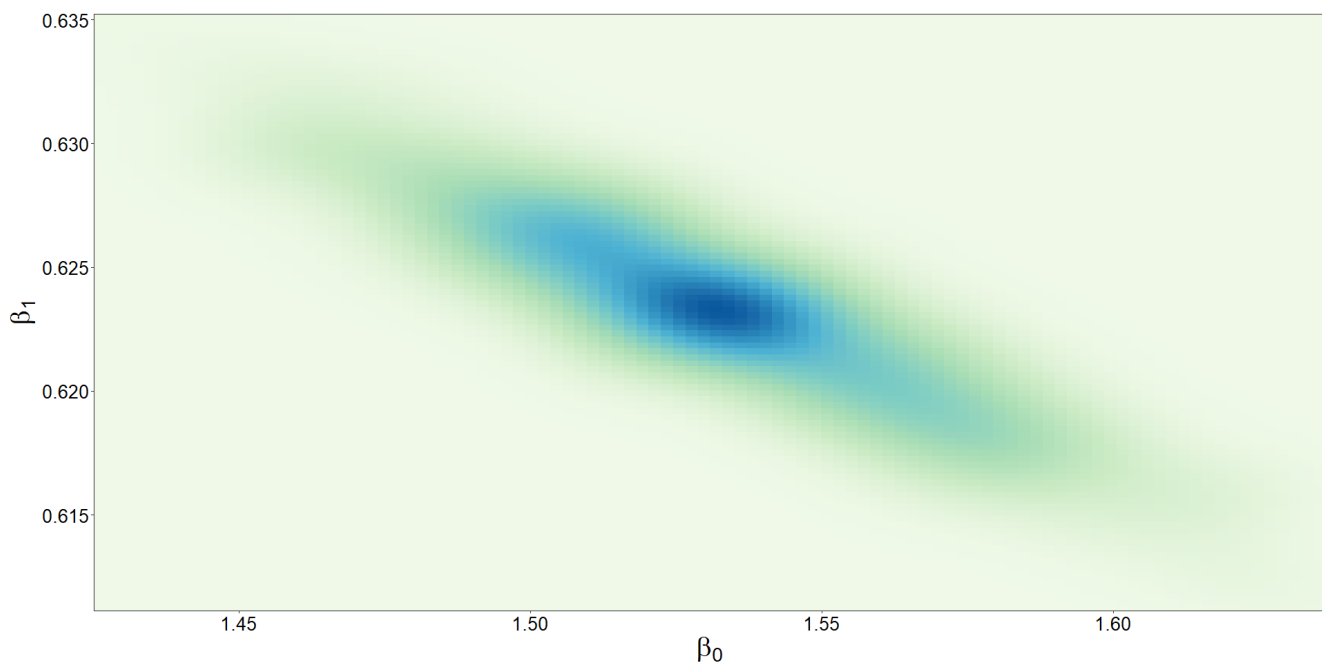


Figure 3.3: The distribution of Beta parameter

```
P$Close.Dist
```

```
P$NoClose.Dist
```

## 4 Complete dataset

```
#####
#                               Complete dataset
#####
Total_Data <- list(
  n = dim(Yts_Xts$Y_ts)[2]
  , Nt = dim(Yts_Xts$Y_ts)[1]
  , N.BAUs = Data_Str$N.BAUs
  , Y_ts = Yts_Xts$Y_ts
  , X_ts = Yts_Xts$X_ts
  , BAUs.Dist = Data_Str$BAUs.Dist
  , Adj.Mat = Data_Str$G
  , Hs = Data_Str$Hs
)
Total_Data$Y_ts = sqrt(Total_Data$Y_ts)
Total_Data$X_ts = sqrt(Total_Data$X_ts)

#####
#                               fit model and prediction
#####
# Total <- spVBEKs(data = Total_Data,
#                   prior = prior,
```

