

First-Order Partial Differential Equations > Quasilinear Equations > Section 2.3

5.
$$\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(y)$$
.

General solution:

$$x = \int_{y_0}^{y} \psi \left(G(t) - G(y) + F(w) \right) dt + \Phi \left(F(w) - G(y) \right),$$

where $G(y)=\int g(y)\,dy, \ F(w)=\int f(w)\,dw,$ and $\Phi(u)$ is an arbitrary function. The function $\psi=\psi(z)$ is defined parametrically by $\psi=\frac{1}{f(w)},\ z=F(w).$

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., Handbook of First Order Partial Differential Equations, Taylor & Francis, London, 2002.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpde2305.pdf