

10. y(x+a) - f(x)y(x) = 0.

Here, f(x) = f(x + a) is a prescribed periodic function with period a. Solution:

$$y(x) = \Theta(x) [f(x)]^{x/a},$$

where $\Theta(x) = \Theta(x+a)$ is an arbitrary periodic function with period a.

For $\Theta(x) \equiv \text{const}$, there is a particular solution $y(x) = C[f(x)]^{x/a}$, where C is an arbitrary constant.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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