

Exact Solutions > First-Order Partial Differential Equations > Linear Partial Differential Equations

1. First-Order Linear Partial Differential Equations

1.1. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y)\frac{\partial w}{\partial y}=0$

1.
$$\frac{\partial w}{\partial x} + [f(x)y + g(x)]\frac{\partial w}{\partial y} = 0$$
.

2.
$$\frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = 0$$
.

3.
$$\frac{\partial w}{\partial x} + \left[f(x)e^{\lambda y} + g(x) \right] \frac{\partial w}{\partial y} = 0$$
.

4.
$$f(x)\frac{\partial w}{\partial x} + g(y)\frac{\partial w}{\partial y} = 0$$
.

5.
$$[f(y) + amx^ny^{m-1}] \frac{\partial w}{\partial x} - [g(x) + anx^{n-1}y^m] \frac{\partial w}{\partial y} = 0.$$

6.
$$\left[e^{\alpha x}f(y)+c\beta\right]\frac{\partial w}{\partial x}-\left[e^{\beta y}g(x)+c\alpha\right]\frac{\partial w}{\partial y}=0.$$

7.
$$\frac{\partial w}{\partial x} + f(ax + by + c) \frac{\partial w}{\partial y} = 0$$
.

8.
$$\frac{\partial w}{\partial x} + f\left(\frac{y}{x}\right) \frac{\partial w}{\partial y} = 0$$
.

9.
$$x \frac{\partial w}{\partial x} + y f(x^n y^m) \frac{\partial w}{\partial y} = 0$$
.

10.
$$\frac{\partial w}{\partial x} + y f(e^{\alpha x} y^m) \frac{\partial w}{\partial y} = 0.$$

11.
$$x\frac{\partial w}{\partial x} + f(x^n e^{\alpha y})\frac{\partial w}{\partial y} = 0.$$

1.2. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y)\frac{\partial w}{\partial y}=h(x,y)$

1.
$$a\frac{\partial w}{\partial x} + b\frac{\partial w}{\partial y} = f(x)$$
.

2.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)y^k$$
.

3.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)e^{\lambda y}$$
.

4.
$$a\frac{\partial w}{\partial x} + b\frac{\partial w}{\partial y} = f(x) + g(y)$$
.

5.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)g(y)$$
.

6.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)$$
.

7.
$$\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)$$
.

8.
$$\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)h(y)$$
.

9.
$$\frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = h(x)$$
.

10.
$$\frac{\partial w}{\partial x} + \left[f(x) + g(x)e^{\lambda y} \right] \frac{\partial w}{\partial y} = h(x).$$

11.
$$ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y)$$
.

12.
$$f(x)\frac{\partial w}{\partial x} + g(y)\frac{\partial w}{\partial y} = h_1(x) + h_2(y)$$
.

13.
$$f(x)\frac{\partial w}{\partial x} + g(y)\frac{\partial w}{\partial y} = h(x, y)$$
.

14.
$$f(y)\frac{\partial w}{\partial x} + g(x)\frac{\partial w}{\partial y} = h(x,y)$$
.

1.3. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y)\frac{\partial w}{\partial y}=h(x,y)w+r(x,y)$

1.
$$a\frac{\partial w}{\partial x} + b\frac{\partial w}{\partial y} = f(x)w$$
.

2.
$$a\frac{\partial w}{\partial x} + b\frac{\partial w}{\partial y} = f(x)w + g(x)$$
.

3.
$$a\frac{\partial w}{\partial x} + b\frac{\partial w}{\partial y} = [f(x) + g(y)]w$$
.

4.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w$$
.

5.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w + g(x, y)$$
.

6.
$$ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x)w + g(x)$$
.

7.
$$ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y)w$$
.

8.
$$x \frac{\partial w}{\partial x} + ay \frac{\partial w}{\partial y} = f(x, y)w + g(x, y)$$
.

9.
$$f(x)\frac{\partial w}{\partial x} + g(y)\frac{\partial w}{\partial y} = [h_1(x) + h_2(y)]w$$
.

10.
$$f_1(x)\frac{\partial w}{\partial x} + f_2(y)\frac{\partial w}{\partial y} = aw + g_1(x) + g_2(y)$$
.

11.
$$f(x)\frac{\partial w}{\partial x} + g(y)\frac{\partial w}{\partial y} = h(x,y)w + r(x,y).$$

12.
$$f(y)\frac{\partial w}{\partial x} + g(x)\frac{\partial w}{\partial y} = h(x,y)w + r(x,y)$$
.

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpdetoc1.pdf