



2. Nonlinear Functional Equations with One Independent Variable

2.1. Functional Equations with Quadratic Nonlinearity

1. $y(x+1) - ay^2(x) = f(x).$
2. $y(2x) - ay^2(x) = 0.$
3. $y(2x) - 2y^2(x) + a = 0.$
4. $y(x)y(a-x) = b^2.$
5. $y(x)y(a-x) = f^2(x).$
6. $y^2(x) + y^2(a-x) = b^2.$
7. $y^2(x) + Ay(x)y(a-x) + By^2(a-x) + Cy(x) + Dy(a-x) = f(x).$
8. $y(x)y(ax) = f(x).$
9. $y(x^2) - ay^2(x) = 0.$
10. $y(x)y(x^a) = f(x), \quad a > 0.$
11. $y(x)y(a/x) = b^2.$
12. $y(x)y(a/x) = f^2(x).$
13. $y^2(x) + Ay(x)y(a/x) + By^2(a/x) + Cy(x) + Dy(a/x) = f(x).$
14. $y(x)y\left(\frac{a-x}{1+bx}\right) = A^2.$
15. $y(x)y\left(\frac{a-x}{1+bx}\right) = f^2(x).$
16. $y^2(x) + Ay(x)y\left(\frac{a-x}{1+bx}\right) + By(x) = f(x).$
17. $y(x)y(\sqrt{a^2-x^2}) = b^2, \quad 0 \leq x \leq a.$
18. $y(x)y(\sqrt{a^2-x^2}) = f^2(x), \quad 0 \leq x \leq a.$
19. $y(\sin x)y(\cos x) = a^2.$
20. $y(\sin x)y(\cos x) = f^2(x).$
21. $y(x)y(\omega(x)) = b^2, \quad \text{where } \omega(\omega(x)) = x.$
22. $y(x)y(\omega(x)) = f^2(x), \quad \text{where } \omega(\omega(x)) = x.$

2.2. Functional Equations with Power-Law Nonlinearity

1. $y(x+a) - by^\lambda(x) = f(x).$
2. $y^\lambda(x)y(a-x) = f(x).$
3. $y^{2n+1}(x) + y^{2n+1}(a-x) = b, \quad n = 1, 2, \dots$
4. $y^\lambda(x)y(a/x) = f(x).$
5. $y^\lambda(x)y\left(\frac{a-x}{1+bx}\right) = f(x).$
6. $y^\lambda(x)y\left(\frac{ax-\beta}{x+b}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
7. $y^\lambda(x)y\left(\frac{bx+\beta}{a-x}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
8. $y^\lambda(x)y(x^a) = f(x).$
9. $y^\lambda(x)y(\sqrt{a^2-x^2}) = f(x).$
10. $y^\lambda(\sin x)y(\cos x) = f(x).$

2.3. Nonlinear Functional Equations of General Form

1. $F(x, y(x), y(x+a)) = 0.$
2. $F(x, y(x), y(a-x)) = 0.$
3. $F(x, y(x), y(ax)) = 0, \quad a > 0.$
4. $F(x, y(x), y(a/x)) = 0.$
5. $F\left(x, y(x), y\left(\frac{a-x}{1+bx}\right)\right) = 0.$
6. $F\left(x, y(x), y\left(\frac{ax-\beta}{x+b}\right)\right) = 0, \quad \beta = a^2 + ab + b^2.$
7. $F\left(x, y(x), y\left(\frac{bx+\beta}{a-x}\right)\right) = 0, \quad \beta = a^2 + ab + b^2.$
8. $F(x, y(x), y(x^a)) = 0.$
9. $F(x, y(x), y(\sqrt{a^2-x^2})) = 0, \quad 0 \leq x \leq a.$
10. $F(x, y(\sin x), y(\cos x)) = 0.$
11. $F(x, y(x), y(\omega(x))) = 0, \quad \text{where } \omega(\omega(x)) = x.$
12. $F(x, y(x), y(x+1), y(x+2)) = 0.$

$$13. \quad F\left(x, y(x), y\left(\frac{ax - \beta}{x + b}\right), y\left(\frac{bx + \beta}{a - x}\right)\right) = 0, \quad \beta = a^2 + ab + b^2.$$

$$14. \quad F(x, y(x), y(x + 1), \dots, y(x + n)) = 0.$$

$$15. \quad F(x, y(x), y^{[2]}(x), \dots, y^{[n]}(x)) = 0, \quad y^{[n]}(x) = y(y^{[n-1]}(x)).$$

$$16. \quad F(x, y(\theta_0(x)), y(\theta_1(x)), \dots, y(\theta_{n-1}(x))) = 0.$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

Copyright © 2004–2005 Andrei D. Polyanin

<http://eqworld.ipmnet.ru/en/solutions/fe/fe-toc2.pdf>