

$$6. \quad \int_{-\infty}^{\infty} \frac{\operatorname{sign}(x-t)}{|x-t|^{1-\lambda}} y(t) \, dt = f(x), \qquad 0 < \lambda < 1.$$

Solution:

$$y(x) = \frac{\lambda}{2\pi} \cot\left(\frac{\pi\lambda}{2}\right) \int_{-\infty}^{\infty} \frac{f(x) - f(t)}{|x - t|^{1+\lambda}} \operatorname{sign}(x - t) dt.$$

References

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