

2. 
$$y(x) + A \int_{a}^{b} |x - t| y(t) dt = f(x)$$
.

 $1^{\circ}$ . For A < 0, the solution is given by

$$y(x) = C_1 \cosh(kx) + C_2 \sinh(kx) + f(x) + k \int_a^x \sinh[k(x-t)]f(t) dt, \quad k = \sqrt{-2A}, \quad (1)$$

where the constants  $C_1$  and  $C_2$  are determined by conditions:

$$y'_{x}(a) + y'_{x}(b) = f'_{x}(a) + f'_{x}(b),$$
  

$$y(a) + y(b) + (b - a)y'_{x}(a) = f(a) + f(b) + (b - a)f'_{x}(a).$$
(2)

For A > 0, the solution is given by

$$y(x) = C_1 \cos(kx) + C_2 \sin(kx) + f(x) - k \int_a^x \sin[k(x-t)]f(t) dt, \quad k = \sqrt{2A},$$
 (3)

where the constants  $C_1$  and  $C_2$  are determined by conditions (2).

 $3^{\circ}$ . In the special case a = 0 and A > 0, the solution of the integral equation is given by formula (3) with

$$\begin{split} C_1 &= k \frac{I_{\rm s}(1+\cos\lambda) - I_{\rm c}(\lambda+\sin\lambda)}{2+2\cos\lambda + \lambda\sin\lambda}, \qquad C_2 = k \frac{I_{\rm s}\sin\lambda + I_{\rm c}(1+\cos\lambda)}{2+2\cos\lambda + \lambda\sin\lambda}, \\ k &= \sqrt{2A}, \ \lambda = bk, \ I_{\rm s} = \int_0^b \sin[k(b-t)]f(t)\,dt, \ I_{\rm c} = \int_0^b \cos[k(b-t)]f(t)\,dt. \end{split}$$

## Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/ie/ie0402.pdf