

First-Order Partial Differential Equations > Linear Equations > Section 1.3

6. 
$$ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x)w + g(x)$$
.

General solution:

$$w = \exp \left[ \frac{1}{a} \int \frac{f(x) \, dx}{x} \right] \left\{ \Phi \left( x^{-b/a} y \right) + \frac{1}{a} \int \frac{g(x)}{x} \exp \left[ -\frac{1}{a} \int \frac{f(x) \, dx}{x} \right] dx \right\},$$

where  $\Phi(u)$  is an arbitrary function.

## Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., Handbook of First Order Partial Differential Equations, Taylor & Francis, London, 2002.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpde1306.pdf