

Exact Solutions > Nonlinear Partial Differential Equations > Second-Order Hyperbolic Partial Differential Equations > Klein-Gordon Equation with a Power-Law Nonlinearity - 1

1. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + aw + bw^n$$
.

Klein-Gordon equation with a power-law nonlinearity - 1.

1°. Traveling-wave solutions for a > 0:

$$w(x,t) = \left[\frac{2b \sinh^2 z}{a(n+1)}\right]^{\frac{1}{1-n}}, \quad z = \frac{1}{2}\sqrt{a}(1-n)(x \sinh C_1 \pm t \cosh C_1) + C_2 \quad \text{if } b(n+1) > 0,$$

$$w(x,t) = \left[-\frac{2b \cosh^2 z}{a(n+1)}\right]^{\frac{1}{1-n}}, \quad z = \frac{1}{2}\sqrt{a}(1-n)(x \sinh C_1 \pm t \cosh C_1) + C_2 \quad \text{if } b(n+1) < 0,$$

where  $C_1$  and  $C_2$  are arbitrary constants.

 $2^{\circ}$ . Traveling-wave solutions for a < 0 and b(n + 1) > 0:

$$w(x,t) = \left[ -\frac{2b\cos^2 z}{a(n+1)} \right]^{\frac{1}{1-n}}, \quad z = \frac{1}{2}\sqrt{|a|}(1-n)(x\sinh C_1 \pm t\cosh C_1) + C_2.$$

- 3°. For a=0, there is a self-similar solution of the form  $w=t^{\frac{2}{1-n}}F(z)$ , where z=x/t.
- 4°. For other exact solutions of this equation, see the nonlinear Klein–Gordon equation with  $f(w) = aw + bw^n$ .

## Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton. 2004.

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