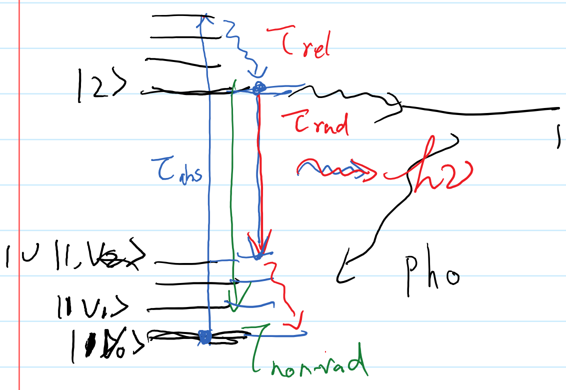
2015_Ch7_Light matter interaction

2015年12月2日 14:42

Quantum emitter:

Single molecule fluorescence

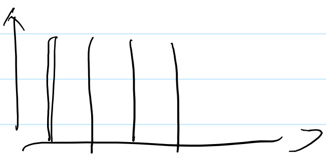


$\tau_{abs} \sim$ instant

$\tau_{rel} \sim 1\,ps$

τ_{rad} (flu) $\sim 1\,ns$

$\tau_{pho} \sim \rightarrow$



$\sigma_{abs,\,dye} \simeq 10^{-15}\,cm^2$ laser dye.

GFP $10^{-18}\,cm^2$

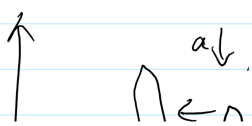
$$\sigma_{abs,max} = \frac{3\lambda^2}{2\pi}$$

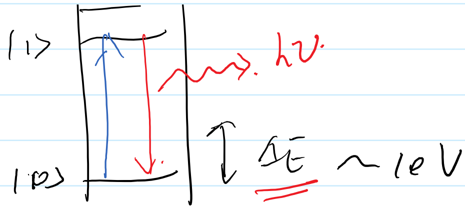
QDs:

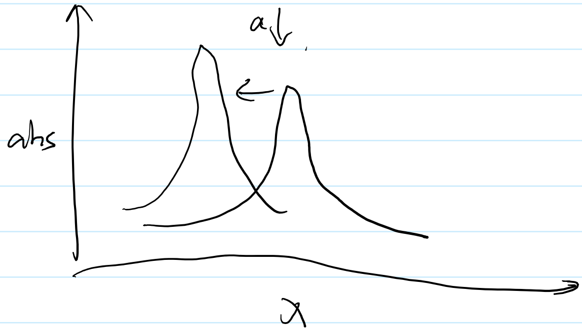
nanocrystal.

$$\hat{H} = -\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 - \frac{e^2}{\varepsilon|r_e - r_h|}$$

$$\hat{H}\psi = V(r) \quad \begin{cases} V(r) = \infty & |r| \geq \frac{a}{2} \\ V(r) = 0 & |r| < \frac{a}{2} \end{cases}$$





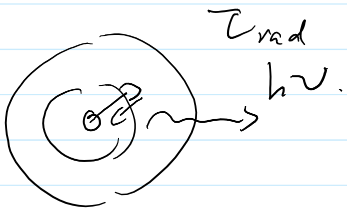


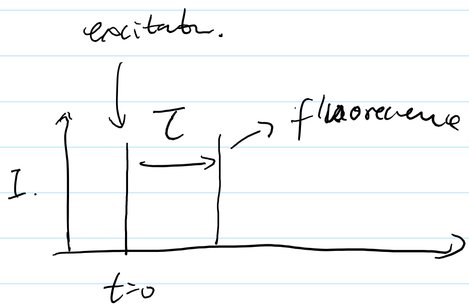
$$E_{nl} = \frac{\hbar^2 \alpha_{nl}^2}{2 m a^2}$$

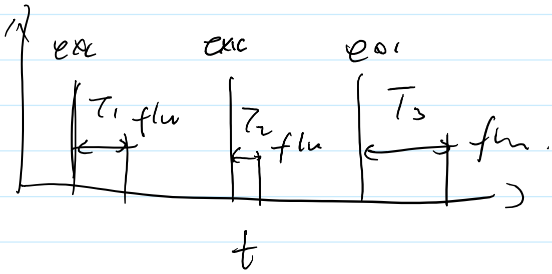
NV center in nanodiamond. → extremely narrow line

