Exact Solutions > Nonlinear Partial Differential Equations > Second-Order Hyperbolic Partial Differential Equations

## 2. Nonlinear Hyperbolic Equations

## 2.1. Nonlinear Wave Equations of the Form $\frac{\partial^2 w}{\partial t^2} = a \frac{\partial^2 w}{\partial x^2} + f(w)$

1. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + aw + bw^n$$
. Klein-Gordon equation with a power-law nonlinearity.

2. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + aw^n + bw^{2n-1}$$
. Klein-Gordon equation with a power-law nonlinearity.

3. 
$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2} + be^{\beta w}$$
. Modified Liouville equation.

4. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + ae^{\beta w} + be^{2\beta w}$$
. Klein–Gordon equation with a exponential nonlinearity.

5. 
$$\frac{\partial^2 w}{\partial t^2} = a \frac{\partial^2 w}{\partial x^2} + b \sinh(\lambda w)$$
. Sinh-Gordon equation.

6. 
$$\frac{\partial^2 w}{\partial t^2} = a \frac{\partial^2 w}{\partial x^2} + b \sin(\lambda w)$$
. Sine-Gordon equation.

7. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + f(w)$$
. Nonlinear Klein-Gordon equation.

## 2.2. Other Nonlinear Hyperbolic Equations

1. 
$$\frac{\partial^2 w}{\partial t^2} = a \frac{\partial}{\partial x} \left( w \frac{\partial w}{\partial x} \right)$$
.

$$2. \quad \frac{\partial^2 w}{\partial t^2} = a \frac{\partial}{\partial x} \left( w^n \frac{\partial w}{\partial x} \right) + b w^k.$$

$$3. \quad \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \bigg( a e^{\lambda w} \frac{\partial w}{\partial x} \bigg).$$

4. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial w}{\partial x} \right) + f(w)$$
.

5. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \left[ a(x+b)^n \frac{\partial w}{\partial x} \right] + f(w)$$
.

6. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \left( ae^{\lambda x} \frac{\partial w}{\partial x} \right) + f(w)$$
.

7. 
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \left[ f(w) \frac{\partial w}{\partial x} \right]$$
.

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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