Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

18. $\quad x''_{tt} = x'_t \Phi(x, y, t, x'_t, y'_t) + f(y), \quad y''_{tt} = -y'_t \Phi(x, y, t, x'_t, y'_t) + g(x).$

First integral:

$$x'_t y'_t - \int f(y)\, dy - \int g(x)\, dx = C,$$

where C is an arbitrary constant.

$Remark.$ The function Φ can also depend on the second and higher derivatives with respect to t.

http://eqworld.ipmnet.ru/en/solutions/sysode/sode0318.pdf