

11. M(f(x), f(y)) = f(M(x, y)).

Here,  $M(x,y) = \varphi^{-1}\left(\frac{\varphi(x) + \varphi(y)}{2}\right)$  is the quasi-arithmetic mean for a continuous strictly monotonic function  $\varphi$ , and  $\varphi^{-1}$  is the inverse of  $\varphi$ .

Solution:

$$f(x) = \varphi^{-1} (a\varphi(x) + b),$$

where a and b are arbitrary constants.

## Reference

Aczél, J. and Dhombres, J., Functional Equations in Several Variables, Cambridge Univ. Press, Cambridge, 1989.

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