非线性 角谱理论

The Nonlinear AST

一． 麦氏方程组

二． 含时复色矢量非齐次电场波动方程

$$\nabla\times\left(\nabla\times\widetilde{\boldsymbol{E}}\right)+\frac{1}{c^2}\frac{\partial^2\widetilde{\boldsymbol{E}}}{\partial t^2}=-\frac{1}{\varepsilon_0 c^2}\left[\frac{\partial^2\widetilde{\boldsymbol{P}}}{\partial t^2}+\frac{\partial}{\partial t}\left(\nabla\times\widetilde{\boldsymbol{M}}\right)\right]$$

三． 单色定态标量非齐次非线性电场波动方程（空域三维）

$$\left(\nabla^2+k_n^2\right)E_n\left(\boldsymbol{r}\right)=-\frac{k_n^2}{\varepsilon_n^{(1)}}P_n^{NL}\left(\boldsymbol{r}\right)$$

四． 考虑二阶非线性，以和频为例（空域三维）

$$\left(\nabla^2+k_3^2\right)E_3\left(\boldsymbol{r}\right)=-\frac{k_3^2}{\varepsilon_3^{(1)}}P_3^{(2)}\left(\boldsymbol{r}\right)$$

四. 考虑二阶非线性，以和频为例（空域三维）

$$\left(\nabla^2+k_3^2\right)E_3\left(\boldsymbol{r}\right)=-\frac{k_3^2}{\varepsilon_3^{(1)}}\left[\varepsilon_0\chi_{\mathrm{eff}}\left(\boldsymbol{r}\right)\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]$$

五. 从频域考察该方程（频域二维 · 空域三维）

$$\left(\nabla^2+k_3^2\right)\mathcal{F}^{-1}\left[G_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}=-\frac{k_3^2}{\varepsilon_{3\mathrm{r}}^{(1)}}\mathcal{F}^{-1}\left[Q_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}$$

$$\left(\nabla^2+k_3^2\right)\iint G_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}\mathrm{d}k_{3x}\mathrm{d}k_{3y}=-\frac{k_3^2}{n_3^2}\iint Q_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}\mathrm{d}k_{3x}\mathrm{d}k_{3y}$$

$$\left(\nabla^2+k_3^2\right)\left[G_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}\right]=-\frac{k_3^2}{n_3^2}Q_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}$$

五． 从频域考察该方程（频域二维 · 空域三维）

$$\left(\frac{\partial^2}{\partial z^2}+\nabla_{\mathrm{T}}^2+k_3^2\right)\left[G_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}\right]=-\frac{k_3^2}{n_3^2}Q_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{i\left(k_{3x}x+k_{3y}y\right)}$$

六． 先处理左侧，设 $G_{3z}\left(k_{3x},k_{3y}\right)$ 不含 x, y ，得

$$\left(\frac{\partial^2}{\partial z^2}+k_3^2-k_{3x}^2-k_{3y}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{k_3^2}{n_3^2}Q_{3z}\left(k_{3x},k_{3y}\right)$$

设 $k_{3z}:=\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}$ ，则有 $\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{k_3^2}{n_3^2}Q_{3z}\left(k_{3x},k_{3y}\right)$

其中，$Q_{3z}\left(k_{3x},k_{3y}\right)=\mathcal{F}\left[\chi_{\mathrm{eff}}\left(\boldsymbol{r}\right)\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|^{x,y}_{k_{3x},k_{3y}}$

七． 该方程的解，与 右侧非齐次项 关于 z 的函数形式 有关

1． 若 双泵浦 均未耗尽，即 $E_1(\boldsymbol{r}),E_2(\boldsymbol{r})$ 只线性衍射 · 低效和频

则可获知 $Q_{3z}(k_{3x},k_{3y})$ 关于 z 的函数，且不包含 $E_3(\boldsymbol{r})$ 本身：

$$
\begin{aligned}
Q_{3z}\left(k_{3x},k_{3y}\right) &= \mathcal{F}\left[\chi_{\text{eff}}\left(\boldsymbol{r}\right)\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\
&= \mathcal{F}\left[\chi_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} * \mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}
\end{aligned}
$$

I. 关键在于，将 $Q_{3z}\left(k_{3x},k_{3y}\right)$ 中的 z 分离出来：

$$Q_{3z}\left(k_{3x},k_{3y}\right)=\mathcal{F}\left[\chi_{\text{eff}}\left(\boldsymbol{r}\right)\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

$$=\mathcal{F}\left[\chi_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}*\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

II. 对于 $\mathcal{F}\left[\chi_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}$ ，设 $\chi_{\text{eff}}\left(\boldsymbol{r}\right)=\chi_{\text{eff}}\cdot M_{\text{eff}}\left(\boldsymbol{r}\right)$ ，其中 $M_{\text{eff}}\left(\boldsymbol{r}\right)$ 为实际调制函数 $M\left(\boldsymbol{r}\right)$ 与基波交叠区域 $\{\boldsymbol{r}\}=V_{\text{overlap}}$ 的交集 $\{\boldsymbol{r}\}=V_{\text{eff}}$ 内的，调制函数 $M_{\text{overlap}}\left(\boldsymbol{r}\right)$ 的三维周期延拓。

II. 对于 $\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$ ，设 $\chi_{\text{eff}}(\boldsymbol{r})=\chi_{\text{eff}}\cdot M_{\text{eff}}(\boldsymbol{r})$ ，其中 $M_{\text{eff}}(\boldsymbol{r})$ 为实际调制函数 $M(\boldsymbol{r})$ 与基波交叠区域 $\{\boldsymbol{r}\}=V_{\text{overlap}}$ 的交集 $\{\boldsymbol{r}\}=V_{\text{eff}}$ 内的，调制函数 $M_{\text{overlap}}(\boldsymbol{r})$ 的三维周期延拓。

1. 则三维周期函数 $M_{\text{eff}}(\boldsymbol{r})$ 可展为三维傅立叶级数

$$M_{\text{eff}}(\boldsymbol{r})=\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot e^{i\boldsymbol{g}_{l_x}\cdot\boldsymbol{r}}=\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot e^{i\left(g_{l_x}x+g_{l_y}y+g_{l_z}z\right)}$$

$$=\sum_{m_x,m_y,m_z=-\infty}^{+\infty}\sum_{n_x,n_y,n_z\in\Omega}C_{m_x,m_y,m_z;n_x,n_y,n_z}\cdot e^{i\left(\boldsymbol{G}_{m_x,m_y,m_z}+\boldsymbol{q}_{n_x,n_y,n_z}\right)\cdot\boldsymbol{r}}$$

1. 则三维周期函数 $M_{\text{eff}}(\boldsymbol{r})$ 可展为三维傅立叶级数

2. 则

$$\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \chi_{\text{eff}}\cdot\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \chi_{\text{eff}}\cdot\mathcal{F}\left[\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot e^{i\left(g_{l_x}x+g_{l_y}y+g_{l_z}z\right)}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}$$

$$= \chi_{\text{eff}}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[e^{i\left(g_{l_x}x+g_{l_y}y\right)}\right]\Big|_{k_{3x},k_{3y}}^{x,y} e^{ig_{l_z}z}$$

$$= \chi_{\text{eff}}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot\delta\left(k_{3x}-g_{l_x},k_{3y}-g_{l_y}\right)e^{ig_{l_z}z}$$

III.

$$\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \mathcal{F}\left[E_1(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} * \mathcal{F}\left[E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

$$= G_{1z}\left(k_{3x},k_{3y}\right)*G_{2z}\left(k_{3x},k_{3y}\right)$$

$$= \iint G_{1z}\left(k_x,k_y\right)\cdot G_{2z}\left(k_{3x}-k_x,k_{3y}-k_y\right)\mathrm{d}k_x\mathrm{d}k_y$$

$$= \iint g_1\left(k_x,k_y\right)H_1\left(k_x,k_y\right)\cdot g_2\left(k_{3x}-k_x,k_{3y}-k_y\right)H_2\left(k_{3x}-k_x,k_{3y}-k_y\right)\mathrm{d}k_x\mathrm{d}k_y$$

$$= \iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y} e^{i\sqrt{k_1^2-k_x^2-k_y^2}\,z}\cdot\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-k_x,k_{3y}-k_y}^{x,y} e^{i\sqrt{k_2^2-\left(k_{3x}-k_x\right)^2-\left(k_{3y}-k_y\right)^2}\,z}\mathrm{d}k_x\mathrm{d}k_y$$

IV.

$$\begin{cases} \mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \chi_{\text{eff}} \cdot \sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z} \cdot \delta\left(k_{3x}-g_{l_x}, k_{3y}-g_{l_y}\right) e^{ig_{l_z} z} \\ \mathcal{F}\left[E_1(\boldsymbol{r}) E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y} e^{i\sqrt{k_1^2-k_x^2-k_y^2}z} \cdot \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-k_x,k_{3y}-k_y}^{x,y} e^{i\sqrt{k_2^2-(k_{3x}-k_x)^2-(k_{3y}-k_y)^2}z} \mathrm{d}k_x \mathrm{d}k_y \end{cases}$$

代入

$$Q_{3z}\left(k_{3x},k_{3y}\right) = \mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} * \mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

$$= \chi_{\text{eff}} \cdot \sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z} \cdot \iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_{l_x}-k_x,k_{3y}-g_{l_y}-k_y}^{x,y} e^{i\left(\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_{l_x}-k_x\right)^2-\left(k_{3y}-g_{l_y}-k_y\right)^2}+g_{l_z}\right)z} \mathrm{d}k_x \mathrm{d}k_y$$

代入

$$\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right) G_{3z}\left(k_{3x},k_{3y}\right) = -\frac{k_3^2}{n_3^2} Q_{3z}\left(k_{3x},k_{3y}\right)$$

V. 和频 传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{\chi_{\text{eff}}k_3^2}{n_3^2}\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}^{x,y}e^{ik_{zQ}z}\,\mathrm{d}k_x\mathrm{d}k_y$$

其中，

$$k_{zQ}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_{l_x}-k_x\right)^2-\left(k_{3y}-g_{l_y}-k_y\right)^2}+g_{l_z}$$

VI. 双泵浦 均未耗尽 时，频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 和 空域解 $E_3(\boldsymbol{r})$

$$\begin{cases} G_{3z}\left(k_{3x},k_{3y}\right)=g_3^+\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}z}+g_3^-\left(k_{3x},k_{3y}\right)\cdot e^{-ik_{3z}z} \\ \qquad +\dfrac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}^{x,y}\frac{e^{ik_{zQ}z}}{k_{zQ}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y \\ E_3(x,y,z)=\mathcal{F}^{-1}\left[G_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}} \end{cases}$$

1. 只考虑 前向传播 的 和频光 $g_3^+(k_{3x},k_{3y}) = g_3(k_{3x},k_{3y}),\ g_3^-(k_{3x},k_{3y}) = 0$

$$G_{3z}(k_{3x},k_{3y}) = g_3(k_{3x},k_{3y})\cdot e^{ik_{3z}z} + \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}\ \frac{e^{ik_{zQ}z}}{k_{zQ}^2-k_{3z}^2}\,\mathrm{d}k_x\mathrm{d}k_y$$

2. 引入 边界条件 $G_{30}(k_{3x},k_{3y}) = \mathbf{0}$

$$\frac{2}{k_{zQ}/k_{3z}+1}$$

$$\left\{\begin{aligned}
G_{3z}(k_{3x},k_{3y}) &= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}\ \frac{e^{ik_{zQ}z}-e^{ik_{3z}z}}{k_{zQ}^2-k_{3z}^2}\,\mathrm{d}k_x\mathrm{d}k_y \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty} C_{l_x,l_y,l_z}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_{l_x}-k_x,\,k_{3y}-g_{l_y}-k_y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\boxed{\frac{2}{\Delta k_{zQ}/k_{3z}+2}}\,\mathrm{d}k_x\mathrm{d}k_y \\
&\text{其中，}\quad \Delta k_{zQ} = k_{zQ}-k_{3z} = \sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_{l_x}-k_x\right)^2-\left(k_{3y}-g_{l_y}-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_{l_z} \\
E_3(x,y,z) &= \mathcal{F}^{-1}\left[G_{3z}(k_{3x},k_{3y})\right]\Big|^{k_{3x},k_{3y}}_{x,y}
\end{aligned}\right.$$

V. 倍频 传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{2z}^2\right)G_{2z}\left(k_{2x},k_{2y}\right)=-\frac{\chi_{\text{eff}}k_2^2}{n_2^2}\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_{l_x}-k_x,k_{2y}-g_{l_y}-k_y}^{x,y}e^{ik_{zQ}z}\mathrm{d}k_x\mathrm{d}k_y$$

VI. 泵浦 未耗尽 时，频域解 $G_{2z}\left(k_{2x},k_{2y}\right)$ 和 空域解 $E_2(\boldsymbol{r})$

$$\begin{cases}G_{2z}\left(k_{2x},k_{2y}\right)=\dfrac{\chi_{\text{eff}}\omega_2^2}{c^2}\cdot\displaystyle\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_{l_x}-k_x,k_{2y}-g_{l_y}-k_y}^{x,y}\dfrac{e^{ik_{zQ}z}-e^{ik_{2z}z}}{k_{zQ}^2-k_{2z}^2}\mathrm{d}k_x\mathrm{d}k_y\\ \qquad=\dfrac{d_{\text{eff}}\omega_2^2}{c^2}\dfrac{e^{ik_{2z}z}}{k_{2z}}\cdot\displaystyle\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_{l_x}-k_x,k_{2y}-g_{l_y}-k_y}^{x,y}\dfrac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\dfrac{2}{\Delta k_{zQ}/k_{2z}+2}\mathrm{d}k_x\mathrm{d}k_y\\ \text{其中，}\ \Delta k_{zQ}=k_{zQ}-k_{2z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_1^2-\left(k_{2x}-g_{l_x}-k_x\right)^2-\left(k_{2y}-g_{l_y}-k_y\right)^2}-\sqrt{k_2^2-k_{2x}^2-k_{2y}^2}+g_{l_z}\\ E_2\left(x,y,z\right)=\mathcal{F}^{-1}\left[G_{2z}\left(k_{2x},k_{2y}\right)\right]\Big|_{x,y}^{k_{2x},k_{2y}}\end{cases}$$

II. 第二种对 $\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$ 中的 $\chi_{\text{eff}}(\boldsymbol{r})=\chi_{\text{eff}}\cdot M_{\text{eff}}(\boldsymbol{r})$ 中的 $M_{\text{eff}}(\boldsymbol{r})$ 的处理方法：

$$\begin{aligned}\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} &=\chi_{\text{eff}}\cdot\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\chi_{\text{eff}}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot e^{ig_{l_z}z}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\\ &=\chi_{\text{eff}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\ &=\chi_{\text{eff}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)\end{aligned}$$

$\Downarrow$ 代入

$$\begin{aligned}Q_{3z}\left(k_{3x},k_{3y}\right) &=\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\ &=\chi_{\text{eff}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{lz}z}\cdot C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\ &=\chi_{\text{eff}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{lz}z}\cdot\iint C\left(g_x,g_y\right)\cdot\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\\ &=\chi_{\text{eff}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{i\left(\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}+g_{lz}\right)z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{aligned}$$

V. 和频传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{\chi_{\text{eff}}k_3^2}{n_3^2}\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{ik_{zQ}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

VI. 泵浦未耗尽时，频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 和空域解 $E_3(\boldsymbol{r})$

$$\begin{cases}G_{3z}\left(k_{3x},k_{3y}\right)=\dfrac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\dfrac{e^{ik_{zQ}z}-e^{ik_{3z}z}}{k_{zQ}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\ \qquad=\dfrac{\chi_{\text{eff}}\omega_3^2}{2c^2}\dfrac{e^{ik_{3z}z}}{k_{3z}}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\dfrac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\dfrac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\ \text{其中，}\quad\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_{lz}\\ E_3\left(x,y,z\right)=\mathcal{F}^{-1}\left[G_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}\end{cases}$$

V. 倍频传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{2z}^2\right)G_{2z}\left(k_{2x},k_{2y}\right)=-\frac{\chi_{\mathrm{eff}}k_2^2}{n_2^2}\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{2x}-g_x-k_x,\,k_{2y}-g_y-k_y}^{x,y}e^{ik_{zQ}z}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

VI. 泵浦未耗尽时，频域解 $G_{2z}\left(k_{2x},k_{2y}\right)$ 和空域解 $E_2(\boldsymbol{r})$

$$\left\{\begin{aligned}
G_{2z}\left(k_{2x},k_{2y}\right)&=\frac{\chi_{\mathrm{eff}}\omega_2^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{2x}-g_x-k_x,\,k_{2y}-g_y-k_y}^{x,y}\frac{e^{ik_{zQ}z}-e^{ik_{2z}z}}{k_{zQ}^2-k_{2z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\
&=\frac{d_{\mathrm{eff}}\omega_2^2}{c^2}\frac{e^{ik_{2z}z}}{k_{2z}}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{2x}-g_x-k_x,\,k_{2y}-g_y-k_y}^{x,y}\frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{2z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\
&\text{其中，}\quad\Delta k_{zQ}=k_{zQ}-k_{2z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_1^2-\left(k_{2x}-g_x-k_x\right)^2-\left(k_{2y}-g_y-k_y\right)^2}-\sqrt{k_2^2-k_{2x}^2-k_{2y}^2}+g_{l_z}\\
E_2(x,y,z)&=\mathcal{F}^{-1}\left[G_{2z}\left(k_{2x},k_{2y}\right)\right]\Big|_{x,y}^{k_{2x},k_{2y}}
\end{aligned}\right.$$

II. 第三种对 $\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$ 中的 $\chi_{\text{eff}}(\boldsymbol{r})=\chi_{\text{eff}}\cdot M_{\text{eff}}(\boldsymbol{r})$ 中的 $M_{\text{eff}}(\boldsymbol{r})$ 的处理方法：

$$\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\chi_{\text{eff}}\cdot\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\chi_{\text{eff}}\cdot\int\mathcal{F}\left[M_{\text{eff}}(x,y,z)\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}e^{ig_{lz}z}\mathrm{d}z$$

$$=\chi_{\text{eff}}\cdot\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z$$

⇓ 代入

$$Q_{3z}\left(k_{3x},k_{3y}\right)=\mathcal{F}\left[\chi_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

$$=\chi_{\text{eff}}\cdot\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

$$=\chi_{\text{eff}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}e^{ig_z z}\,\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\chi_{\text{eff}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{i\left(\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}+g_z\right)z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

