

3.
$$\int_a^b \frac{y(t)}{|x-t|^k} dt = f(x), \quad 0 < k < 1.$$

It is assumed that $|a| + |b| < \infty$. Solution:

$$y(x) = \frac{1}{2\pi} \cot(\frac{1}{2}\pi k) \frac{d}{dx} \int_{a}^{x} \frac{f(t) dt}{(x-t)^{1-k}} - \frac{1}{\pi^{2}} \cos^{2}(\frac{1}{2}\pi k) \int_{a}^{x} \frac{Z(t)F(t)}{(x-t)^{1-k}} dt,$$

where

$$Z(t) = (t - a)^{\frac{1+k}{2}} (b - t)^{\frac{1-k}{2}}, \quad F(t) = \frac{d}{dt} \left[\int_a^t \frac{d\tau}{(t - \tau)^k} \int_\tau^b \frac{f(s) \, ds}{Z(s)(s - \tau)^{1-k}} \right].$$

References

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