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5. $\left(\dfrac{\partial w}{\partial x}\right)^2 + \left(\dfrac{\partial w}{\partial y}\right)^2 = f(x) + g(y).$

A separable equation. Complete integral:

$$w = \pm \int \sqrt{f(x) + C_1}\, dx \pm \int \sqrt{g_2(y) - C_1}\, dy + C_2,$$

where C_1 and C_2 are arbitrary constants. The signs before each of the integrals can be chosen independently of each other.

References

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