Exact Solutions > Integral Equations > Linear Volterra Integral Equations of the First Kind and Related Integral Equations with Variable Limit of Integration

1. Volterra Integral Equations of the First Kind

1-1. Integral equations whose kernels contain power-law functions

1.
$$\int_a^x (x-t)y(t) dt = f(x).$$

2.
$$\int_{a}^{x} (Ax + Bt + C)y(t) dt = f(x).$$

3.
$$\int_{a}^{x} (x-t)^{n} y(t) dt = f(x)$$
.

4.
$$\int_a^x \sqrt{x-t} \, y(t) \, dt = f(x).$$

5.
$$\int_{a}^{x} \frac{y(t) dt}{\sqrt{x-t}} = f(x)$$
. Abel equation.

6.
$$\int_a^x (x-t)^{\lambda} y(t) dt = f(x).$$

7.
$$\int_{a}^{x} \frac{y(t) dt}{(x-t)^{\lambda}} = f(x).$$
 Generalized Abel equation.

1-2. Integral equations whose kernels contain exponential functions

8.
$$\int_{a}^{x} e^{\lambda(x-t)} y(t) dt = f(x).$$

9.
$$\int_a^x e^{\lambda x + \beta t} y(t) dt = f(x).$$

10.
$$\int_a^x \left[e^{\lambda(x-t)}-1\right]y(t)\,dt=f(x).$$

11.
$$\int_a^x \left[e^{\lambda(x-t)} + b\right] y(t) dt = f(x).$$

12.
$$\int_{a}^{x} \left[e^{\lambda(x-t)} - e^{\mu(x-t)} \right] y(t) dt = f(x).$$

13.
$$\int_a^x \frac{y(t) dt}{\sqrt{e^{\lambda x} - e^{\lambda t}}} = f(x).$$

1-3. Integral equations whose kernels contain hyperbolic functions

14.
$$\int_{a}^{x} \cosh[\lambda(x-t)]y(t) dt = f(x).$$

15.
$$\int_a^x \left\{ \cosh[\lambda(x-t)] - 1 \right\} y(t) dt = f(x).$$

16.
$$\int_{a}^{x} \left\{ \cosh[\lambda(x-t)] + b \right\} y(t) dt = f(x).$$

17.
$$\int_a^x \cosh^2[\lambda(x-t)]y(t) dt = f(x).$$

18.
$$\int_{a}^{x} \sinh[\lambda(x-t)]y(t) dt = f(x).$$

19.
$$\int_a^x \left\{ \sinh[\lambda(x-t)] + b \right\} y(t) dt = f(x).$$

20.
$$\int_{a}^{x} \sinh(\lambda \sqrt{x-t}) y(t) dt = f(x).$$

1-4. Integral equations whose kernels contain logarithmic functions

$$21. \quad \int_0^x \ln(x-t)y(t) dt = f(x).$$

22.
$$\int_{a}^{x} [\ln(x-t) + A]y(t) dt = f(x)$$
.

23.
$$\int_{a}^{x} (x-t) [\ln(x-t) + A] y(t) dt = f(x).$$

1-5. Integral equations whose kernels contain trigonometric functions

24.
$$\int_{a}^{x} \cos[\lambda(x-t)]y(t) dt = f(x).$$

25.
$$\int_a^x \left\{ \cos[\lambda(x-t)] - 1 \right\} y(t) dt = f(x).$$

26.
$$\int_a^x \left\{ \cos[\lambda(x-t)] + b \right\} y(t) dt = f(x).$$

27.
$$\int_a^x \sin[\lambda(x-t)]y(t) dt = f(x).$$

28.
$$\int_{a}^{x} \sin(\lambda \sqrt{x-t}) y(t) dt = f(x).$$

1-6. Integral equations whose kernels contain special functions

29.
$$\int_a^x J_0(\lambda(x-t))y(t) dt = f(x).$$

30.
$$\int_a^x J_0(\lambda \sqrt{x-t}) y(t) dt = f(x).$$

31.
$$\int_a^x I_0(\lambda(x-t))y(t) dt = f(x).$$

32.
$$\int_{a}^{x} I_{0}(\lambda \sqrt{x-t}) y(t) dt = f(x).$$

1-7. Integral equations whose kernels contain arbitrary functions

33.
$$\int_{a}^{x} [g(x) - g(t)]y(t) dt = f(x)$$
.

34.
$$\int_{a}^{x} [g(x) - g(t) + b] y(t) dt = f(x).$$

35.
$$\int_{a}^{x} [g(x) + h(t)]y(t) dt = f(x)$$
.

$$36. \quad \int_a^x K(x-t)y(t)\,dt = f(x).$$

37.
$$\int_{a}^{x} \sqrt{g(x)-g(t)} y(t) dt = f(x).$$

38.
$$\int_{a}^{x} \frac{y(t) dt}{\sqrt{q(x) - q(t)}} = f(x)$$
.

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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