

7. f(x)g(y) + h(y) = f(x + y).

Here, f(x), g(y), and h(z) are unknown functions. Solutions:

$$f(x) = C_1 x + C_2$$
, $g(x) = 1$, $h(x) = C_1 x$ (first solution);
 $f(x) = C_1 e^{ax} + C_2$, $g(x) = e^{ax}$, $h(x) = C_2 (1 - e^{ax})$ (second solution),

where a, C_1 , and C_2 are arbitrary constants.

Reference

Aczél, J. and Dhombres, J., Functional Equations in Several Variables, Cambridge Univ. Press, Cambridge, 1989.

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