

Exact Solutions > Functional Equations > Linear Difference and Functional Equations with One Independent Variable > First-Order Constant-Coefficient Linear Homogeneous Difference Equation

1. y(x+1) - ay(x) = 0.

First-order constant-coefficient linear homogeneous difference equation.

Solution:

$$y(x) = \Theta(x)a^x$$
,

where $\Theta(x) = \Theta(x+1)$ is an arbitrary periodic function with unit period. If $\Theta(x) \equiv \text{const}$, there is a particular solution $y(x) = Ca^x$, where C is an arbitrary constant.

Remark. Here and henceforth in similar cases, $\Theta(x)$ can be an arbitrary periodic function with period $T=\frac{1}{n}$, where n is any positive integer. To the minimum value n=1 there corresponds a function with maximum period $T_{\max}=1$.

References

Kuczma, M., Functional Equations in a Single Variable, Polish Scientific Publishers, 1968.

Mirolyubov, A. A., and Soldatov, M. A., Linear Homogeneous Difference Equations [in Russian], Nauka, Moscow, 1981. Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional

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First-Order Constant-Coefficient Linear Difference Equation

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