V. 和频 传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{\chi_{\mathrm{eff}}k_3^2}{n_3^2}\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_{l_x}-k_x,k_{3y}-g_{l_y}-k_y}^{x,y}e^{ik_{zQ}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

VI. 泵浦 未耗尽 时，频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 和 空域解 $E_3\left(\boldsymbol{r}\right)$

$$\begin{cases}G_{3z}\left(k_{3x},k_{3y}\right)=\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\dfrac{e^{ik_{zQ}z}-e^{ik_{3z}z}}{k_{zQ}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\\ \qquad=\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\dfrac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\dfrac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\dfrac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\\ \text{其中，}\quad\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z\\ E_3\left(x,y,z\right)=\mathcal{F}^{-1}\left[G_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}\end{cases}$$

V. 倍频传播方程（频域）

$$\left(\frac{\partial^2}{\partial z^2}+k_{2z}^2\right)G_{2z}\left(k_{2x},k_{2y}\right)=-\frac{\chi_{\text{eff}}k_2^2}{n_2^2}\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_x-k_x,k_{2y}-g_y-k_y}^{x,y}\ e^{ik_{zQ}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

VI. 泵浦未耗尽时，频域解 $G_{2z}\left(k_{2x},k_{2y}\right)$ 和空域解 $E_2(\boldsymbol{r})$

$$G_{2z}\left(k_{2x},k_{2y}\right)=\frac{\chi_{\text{eff}}\omega_2^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_x-k_x,k_{2y}-g_y-k_y}^{x,y}\ \frac{e^{ik_{zQ}z}-e^{ik_{2z}z}}{k_{zQ}^2-k_{2z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{d_{\text{eff}}\omega_2^2}{c^2}\frac{e^{ik_{2z}z}}{k_{2z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_{2x}-g_y-k_x,k_{2y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{2z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

其中， $\Delta k_{zQ}=k_{zQ}-k_{2z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_1^2-\left(k_{2x}-g_x-k_x\right)^2-\left(k_{2y}-g_y-k_y\right)^2}-\sqrt{k_2^2-k_{2x}^2-k_{2y}^2}+g_z$

$$E_2\left(x,y,z\right)=\mathcal{F}^{-1}\left[G_{2z}\left(k_{2x},k_{2y}\right)\right]\Big|_{x,y}^{k_{2x},k_{2y}}$$

VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解

$$G_{3z}(k_{3x},k_{3y})=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$\approx\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{1}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\left(e^{i\Delta k_{zQ}z}-1\right)\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{1}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\cdot\begin{bmatrix}e^{ig_zz}\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{i(k_{zQ}-g_z)z}\mathrm{d}k_x\mathrm{d}k_y\\-e^{ik_{3z}z}\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\mathrm{d}k_x\mathrm{d}k_y\end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{1}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\cdot\begin{bmatrix}e^{ig_zz}\cdot G_{1z}(k_{3x}-g_x,k_{3y}-g_y)*G_{2z}(k_{3x}-g_x,k_{3y}-g_y)\\-e^{ik_{3z}z}\cdot g_1(k_{3x}-g_x,k_{3y}-g_y)*g_z(k_{3x}-g_x,k_{3y}-g_y)\end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

其中，$\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-(k_{3x}-g_x-k_x)^2-(k_{3y}-g_y-k_y)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

$$\Delta k'_{zQ}=\Delta k_{zQ}\Big|_{k_x,k_y\to0,0}=k'_{zQ}-k_{3z}=k_{zQ}\Big|_{k_x,k_y\to0,0}-k_{3z}=k_1+\sqrt{k_2^2-(k_{3x}-g_x)^2-(k_{3y}-g_y)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$$

要想将分母提出来，本并不需要对 k_{3x},k_{3y} 甚至 k_{3z} 有任何限制，只需要对 k_x,k_y 限制即可

尽管被积表达式得是个纯粹的关于 $k_{3x}-g_x-k_x$ 的杂 k_{3x}，不能是脱离 $-g_x-k_x$ 的纯 k_{3x}，只要无 k_x,k_y，把它从对 k_x,k_y 的积分中提出来就行

VII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 3D & 1D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\cdot\begin{bmatrix} e^{ig_z z}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right) \\ -e^{ik_{3z}z}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_z\left(k_{3x}-g_x,k_{3y}-g_y\right)\end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

均一结构： $$\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{\Delta k'_z}\cdot\frac{2}{\Delta k'_z/k_{3z}+2}$$

其中，$$\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$$

$$\Delta k'_{zQ}=\Delta k_{zQ}\Big|_{k_x,k_y\to 0,0}=k'_{zQ}-k_{3z}=k_{zQ}\Big|_{k_x,k_y\to 0,0}-k_{3z}=k_1+\sqrt{k_2^2-\left(k_{3x}-g_x\right)^2-\left(k_{3y}-g_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$$

$$\Delta k''_z=k''_z-k_{3z}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}$$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 近似解 new 3D

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}[E_{10}E_{20}]\big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{\Delta k'_{zQ}}\cdot\frac{2}{\Delta k'_{zQ}/k_{3z}+2}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}[E_{10}E_{20}]\big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{k'^2_{zQ}-k^2_{3z}}\cdot\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left[C(k_{3x},k_{3y},g_z)*\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\big|_{k_{3x},k_{3y}}^{x,y}}{k''^2_{zQ}-k^2_{3z}}\cdot e^{ig_z z}-C(k_{3x},k_{3y},g_z)*\frac{\mathcal{F}[E_{10}E_{20}]\big|_{k_{3x},k_{3y}}^{x,y}}{k''^2_{zQ}-k^2_{3z}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

k_{3z} 卷不得

其中，$k'_{zQ}=k_{zQ}\big|_{k_x,k_y\to 0,0}=k_1+\sqrt{k_2^2-(k_{3x}-g_x)^2-(k_{3y}-g_y)^2}+g_z$

$$k''_{zQ}=k'_{zQ}\big|_{k_{3x}-g_x,k_{3y}-g_y\to k_{3x},k_{3y}}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_z$$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的近似解 new 2D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left[C\left(k_{3x},k_{3y},g_z\right)*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{zQ}-k^2_{3z}}\cdot e^{ig_z z}-C\left(k_{3x},k_{3y},g_z\right)*\frac{\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{zQ}-k^2_{3z}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

通光方向均一时：

$\mathrm{T_z}\ \longrightarrow\ \infty$

$$\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}-\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z*\frac{\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}\cdot e^{ik_{3z}z}\right]$$

$C\left(k_{3x},k_{3y},g_z\neq 0\right)\rightarrow 0$

$g_z\rightarrow 0$

$k''_{zQ}\rightarrow k''_z$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\mathcal{F}\left[M_{\mathrm{eff}}(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}-\mathcal{F}\left[M_{\mathrm{eff}}(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}*\frac{\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}\cdot e^{ik_{3z}z}\right]$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\mathcal{F}\left[M_{\mathrm{eff}}(\boldsymbol{r})\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}-\mathcal{F}\left[M_{\mathrm{eff}}(\boldsymbol{r})\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k''^2_{z}-k^2_{3z}}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right\}$$

其中，$k''_{zQ}=k''_z+g_z=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_z$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 近似解 new 1D

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}[E_1(\mathbf{r})E_2(\mathbf{r})]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y} - \mathcal{F}\left[M_{\text{eff}}(x,y)\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right\}$$

结构均一时：

$M_{\text{eff}}(x,y)=1$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\frac{\mathcal{F}[E_1(\mathbf{r})E_2(\mathbf{r})]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_{3z}^2}-\frac{\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_{3z}^2}\cdot e^{ik_{3z}z}\right\}$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\mathcal{F}[E_1(\mathbf{r})E_2(\mathbf{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\frac{\mathcal{F}[E_1(\mathbf{r})E_2(\mathbf{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{\Delta k_z''}\cdot\frac{2}{\Delta k_z'/k_{3z}+2}$$

其中，

$$k_z''=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}$$

$$\Delta k_z''=k_z''-k_{3z}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}$$

0.3

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 New 3D

k3z − gz 卷不得

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}'^2-k_{3z}^2}\cdot \mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\left[\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}\cdot \mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left[C(k_{3x},k_{3y},g_z)*\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}\right]\cdot e^{ig_z z}\cdot \mathrm{d}g_z$$

k3z 卷不得

其中，

$$k_{zQ}'=k_{zQ}\Big|_{k_x,k_y\to 0,0}=k_1+\sqrt{k_2^2-(k_{3x}-g_x)^2-(k_{3y}-g_y)^2}+g_z$$

$$k_{zQ}''=k_{zQ}'\Big|_{k_{3x}-g_x,k_{3y}-g_y\to k_{3x},k_{3y}}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_z$$

0.3

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 New 2D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left[C\left(k_{3x},k_{3y},g_z\right)*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}\right]\cdot e^{ig_z z}\cdot \mathrm{d}g_z$$

通光方向均一时：
$\mathrm{T_z}\ \rightarrow\ \infty$

$$\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}$$

$C\left(k_{3x},k_{3y},g_z\neq 0\right)\rightarrow 0$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}*\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}$$

$g_z\rightarrow 0$

$k_{zQ}''\rightarrow k_z''$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}$$

其中，$k_{zQ}''=k_z''+g_z=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_z$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的近似解 New 1D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}$$

结构均一时：

$M_{\text{eff}}\left(x,y\right)=1$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_z''^2-k_{3z}^2}$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{1}{k_{3z}}\cdot\frac{\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{\Delta k_z''}\cdot\frac{2}{\Delta k_z'/k_{3z}+2}$$

其中，

$$k_z''=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}$$

$$\Delta k_z''=k_z''-k_{3z}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}$$

0.1

VII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的近似解 NEW

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{\substack{x,y\\k_x,k_y}} \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}} \frac{e^{ik_{zQ}z}-e^{ik_{3z}z}}{k_{zQ}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&\approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_3^2}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{\substack{x,y\\k_x,k_y}} \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}} \left(e^{ik_{zQ}z}-e^{ik_{3z}z}\right)\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_3^2}\cdot\begin{bmatrix} e^{ig_z z}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{\substack{x,y\\k_x,k_y}} \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}} e^{i\left(k_{zQ}-g_z\right)z}\mathrm{d}k_x\mathrm{d}k_y \\ -e^{ik_{3z}z}\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{\substack{x,y\\k_x,k_y}} \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}} \mathrm{d}k_x\mathrm{d}k_y \end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_3^2}\cdot\begin{bmatrix} e^{ig_z z}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right) \\ -e^{ik_{3z}z}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_z\left(k_{3x}-g_x,k_{3y}-g_y\right) \end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z
\end{aligned}
$$

其中，
$$
k_{zQ}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}+g_z
$$

$$
k_{zQ}'=k_{zQ}\Big|_{\substack{k_x,k_y\to 0,0\\k_{3x},k_{3y}\to 0,0}}=k_1+\sqrt{k_2^2-g_x^2-g_y^2}+g_z
$$

要想将分母提出来，本并不需要对 k_{3x},k_{3y} 甚至 k_{3z} 有任何限制，只需要对 k_x,k_y 限制即可

尽管被积表达式得是个纯粹的关于 $k_{3x}-g_x-k_x$ 的杂 k_{3x}，不能是脱离 $-g_x-k_x$ 的纯 k_{3x}，只要无 k_x,k_y，把它从对 k_x,k_y 的积分中提出来就行

0.1

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 NEW 3D

$k_{3z}-g_z$ 卷不得

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}'^2-k_3^2}\cdot \mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{C(k_{3x},k_{3y},g_z)}{k_{zQ}''^2-k_3^2}*\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-\frac{C(k_{3x},k_{3y},g_z)}{k_{zQ}''^2-k_3^2}*\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\int\frac{C(k_{3x},k_{3y},g_z)\cdot e^{ig_z z}}{k_{zQ}''^2-k_3^2}\mathrm{d}g_z*\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\int\frac{C(k_{3x},k_{3y},g_z)}{k_{zQ}''^2-k_3^2}\mathrm{d}g_z*\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]$$

二维结构时：

$$\Leftrightarrow\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\frac{C(k_{3x},k_{3y})}{k_z''^2-k_3^2}*\mathcal{F}[E_1(\boldsymbol{r})E_2(\boldsymbol{r})]\Big|_{k_{3x},k_{3y}}^{x,y}-\frac{C(k_{3x},k_{3y})}{k_z''^2-k_3^2}*\mathcal{F}[E_{10}E_{20}]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]$$

$\mathrm{T}_z = \infty$

$C(k_{3x},k_{3y},g_z)=C(k_{3x},k_{3y})$

$g_z=0$

$k_{zQ}''=k_z''$

其中，

$$k_{zQ}'=k_{zQ}\Big|_{\substack{k_x,k_y\to 0,0\\ k_{3x},k_{3y}\to 0,0}}=k_1+\sqrt{k_2^2-g_x^2-g_y^2}+g_z$$

$$k_{zQ}''=k_{zQ}'\Big|_{g_x,g_y\to k_{3x},k_{3y}}=k_z''+g_z=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_z$$

0.1

VIII. 泵浦 未耗尽 时， 和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 NEW 2D

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &\approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\frac{C\left(k_{3x},k_{3y}\right)}{k_z''^2-k_3^2} * \mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}} - \frac{C\left(k_{3x},k_{3y}\right)}{k_z''^2-k_3^2} * \mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}} \cdot e^{ik_{3z}z}\right] \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2} * \mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}} - \frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2} * \mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}} \cdot e^{ik_{3z}z}\right] \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Bigg|^{x,y}_{k_{3x},k_{3y}} - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_{10}E_{20}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}} \cdot e^{ik_{3z}z}\right\}
\end{aligned}
$$

其中， $k_z'' = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$

0.1

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 NEW 1D

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}-\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_{10}E_{20}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right\}$$

$$=\frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\delta(k_{3x},k_{3y})}{k_{SUM}^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}-\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\delta(k_{3x},k_{3y})}{k_{SUM}^2-k_3^2}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_{10}E_{20}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right\}$$

$$=\frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}}{k_{SUM}^2-k_3^2}$$

其中，$k_z''=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}$

$$k_{SUM}=k_z''\Big|_{k_{3x},k_{3y}\to 0,0}=k_1+k_2$$

0.1

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 NEW 2D$^+$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{C\left(k_{3x},k_{3y},g_z\right)}{k''^2_{zQ}-k_3^2}*\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}-\frac{C\left(k_{3x},k_{3y},g_z\right)}{k''^2_{zQ}-k_3^2}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

结构 x,y 分布与 z 无关 时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\left[\frac{C\left(k_{3x},k_{3y}\right)}{k''^2_{zQ}-k_3^2}*\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\frac{C\left(k_{3x},k_{3y}\right)}{k''^2_{zQ}-k_3^2}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right]$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{k''^2_{zQ}-k_3^2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{k''^2_{zQ}-k_3^2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{10}E_{20}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right\}$$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

其中， $k''_{zQ}=k'_{zQ}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=k''_z+g_{l_z}=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}+g_{l_z}$

$$
\begin{aligned}
k_{zQ}^2 - k_{3z}^2 &= \left(\sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2} + g_z\right)^2 - \left(k_3^2 - k_{3x}^2 - k_{3y}^2\right) \\
&\approx \left(k_1 - \frac{k_x^2 + k_y^2}{2k_1} + \sqrt{k_2^2 - g_x^2 - g_y^2 + 2\left(k_{3x} - k_x\right)g_x + 2\left(k_{3y} - k_y\right)g_y} + g_z\right)^2 - k_3^2 + k_{3x}^2 + k_{3y}^2 \\
&\approx \left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z + \frac{2\left(k_{3x} - k_x\right)g_x + 2\left(k_{3y} - k_y\right)g_y}{2\sqrt{k_2^2 - g_x^2 - g_y^2}} - \frac{k_x^2 + k_y^2}{2k_1}\right)^2 - k_3^2 + k_{3x}^2 + k_{3y}^2 \\
&\approx \left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z\right)^2 + 2\left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z\right)\left(\frac{\left(k_{3x} - k_x\right)g_x + \left(k_{3y} - k_y\right)g_y}{\sqrt{k_2^2 - g_x^2 - g_y^2}} - \frac{k_x^2 + k_y^2}{2k_1}\right) - k_3^2 + k_{3x}^2 + k_{3y}^2 \\
&\approx \left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z\right)^2 - k_3^2 \\
&\quad + \frac{2\left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z\right)g_x}{\sqrt{k_2^2 - g_x^2 - g_y^2}}\left(k_{3x} - k_x\right) + \frac{2\left(k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z\right)g_y}{\sqrt{k_2^2 - g_x^2 - g_y^2}}\left(k_{3y} - k_y\right) \\
&\quad + \left(k_{3x}^2 + k_{3y}^2\right) - \frac{k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} + g_z}{k_1}\left(k_x^2 + k_y^2\right)
\end{aligned}
$$

1.1

VII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 final

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{ik_{zQ}z}-e^{ik_{3z}z}}{k_{zQ}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_{3z}^2}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\left(e^{ik_{zQ}z}-e^{ik_{3z}z}\right)\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_{3z}^2}\cdot\begin{bmatrix}e^{ig_zz}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{ik_{zq}z}\mathrm{d}k_x\mathrm{d}k_y\\-e^{ik_{3z}z}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\mathrm{d}k_x\mathrm{d}k_y\end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{k_{zQ}'^2-k_{3z}^2}\cdot\begin{bmatrix}e^{ig_zz}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\\-e^{ik_{3z}z}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\end{bmatrix}\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

其中， $k_{zQ}=k_{zq}+g_z=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}+g_z$

$$k_{zQ}'=k_{zQ}\Big|_{k_x,k_y\to K_{1x},K_{1y}}=\sqrt{k_1^2-K_{1x}^2-K_{1y}^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-K_{1x}\right)^2-\left(k_{3y}-g_y-K_{1y}\right)^2}+g_z$$

要想将 分母 提出来，只需要对 k_x,k_y 限制。而从交叠积分的角度， 几乎只有特定 $\{k_{1x},k_{1y}\}$ 的地方， g_1 的值才非零，或比较大。
因此 k_x,k_y 只需要在 $g(\{k_{1x},k_{1y}\})$ 较大的 $\{k_{1x},k_{1y}\}$ 处，保证取值准确即可，在其他地方取什么值都影响不大，毕竟在那些地方 $g_1\approx 0$。
而且 k_x,k_y 也不必遍历 $\{k_{1x},k_{1y}\}$ 这个集合，而只需保证 $k_{1z}(k_x,k_y)\approx k_{1z}(\{k_{1x},k_{1y}\})$ 即可，那么只需保证所选的 $k_{1z}(K_x,K_y)$ 可代表 k_{1z} 的加权均值即可。

