

11.
$$y_x^{(n)} = axy + b$$
, $a > 0$.

Solution:

$$y = \sum_{\nu=0}^n C_\nu \varepsilon_\nu \int_0^\infty \exp \left[\varepsilon_\nu x t - \frac{t^{n+1}}{a(n+1)} \right] dt, \quad \ \varepsilon_\nu = \exp \left(\frac{2\pi \nu i}{n+1} \right),$$

where
$$\sum_{\nu=0}^{n} C_{\nu} = \frac{b}{a}$$
 and $i^2 = -1$.

References

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