

Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

8. 
$$x_{tt}'' = kxr^{-3}$$
,  $y_{tt}'' = kyr^{-3}$ , where  $r = \sqrt{x^2 + y^2}$ .

Equation of motion of point mass in the xy-plane under gravitational force.

On proceeding to polar coordinates by the formulas

$$x = r \cos \varphi$$
,  $y = r \sin \varphi$ ,  $r = r(t)$ ,  $\varphi = \varphi(t)$ ,

one can obtain the first integrals

$$r^2 \varphi_t' = C_1$$
,  $(r_t')^2 + r^2 (\varphi_t')^2 = -2kr^{-1} + C_2$ ,

where  $C_1$  and  $C_2$  are arbitrary constants. Integrating further, with  $C_1 \neq 0$ , yields

$$r[C\cos(\varphi - \varphi_0) - k] = C_1^2,$$
  $C^2 = C_1^2 C_2 + k^2.$ 

This equation describes conical sections. The function  $\varphi(t)$  can be determined using either first integral.

## Reference

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen, B. G. Teubner, Leipzig, 1977.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/sysode/sode0308.pdf