39. $yy''_{xx} = f(x)(y'_x)^2.$

The substitution $w(x) = xy'_x/y$ leads to a Bernoulli equation 1.4: $xw'_x = w + [f(x) - 1]w^2$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

http://eqworld.ipmnet.ru/en/solutions/ode/ode0339.pdf