



## 2. First-Order Quasilinear Partial Differential Equations

### 2.1. Equations of the Form $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y, w)$

1.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)w + g(x)w^k.$

2.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x) + g(x)e^{\lambda w}.$

3.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(w).$

4.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)g(w).$

5.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)g(y)h(w).$

6.  $ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(w).$

7.  $ay \frac{\partial w}{\partial x} + bx \frac{\partial w}{\partial y} = f(w).$

8.  $ax^n \frac{\partial w}{\partial x} + by^k \frac{\partial w}{\partial y} = f(w).$

9.  $ay^n \frac{\partial w}{\partial x} + bx^k \frac{\partial w}{\partial y} = f(w).$

10.  $ae^{\lambda x} \frac{\partial w}{\partial x} + be^{\beta y} \frac{\partial w}{\partial y} = f(w).$

11.  $ae^{\lambda y} \frac{\partial w}{\partial x} + be^{\beta x} \frac{\partial w}{\partial y} = f(w).$

12.  $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(w).$

13.  $f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(w).$

### 2.2. Equations of the Form $\frac{\partial w}{\partial x} + f(x, y, w) \frac{\partial w}{\partial y} = 0$

1.  $\frac{\partial w}{\partial x} + [aw + yf(x)] \frac{\partial w}{\partial y} = 0.$

2.  $\frac{\partial w}{\partial x} + [aw + f(y)] \frac{\partial w}{\partial y} = 0.$
3.  $\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = 0.$
4.  $\frac{\partial w}{\partial x} + [f(w) + ax] \frac{\partial w}{\partial y} = 0.$
5.  $\frac{\partial w}{\partial x} + [f(w) + ay] \frac{\partial w}{\partial y} = 0.$
6.  $\frac{\partial w}{\partial x} + [f(w) + g(x)] \frac{\partial w}{\partial y} = 0.$
7.  $\frac{\partial w}{\partial x} + [f(w) + g(y)] \frac{\partial w}{\partial y} = 0.$
8.  $\frac{\partial w}{\partial x} + [yf(w) + g(x)] \frac{\partial w}{\partial y} = 0.$
9.  $\frac{\partial w}{\partial x} + [xf(w) + yg(w) + h(w)] \frac{\partial w}{\partial y} = 0.$
10.  $\frac{\partial w}{\partial x} + f(x)g(y)h(w) \frac{\partial w}{\partial y} = 0.$

### 2.3. Equations of the Form $\frac{\partial w}{\partial x} + f(x, y, w) \frac{\partial w}{\partial y} = g(x, y, w)$

1.  $\frac{\partial w}{\partial x} + aw \frac{\partial w}{\partial y} = f(x).$
2.  $\frac{\partial w}{\partial x} + aw \frac{\partial w}{\partial y} = f(y).$
3.  $\frac{\partial w}{\partial x} + [aw + f(x)] \frac{\partial w}{\partial y} = g(x).$
4.  $\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(x).$
5.  $\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(y).$
6.  $\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(w).$
7.  $\frac{\partial w}{\partial x} + [f(w) + g(x)] \frac{\partial w}{\partial y} = h(x).$
8.  $\frac{\partial w}{\partial x} + [f(w) + g(x)] \frac{\partial w}{\partial y} = h(w).$

$$9. \quad \frac{\partial w}{\partial x} + [f(w) + yg(x)] \frac{\partial w}{\partial y} = h(x).$$

$$10. \quad \frac{\partial w}{\partial x} + f(x, w) \frac{\partial w}{\partial y} = g(x).$$

$$11. \quad \frac{\partial w}{\partial x} + f(x, w) \frac{\partial w}{\partial y} = g(w).$$

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The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/fpde/fpdetoc2.pdf>