1.1

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 近似解 final 3D

$k_{3z}-g_z$ 卷不得

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\ k_{3x}-g_x,k_{3y}-g_y}}\cdot e^{ig_zz}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x}-g_x,k_{3y}-g_y}}\cdot e^{ik_{3z}z}}{k_{zQ}'^2-k_{3z}^2}\cdot \mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\left[\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\ k_{3x}-g_x,k_{3y}-g_y}}\cdot e^{ig_zz}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x}-g_x,k_{3y}-g_y}}\cdot e^{ik_{3z}z}\right]\cdot \mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{k_{zQ}''^2-k_{3z}^2}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\frac{1}{k_{zQ}''^2-k_{3z}^2}\cdot\left[C(k_{3x},k_{3y},g_z)*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}\cdot e^{ig_zz}-C(k_{3x},k_{3y},g_z)*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$G_{3z}(k_{3x},k_{3y})=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\left[\frac{\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\ k_{3x}-g_{l_x},k_{3y}-g_{l_y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x}-g_{l_x},k_{3y}-g_{l_y}}}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}\right]$$

其中，$k_{zQ}'=k_{zQ}\Big|_{k_x,k_y\to K_{1x},K_{1y}}=k_{zQ}=\sqrt{k_1^2-K_{1x}^2-K_{1y}^2}+\sqrt{k_2^2-(k_{3x}-g_x-K_{1x})^2-(k_{3y}-g_y-K_{1y})^2}+g_z$

$$k_{zQ}''=k_{zQ}'\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$$

为可卷积，k_{zQ} 必须 或包含 g_x,g_y，或包含 $k_{3x}-g_x,k_{3y}-g_y$，且二者可分离；且如果包含了 k_{3x},k_{3y}，则必须三者可两两分离。另一方面，这里分母也最好不参与卷积，否则又是单独算完每一项（除以了分母再卷积）之后再做差，而不是做了差之后再除以分母。这样就会导致遇到非零分子，除以零分母的错误。
因此，分母直接弄成与 g_x,g_y 无关，并从积分中提出来；依据同样是只有特定 $\{k_{2x},k_{2y}\}$ 处，g_2 值才非零，只需保证 $k_{2z}(K_{2x},K_{2y})$ 可代表 k_{2z} 的加权均值即可。

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 final 2D⁺

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\frac{1}{k_{zQ}''^2-k_{3z}^2}\cdot\left[C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}-C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

结构 x,y 分布与 z 无关时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{1}{k_{zQ}''^2-k_{3z}^2}\cdot\left[C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right]$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}$$

其中，$k_{zQ}''=k_{zQ}'\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

$$K_{1z}=\sum_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}k_{1z}\left(k_{1x},k_{1y}\right)$$

2.0

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 final 3D+

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\frac{1}{k''^2_{zQ}-k^2_{3z}}\cdot\left[C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\frac{1}{k''^2_{zQ}-k^2_{3z}}\cdot\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\int\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z}\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-\mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z}\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right\}\cdot\frac{1}{k''^2_{zQ}-k^2_{3z}}\cdot\mathrm{d}g_z$$

其中，$k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

$$K_{1z}=\sum_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}k_{1z}\left(k_{1x},k_{1y}\right)$$

2.0

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 Final 3D

$k_{3z}-g_z$ 卷不得

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{\mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}'^2-k_{3z}^2}\cdot \mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint\frac{C\left(g_x,g_y,g_z\right)}{k_{zQ}''^2-k_{3z}^2}\cdot\left[\mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ig_z z}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot e^{ik_{3z}z}\right]\cdot \mathrm{d}g_x\mathrm{d}g_y\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{C\left(k_{3x},k_{3y},g_z\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-\frac{C\left(k_{3x},k_{3y},g_z\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\left[\frac{\mathcal{F}\left[E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}\right]$$

其中，$k_{zQ}'=k_{zQ}\Big|_{k_x,k_y\to K_{1x},K_{1y}}=k_{zQ}=\sqrt{k_1^2-K_{1x}^2-K_{1y}^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-K_{1x}\right)^2-\left(k_{3y}-g_y-K_{1y}\right)^2}+g_z$

$$k_{zQ}''=k_{zQ}'\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z \qquad k_{3z}=k_{3z}\Big|_{k_{3x},k_{3y}\to K_{3x},K_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+g_x\right)^2-\left(K_{1y}+K_{2y}+g_y\right)^2}$$

$$k_{3z}''=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$$

为可卷积，k_{zQ} 必须 或包含 g_x,g_y，或包含 $k_{3x}-g_x,k_{3y}-g_y$，且二者可分离；且如果包含了 k_{3x},k_{3y}，则必须三者可两两分离。另一方面，这里分母也最好不参与卷积，否则又是单独算完每一项（除以了分母再卷积）之后再做差，而不是做了差之后再除以分母。这样就会导致遇到非零分子，除以零分母的错误。
因此，分母直接弄成与 g_x,g_y 无关，并从积分中提出来；依据同样是只有特定 $\{k_{2x},k_{2y}\}$ 处，g_2 值才非零，只需保证 $k_{2z}(K_{2x},K_{2y})$ 可代表 k_{2z} 的加权均值即可。

2.0

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 近似解 Final 2D^{+}

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{C\left(k_{3x},k_{3y},g_z\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}-\frac{C\left(k_{3x},k_{3y},g_z\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

结构 x,y 分布
与 z 无关 时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\left[\frac{C\left(k_{3x},k_{3y}\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z}-\frac{C\left(k_{3x},k_{3y}\right)}{k_{zQ}''^2-k_{3z}''^2}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right]$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_{zQ}''^2-k_{3z}''^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_{zQ}''^2-k_{3z}''^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\cdot E_{10}E_{20}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}\right\}$$

其中，$k_{zQ}''=k_{zQ}'\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

$$k_{3z}''=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$$

$$K_{1z}=\sum\nolimits_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum\nolimits_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}k_{1z}\left(k_{1x},k_{1y}\right)\qquad\left(K_{1x},K_{1y}\right)=\sum\nolimits_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum\nolimits_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}\left(k_{1x},k_{1y}\right)$$

2.0

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的近似解 Final 3D$^+$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{C\left(k_{3x},k_{3y},g_z\right)}{k''^2_{zQ}-k''^2_{3z}}*\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ig_z z}-\frac{C\left(k_{3x},k_{3y},g_z\right)}{k''^2_{zQ}-k''^2_{3z}}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y,z}_{k_{3x},k_{3y},g_z}}{k''^2_{zQ}-k''^2_{3z}}*\mathcal{F}\left[E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ig_z z}-\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y,z}_{k_{3x},k_{3y},g_z}}{k''^2_{zQ}-k''^2_{3z}}*\mathcal{F}\left[E_{10}E_{20}\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right]\mathrm{d}g_z$$

$$\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y}_{k_{3x},k_{3y}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\int\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y,z}_{k_{3x},k_{3y},g_z}e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y,z}_{k_{3x},k_{3y},g_z}}{k''^2_{zQ}-k''^2_{3z}}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ig_z z}-\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|^{x,y,z}_{k_{3x},k_{3y},g_z}}{k''^2_{zQ}-k''^2_{3z}}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_{10}E_{20}\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ik_{3z}z}\right\}\mathrm{d}g_z$$

其中，$k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

$$k''_{3z}=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$$

$$K_{1z}=\sum\nolimits_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum\nolimits_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}k_{1z}\left(k_{1x},k_{1y}\right)\qquad\left(K_{1x},K_{1y}\right)=\sum\nolimits_{k_{1x},k_{1y}}\frac{g_1^2\left(k_{1x},k_{1y}\right)}{\sum\nolimits_{k_{1x},k_{1y}}g_1^2\left(k_{1x},k_{1y}\right)}\left(k_{1x},k_{1y}\right)$$

VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的匹配解

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\,\mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}\frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\,\mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y} iz\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&\approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\,\mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y} e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot i\frac{z}{2}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|^{x,y}_{k_x,k_y}\,\mathcal{F}\left[E_{20}(x,y)\right]\Big|^{x,y}_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y} e^{ik_{zq}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot i\frac{z}{2}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z
\end{aligned}
$$

其中，$\Delta k_{zQ} = k_{zQ} - k_{3z} = \sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 匹配解 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot i\frac{z}{2}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot i\frac{z}{2}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot e^{i\frac{g_z}{2}z}\cdot\mathrm{d}\frac{g_z}{2}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_zz}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_zz}\right\}\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot e^{ig_{l_z}z}\right\}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的匹配解 2D^{+}

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z$$

结构 x,y 分布与 z 无关时：

$$\Leftrightarrow\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\left\{C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\right\}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}$$

$$\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

3.1

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 3D⁺

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &\approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\int\left\{\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z
\end{aligned}
$$

$$
\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}} = \int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z = \int\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}} e^{ig_z z}\mathrm{d}g_z
$$

$$
G_{3z}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}\left[M_{\mathrm{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{z\\g_z}} E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz
$$

VII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的匹配解 final

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\mathrm{sinhc}\left(i\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{1}{\Delta k''_{zQ}+2k_{3z}}\cdot\mathrm{sinhc}\left(i\Delta k''_{zQ}\frac{z}{2}\right)\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{\mathrm{sinhc}\left(i\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\iint\mathcal{F}\left[E_{10}\left(x,y\right)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\left(x,y\right)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{ik_{zq}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

其中，$\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

$$\Delta k''_{zQ}=k''_{zQ}-k_{3z}$$

$$k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z\qquad k_{3z}=k_{3z}\Big|_{k_{3x},k_{3y}\to K_{3x},K_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+g_x\right)^2-\left(K_{1y}+K_{2y}+g_y\right)^2}$$

3.2

VIII. 泵浦 未耗尽 时， 和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 final 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\frac{2\cdot\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot e^{i\frac{g_z}{2}z}\cdot\mathrm{d}\frac{g_z}{2}$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot iz\cdot e^{ik_{3z}\frac{z}{2}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z z}\cdot\mathrm{d}g_z$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[\left[C\left(k_{3x},k_{3y},g_z\right)\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\frac{\sin\left[z\cdot\left(k''_{zQ}-k''_{3z}\right)/2\right]}{\left(k''^2_{zQ}-k''^2_{3z}\right)/4}\cdot\frac{1}{z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\boxed{\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}}\cdot e^{ig_{l_z}z}\right\}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\Delta k''_{zQ}=k''_{zQ}-k''_{3z}$ $k''_{3z}=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$

3.2

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 final 2D$^+$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[\left[C\left(k_{3x},k_{3y},g_z\right)\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

结构 x,y 分布
与 z 无关 时：

$$\Leftrightarrow\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\left[\left[C\left(k_{3x},k_{3y}\right)\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\right\}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}\left[M_{\mathrm{eff}}\left(\mathbf{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{\sin\left[z\cdot\left(k''_{zQ}-k''_{3z}\right)/2\right]}{\left(k''^2_{zQ}-k''^2_{3z}\right)/4}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}\cdot i$$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的匹配解 final 3D⁺

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[\left[C\left(k_{3x},k_{3y},g_z\right)\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\right|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[\left[\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\right|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\int\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}\cdot\frac{\sin\left[z\cdot\left(k''_{zQ}-k''_{3z}\right)/2\right]}{\left(k''^2_{zQ}-k''^2_{3z}\right)/4}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot i$$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 反向解 final 2D$^+$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z} e^{ig_{l_z}z}\cdot \mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}^{-1}\left[\frac{G_{3z}(k_{3x},k_{3y})}{\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z} e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}} \approx \mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)$$

$$E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right) \approx \mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_3(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z} e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}} \Bigg/ \mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k''_{3z}\right)/2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}$$

$$k''_{zQ} = k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}} = K_{1z}+K_{2z}+g_{l_z} \qquad k''_{3z} = k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}} = \sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$$

结构 x,y 分布 与 z 无关 时：

$$\mathcal{F}\left[M_{\text{eff}}(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \int C(k_{3x},k_{3y},g_z)e^{ig_z z}\mathrm{d}g_z = \sum_{l_z=-\infty}^{+\infty} C_{l_z}e^{ig_{l_z}z}\cdot C(k_{3x},k_{3y})$$

VII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 匹配解 Final

$$
\begin{aligned}
G_{3z}(k_{3x},k_{3y}) &= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\frac{e^{ik_{3z}z}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \operatorname{sinc}\left(\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&\approx \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{ik_{zq}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C(g_x,g_y,g_z)\cdot\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)*G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z
\end{aligned}
$$

其中，

$$\Delta k_{zQ} = k_{zQ} - k_{3z} = \sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-(k_{3x}-g_x-k_x)^2-(k_{3y}-g_y-k_y)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$$

$$\Delta k''_{zQ} = k''_{zQ} - k_{3z}$$

$$k''_{zQ} = k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}} = K_{1z}+K_{2z}+g_z \qquad k_{3z} = k_{3z}\Big|_{k_{3x},k_{3y}\to K_{3x},K_{3y}} = \sqrt{k_3^2-(K_{1x}+K_{2x}+g_x)^2-(K_{1y}+K_{2y}+g_y)^2}$$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 Final 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot G_{1_{\frac{z}{2}}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2_{\frac{z}{2}}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{dg}_x\mathrm{dg}_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{dg}_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x}-g_x,k_{3y}-g_y}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\mathrm{dg}_x\mathrm{dg}_y\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{dg}_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x}-g_x,k_{3y}-g_y}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\mathrm{dg}_x\mathrm{dg}_y\cdot e^{i\frac{g_z}{2}z}\cdot\mathrm{d}\frac{g_z}{2}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot iz\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x}-g_x,k_{3y}-g_y}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\mathrm{dg}_x\mathrm{dg}_y\cdot e^{ig_z z}\cdot\mathrm{dg}_z$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[C\left(k_{3x},k_{3y},g_z\right)\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot e^{ig_z z}\right\}\mathrm{dg}_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot e^{ig_{l_z}z}\right\}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\quad\Delta k''_{zQ}=k''_{zQ}-k''_{3z}$ $\quad k''_{3z}=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的匹配解 Final 2D$^+$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\left[C\left(k_{3x},k_{3y},g_z\right)\cdot\text{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}\right\}\text{d}g_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

结构 x,y 分布与 z 无关时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\left[C\left(k_{3x},k_{3y}\right)\cdot\text{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\right\}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\text{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot\text{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k''_{3z}$

$$k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}\qquad k''_{3z}=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$$

3.3

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的匹配解 Final 3D$^+$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\rm eff}\omega_3^2}{c^2}\cdot\int\left\{\left[C(k_{3x},k_{3y},g_z)\cdot \mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right] * \mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

$$= \frac{\chi_{\rm eff}\omega_3^2}{c^2}\cdot\int\left\{\left[\mathcal{F}\left[M_{\rm eff}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}\cdot \mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right] * \mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

$$\mathcal{F}\left[M_{\rm eff}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \int C(k_{3x},k_{3y},g_z)e^{ig_z z}\mathrm{d}g_z = \int \mathcal{F}\left[M_{\rm eff}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\rm eff}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\rm eff}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z}\cdot \mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\Bigg|_{x,y}^{k_{3x},k_{3y}} E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

其中， $\Delta k''_{zQ} = k''_{zQ} - k''_{3z}$

$$k''_{zQ} = k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}} = K_{1z} + K_{2z} + g_z \qquad k''_{3z} = k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}} = \sqrt{k_3^2 - (K_{1x}+K_{2x}+k_{3x})^2 - (K_{1y}+K_{2y}+k_{3y})^2}$$

3.3

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 反向解 Final 2D^{+}

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot C_{l_z}e^{ig_{l_z}z}\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|^{x,y}_{k_{3x},k_{3y}}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz$$

$$\mathcal{F}^{-1}\left[\frac{G_{3z}\left(k_{3x},k_{3y}\right)}{\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot C_{l_z}e^{ig_{l_z}z}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\approx\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)$$

$$E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\approx\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_3(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot C_{l_z}e^{ig_{l_z}z}\cdot\frac{e^{ik_{3z}\frac{z}{2}}}{k_{3z}}\cdot iz}\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}\Bigg/\mathcal{F}^{-1}\left[\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}\cdot\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\Bigg|^{k_{3x},k_{3y}}_{x,y}$$

其中， $k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$ $\quad k''_{3z}=k_{3z}\Big|_{g_x,g_y\to k_{3x},k_{3y}}=\sqrt{k_3^2-\left(K_{1x}+K_{2x}+k_{3x}\right)^2-\left(K_{1y}+K_{2y}+k_{3y}\right)^2}$

结构 x,y 分布 与 z 无关 时： $\mathcal{F}\left[M_{\mathrm{eff}}(\boldsymbol{r})\right]\Big|^{x,y}_{k_{3x},k_{3y}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$

3.4

VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的匹配解 King

$$
\begin{aligned}
G_{3z}(k_{3x},k_{3y}) &= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}} \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \operatorname{sinc}\left(\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}} \\
&\approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\Delta k''_{zQ}+2k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{ik_{zq}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)*G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz
\end{aligned}
$$

其中，$\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-(k_{3x}-g_x-k_x)^2-(k_{3y}-g_y-k_y)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

$$\Delta k''_{zQ}=k''_{zQ}-k_{3z}$$

$$k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 匹配解 King 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{i\frac{g_z}{2}z}\cdot\mathrm{d}\frac{g_z}{2}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\quad k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

3.4

VIII. 泵浦 未耗尽 时， 和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 匹配解 King 2D$^+$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C(k_{3x},k_{3y},g_z)*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{(k''_{zQ}+k_{3z})/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

结构 x,y 分布
与 z 无关 时：

$$\Leftrightarrow \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{C(k_{3x},k_{3y})*\mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\right\}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{(k''_{zQ}+k_{3z})/2}\cdot e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\int C(k_{3x},k_{3y},g_z)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C(k_{3x},k_{3y})$$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{(k''_{zQ}+k_{3z})/2}\cdot e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\qquad k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 匹配解 King 3D$^+$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C(k_{3x},k_{3y},g_z) * \mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} * \mathcal{F}\left[E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \int C(k_{3x},k_{3y},g_z)e^{ig_z z}\mathrm{d}g_z = \int \mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z} E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_z z}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $\Delta k''_{zQ} = k''_{zQ} - k_{3z}$ $\quad k''_{zQ} = k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}} = K_{1z}+K_{2z}+g_z$

