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2.
$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = A$$
.

Nonhomogeneous Monge-Ampere equation (Monge-Ampère equation).

1°. General solution in parametric form for $A = a^2 > 0$:

$$x = \frac{\beta - \lambda}{2a}, \quad y = \frac{\psi'(\lambda) - \varphi'(\beta)}{2a}, \quad w = \frac{(\beta + \lambda)[\psi'(\lambda) - \varphi'(\beta)] + 2\varphi(\beta) - 2\psi(\lambda)}{4a},$$

where β and λ are the parameters, $\varphi = \varphi(\beta)$ and $\psi = \psi(\lambda)$ are arbitrary functions.

2°. Solutions:

$$\begin{split} w(x,y) &= \pm \frac{\sqrt{A}}{C_2} x (C_1 x + C_2 y) + \varphi(C_1 x + C_2 y) + C_3 x + C_4 y, \\ w(x,y) &= C_1 y^2 + C_2 x y + \frac{1}{4C_1} (C_2^2 - A) x^2 + C_3 y + C_4 x + C_5, \\ w(x,y) &= \frac{1}{x + C_1} \left(C_2 y^2 + C_3 y + \frac{C_3^2}{4C_2} \right) - \frac{A}{12C_2} (x^3 + 3C_1 x^2) + C_4 y + C_5 x + C_6, \\ w(x,y) &= \pm \frac{2\sqrt{A}}{3C_1C_2} (C_1 x - C_2^2 y^2 + C_3)^{3/2} + C_4 x + C_5 y + C_6, \end{split}$$

where C_1, \ldots, C_6 are arbitrary constants and $\varphi = \varphi(z)$ is an arbitrary function.

References

Goursat, E., A Course of Mathematical Analysis. Vol 3. Part 1 [Russian translation], Gostekhizdat, Moscow, 1933.
Polyanin, A. D. and Zaitsev, V. F., Handbook of Nonlinear Partial Differential Equations, Chapman & Hall/CRC, Boca Raton, 2004.

Nonhomogeneous Monge-Ampere Equation

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