

 $5. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u^k f\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w^k g\left(\frac{u}{w}\right).$

Self-similar solution:

$$u = (C_1 t + C_2)^{\frac{2}{1-k}} y(\xi), \quad w = (C_1 t + C_2)^{\frac{2}{1-k}} z(\xi), \quad \xi = \frac{x}{C_1 t + C_2},$$

where C_1 and C_2 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$C_1^2 \xi^2 y_{\xi\xi}^{\prime\prime} + \frac{2C_1^2(k+1)}{k-1} \xi y_{\xi}^{\prime} + \frac{C_1^2(k+1)}{(k-1)^2} y = \frac{a}{\xi^n} (\xi^n y_{\xi}^{\prime})_{\xi}^{\prime} + y^k f\left(\frac{y}{z}\right),$$

$$C_1^2 \xi^2 z_{\xi\xi}^{"} + \frac{2C_1^2(k+1)}{k-1} \xi z_{\xi}^{'} + \frac{C_1^2(k+1)}{(k-1)^2} z = \frac{b}{\xi^n} (\xi^n z_{\xi}^{'})_{\xi}^{'} + z^k g\left(\frac{y}{z}\right).$$

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