

Exact Solutions > Ordinary Differential Equations > Second-Order Linear Ordinary Differential Equations > Constant Coefficient Linear Differential Equation

3.
$$y_{xx}'' + ay_x' + by = 0$$
.

Second-order constant coefficient linear differential equation. In physics this equation is called an equation of damped vibrations.

Solution:
$$y = \begin{cases} \exp\left(-\frac{1}{2}ax\right)\left[C_1\exp\left(\frac{1}{2}\lambda x\right) + C_2\exp\left(-\frac{1}{2}\lambda x\right)\right] & \text{if } \lambda^2 = a^2 - 4b > 0, \\ \exp\left(-\frac{1}{2}ax\right)\left[C_1\sin\left(\frac{1}{2}\lambda x\right) + C_2\cos\left(\frac{1}{2}\lambda x\right)\right] & \text{if } \lambda^2 = 4b - a^2 > 0, \\ \exp\left(-\frac{1}{2}ax\right)\left(C_1x + C_2\right) & \text{if } a^2 = 4b. \end{cases}$$

References

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen, B. G. Teubner, Leipzig, 1977.

Boyce, W. E. and DiPrima, R. C., Elementary Differential Equations, 7th Edition, Wiley, New York, 2000.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

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