1. $y''_{xx} = f(y).$

Second-order autonomous equation.

Solution:

$$\int \left[C_1 + 2 \int f(y)\, dy \right]^{-1/2} dy = C_2 \pm x,$$

where C_1 and C_2 are arbitrary constants.

Particular solutions: $y = A_k$, where A_k are roots of the algebraic (transcendental) equation $f(A_k) = 0$.

References

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