



3. Нелинейные дифференциальные уравнения в частных производных второго порядка эллиптического типа

3.1. Нелинейные уравнения стационарной теплопроводности вида

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(w)$$

1. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = aw + bw^n.$
2. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = aw^n + bw^{2n-1}.$
3. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = ae^{\beta w}.$
4. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = ae^{\beta w} + be^{2\beta w}.$
5. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha w \ln(\beta w).$
6. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha \sin(\beta w).$
7. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(w).$

3.2. Нелинейные уравнения теплопроводности для анизотропной

среды вида $\frac{\partial}{\partial x} \left[f(x) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[g(y) \frac{\partial w}{\partial y} \right] = f(w)$

1. $\frac{\partial}{\partial x} \left(ax^n \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(by^m \frac{\partial w}{\partial y} \right) = f(w).$
2. $a \frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left(be^{\mu y} \frac{\partial w}{\partial y} \right) = f(w).$
3. $\frac{\partial}{\partial x} \left(ae^{\beta x} \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(be^{\mu y} \frac{\partial w}{\partial y} \right) = f(w).$
4. $\frac{\partial}{\partial x} \left[f(x) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[g(y) \frac{\partial w}{\partial y} \right] = kw \ln w.$

3.3. Нелинейные уравнения теплопроводности для анизотропной

среды вида $\frac{\partial}{\partial x} \left[f(w) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[g(w) \frac{\partial w}{\partial y} \right] = h(w)$

1. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left[(\alpha w + \beta) \frac{\partial w}{\partial y} \right] = 0.$ Стационарное уравнение Хохлова–Заболоцкой.

2. $\frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left(a e^{\beta w} \frac{\partial w}{\partial y} \right) = 0, \quad a > 0.$

3. $\frac{\partial}{\partial x} \left[f(w) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[g(w) \frac{\partial w}{\partial y} \right] = 0.$

Нелинейное уравнение диффузии (теплопроводности) для анизотропной среды.