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2. $a\dfrac{\partial w}{\partial x} + b\dfrac{\partial w}{\partial y} = f(x)w + g(x).$

General solution:

$$w = \exp\left[\frac{1}{a}\int f(x)\,dx\right]\left\{\Phi(bx - ay) + \frac{1}{a}\int g(x)\exp\left[-\frac{1}{a}\int f(x)\,dx\right]dx\right\},$$

where $\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.