First-Order Partial Differential Equations > Nonlinear Equations > Section 3.2

2. $\left(\dfrac{\partial w}{\partial x}\right)^2 + \left(\dfrac{\partial w}{\partial y}\right)^2 = a - 2by.$

This equation governs parabolic motion of a point mass in vacuum (the coordinate x is measured along the Earth's surface, the coordinate y is measured vertically upward from the Earth's surface, and a is the gravitational acceleration).

Complete integral:

$$w = C_1 x \pm \frac{1}{3b}(a - C_1^2 - 2by)^{3/2} + C_2,$$

where C_1 and C_2 are arbitrary constants.

References

Appell, P., *Traité de Mécanique Rationnelle, T. 1: Statique. Dynamique du Point (Ed. 6)*, Gauthier-Villars, Paris, 1953.

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde3202.pdf