

13.
$$y(x) - \lambda \int_0^x J_0(x-t)y(t) dt = f(x)$$
.

Here, $J_0(z)$ is the Bessel function of the first kind.

Solution:

$$y(x) = f(x) + \int_0^x R(x - t)f(t) dt,$$

where

$$R(x) = \lambda \cos\left(\sqrt{1-\lambda^2}\,x\right) + \frac{\lambda^2}{\sqrt{1-\lambda^2}} \sin\left(\sqrt{1-\lambda^2}\,x\right) + \frac{\lambda}{\sqrt{1-\lambda^2}} \int_0^x \sin\left[\sqrt{1-\lambda^2}\,(x-t)\right] \frac{J_1(t)}{t} \, dt.$$

References

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