

$$\mathcal{F}[E_{3z}] \approx \frac{\chi_{\text{eff}} \omega_3^2}{2c^2} \cdot \sum_j a_j \int \mathcal{F} \left[\mathcal{F}_z [M_{\text{eff}}(\mathbf{r})] \cdot E_{1\frac{b_j \pm 1}{2b_j}z} E_{2\frac{b_j \pm 1}{2b_j}z} \right] \cdot \frac{e^{\frac{ig_z}{2b_j}z}}{k_{1z} + k_{2z} + g_z + k_{3z}} \cdot dg_z \cdot e^{ik_{3z}\frac{b_j \mp 1}{2b_j}z} \cdot iz$$

$$\mathcal{F}[E_{3z}] \approx \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \int \mathcal{F} \left[\mathcal{F}_z [M_{\text{eff}}(\mathbf{r})] \cdot E_{1\frac{z}{2}} E_{2\frac{z}{2}} \right] \cdot e^{ig_z \frac{z}{2}} \cdot \frac{\text{sinc} \left[\left(\overline{k_{1z}} + \overline{k_{2z}} + g_z - k_{3z} \right) \frac{z}{2} \right]}{k_{1z} + k_{2z} + g_z + k_{3z}} \cdot dg_z \cdot e^{ik_{3z} \frac{z}{2}} \cdot iz$$

$$\mathcal{F}[E_{3z}] \approx \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \int \frac{\mathcal{F} \left[\mathcal{F}_z [M_{\text{eff}}(\mathbf{r})] \cdot E_{1z} E_{2z} \right] \cdot e^{ig_z z} - \mathcal{F} \left[\mathcal{F}_z [M_{\text{eff}}(\mathbf{r})] \cdot E_{10} E_{20} \right] \cdot e^{ik_{3z} z}}{\left(\overline{k_{1z}} + \overline{k_{2z}} + g_z \right)^2 - k_{3z}^2} \cdot dg_z$$

$$\mathcal{F}[E_{3z}] = \frac{\chi_{\text{eff}} \omega_3^2}{c^2} \cdot \iiint \mathcal{F}_{3D} [M_{\text{eff}}(\mathbf{r})] \cdot \left\{ \right\} \cdot dg_x dg_y dg_z$$

$$\iint \mathcal{F}[E_{10}] \cdot \mathcal{F}[E_{20}] \cdot \frac{e^{i(k_{1z} + k_{2z} + g_z)z} - e^{ik_{3z}z}}{\left(k_{1z} + k_{2z} + g_z \right)^2 - k_{3z}^2} \cdot dk_{1x} dk_{1y}$$

- ◆ 线性卷积 (LC): 可以分离变量 ==> 可以写成 FFT,
- ◆ 非线性卷积 (NC): 无法解关联 ==> 无法写成 FFT.

$$\begin{cases} k_{ix}^2 + k_{iy}^2 + k_{iz}^2 = k_i^2, & i = 1, 2, 3 \\ k_{1j} + k_{2j} + g_j = k_{3j}, & j = xyz \end{cases}$$

分数阶 Cos 级数版 非线性角谱 - 广义版 NLAST

Sinc 版 非线性角谱 - 匹配版 NLAST

非线性角谱 - 失配版 NLAST

非线性卷积 NLCOV

空域 SFG

空域混频 = {k_i} - NC

脉冲 => 时域混频 = {ω_i} - NC

光学非线性 = 能量 {ω_i}、动量 {k_i} - NC

自然界的本质 = A Set Of - 非线性卷积 (NC) 过程

但, 通过
1. 近似、
2. 级数,
可以写成 FFT

NLAST 的传承与发展

The preservation and innovative development of NLAST