3.4

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 反向解 King 2D+

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$\mathcal{F}^{-1}\left[\frac{G_{3z}\left(k_{3x},k_{3y}\right)}{\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\approx M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)$$

$$E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\approx\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_3(\mathbf{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_{l_z}z}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\Bigg/M_{\text{eff}}(x,y)$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\quad k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

结构 x,y 分布与 z 无关时： $\mathcal{F}\left[M_{\text{eff}}(\mathbf{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$

3.5

对比 3.4 与 1.1 的 2D⁺，可得 3.4 sinc 内 与 1.1 分母 的 精确表达式

$$
\begin{cases}
G_{3z}\left(k_{3x},k_{3y}\right)\approx\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\dfrac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{i\frac{g_{l_z}}{2}z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz \\[2ex]
G_{3z}\left(k_{3x},k_{3y}\right)\approx\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\dfrac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k''^2_{zQ}-k^2_{3z}}
\end{cases}
$$

$$
\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{i\frac{g_{l_z}}{2}z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz=\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k''^2_{zQ}-k^2_{3z}}
$$

$$
\mathrm{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)=\mathrm{sinhc}\left(\frac{i\Delta k''_{zQ}z}{2}\right)=\frac{\dfrac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}}{i\Delta k''_{zQ}z}
$$

进一步对比两种形式的 sinc，可得 $i\Delta k''_{zQ}z$ 的精确表达式

$$
\begin{cases}
\operatorname{sinc}\left(\dfrac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\dfrac{i\Delta k''_{zQ}z}{2}\right)=\dfrac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z} \\[3ex]
\operatorname{sinc}\left(\dfrac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\dfrac{i\Delta k''_{zQ}z}{2}\right)=\dfrac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}-1\Bigg/\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}}{i\Delta k''_{zQ}z}
\end{cases}
$$

$$
i\Delta k''_{zQ}z=\frac{i\Delta k''_{zQ}z}{2}-\left(-\frac{i\Delta k''_{zQ}z}{2}\right)=\ln\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\ln\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)
$$

$$
=\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)
$$

其实无论是 arg 还是 ln，都无法取出指数上的相位值 φ，因为 e^(iφ) 是个多对一的函数。

3.5

进一步 对比 两种形式 的 sinc，可得 $\Delta k''_{zQ}z$ 的 精确表达式

$$\Delta k''_{zQ}z=\frac{1}{i}\left[\frac{i\Delta k''_{zQ}z}{2}-\left(-\frac{i\Delta k''_{zQ}z}{2}\right)\right]=\frac{1}{i}\left[\ln\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\ln\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)\right]$$

$$=\frac{1}{i}\left\{\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)\right\}$$

✕

$$\Delta k''_{zQ}z=\arg\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\arg\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)$$

$$=\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)$$

arg 有取值限制，导致 $\Delta k''_{zQ}z$ 有取值限制，所以似乎不如 ln。

3.5

进一步对比两种形式的 sinc，可得 $\Delta k''_{zQ}$ 的精确表达式

$$\Delta k''_{zQ}=\frac{1}{iz}\left[\frac{i\Delta k''_{zQ}z}{2}-\left(-\frac{i\Delta k''_{zQ}z}{2}\right)\right]=\frac{1}{iz}\left[\ln\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\ln\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)\right]$$

$$=\frac{1}{iz}\left\{\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)\right\}$$

✕

$$\Delta k''_{zQ}=\frac{1}{z}\left[\arg\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\arg\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)\right]$$

$$=\frac{1}{z}\left[\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)\right]$$

arg 有取值限制，导致 $\Delta k''_{zQ}z$ 有取值限制，所以似乎不如 ln。

进一步对比两种形式的sinc，可得 k''_{zQ} 的精确表达式

$$k''_{zQ}=\Delta k''_{zQ}+k_{3z}=k_{3z}+\frac{1}{iz}\left[\frac{i\Delta k''_{zQ}z}{2}-\left(-\frac{i\Delta k''_{zQ}z}{2}\right)\right]=k_{3z}+\frac{1}{iz}\left[\ln\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\ln\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)\right]$$

$$=k_{3z}+\frac{1}{iz}\left\{\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\ln\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)\right\}$$

✕

$$k''_{zQ}=\Delta k''_{zQ}+k_{3z}=k_{3z}+\frac{1}{z}\left[\arg\left(e^{\frac{i\Delta k''_{zQ}z}{2}}\right)-\arg\left(e^{-\frac{i\Delta k''_{zQ}z}{2}}\right)\right]$$

$$=k_{3z}+\frac{1}{z}\left[\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}\right)-\arg\left(\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{k_{3z}-g_{l_z}}{2}z}\right)\right]$$

arg 有取值限制，导致 $\Delta k''_{zQ}z$ 有取值限制，所以似乎不如 ln。

3.5.1

对比 两种形式 的 sinc，找出 使得 分子相同的 $e^{i(K_{1z}+K_{2z})z}$ 值

尽可能地 趋近

$$\begin{cases} \operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\frac{i\Delta k''_{zQ}z}{2}\right)=\frac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z} \\ \operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\frac{i\Delta k''_{zQ}z}{2}\right)=\frac{\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z} \end{cases}$$

$$e^{i\frac{\Delta k''_{zQ}}{2}z}-e^{-i\frac{\Delta k''_{zQ}}{2}z}=\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}$$

$$e^{i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-e^{-i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}=\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}$$

$2i\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)$ 左边实部为 0 ，而右边不一定，所以要想严格实虚部相等，不太可能；除非 sin 里是 复数。

3.5.1

对比 两种形式 的 sinc，找出 使得 分子相同的 $e^{i(K_{1z}+K_{2z})z}$ 值

$$x\cdot a-\frac{1}{x}\cdot\frac{1}{a}=B\cdot a-C\cdot\frac{1}{a}$$

$$x^2\cdot a^2-1=\left(B\cdot a^2-C\right)\cdot x$$

$$a^2x^2-\left(B\cdot a^2-C\right)\cdot x-1=0$$

$$x=\frac{B\cdot a^2-C\pm\sqrt{\left(B\cdot a^2-C\right)^2+4a^2}}{2a^2}$$

$$e^{i\frac{K_{1z}+K_{2z}}{2}z}=\frac{b\pm\sqrt{b^2+4e^{i\left(g_{l_z}-k_{3z}\right)z}}}{2e^{i\left(g_{l_z}-k_{3z}\right)z}}$$

$$b=\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\left(g_{l_z}-k_{3z}\right)z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}$$

复系数 的 一元二次求根公式 可能不长这样…

$$2i\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)$$

左边纯相位，而右边不一定。要想相等，除非 sin 里是 复数。

给 3.4 整体，乘上 一个修正因子 呢？

尽可能地 趋近

$$
\begin{cases}
\operatorname{sinc}\left(\dfrac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\dfrac{i\Delta k''_{zQ}z}{2}\right)=\dfrac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z}\\[2ex]
\operatorname{sinc}\left(\dfrac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\dfrac{i\Delta k''_{zQ}z}{2}\right)=\dfrac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}
\end{cases}
$$

乘以

$$
\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}
$$

即

$$
\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}e^{i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}e^{-i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}
$$

不就行了

这其实 又是 另一种 sinc，不知道 会不会 更好

给 3.4 整体，乘上 一个修正因子 呢？

$$
\begin{cases}
G_{3z}\left(k_{3x},k_{3y}\right)\approx\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum\limits_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\dfrac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot e^{i\frac{g_{l_z}}{2}z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz \\
G_{3z}\left(k_{3x},k_{3y}\right)\approx\dfrac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum\limits_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\dfrac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{k''^2_{zQ}-k^2_{3z}}
\end{cases}
$$

$$
G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}\cdot iz
$$

$$
G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathrm{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{2\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)}\cdot z
$$

实际上 又回到 1.1 了…

3.5.3

对比 两种 的 sinc，找出 使得 分子相近的 纯相位全息图 $\left(K_{1z}+K_{2z}\right)\left(k_{3x},k_{3y}\right)$

尽可能地 趋近

$$\left\{\begin{aligned}&\operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\frac{i\Delta k''_{zQ}z}{2}\right)=\frac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z}\\&\operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)=\operatorname{sinhc}\left(\frac{i\Delta k''_{zQ}z}{2}\right)=\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}\end{aligned}\right.$$

$$e^{i\frac{\Delta k''_{zQ}}{2}z}-e^{-i\frac{\Delta k''_{zQ}}{2}z}=\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}$$

$$e^{i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-e^{-i\frac{K_{1z}+K_{2z}}{2}z}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}=\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}$$

$2i\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)$ 左边实部为 0 ，而右边不一定，所以要想严格实虚部相等，不太可能；除非 sin 里是 复数。

3.5.3

对比 两种 的 sinc，找出 使得 分子相近的 纯相位全息图 $(K_{1z}+K_{2z})(k_{3x},k_{3y})$

$$e^{i\frac{K_{1z}+K_{2z}}{2}z}e^{i\delta(k_{3x},k_{3y})z}=\rightarrow\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}$$

$$e^{-i\frac{K_{1z}+K_{2z}}{2}z}e^{-i\delta(k_{3x},k_{3y})z}\rightarrow\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}$$

$$2i\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)=\mathrm{Im}\left\{\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\right\}$$

$$\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)=\mathrm{Re}\left\{\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{2i\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\right\}$$

$$\frac{\Delta k''_{zQ}z}{2}=\arcsin\left(\mathrm{Re}\left\{\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{2i\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\right\}\right)$$

右侧 实部，在 匹配 和 不匹配 时，均 → 1.0x > 1 ；虚部均 → 0
所以 无法 仅靠 修改 sin 内 自变量，来做到 非匹配时的逼近？
但 sin 的 分母，是 随 sin 内 自变量 而变的，而后者 只是 分母 > 1，其分子 是不变的。
所以 在远离 匹配时，似乎仍可以 人为地 更改 sinc 的值，以使 整体与右侧相等：通过 整体修改 sinc 自变量 随 k_{3x}，k_{3y} 的新关系。

$2i\cdot\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)$ 左边实部为 0 ，而右边不一定，所以要想严格实虚部相等，不太可能；除非 sin 里是 复数。

修正 3.4 ：对比 两种 sinc，找出 远离零点时 整体相近的 $\Delta k''_{zQ}(k_{3x},k_{3y})$

$$\frac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z}\overset{|\Delta k''_{zQ}|\gg 0}{=}\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}$$

$$\frac{\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)}{\frac{\Delta k''_{zQ}z}{2}}\overset{|\Delta k''_{zQ}|\gg 0}{=}\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}$$

$$\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)\cdot\frac{\Delta k''_{zQ}}{\Delta k''_{zQ}}\overset{|\Delta k''_{zQ}|\gg 0}{=}\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{2i\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\approx 1.\text{xxx}+0.\text{xxx}\cdot i$$

是个 数组大小 个 超越方程 们，每个都可解；但解那么多个，所用的资源，不如直接用 1.1 版

尽管可将 左侧近似为 e^(-x^2/3) 或 (60-7x^2)/(60+3x^2)，但这种近似零点 才有效，然而方程解的是 远离零点的。

修正 1.1 ： 对比 两种 sinc， 找出 接近零点时 整体相近的 $\Delta k''_{zQ}\left(k_{3x},k_{3y}\right)$

$$\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(\mathbf{r}\right)E_2\left(\mathbf{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}\overset{\Delta k''_{zQ}\to 0}{=}\frac{e^{\frac{i\Delta k''_{zQ}z}{2}}-e^{-\frac{i\Delta k''_{zQ}z}{2}}}{i\Delta k''_{zQ}z}$$

$$\frac{\dfrac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(\mathbf{r}\right)E_2\left(\mathbf{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\dfrac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{i\Delta k''_{zQ}z}\overset{\Delta k''_{zQ}\to 0}{=}\frac{\sin\left(\frac{\Delta k''_{zQ}z}{2}\right)}{\frac{\Delta k''_{zQ}z}{2}}$$

$$\Delta k''_{zQ}\overset{\Delta k''_{zQ}\to 0}{=}\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(\mathbf{r}\right)E_2\left(\mathbf{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{i\frac{g_{l_z}-k_{3z}}{2}z}-\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{-i\frac{g_{l_z}-k_{3z}}{2}z}}{\operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)\cdot iz\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}\approx\left(1.\text{xxx}+0.\text{xxx}\cdot i\right)\cdot\frac{1}{\operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)\cdot\frac{z}{2}}\approx\frac{1}{\operatorname{sinc}\left(\frac{\Delta k''_{zQ}z}{2}\right)\cdot\frac{z}{2}}$$

但似乎那个 接近 1 的系数， 仍不可忽略。Sinc 还只能和它在一起。

1.3

修正 1.1：对比两种 sinc，找出接近零点时整体相近的 $\Delta k''_{zQ}(k_{3x},k_{3y})$

$$
\begin{cases}
G_{3z}(k_{3x},k_{3y}) \approx \dfrac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\dfrac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{k''_{zQ}+k_{3z}}\cdot\boxed{e^{i\frac{g_{l_z}}{2}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}}\cdot iz \\[2em]
G_{3z}(k_{3x},k_{3y}) \approx \dfrac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\displaystyle\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\dfrac{\boxed{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}}{k''^2_{zQ}-k^2_{3z}}
\end{cases}
$$

（ 差个 i 或 2i，二者
就可以相互转换 ）

$$
\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{2i\cdot e^{i\frac{g_{l_z}}{2}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}}\approx(1.\text{xxx}+0.\text{xxx}\cdot i)
$$

$$
G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)\cdot\frac{z}{2}}{k''_{zQ}+k_{3z}}\cdot\left\{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1(\mathbf{r})E_2(\mathbf{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}\right\}
$$

但似乎那个接近 1 的系数，仍不可忽略。Sinc 还只能和它在一起。

4.0

VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的指数解

$$G_{3z}(k_{3x},k_{3y})=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}[E_{20}(x,y)]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}}$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}[E_{20}(x,y)]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \operatorname{sinc}\left(\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}}$$

$$\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}[E_{20}(x,y)]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{-\frac{\left(\Delta k_{zQ}\frac{z}{2}\right)^2}{6}}\cdot e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{\Delta k''_{zQ}+2k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}[E_{20}(x,y)]\Big|_{k_{2x},k_{2y}}^{x,y}\ e^{-\frac{(\Delta k_{zQ}z)^2}{24}}\cdot e^{i(k_{1z}+k_{2z})\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}[E_{20}(x,y)]\Big|_{k_{2x},k_{2y}}^{x,y}\ e^{-\frac{\left[k_{1z}^2+k_{2z}^2+2k_{1z}k_{2z}+2k_{1z}(g_z-k_{3z})+2k_{2z}(g_z-k_{3z})+(g_z-k_{3z})^2\right]z^2}{24}}\cdot e^{i(k_{1z}+k_{2z})\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}[E_{10}(x,y)]\Big|_{k_{1x},k_{1y}}^{x,y}\cdot e^{ik_{1z}\frac{z}{2}}e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\cdot\mathcal{F}[E_{20}(x,y)]\Big|_{k_{2x},k_{2y}}^{x,y}\cdot e^{ik_{2z}\frac{z}{2}}e^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{z}{2}}e^{-\frac{(k_{3z}-g_z)^2z^2}{24}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-(K_{1z}+K_{2z})k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\left\{G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right\}*\left\{G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right\}\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{z}{2}}e^{-\frac{(k_{3z}-g_z)^2z^2}{24}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-(K_{1z}+K_{2z})k_{3z}\right]z^2}{12}}\cdot iz$$

其中，$\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-(k_{3x}-g_x-k_x)^2-(k_{3y}-g_y-k_y)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

$\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\qquad k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

4.0

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的指数解 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\left\{G_{1\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right\}*\left\{G_{2\frac{z}{2}}\left(k_{3x}-g_x,k_{3y}-g_y\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right\}\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{z}{2}}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{z}{2}}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{i\frac{g_z}{2}z}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot\mathrm{d}\frac{g_z}{2}\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\cdot\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_zz}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ig_zz}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_{l_z}\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_{l_z}\right]z^2}{24}}\right]\Bigg|_{x,y}^{k_{1x},k_{1y}}\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{e^{ig_{l_z}z}e^{-\frac{\left(k_{3z}-g_{l_z}\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

其中， $k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

VIII. 泵浦 未耗尽 时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 指数解 2D$^+$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{e^{ig_zz}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

结构 x,y 分布
与 z 无关 时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\right\}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_zz}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_zz}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}z}e^{-\frac{\left(k_{3z}-g_{l_z}\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_{l_z}\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_{l_z}\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

其中， $k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_{l_z}$

e 指数 粉框内 很小，
粉框外 很大 的 问题，
无法解决。

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的指数解 3D^{+}

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{e^{ig_z z}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}*\mathcal{F}\left[\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{e^{ig_z z}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\int\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{x,y,z\\k_{3x},k_{3y},g_z}}e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\int\left\{\mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{\substack{z\\g_z}}\mathcal{F}^{-1}\left[G_{1\frac{z}{2}}\left(k_{1x},k_{1y}\right)e^{-\frac{\left[k_{1z}^2+2k_{1z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\mathcal{F}^{-1}\left[G_{2\frac{z}{2}}\left(k_{2x},k_{2y}\right)^{-\frac{\left[k_{2z}^2+2k_{2z}g_z\right]z^2}{24}}\right]\Bigg|_{\substack{k_{1x},k_{1y}\\x,y}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot\frac{e^{ig_z z}e^{-\frac{\left(k_{3z}-g_z\right)^2z^2}{24}}}{\left(k''_{zQ}+k_{3z}\right)/2}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}e^{-\frac{\left[K_{1z}K_{2z}-\left(K_{1z}+K_{2z}\right)k_{3z}\right]z^2}{12}}\cdot iz$$

其中， $\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\qquad$ $k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

