

$4. \quad \int_0^b \frac{y(t)}{|x^\lambda - t^\lambda|^k} \ dt = f(x), \qquad 0 < k < 1, \quad \lambda > 0.$

Solution

$$y(x) = -Ax^{\frac{\lambda(k-1)}{2}} \frac{d}{dx} \left[\int_{x}^{b} \frac{t^{\frac{\lambda(3-2k)-2}{2}}}{(t^{\lambda} - x^{\lambda})^{\frac{1-k}{2}}} \int_{0}^{t} \frac{s^{\frac{\lambda(k+1)-2}{2}}}{(t^{\lambda} - s^{\lambda})^{\frac{1-k}{2}}} \right],$$
$$A = \frac{\lambda^{2}}{2\pi} \cos\left(\frac{\pi k}{2}\right) \Gamma(k) \left[\Gamma\left(\frac{1+k}{2}\right) \right]^{-2},$$

where $\Gamma(k)$ is the gamma function.

Reference

Polyanin, A. D. and Manzhirov, A. V., Handbook of Integral Equations, CRC Press, Boca Raton, 1998.

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