

4.
$$(ax^2 + bx + c)^3 y_{xxx}^{""} = ky$$
.

The transformation

$$\xi = \int \frac{dx}{ax^2 + bx + c}, \quad w = \frac{y}{ax^2 + bx + c}$$

leads to a constant coefficient linear equation:

$$w'''_{\xi\xi\xi} + (4ac - b^2)w'_{\xi} = kw.$$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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http://eqworld.ipmnet.ru/en/solutions/ode/ode0404.pdf