

$$7. \quad \int_{-\infty}^{\infty} \frac{a+b \operatorname{sign}(x-t)}{|x-t|^{1-\lambda}} y(t) \ dt = f(x), \qquad 0 < \lambda < 1.$$

Solution:

$$y(x) = \frac{\lambda \sin(\pi \lambda)}{4\pi \left[a^2 \cos^2\left(\frac{1}{2}\pi\lambda\right) + b^2 \sin^2\left(\frac{1}{2}\pi\lambda\right)\right]} \int_{-\infty}^{\infty} \frac{a + b \operatorname{sign}(x - t)}{|x - t|^{1 + \lambda}} \left[f(x) - f(t)\right] dt.$$

References

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