

1.
$$\int_a^b g(t)y(x)y(t) dt = f(x).$$

Solutions:

$$y(x) = \pm \lambda f(x), \qquad \lambda = \left[\int_a^b f(t)g(t) dt \right]^{-1/2}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/ie/ie0601.pdf