

14. y(x) + y(a - x) = 0.

Solution:

$$y(x) = \Phi(x, a - x),$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function with two arguments.

As $\Phi(x,z)$, one may take $\Phi(x,z) = \varphi(x,z) - \varphi(z,x)$, where $\varphi(x,z)$ is any function of two arguments. A special case of this formula is $\Phi(x,z) = \varphi(x) - \varphi(z)$, where $\varphi(x)$ is an arbitrary function of a single argument.

Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations) [in Russian], Faktorial, Moscow, 1998.

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