

EqWorld

5.
$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha w \ln(\beta w)$$
.

1°. Solutions:

$$w(x,y) = \frac{1}{\beta} \exp\left[\frac{1}{4}\alpha(x+A)^2 + \frac{1}{4}\alpha(y+B)^2 + 1\right],$$

$$w(x,y) = \frac{1}{\beta} \exp\left[A(x+B)^2 \pm \sqrt{A\alpha - 4A^2}(x+B)(y+C) + (\frac{1}{4}\alpha - A)(y+C)^2 + \frac{1}{2}\right],$$

where A, B, and C are arbitrary constants.

 2° . There are exact solutions of the following forms:

$$\begin{split} & w(x,y) = F(z), \quad z = Ax + By, \\ & w(x,y) = G(r), \quad r = \sqrt{(x+C_1)^2 + (y+C_2)^2}, \\ & w(x,y) = f(x)g(y). \end{split}$$

References

Shercliff, J. A., Simple rotational flows, J. Fluid Mech., Vol. 82, No. 4, pp. 687–703, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

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