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5. $\quad \dfrac{\partial w}{\partial x} + a\left(\dfrac{\partial w}{\partial y}\right)^2 = f(x)w + g(x).$

Complete integral:

$$w = F(x)(C_1 + C_2 y) + F(x)\int \left[g(x) - aC_2^2 F^2(x)\right]\frac{dx}{F(x)}, \quad \text{where} \quad F(x) = \exp\left[\int f(x)\,dx\right],$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde3105.pdf