9. $\left(\dfrac{f^2(x) + f^2(y)}{2} \right)^{1/2} = f\left(\left(\dfrac{x^2 + y^2}{2} \right)^{1/2} \right).$

Solution:

$$f(x) = (ax^2 + b)^{1/2},$$

where $a > 0$ and $b > 0$ are arbitrary constants.

Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.