

6.
$$y''''_{xxxx} = f(y)$$
.

Autonomous equation. By integrating, we obtain

$$2y'_x y'''_{xxx} - (y''_{xx})^2 = 2 \int f(y) \, dy + 2C.$$

The substitution $w(y) = |y'_x|^{3/2}$ leads to a second-order equation:

$$w_{yy}'' = \frac{3}{2} \left[\int f(y) \, dy + C \right] w^{-5/3}.$$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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http://eqworld.ipmnet.ru/en/solutions/ode/ode0506.pdf