

$$\mathbf{4.} \quad \frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + f(w), \qquad a > 0.$$

To n=1 and n=2 there correspond nonlinear waves with axial and central symmetry, respectively. Functional separable solution:

$$w = w(\xi), \quad \xi = \sqrt{ak(t+C)^2 - kx^2},$$

where the function $w(\xi)$ is determined by the ordinary differential equation $w''_{\xi\xi} + (1+n)\xi^{-1}w'_{\xi} = (ak)^{-1}f(w)$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

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http://eqworld.ipmnet.ru/en/solutions/npde/npde2204.pdf