First-Order Partial Differential Equations > Quasilinear Equations > Section 2.1

2. $\dfrac{\partial w}{\partial x} + a\dfrac{\partial w}{\partial y} = f(x) + g(x)e^{\lambda w}.$

General solution:

$$e^{-\lambda w} = F(x)\Phi(y - ax) - \lambda F(x) \int \frac{g(x)}{F(x)}\,dx, \ \ \text{where } F(x) = \exp\left[-\lambda \int f(x)\,dx\right],$$

$\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde2102.pdf