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Bernoulli Equation

4. $g(x)y'_x = f_1(x)y + f_n(x)y^n.$

Bernoulli equation. Here, n is an arbitrary number.

$1°$. For $n \neq 1$, the substitution $w(x) = y^{1-n}$ leads to a linear equation:

$$g(x)w'_x = (1-n)f_1(x)w + (1-n)f_n(x).$$

$2°$. Solution:

$$y^{1-n} = Ce^F + (1-n)e^F \int e^{-F} \frac{f_n(x)}{g(x)}\, dx, \quad \text{where} \quad F(x) = (1-n)\int \frac{f_1(x)}{g(x)}\, dx,$$

C is an arbitrary constant.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Boyce, W. E. and DiPrima, R. C., *Elementary Differential Equations, 7th Edition*, Wiley, New York, 2000.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition* , Chapman & Hall/CRC, Boca Raton, 2003.

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