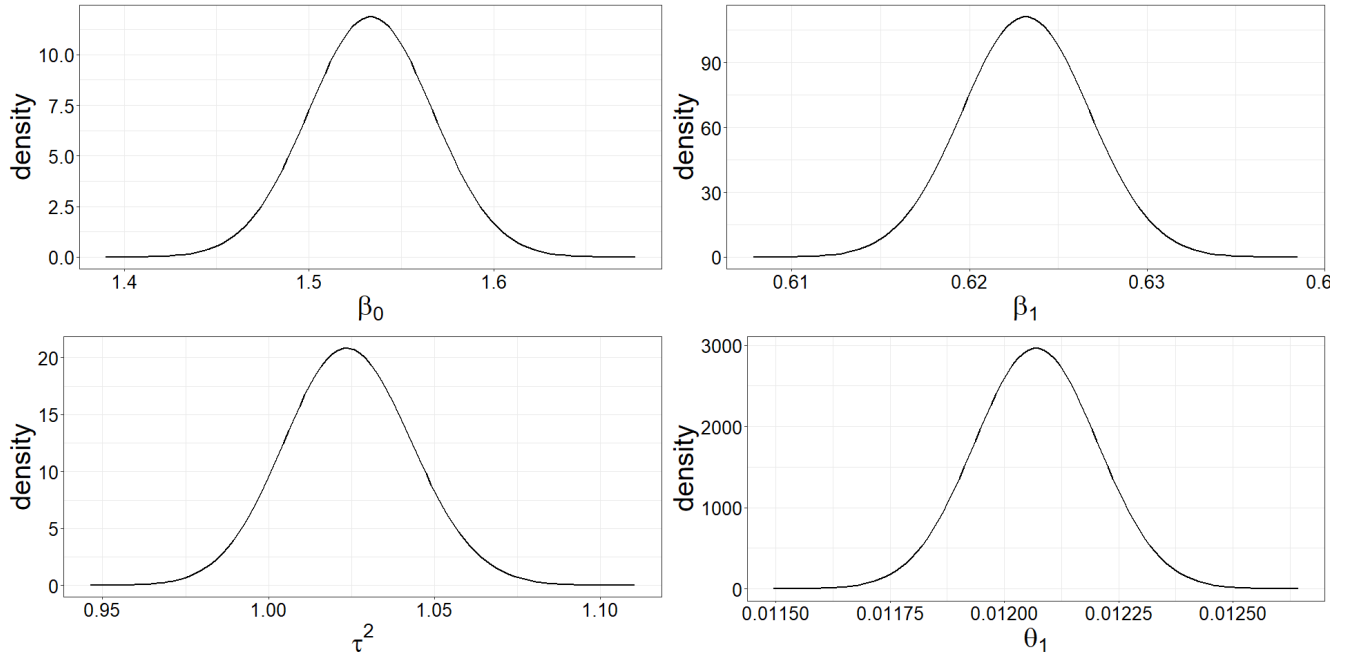


Figure 3.4: The distributions of parameters of close form

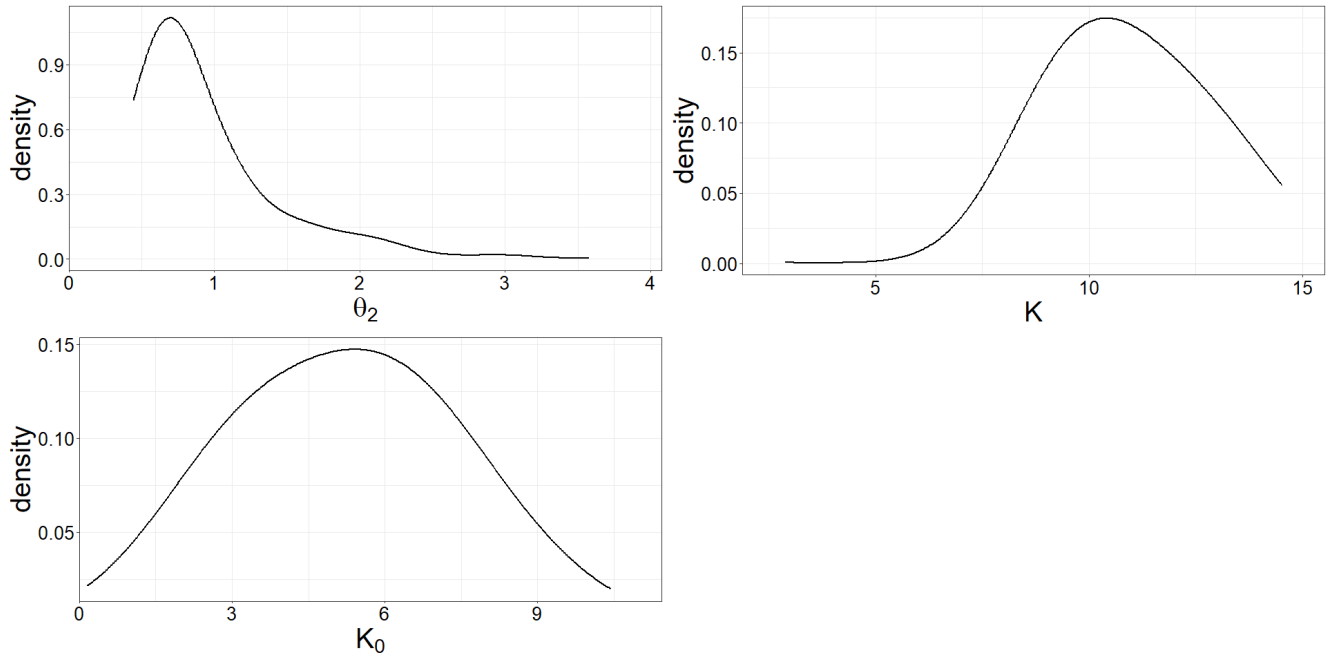


Figure 3.5: The distributions of parameters of no close form

```

#           IS = IS,
#           para = initial.para,
#           true.para = NULL,
#           parallel = TRUE,
#           verbose = TRUE,
#           verbose.VB = TRUE,
#           Ensemble.size = 50,
#           cs = 0.4,
#           ct = 1,
#           Remove.CPU.Count = 2,
#           N.Chunk = 1,
#           itMax = 1e2,
#           tol.vb = 1e-3,
#           tol.real = 1e-3)
load("./data/Generate_Data/Total/Total.Rdata")
print(Total)

##
## Call: spVBEnKs(data = Total_Data, prior = prior, IS = IS, para = initial.para,
##   true.para = NULL, parallel = TRUE, verbose = TRUE, verbose.VB = TRUE,
##   Ensemble.size = 50, cs = 0.4, ct = 1, Remove.CPU.Count = 2,
##   N.Chunk = 1, itMax = 100, tol.vb = 0.01, tol.real = 0.001)
##
## Iterations: 7
##
## Log-likelihood: 179093.444
##
## Parameter estimation:
##      beta0 beta1 tau_2 theta1 theta2      k      k0 iter
## True:   NA    NA    NA     NA     NA     NA     NA    0
## Init:  2.00 0.500 1.000  0.005  1.000  5.000  5.000   0
## Esti:  1.49 0.623 1.039  0.012  0.902 10.722  5.043   7
##
## (...output suppressed due to large dimension!)

```

## 4.1 The distributions of parameters

```

#####
#           the distributions of parameters
#####
P <- plot(Total, n = 500, title.size = 30, text.size = 20, line.size = 1)
P$Beta.plot

P$Close.Dist

P$NoClose.Dist

