计算机科学中的数学基础

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Basics 15

15 Evaluate $\boxdot_n = \sum_{k=1}^{n} k^3$ by the text's Method 5 as follows: First write $\boxdot_n + \square_n = 2\sum_{1\leqslant j\leqslant k\leqslant n} jk$; then apply (2.33).

由 2-33，我们令 $a_k = k$，于是有：

$$\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2 = 2(\sum_{1\leq j\leq k\leq n} jk) \tag{1}$$

$$= (\sum_{k=1}^{n} k)^2 + \sum_{k=1}^{n} k^2 \tag{2}$$

$$= \frac{n^2(n+1)^2}{4} + \sum_{k=1}^{n} k^2 \tag{3}$$

整理等式，两侧消去相同项，得到:

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \tag{4}$$

Basics 16

16 Prove that $x^{\underline{m}}/(x-n)^{\underline{m}} = x^{\underline{n}}/(x-m)^{\underline{n}}$, unless one of the denominators is zero.

若两式中有一者分母为零，则显然不成立，下面讨论分母不为零的情况。

$$\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x(x-1)\cdots(x-m+1)}{(x-n)(x-n-1)\cdots(x-n-m+1)} \tag{5}$$

若 m=n，原式显然成立，不妨令 m<n；

$$\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x(x-1)\cdots(x-m+1)}{(x-n)(x-n-1)\cdots(x-n-m+1)} \tag{6}$$

$$= \frac{x(x-1)\cdots(x-m+1)(x-m)\cdots(x-n+1)}{(x-m)(x-m-1)\cdots(x-n+1)(x-n)(x-n-1)\cdots(x-n-m+1)} \tag{7}$$

$$= \frac{x^{\underline{n}}}{(x-m)^{\underline{n}}} \tag{8}$$

Basics 17

17 Show that the following formulas can be used to convert between rising and falling factorial powers, for all integers m:

$$\begin{aligned} x^{\overline{m}} &= (-1)^m(-x)^{\underline{m}} = (x+m-1)^{\underline{m}} = 1/(x-1)^{\underline{-m}}; \\ x^{\underline{m}} &= (-1)^m(-x)^{\overline{m}} = (x-m+1)^{\overline{m}} = 1/(x+1)^{\overline{-m}}. \end{aligned}$$

(The answer to exercise 9 defines $x^{\overline{-m}}$.)

从左到右进行等价变换，有：

$$
\begin{aligned}
x^{\overline{m}} &= x(x+1)\cdots(x+m-1) \qquad (9)\\
&= (-1)^m(-x)(-x-1)\cdots(-x-m+1) \qquad (10)\\
&= (-x)^{\underline{m}} \qquad (11)\\
&= (x+m-1)(x+m-1-1)\cdots(x+m-1-m+1) \qquad (12)\\
&= (x+m-1)^{\underline{m}} \qquad (13)\\
& \qquad (14)
\end{aligned}
$$

由习题 9 给出的定义：

$$x^{\overline{m}} = \frac{1}{(x-1)^{\underline{-m}}} \tag{15}$$

原式得证。对于第二个式子，有

$$
\begin{aligned}
x^{\underline{m}} &= x(x-1)\cdots(x-m+1) \qquad (16)\\
&= (-1)^m(-x)(-x+1)\cdots(-x+m-1) \qquad (17)\\
&= (-x)^{\overline{m}} \qquad (18)\\
&= (x-m+1)(x-m+1+1)\cdots(x-m+1=m-1) \qquad (19)\\
&= (x+m-1)^{\overline{m}} \qquad (20)\\
& \qquad (21)
\end{aligned}
$$

由习题 9 给出的定义：

$$x^{\underline{m}} = \frac{1}{(x+1)^{\overline{-m}}} \tag{22}$$

原式得证。

Basics 18

19 Use a summation factor to solve the recurrence

$$
\begin{aligned}
T_0 &= 5;\\
2T_n &= nT_{n-1} + 3 \cdot n!\,, \qquad \text{for } n > 0.
\end{aligned}
$$

由求和因子法，不妨令，$a_n = 2$,$b_n = n$, 我们需要求 s_n, 使得:

$$a_{n-1}s_{n-1} = a_n b_n \tag{23}$$

得到：

$$\frac{s_n}{s_{n-1}} = \frac{2}{n} \tag{24}$$

使用累乘，得到:

$$s_n = \frac{2^{n-1}}{n!} \tag{25}$$

故，不妨令 $A_n = a_n s_n T_n$，得到:

$$A_n = A_{n-1} + 3 \times 2^{n-1} \tag{26}$$

累加得到:

$$A_n = 3 \times 2^n + 5 \tag{27}$$

最终得到：

$$T_n = 3 \times n! + \frac{n!}{2^{n-1}} \tag{28}$$