

15.
$$y_{xx}^{\prime\prime} = \frac{\psi_{xx}^{\prime\prime}}{\psi} y + \psi^{-3} f\left(\frac{y}{\psi}\right), \qquad \psi = \psi(x).$$

The transformation $\xi = \int \frac{dx}{\psi^2}$, $w = \frac{y}{\psi}$ leads to an autonomous equation of the form 3.1: $w_{\xi\xi}'' = f(w)$.

$$\int \left[C_1 + 2 \int f(w) \, dw \right]^{-1/2} dw = C_2 \pm \int \frac{dx}{\psi^2(x)},$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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