Systems of Ordinary Differential Equations > Linear Systems of Two Equations

13. $x''_{tt} = f(t)(a_1 x + b_1 y), \quad y''_{tt} = f(t)(a_2 x + b_2 y).$

Let k_1 and k_2 are roots of the quadratic equation

$$k^2 - (a_1 + b_2)k + a_1 b_2 - a_2 b_1 = 0.$$

Then one can add together the original equations multiplied by appropriate constants to obtain the following two independent equations:

$$z''_1 = k_1 f(t) z_1, \quad z_1 = a_2 x + (k_1 - a_1)y;$$
$$z''_2 = k_2 f(t) z_2, \quad z_2 = a_2 x + (k_2 - a_1)y.$$

A prime denotes a derivative with respect to t.

http://eqworld.ipmnet.ru/en/solutions/sysode/sode0113.pdf