

Exact Solutions > Basic Handbooks > A. D. Polyanin, V. F. Zaitsev, and A. Moussiaux, *Handbook of First Order Differential Equations*, Taylor & Francis, London, 2002

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