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## 1. Линейные дифференциальные уравнения в частных производных

### 1.1. Уравнение вида $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = 0$

1.  $\frac{\partial w}{\partial x} + [f(x)y + g(x)] \frac{\partial w}{\partial y} = 0.$
2.  $\frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = 0.$
3.  $\frac{\partial w}{\partial x} + [f(x)e^{\lambda y} + g(x)] \frac{\partial w}{\partial y} = 0.$
4.  $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = 0.$
5.  $[f(y) + amx^n y^{m-1}] \frac{\partial w}{\partial x} - [g(x) + anx^{n-1} y^m] \frac{\partial w}{\partial y} = 0.$
6.  $[e^{\alpha x} f(y) + c\beta] \frac{\partial w}{\partial x} - [e^{\beta y} g(x) + c\alpha] \frac{\partial w}{\partial y} = 0.$
7.  $\frac{\partial w}{\partial x} + f(ax + by + c) \frac{\partial w}{\partial y} = 0.$
8.  $\frac{\partial w}{\partial x} + f\left(\frac{y}{x}\right) \frac{\partial w}{\partial y} = 0.$
9.  $x \frac{\partial w}{\partial x} + y f(x^n y^m) \frac{\partial w}{\partial y} = 0.$
10.  $\frac{\partial w}{\partial x} + y f(e^{\alpha x} y^m) \frac{\partial w}{\partial y} = 0.$
11.  $x \frac{\partial w}{\partial x} + f(x^n e^{\alpha y}) \frac{\partial w}{\partial y} = 0.$

### 1.2. Уравнения вида $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y)$

1.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x).$
2.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)y^k.$
3.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)e^{\lambda y}.$
4.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x) + g(y).$

5.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)g(y).$
6.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y).$
7.  $\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x).$
8.  $\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)h(y).$
9.  $\frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = h(x).$
10.  $\frac{\partial w}{\partial x} + [f(x) + g(x)e^{\lambda y}] \frac{\partial w}{\partial y} = h(x).$
11.  $ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y).$
12.  $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h_1(x) + h_2(y).$
13.  $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(x, y).$
14.  $f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(x, y).$

**1.3. Уравнения вида  $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y)$**

1.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)w.$
2.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)w + g(x).$
3.  $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = [f(x) + g(y)]w.$
4.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w.$
5.  $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w + g(x, y).$
6.  $ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x)w + g(x).$
7.  $ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y)w.$
8.  $x \frac{\partial w}{\partial x} + ay \frac{\partial w}{\partial y} = f(x, y)w + g(x, y).$
9.  $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = [h_1(x) + h_2(y)]w.$

$$10. \quad f_1(x) \frac{\partial w}{\partial x} + f_2(y) \frac{\partial w}{\partial y} = aw + g_1(x) + g_2(y).$$

$$11. \quad f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y).$$

$$12. \quad f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y).$$