习题二

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$$A=\begin{pmatrix}1&1&1\\1&1&-1\\1&-1&1\end{pmatrix},\quad B=\begin{pmatrix}1&2&3\\-1&-2&4\\0&5&1\end{pmatrix}$$

解:

$$3AB-2A=3\begin{pmatrix}1&1&1\\1&1&-1\\1&-1&1\end{pmatrix}\begin{pmatrix}1&2&3\\-1&-2&4\\0&5&1\end{pmatrix}-2\begin{pmatrix}1&1&1\\1&1&-1\\1&-1&1\end{pmatrix}$$

$$=3\begin{pmatrix}0&5&8\\0&-5&6\\2&9&0\end{pmatrix}-2\begin{pmatrix}1&1&1\\1&1&-1\\1&-1&1\end{pmatrix}$$

$$=\begin{pmatrix}-2&13&22\\-2&-17&20\\4&29&-2\end{pmatrix}$$

$$A^TB=AB=\begin{pmatrix}1&1&1\\1&1&-1\\1&-1&1\end{pmatrix}\begin{pmatrix}1&2&3\\-1&-2&4\\0&5&1\end{pmatrix}=\begin{pmatrix}0&5&8\\0&-5&6\\2&9&0\end{pmatrix}$$

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$$\begin{cases}x_1=2y_1 & & +y_3,\\x_2=-2y_1 & +3y_2 & +2y_3,\\x_3=4y_1 & +y_2 & +5y_3,\end{cases}$$

$$\begin{cases}y_1=-3z_1 & +z_2, & \\y_2=2z_1 & & +z_3,\\y_3= & -z_2 & +3z_3.\end{cases}$$

解:

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2&0&1\\-2&3&2\\4&1&5\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix},\quad \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}-3&1&0\\2&0&1\\0&-1&3\end{pmatrix}\begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2&0&1\\-2&3&2\\4&1&5\end{pmatrix}\begin{pmatrix}-3&1&0\\2&0&1\\0&-1&3\end{pmatrix}\begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}$$

$$=\begin{pmatrix}-6&1&3\\12&-4&6\\10&-1&16\end{pmatrix}\begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}$$

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$$A=\begin{pmatrix}3&1\\1&-3\end{pmatrix}$$

解:

$$A^2=\begin{pmatrix}3&1\\1&-3\end{pmatrix}\begin{pmatrix}3&1\\1&-3\end{pmatrix}=\begin{pmatrix}10&0\\0&10\end{pmatrix}=10\begin{pmatrix}1&0\\0&1\end{pmatrix}A^{50}=A^{2\times 25}=10^{25}\begin{pmatrix}1&0\\0&1\end{pmatrix}A^{51}=A^{50}A=10^{25}\begin{pmatrix}3&1\\1&-$$

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$$(B^TAB)^T=B^TA^TB\because A^T=A\therefore B^TA^TB=B^TAB\therefore B^TA^TB\text{为对称矩阵}$$

若AB为对称矩阵，则$(AB)^T=AB$

$\implies B^TA^T=AB=BA$

若$AB=BA$，则$(AB)^T=(BA)^T$

$\implies AB=(AB)^T$

$\therefore AB=(AB)^T\iff AB=BA$

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$$\begin{cases}x_1=2y_1+2y_2+y_3,\\x_2=3y_1+y_2+5y_3,\\x_3=3y_1+2y_2+3y_3.\end{cases}$$

可得:

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2&2&1\\3&1&5\\3&2&3\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$$

$\therefore$

$$\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}2&2&1\\3&1&5\\3&2&3\end{pmatrix}^{-1}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

解得:

$$\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}2&2&1\\3&1&5\\3&2&3\end{pmatrix}^{-1}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$\because$

$$\begin{pmatrix}2&2&1\\3&1&5\\3&2&3\end{pmatrix}^{-1}=\begin{pmatrix}-7&-4&9\\6&3&-7\\3&2&-4\end{pmatrix}$$

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$$E-J=\begin{pmatrix}0&-1&\cdots&-1\\-1&0&\cdots&-1\\\vdots&\vdots&\ddots&\vdots\\-1&-1&\cdots&0\end{pmatrix}=\begin{pmatrix}1&0&\cdots&-1\\-1&1&\cdots&-1\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{pmatrix}=\begin{pmatrix}1&0&\cdots&-1\\0&1&\cdots&-2\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&-(n-1)\end{pmatrix}=-n+1$$

$\therefore$

$$|E-J|=\begin{vmatrix}1&0&\cdots&-1\\0&1&\cdots&-2\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&-(n-1)\end{vmatrix}=-n+1\neq 0,\quad (n\geq 2)$$

$\therefore$ $E-J$是可逆矩阵 $\because$

$$(E-J)(E-\frac{1}{n-1}J)=E-(1-\frac{1}{n-1})J+\frac{1}{n-1}J^2$$

又 $\because$

$$J^2=\begin{pmatrix}1&1&\cdots&1\\1&1&\cdots&1\\\vdots&\vdots&\ddots&\vdots\\1&1&\cdots&1\end{pmatrix}\begin{pmatrix}1&1&\cdots&1\\1&1&\cdots&1\\\vdots&\vdots&\ddots&\vdots\\1&1&\cdots&1\end{pmatrix}=n\begin{pmatrix}1&1&\cdots&1\\1&1&\cdots&1\\\vdots&\vdots&\ddots&\vdots\\1&1&\cdots&1\end{pmatrix}=nJ$$

$\therefore$

$$\begin{aligned}&(E-J)(E-\frac{1}{n-1}J)\\=&E-(1-\frac{1}{n-1})J+\frac{1}{n-1}J^2\\=&E-\frac{n}{n-1}J+\frac{1}{n-1}nJ\\=&E\end{aligned}$$

即：$(E-J)^{-1}=E-\dfrac{1}{n-1}J$

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$$A=\begin{pmatrix}0&3&3\\1&1&0\\-1&2&3\end{pmatrix}$$

$$AB=A+2B$$

$$(A-2E)B=A$$

$$\begin{pmatrix}-2&1&1\\-1&-1&-2\\-3&0&1\end{pmatrix}B=\begin{pmatrix}0&3&3\\1&1&0\\-1&2&3\end{pmatrix}$$

$$B=\begin{pmatrix}0&3&3\\1&1&0\\-1&2&3\end{pmatrix}\begin{pmatrix}-2&1&1\\-1&-1&-2\\-3&0&1\end{pmatrix}^{-1}$$

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$$A=\begin{pmatrix}1&0&0\\0&-2&0\\1&0&1\end{pmatrix}$$

$$A^*BA=2BA-8E$$

$$|A|A^{-1}BA=2BA-8E$$

$$|A|A^{-1}B=2B-8A^{-1}$$

$$(|A|A^{-1}-2E)B=-8A^{-1}$$

$$B=-8A^{-1}(|A|A^{-1}-2E)^{-1}$$

$$B=-8(A^*-2E)A$$

将$\begin{pmatrix}1&0&0\\0&-2&0\\1&0&1\end{pmatrix}$代入上式，解得：

$$B=16\begin{pmatrix}2&0&0\\0&-1&0\\0&0&2\end{pmatrix}$$