VII. 泵浦未耗尽时，和频频域解 $G_{3z}(k_{3x},k_{3y})$ 的级数解

$$
\begin{aligned}
G_{3z}(k_{3x},k_{3y}) &= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}}\\
&= \frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \operatorname{sinc}\left(\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}}\\
&\approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1-\frac{7}{60}\cdot\left(\Delta k_{zQ}\frac{z}{2}\right)^2}{1+\frac{3}{60}\cdot\left(\Delta k_{zQ}\frac{z}{2}\right)^2}\cdot e^{ik_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{\Delta k''_{zQ}+2k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz\\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\left[1+\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]\cdot e^{i(k_{1z}+k_{2z})\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{1}{k''_{zQ}+k_{3z}}\cdot e^{ig_z\frac{z}{2}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz\\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\left[1+\sqrt{\frac{7}{60}}\cdot\left((k_{1z}+k_{2z}+g_z-k_{3z})\frac{z}{2}\right)\right]\cdot e^{i(k_{1z}+k_{2z})\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz\\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\left[1+\sqrt{\frac{7}{60}}\cdot(g_z-k_{3z})\frac{z}{2}+\sqrt{\frac{7}{60}}\cdot(k_{1z}+k_{2z})\frac{z}{2}\right]\cdot e^{i(k_{1z}+k_{2z})\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz\\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\iiint C(g_x,g_y,g_z)\cdot\left[\begin{aligned}
&G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)*G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{\left[1+\sqrt{\frac{7}{60}}\cdot(g_z-k_{3z})\frac{z}{2}\right]\cdot\left[1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)\right]}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\\
&+\left\{G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\cdot k_{1z}\right\}*G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\cdot\sqrt{\frac{7}{60}}\frac{z}{2}\\
&+G_{1\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)*\left\{G_{2\frac{z}{2}}(k_{3x}-g_x,k_{3y}-g_y)\cdot k_{2z}\right\}\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1-\sqrt{\frac{7}{60}}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)}{1+\frac{1}{20}\cdot\left(\Delta k''_{zQ}\frac{z}{2}\right)^2}\cdot\frac{e^{ig_z\frac{z}{2}}}{k''_{zQ}+k_{3z}}\cdot\sqrt{\frac{7}{60}}\frac{z}{2}
\end{aligned}\right]\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz
\end{aligned}
$$

VII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 cos 解

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\Delta k_{zQ}z}-1}{\Delta k_{zQ}}\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}} \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \mathrm{sinc}\left(\Delta k_{zQ}\frac{z}{2}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\cdot iz\cdot\frac{2}{\Delta k_{zQ}/k_{3z}+2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\mathrm{d}g_z\cdot\frac{e^{ik_{3z}z}}{k_{3z}} \\
&\approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \cos\left(\Delta k_{zQ}\frac{z}{2}\cdot\frac{1}{\sqrt{3}}\right)\cdot e^{i\Delta k_{zQ}\frac{z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{\Delta k''_{zQ}+2k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}z}\cdot iz \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \left(e^{\frac{i\Delta k_{zQ}z}{2\sqrt{3}}}+e^{-\frac{i\Delta k_{zQ}z}{2\sqrt{3}}}\right)\cdot e^{\frac{i\Delta k_{zQ}z}{2}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}z}\cdot iz \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\left(e^{i\left(k_{1z}+k_{2z}+g_z-k_{3z}\right)\frac{\sqrt{3}+1}{2\sqrt{3}}z}+e^{i\left(k_{1z}+k_{2z}+g_z-k_{3z}\right)\frac{\sqrt{3}-1}{2\sqrt{3}}z}\right)\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{1}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}z}\cdot iz \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{aligned}&\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\ e^{i\left(k_{1z}+k_{2z}\right)\frac{\sqrt{3}+1}{2\sqrt{3}}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\ &+\iiint C\left(g_x,g_y,g_z\right)\cdot\iint \mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\ \mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\ e^{i\left(k_{1z}+k_{2z}\right)\frac{\sqrt{3}-1}{2\sqrt{3}}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{aligned}\right\}\cdot iz
\end{aligned}
$$

其中， $\Delta k_{zQ}=k_{zQ}-k_{3z}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}-\sqrt{k_3^2-k_{3x}^2-k_{3y}^2}+g_z$

$\Delta k''_{zQ}=k''_{zQ}-k_{3z}$ $\quad$ $k''_{zQ}=k'_{zQ}\Big|_{k_{2x},k_{2y}\to K_{2x},K_{2y}}=K_{1z}+K_{2z}+g_z$

VIII. 泵浦未耗尽时，和频频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 cos 解 3D

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{aligned}&\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{\sqrt{3}+1}{2\sqrt{3}}z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{\sqrt{3}+1}{2\sqrt{3}}z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\&+\iiint C\left(g_x,g_y,g_z\right)\cdot G_{1\frac{\sqrt{3}-1}{2\sqrt{3}}z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2\frac{\sqrt{3}-1}{2\sqrt{3}}z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{aligned}\right\}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{aligned}&\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\&+\iiint C\left(g_x,g_y,g_z\right)\cdot\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot\frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{aligned}\right\}\cdot iz$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{aligned}&\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\&+\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{aligned}\right\}\cdot iz$$

$\sqrt{3}$ 可被替换为任意 >1 的值，甚至 $+\infty$

$$\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left[\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{1}{2}z\right)E_2\left(x,y;\frac{1}{2}z\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{e^{ig_{l_z}\frac{1}{2}z}}{k''_{zQ}+k_{3z}}\right]\cdot e^{ik_{3z}\frac{1}{2}z}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{aligned}&\left[\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right]\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\&+\left[\sum_{l_x,l_y,l_z=-\infty}^{+\infty}C_{l_x,l_y,l_z}\cdot\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x}-g_{l_x},k_{3y}-g_{l_y}}^{x,y}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right]\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{aligned}\right\}\cdot iz$$

VIII. 泵浦未耗尽时，和频 频域解 $G_{3z}\left(k_{3x},k_{3y}\right)$ 的 cos 解 2D^{+}

4.2

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{array}{l}\displaystyle\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\ \displaystyle+\int\left\{C\left(k_{3x},k_{3y},g_z\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\right\}\mathrm{d}g_z\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{array}\right\}\cdot iz$$

结构 x,y 分布与 z 无关时：

$$\Leftrightarrow\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{array}{l}\displaystyle C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\ \displaystyle+C\left(k_{3x},k_{3y}\right)*\mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{array}\right\}\cdot iz$$

$\sqrt{3}$ 可被替换为任意 >1 的值，甚至 $+\infty$

$$\mathcal{F}\left[M_{\text{eff}}\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\int C\left(k_{3x},k_{3y},g_z\right)e^{ig_z z}\mathrm{d}g_z=\sum_{l_z=-\infty}^{+\infty}C_{l_z}e^{ig_{l_z}z}\cdot C\left(k_{3x},k_{3y}\right)$$

$$\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}\frac{1}{2}z}}{k''_{zQ}+k_{3z}}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{1}{2}z\right)E_2\left(x,y;\frac{1}{2}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{1}{2}z}\cdot iz$$

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{2c^2}\cdot\left\{\begin{array}{l}\displaystyle\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}\\ \displaystyle+\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{e^{ig_{l_z}\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z}\end{array}\right\}\cdot iz$$

VIII. 泵浦未耗尽时，和频 频域解 $G_{3z}(k_{3x},k_{3y})$ 的 cos 解 3D⁺

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{2c^2} \cdot \left\{ \begin{array}{l} \int \left\{ \mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} * \mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \cdot \frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}} \right\} \mathrm{d}g_z \cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z} \\ +\int \left\{ \mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} * \mathcal{F}\left[E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \cdot \frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}} \right\} \mathrm{d}g_z \cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z} \end{array} \right\} \cdot iz$$

$$\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} = \int C(k_{3x},k_{3y},g_z)e^{ig_z z}\mathrm{d}g_z = \int \mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y},g_z}^{x,y,z} e^{ig_z z}\mathrm{d}g_z$$

$$G_{3z}(k_{3x},k_{3y}) \approx \frac{\chi_{\text{eff}}\omega_3^2}{2c^2} \cdot \left\{ \begin{array}{l} \int \left\{ \mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z} E_1\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}+1}{2\sqrt{3}}z\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \cdot \frac{e^{ig_z\frac{\sqrt{3}+1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}} \right\} \mathrm{d}g_z \cdot e^{ik_{3z}\frac{\sqrt{3}-1}{2\sqrt{3}}z} \\ +\int \left\{ \mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z} E_1\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)E_2\left(x,y;\frac{\sqrt{3}-1}{2\sqrt{3}}z\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \cdot \frac{e^{ig_z\frac{\sqrt{3}-1}{2\sqrt{3}}z}}{k''_{zQ}+k_{3z}} \right\} \mathrm{d}g_z \cdot e^{ik_{3z}\frac{\sqrt{3}+1}{2\sqrt{3}}z} \end{array} \right\} \cdot iz$$

√3 可被替换为任意 > 1 的值，甚至 +∞

$$\frac{\chi_{\text{eff}}\omega_3^2}{c^2} \cdot \int \left\{ \mathcal{F}\left[\mathcal{F}\left[M_{\text{eff}}(\boldsymbol{r})\right]\Big|_{g_z}^{z} E_1\left(x,y;\frac{1}{2}z\right)E_2\left(x,y;\frac{1}{2}z\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \cdot \frac{e^{ig_z\frac{1}{2}z}}{k''_{zQ}+k_{3z}} \right\} \mathrm{d}g_z \cdot e^{ik_{3z}\frac{1}{2}z} \cdot iz$$

A. 上述方法，由于已对 z 积分完全，因而完全在 频域 操作

a. 其 好处 在于

a) 由于整体完全满足波动方程，理论精度只取决于程序计算误差。
b) 无细节倒格矢且 z 不小的情况下，计算速度和精度优于任何方案。
c) 弥补了只能模拟近场 SSI 与 只能模拟远场 Green 间的空白，并且适用于任意 z，计算量与 z 无关。
d) 公式本身就是非线性过程的完全描述，每一个基波分量、每一个倒格矢，均对每一个和频分量有贡献。
e) 公式还暗示了：和频的衍射、基波的衍射、基波到和频的转换、衍射的和频与产生的和频的相干叠加，四者完全交织在一起，任意两者间均无法解耦，是非线性过程的真正含义。

A. 上述方法，由于已对 z 积分完全，因而完全在 频域 操作

a. 其 坏处 在于

a) 计算量虽与 z 无关，但既包含了 FDTD 的 FFT，又包含了 Green 的体积分，同时还包含了无法用卷积表示的二维频域积分，导致计算量在小 z 时比 SSI 大；且在有结构时，比 Green 计算量大。
b) 一次计算只能给出某一 z 截面场分布，而 SSI 可以算全场分布。
c) 无法仅计算远场某一场点的复振幅分部，而 Green 可以做到。
d) 对于复杂的 χ_2 分布，既不像 SSI 一样，可以一次算所有衍射级分布，该方法对频域体积分比较耗时；也不像 Green 可单独计算复杂 χ_2 分布在远场的点分布；面分布 也没有 Green 算得快。
e) 与 SSI 一样，对于大衍射角的情况，计算量巨大，因为均用到了二维 FFT；而 Green 可以计算大衍射角下的远场面分布，因为 Green 可单独算任一场点。

B. 正在开发 SSI 版的 非线性角谱 理论，目的是 z 向划分后，可以将每个截面的结构与场分布整体处理；并且由于 z 向微分相对于 z 向积分，可以写成卷积，并因此可以用 FFT 来加速计算过程，以致于对于任何结构，都能得到高精度和高速度的全场分布解，不局限于一个 z 截面。

B. SSI 版 非线性角谱 理论

a. 放宽已知条件，充分利用每一步长后端面可获取的一切信息

分步迭代求解 $$\left(\frac{\partial^2}{\partial z^2}+k_{3z}^2\right)G_{3z}\left(k_{3x},k_{3y}\right)=-\frac{k_3^2}{n_3^2}Q_{3z}\left(k_{3x},k_{3y}\right)$$

设 [z, z + dz) 范围内 $Q_{3z}\left(k_{3x},k_{3y}\right)$ 、$g_{3z}\left(k_{3x},k_{3y}\right)$ 近似不变

该范围内 正向传播解为 $$G_{3z}\left(k_{3x},k_{3y}\right)=g_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}z}-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}\left(k_{3x},k_{3y}\right)$$

亦 $$G_{3,z+\mathrm{d}z}\left(k_{3x},k_{3y}\right)=g_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}(z+\mathrm{d}z)}-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}\left(k_{3x},k_{3y}\right)$$

b. 下面推导递推关系

B. SSI 版 非线性角谱 理论

b. 下面推导递推关系，设 [z, z + dz) 范围内 $\chi_{\text{eff}}(\boldsymbol{r})$ 分布近似不变

a) z = 0 已知

$$\begin{cases} G_{30}\left(k_{3x},k_{3y}\right)=0 \\ Q_{30}\left(k_{3x},k_{3y}\right)=\mathcal{F}\left[\chi_{\text{eff}}\left(x,y,0\right)\cdot E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \end{cases}$$

$$\begin{cases} \chi_{\text{eff}}\left(x,y,0\right) \\ E_{10}\left(x,y\right)=E_{1}\left(x,y,0\right) \\ E_{20}\left(x,y\right)=E_{2}\left(x,y,0\right) \end{cases}$$

代入 正向解

$$G_{30}\left(k_{3x},k_{3y}\right)=g_{30}\left(k_{3x},k_{3y}\right)-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)=0$$

$$g_{30}\left(k_{3x},k_{3y}\right)=\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)$$

b) z = dz 可知

$$G_{3,\text{dz}}\left(k_{3x},k_{3y}\right)=g_{30}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\text{dz}}-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)$$

$$=\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\text{dz}}-1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)$$

B. SSI 版 非线性角谱 理论

c. 单线程 迭代 $Q_{3z}(k_{3x},k_{3y})$：每算一个 $Q_{3z}(k_{3x},k_{3y})$ 就算一个 $G_{3,z+\mathrm{dz}}(k_{3x},k_{3y})$

a) z 已知

$$\begin{cases} g_{3z}(k_{3x},k_{3y}) = \left[G_{3z}(k_{3x},k_{3y}) + \frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\right]\cdot\frac{1}{e^{ik_{3z}z}} \\ Q_{3z}(k_{3x},k_{3y}) = \mathcal{F}\left[\chi_{\mathrm{eff}}(\boldsymbol{r})\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y} \quad \begin{cases} \chi_{\mathrm{eff}}(x,y,z) \\ E_1(x,y,z) = \mathcal{F}^{-1}\left[\mathcal{F}\left[E_{1,z-\mathrm{dz}}(x,y)\right]\Big|_{k_{1x},k_{1y}}^{x,y}\cdot e^{ik_{1z}\mathrm{dz}}\right]\Big|_{x,y}^{k_{1x},k_{1y}} \\ E_2(x,y,z) = \mathcal{F}^{-1}\left[\mathcal{F}\left[E_{2,z-\mathrm{dz}}(x,y)\right]\Big|_{k_{2x},k_{2y}}^{x,y}\cdot e^{ik_{2z}\mathrm{dz}}\right]\Big|_{x,y}^{k_{2x},k_{2y}} \end{cases} \end{cases}$$

b) z + dz 可知

$$\begin{aligned} G_{3,z+\mathrm{dz}}(k_{3x},k_{3y}) &= g_{3z}(k_{3x},k_{3y})\cdot e^{ik_{3z}(z+\mathrm{dz})} - \frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y}) \\ &= \left[G_{3z}(k_{3x},k_{3y}) + \frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\right]\cdot e^{ik_{3z}\mathrm{dz}} - \frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y}) \\ &= G_{3z}(k_{3x},k_{3y})\cdot e^{ik_{3z}\mathrm{dz}} + \frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y}) \end{aligned}$$

B. SSI 版 非线性角谱 理论

d. 多线程 并行计算 $Q_{3z}(k_{3x},k_{3y})$：先算所有 $Q_{3z}(k_{3x},k_{3y})$，再迭代 $G_{3,z+\mathrm{dz}}(k_{3x},k_{3y})$

a) z 已知

$$g_{3z}(k_{3x},k_{3y})=\left[G_{3z}(k_{3x},k_{3y})+\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\right]\cdot\frac{1}{e^{ik_{3z}z}}$$

$$Q_{3z}(k_{3x},k_{3y})=\mathcal{F}\left[\chi_{\mathrm{eff}}(\boldsymbol{r})\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}$$

$$\begin{cases}\chi_{\mathrm{eff}}(x,y,z)\\ E_1(x,y,z)=\mathcal{F}^{-1}\left[\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{\substack{x,y\\k_{1x},k_{1y}}}\cdot e^{ik_{1z}z}\right]\Big|_{\substack{k_{1x},k_{1y}\\x,y}}\\ E_2(x,y,z)=\mathcal{F}^{-1}\left[\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{\substack{x,y\\k_{2x},k_{2y}}}\cdot e^{ik_{2z}z}\right]\Big|_{\substack{k_{2x},k_{2y}\\x,y}}\end{cases}$$

b) z + dz 可知

$$\begin{aligned}G_{3,z+\mathrm{dz}}(k_{3x},k_{3y})&=g_{3z}(k_{3x},k_{3y})\cdot e^{ik_{3z}(z+\mathrm{dz})}-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\\&=\left[G_{3z}(k_{3x},k_{3y})+\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\right]\cdot e^{ik_{3z}\mathrm{dz}}-\frac{k_3^2}{n_3^2}\frac{1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\\&=G_{3z}(k_{3x},k_{3y})\cdot e^{ik_{3z}\mathrm{dz}}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}Q_{3z}(k_{3x},k_{3y})\end{aligned}$$

根据递推公式 $$G_{3,z+\mathrm{d}z}\left(k_{3x},k_{3y}\right)=G_{3z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\mathrm{d}z}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}Q_{3z}\left(k_{3x},k_{3y}\right)$$

可得求和版本
$$\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right)&=G_{3,z-\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\mathrm{d}z}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}Q_{3,z-\mathrm{d}z}\left(k_{3x},k_{3y}\right)\\
&=G_{3,z-2\cdot\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}2\cdot\mathrm{d}z}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}Q_{3,z-2\cdot\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\mathrm{d}z}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}Q_{3,z-\mathrm{d}z}\left(k_{3x},k_{3y}\right)\\
&=\dots\\
&=G_{30}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}z}+\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}\left[Q_{3,z-\mathrm{d}z}\left(k_{3x},k_{3y}\right)+Q_{3,z-2\cdot\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\mathrm{d}z}+\dots\right]\\
&=\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{d}z}-1}{k_{3z}^2}\sum_{j=1}^{z/\mathrm{d}z}Q_{3,z-j\cdot\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot(j-1)\cdot\mathrm{d}z}\\
&=\frac{k_3^2}{n_3^2}\frac{1-e^{-ik_{3z}\mathrm{d}z}}{k_{3z}^2}\sum_{j=1}^{z/\mathrm{d}z}Q_{3,z-j\cdot\mathrm{d}z}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot j\cdot\mathrm{d}z}
\end{aligned}$$

