

$$\begin{aligned} \textbf{4.} \quad & \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f \left(\frac{u}{w} \right) + \frac{u}{w} h \left(\frac{u}{w} \right), \\ & \frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial w} \right) + w g \left(\frac{u}{w} \right) + h \left(\frac{u}{w} \right). \end{aligned}$$

Solution:

$$u = k\theta(x, t), \quad w = \theta(x, t),$$

where k is a root of the algebraic (transcendental) equation f(k) = g(k), and the function $\theta = \theta(x, t)$ satisfies the linear equation

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial \theta}{\partial x} \right) + f(k) \theta + h(k).$$

1

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/syspde/spde4104.pdf