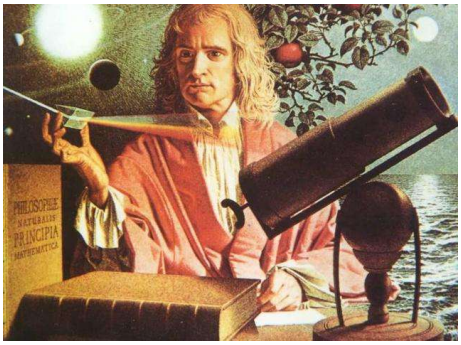
Observational Cosmology: Homework No.2

*Student ID:*_____ *Name:*_____

Nature and nature's laws lay hid in night,
God said "Let Newton be" and all was light.



A. Short questions

1. What are Newton's three laws of motion? What are Kepler's three laws of planetary motion?
2. An astronaut (who weighs 600N on Earth) orbits the Earth at an altitude of 6400 km. If the Earth has a radius of 6400 km, what does the astronaut weigh while in orbit?
3. Saturn is on average 10 AU from the sun. What is the approximate orbital period of Saturn?
4. According to Newton's gravitational force equation, how much smaller or larger is the Sun's gravitational force on Jupiter than the Sun's gravitational force on Earth? Jupiter is 300 times more massive than the Earth and 5 times as far away from the Sun.
5. The Kepler's second law says that a line joining a planet and the Sun sweeps out equal areas during equal intervals of time. Please derive it with the conservation of angular momentum for the circular orbit, $L = m \times v \times r$.(Note: this formula for angular momentum is only for the case that $\vec{r}$ and $\vec{v}$ are perpendicular.)
6. Are Kepler's laws of planetary motion results of Newton's law of gravity? Explain and prove it (at least for circular motion).
7. Newton's law of gravity $F = G\frac{m_1 m_2}{r^2}$ gives the universal formula of gravitational interaction between two mass points.

   (1) Calculate the gravity between the Earth and the Moon.

   (2) Calculate the gravity between the Sun and the Moon.

   (3) Compare the magnitudes of the two forces obtained in the above questions, and explain why the Moon can keep to circle around the Earth.

B. Questions that need more thinking

1. Explain and prove why each year the vernal equinox moves about 50 arc-seconds westwards along the ecliptic.
2. You and your best friend are both astronauts working on a space station. Once you and your friend were asked to go outside of the station to make some repairs, and your friend needed to carry a lot of metal tools. Once outside your friend held you tightly, as he was afraid that he will fall down toward the earth fast because of the metal tools he was carrying. Use what you learned about gravity to convince your friend that he does not need to worry. How can you use such examples to understand gravity is equivalent to curved space, as in Einstein's General Relativity?

3. Is an elliptical orbit a **necessary consequence** of the universal gravitational force, i.e. are circular orbits allowed in the real Universe? Can the central star be located at the center of the orbit if its planets move in elliptical orbits?

4. (Open question: Modify Newton's law of gravity) Newton's law of gravity gives the universal formula of gravity, which has been verified by many physical experiments in the history. Now consider some different cases that the gravity slightly deviates from the standard inverse-square law, such as

$$F = G\frac{m_1 m_2}{r^{2+\delta}},$$

or

$$F = G\frac{m_1 m_2}{r^2} + G'\frac{m_1 m_2}{r^4},$$

where δ and G' are very small quantities. What would happen in these cases? (Use your reasoning and consideration to write down any possible answers)