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8. $\dfrac{\partial w}{\partial x} + \big[ay + f(x)\big]\dfrac{\partial w}{\partial y} = g(x)h(y).$

General solution:

$$w = \int g(x)\, h\left(e^{ax}u + e^{ax}\int f(x)e^{-ax}\, dx \right) dx + \Phi(u), \quad \text{where} \ \ u = e^{-ax}y - \int f(x)e^{-ax}\, dx.$$

Here, $\Phi(u)$ is an arbitrary function; in the integration, u is considered a parameter.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.