1. $y'''_{xxx} + \lambda y = 0.$

Solution:

$$y = \begin{cases} C_1 + C_2 x + C_3 x^2 & \text{if } \lambda = 0, \\ C_1 \exp(-kx) + C_2 \exp\left(\frac{1}{2}kx\right) \cos\left(\frac{\sqrt{3}}{2}kx\right) + C_3 \exp\left(\frac{1}{2}kx\right) \sin\left(\frac{\sqrt{3}}{2}kx\right) & \text{if } \lambda \neq 0, \end{cases}$$

where $k = \lambda^{1/3}$.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

http://eqworld.ipmnet.ru/en/solutions/ode/ode0401.pdf