

First-Order Partial Differential Equations > Nonlinear Equations > Section 3.3

11.
$$F_1\left(x, \frac{\partial w}{\partial x}\right) + F_2\left(y, \frac{\partial w}{\partial y}\right) + aw = 0.$$

A separable equation.

Complete integral:

$$w = \varphi(x) + \psi(y),$$

where the functions $\varphi = \varphi(x)$ and $\psi = \psi(y)$ are determined from the ordinary differential equations

$$F_1(x, \varphi_x') + a\varphi = C_1, \quad F_2(y, \psi_y') + a\psi = -C_1,$$

where C_1 is an arbitrary constant. If $a \neq 0$, one can set $C_1 = 0$ in these equations.

References

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, II, Partielle Differentialgleichungen Erster Ordnung für eine gesuchte Funktion, Akad. Verlagsgesellschaft Geest & Portig, Leipzig, 1965.

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