5. $y_{xxxx}''''= Ay^{-5/3}$.

Solution:

$$\int \left[9(C_1^2 - 4C_0C_2) + 54Aw - 2w^3\right]^{-1/2} \frac{dw}{w} \pm \int \frac{dx}{3P(x)} = C_3,$$

where

$$w = P(x)y^{-2/3}, \qquad P(x) = C_2x^2 + C_1x + C_0,$$

and C_0, C_1, C_2, and C_3 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

http://eqworld.ipmnet.ru/en/solutions/ode/ode0505.pdf