

7.
$$x^2 y_{xxxx}^{\prime\prime\prime\prime} + 6x y_{xxx}^{\prime\prime\prime} + 6y_{xx}^{\prime\prime} - a^2 y = 0$$
.

Equation of transverse vibrations of a pointed bar.

Solution

$$y = \frac{1}{\sqrt{x}} \left[C_1 J_1 \left(2 \sqrt{ax} \right) + C_2 Y_1 \left(2 \sqrt{ax} \right) + C_3 I_1 \left(2 \sqrt{ax} \right) + C_4 K_1 \left(2 \sqrt{ax} \right) \right],$$

where $J_1(z)$ and $Y_1(z)$ are the Bessel functions, and $I_1(z)$ and $K_1(z)$ are the modified Bessel functions.

References

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