



4. Nonlinear Systems of Two Hyperbolic Partial Differential Equations

1.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(bu - cw) + g(bu - cw),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(bu - cw) + h(bu - cw).$$
2.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + e^{\lambda u} f(\lambda u - \sigma w),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + e^{\sigma w} g(\lambda u - \sigma w).$$
3.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w g\left(\frac{u}{w}\right).$$
4.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f\left(\frac{u}{w}\right) + \frac{u}{w} h\left(\frac{u}{w}\right),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w g\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$$
5.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u^k f\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w^k g\left(\frac{u}{w}\right).$$
6.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(x, u^k w^m), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w g(x, u^k w^m).$$
7.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g(u^2 + w^2),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g(u^2 + w^2).$$
8.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 - w^2) + w g(u^2 - w^2),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 - w^2) + u g(u^2 - w^2).$$