

$$\mathbf{9.}\quad \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u^{1+kn} f \left(u^n w^m \right), \ \frac{\partial w}{\partial t} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w^{1-km} g \left(u^n w^m \right).$$

Self-similar solution:

$$u = (C_1 t + C_2)^{-\frac{1}{kn}} y(\xi), \quad w = (C_1 t + C_2)^{\frac{1}{km}} z(\xi), \quad \xi = \frac{x}{\sqrt{C_1 t + C_2}},$$

where C_1 and C_2 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$a\xi^{-n}(\xi^n y'_{\xi})'_{\xi} + \frac{1}{2}C_1\xi y'_{\xi} + \frac{C_1}{kn}y + y^{1+kn}f(y^n z^m) = 0,$$

$$b\xi^{-n}(\xi^n z'_{\xi})'_{\xi} + \frac{1}{2}C_1\xi z'_{\xi} - \frac{C_1}{km}z + z^{1-km}g(y^n z^m) = 0.$$

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