

4.
$$y(x) + \lambda \int_{a}^{x} (x-t)^{3} y(t) dt = f(x)$$
.

Solution:

$$y(x) = f(x) - \int_{a}^{x} R(x - t)f(t) dt,$$

where

$$R(x) = \begin{cases} k \left[\cosh(kx) \sin(kx) - \sinh(kx) \cos(kx) \right], & k = \left(\frac{3}{2} \lambda \right)^{1/4} & \text{for } \lambda > 0, \\ \frac{1}{2} s \left[\sin(sx) - \sinh(sx) \right], & s = (-6\lambda)^{1/4} & \text{for } \lambda < 0. \end{cases}$$

Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/ie/ie0204.pdf