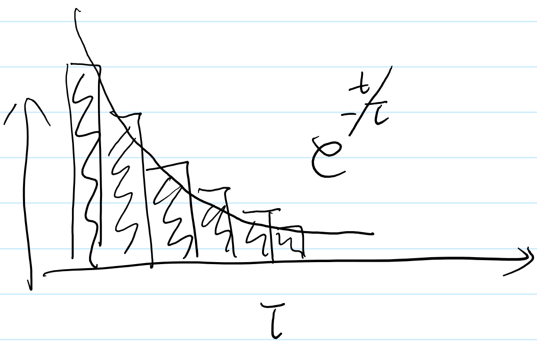
Rare earth elements

Atom vapor









histogram.

|1⟩ —— P

|0⟩ ——

$$\frac{dP}{dt} = -\frac{1}{\tau} P + B_{abs} I_{exc}$$

$B_{abs} I_{exc} = 0$

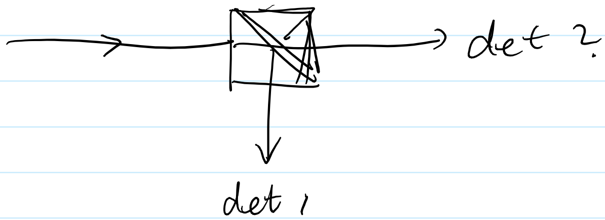
$$P \sim e^{-\frac{t}{\tau}}$$

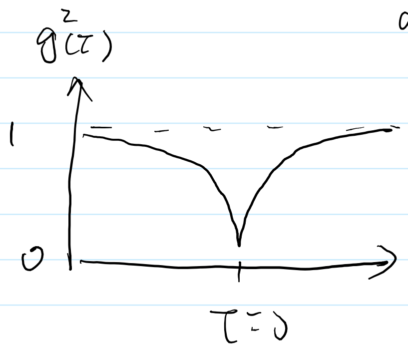
$|0\rangle$ ——— ——— $\overset{\shortparallel}{o}$.

$$p \sim e^{-\frac{t}{\tau}}$$

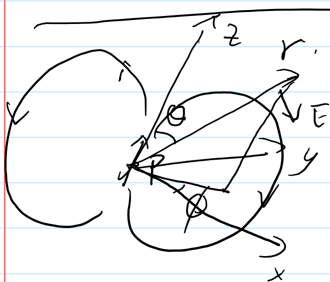
Antibunching:

$$g^{(2)}(\tau) = \frac{\langle I_1(t)\rangle\langle I_2(t+\tau)\rangle}{\langle I(t)\rangle^2}.$$





single photon source



$$\overline{\overline{G}} = \frac{e^{ikR}}{4\pi R}\left[\left(1+\frac{ikR-1}{k^2R^2}\right)\overline{\overline{I}} + \frac{3-3ikR-k^2R^2}{k^2R^2}\frac{\vec{R}\vec{R}}{R^2}\right]$$

$// R = |\vec{r}-\vec{r}_0|$

$$\vec{E}(r) = \omega^2\mu\mu_0\,\overline{\overline{G}}(\vec{r},\vec{r}_0)\,\underline{\underline{P}}$$

dipole.

$$\vec{E}_r = \frac{|P|\cos\theta}{4\pi\varepsilon_0\varepsilon}\frac{e^{ikr}}{r}\,k^2\left[\frac{2}{k^2r^2}-\frac{2i}{kr}\right]$$

$$\vec{E}_\theta = \frac{|P|\sin\theta}{4\pi\varepsilon_0\varepsilon}\frac{e^{ikr}}{r}\,k^2\left[\frac{1}{k^2r^2}-\frac{i}{k}-1\right]$$

$\vec{H}_\phi = |P|\sin\theta\ e^{ikr}$ ...

$$\vec{H}_\phi = \frac{1\, p \sin\theta}{4\pi \varepsilon_0 \varepsilon} \frac{e^{ikr}}{r} k^2 \left[-\frac{i}{kr} - 1 \right] \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}$$

