4. $\quad y(x) + \lambda \displaystyle\int_a^x (x-t)^3 y(t)\, dt = f(x).$

Solution:

$$y(x) = f(x) - \int_a^x R(x-t) f(t)\, dt,$$

where

$$R(x) = \begin{cases} k\big[\cosh(kx)\sin(kx) - \sinh(kx)\cos(kx)\big], & k = \left(\tfrac{3}{2}\lambda\right)^{1/4} & \text{for } \lambda > 0, \\ \tfrac{1}{2} s\big[\sin(sx) - \sinh(sx)\big], & s = (-6\lambda)^{1/4} & \text{for } \lambda < 0. \end{cases}$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.