

11.
$$\frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g(u^2 + w^2),$$
$$\frac{\partial w}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g(u^2 + w^2).$$

Periodic (in time) solution:

$$u = r(x)\cos\left[\theta(x) + C_1t + C_2\right], \quad w = r(x)\sin\left[\theta(x) + C_1t + C_2\right],$$

where C_1 and C_2 are arbitrary constants, and the function r = r(x) are determined by the system of ordinary differential equations

$$ar''_{xx} - ar(\theta'_x)^2 + \frac{an}{x}r'_x + rf(r^2) = 0,$$

$$ar\theta''_{xx} + 2ar'_x\theta'_x + \frac{an}{x}r\theta'_x + rg(r^2) - C_1r = 0.$$

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/syspde/spde2211.pdf