FF

Power dissipation

$$\frac{dW}{dt} = -\frac{1}{2} \int \mathrm{Re}(j^* \cdot E)\, dV \quad ①$$

$$j = -i\omega \vec{p}\, \delta(\vec{r} - \vec{r}_0) \quad ②$$

$$\int_V f(r)\, \delta(r - r_0)\, dV = f(r_0)$$

$$\Rightarrow \frac{dW}{dt} = \frac{\omega}{2} \mathrm{Im}(p^* \cdot E(r_0)) \quad ③$$

$$E = \omega^2 \mu \mu_0 \overline{\overline{G}}(r, r_0)\, \bar{P}(r_0) \quad ④$$

$$\frac{dW}{dt} = \frac{\omega^3 |p|^2}{2c^2 \varepsilon_0 \varepsilon} \left[\hat{n}_p \cdot \mathrm{Im}\, \overline{\overline{G}}(r, r, \omega) \cdot \hat{n}_p \right] \quad ⑤$$

LDOS

local density of state

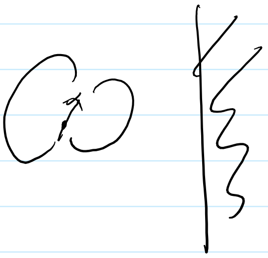
③

$$\frac{dW}{dt} = \lim_{R\to 0} \frac{\omega}{2} |P|\, \mathrm{Im}(E_z) = \frac{\omega}{8\pi} \frac{|P|^2}{\varepsilon_0 \varepsilon} \lim_{R\to 0} \left\{ \frac{2}{3} k^3 + R^2(\dots)_r \dots \right\}$$

$$= \frac{P^2}{12\pi} \frac{\omega}{\varepsilon_0 \varepsilon} k^3$$

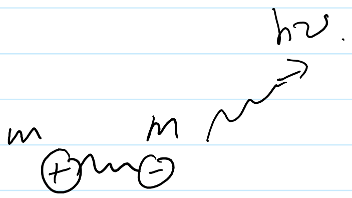
dissipation rate of a photon emitter in free space

(homogeneous environment)



$$E = E_0 + E_s$$

$$\frac{P}{P_0} = 1 + \frac{6\pi\varepsilon_0\varepsilon}{|\vec{p}|^2}\frac{1}{k^3}\,\mathrm{Im}\,(\vec{p}^* \cdot \vec{E}_s(r_0))$$



$$m\ddot{r} + \omega_0^2 m \vec{r} = F_r \quad (1)$$ radiation reaction force.

$$p(t) = \int_{\partial V} \vec{S}\cdot\hat{n}\,da = \frac{1}{4\pi\varepsilon_0\varepsilon}\frac{2n^3}{3c^3}\left[\frac{d^2\vec{p}}{dt^2}\right]^2 \quad (2)$$

$$\int_{t_1}^{t_2}\left[F_r\cdot\dot{r} + \frac{q^2(\ddot{r}\cdot\ddot{r})}{6\pi\varepsilon_0 c^3}\right]dt = 0 \quad (3)$$

$$\frac{d}{dt}(\dot{r}\cdot\ddot{r}) = \ddot{r}\cdot\ddot{r} + \dot{r}\cdot\dddot{r} \quad (4)$$

$\Rightarrow$

$$\int_{t_1}^{t_2}\left(F_r\cdot\dot{r} - \frac{q^2\,\dddot{r}\cdot\dot{r}}{6\pi\varepsilon_0 c}\right)dt + \left.\frac{q^2\,\ddot{r}\cdot\dot{r}}{6\pi\varepsilon_0 c^3}\right|_{t_1}^{t_2} = 0 \quad (5)$$

$$F_r\cdot\dot{r} = \frac{q^2\,\dddot{r}\cdot\dot{r}}{6\pi\varepsilon_0 c^3} \quad (6)$$

(6), (1)
$$\ddot{r} - \frac{q^2}{6\pi\varepsilon_0 c^3 m}\dddot{r} + \omega_0^2 r = 0$$

