

$$7. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g(u^2 + w^2),$$
 
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g(u^2 + w^2).$$

 $1^{\circ}$ . Periodic solution in t:

$$u = r(x)\cos\left[\theta(x) + C_1t + C_2\right], \quad w = r(x)\sin\left[\theta(x) + C_1t + C_2\right],$$

where  $C_1$  and  $C_2$  are arbitrary constants, and the functions r = r(x) and  $\theta(x)$  are determined by the system of ordinary differential equations

$$ar''_{xx} - ar(\theta'_x)^2 + \frac{an}{x}r'_x + C_1^2r + rf(r^2) = 0,$$
  
$$ar\theta''_{xx} + 2ar'_x\theta'_x + \frac{an}{x}r\theta'_x + rg(r^2) = 0.$$

 $2^{\circ}$ . For n = 0, there is an exact solution of the form

$$u = r(z)\cos\left[\theta(z) + C_1t + C_2\right], \quad w = r(z)\sin\left[\theta(z) + C_1t + C_2\right], \quad z = kx - \lambda t.$$

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