

$$\begin{aligned} \textbf{8.} \quad & \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 - w^2) + w g(u^2 - w^2), \\ & \frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 - w^2) + u g(u^2 - w^2). \end{aligned}$$

1°. Solution:

$$u=r(x)\cosh\big[\theta(x)+C_1t+C_2\big], \qquad w=r(x)\sinh\big[\theta(x)+C_1t+C_2\big],$$

where C_1 and C_2 are arbitrary constants, and the functions r = r(x) and $\theta(x)$ are determined by the system of ordinary differential equations

$$\begin{split} ar_{xx}^{\prime\prime} + ar(\theta_x^\prime)^2 + \frac{an}{x}r_x^\prime - C_1^2r + rf(r^2) &= 0, \\ ar\theta_{xx}^{\prime\prime} + 2ar_x^\prime\theta_x^\prime + \frac{an}{x}r\theta_x^\prime + rg(r^2) &= 0. \end{split}$$

 2° . For n = 0, there is an exact solution of the form

$$u = r(z)\cosh\left[\theta(z) + C_1t + C_2\right], \quad w = r(z)\sinh\left[\theta(z) + C_1t + C_2\right], \quad z = kx - \lambda t.$$

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