First-Order Partial Differential Equations > Nonlinear Equations > Section 3.3

12. $F_1\left(x, \dfrac{1}{w}\dfrac{\partial w}{\partial x}\right) + w^k F_2\left(y, \dfrac{1}{w}\dfrac{\partial w}{\partial y}\right) = 0.$

Complete integral:

$$w(x, y) = \varphi(x)\psi(y).$$

The functions $\varphi = \varphi(x)$ and $\psi = \psi(y)$ are determined by solving the ordinary differential equations

$$\varphi^{-k}F_1\left(x, \varphi'_x/\varphi\right) = C, \qquad \psi^k F_2\left(y, \psi'_y/\psi\right) = -C,$$

where C is an arbitrary constant.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

http://eqworld.ipmnet.ru/en/solutions/fpde/fpde3312.pdf