

4.
$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = ae^{\beta w} + be^{2\beta w}.$$

1°. Traveling-wave solutions:

$$\begin{split} w(x,y) &= -\frac{1}{\beta} \ln \left[-\frac{a\beta}{C_1^2 + C_2^2} + C_3 \exp(C_1 x + C_2 y) + \frac{a^2 \beta^2 - b\beta(C_1^2 + C_2^2)}{4C_3(C_1^2 + C_2^2)^2} \exp(-C_1 x - C_2 y) \right], \\ w(x,y) &= -\frac{1}{\beta} \ln \left[\frac{a\beta}{C_1^2 + C_2^2} + \frac{\sqrt{a^2 \beta^2 + b\beta(C_1^2 + C_2^2)}}{C_1^2 + C_2^2} \right] \sin(C_1 x + C_2 y + C_3) \right], \end{split}$$

where C_1 , C_2 , and C_3 are arbitrary constants.

 2° . For other exact solutions of this equation, see equation 3.1.7 with $f(w) = ae^{\beta w} + be^{2\beta w}$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

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