



## 3. Linear Functional Equations with Several Independent Variables

1.  $f(x + y) = f(x) + f(y)$ . *Cauchy equation.*
2.  $f(xy) = f(x) + f(y)$ . *Logarithmic Cauchy equation.*
3.  $2f(x + y) = f(2x) + f(2y)$ . *Jensen equation.*
4.  $f(x + y) + f(x - y) = 2f(x) \cosh y$ .
5.  $f(x + y) + f(x - y) = 2f(x) \cos y$ .
6.  $f(\sqrt{x^2 + y^2}) = f(x)f(y)$ . *Gauss equation.*
7.  $f((x^n + y^n)^{1/n}) = f(x) + f(y)$ .
8.  $f(x) + g(y) = h(x + y)$ . *Pexider's equation.*
9.  $f(x) + (1 - x)f\left(\frac{y}{1 - x}\right) = f(y) + (1 - y)f\left(\frac{x}{1 - y}\right)$ .  
*Basic equation of information theory.*
10.  $f(1 - x) + (1 - x)^\alpha f\left(\frac{y}{1 - x}\right) = f(y) + (1 - y)^\alpha f\left(\frac{x}{1 - y}\right)$ .
11.  $f(ax, ay) = f(x, y)$ .
12.  $f(ax, ay) = a^\beta f(x, y)$ . *Homogeneity equation.*
13.  $f(ax, a^\beta y) = f(x, y)$ .
14.  $f(ax, a^\beta y) = a^\gamma f(x, y)$ .
15.  $f(x, y) + f(y, z) = f(x, z)$ .