16. $F\big(x,\, y(\theta_0(x)),\, y(\theta_1(x)),\, \dots,\, y(\theta_{n-1}(x))\big) = 0.$

Notation: $\theta_k(x) \equiv \theta\big(x + \frac{k}{n}T\big)$ with $k = 0,\, 1,\, \dots,\, n-1$. It is assumed that $\theta(x) = \theta(x+T)$ is a periodic function with period T, and the left-hand side of the equation satisfies the condition $F(x, \dots) = F(x+T, \dots)$.

Let us substitute $x + \frac{k}{n}T$, $k = 0,\, 1,\, \dots,\, n-1$, in the original equation for x to obtain the system (the original equation comes first):

$$
\begin{aligned}
&F\big(x,\, y_0,\, y_1,\, \dots,\, y_{n-1}\big) = 0,\\
&F\big(x + \tfrac{1}{n}T,\, y_1,\, y_2,\, \dots,\, y_0\big) = 0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots ,\\
&F\big(x + \tfrac{n-1}{n}T,\, y_{n-1},\, y_0,\, \dots,\, y_{n-2}\big) = 0,
\end{aligned}
\tag{1}
$$

where the short notation $y_k \equiv y\big(\theta_k(x)\big)$ is used.

On eliminating $y_1,\, y_2,\, \dots,\, y_{n-1}$ from the system of nonlinear algebraic (or transcendental) equations (1), one arrives at the solution of the functional equation in implicit form: $\Psi\big(x,\, y_0\big) = 0$, where $y_0 = y\big(\theta(x)\big)$.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