VIII. 根据 近似解 NEW 2D，导出 求和版

a. 设 $[z_j, z_j + dz_j)$ 范围内 $\chi_{\mathrm{eff},z_j}(x,y)$ 分布不变 $\begin{cases} z_0 = 0, \quad dz_j \neq 0 \\ z_{J+1} = z_J + dz_J = \Sigma_{j \in [0,J)}\, dz_j \end{cases}$

$$\mathrm{d}G_{3z_{j+1}}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \left\{ \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2 - k_3^2} \right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_{1z_{j+1}}(x,y) E_{2z_{j+1}}(x,y) \right]\Bigg|^{x,y}_{k_{3x},k_{3y}} - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2 - k_3^2} \right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_{1z_j} E_{2z_j} \right]\Bigg|^{x,y}_{k_{3x},k_{3y}} \cdot e^{ik_{3z}\mathrm{d}z_{j+1}} \right\}$$

$$\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2} \cdot \left\{ \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2 - k_3^2} \right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_{1z_j}(x,y) E_{2z_j}(x,y) \right]\Bigg|^{x,y}_{k_{3x},k_{3y}} - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|^{x,y}_{k_{3x},k_{3y}}}{k_z''^2 - k_3^2} \right]\Bigg|^{k_{3x},k_{3y}}_{x,y} \cdot E_{1z_{j-1}} E_{2z_{j-1}} \right]\Bigg|^{x,y}_{k_{3x},k_{3y}} \cdot e^{ik_{3z}\mathrm{d}z_j} \right\}$$

其中， $k_z'' = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$

VIII. 根据 近似解 NEW 2D， 导出 求和版

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j$

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right) + \mathrm{d}G_{3z_J}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot \mathrm{d}z_J} + \mathrm{d}G_{3z_{J-1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\mathrm{d}z_{J-1}+\mathrm{d}z_J\right)} + \ldots + \mathrm{d}G_{3z_1}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{j=1}^{J}\mathrm{d}z_j} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right) + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{i=j}^{J}\mathrm{d}z_i} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)} + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\sum_{i=0}^{J}\mathrm{d}z_i-\sum_{i=0}^{j-1}\mathrm{d}z_i\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)} + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \sum_{j=1}^{J+1}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}
\end{aligned}
$$

搞错了

$$
= \frac{\chi_{\mathrm{eff}}\,\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\left\{
\begin{aligned}
&\mathcal{F}\left[\left.\mathcal{F}^{-1}\left[\frac{\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_3^2}\right]\right|_{x,y}^{k_{3x},k_{3y}}\cdot E_{1z_j}(x,y)\,E_{2z_j}(x,y)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&-\mathcal{F}\left[\left.\mathcal{F}^{-1}\left[\frac{\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_3^2}\right]\right|_{x,y}^{k_{3x},k_{3y}}\cdot E_{1z_{j-1}}E_{2z_{j-1}}\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\cdot z_{J+1}}
\end{aligned}
\right\}
$$

其中， $k_z'' = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$

VIII. 根据 近似解 NEW 2D，导出 SSI 迭代版

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j \in [0,\, J)}\, dz_j$

$$\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \sum_{j=1}^{J+1} \mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\sum_{j=1}^{J} \mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\sum_{j=1}^{J} \mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_J-z_j\right)}\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_J\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+G_{3,z-\mathrm{d}z_J}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot \mathrm{d}z_J}
\end{aligned}$$

$$G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)=G_{3z_J}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot \mathrm{d}z_J}+\mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)$$

VIII. 试一试 近似解 NEW 2D 的 求和版 能导出什么 NEW 3D

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{aligned}&\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_3^2}*\mathcal{F}\left[E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}\\&-\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}}{k_z''^2-k_3^2}*\mathcal{F}\left[E_{1z_{j-1}}E_{2z_{j-1}}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\cdot z_{J+1}}\end{aligned}\right\}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\begin{aligned}&\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)z_j}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\&-\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}e^{ik_{zq}z_{j-1}}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}\cdot e^{ik_{3z}\cdot z_{J+1}}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{aligned}&\iint\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\mathcal{F}\left[E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\,\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}\\&-\iint\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\mathcal{F}\left[E_{1z_{j-1}}E_{2z_{j-1}}\right]\Big|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\,\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot z_{J+1}}\end{aligned}\right\}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{aligned}&\iint\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,e^{ik_{zq}z_j}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}\\&-\iint\frac{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,e^{ik_{zq}z_{j-1}}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot z_{J+1}}\end{aligned}\right\}$$

$$k_z''=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}$$

$$k_{zq}'=k_{zQ}'\Big|_{g_z\to0}=k_1+\sqrt{k_2^2-g_x^2-g_y^2}$$

$$k_{zq}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}$$

0.1

VIII. 试一试 近似解 NEW 2D 的 求和版 能导出什么 NEW 3D

a. 设 $dz_j = dz$, $z_j = \Sigma_{i\in[0,\,j-1)} dz_i = j\cdot dz$, $z = (J+1)\cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\rm eff}\omega_3^2}{c^2}\cdot\left\{\begin{aligned}&\iint\frac{\mathcal{F}\left[M_{\rm eff}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot{\rm dz}}{\rm d}k_x{\rm d}k_y{\rm d}g_x{\rm d}g_y\\&-\iint\frac{\mathcal{F}\left[M_{\rm eff}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{1}{e^{ik_{zq}\cdot{\rm dz}}}\sum_{j=1}^{J+1}(-1)^{j-1}e^{ik_{zq}j\cdot{\rm dz}}{\rm d}k_x{\rm d}k_y{\rm d}g_x{\rm d}g_y\end{aligned}\right\}\cdot e^{ik_{3z}\cdot z}$$

还是 涉及到 分母含 k_x, k_y 的情况，只要有这个，且只要 忽略了，就无法一步 准确计算 含 Tz 的 3D NPC

z 大即 J 很大 的 时候，匹配带宽很窄，相位 和 振幅 的 振荡 非常密集，对相位的精度要求非常高

其中 $\displaystyle\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{i\cdot C}}e^{i\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{-i\cdot C}}$ ， 则 $\displaystyle\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot k\cdot{\rm dz}}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{i\cdot k\cdot{\rm dz}}}e^{i\cdot k\cdot{\rm dz}}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{-i\cdot k\cdot{\rm dz}}}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\rm eff}\omega_3^2}{c^2}\cdot\left\{\begin{aligned}&\iint\frac{\mathcal{F}\left[M_{\rm eff}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{1+(-1)^J e^{i\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot{\rm dz}}}{\rm d}k_x{\rm d}k_y{\rm d}g_x{\rm d}g_y\\&-\iint\frac{\mathcal{F}\left[M_{\rm eff}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{1+(-1)^J e^{i\cdot k_{zq}\cdot z}}{1+e^{i\cdot k_{zq}\cdot{\rm dz}}}{\rm d}k_x{\rm d}k_y{\rm d}g_x{\rm d}g_y\end{aligned}\right\}\cdot e^{ik_{3z}\cdot z}$$

$$k_z''=k_1+\sqrt{k_2^2-k_{3x}^2-k_{3y}^2}$$

$$k_{zq}'=k_{zQ}'\Big|_{g_z\to0}=k_1+\sqrt{k_2^2-g_x^2-g_y^2}$$

$$k_{zq}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}$$

VIII. 试一试 近似解 NEW 2D 的 求和版 能导出什么 NEW 3D

$$k_z'' = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$$

a. 分母可不含 k_x, k_y

$$k_{zq}' = k_{zQ}'\big|_{g_z \to 0} = k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2} \qquad K_{zq}' = k_{zQ}\big|_{g_z \to 0} = k_1 + \sqrt{k_2^2 - \left(k_{3x} - g_x\right)^2 - \left(k_{3y} - g_y\right)^2}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2} \qquad \Delta K_{zq}' = K_{zq}' - k_{3z}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2} \cdot \left\{ \begin{array}{l} \displaystyle \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \frac{1}{1 + e^{-i\cdot\Delta K_{zq}'\cdot dz}} \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y} \left[1 + (-1)^J e^{i\cdot\left(k_{zq} - k_{3z}\right)\cdot z}\right] dk_x dk_y dg_x dg_y \\ \displaystyle - \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \frac{1}{1 + e^{i\cdot K_{zq}'\cdot dz}} \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y} \left[1 + (-1)^J e^{i\cdot k_{zq}\cdot z}\right] dk_x dk_y dg_x dg_y \end{array} \right\} \cdot e^{ik_{3z}\cdot z}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2} \cdot \left\{ \begin{array}{l} \displaystyle \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \left(\frac{1}{1 + e^{-i\cdot\Delta K_{zq}'\cdot dz}} - \frac{1}{1 + e^{i\cdot K_{zq}'\cdot dz}}\right) \cdot e^{ik_{3z}\cdot z} \cdot \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y} dk_x dk_y dg_x dg_y \\ \displaystyle + \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \left(\frac{1}{1 + e^{-i\cdot\Delta K_{zq}'\cdot dz}} - \frac{e^{ik_{3z}\cdot z}}{1 + e^{i\cdot K_{zq}'\cdot dz}}\right) \cdot (-1)^J \cdot \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y} e^{i\cdot k_{zq}\cdot z} dk_x dk_y dg_x dg_y \end{array} \right\}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2} \cdot \left\{ \begin{array}{l} \displaystyle \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \left(\frac{1}{1 + e^{-i\cdot\Delta K_{zq}'\cdot dz}} - \frac{1}{1 + e^{i\cdot K_{zq}'\cdot dz}}\right) \cdot e^{ik_{3z}\cdot z} \cdot g_1\left(k_{3x} - g_x, k_{3y} - g_y\right) * g_2\left(k_{3x} - g_x, k_{3y} - g_y\right) dg_x dg_y \\ \displaystyle + \iint \frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2 - k_3^2} \cdot \left(\frac{1}{1 + e^{-i\cdot\Delta K_{zq}'\cdot dz}} - \frac{e^{ik_{3z}\cdot z}}{1 + e^{i\cdot K_{zq}'\cdot dz}}\right) \cdot (-1)^J \cdot G_{1z}\left(k_{3x} - g_x, k_{3y} - g_y\right) * G_{2z}\left(k_{3x} - g_x, k_{3y} - g_y\right) dg_x dg_y \end{array} \right\}$$

0.1

VIII. 试一试 近似解 NEW 2D 的 求和版 能导出什么 NEW 3D

a. 分母 $\Delta K'_{zq} = K'_{zq} - k_{3z}$ 对于卷积不纯，扔掉 k_{3x}, k_{3y} 成为 $\Delta k'_{zq} = k'_{zq} - k_3$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\begin{array}{l}\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\left(\frac{1}{1+e^{-i\cdot\Delta k'_{zq}\cdot\mathrm{d}z}}-\frac{1}{1+e^{i\cdot K'_{zq}\cdot\mathrm{d}z}}\right)\cdot e^{ik_{3z}\cdot z}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\+\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\left(\frac{1}{1+e^{-i\cdot\Delta k'_{zq}\cdot\mathrm{d}z}}-\frac{e^{ik_{3z}\cdot z}}{1+e^{i\cdot K'_{zq}\cdot\mathrm{d}z}}\right)\cdot(-1)^J\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}$$

$$k''_z = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$$

$$k'_{zq} = k'_{zQ}\Big|_{g_z\to 0} = k_1 + \sqrt{k_2^2 - g_x^2 - g_y^2}$$

b. 或者将 分母 都改造为 k'_{zq} 相关

$$K'_{zq} = k_{zQ}\Big|_{g_z\to 0} = k_1 + \sqrt{k_2^2 - \left(k_{3x} - g_x\right)^2 - \left(k_{3y} - g_y\right)^2}$$

$$\Delta K'_{zq} = K'_{zq} - k_{3z}$$

$$\Delta k'_{zq} = k'_{zq} - k_3$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\left\{\begin{array}{l}\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\left(\frac{1}{1+e^{-i\cdot\Delta k'_{zq}\cdot\mathrm{d}z}}-\frac{1}{1+e^{i\cdot k'_{zq}\cdot\mathrm{d}z}}\right)\cdot e^{ik_{3z}\cdot z}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\+\iint\frac{\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}}{k_{zq}'^2-k_3^2}\cdot\left(\frac{1}{1+e^{-i\cdot\Delta k'_{zq}\cdot\mathrm{d}z}}-\frac{e^{ik_{3z}\cdot z}}{1+e^{i\cdot k'_{zq}\cdot\mathrm{d}z}}\right)\cdot(-1)^J\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}$$

0.1

VIII. 试一试 近似解 NEW 2D 的 求和版 能导出什么 NEW 3D

a. 版本一（少近似）

$$k_z'' = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2}$$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\begin{aligned} &\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\left(k_z''^2-k_3^2\right)\left(1+e^{-i\cdot\Delta k_z''\cdot \mathrm{d}z}\right)}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{10}E_{20}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\cdot z} - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{k_z''^2-k_3^2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot\frac{E_{10}E_{20}}{1+e^{i\cdot k_z''\cdot \mathrm{d}z}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\cdot z}\\ &+\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\left(k_z''^2-k_3^2\right)\left(1+e^{-i\cdot\Delta k_z''\cdot \mathrm{d}z}\right)}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{1z}E_{2z}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot(-1)^J - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{k_z''^2-k_3^2}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot\frac{E_{1z}E_{2z}}{1+e^{i\cdot k_z''\cdot \mathrm{d}z}}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot(-1)^J\cdot e^{ik_{3z}\cdot z}\end{aligned}\right\}$$

b. 版本二（更自洽）

$$\Delta k_z'' = k_z'' - k_3 = k_1 + \sqrt{k_2^2 - k_{3x}^2 - k_{3y}^2} - k_3$$

$$G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\left\{\begin{aligned} &\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\left(k_z''^2-k_3^2\right)}\left(\frac{1}{1+e^{-i\cdot\Delta k_z''\cdot \mathrm{d}z}}-\frac{1}{1+e^{i\cdot k_z''\cdot \mathrm{d}z}}\right)\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{10}E_{20}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot e^{ik_{3z}\cdot z}\\ &+\mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\left(k_z''^2-k_3^2\right)\left(1+e^{-i\cdot\Delta k_z''\cdot \mathrm{d}z}\right)}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{1z}E_{2z}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot(-1)^J - \mathcal{F}\left[\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}}{\left(k_z''^2-k_3^2\right)\left(1+e^{i\cdot k_z''\cdot \mathrm{d}z}\right)}\right]\Bigg|_{\substack{k_{3x},k_{3y}\\x,y}}\cdot E_{1z}E_{2z}\right]\Bigg|_{\substack{x,y\\k_{3x},k_{3y}}}\cdot(-1)^J\cdot e^{ik_{3z}\cdot z}\end{aligned}\right\}$$

I. 根据 失配解 1.1，导出 求和版

a. 设 $[z_j, z_j + dz_j)$ 范围内 $\chi_{\mathrm{eff},z_j}(x,y)$ 分布不变 $\begin{cases} z_0 = 0, \quad dz_j \neq 0 \\ z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j \end{cases}$

$$\mathrm{d}G_{3z_{j+1}}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\cdot E_{1z_{j+1}}(x,y)E_{2z_{j+1}}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y} - \mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\cdot E_{1z_j}E_{2z_j}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\mathrm{d}z_j}\right\}$$

$$\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y} - \mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_{j-1}}E_{2z_{j-1}}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\mathrm{d}z_{j-1}}\right\}$$

原始： $G_{3z}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\frac{\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1(\boldsymbol{r})E_2(\boldsymbol{r})\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z} - \mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}$

其中， $k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$

1.1

I. 根据 失配解 1.1，导出 求和版

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)} dz_j$

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\mathrm{d}G_{3z_J}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot \mathrm{d}z_J}+\mathrm{d}G_{3z_{J-1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\mathrm{d}z_{J-1}+\mathrm{d}z_J\right)}+\ldots+\mathrm{d}G_{3z_1}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{j=1}^{J}\mathrm{d}z_j} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{i=j}^{J}\mathrm{d}z_i} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)}+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\sum_{i=0}^{J}\mathrm{d}z_i-\sum_{i=0}^{j-1}\mathrm{d}z_i\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)}+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \sum_{j=1}^{J+1}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{array}{l}\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ -\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_{j-1}}E_{2z_{j-1}}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\mathrm{d}z_{j-1}}\end{array}\right\}\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}
\end{aligned}
$$

其中， $k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$

I. 试一试 失配解 1.1 的 求和版 能导出什么 3D

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j$

$$k_z'' = \left. k_{zQ}'' \right|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{array}{l} \left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\right|_{k_{3x},k_{3y}}^{x,y} \\ -\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1z_{j-1}}E_{2z_{j-1}}\right]\right|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\mathrm{d}z_{j-1}} \end{array}\right\}\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}$$

$$= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{array}{l} \iint\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\left.\mathcal{F}\left[E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\right|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y} e^{-ik_{3z}\cdot z_j}\mathrm{d}g_x\mathrm{d}g_y \\ -\iint\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\left.\mathcal{F}\left[E_{1z_{j-1}}E_{2z_{j-1}}\right]\right|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y} e^{ik_{3z}\mathrm{d}z_{j-1}}\cdot e^{-ik_{3z}\cdot z_j}\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}\cdot e^{ik_{3z}\cdot z_{J+1}}$$

$$= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{array}{l} \iint\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\iint\left.\mathcal{F}\left[E_{10}\right]\right|_{k_x,k_y}^{x,y}\left.\mathcal{F}\left[E_{20}\right]\right|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} e^{ik_{zq}z_j}e^{-ik_{3z}\cdot z_j}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ -\iint\left.\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\iint\left.\mathcal{F}\left[E_{10}\right]\right|_{k_x,k_y}^{x,y}\left.\mathcal{F}\left[E_{20}\right]\right|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} e^{ik_{zq}z_{j-1}}\cdot e^{-ik_{3z}\cdot z_{j-1}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}\cdot e^{ik_{3z}\cdot z_{J+1}}$$

$$= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{array}{l} \iint\left.\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\iint\left.\mathcal{F}\left[E_{10}\right]\right|_{k_x,k_y}^{x,y}\left.\mathcal{F}\left[E_{20}\right]\right|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)z_j}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ -\iint\left.\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\right|_{g_x,g_y}^{x,y}\cdot\iint\left.\mathcal{F}\left[E_{10}\right]\right|_{k_x,k_y}^{x,y}\left.\mathcal{F}\left[E_{20}\right]\right|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)z_{j-1}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}\cdot e^{ik_{3z}\cdot z_{J+1}}$$

