

Exact Solutions > Functional Equations > Linear Difference and Functional Equations with One Independent Variable > nth-Order Constant-Coefficient Linear Nonhomogeneous Difference Equation

16. $y(x+n) + a_{n-1}y(x+n-1) + \ldots + a_1y(x+1) + a_0y(x) = f(x)$.

nth-order constant-coefficient linear nonhomogeneous difference equation.

1°. Solution:

$$y(x) = Y(x) + \bar{y}(x),$$

where Y(x) is the general solution of the homogeneous equation,

$$Y(x+n) + a_{n-1}Y(x+n-1) + \dots + a_1Y(x+1) + a_0Y(x) = 0$$

(see the preceding equation), and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

- 2°. For $f(x) = \sum_{k=0}^{n} A_k x^n$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=0}^{n} B_k x^n$, where the constants B_k are found by the method of undetermined coefficients.
- 3°. For $f(x) = \sum_{k=1}^{n} A_k \exp(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \exp(\lambda_k x)$, where the constants B_k are found by the method of undetermined coefficients.
- 4° . For $f(x) = \sum_{k=1}^{n} A_k \cos(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants B_k and D_k are found by the method of undetermined coefficients.
- 5°. For $f(x) = \sum_{k=1}^{n} A_k \sin(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants B_k and D_k are found by the method of undetermined coefficients.

Reference

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