

$$\mathbf{2.} \quad \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial u}{\partial x} \right) + e^{\lambda u} f(\lambda u - \sigma w), \quad \frac{\partial w}{\partial t} = \frac{b}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial w}{\partial x} \right) + e^{\sigma w} g(\lambda u - \sigma w).$$

Solution:

$$u = y(\xi) - \frac{1}{\lambda} \ln(C_1 t + C_2), \quad w = z(\xi) - \frac{1}{\sigma} \ln(C_1 t + C_2), \quad \xi = \frac{x}{\sqrt{C_1 t + C_2}},$$

where  $C_1$  and  $C_2$  are arbitrary constants, and the functions  $y = y(\xi)$  and  $z = z(\xi)$  are determined by the system of ordinary differential equations

$$a\xi^{-n}(\xi^{n}y'_{\xi})'_{\xi} + \frac{1}{2}C_{1}\xi y'_{\xi} + \frac{C_{1}}{\lambda} + e^{\lambda y}f(\lambda y - \sigma z) = 0,$$
  

$$b\xi^{-n}(\xi^{n}z'_{\xi})'_{\xi} + \frac{1}{2}C_{1}\xi z'_{\xi} + \frac{C_{1}}{\sigma} + e^{\sigma z}g(\lambda y - \sigma z) = 0.$$

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