

14. $\int_{-a}^{a} \left(\ln \frac{A}{|x-t|} \right) y(t) dt = f(x), \quad -a \le x \le a.$

Solution for 0 < a < 2A:

$$\begin{split} y(x) &= \frac{1}{2M'(a)} \left[\frac{d}{da} \int_{-a}^{a} w(t,a) f(t) \, dt \right] w(x,a) \\ &- \frac{1}{2} \int_{|x|}^{a} w(x,\xi) \frac{d}{d\xi} \left[\frac{1}{M'(\xi)} \frac{d}{d\xi} \int_{-\xi}^{\xi} w(t,\xi) f(t) \, dt \right] d\xi \\ &- \frac{1}{2} \frac{d}{dx} \int_{|x|}^{a} \frac{w(x,\xi)}{M'(\xi)} \left[\int_{-\xi}^{\xi} w(t,\xi) \, df(t) \right] d\xi, \end{split}$$

where the prime stands for the derivative and

$$M(\xi) = \left(\ln \frac{2A}{\xi}\right)^{-1}, \quad w(x,\xi) = \frac{M(\xi)}{\pi \sqrt{\xi^2 - x^2}},$$

References

Gohberg, I. C. and Krein, M. G., The Theory of Volterra Operators in a Hilbert Space and Its Applications [in Russian], Nauka, Moscow, 1967.

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

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