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8.　$F\big(y,\, y_x',\, y_{xx}'',\, \ldots,\, y_x^{(n)}\big) = 0.$

Autonomous equation. It does not depend on x explicitly. The substitution $w(y) = y_x'$ leads to an
$(n-1)$st-order equation. The derivatives of the original equation and the transformed one are related
by

$$y_{xx}'' = w w_y', \quad y_{xxx}''' = w^2 w_{yy}'' + w(w_y')^2, \quad \ldots, \quad y_x^{(n)} = w\big(y_x^{(n-1)}\big)_y'.$$

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