

First-Order Partial Differential Equations > Nonlinear Equations > Section 3.2

1.
$$a\left(\frac{\partial w}{\partial x}\right)^2 + b\left(\frac{\partial w}{\partial y}\right)^2 = c$$
.

For a = b, this is a differential equation of light rays.

1°. Complete integral:

$$w = C_1 x + C_2 y + C_3$$
, where $aC_1^2 + bC_2^2 = c$,

 C_1 and C_3 are arbitrary constants.

2°. An alternative form of the complete integral:

$$\frac{w^2}{c} = \frac{(x - C_1)^2}{a} + \frac{(y - C_2)^2}{b},$$

where C_1 and C_2 are arbitrary constants.

References

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, II, Partielle Differentialgleichungen Erster Ordnung für eine gesuchte Funktion, Akad. Verlagsgesellschaft Geest & Portig, Leipzig, 1965.

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., Handbook of First Order Partial Differential Equations, Taylor & Francis, London, 2002.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpde3201.pdf