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4. Nonlinear Systems of Two Hyperbolic Partial Differential Equations

$$\begin{aligned} \mathbf{1.} \quad & \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(bu - cw) + g(bu - cw), \\ & \frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(bu - cw) + h(bu - cw). \end{aligned}$$

2.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + e^{\lambda u} f(\lambda u - \sigma w),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + e^{\sigma w} g(\lambda u - \sigma w).$$

$$3. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f \left(\frac{u}{w} \right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w g \left(\frac{u}{w} \right).$$

$$4. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f \left(\frac{u}{w} \right) + \frac{u}{w} h \left(\frac{u}{w} \right),$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial w} \right) + w g \left(\frac{u}{w} \right) + h \left(\frac{u}{w} \right).$$

$$5. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u^k f\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w^k g\left(\frac{u}{w}\right).$$

$$\mathbf{6.} \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(x, u^k w^m), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w g(x, u^k w^m).$$

7.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g(u^2 + w^2),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g(u^2 + w^2).$$

8.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 - w^2) + w g(u^2 - w^2),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 - w^2) + u g(u^2 - w^2).$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.