

First-Order Partial Differential Equations > Linear Equations > Section 1.2

8.
$$\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)h(y)$$
.

General solution:

$$w = \int g(x) \, h\bigg(e^{ax}u + e^{ax} \int f(x)e^{-ax} \, dx\bigg) \, dx + \Phi(u), \quad \text{where} \quad u = e^{-ax}y - \int f(x)e^{-ax} \, dx.$$

Here, $\Phi(u)$ is an arbitrary function; in the integration, u is considered a parameter.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpde1208.pdf