

$$2. \quad \frac{\partial^2 u}{\partial t^2} = L[u] + a_1 u + b_1 w, \quad \frac{\partial^2 w}{\partial t^2} = L[w] + a_2 u + b_2 w.$$

Here, L is an arbitrary linear differential operator in the coordinates x_1, \ldots, x_n (of any order in derivatives).

Solution:

$$u = \frac{a_1 - \lambda_2}{a_2(\lambda_1 - \lambda_2)}U - \frac{a_1 - \lambda_1}{a_2(\lambda_1 - \lambda_2)}W, \quad w = \frac{1}{\lambda_1 - \lambda_2}(U - W),$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - (a_1 + b_2)\lambda + a_1b_2 - a_2b_1 = 0,$$

and the functions $U = U(x_1, \dots, x_n, t)$ and $W = W(x_1, \dots, x_n, t)$ satisfy the independent linear equations

$$\frac{\partial^2 U}{\partial t^2} = L[U] + \lambda_1 U, \quad \frac{\partial^2 W}{\partial t^2} = L[W] + \lambda_2 W.$$

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