5. $\quad y(x+1) - R(x)y(x) = 0, \qquad R(x) = a\,\dfrac{(x-\lambda_1)(x-\lambda_2)\dots(x-\lambda_n)}{(x-\mu_1)(x-\mu_2)\dots(x-\mu_m)}.$

Solution:

$$y(x) = \Theta(x)a^x\,\frac{\Gamma(x-\lambda_1)\Gamma(x-\lambda_2)\dots\Gamma(x-\lambda_n)}{\Gamma(x-\mu_1)\Gamma(x-\mu_2)\dots\Gamma(x-\mu_m)},$$

where $\Gamma(x)$ is the gamma function, $\Theta(x) = \Theta(x+1)$ is an arbitrary periodic function with unit period.

The simplest particular solution corresponds to $\Theta(x) \equiv 1$.

References

Mirolyubov, A. A., and Soldatov, M. A., *Linear Homogeneous Difference Equations* [in Russian], Nauka, Moscow, 1981 (page 52).

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.