Systems of Ordinary Differential Equations > Linear Systems of Three and More Equations

3. $ax'_t = bc(y - z),$ $by'_t = ac(z - x),$ $cz'_t = ab(x - y).$

$1°$. First integral:
$$a^2 x + b^2 y + c^2 z = A,$$
where A is an arbitrary constant. It follows that the integral lines are plane curves.

$2°$. Solution:
$$x = C_0 + kC_1 \cos(kt) + a^{-1}bc(C_2 - C_3)\sin(kt),$$
$$y = C_0 + kC_2 \cos(kt) + ab^{-1}c(C_3 - C_1)\sin(kt),$$
$$z = C_0 + kC_3 \cos(kt) + abc^{-1}(C_1 - C_2)\sin(kt),$$

where $k = \sqrt{a^2 + b^2 + c^2}$ and the four constants of integration, $C_1, \ldots, C_4$, are constrained by a single relation,
$$a^2 C_1 + b^2 C_2 + c^2 C_3 = 0.$$

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.