



3. Nonlinear Systems of Two Ordinary Differential Equations

3.1. Systems of First-Order Ordinary Differential Equations

1. $x'_t = x^n F(x, y), \quad y'_t = g(y)F(x, y).$
2. $x'_t = e^{\lambda x} F(x, y), \quad y'_t = g(y)F(x, y).$
3. $x'_t = F(x, y), \quad y'_t = G(x, y).$ *Autonomous system of general form.*
4. $x'_t = f_1(x)g_1(y)\Phi(x, y, t), \quad y'_t = f_2(x)g_2(y)\Phi(x, y, t).$
5. $x = tx'_t + F(x'_t, y'_t), \quad y = ty'_t + G(x'_t, y'_t).$ *Clairaut system.*

3.2. Systems of Second-Order Ordinary Differential Equations

6. $x''_{tt} = x f(ax - by) + g(ax - by), \quad y''_{tt} = y f(ax - by) + h(ax - by).$
7. $x''_{tt} = x f(y/x), \quad y''_{tt} = y g(y/x).$
8. $x''_{tt} = kxr^{-3}, \quad y''_{tt} = kyr^{-3},$ **where** $r = \sqrt{x^2 + y^2}.$
Equation of motion of a point mass in gravitational field.
9. $x''_{tt} = x f(r), \quad y''_{tt} = y f(r),$ **where** $r = \sqrt{x^2 + y^2}.$
Equation of motion of a point mass in central force field.
10. $x''_{tt} = x f(x^2 + y^2, y/x) - yg(y/x), \quad y''_{tt} = y f(x^2 + y^2, y/x) + xg(y/x).$
11. $x''_{tt} = -f(y)g(v)x'_t, \quad y''_{tt} = -f(y)g(v)y'_t - a,$ **where** $v = \sqrt{(x'_t)^2 + (y'_t)^2}.$
Equation of motion of a projectile.
12. $x''_{tt} + a(t)x = x^{-3}f(y/x), \quad y''_{tt} + a(t)y = y^{-3}g(y/x).$
Generalized Ermakov (Yermakov) system.
13. $x''_{tt} = \frac{1}{x^3}F\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad y''_{tt} = \frac{1}{y^3}G\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad \varphi(t) = \sqrt{at^2 + bt + c}.$
14. $x''_{tt} = f(y'_t/x'_t), \quad y''_{tt} = g(y'_t/x'_t).$
15. $x''_{tt} = x\Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} = y\Phi(x, y, t, x'_t, y'_t).$
16. $x''_{tt} + x^{-3}f(y/x) = x\Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} + y^{-3}g(y/x) = y\Phi(x, y, t, x'_t, y'_t).$
17. $x''_{tt} = F(t, tx'_t - x, ty'_t - y), \quad y''_{tt} = G(t, tx'_t - x, ty'_t - y).$
18. $x''_{tt} = x'_t\Phi(x, y, t, x'_t, y'_t) + f(y), \quad y''_{tt} = -y'_t\Phi(x, y, t, x'_t, y'_t) + g(x).$

$$19. \quad x''_{tt} = ay'_t\Phi(x, y, t, x'_t, y'_t) + f(x), \quad y''_{tt} = bx'_t\Phi(x, y, t, x'_t, y'_t) + g(y).$$

$$20. \quad x''_{tt} = f(y'_t)\Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} = g(x'_t)\Phi(x, y, t, x'_t, y'_t).$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/sysode/sode-toc3.pdf>