

$25. \quad \frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + uf\left(u^2 - w^2, \frac{w}{u}\right) + wg\left(\frac{w}{u}\right), \quad \frac{\partial w}{\partial t} = a\frac{\partial^2 w}{\partial x^2} + wf\left(u^2 - w^2, \frac{w}{u}\right) + ug\left(\frac{w}{u}\right).$

Solution:

$$u = r(x, t) \cosh \varphi(t), \quad w = r(x, t) \sinh \varphi(t),$$

where the function $\varphi = \varphi(t)$ is determined by the separable first-order ordinary differential equation

$$\varphi_t' = g(\tanh \varphi),$$

and the function r = r(x, t) is determined by the differential equation

$$\frac{\partial r}{\partial t} = a \frac{\partial^2 r}{\partial x^2} + r f(r^2, \tanh \varphi).$$

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/syspde/spde2125.pdf