VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 King 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{i\frac{g_z}{2}z}\cdot\mathrm{d}\frac{g_z}{2}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\quad k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$