13. $F\left(x,\ y(x),\ y\left(\dfrac{ax-\beta}{x+b}\right),\ y\left(\dfrac{bx+\beta}{a-x}\right)\right) = 0, \qquad \beta = a^2 + ab + b^2.$

Let us substitute first $\dfrac{ax-\beta}{x+b}$ for x and then $\dfrac{bx+\beta}{a-x}$ for x to obtain the system of equations (the original equation comes first)

$$\begin{aligned} &F\bigl(x,\ y(x),\ y(u),\ y(w)\bigr) = 0,\\ &F\bigl(u,\ y(u),\ y(w),\ y(x)\bigr) = 0,\\ &F\bigl(w,\ y(w),\ y(x),\ y(u)\bigr) = 0. \end{aligned} \tag{1}$$

The arguments u and w are expressed in terms of x as

$$u = \frac{ax-\beta}{x+b}, \qquad w = \frac{bx+\beta}{a-x}.$$

On eliminating $y(u)$ and $y(w)$ from the system of nonlinear algebraic (or transcendental) equations (1), one arrives at the solution, $y = y(x)$, of the original equation.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

