First-Order Partial Differential Equations > Nonlinear Equations > Section 3.1

8. $\dfrac{\partial w}{\partial x} + a\left(\dfrac{\partial w}{\partial y}\right)^2 + b\dfrac{\partial w}{\partial y} = f(x) + g(y).$

Complete integral:

$$w = -C_1 x + C_2 + \int f(x)\,dx - \frac{b}{2a}y \pm \frac{1}{2a}\int \sqrt{4ag(y) + b^2 + 4aC_1}\,dy,$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde3108.pdf