

9. $xy_{xx}'' + ay_x' + bx^n y = 0$.

If n = -1 and b = 0, we have the Euler equation 2.12. If $n \neq -1$ and $b \neq 0$, the solution is expressed in terms of Bessel functions:

$$y = x^{\frac{1-a}{2}} \left[C_1 J_{\nu} \left(\frac{2\sqrt{b}}{n+1} x^{\frac{n+1}{2}} \right) + C_2 Y_{\nu} \left(\frac{2\sqrt{b}}{n+1} x^{\frac{n+1}{2}} \right) \right], \quad \text{ where } \quad \nu = \frac{|1-a|}{n+1},$$

 C_1 and C_2 are arbitrary constants.

References

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