9. $y_x^{(6)} + ay = 0.$

$1°$. Solution for $a = 0$:

$$y = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 x^4 + C_6 x^5.$$

$2°$. Solution for $a = k^6 > 0$:

$$y = C_1 \cos kx + C_2 \sin kx + \cos\left(\tfrac{1}{2}kx\right)\left(C_3 \cosh \xi + C_4 \sinh \xi\right)$$
$$+ \sin\left(\tfrac{1}{2}kx\right)\left(C_5 \cosh \xi + C_6 \sinh \xi\right), \quad \text{where} \quad \xi = \tfrac{\sqrt{3}}{2}kx.$$

$3°$. Solution for $a = -k^6 < 0$:

$$y = C_1 \cosh kx + C_2 \sinh kx + \cosh\left(\tfrac{1}{2}kx\right)\left(C_3 \cos \xi + C_4 \sin \xi\right)$$
$$+ \sinh\left(\tfrac{1}{2}kx\right)\left(C_5 \cos \xi + C_6 \sin \xi\right), \quad \text{where} \quad \xi = \tfrac{\sqrt{3}}{2}kx.$$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

http://eqworld.ipmnet.ru/en/solutions/ode/ode0409.pdf