

8. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = uf(u^2 - w^2) + wg(u^2 - w^2), \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = wf(u^2 - w^2) + ug(u^2 - w^2).$

Solution:

$$u = r(z)\cosh\left[\theta(z) + C_1y + C_2\right], \quad w = r(z)\sinh\left[\theta(z) + C_1y + C_2\right], \quad z = k_1x + k_2y,$$

where C_1 , C_2 , k_1 , and k_2 are arbitrary constants, and the functions r = r(z), $\theta = \theta(z)$ are determined by the system of ordinary differential equations

$$\begin{split} &(k_1^2+k_2^2)r_{zz}^{\prime\prime}+k_1^2r(\theta_z^\prime)^2+r(k_2\theta_z^\prime+C_1)^2=rf(r^2),\\ &(k_1^2+k_2^2)r\theta_{zz}^{\prime\prime}+2\big[(k_1^2+k_2^2)\theta_z^\prime+C_1k_2\big]r_z^\prime=rg(r^2). \end{split}$$

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