38. $xy''_{xx} + (xf + a)y'_x + (a-1)fy = 0, \qquad f = f(x).$

Particular solution: $y_0 = x^{1-a}$.

Solution:

$$y = y_0 \left(C_1 + C_2 \int \frac{e^{-F}}{y_0^2}\, dx \right), \qquad \text{where} \quad F = a \ln|x| + \int f\, dx,$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.