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Laplace Transforms: Expressions with Logarithmic Functions

No	**Original function**, $f(x)$	**Laplace transform**, $\widetilde{f}(p) = \int_0^\infty e^{-px} f(x)\, dx$
1	$\ln x$	$-\dfrac{1}{p}(\ln p + \mathcal{C})$
2	$\ln(1 + ax)$	$-\dfrac{1}{p} e^{p/a}\, \mathrm{Ei}(-p/a)$
3	$\ln(x + a)$	$\dfrac{1}{p}\left[\ln a - e^{ap}\, \mathrm{Ei}(-ap)\right]$
4	$x^n \ln x, \qquad n = 1,\, 2,\, \ldots$	$\dfrac{n!}{p^{n+1}}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln p - \mathcal{C}\right)$
5	$\dfrac{1}{\sqrt{x}} \ln x$	$-\sqrt{\pi/p}\left[\ln(4p) + \mathcal{C}\right]$
6	$x^{n-1/2} \ln x, \qquad n = 1,\, 2,\, \ldots$	$\dfrac{k_n}{p^{n+1/2}}\left[2 + \frac{2}{3} + \frac{2}{5} + \cdots + \frac{2}{2n-1} - \ln(4p) - \mathcal{C}\right],$ $k_n = 1 \cdot 3 \cdot 5 \ldots (2n-1)\dfrac{\sqrt{\pi}}{2^n}$
7	$x^{\nu-1} \ln x, \quad \nu > 0$	$\Gamma(\nu) p^{-\nu}\left[\psi(\nu) - \ln p\right]$
8	$(\ln x)^2$	$\dfrac{1}{p}\left[(\ln x + \mathcal{C})^2 + \frac{1}{6}\pi^2\right]$
9	$e^{-ax} \ln x$	$-\dfrac{\ln(p + a) + \mathcal{C}}{p + a}$

Notation: $\mathcal{C} = 0.5772\ldots$ is the Euler constant, $\mathrm{Ei}(z)$ is the integral exponent, $\Gamma(\nu)$ is the gamma function, $\psi(\nu)$ is the logarithmic derivative of the gamma function.

References

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