3. $F\big(x,\, y(x),\, y(ax)\big) = 0, \qquad a > 0.$

The transformation $z = \ln x$, $w(z) = y(x)$ leads to the equation

$$F\big(e^z,\, w(z),\, w(z+b)\big) = 0, \qquad b = \ln a.$$

See equation 1 in the current subsection.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.