Systems of Ordinary Differential Equations > Nonlinear Systems of Three and More Equations

9. $x''_{tt} = xF, \ \ y''_{tt} = yF, \ \ z''_{tt} = zF,$ **where** $F = F(x, y, z, t, x'_t, y'_t, z'_t).$

First integrals (laws of conservation of areas):

$$zy'_t - yz'_t = C_1,$$
$$xz'_t - zx'_t = C_2,$$
$$yx'_t - xy'_t = C_3,$$

where C_1, C_2, and C_3 are arbitrary constants.

Corollary of the conservation laws:

$$C_1 x + C_2 y + C_3 z = 0.$$

Consequently, all integral lines are plane curves.

Remark. The function Φ may also depend on the second- and higher-order derivatives with respect to t.

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0409.pdf