$$m\ddot{x} = F = -kx$$

$$\begin{cases} \dot{x} = v \\ \dot{v} = -\frac{kx}{m} \end{cases}$$

be

$$\begin{cases} x^{n+1} = x^n + \Delta t\, v^{n+1} \\ v^{n+1} = v^n - \Delta t \frac{k}{m} x^{n+1} \end{cases}$$

$$\frac{1}{1+\Delta t^2 \frac{k}{m}} \begin{bmatrix} 1 & +\Delta t \\ -\Delta t \frac{k}{m} & 1 \end{bmatrix}$$

$$\begin{cases} x^{n+1} - \Delta t\, v^{n+1} = x^n \\ \Delta t \frac{k}{m} x^{n+1} + v^{n+1} = v^n \end{cases}$$

$$\begin{bmatrix} 1 & -\Delta t \\ \Delta t \frac{k}{m} & 1 \end{bmatrix} \begin{bmatrix} x^{n+1} \\ v^{n+1} \end{bmatrix} = \begin{bmatrix} x^n \\ v^n \end{bmatrix}$$

$$\begin{bmatrix} x^{n+1} \\ v^{n+1} \end{bmatrix} = \frac{1}{1+\Delta t^2 \frac{k}{m}} \begin{bmatrix} 1 & +\Delta t \\ -\Delta t \frac{k}{m} & 1 \end{bmatrix} \begin{bmatrix} x^n \\ v^n \end{bmatrix}$$