EqWorld http://eqworld.ipmnet.ru

Exact Solutions > Nonlinear Partial Differential Equations >
Second-Order Parabolic (Evolution) Partial Differential Equations

1. Nonlinear Parabolic Equations

1.1. Nonlinear Heat Equations of the Form $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + f(w)$

1. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw(1 - w).$ *Fisher equation.*

2. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw - bw^3.$ *Newell–Whitehead equation.*

3. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} - w(1 - w)(a - w).$ *FitzHugh–Nagumo equation.*

4. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw + bw^m.$

5. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + a + be^{\lambda w}.$

6. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw \ln w.$

1.2. Nonlinear Heat Equations of the Form $\dfrac{\partial w}{\partial t} = \dfrac{\partial}{\partial x}\left[f(w)\dfrac{\partial w}{\partial x}\right] + g(w)$

1. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^m \dfrac{\partial w}{\partial x}\right).$ *Heat equation with a power-law nonlinearity.*

2. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^m \dfrac{\partial w}{\partial x}\right) + bw.$

3. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^m \dfrac{\partial w}{\partial x}\right) + bw^{m+1}.$

4. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^m \dfrac{\partial w}{\partial x}\right) + bw^{1-m}.$

5. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^{2n} \dfrac{\partial w}{\partial x}\right) + bw^{1-n}.$

6. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^n \dfrac{\partial w}{\partial x}\right) + bw + c_1 w^m + c_2 w^k.$

7. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(e^{\lambda w} \dfrac{\partial w}{\partial x}\right).$ *Heat equation with a exponential nonlinearity.*

8. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(e^{\lambda w} \dfrac{\partial w}{\partial x}\right) + b + c_1 e^{\beta w} + c_2 e^{\gamma w}.$

9. $\dfrac{\partial w}{\partial t} = \dfrac{\partial}{\partial x}\left[f(w)\dfrac{\partial w}{\partial x}\right].$ *Nonlinear heat equation of general form.*

10. $\dfrac{\partial w}{\partial t} = \dfrac{\partial}{\partial x}\left[f(w)\dfrac{\partial w}{\partial x}\right] + g(w).$

1.3. Other Nonlinear Parabolic Equations

1. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + w\dfrac{\partial w}{\partial x}.$ *Burgers equation.*

2. $\dfrac{\partial w}{\partial t} + \sigma w\dfrac{\partial w}{\partial x} = a\dfrac{\partial^2 w}{\partial x^2} + b_0 + b_1 w + b_2 w^2 + b_3 w^3.$

3. $\dfrac{\partial w}{\partial t} = \dfrac{1}{x^n}\dfrac{\partial}{\partial x}\left[x^n f(w)\dfrac{\partial w}{\partial x}\right] + g(w).$

4. $\dfrac{\partial w}{\partial t} = \dfrac{\partial}{\partial x}\left[f(w)\left(\dfrac{\partial w}{\partial x}\right)^n\right] + g(w).$

1.4. Nonlinear Schrodinger Equations

1. $i\dfrac{\partial w}{\partial t} + \dfrac{\partial^2 w}{\partial x^2} + k|w|^2 w = 0.$ *Schrodinger equation with a cubic nonlinearity.*

2. $i\dfrac{\partial w}{\partial t} + \dfrac{\partial^2 w}{\partial x^2} + A|w|^{2n} w = 0.$ *Schrodinger equation with a power-law nonlinearity.*

3. $i\dfrac{\partial w}{\partial t} + \dfrac{\partial^2 w}{\partial x^2} + f(|w|)w = 0.$ *Schrodinger equation of general form.*

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.