First-Order Partial Differential Equations > Quasilinear Equations > Section 2.3

8. $\dfrac{\partial w}{\partial x} + \big[\, f(w) + g(x)\big]\dfrac{\partial w}{\partial y} = h(w).$

General solution:

$$y = \int \frac{f(w)}{h(w)}\,dw + \int_{w_0}^{w} \frac{g\big(H(t) - H(w) + x\big)}{h(t)}\,dt + \Phi\big(x - H(w)\big),$$

where $H(x) = \displaystyle\int \frac{dw}{h(w)}$ and $\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde2308.pdf