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2.
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$
, $m = 1, 2, \ldots, n$.

System of n linear algebraic equations in n unknowns.

A system of n linear equations in n unknowns can be rewritten in unfolded form as

where $a_{11}, a_{12}, \ldots, a_{nn}$ are coefficients; b_1, b_2, \ldots, b_n are free terms; and x_1, x_2, \ldots, x_n are unknowns.

A set of n numbers x_1, \ldots, x_n that, when substituted for the unknowns in system (1), turn the equations of (1) into identities is called a solution of system (1). A system of equations is called compatible if it has at least one solution. A system that has no solution is called incompatible.

Kramer's rule. If the determinant of system (1) is nonzero,

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2k} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nk} & \dots & a_{nn} \end{pmatrix} \neq 0,$$
 (2)

the system has a unique solution given by Kramer's rule:

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \quad \dots, \quad x_k = \frac{\Delta_k}{\Delta}, \quad \dots, \quad x_n = \frac{\Delta_n}{\Delta},$$

where Δ_k is the determinant obtained by replacing the kth column in the determinant (2) by the column of free terms:

$$\Delta_k = \det \begin{pmatrix} a_{11} & a_{12} & \dots & b_1 & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & b_2 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & b_n & \dots & a_{nn} \end{pmatrix}.$$

References

Kurosh, A. G., Lectures on General Algebra, Chelsea Publ., New York, 1965.

Mishina, A. P. and Proskuryakov, I. V., Higher Algebra, Pergamon Press, New York, 1965.

Korn, G. A. and Korn, T. M., Mathematical Handbook for Scientists and Engineers, 2nd Edition, Dover, New York, 2000.

Weisstein, E. W., CRC Concise Encyclopedia of Mathematics, 2nd Edition, Chapman & Hall/CRC, Boca Raton, 2003.

Bronshtein, I.N. and Semendyayev, K.A., Handbook of Mathematics, 4th Edition, Springer-Verlag, Berlin, 2004.

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