

$$\begin{aligned} \mathbf{5.} \quad & \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \bigg(x^n \frac{\partial u}{\partial x} \bigg) + u f \bigg(\frac{u}{w} \bigg) + \frac{u}{w} h \bigg(\frac{u}{w} \bigg), \\ & \frac{\partial w}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \bigg(x^n \frac{\partial u}{\partial w} \bigg) + w g \bigg(\frac{u}{w} \bigg) + h \bigg(\frac{u}{w} \bigg). \end{aligned}$$

Solution:

$$u = \varphi(t)G(t)\left[\theta(x,t) + \int \frac{h(\varphi)}{G(t)}\,dt\right], \quad w = G(t)\left[\theta(x,t) + \int \frac{h(\varphi)}{G(t)}\,dt\right], \quad G(t) = \exp\left[\int g(\varphi)\,dt\right],$$

where the function $\varphi = \varphi(t)$ is determined by the separable nonlinear first-order ordinary differential equation

$$\varphi_t' = [f(\varphi) - g(\varphi)]\varphi,\tag{1}$$

and the function $\theta = \theta(x, t)$ satisfies the linear heat equation

$$\frac{\partial \theta}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial \theta}{\partial x} \right).$$

The general solution of equation (1) is written out in implicit form as

$$\int \frac{d\varphi}{[f(\varphi) - g(\varphi)]\varphi} = t + C.$$

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