

Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

9.
$$x''_{tt} = xf(r)$$
, $y''_{tt} = yf(r)$, where $r = \sqrt{x^2 + y^2}$.

Equation of motion of a point mass in the xy-plane under central force.

On proceeding to polar coordinates by the formulas

$$x = r \cos \varphi$$
, $y = r \sin \varphi$, $r = r(t)$, $\varphi = \varphi(t)$,

one can obtain the first integrals

$$r^2 \varphi_t' = C_1, \quad (r_t')^2 + r^2 (\varphi_t')^2 = 2 \int r f(r) dr + C_2,$$

where C_1 and C_2 are arbitrary constants. Integrating further yields

$$t + C_3 = \pm \int \frac{r \, dr}{\sqrt{2r^2 F(r) + r^2 C_2 - C_1^2}}, \quad \varphi = C_1 \int \frac{dt}{r} + C_4, \tag{*}$$

where C_3 and C_4 are arbitrary constants, and

$$F(r) = \int r f(r) \, dr$$

In the last relation of (*), it is assumed that the dependence r = r(t) is obtained by solving the first equation of (*) for r.

Reference

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen, B. G. Teubner, Leipzig, 1977.

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0309.pdf