

$$\mathbf{4.} \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = uf\left(\frac{u}{w}\right) + \frac{u}{w}h\left(\frac{u}{w}\right), \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = wg\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$$

Solution:

$$u = kw, \quad w = \theta(x, y) - \frac{h(k)}{f(k)},$$

where k is a root of the algebraic (transcendental) equation

$$f(k) = g(k),$$

and the function $\theta = \theta(x, y)$ satisfies the linear Helmholtz equation

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = f(k)w.$$

For its exact solutions see the "Handbook of Linear Partial Differential Equations for Engineers and Scientists" by A. D. Polyanin (2002).

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