

4. F(x,y(x),y(a/x)) = 0.

On substituting a/x for x, we obtain F(a/x, y(a/x), y(x)) = 0. On eliminating y(a/x) from this equation and the original one, we arrive at an ordinary algebraic (or transcendental) equation of the form $\Psi(x, y(x)) = 0$.

In other words, the solution of the original functional equation, y = y(x), is determined parametrically by the system of two algebraic (transcendental) equations

$$F(x, y, t) = 0,$$
 $F(a/x, t, y) = 0,$

where t is the parameter.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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