

12.
$$y(x) + A \int_a^x \sinh[\lambda(x-t)]y(t) dt = f(x)$$
.

1°. Solution for $\lambda(A - \lambda) > 0$:

$$y(x) = f(x) - \frac{A\lambda}{k} \int_{a}^{x} \sin[k(x-t)]f(t) dt$$
, where $k = \sqrt{\lambda(A-\lambda)}$.

 2° . Solution for $\lambda(A - \lambda) < 0$:

$$y(x) = f(x) - \frac{A\lambda}{k} \int_{a}^{x} \sinh[k(x-t)]f(t) dt, \quad \text{where} \quad k = \sqrt{\lambda(\lambda - A)}.$$

 3° . Solution for $A = \lambda$:

$$y(x) = f(x) - \lambda^2 \int_a^x (x - t) f(t) dt.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

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