

50.
$$gy''_{xx} + \frac{1}{2}g'_xy'_x = f(y)h(y'_x\sqrt{g}), \qquad g = g(x).$$

The substitution $w(y) = y'_x \sqrt{g}$ leads to a first-order separable equation: $ww'_y = f(y)h(w)$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/ode/ode0350.pdf