22. $\displaystyle\int_a^x [\ln(x-t) + A]y(t)\,dt = f(x).$

Solution:

$$y(x) = -\frac{d}{dx}\int_a^x \nu_A(x-t)f(t)\,dt, \qquad \nu_A(x) = \frac{d}{dx}\int_0^\infty \frac{x^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz,$$

where $\mathcal{C} = 0.5772\ldots$ is the Euler constant and $\Gamma(z)$ is the gamma function.

For $a = 0$, the solution can be written in the form

$$y(x) = -\int_0^x f_{tt}''(t)\,dt \int_0^\infty \frac{(x-t)^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz - f_x'(0)\int_0^\infty \frac{x^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz.$$

References

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