15. $x^2 y''_{xx} + axy'_x + (bx^n + c)y = 0, \qquad n \neq 0.$

The case $b = 0$ corresponds to the Euler equation 2.12.

For $b \neq 0$, the solution is:

$$y = x^{\frac{1-a}{2}} \left[C_1 J_\nu \left(\frac{2}{n} \sqrt{b}\, x^{\frac{n}{2}} \right) + C_2 Y_\nu \left(\frac{2}{n} \sqrt{b}\, x^{\frac{n}{2}} \right) \right],$$

where $\nu = \frac{1}{n} \sqrt{(1-a)^2 - 4c}$; C_1 and C_2 are arbitrary constants; $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions of the first and second kind.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.