

30.
$$gy_{xx}'' + \frac{1}{2}g_x'y_x' = f(y)$$
, $g = g(x)$.

Integrating yields a first-order separable equation: $g(x)(y'_x)^2 = 2 \int f(y) dy + C_1$. Solution for $g(x) \ge 0$:

$$\int \left[C_1 + 2 \int f(y) \, dy \right]^{-1/2} dy = C_2 \pm \int \frac{dx}{\sqrt{g(x)}},$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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