

Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

17.
$$x_{tt}'' = F(t, tx_t' - x, ty_t' - y), \quad y_{tt}'' = G(t, tx_t' - x, ty_t' - y).$$

1°. The transformation

$$u = tx_t - x, \quad v = ty_t' - y \tag{1}$$

leads to the system of first-order equations

$$u'_t = tF(t, u, v), \quad v'_t = tG(t, u, v).$$
 (2)

2°. Suppose a solution of system (2),

$$u = u(t, C_1, C_2), \quad v = v(t, C_1, C_2),$$
 (3)

where C_1 and C_2 are arbitrary constants, is known. Then, on substituting (3) into (1) and integrating, one obtains a solution of the original system in the form

$$x = C_3 t + t \int \frac{u(t, C_1, C_2)}{t^2} \, dt, \quad \ y = C_4 t + t \int \frac{v(t, C_1, C_2)}{t^2} \, dt.$$

 3° . If the functions F and G are independent of t, then, having eliminated t from system (2), one arrives at the first-order equation

$$g(u, v)u'_v = F(u, v).$$

• Reference: A. D. Polyanin, EqWorld, 2004 (Private communication, received 23 April 2004).

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http://eqworld.ipmnet.ru/en/solutions/sysode/sode0317.pdf