

$$\begin{aligned} \mathbf{13.} \quad & \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g\left(\frac{w}{u} \right), \\ & \frac{\partial w}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g\left(\frac{w}{u} \right). \end{aligned}$$

Solution:

$$u = r(x, t) \cos \varphi(t), \quad w = r(x, t) \sin \varphi(t),$$

where the function $\varphi = \varphi(t)$ is determined by the autonomous ordinary differential equation

$$\varphi_t' = g(\tan \varphi),\tag{1}$$

and the function r = r(x, t) is determined by the differential equation

$$\frac{\partial r}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial r}{\partial x} \right) + r f(r^2). \tag{2}$$

The general solution of equation (1) is expressed in implicit form as

$$\int \frac{d\varphi}{g(\tan\varphi)} = t + C.$$

Equation (2) admits a stationary exact solution r = r(x).

Remark. The function f can also be explicitly dependent on x.

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