5. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = \alpha w \ln(\beta w).$

$1°.$ Solutions:

$$w(x,y) = \frac{1}{\beta} \exp\left[\tfrac{1}{4}\alpha(x+A)^2 + \tfrac{1}{4}\alpha(y+B)^2 + 1 \right],$$

$$w(x,y) = \frac{1}{\beta} \exp\left[A(x+B)^2 \pm \sqrt{A\alpha - 4A^2}\,(x+B)(y+C) + (\tfrac{1}{4}\alpha - A)(y+C)^2 + \tfrac{1}{2} \right],$$

where A, B, and C are arbitrary constants.

$2°.$ There are exact solutions of the following forms:

$$w(x,y) = F(z), \quad z = Ax + By,$$
$$w(x,y) = G(r), \quad r = \sqrt{(x+C_1)^2 + (y+C_2)^2},$$
$$w(x,y) = f(x)g(y).$$

References

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