

1.
$$\int_0^x y(t)y(x-t) dt = (Ax+B)e^{\lambda x}, \qquad A, B > 0.$$

Solutions:

$$y(x) = \pm \sqrt{B} \, e^{\lambda x} \left[\frac{1}{\sqrt{\pi x}} \exp \left(-\frac{A}{B} x \right) + \sqrt{\frac{A}{B}} \, \operatorname{erf} \left(\sqrt{\frac{A}{B} \, x} \, \right) \right],$$

where erf $z=\frac{2}{\sqrt{\pi}}\int_0^z \exp\left(-t^2\right)\,dt$ is the error function.

Reference

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Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

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http://eqworld.ipmnet.ru/en/solutions/ie/ie0501.pdf