

## VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x}, k_{3y})$ 的级数解

$$\begin{aligned}
 G_{3z}(k_{3x}, k_{3y}) &= \frac{\chi_{\text{eff}} \omega_3^2}{2c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{3x}, k_{3y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{3x}-g_x, k_{3y}-g_y} \frac{e^{i\Delta k_{zQ} z} - 1}{\Delta k_{zQ}} \frac{2}{\Delta k_{zQ}/k_{3z} + 2} dk_x dk_y dg_x dg_y dg_z \cdot \frac{e^{ik_{3z} z}}{k_{3z}} \\
 &= \frac{\chi_{\text{eff}} \omega_3^2}{2c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{3x}, k_{3y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{3x}-g_x, k_{3y}-g_y} \text{sinc}\left(\Delta k_{zQ} \frac{z}{2}\right) \cdot e^{i\Delta k_{zQ} \frac{z}{2}} \cdot iz \cdot \frac{2}{\Delta k_{zQ}/k_{3z} + 2} dk_x dk_y dg_x dg_y dg_z \cdot \frac{e^{ik_{3z} z}}{k_{3z}} \\
 &\approx \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{3x}, k_{3y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{3x}-g_x, k_{3y}-g_y} \frac{1 - \frac{7}{60} \left(\Delta k_{zQ} \frac{z}{2}\right)^2}{1 + \frac{3}{60} \left(\Delta k_{zQ} \frac{z}{2}\right)^2} \cdot e^{ik_{zQ} \frac{z}{2}} dk_x dk_y dg_x dg_y \cdot \frac{1}{\Delta k_{zQ}'' + 2k_{3z}} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz \\
 &= \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{1x}, k_{1y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{2x}, k_{2y}} \left[1 + \sqrt{\frac{7}{60}} \cdot \left(\Delta k_{zQ}'' \frac{z}{2}\right)\right] \cdot e^{i(k_{1z} + k_{2z}) \frac{z}{2}} dk_x dk_y dg_x dg_y \cdot \frac{1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{1}{k_{zQ}'' + k_{3z}} \cdot e^{ig_z \frac{z}{2}} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz \\
 &= \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{1x}, k_{1y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{2x}, k_{2y}} \left[1 + \sqrt{\frac{7}{60}} \cdot \left((k_{1z} + k_{2z} + g_z - k_{3z}) \frac{z}{2}\right)\right] \cdot e^{i(k_{1z} + k_{2z}) \frac{z}{2}} dk_x dk_y dg_x dg_y \cdot \frac{1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{e^{ig_z \frac{z}{2}}}{k_{zQ}'' + k_{3z}} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz \\
 &= \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \iint \mathcal{F}[E_{10}(x, y)] \Big|_{k_{1x}, k_{1y}} \mathcal{F}[E_{20}(x, y)] \Big|_{k_{2x}, k_{2y}} \left[1 + \sqrt{\frac{7}{60}} \cdot (g_z - k_{3z}) \frac{z}{2} + \sqrt{\frac{7}{60}} \cdot (k_{1z} + k_{2z}) \frac{z}{2}\right] \cdot e^{i(k_{1z} + k_{2z}) \frac{z}{2}} dk_x dk_y dg_x dg_y \cdot \frac{1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{e^{ig_z \frac{z}{2}}}{k_{zQ}'' + k_{3z}} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz \\
 &= \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint C(g_x, g_y, g_z) \cdot \left\{ G_{\frac{1}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) * G_{\frac{2}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) dg_x dg_y \cdot \frac{\left[1 + \sqrt{\frac{7}{60}} (g_z - k_{3z}) \frac{z}{2}\right] \left[1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)\right]}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{e^{ig_z \frac{z}{2}}}{k_{zQ}'' + k_{3z}} \right. \\
 &\quad + \left\{ G_{\frac{1}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) \cdot k_{1z} \right\} * G_{\frac{2}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) dg_x dg_y \cdot \frac{1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{e^{ig_z \frac{z}{2}}}{k_{zQ}'' + k_{3z}} \cdot \sqrt{\frac{7}{60}} \frac{z}{2} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz \\
 &\quad \left. + G_{\frac{1}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) * \left\{ G_{\frac{2}{2} \frac{z}{2}}(k_{3x} - g_x, k_{3y} - g_y) \cdot k_{2z} \right\} dg_x dg_y \cdot \frac{1 - \sqrt{\frac{7}{60}} \left(\Delta k_{zQ}'' \frac{z}{2}\right)}{1 + \frac{1}{20} \left(\Delta k_{zQ}'' \frac{z}{2}\right)^2} \cdot \frac{e^{ig_z \frac{z}{2}}}{k_{zQ}'' + k_{3z}} \cdot \sqrt{\frac{7}{60}} \frac{z}{2} \right\} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz
 \end{aligned}$$