4.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f\left(\frac{u}{w} \right) + \frac{u}{w} h\left(\frac{u}{w} \right),$$
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial w} \right) + w g\left(\frac{u}{w} \right) + h\left(\frac{u}{w} \right).$$

Solution:
$$u = k\theta(x, t), \qquad w = \theta(x, t),$$

where k is a root of the algebraic (transcendental) equation $f(k) = g(k)$, and the function $\theta = \theta(x, t)$ satisfies the linear equation

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial \theta}{\partial x} \right) + f(k)\theta + h(k).$$