

6.
$$y^2(x) + y^2(a-x) = b^2$$
.

Particular solutions:

$$y_{1,2}(x) = \pm b \sin\left(\frac{\pi x}{2a}\right), \qquad y_{3,4}(x) = \pm b \cos\left(\frac{\pi x}{2a}\right).$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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http://eqworld.ipmnet.ru/en/solutions/fe/fe2106.pdf