

51.
$$y_{xx}'' = f(y_x'^2 + ay)$$
.

The substitution $w(y) = (y'_x)^2 + ay$ leads to a first-order separable equation: $w'_y = 2f(w) + a$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition,* Chapman & Hall/CRC, Boca Raton, 2003.

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http://eqworld.ipmnet.ru/en/solutions/ode/ode0351.pdf