52. $\quad y(x) - y\big(\omega(x)\big) = f(x), \qquad$ **where** $\quad \omega\big(\omega(x)\big) = x.$

The function $f(x)$ is assumed to satisfy the condition $f(x) = -f\big(\omega(x)\big)$.

Solution:

$$y(x) = \tfrac{1}{2} f(x) + \Phi\big(x,\, \omega(x)\big),$$

where $\Phi(x, z) = \Phi(z, x)$ is any symmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.