

First-Order Partial Differential Equations > Linear Equations > Section 1.3

8.
$$x \frac{\partial w}{\partial x} + ay \frac{\partial w}{\partial y} = f(x, y)w + g(x, y)$$
.

General solution:

$$w = F(x,u) \bigg[\Phi(u) + \int \frac{g(x,\,ux^a)}{x F(x,u)} \, dx \bigg], \quad F(x,u) = \exp \bigg[\int \frac{1}{x} f(x,\,ux^a) \, dx \bigg],$$

where $u = yx^{-a}$ and $\Phi(u)$ is an arbitrary function. In the integration, u is considered a parameter.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., Handbook of First Order Partial Differential Equations, Taylor & Francis, London, 2002.

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http://eqworld.ipmnet.ru/en/solutions/fpde/fpde1308.pdf