12. $y(x)y(a/x) = f^2(x).$

The right-hand side function is assumed to satisfy the condition $f(x) = \pm f(a/x)$. To be specific, we assume $f(x) = f(a/x)$.

Solution:

$$y(x) = \pm f(x) \exp\left[\Phi(x, a/x)\right],$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

For $\Phi(x, z) = C(\ln x - \ln z)$, there are particular solutions of the form

$$y = \pm a^{-C} x^{2C} f(x),$$

where C is an arbitrary constant.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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