40. $y(x) - y\big(\sqrt{a^2 - x^2}\,\big) = f(x), \qquad 0 \le x \le a.$

The function $f(x)$ is assumed to satisfy the condition $f(x) = -f\big(\sqrt{a^2 - x^2}\,\big)$.

Solution:

$$y(x) = \tfrac{1}{2} f(x) + \Phi\big(x,\ \sqrt{a^2 - x^2}\,\big),$$

where $\Phi(x, z) = \Phi(z, x)$ is any symmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

http://eqworld.ipmnet.ru/en/solutions/fe/fe1140.pdf