Systems of Ordinary Differential Equations > Nonlinear Systems of Three and More Equations

7. $\quad x'_t = h(z)F_2 - g(y)F_3, \quad y'_t = f(x)F_3 - h(z)F_1, \quad z'_t = g(y)F_1 - f(x)F_2.$

Here, $F_n = F_n(x, y, z, t)$ are arbitrary functions.

First integral:

$$\int f(x)\,dx + \int g(y)\,dy + \int h(z)\,dz = C_1,$$

where C is an arbitrary constant. If the function F_n is independent of t, then, by eliminating t and z from the first two equations of the system (with the above integral), one arrives at a first-order equation.