

39.
$$xy_{xx}'' + [(ax+1)f + ax - 1]y_x' + a^2xfy = 0, f = f(x).$$

Particular solution: $y_0 = (ax + 1)e^{-ax}$.

Solution:

$$y = y_0 \left(C_1 + C_2 \int \frac{e^{-F}}{y_0^2} dx \right), \quad \text{where} \quad F = ax - \ln|x| + \int (a + x^{-1}) f dx,$$

 C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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