

3.
$$\frac{\partial}{\partial x}\left(ae^{\beta x}\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(be^{\mu y}\frac{\partial w}{\partial y}\right) = f(w), \qquad ab > 0.$$

Heat/mass transfer equation for inhomogeneous anisotropic media with volume reaction. Functional separable solution for $\beta\mu \neq 0$:

$$w = w(\xi),$$
 $\xi = (b\mu^2 e^{-\beta x} + a\beta^2 e^{-\mu y})^{1/2},$

where the function $w(\xi)$ is determined by the ordinary differential equation $w_{\xi\xi}'' - \xi^{-1}w_{\xi}' = Af(w)$, $A = 4/(ab\beta^2\mu^2)$.

References

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