

$$\begin{aligned} \textbf{13.} \quad & \frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + u f \left( \frac{w}{u} \right) + w g \left( \frac{w}{u} \right) + \frac{u}{\sqrt{u^2 - w^2}} h \left( \frac{w}{u} \right), \\ & \frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + w f \left( \frac{w}{u} \right) + u g \left( \frac{w}{u} \right) + \frac{w}{\sqrt{u^2 - w^2}} h \left( \frac{w}{u} \right). \end{aligned}$$

Solution:

$$u = r(x, t) \cosh \varphi(t), \quad w = r(x, t) \sinh \varphi(t),$$

where the function  $\varphi = \varphi(t)$  is determined by the separable first-order ordinary differential equation

$$\varphi_t' = g(\tanh \varphi),$$

and the function r = r(x, t) satisfies the linear equation

$$\frac{\partial r}{\partial t} = a \frac{\partial^2 r}{\partial x^2} + r f(\tanh \varphi) + h(\tanh \varphi). \tag{1}$$

The change of variable

$$r = F(t) \Big[ Z(x,t) + \int \frac{h(\tanh\varphi) \, dt}{F(t)} \Big], \qquad F(t) = \exp\Big[ \int f(\tanh\varphi) \, dt \Big]$$

brings (1) to the linear heat equation

$$\frac{\partial Z}{\partial t} = a \frac{\partial^2 Z}{\partial x^2}.$$

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