```

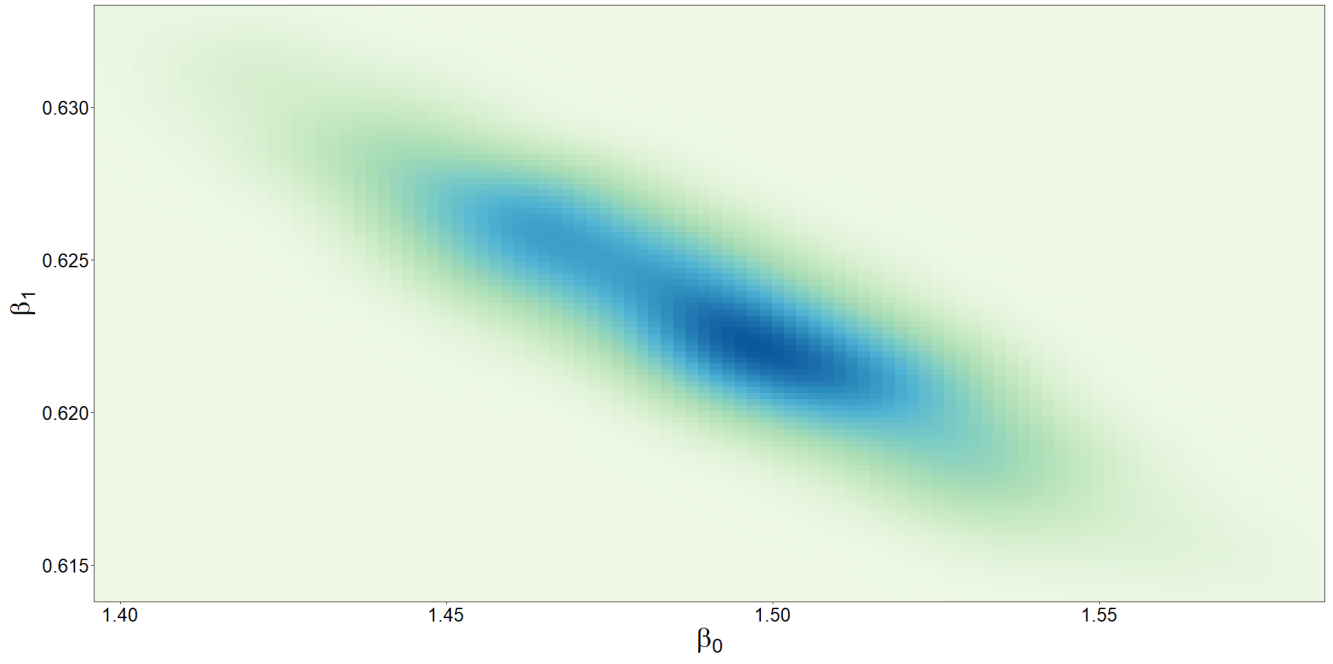


Figure 4.1: The distribution of Beta parameter

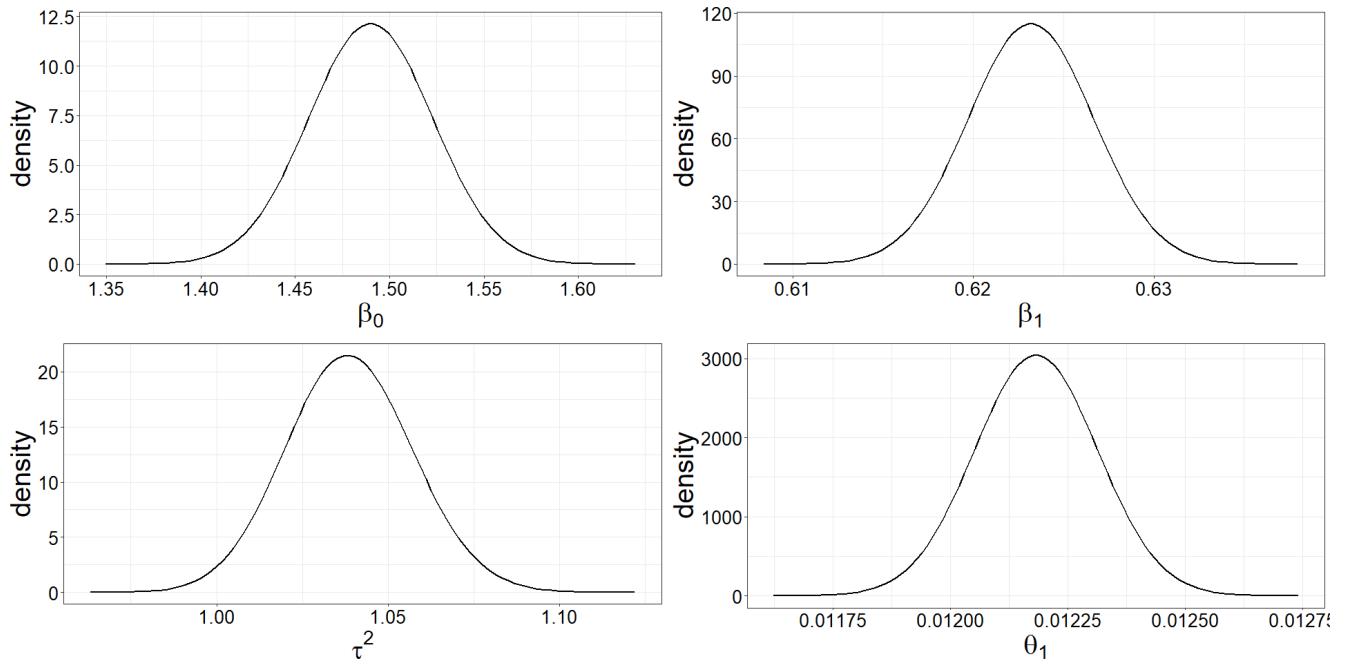


