Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

1. $x'_t = x^n F(x, y), \quad y'_t = g(y) F(x, y).$

Solution:

$$x = \varphi(y), \quad \int \frac{dy}{g(y) F(\varphi(y), y)} = t + C_2,$$

where

$$\varphi(y) = \begin{cases} \left[C_1 + (1 - n) \displaystyle\int \frac{dy}{g(y)} \right]^{\frac{1}{1-n}} & \text{if } n \neq 1, \\[2mm] C_1 \exp\left[\displaystyle\int \frac{dy}{g(y)} \right] & \text{if } n = 1, \end{cases}$$

C_1 and C_2 are arbitrary constants.