

Systems of Ordinary Differential Equations > Linear Systems of Two Equations

15.
$$x_{tt}'' = af(t)(ty_t' - y), \quad y_{tt}'' = bf(t)(tx_t' - x).$$

The transformation

$$u = tx_t - x, \quad v = ty_t' - y \tag{1}$$

leads to the system of first-order equations

$$u'_t = atf(t)v, \quad v'_t = btf(t)u.$$

The general solution of this system has the form

with
$$ab > 0$$
:
$$\begin{cases} u(t) = C_1 a \exp\left(\sqrt{ab} \int t f(t) dt\right) + C_2 a \exp\left(-\sqrt{ab} \int t f(t) dt\right), \\ v(t) = C_1 \sqrt{ab} \exp\left(\sqrt{ab} \int t f(t) dt\right) - C_2 \sqrt{ab} \exp\left(-\sqrt{ab} \int t f(t) dt\right), \end{cases}$$
with $ab < 0$:
$$\begin{cases} u(t) = C_1 a \cos\left(\sqrt{|ab|} \int t f(t) dt\right) + C_2 a \sin\left(\sqrt{|ab|} \int t f(t) dt\right), \\ v(t) = -C_1 \sqrt{|ab|} \sin\left(\sqrt{|ab|} \int t f(t) dt\right) + C_2 \sqrt{|ab|} \cos\left(\sqrt{|ab|} \int t f(t) dt\right), \end{cases}$$

$$(2)$$

where C_1 and C_2 are arbitrary constants. On substituting (2) into (1) and on integrating the resulting relation, one arrives at the solution of the original system:

$$x = C_3 t + t \int \frac{u(t)}{t^2} dt$$
, $y = C_4 t + t \int \frac{v(t)}{t^2} dt$,

where C_3 and C_4 are arbitrary constants.

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0115.pdf