Figure 4.2: The distributions of parameters of close form

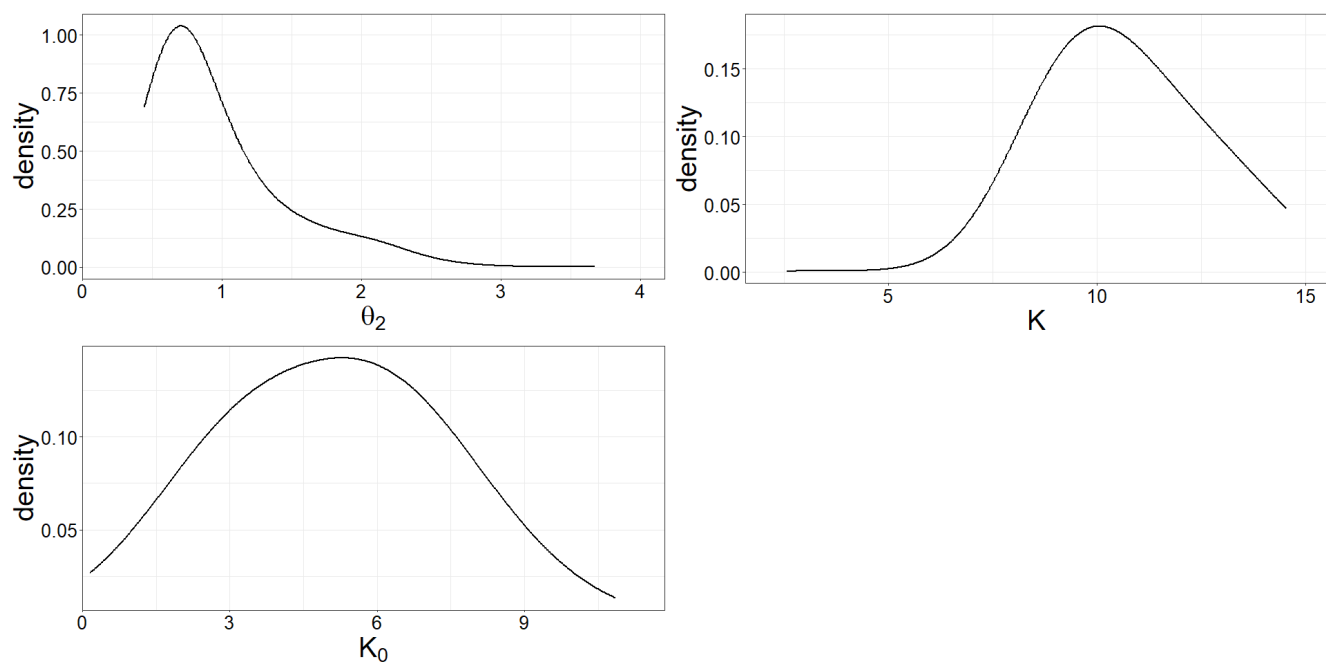


Figure 4.3: The distributions of parameters of no close form