1.1

VIII. 试一试 失配解 1.1 的 求和版 能导出什么 3D

a. 设 $dz_j = dz$, $z_j = \Sigma_{i \in [0,\, j-1)} dz_i = j \cdot dz$, $z = (J+1) \cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l}\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{d}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\-\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{-i\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{d}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}\cdot e^{ik_{3z}\cdot z}$$

可提出来个

$$1-\frac{1}{e^{i\left(k_{zq}-k_{3z}\right)\mathrm{d}z}}$$

其中 $\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{i\cdot C}}e^{i\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{-i\cdot C}}$ ，则 $\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot k\cdot\mathrm{d}z}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{i\cdot k\cdot\mathrm{d}z}}e^{i\cdot k\cdot\mathrm{d}z}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{-i\cdot k\cdot\mathrm{d}z}}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l}\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\-\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}\cdot e^{ik_{3z}\cdot z}$$

$$k_z''=k_{zQ}''\Big|_{g_{l_z}\to 0}=K_{1z}+K_{2z}$$

$$k_{zq}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}$$

VIII. 试一试 失配解 1.1 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{aligned} &\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{d}z}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ -&\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{d}z}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{aligned}\right\}\cdot e^{ik_{3z}\cdot z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{aligned} &\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}} - \frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot e^{ik_{3z}\cdot z}\cdot\iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ +&\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}} - \frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot(-1)^J\cdot\iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{i\cdot k_{zq}\cdot z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}} - \frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot\left\{\begin{aligned} &e^{ik_{3z}\cdot z}\cdot\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right) * g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y \\ +&(-1)^J\cdot\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right) * G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y \end{aligned}\right\}$$

VIII. 试一试 失配解 1.1 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot \mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot \mathrm{d}z}}\right]\cdot\left\{\begin{aligned}&e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\&+\left(-1\right)^J\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot \mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot \mathrm{d}z}}\right]\cdot\left\{\begin{aligned}&e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\&+\left(-1\right)^J\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{aligned}\right\}$$

对比：

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(\boldsymbol{r}\right)E_2\left(\boldsymbol{r}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}$$

VIII. 试一试 失配解 1.1 的 bulk 求和版

a. 设 $dz_j = dz$, $z_j = \Sigma_{i \in [0,\, j-1)} dz_i = j \cdot dz$, $z = (J+1) \cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l}\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{dz}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\-\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{-i\left(k_{zq}-k_{3z}\right)\cdot\mathrm{dz}}\sum_{j=1}^{J+1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{dz}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}\cdot e^{ik_{3z}\cdot z}$$

可提出来个

$$1-\frac{1}{e^{i\left(k_{zq}-k_{3z}\right)\cdot\mathrm{dz}}}$$

其中 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot C}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}$ ，则 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot k\cdot\mathrm{dz}}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}=\frac{e^{i\cdot k\cdot z}-1}{e^{i\cdot k\cdot\mathrm{dz}}-1}e^{i\cdot k\cdot\mathrm{dz}}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l}\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{dz}}-1}\cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{dz}}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\-\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{dz}}-1}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}\cdot e^{ik_{3z}\cdot z}$$

$$k_z''=k_{zQ}''\Big|_{g_{l_z}\to 0}=K_{1z}+K_{2z}$$

$$k_{zq}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}$$

1.1

VIII. 试一试 失配解 1.1 的 bulk 求和版

a. 分母可不含 k_x, k_y

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{aligned}&\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{d}z}-1}\cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\ &-\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{d}z}-1}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}\cdot e^{ik_{3z}\cdot z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \left[e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\right\}\cdot e^{ik_{3z}\cdot z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{aligned}&\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{i\cdot k_{zq}\cdot z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\ &-e^{ik_{3z}\cdot z}\cdot\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2 - k_{3z}^2}\cdot\left\{\begin{aligned}&\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\ &-e^{ik_{3z}\cdot z}\cdot\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

VIII. 试一试 失配解 1.1 的 bulk 求和版

a. 分母可不含 k_x, k_y

$$k''_z = k''_{zQ}\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l}\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\ g_x,g_y}}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\ -e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\ g_x,g_y}}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{array}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}-\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}\cdot e^{ik_{3z}\cdot z}}{k_z''^2-k_{3z}^2}$$

与 J、dz 无关，合理

对比（不能说很像，只能说一模一样）：

$$G_{3z}\left(k_{3x},k_{3y}\right)\approx\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(\mathbf{r}\right)E_2\left(\mathbf{r}\right)\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}\cdot e^{ig_{l_z}z}-\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\ k_{3x},k_{3y}}}\cdot e^{ik_{3z}z}}{k_{zQ}''^2-k_{3z}^2}$$

3.4

I. 根据 匹配解 3.4，导出 求和版

a. 设 $[z_j, z_j + dz_j)$ 范围内 $\chi_{\mathrm{eff},z_j}(x,y)$ 分布不变 $\begin{cases} z_0 = 0, \quad dz_j \neq 0 \\ z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j \end{cases}$

$$dG_{3_{z_{j+1}}}(k_{3x},k_{3y}) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{\mathrm{sinc}\left(\Delta k_z''\frac{dz_j}{2}\right)}{(k_z''+k_{3z})/2}\cdot\mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\cdot E_{1\left(z_j+\frac{dz_j}{2}\right)}(x,y)E_{2\left(z_j+\frac{dz_j}{2}\right)}(x,y)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{dz_j}{2}}\cdot i dz_j$$

$$dG_{3_{z_j}}(k_{3x},k_{3y}) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{\mathrm{sinc}\left(\Delta k_z''\frac{dz_{j-1}}{2}\right)}{(k_z''+k_{3z})/2}\cdot\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1\left(z_{j-1}+\frac{dz_{j-1}}{2}\right)}(x,y)E_{2\left(z_{j-1}+\frac{dz_{j-1}}{2}\right)}(x,y)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{dz_{j-1}}{2}}\cdot i dz_{j-1}$$

原始： $$G_{3_z}(k_{3x},k_{3y}) \approx \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty}C_{l_z}\cdot\frac{\mathrm{sinc}\left(\Delta k_{zQ}''\frac{z}{2}\right)}{(k_{zQ}''+k_{3z})/2}\cdot e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

其中， $k_z'' = k_{zQ}''\big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k_z'' = k_z'' - k_{3z}$

I. 根据 匹配解 3.4， 导出 求和版

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j$

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\mathrm{d}G_{3z_J}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot \mathrm{d}z_J}+\mathrm{d}G_{3z_{J-1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\mathrm{d}z_{J-1}+\mathrm{d}z_J\right)}+\ldots+\mathrm{d}G_{3z_1}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{j=1}^{J}\mathrm{d}z_j} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\sum_{i=j}^{J}\mathrm{d}z_i} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)}+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(\sum_{i=0}^{J}\mathrm{d}z_i-\sum_{i=0}^{j-1}\mathrm{d}z_i\right)} \\
&= \mathrm{d}G_{3z_{J+1}}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_{J+1}\right)}+\sum_{j=1}^{J}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \sum_{j=1}^{J+1}\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)} \\
&= \frac{\chi_{\mathrm{eff}}\,\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{\left(k_z''+k_{3z}\right)/2}\cdot\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}\left(x,y\right)\cdot E_{1\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}\left(x,y\right)E_{2\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}\left(x,y\right)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{\mathrm{d}z_{j-1}}{2}}e^{-ik_{3z}z_j}\cdot i\mathrm{d}z_{j-1}
\end{aligned}
$$

其中， $k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z}+K_{2z}$ $\qquad \Delta k_z'' = k_z'' - k_{3z}$

I. 试一试 匹配解 3.4 的 求和版 能导出什么 3D

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)} dz_j$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{\left(k_z''+k_{3z}\right)/2}\cdot\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\cdot E_{1\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)E_{2\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)\right]\Bigg|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{\mathrm{d}z_{j-1}}{2}}e^{-ik_{3z}z_j}\cdot i\mathrm{d}z_{j-1}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{\left(k_z''+k_{3z}\right)/2}\cdot\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot\iint\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\mathcal{F}\left[E_{1\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)E_{2\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)\right]\Bigg|_{k_{3x}-g_x,k_{3y}-g_y}^{x,y}\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\frac{\mathrm{d}z_{j-1}}{2}}e^{-ik_{3z}z_j}\cdot i\mathrm{d}z_{j-1}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{\left(k_z''+k_{3z}\right)/2}\cdot\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot\iint\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{ik_{zq}\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\frac{\mathrm{d}z_{j-1}}{2}}e^{-ik_{3z}z_j}\cdot i\mathrm{d}z_{j-1}$$

$$=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}(-1)^{j-1}\cdot\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot i\mathrm{d}z_{j-1}\cdot e^{i\left(k_{zq}-k_{3z}\right)\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

其中， $k_z''=k_{zQ}''\big|_{g_{l_z}\to0}=K_{1z}+K_{2z}$ $\quad\Delta k_z''=k_z''-k_{3z}$ $\quad k_{zq}=\sqrt{k_1^2-k_x^2-k_y^2}+\sqrt{k_2^2-\left(k_{3x}-g_x-k_x\right)^2-\left(k_{3y}-g_y-k_y\right)^2}$

I. 试一试 匹配解 3.4 的 求和版 能导出什么 3D

a. 设 $dz_j = dz$, $z_j = \Sigma_{i \in [0,\, j-1)} dz_i = j \cdot dz$, $z = (J+1) \cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \sum_{j=1}^{J+1}(-1)^{j-1}\cdot\text{sinc}\left(\Delta k_z''\frac{\text{d}z}{2}\right)\cdot i\text{d}z\cdot e^{i\left(k_{zq}-k_{3z}\right)\left(j-\frac{1}{2}\right)\cdot\text{d}z}\cdot\text{d}k_x\text{d}k_y\text{d}g_x\text{d}g_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\text{sinc}\left(\Delta k_z''\frac{\text{d}z}{2}\right)\cdot i\text{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{-i\left(k_{zq}-k_{3z}\right)\frac{\text{d}z}{2}}\sum_{j=1}^{J+1}(-1)^{j-1}\cdot e^{i\left(k_{zq}-k_{3z}\right)j\cdot\text{d}z}\cdot\text{d}k_x\text{d}k_y\text{d}g_x\text{d}g_y$$

其中 $\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{i\cdot C}}e^{i\cdot C}=\frac{1+(-1)^J e^{i\cdot C(J+1)}}{1+e^{-i\cdot C}}$ ，则 $\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\cdot j\cdot k\cdot \text{d}z}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{i\cdot k\cdot \text{d}z}}e^{i\cdot k\cdot \text{d}z}=\frac{1+(-1)^J e^{i\cdot k\cdot z}}{1+e^{-i\cdot k\cdot \text{d}z}}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\text{sinc}\left(\Delta k_z''\frac{\text{d}z}{2}\right)\cdot i\text{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ e^{-i\left(k_{zq}-k_{3z}\right)\frac{\text{d}z}{2}}\frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\text{d}z}}\cdot\text{d}k_x\text{d}k_y\text{d}g_x\text{d}g_y$$

$$k_z''=k_{zQ}''\Big|_{g_{l_z}\to 0}=K_{1z}+K_{2z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{\text{sinc}\left(\Delta k_z''\frac{\text{d}z}{2}\right)\cdot i\text{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1+(-1)^J e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\text{d}z}{2}}+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\text{d}z}{2}}}\cdot\text{d}k_x\text{d}k_y\text{d}g_x\text{d}g_y$$

$$\Delta k_z''=k_z''-k_{3z}$$

I. 试一试 匹配解 3.4 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k_z'' = k_z'' - k_{3z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{\operatorname{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\,\frac{1+(-1)^J\,e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{\operatorname{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z}{\left(k_z''+k_{3z}\right)/2}\cdot\left\{\begin{aligned}&\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\frac{1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\,\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\&+\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\frac{1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}}\cdot(-1)^J\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\,e^{i\cdot k_{zq}\cdot z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{\operatorname{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z}{\left(k_z''+k_{3z}\right)/2}\cdot\frac{1}{e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}}\cdot\left\{\begin{aligned}&e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\&+(-1)^J\cdot\iint\mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{\operatorname{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z}{k_z''+k_{3z}}\cdot\frac{1}{\cosh\left(i\Delta k_z''\frac{\mathrm{d}z}{2}\right)}\cdot\left\{\begin{aligned}&e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\&+(-1)^J\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{aligned}\right\}$$

I. 试一试 匹配解 3.4 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k_z'' = k_z'' - k_{3z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{i}{k_z''+k_{3z}}\cdot\frac{\operatorname{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot\mathrm{d}z}{\cos\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)}\cdot\left\{\begin{array}{l} e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ +\left(-1\right)^J\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{array}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\frac{2i\cdot\tan\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)}{k_z''^2-k_{3z}^2}\cdot\left\{\begin{array}{l} e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ +\left(-1\right)^J\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{array}\right\}$$

$$\frac{1}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}}$$

dz = l_c = π/△k 时
会使分母 cos（π/2）→ 0…

反而 3.4 从匹配解，
变得不能算匹配了…

$$\frac{1}{1+e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}} - \frac{1}{1+e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\mathrm{d}z}}$$

同理，1.1 分母也会趋于 1-1 ≈ 0，
所以似乎只有 -.- 还有希望？
但仿真时 它在匹配时 也出现了 除零错误。

对比：

$$G_{3z}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\text{eff}}\,\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\frac{\operatorname{sinc}\left(\Delta k_{zQ}''\frac{z}{2}\right)}{\left(k_{zQ}''+k_{3z}\right)/2}\cdot e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\text{eff}}\left(x,y\right)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

I. 试一试 匹配解 3.4 的 bulk 求和版

a. 设 $dz_j = dz,\ z_j = \Sigma_{i \in [0,\ j-1)}\, dz_i = j \cdot dz,\ z = (J+1) \cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z\cdot e^{i\left(k_{zq}-k_{3z}\right)\left(j-\frac{1}{2}\right)\cdot\mathrm{d}z}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{-i\left(k_{zq}-k_{3z}\right)\frac{\mathrm{d}z}{2}}\sum_{j=1}^{J+1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{d}z}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

其中 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot C}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}$ ，则 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot k\cdot\mathrm{d}z}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}=\frac{e^{i\cdot k\cdot z}-1}{e^{i\cdot k\cdot\mathrm{d}z}-1}e^{i\cdot k\cdot\mathrm{d}z}$

又设 $T_z/2 = dz = z/(J+1)$，则 $J = z/dz - 1 = 2z/T_z - 1$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}e^{-i\left(k_{zq}-k_{3z}\right)\frac{\mathrm{d}z}{2}}\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}-1}\cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$k_z''=k_{zQ}''\Big|_{g_{l_z}\to 0}=K_{1z}+K_{2z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z}{2}\right)\cdot i\mathrm{d}z\cdot e^{ik_{3z}\cdot z}}{\left(k_z''+k_{3z}\right)/2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}-1}\cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$\Delta k_z''=k_z''-k_{3z}$$

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$$k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k_z'' = k_z'' - k_{3z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2} \cdot \frac{\operatorname{sinc}\left(\Delta k_z'' \frac{\mathrm{d}z}{2}\right) \cdot i\mathrm{d}z \cdot e^{ik_{3z}\cdot z}}{\left(k_z'' + k_{3z}\right)/2} \cdot \iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} \frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z} - 1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{d}z} - 1} \cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}} \cdot \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}g_x \mathrm{d}g_y$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2} \cdot \frac{\operatorname{sinc}\left(\Delta k_z'' \frac{\mathrm{d}z}{2}\right) \cdot i\mathrm{d}z}{\left(k_z'' + k_{3z}\right)/2} \cdot \frac{1}{e^{i\cdot\left(k_z''-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}} + e^{-i\cdot\left(k_z''-k_{3z}\right)\cdot\frac{\mathrm{d}z}{2}}} \cdot \left\{\begin{aligned} &\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} e^{i\cdot k_{zq}\cdot z} \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}g_x \mathrm{d}g_y \\ &-e^{ik_{3z}\cdot z} \cdot \iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y} \cdot \iint \mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y} \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y} \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}g_x \mathrm{d}g_y \end{aligned}\right\}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2} \cdot \frac{\operatorname{sinc}\left(\Delta k_z'' \frac{\mathrm{d}z}{2}\right) \cdot i\mathrm{d}z}{k_z'' + k_{3z}} \cdot \frac{1}{\cosh\left(i\Delta k_z'' \frac{\mathrm{d}z}{2}\right)} \cdot \left\{\begin{aligned} &\iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y} \cdot G_{1z}\left(k_{3x}-g_x, k_{3y}-g_y\right) * G_{2z}\left(k_{3x}-g_x, k_{3y}-g_y\right) \mathrm{d}g_x \mathrm{d}g_y \\ &-e^{ik_{3z}\cdot z} \cdot \iint \mathcal{F}\left[M_{\text{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y} \cdot g_1\left(k_{3x}-g_x, k_{3y}-g_y\right) * g_2\left(k_{3x}-g_x, k_{3y}-g_y\right) \mathrm{d}g_x \mathrm{d}g_y \end{aligned}\right\}$$

$$G_{3z}\left(k_{3x}, k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2} \cdot \frac{i}{k_z'' + k_{3z}} \cdot \frac{\operatorname{sinc}\left(\Delta k_z'' \frac{\mathrm{d}z}{2}\right) \cdot \mathrm{d}z}{\cos\left(\Delta k_z'' \frac{\mathrm{d}z}{2}\right)} \cdot \left\{\begin{aligned} &\mathcal{F}\left[M_{\text{eff}}(x,y) \cdot E_{1z} E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ &-e^{ik_{3z}\cdot z} \cdot \mathcal{F}\left[M_{\text{eff}}(x,y) \cdot E_{10} E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \end{aligned}\right\}$$

I. 试一试 匹配解 3.4 的 bulk 求和版

a. 分母可不含 k_x, k_y

$$k''_z = k''_{zQ}\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k''_z = k''_z - k_{3z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{i}{k''_z + k_{3z}}\cdot\frac{\text{sinc}\left(\Delta k''_z \frac{\text{d}z}{2}\right)\cdot\text{d}z}{\cos\left(\Delta k''_z \frac{\text{d}z}{2}\right)}\cdot\left\{\begin{matrix}\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ -e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{matrix}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{2i\cdot\tan\left(\Delta k''_z \frac{\text{d}z}{2}\right)}{k''^2_z - k^2_{3z}}\cdot\left\{\begin{matrix}\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y} \\ -e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\end{matrix}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot 2i\cdot\tan\left(\Delta k''_z \frac{\text{d}z}{2}\right)\cdot\frac{\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{1z}E_{2z}\right]\Big|_{k_{3x},k_{3y}}^{x,y} - \mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_{10}E_{20}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\cdot z}}{k''^2_z - k^2_{3z}}$$

