

$$\mathbf{24.} \quad \frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + u f\left(u^2 + w^2, \frac{w}{u}\right) - w g\left(\frac{w}{u}\right), \quad \frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + w f\left(u^2 + w^2, \frac{w}{u}\right) + u g\left(\frac{w}{u}\right).$$

Solution:

$$u = r(x, t) \cos \varphi(t), \quad w = r(x, t) \sin \varphi(t),$$

where the function  $\varphi = \varphi(t)$  is determined by the separable first-order ordinary differential equation

$$\varphi_t' = g(\tan \varphi),$$

and the function r = r(x, t) is determined by the differential equation

$$\frac{\partial r}{\partial t} = a \frac{\partial^2 r}{\partial x^2} + r f(r^2, \tan \varphi).$$

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