First-Order Partial Differential Equations > Quasilinear Equations > Section 2.3

4. $\dfrac{\partial w}{\partial x} + f(w)\dfrac{\partial w}{\partial y} = g(x).$

General solution:

$$y = \int_{x_0}^{x} f\big(G(t) - G(x) + w\big)\, dt + \Phi\big(w - G(x)\big), \qquad G(x) = \int g(x)\, dx,$$

where $\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.