19. $\displaystyle\int_a^x \big\{ \sinh[\lambda(x - t)] + b \big\} y(t)\,dt = f(x), \qquad f(a) = 0.$

For $b = 0$, see equation 1.18.

Solution:

$$y(x) = \frac{1}{b} f_x'(x) + \int_a^x R(x - t) f_t'(t)\,dt,$$

$$R(x) = \frac{\lambda}{b^2} \exp\left(-\frac{\lambda x}{2b} \right) \left[\frac{\lambda}{2bk} \sinh(kx) - \cosh(kx) \right], \quad k = \frac{\lambda\sqrt{1 + 4b^2}}{2b}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

http://eqworld.ipmnet.ru/en/solutions/ie/ie0119.pdf