5. $\boldsymbol{F\left(x, y(x), y\left(\dfrac{a-x}{1+bx}\right)\right) = 0.}$

On substituting $\dfrac{a-x}{1+bx}$ for x, we obtain

$$F\left(\frac{a-x}{1+bx},\ y\left(\frac{a-x}{1+bx}\right),\ y(x)\right) = 0.$$

On eliminating $y\left(\dfrac{a-x}{1+bx}\right)$ from this equation and the original one, we arrive at an ordinary algebraic (or transcendental) equation of the form $\Psi\bigl(x, y(x)\bigr) = 0$.

In other words, the solution of the original functional equation, $y = y(x)$, is determined parametrically by the system of two algebraic (transcendental) equations

$$F(x,\ y,\ t) = 0, \qquad F\left(\frac{a-x}{1+bx},\ t,\ y\right) = 0,$$

where t is the parameter.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

