Evolutionary Outcomes

$$\left(\nabla^2 + k_3^2\right) E_3\left(\boldsymbol{r}\right) = -\frac{k_3^2}{\varepsilon_3^{(1)}}\left[\varepsilon_0 \chi_{\text{eff}}\left(\boldsymbol{r}\right) \cdot E_1\left(\boldsymbol{r}\right) E_2\left(\boldsymbol{r}\right)\right]$$

$$\left(\frac{\partial^2}{\partial z^2} + k_{3z}^2\right) G_{3z}\left(k_{3x}, k_{3y}\right) = -\frac{k_3^2}{n_3^2} Q_{3z}\left(k_{3x}, k_{3y}\right)$$

$$E_3\left(x, y, z\right) = \mathcal{F}^{-1}\left[G_{3z}\left(k_{3x}, k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\frac{e^{ik_{3z}z}}{k_{3z}} \cdot \sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z} \cdot \iint \mathcal{F}\left[E_{10}\left(x, y\right)\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\left(x, y\right)\right]\Big|_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}^{x,y} \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{1}{\Delta k_{zQ}/k_{3z}+2}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$\Delta k_{zQ} = k_{zQ} - k_{3z} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_{l_x} - k_x\right)^2 - \left(k_{3y} - g_{l_y} - k_y\right)^2} - \sqrt{k_3^2 - k_{3x}^2 - k_{3y}^2} + g_{lz}$$