3.4

I. 试一试 匹配解 3.4 的 求和版 能导出什么 3D

a. 设 $dz_j = dz$, $z_j = \Sigma_{i \in [0,\, j-1)}\, dz_i = j \cdot dz$, $z = (J+1) \cdot dz$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \frac{e^{ik_{3z}\cdot z}}{(k_z''+k_{3z})/2} \cdot \iint \mathcal{F}[M_{\mathrm{eff}}(x,y)]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}[E_{10}]\Big|_{k_x,k_y}^{x,y}\, \mathcal{F}[E_{20}]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} \sum_{j=1}^{J+1}(-1)^{j-1}\cdot \mathrm{sinc}\left(\Delta k_z''\frac{dz}{2}\right)\cdot i dz \cdot e^{i(k_{zq}-k_{3z})\left(j-\frac{1}{2}\right)\cdot dz} \cdot dk_x dk_y dg_x dg_y$$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \frac{\mathrm{sinc}\left(\Delta k_z''\frac{dz}{2}\right)\cdot i dz \cdot e^{ik_{3z}\cdot z}}{(k_z''+k_{3z})/2} \cdot \iint \mathcal{F}[M_{\mathrm{eff}}(x,y)]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}[E_{10}]\Big|_{k_x,k_y}^{x,y}\, \mathcal{F}[E_{20}]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} e^{-i(k_{zq}-k_{3z})\frac{dz}{2}} \sum_{j=1}^{J+1}(-1)^{j-1}\cdot e^{i(k_{zq}-k_{3z})j\cdot dz} \cdot dk_x dk_y dg_x dg_y$$

其中 $\sum_{j=1}^{J+1}(-1)^{j-1} e^{i\cdot j\cdot C} = \frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{i\cdot C}} e^{i\cdot C} = \frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{-i\cdot C}}$ ，则 $\sum_{j=1}^{J+1}(-1)^{j-1} e^{i\cdot j\cdot k\cdot dz} = \frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{i\cdot k\cdot dz}} e^{i\cdot k\cdot dz} = \frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{-i\cdot k\cdot dz}}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \frac{\mathrm{sinc}\left(\Delta k_z''\frac{dz}{2}\right)\cdot i dz \cdot e^{ik_{3z}\cdot z}}{(k_z''+k_{3z})/2} \cdot \iint \mathcal{F}[M_{\mathrm{eff}}(x,y)]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}[E_{10}]\Big|_{k_x,k_y}^{x,y}\, \mathcal{F}[E_{20}]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} e^{-i(k_{zq}-k_{3z})\frac{dz}{2}} \frac{1+(-1)^J e^{i\cdot(k_{zq}-k_{3z})\cdot z}}{1+e^{-i\cdot(k_{zq}-k_{3z})\cdot dz}} \cdot dk_x dk_y dg_x dg_y$$

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z}+K_{2z}$$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \frac{\mathrm{sinc}\left(\Delta k_z''\frac{dz}{2}\right)\cdot i dz \cdot e^{ik_{3z}\cdot z}}{(k_z''+k_{3z})/2} \cdot \iint \mathcal{F}[M_{\mathrm{eff}}(x,y)]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}[E_{10}]\Big|_{k_x,k_y}^{x,y}\, \mathcal{F}[E_{20}]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} \frac{1+(-1)^J e^{i\cdot(k_{zq}-k_{3z})\cdot z}}{e^{i\cdot(k_{zq}-k_{3z})\cdot\frac{dz}{2}}+e^{-i\cdot(k_{zq}-k_{3z})\cdot\frac{dz}{2}}} \cdot dk_x dk_y dg_x dg_y$$

$$\Delta k_z'' = k_z'' - k_{3z}$$