5. Systems of Partial Differential Equations of General Form

Notation: L **is an arbitrary linear operator.**

5.1. Linear Systems of Partial Differential Equations

1. $\dfrac{\partial u}{\partial t} = L[u] + f_1(t)u + g_1(t)w, \quad \dfrac{\partial w}{\partial t} = L[w] + f_2(t)u + g_2(t)w.$

2. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + a_1 u + b_1 w, \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + a_2 u + b_2 w.$

5.2. Nonlinear Systems of Two Partial Differential Equations Containing the First Derivatives in t

1. $\dfrac{\partial u}{\partial t} = L[u] + uf(t, au-cw) + g(t, au-cw), \quad \dfrac{\partial w}{\partial t} = L[w] + wf(t, au-cw) + h(t, au-cw).$

2. $\dfrac{\partial u}{\partial t} = L_1[u] + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L_2[w] + wg\left(\dfrac{u}{w}\right).$

3. $\dfrac{\partial u}{\partial t} = L[u] + uf\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L[w] + wg\left(t, \dfrac{u}{w}\right).$

4. $\dfrac{\partial u}{\partial t} = L[u] + uf\left(\dfrac{u}{w}\right) + g\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L[w] + wf\left(\dfrac{u}{w}\right) + h\left(\dfrac{u}{w}\right).$

5. $\dfrac{\partial u}{\partial t} = L[u] + uf\left(t, \dfrac{u}{w}\right) + \dfrac{u}{w}h\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L[w] + wg\left(t, \dfrac{u}{w}\right) + h\left(t, \dfrac{u}{w}\right).$

6. $\dfrac{\partial u}{\partial t} = L[u] + uf\left(t, \dfrac{u}{w}\right)\ln u + ug\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} =$
$L[w] + wf\left(t, \dfrac{u}{w}\right)\ln w + wh\left(t, \dfrac{u}{w}\right).$

5.3. Nonlinear Systems of Two Partial Differential Equations Containing the Second Derivatives in t

1. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + uf(t, au - bw) + g(t, au - bw), \quad \dfrac{\partial^2 w}{\partial t^2} =$
$L[w] + wf(t, au - bw) + h(t, au - bw).$

2. $\dfrac{\partial^2 u}{\partial t^2} = L_1[u] + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L_2[w] + wg\left(\dfrac{u}{w}\right).$

3. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + uf\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + wg\left(t, \dfrac{u}{w}\right).$

4. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + uf\left(\dfrac{u}{w}\right) + g\left(\dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + wf\left(\dfrac{u}{w}\right) + h\left(\dfrac{u}{w}\right).$

5. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + au\ln u + uf\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + aw\ln w + wg\left(t, \dfrac{u}{w}\right).$

5.4. Nonlinear Systems of Many Partial Differential Equations Containing the First Derivatives in t

1. $$\frac{\partial u_m}{\partial t} = L[u_m] + u_m f(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n) + g_m(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n), \qquad m = 1, \dots, n.$$

2. $$\frac{\partial u_m}{\partial t} = L[u_m] + u_m f_m\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right) + \frac{u_m}{u_n} g\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right),$$
$$\frac{\partial u_n}{\partial t} = L[u_n] + u_n f_n\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right) + g\left(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\right), \qquad m = 1, \dots, n-1.$$