

$$4. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a_1 u + b_1 w, \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = a_2 u + b_2 w.$$

Second-order constant-coefficient linear elliptic system.

Solution:

$$u=\frac{a_1-\lambda_2}{a_2(\lambda_1-\lambda_2)}\theta_1-\frac{a_1-\lambda_1}{a_2(\lambda_1-\lambda_2)}\theta_2, \quad w=\frac{1}{\lambda_1-\lambda_2}\left(\theta_1-\theta_2\right),$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - (a_1 + b_2)\lambda + a_1b_2 - a_2b_1 = 0,$$

and the functions $\theta_n = \theta_n(x, y)$ satisfies the linear Helmholtz equation

$$\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} = \lambda_1 \theta_1, \quad \frac{\partial^2 \theta_2}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} = \lambda_2 \theta_2.$$

Second-order constant-coefficient linear elliptic system

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