

$$1. \quad \frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + b_1 u + c_1 w, \quad \frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + b_2 u + c_2 w.$$

Second-order constant-coefficient linear parabolic system.

Solution:

$$u = \frac{b_1 - \lambda_2}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_1 t} \theta_1 - \frac{b_1 - \lambda_1}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_2 t} \theta_2,$$
  
$$w = \frac{1}{\lambda_1 - \lambda_2} \left( e^{\lambda_1 t} \theta_1 - e^{\lambda_2 t} \theta_2 \right),$$

where  $\lambda_1$  and  $\lambda_2$  are roots of the quadratic equation

$$\lambda^2 - (b_1 + c_2)\lambda + b_1c_2 - b_2c_1 = 0,$$

and the functions  $\theta_n = \theta_n(x, t)$  satisfy the linear heat equations

$$\frac{\partial \theta_1}{\partial t} = a \frac{\partial^2 \theta_1}{\partial x^2}, \qquad \frac{\partial \theta_2}{\partial t} = a \frac{\partial^2 \theta_2}{\partial x^2}.$$

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