2. First-Order Quasilinear Partial Differential Equations

2.1. Equations of the Form $f(x,y)\dfrac{\partial w}{\partial x}+g(x,y)\dfrac{\partial w}{\partial y}=h(x,y,w)$

1. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)w+g(x)w^k.$

2. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)+g(x)e^{\lambda w}.$

3. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(w).$

4. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(x)g(w).$

5. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)g(y)h(w).$

6. $ax\dfrac{\partial w}{\partial x}+by\dfrac{\partial w}{\partial y}=f(w).$

7. $ay\dfrac{\partial w}{\partial x}+bx\dfrac{\partial w}{\partial y}=f(w).$

8. $ax^n\dfrac{\partial w}{\partial x}+by^k\dfrac{\partial w}{\partial y}=f(w).$

9. $ay^n\dfrac{\partial w}{\partial x}+bx^k\dfrac{\partial w}{\partial y}=f(w).$

10. $ae^{\lambda x}\dfrac{\partial w}{\partial x}+be^{\beta y}\dfrac{\partial w}{\partial y}=f(w).$

11. $ae^{\lambda y}\dfrac{\partial w}{\partial x}+be^{\beta x}\dfrac{\partial w}{\partial y}=f(w).$

12. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=h(w).$

13. $f(y)\dfrac{\partial w}{\partial x}+g(x)\dfrac{\partial w}{\partial y}=h(w).$

2.2. Equations of the Form $\dfrac{\partial w}{\partial x}+f(x,y,w)\dfrac{\partial w}{\partial y}=0$

1. $\dfrac{\partial w}{\partial x}+\big[aw+yf(x)\big]\dfrac{\partial w}{\partial y}=0.$

2. $\dfrac{\partial w}{\partial x} + \big[aw + f(y)\big]\dfrac{\partial w}{\partial y} = 0.$

3. $\dfrac{\partial w}{\partial x} + f(w)\dfrac{\partial w}{\partial y} = 0.$

4. $\dfrac{\partial w}{\partial x} + \big[f(w) + ax\big]\dfrac{\partial w}{\partial y} = 0.$

5. $\dfrac{\partial w}{\partial x} + \big[f(w) + ay\big]\dfrac{\partial w}{\partial y} = 0.$

6. $\dfrac{\partial w}{\partial x} + \big[f(w) + g(x)\big]\dfrac{\partial w}{\partial y} = 0.$

7. $\dfrac{\partial w}{\partial x} + \big[f(w) + g(y)\big]\dfrac{\partial w}{\partial y} = 0.$

8. $\dfrac{\partial w}{\partial x} + \big[yf(w) + g(x)\big]\dfrac{\partial w}{\partial y} = 0.$

9. $\dfrac{\partial w}{\partial x} + \big[xf(w) + yg(w) + h(w)\big]\dfrac{\partial w}{\partial y} = 0.$

10. $\dfrac{\partial w}{\partial x} + f(x)g(y)h(w)\dfrac{\partial w}{\partial y} = 0.$

2.3. Equations of the Form $\dfrac{\partial w}{\partial x} + f(x, y, w)\dfrac{\partial w}{\partial y} = g(x, y, w)$

1. $\dfrac{\partial w}{\partial x} + aw\dfrac{\partial w}{\partial y} = f(x).$

2. $\dfrac{\partial w}{\partial x} + aw\dfrac{\partial w}{\partial y} = f(y).$

3. $\dfrac{\partial w}{\partial x} + \big[aw + f(x)\big]\dfrac{\partial w}{\partial y} = g(x).$

4. $\dfrac{\partial w}{\partial x} + f(w)\dfrac{\partial w}{\partial y} = g(x).$

5. $\dfrac{\partial w}{\partial x} + f(w)\dfrac{\partial w}{\partial y} = g(y).$

6. $\dfrac{\partial w}{\partial x} + f(w)\dfrac{\partial w}{\partial y} = g(w).$

7. $\dfrac{\partial w}{\partial x} + \big[f(w) + g(x)\big]\dfrac{\partial w}{\partial y} = h(x).$

8. $\dfrac{\partial w}{\partial x} + \big[f(w) + g(x)\big]\dfrac{\partial w}{\partial y} = h(w).$

9. $\dfrac{\partial w}{\partial x} + \big[f(w) + yg(x)\big]\dfrac{\partial w}{\partial y} = h(x).$

10. $\dfrac{\partial w}{\partial x} + f(x,w)\dfrac{\partial w}{\partial y} = g(x).$

11. $\dfrac{\partial w}{\partial x} + f(x,w)\dfrac{\partial w}{\partial y} = g(w).$