$$9. \quad ax\frac{\partial u}{\partial x} + ay\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - f(u,w), \quad ax\frac{\partial w}{\partial x} + ay\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - g(u,w).$$

Solution:

$$u(x, y) = U(z),$$
 $w(x, y) = W(z),$ $z = k_1 x + k_2 y,$

where k_1 and k_2 are arbitrary constants, and the functions U = U(z) and W = W(z) satisfy the system of ordinary differential equations

$$azU' = (k_1^2 + k_2^2)U'' - f(U, W), \quad azW' = (k_1^2 + k_2^2)W'' - g(U, W).$$

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