

Exact Solutions > Functional Equations > Nonlinear Functional Equations with Several Independent Variables > Bilinear Functional Equation - 2

13.
$$f_1(x)g_1(y) + f_2(x)g_2(y) + f_3(x)g_3(y) + f_4(x)g_4(y) = 0$$
.

Bilinear functional equation - 2.

Equations of this type often arise in generalized separation of variables in nonlinear PDEs.

1°. Solution:

$$f_1(x) = C_1 f_3(x) + C_2 f_4(x),$$
 $f_2(x) = C_3 f_3(x) + C_4 f_4(x),$ $g_3(y) = -C_1 g_1(y) - C_3 g_2(y),$ $g_4(y) = -C_2 g_1(y) - C_4 g_2(y)$

dependent on four arbitrary constants C_1, \ldots, C_4 . The functions on the right-hand sides of the solution are arbitrary.

 2° . The equation has also two other solutions:

$$\begin{split} f_1(x) &= C_1 f_4(x), \quad f_2(x) = C_2 f_4(x), \quad f_3(x) = C_3 f_4(x), \quad g_4(y) = -C_1 g_1(y) - C_2 g_2(y) - C_3 g_3(y); \\ g_1(y) &= C_1 g_4(y), \quad g_2(y) = C_2 g_4(y), \quad g_3(y) = C_3 g_4(y), \quad f_4(x) = -C_1 f_1(x) - C_2 f_2(x) - C_3 f_3(x) \end{split}$$

involving three arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations (Supplement S.4.4)*, Chapman & Hall/CRC Press, Boca Raton, 2004.

Bilinear Functional Equation - 2

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