

20.
$$y^{[n]}(x) + a_{n-1}y^{[n-1]}(x) + \ldots + a_1y(x) + a_0x = 0.$$

Notation:
$$y^{[2]}(x) = y(y(x)), \dots, y^{[n]}(x) = y(y^{[n-1]}(x)).$$

Solutions may be sought in the form

$$x=w(t),\quad y=w(t+1).$$

Then the original equation is reduced to the following nth-order linear finite-difference equation (see equation 15 in the current subsection):

$$w(t+n) + a_{n-1}w(t+n-1) + \ldots + a_1w(t+1) + a_0w(t) = 0.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

Copyright © 2004 Andrei D. Polyanin

http://eqworld.ipmnet.ru/en/solutions/fe/fe1220.pdf