37. $y(x) + y\left(\sqrt{a^2 - x^2}\right) = 0,$ $0 \leq x \leq a.$

Solution:

$$y(x) = \Phi\left(x,\ \sqrt{a^2 - x^2}\right),$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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