GAMES101 Lecture Notes

Zhuo Chen

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2 Review of Linear Algebra

Assume 3-dimensional space. Dot product can be written as:

$$\begin{aligned}\vec{a} \cdot \vec{b} &= \vec{a}^{\mathrm{T}} \vec{b} \\ &= \begin{pmatrix} x_a & y_a & z_a \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \\ &= x_a x_b + y_a y_b + z_a z_b\end{aligned}$$

Cross product can be written as:

$$\vec{a} \times \vec{b} = A^* \vec{b} = \underbrace{\begin{pmatrix} 0 & -z_a & y_a \\ z_a & 0 & -x_a \\ -y_a & x_a & 0 \end{pmatrix}}_{\text{dual matrix of } \vec{a}} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = \begin{pmatrix} y_a z_b - z_a y_b \\ z_a x_b - x_a z_b \\ x_a y_b - y_a x_b \end{pmatrix}$$

3 Transformation

2D point is represented as $(x, y, 1)^{\mathrm{T}}$. 2D vector is represented as $(x, y, 0)^{\mathrm{T}}$. This ensures

- Vector + Vector = Vector
- Point − Point = Vector
- Point + Vector = Point

We further define $(x, y, w)^{\mathrm{T}} = (x/w, y/w, 1)^{\mathrm{T}}, w \neq 0$.

Below are some common 2D transformations under homogeneous coordinates:

- Scale

$$S(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotation

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Translation

$$T(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$