

$$\mathbf{4.} \quad \frac{\partial u}{\partial t} = L[u] + uf\left(\frac{u}{w}\right) + g\left(\frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = L[w] + wf\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$$

Here, L is an arbitrary linear differential operator (of any order) in the coordinates  $x_1, \ldots, x_n$ , whose coefficients can depend on  $x_1, \ldots, x_n, t$ . It is assumed that L[const] = 0.

Let k be a root of the algebraic (transcendental) equation

$$g(k) = kh(k). (1)$$

1°. Solution if  $f(k) \neq 0$ :

$$u = k \left( \exp[f(k)t]\theta(x_1, \dots, x_n, t) - \frac{h(k)}{f(k)} \right), \quad w = \exp[f(k)t]\theta(x_1, \dots, x_n, t) - \frac{h(k)}{f(k)},$$

where the function  $\theta = \theta(x_1, \dots, x_n, t)$  satisfies the linear equation

$$\frac{\partial \theta}{\partial t} = L[\theta]. \tag{2}$$

 $2^{\circ}$ . Solution if f(k) = 0:

$$u = k[\theta(x_1, \dots, x_n, t) + h(k)t], \quad w = \theta(x_1, \dots, x_n, t) + h(k)t,$$

where the function  $\theta = \theta(x_1, \dots, x_n, t)$  satisfies the linear equation (2).

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