



5. Systems of Partial Differential Equations of General Form

Notation: L is an arbitrary linear operator.

5.1. Linear Systems of Partial Differential Equations

$$1. \quad \frac{\partial u}{\partial t} = L[u] + f_1(t)u + g_1(t)w, \quad \frac{\partial w}{\partial t} = L[w] + f_2(t)u + g_2(t)w.$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = L[u] + a_1 u + b_1 w, \quad \frac{\partial^2 w}{\partial t^2} = L[w] + a_2 u + b_2 w.$$

5.2. Nonlinear Systems of Two Partial Differential Equations Containing the First Derivatives in t

$$1. \quad \frac{\partial u}{\partial t} = L[u] + u f(t, au - cw) + g(t, au - cw), \quad \frac{\partial w}{\partial t} = L[w] + w f(t, au - cw) + h(t, au - cw).$$

$$2. \quad \frac{\partial u}{\partial t} = L_1[u] + u f\left(\frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L_2[w] + w g\left(\frac{u}{w}\right).$$

$$3. \quad \frac{\partial u}{\partial t} = L[u] + u f\left(t, \frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L[w] + w g\left(t, \frac{u}{w}\right).$$

$$4. \quad \frac{\partial u}{\partial t} = L[u] + u f\left(\frac{u}{w}\right) + g\left(\frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L[w] + w f\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$$

$$5. \quad \frac{\partial u}{\partial t} = L[u] + u f\left(t, \frac{u}{w}\right) + \frac{u}{w} h\left(t, \frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L[w] + w g\left(t, \frac{u}{w}\right) + h\left(t, \frac{u}{w}\right).$$

$$6. \quad \frac{\partial u}{\partial t} = L[u] + u f\left(t, \frac{u}{w}\right) \ln u + u g\left(t, \frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L[w] + w f\left(t, \frac{u}{w}\right) \ln w + w h\left(t, \frac{u}{w}\right).$$

5.3. Nonlinear Systems of Two Partial Differential Equations Containing the Second Derivatives in t

$$1. \quad \frac{\partial^2 u}{\partial t^2} = L[u] + u f(t, au - bw) + g(t, au - bw), \quad \frac{\partial^2 w}{\partial t^2} = L[w] + w f(t, au - bw) + h(t, au - bw).$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = L_1[u] + u f\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = L_2[w] + w g\left(\frac{u}{w}\right).$$

3. $\frac{\partial^2 u}{\partial t^2} = L[u] + uf\left(t, \frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = L[w] + wg\left(t, \frac{u}{w}\right).$
4. $\frac{\partial^2 u}{\partial t^2} = L[u] + uf\left(\frac{u}{w}\right) + g\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = L[w] + wf\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$
5. $\frac{\partial^2 u}{\partial t^2} = L[u] + au \ln u + uf\left(t, \frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = L[w] + aw \ln w + wg\left(t, \frac{u}{w}\right).$

5.4. Nonlinear Systems of Many Partial Differential Equations Containing the First Derivatives in t

1.
$$\frac{\partial u_m}{\partial t} = L[u_m] + u_m f(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n) + g_m(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n), \quad m = 1, \dots, n.$$
2.
$$\begin{aligned} \frac{\partial u_m}{\partial t} &= L[u_m] + u_m f_m\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right) + \frac{u_m}{u_n} g\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right), \\ \frac{\partial u_n}{\partial t} &= L[u_n] + u_n f_n\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right) + g\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right), \quad m = 1, \dots, n-1. \end{aligned}$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/syspde/spde-toc5.pdf>