

Systems of Ordinary Differential Equations > Linear Systems of Two Equations

2.
$$x'_t = a_1x + b_1y + c_1$$
, $y'_t = a_2x + b_2y + c_2$.

The general solution of this system is given by the sum of its particular solution and the general solution of the corresponding homogeneous system (see equation 1.1).

1°. Let $a_1b_2 - a_2b_1 \neq 0$. A particular solution:

$$x = x_0, \quad y = y_0,$$

where the constants x_0 and y_0 are determined by solving the linear system of equations

$$a_1x_0 + b_1y_0 + c_1 = 0$$
, $a_2x_0 + b_2y_0 + c_2 = 0$.

 2° . Let $a_1b_2 - a_2b_1 = 0$ and $a_1^2 + b_1^2 > 0$. In this case, the system in question becomes

$$x'_t = ax + by + c_1, \quad y'_t = k(ax + by) + c_2.$$

2.1. If $\sigma = a + bk \neq 0$, a particular solution of the original system is given by

$$x = b\sigma^{-1}(c_1k - c_2)t - \sigma^{-2}(ac_1 + bc_2), \quad y = kx + (c_2 - c_1k)t.$$

2.2. If $\sigma = a + bk = 0$, a particular solution of the original system is given by

$$x = \frac{1}{2}b(c_2 - c_1k)t^2 + c_1t$$
, $y = kx + (c_2 - c_1k)t$.

Reference

Kamke, E., Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen, B. G. Teubner, Leipzig, 1977.

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0102.pdf