First-Order Partial Differential Equations > Nonlinear Equations > Section 3.3

7. $\dfrac{\partial w}{\partial x} + F\left(x, \dfrac{\partial w}{\partial y}\right) = g(x)w.$

Complete integral:

$$w = \varphi(x)(C_1 y + C_2) - \varphi(x) \int F\big(x, C_1\varphi(x)\big)\,\frac{dx}{\varphi(x)}, \quad \text{where} \quad \varphi(x) = \exp\left[\int g(x)\,dx\right],$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.