

Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

10.
$$x_{tt}'' = x f(x^2 + y^2, y/x) - y g(y/x)$$
, $y_{tt}'' = y f(x^2 + y^2, y/x) + x g(y/x)$.

Particular solution:

$$u = r(t)\cos\beta, \quad w = r(t)\sin\beta,$$

where the constant β is determined by the transcendental equation

$$g(\tan \beta) = 0$$

and the function r = r(t) satisfies the solvable autonomous second-order equation

$$r_{tt}^{\prime\prime} = rf(r^2, \tan\beta).$$

For $g(z) \equiv 0$, β is an arbitrary constant.

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0310.pdf