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Laplace Transforms: Expressions with Trigonometric Functions

No	Original function, $f(x)$	Laplace transform, $\widetilde{f}(p) = \int_0^\infty e^{-px} f(x)\, dx$
1	$\sin(ax)$	$\dfrac{a}{p^2 + a^2}$
2	$|\sin(ax)|, \qquad a > 0$	$\dfrac{a}{p^2 + a^2} \coth\left(\dfrac{\pi p}{2a}\right)$
3	$\sin^{2n}(ax), \qquad n = 1, 2, \ldots$	$\dfrac{a^{2n}(2n)!}{p\left[p^2 + (2a)^2\right]\left[p^2 + (4a)^2\right] \ldots \left[p^2 + (2na)^2\right]}$
4	$\sin^{2n+1}(ax), \qquad n = 1, 2, \ldots$	$\dfrac{a^{2n+1}(2n+1)!}{\left[p^2 + a^2\right]\left[p^2 + 3^2 a^2\right] \ldots \left[p^2 + (2n+1)^2 a^2\right]}$
5	$x^n \sin(ax), \qquad n = 1, 2, \ldots$	$\dfrac{n!\, p^{n+1}}{\left(p^2 + a^2\right)^{n+1}} \displaystyle\sum_{0 \le 2k \le n} (-1)^k C_{n+1}^{2k+1} \left(\dfrac{a}{p}\right)^{2k+1}$
6	$\dfrac{1}{x} \sin(ax)$	$\arctan\left(\dfrac{a}{p}\right)$
7	$\dfrac{1}{x} \sin^2(ax)$	$\tfrac{1}{4} \ln\left(1 + 4a^2 p^{-2}\right)$
8	$\dfrac{1}{x^2} \sin^2(ax)$	$a \arctan(2a/p) - \tfrac{1}{4} p \ln\left(1 + 4a^2 p^{-2}\right)$
9	$\sin\left(2\sqrt{ax}\right)$	$\dfrac{\sqrt{\pi a}}{p\sqrt{p}} e^{-a/p}$
10	$\dfrac{1}{x} \sin\left(2\sqrt{ax}\right)$	$\pi\, \mathrm{erf}\left(\sqrt{a/p}\right)$
11	$\cos(ax)$	$\dfrac{p}{p^2 + a^2}$
12	$\cos^2(ax)$	$\dfrac{p^2 + 2a^2}{p\left(p^2 + 4a^2\right)}$
13	$x^n \cos(ax), \qquad n = 1, 2, \ldots$	$\dfrac{n!\, p^{n+1}}{\left(p^2 + a^2\right)^{n+1}} \displaystyle\sum_{0 \le 2k \le n+1} (-1)^k C_{n+1}^{2k} \left(\dfrac{a}{p}\right)^{2k}$
14	$\dfrac{1}{x}\left[1 - \cos(ax)\right]$	$\tfrac{1}{2} \ln\left(1 + a^2 p^{-2}\right)$
15	$\dfrac{1}{x}\left[\cos(ax) - \cos(bx)\right]$	$\dfrac{1}{2} \ln \dfrac{p^2 + b^2}{p^2 + a^2}$

No	Original function, $f(x)$	Laplace transform, $\widetilde{f}(p) = \int_0^\infty e^{-px} f(x)\, dx$
16	$\sqrt{x}\cos\big(2\sqrt{ax}\,\big)$	$\frac{1}{2}\pi^{1/2}p^{-5/2}(p-2a)e^{-a/p}$
17	$\dfrac{1}{\sqrt{x}}\cos\big(2\sqrt{ax}\,\big)$	$\sqrt{\pi/p}\,e^{-a/p}$
18	$\sin(ax)\sin(bx)$	$\dfrac{2abp}{\big[p^2+(a+b)^2\big]\big[p^2+(a-b)^2\big]}$
19	$\cos(ax)\sin(bx)$	$\dfrac{b\big(p^2-a^2+b^2\big)}{\big[p^2+(a+b)^2\big]\big[p^2+(a-b)^2\big]}$
20	$\cos(ax)\cos(bx)$	$\dfrac{p\big(p^2+a^2+b^2\big)}{\big[p^2+(a+b)^2\big]\big[p^2+(a-b)^2\big]}$
21	$e^{bx}\sin(ax)$	$\dfrac{a}{(p-b)^2+a^2}$
22	$e^{bx}\cos(ax)$	$\dfrac{p-b}{(p-b)^2+a^2}$

Notation: erfc z is the complementary error function.

References

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