

22.
$$y_{xx}^{\prime\prime} = [e^{\alpha x}f(y) + \alpha]y_x^{\prime}$$
.

The substitution $w(y) = e^{-\alpha x} y_x'$ leads to a first-order separable equation: $w_y' = f(y)$.

$$\int \frac{dy}{F(y) + C_1} = C_2 + \frac{1}{\alpha} e^{\alpha x}, \qquad F(y) = \int f(y) \, dy,$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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http://eqworld.ipmnet.ru/en/solutions/ode/ode0322.pdf