

22.
$$y(x)y(\omega(x)) = f^2(x)$$
, where $\omega(\omega(x)) = x$.

The right-hand side function is assumed to satisfy the condition $f(x) = \pm f(\omega(x))$. To be specific, we take $f(x) = f(\omega(x))$.

Solution:

$$y(x) = \pm f(x) \exp[\Phi(x, \omega(x))],$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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