5. $\quad \dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial}{\partial x}\left[a(x+b)^n \dfrac{\partial w}{\partial x}\right] + f(w), \qquad a > 0.$

This equation describes the propagation of nonlinear waves in an inhomogeneous medium.

Functional separable solution for $n \neq 2$:

$$w = w(r), \qquad r^2 = k\left[\frac{1}{4}(t+C)^2 - \frac{(x+b)^{2-n}}{a(2-n)^2}\right],$$

where k and the expression in square brackets must have like signs, and the function $w(r)$ is determined by the ordinary differential equation

$$w''_{rr} + \frac{2}{2-n}\frac{1}{r}w'_r = \frac{4}{k}f(w).$$

References

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