4. $yy'''_{xxx} = f(x)$.

On integrating the equation, we have

$$yy''_{xx} - \tfrac{1}{2}(y'_x)^2 = \int f(x)\,dx + C,$$

where C is an arbitrary constant.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

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