

Exact Solutions > Ordinary Differential Equations > Higher-Order Linear Ordinary Differential Equations > Euler Equation

16. 
$$a_n x^n y_x^{(n)} + a_{n-1} x^{n-1} y_x^{(n-1)} + \dots + a_1 x y_x' + a_0 y = 0$$
.

## Euler equation.

 $1^{\circ}$ . If all roots  $\lambda_k$  (k = 1, 2, ..., n) of the algebraic equation

$$\sum_{\nu=1}^{n} a_{\nu} \lambda(\lambda - 1) \dots (\lambda - \nu + 1) = -a_0$$

are different, the general solution of the Euler equation is given by:

$$y = C_1 |x|^{\lambda_1} + C_2 |x|^{\lambda_2} + \dots + C_n |x|^{\lambda_n}.$$

 $2^{\circ}$ . In the general case, the substitution  $t = \ln |x|$  leads to a constant coefficient linear equation of the form 4.15:

$$\sum_{\nu=1}^{n} a_{\nu} D(D-1) \dots (D-\nu+1) y = -a_{0} y, \quad \text{where} \quad D = \frac{d}{dx}.$$

## References

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen, B. G. Teubner, Leipzig, 1977.

**Polyanin, A. D. and Zaitsev, V. F.,** *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

Euler Equation

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