计算机科学中的数学基础-Exercise5

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Warmup7

7 Let $\nabla f(x) = f(x) - f(x-1)$. What is $\nabla(x^{\overline{m}})$?

分局 m 次上升幂的定义，将式子展开，有，

$$\bigtriangledown(x^{\bar{m}}) = x^{\bar{m}} - (x-1)^{\bar{m}} \tag{1}$$

展开，

$$=x(x+1)(x+2)\cdots(x+m-1)-(x-1)x(x+1)\cdots(x+m-2) \tag{2}$$
$$=x(x+1)\cdots(x+m-2)\times(x+m-1-(x-1)) \tag{3}$$
$$=mx(x+1)\cdots(x+m-2) \tag{4}$$
$$=m(x-1)^{\overline{m-1}} \tag{5}$$

Warmup8

8 What is the value of $0^{\underline{m}}$, when m is a given integer?

对 m 的取值范围分类讨论，有，
若 m$\geq$ 0,

$$0^{\underline{m}} = 0\times 1\times\cdots(m-1) \tag{6}$$
$$= 0 \tag{7}$$

若 m $<$ 0,

$$0^{\underline{m}} = \frac{1}{1\times 2\times\cdots m} \tag{8}$$
$$= \frac{1}{|m|!} \tag{9}$$

Warmup9

9 What is the law of exponents for rising factorial powers, analogous to (2.52)? Use this to define $x^{\overline{-n}}$.

类比 2.52：

$$x^{\underline{m+n}} = x^{\underline{m}}(x-m)^{\underline{n}} \tag{10}$$

可得：

$$x^{m\bar{+}n} = x^{\bar{m}}(x+m)^{\bar{n}} \tag{11}$$

对上式，令 m=-n, 就有：

$$1 = x^{\overline{-n}} \times (x-n)^{\overline{n}} \tag{12}$$

$$x^{\overline{-n}} = \frac{1}{(x-n)^{\overline{n}}} \tag{13}$$

$$x^{\overline{-n}} = \frac{1}{(x-1)^{\underline{n}}} \tag{14}$$

Warmup10

10 The text derives the following formula for the difference of a product:

$$\Delta(uv) = u\,\Delta v + Ev\,\Delta u.$$

How can this formula be correct, when the left-hand side is symmetric with respect to u and v but the right-hand side is not?

由推导:

$$\begin{aligned}\Delta(u(x)v(x)) =& u(x+1)v(x+1) - u(x)v(x) && (15)\\ =& u(x+1)v(x+1) - u(x)v(x+1) + u(x)v(x+1) - u(x)v(x) && (16)\\ =& u(x)\Delta v(x) + v(x+1)\Delta u(x) && (17)\end{aligned}$$

可知，左侧同样等于:

$$\begin{aligned}\Delta(u(x)v(x)) =& u(x+1)v(x+1) - u(x)v(x) && (18)\\ =& u(x+1)v(x+1) - u(x+1)v(x) + u(x+1)v(x) - u(x)v(x) && (19)\\ =& u(x+1)\Delta v(x) + v(x)\Delta u(x) && (20)\end{aligned}$$

即，

$$\Delta(uv) = Eu\Delta v + v\Delta u \tag{21}$$

Basic14

14 Evaluate $\sum_{k=1}^{n} k2^k$ by rewriting it as the multiple sum $\sum_{1 \leqslant j \leqslant k \leqslant n} 2^k$.

显然，

$$k = \sum_{j=1}^{k} \tag{22}$$

故，原式等于，

$$\begin{aligned}\sum_{k=1}^{n} k2^k &= \sum_{k=1}^{n}\sum_{j=1}^{k} 2^k && (23)\\ &= \sum_{1\le j\le n} (2^{n+1} - 2^j) && (24)\\ &= n2^{n+1} - (2^{n+1} - 2) && (25)\\ &= (n-1)2^{n+1} - 2 && (26)\end{aligned}$$