

$$\begin{aligned} \textbf{10.} \quad & \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + cu \ln u + u f(x, u^k w^m), \\ & \frac{\partial w}{\partial t} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + cw \ln w + w g(x, u^k w^m). \end{aligned}$$

Multiplicative separable solution:

$$u = \exp(Ame^{ct})y(x), \quad w = \exp(-Ake^{ct})z(x),$$

where A is an arbitrary constant, and the functions y = y(x) and z = z(x) are determined by the system of ordinary differential equations

$$ax^{-n}(x^ny'_x)'_x + cy \ln y + yf(x, y^k z^m) = 0,$$

$$bx^{-n}(x^nz'_x)'_x + cz \ln z + zg(x, y^k z^m) = 0.$$

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