



15. $F(x, y(x), y^{[2]}(x), \dots, y^{[n]}(x)) = 0.$

Notation: $y^{[2]}(x) = y(y(x)), \dots, y^{[n]}(x) = y(y^{[n-1]}(x)).$

Solutions can be sought in the parametric form

$$x = w(t), \quad y = w(t+1). \quad (1)$$

The original equation is then reduced to the following n th-order finite-difference equation (see the preceding equation):

$$F(w(t), w(t+1), w(t+2), \dots, w(t+n)) = 0. \quad (2)$$

The general solution of equation (2) has the structure

$$\begin{aligned} x = w(t) &= \varphi(t; C_1, \dots, C_n), \\ y = w(t+1) &= \varphi(t+1; C_1, \dots, C_n), \end{aligned}$$

where $C_1 = C_1(t), \dots, C_n = C_n(t)$ are arbitrary periodic functions with unit period, $C_k(t) = C_k(t+1)$, $k = 1, 2, \dots, n$.

References

Mathematical Encyclopedia, Vol. 5 [in Russian], Sovetskaya Entsiklopediya, Moscow, 1985 (page 703).

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.