与 dz 有关，不合理

对比：

$$G_{3z}\left(k_{3x},k_{3y}\right) \approx \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{l_z=-\infty}^{+\infty} C_{l_z}\cdot\frac{\text{sinc}\left(\Delta k''_{zQ}\frac{z}{2}\right)}{\left(k''_{zQ}+k_{3z}\right)/2}\cdot e^{ig_{l_z}z}\cdot\mathcal{F}\left[M_{\text{eff}}(x,y)\cdot E_1\left(x,y;\frac{z}{2}\right)E_2\left(x,y;\frac{z}{2}\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{z}{2}}\cdot iz$$

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① 直接基于 非线性卷积（交叠积分），导出 求和版

a. 设 $[z_j, z_j + dz_j)$ 范围内 $\chi_{\mathrm{eff},z_j}(x,y)$ 分布不变 $\begin{cases} z_0 = 0, \quad dz_j \neq 0 \\ z_{J+1} = z_J + dz_J = \Sigma_{j \in [0, J)}\, dz_j \end{cases}$

$$\mathrm{d}G_{3z_{j+1}}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff},z_j}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{1z_j}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{2z_j}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{ik_{zq}\mathrm{d}z_j}-e^{ik_{3z}\mathrm{d}z_j}}{k_{zq}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$\mathrm{d}G_{3z_j}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{1z_{j-1}}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{2z_{j-1}}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{ik_{zq}\mathrm{d}z_{j-1}}-e^{ik_{3z}\mathrm{d}z_{j-1}}}{k_{zq}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\iint C\left(g_x,g_y\right)\cdot\iint\mathcal{F}\left[E_{10}(x,y)\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{20}(x,y)\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{ik_{zq}z}-e^{ik_{3z}z}}{k_{zq}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

其中， $k_z'' = k_{zQ}''\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$ $\quad \Delta k_z'' = k_z'' - k_{3z}$

-.-

① 直接基于 非线性卷积（交叠积分），导出 求和版

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j\in[0,J)}\, dz_j$

$$
\begin{aligned}
G_{3z}(k_{3x},k_{3y}) &= \mathrm{d}G_{3z_{J+1}}(k_{3x},k_{3y}) + \mathrm{d}G_{3z_J}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot \mathrm{d}z_J} + \mathrm{d}G_{3z_{J-1}}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot(\mathrm{d}z_{J-1}+\mathrm{d}z_J)} + \ldots + \mathrm{d}G_{3z_1}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot\sum_{j=1}^{J}\mathrm{d}z_j} \\
&= \mathrm{d}G_{3z_{J+1}}(k_{3x},k_{3y}) + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot\sum_{i=j}^{J}\mathrm{d}z_i} \\
&= \mathrm{d}G_{3z_{J+1}}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot(z_{J+1}-z_{J+1})} + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot\left(\sum_{i=0}^{J}\mathrm{d}z_i-\sum_{i=0}^{j-1}\mathrm{d}z_i\right)} \\
&= \mathrm{d}G_{3z_{J+1}}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot(z_{J+1}-z_{J+1})} + \sum_{j=1}^{J}\mathrm{d}G_{3z_j}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot(z_{J+1}-z_j)} \\
&= \sum_{j=1}^{J+1}\mathrm{d}G_{3z_j}(k_{3x},k_{3y})\cdot e^{ik_{3z}\cdot(z_{J+1}-z_j)} \\
&= \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\iint \mathcal{F}\left[M_{\text{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{1z_{j-1}}\right]\Big|_{k_x,k_y}^{x,y}\mathcal{F}\left[E_{2z_{j-1}}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\frac{e^{ik_{zq}\mathrm{d}z_{j-1}}-e^{ik_{3z}\mathrm{d}z_{j-1}}}{k_{zq}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot(z_{J+1}-z_j)}
\end{aligned}
$$

$$
G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k_z''^2-k_{3z}^2}\cdot\sum_{j=1}^{J+1}\left\{\begin{aligned}&\mathcal{F}\left[M_{\text{eff},z_{j-1}}(x,y)\cdot E_{1z_j}(x,y)E_{2z_j}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\\ &-\mathcal{F}\left[M_{\text{eff},z_{j-1}}(x,y)\cdot E_{1z_{j-1}}E_{2z_{j-1}}\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\mathrm{d}z_{j-1}}\end{aligned}\right\}\cdot e^{ik_{3z}\cdot(z_{J+1}-z_j)}
$$

$$
G_{3z}(k_{3x},k_{3y}) = \frac{\chi_{\text{eff}}\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z_{J+1}}}{(k_z''+k_{3z})/2}\cdot\sum_{j=1}^{J+1}\mathrm{sinc}\left(\Delta k_z''\frac{\mathrm{d}z_{j-1}}{2}\right)\cdot\mathcal{F}\left[M_{\text{eff},z_{j-1}}(x,y)\cdot E_{1\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)E_{2\left(z_{j-1}+\frac{\mathrm{d}z_{j-1}}{2}\right)}(x,y)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot e^{ik_{3z}\frac{\mathrm{d}z_{j-1}}{2}}e^{-ik_{3z}z_j}\cdot i\mathrm{d}z_{j-1}
$$

其中，$k_z'' = k_{zQ}''\big|_{g_{l_z}\to 0} = K_{1z}+K_{2z}$ $\quad \Delta k_z'' = k_z'' - k_{3z}$

-.-

I. 试一试 非线性卷积 的 求和版 能导出什么 3D

a. 设 $z = z_{J+1} = z_J + dz_J = \Sigma_{j \in [0, J)} dz_j$

$$
\begin{aligned}
G_{3z}\left(k_{3x},k_{3y}\right) &= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\sum_{j=1}^{J+1}\iint\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{1z_{j-1}}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{2z_{j-1}}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,\frac{e^{ik_{zq}\mathrm{d}z_{j-1}}-e^{ik_{3z}\mathrm{d}z_{j-1}}}{k_{zq}^2-k_{3z}^2}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\cdot e^{ik_{3z}\cdot\left(z_{J+1}-z_j\right)}\\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z_{J+1}}\cdot\sum_{j=1}^{J+1}\iint\mathcal{F}\left[M_{\mathrm{eff},z_{j-1}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,e^{ik_{zq}z_{j-1}}\cdot\frac{e^{ik_{zq}\mathrm{d}z_{j-1}}-e^{ik_{3z}\mathrm{d}z_{j-1}}}{k_{zq}^2-k_{3z}^2}\cdot e^{-ik_{3z}z_j}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z_{J+1}}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,\sum_{j=1}^{J+1}(-1)^{j-1}\frac{e^{ik_{zq}z_j}-e^{i\left(k_{zq}-k_{3z}\right)z_{j-1}}}{k_{zq}^2-k_{3z}^2}\cdot\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y\\
&= \frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z_{J+1}}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}(x,y)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\,\frac{1}{k_{zq}^2-k_{3z}^2}\left[\sum_{j=1}^{J+1}(-1)^{j-1}e^{ik_{zq}z_j}-\sum_{j=1}^{J+1}(-1)^{j-1}e^{i\left(k_{zq}-k_{3z}\right)z_{j-1}}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y
\end{aligned}
$$

其中， $k''_z = k''_{zQ}\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$ $\quad \Delta k''_z = k''_z - k_{3z}$ $\quad k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$

-.-

I. 试一试 非线性卷积 的 求和版 能导出什么 3D

a. 设 $\mathrm{d}z_j = \mathrm{d}z,\ z_j = \Sigma_{i\in[0,\ j-1)}\ \mathrm{d}z_i = j\cdot \mathrm{d}z,\ z = (J+1)\cdot \mathrm{d}z$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1}{k_{zq}^2-k_{3z}^2}\left[\sum_{j=1}^{J+1}(-1)^{j-1}\ e^{ik_{zq}j\cdot\mathrm{d}z}-e^{-i\left(k_{zq}-k_{3z}\right)\mathrm{d}z}\sum_{j=1}^{J+1}(-1)^{j-1}\ e^{i\left(k_{zq}-k_{3z}\right)j\cdot\mathrm{d}z}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

其中 $\displaystyle\sum_{j=1}^{J+1}(-1)^{j-1}\ e^{i\cdot j\cdot C}=\frac{1+(-1)^J\ e^{i\cdot C(J+1)}}{1+e^{i\cdot C}}e^{i\cdot C}=\frac{1+(-1)^J\ e^{i\cdot C(J+1)}}{1+e^{-i\cdot C}}$ ，则 $\displaystyle\sum_{j=1}^{J+1}(-1)^{j-1}\ e^{i\cdot j\cdot k\cdot\mathrm{d}z}=\frac{1+(-1)^J\ e^{i\cdot k\cdot z}}{1+e^{i\cdot k\cdot\mathrm{d}z}}e^{i\cdot k\cdot\mathrm{d}z}=\frac{1+(-1)^J\ e^{i\cdot k\cdot z}}{1+e^{-i\cdot k\cdot\mathrm{d}z}}$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1}{k_{zq}^2-k_{3z}^2}\left[\frac{1+(-1)^J\ e^{i\cdot k_{zq}\cdot z}}{1+e^{-i\cdot k_{zq}\cdot\mathrm{d}z}}-e^{-i\left(k_{zq}-k_{3z}\right)\mathrm{d}z}\frac{1+(-1)^J\ e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{-i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\ \mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}^{x,y}\ \frac{1}{k_{zq}^2-k_{3z}^2}\left[\frac{1+(-1)^J\ e^{i\cdot k_{zq}\cdot z}}{1+e^{-i\cdot k_{zq}\cdot\mathrm{d}z}}-\frac{1+(-1)^J\ e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}}{1+e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot\mathrm{d}z}}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y$$

-.-

I. 试一试 非线性卷积 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k''_z = k''_{zQ}\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z} \qquad \Delta k''_z = k''_z - k_{3z}$$

$$k_{zq} = \sqrt{k_1^2 - k_x^2 - k_y^2} + \sqrt{k_2^2 - \left(k_{3x} - g_x - k_x\right)^2 - \left(k_{3y} - g_y - k_y\right)^2}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\mathrm{eff}}\,\omega_3^2}{c^2}\cdot\frac{e^{ik_{3z}\cdot z}}{k''^2_z - k_{3z}^2}\cdot\left\{\begin{array}{l} \dfrac{1}{1+e^{-i\cdot k''_z\cdot \mathrm{d}z}}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\left[1+(-1)^J\,e^{i\cdot k_{zq}\cdot z}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ -\dfrac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\left[1+(-1)^J\,e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\mathrm{eff}}\,\omega_3^2}{c^2}\cdot\frac{1}{k''^2_z - k_{3z}^2}\cdot\left\{\begin{array}{l} \left[\dfrac{1}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\dfrac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \\ +\left[\dfrac{e^{ik_{3z}\cdot z}}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\dfrac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot(-1)^J\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{k_x,k_y}^{x,y}\,\mathcal{F}\left[E_{20}\right]\Big|_{k_{3x}-g_x-k_x,\,k_{3y}-g_y-k_y}^{x,y}e^{i\cdot k_{zq}\cdot z}\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right) = \frac{\chi_{\mathrm{eff}}\,\omega_3^2}{c^2}\cdot\frac{1}{k''^2_z - k_{3z}^2}\cdot\left\{\begin{array}{l} \left[\dfrac{1}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\dfrac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y \\ +\left[\dfrac{e^{ik_{3z}\cdot z}}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\dfrac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot(-1)^J\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{g_x,g_y}^{x,y}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y \end{array}\right\}$$

I. 试一试 非线性卷积 的 求和版 能导出什么 3D

a. 分母可不含 k_x, k_y

$$k''_z = k''_{zQ}\Big|_{g_{l_z}\to 0} = K_{1z} + K_{2z}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k''^2_z-k_{3z}^2}\cdot\left\{\begin{aligned}&\left[\frac{1}{1+e^{-i\cdot k''_z\cdot \mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot g_1\left(k_{3x}-g_x,k_{3y}-g_y\right)*g_2\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\\&+\left[\frac{e^{ik_{3z}\cdot z}}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot(-1)^J\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot G_{1z}\left(k_{3x}-g_x,k_{3y}-g_y\right)*G_{2z}\left(k_{3x}-g_x,k_{3y}-g_y\right)\mathrm{d}g_x\mathrm{d}g_y\end{aligned}\right\}$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k''^2_z-k_{3z}^2}\cdot\left\{\begin{aligned}&\left[\frac{1}{1+e^{-i\cdot k''_z\cdot \mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\\&+\left[\frac{e^{ik_{3z}\cdot z}}{1+e^{-i\cdot k''_z\cdot\mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot(-1)^J\cdot\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\end{aligned}\right\}$$

对比 1.1 的 求和版：

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot\frac{1}{k''^2_z-k_{3z}^2}\cdot\left[\frac{1}{1+e^{-i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}-\frac{1}{1+e^{i\cdot\left(k''_z-k_{3z}\right)\cdot\mathrm{d}z}}\right]\cdot\left\{\begin{aligned}&e^{ik_{3z}\cdot z}\cdot\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\cdot E_{10}E_{20}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\\&+(-1)^J\cdot\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\cdot E_{1z}E_{2z}\right]\Big|_{\substack{x,y\\k_{3x},k_{3y}}}\end{aligned}\right\}$$

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I. 试一试 非线性卷积 的 bulk 求和版

a. 设 $dz_j = dz$, $z_j = \Sigma_{i \in [0,\, j-1)} dz_i = j \cdot dz$, $z = (J+1) \cdot dz$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{\substack{x,y\\k_x,k_y}}\mathcal{F}\left[E_{20}\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}}\frac{1}{k_{zq}^2-k_{3z}^2}\left[\sum_{j=1}^{J+1}e^{ik_{zq}j\cdot \mathrm{dz}}-e^{-i\left(k_{zq}-k_{3z}\right)\mathrm{dz}}\sum_{j=1}^{J+1}e^{i\left(k_{zq}-k_{3z}\right)j\cdot \mathrm{dz}}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{dg}_x\mathrm{dg}_y$$

其中 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot C}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}$ ，则 $\sum_{j=1}^{J+1}e^{i\cdot j\cdot k\cdot \mathrm{dz}}=\frac{e^{i\cdot C(J+1)}-1}{e^{i\cdot C}-1}e^{i\cdot C}=\frac{e^{i\cdot k\cdot z}-1}{e^{i\cdot k\cdot \mathrm{dz}}-1}e^{i\cdot k\cdot \mathrm{dz}}$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{\substack{x,y\\k_x,k_y}}\mathcal{F}\left[E_{20}\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}}\frac{1}{k_{zq}^2-k_{3z}^2}\left[\frac{e^{i\cdot k_{zq}\cdot z}-1}{e^{i\cdot k_{zq}\cdot \mathrm{dz}}-1}\cdot e^{i\cdot k_{zq}\cdot \mathrm{dz}}-e^{-i\left(k_{zq}-k_{3z}\right)\mathrm{dz}}\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{dz}}-1}\cdot e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{dz}}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{dg}_x\mathrm{dg}_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{\substack{x,y\\k_x,k_y}}\mathcal{F}\left[E_{20}\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}}\frac{1}{k_{zq}^2-k_{3z}^2}\left[\frac{e^{i\cdot k_{zq}\cdot z}-1}{e^{i\cdot k_{zq}\cdot \mathrm{dz}}-1}\cdot e^{i\cdot k_{zq}\cdot \mathrm{dz}}-\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{dz}}-1}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{dg}_x\mathrm{dg}_y$$

$$G_{3z}\left(k_{3x},k_{3y}\right)=\frac{\chi_{\mathrm{eff}}\omega_3^2}{c^2}\cdot e^{ik_{3z}\cdot z}\cdot\iint\mathcal{F}\left[M_{\mathrm{eff}}\left(x,y\right)\right]\Big|_{\substack{x,y\\g_x,g_y}}\cdot\iint\mathcal{F}\left[E_{10}\right]\Big|_{\substack{x,y\\k_x,k_y}}\mathcal{F}\left[E_{20}\right]\Big|_{\substack{x,y\\k_{3x}-g_x-k_x,k_{3y}-g_y-k_y}}\frac{1}{k_{zq}^2-k_{3z}^2}\left[\frac{e^{i\cdot k_{zq}\cdot z}-1}{1-e^{-i\cdot k_{zq}\cdot \mathrm{dz}}}-\frac{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot z}-1}{e^{i\cdot\left(k_{zq}-k_{3z}\right)\cdot \mathrm{dz}}-1}\right]\mathrm{d}k_x\mathrm{d}k_y\mathrm{dg}_x\mathrm{dg}_y$$

D. 理论分析：小 z 、无 χ 、有图时，可否近似为 空域 点到点 和频

a. 在 B.b 中，已得到 $G_{3,\mathrm{dz}}\left(k_{3x},k_{3y}\right)=\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)$

其中 $Q_{30}\left(k_{3x},k_{3y}\right)=\mathcal{F}\left[\chi_{\mathrm{eff}}\left(x,y,0\right)\cdot E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}=\chi_{\mathrm{eff}}\cdot\mathcal{F}\left[E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}$

b. 那么

$$G_{3,\mathrm{dz}}\left(k_{3x},k_{3y}\right)=\frac{k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}Q_{30}\left(k_{3x},k_{3y}\right)$$

$$=\frac{\chi_{\mathrm{eff}}k_3^2}{n_3^2}\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}\cdot\mathcal{F}\left[E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}$$

c. 最终

$$E_3\left(x,y,\mathrm{dz}\right)=\mathcal{F}^{-1}\left[G_{3z}\left(k_{3x},k_{3y}\right)\right]\Big|_{x,y}^{k_{3x},k_{3y}}=\frac{\chi_{\mathrm{eff}}k_3^2}{n_3^2}\cdot\mathcal{F}^{-1}\left[\mathcal{F}\left[E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]\Big|_{k_{3x},k_{3y}}^{x,y}\cdot\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}$$

$$=\frac{\chi_{\mathrm{eff}}k_3^2}{n_3^2}\cdot\frac{1}{(2\pi)^2}\left[E_{10}\left(x,y\right)E_{20}\left(x,y\right)\right]*\mathcal{F}^{-1}\left[\frac{e^{ik_{3z}\mathrm{dz}}-1}{k_{3z}^2}\right]\Bigg|_{x,y}^{k_{3x},k_{3y}}$$

SSI 版

非线性 角谱理论 的 应用

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