

$$\mathbf{10.} \quad \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left[f\left(t, \frac{u}{w}\right) \frac{\partial u}{\partial x} \right] + ug\left(t, \frac{u}{w}\right), \quad \frac{\partial w}{\partial t} = \frac{\partial}{\partial x} \left[f\left(t, \frac{u}{w}\right) \frac{\partial w}{\partial x} \right] + wh\left(t, \frac{u}{w}\right).$$

Solution:

$$u = \varphi(t) \exp \left[\int h(t, \varphi(t)) \, dt \right] \theta(x, \tau), \quad w = \exp \left[\int h(t, \varphi(t)) \, dt \right] \theta(x, \tau), \quad \tau = \int f(t, \varphi(t)) \, dt,$$

where the function $\varphi = \varphi(t)$ is determined by the ordinary differential equation

$$\varphi_t' = [g(t, \varphi) - h(t, \varphi)]\varphi,$$

and the function $\theta = \theta(x, \tau)$ satisfies the linear heat equation

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2}.$$

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