

19.
$$y(x^a) - by(x) = 0$$
, $a, b > 0$.

Solution:

$$y(x) = \Theta(\ln|\ln x|) |\ln x|^p, \qquad p = \frac{\ln b}{\ln a},$$

where $\Theta(z) = \Theta(z + \ln a)$ is an arbitrary periodic function with period $|\ln a|$.

For $\Theta(z) \equiv \text{const}$, there is a particular solution $y(x) = C|\ln x|^p$, where C is an arbitrary constant.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

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