

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u f(u^n w^m), \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = w g(u^n w^m).$

Solution in the form of the product of functions with different arguments:

$$u = e^{m(a_1x + b_1y)}\xi(z), \quad w = e^{-n(a_1x + b_1y)}\eta(z), \quad z = a_2x + b_2y,$$

where a_1 , a_2 , b_1 , b_2 are arbitrary constants, and the functions $\xi = \xi(z)$, $\eta = \eta(z)$ are determined by the system of ordinary differential equations

$$(a_2^2 + b_2^2)\xi_{zz}'' + 2m(a_1a_2 + b_1b_2)\xi_z' + m^2(a_1^2 + b_1^2)\xi = \xi f(\xi^n \eta^m),$$

$$(a_2^2 + b_2^2)\eta_{zz}'' - 2n(a_1a_2 + b_1b_2)\eta_z' + n^2(a_1^2 + b_1^2)\eta = \eta g(\xi^n \eta^m).$$

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