$\sim \omega_0 \qquad \dddot{r} = -\omega_0^2\dot{r}$

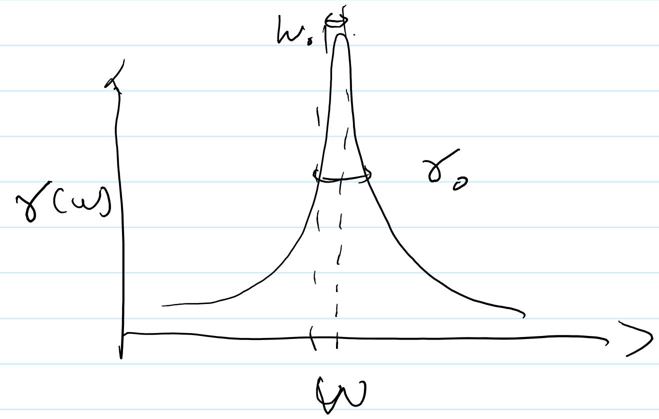
$\Rightarrow$

$$\ddot{r} + \gamma_0\dot{r} + \omega_0^2 r = 0.$$

$\Rightarrow$ $P \ldots = P\,[\,P\, e^{-i\omega_0\sqrt{1-\gamma_0^2/(4\omega_0^2)}\,t - \gamma t/\ldots}]$

$$r + \gamma_0 r + \omega_0 r = 0$$

$$\gamma_0 = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3c^3}\frac{\omega_0^2}{m} \quad \Rightarrow \quad P(t) = \mathrm{Re}\left\{P_0\, e^{-i\omega_0\sqrt{1-\gamma_0^2/(4\omega_0^2)}\,t}\, e^{-\gamma t/2}\right\}$$



Averge energy of the harmonic oscillator is sum of the kinetic and potential energy

$$\overline{W}(t) = \frac{m}{2q^2}\left[\omega_0^2 p^2 + \dot{p}^2\right] = \frac{m\omega_0^2}{2q^2}|P_0|^2 e^{-\gamma_0 t}$$

$$\tau_{rad} = 1/\gamma_0$$

averge radiation power:

$$P_0(t) = \frac{|p(t)|^2}{4\pi\varepsilon_0}\frac{\omega_0^4}{3c^3}$$

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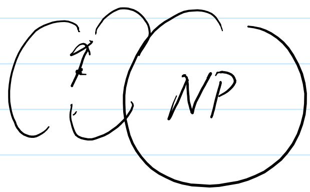
radiation in inhomogeneous environment

$$\frac{d^2}{dt^2}P(t) + \underbrace{\gamma_0\frac{d}{dt}P(t)}_{F_r} + \omega_0^2 P = \frac{q^2}{m}E_s(t)$$



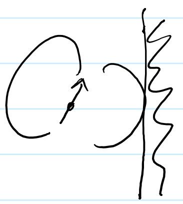
Trial solution:

$$p(t) = \mathrm{Re}\left(P_0\, e^{-i\omega t} e^{-\gamma t/2}\right)$$

$$E_s = \mathrm{Re}\left\{ E_0 e^{-i\omega t} e^{-\gamma t/2} \right\}$$

$$\frac{\gamma}{\gamma_0} = 1 + \frac{6\pi\varepsilon_0}{|p_0|}\frac{1}{k^3}\,\mathrm{Im}\left[p_0^* \cdot \vec{E}_s(r_0) \right]$$

$$\frac{\gamma}{\gamma_0} = \frac{P}{P_0}$$ // useful for numerical simulation



$$\left.\begin{aligned} &\ddot{P} + \gamma \dot{P} + \omega_0^2 P = \frac{e^2}{m} E_{refl} \\ &P = P_0 e^{-i\Omega t} \end{aligned}\right\} \Rightarrow$$

// $\Omega = \omega t - i\frac{b}{2} t$

$$P_0(-i\Omega)^2 + \gamma(-i\Omega b)P_0 + \omega_0^2 P_0^2 = \frac{e^2}{m} E_{refl}$$

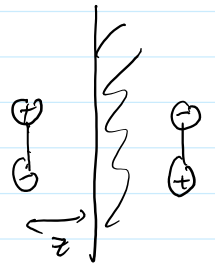
Real: $P_0\left[\omega^2 - \frac{b^2}{4} + \frac{\gamma b}{2} - \omega_0^2\right] = -\frac{e^2}{m}\mathrm{Re}(E_{refl})$

Imagine: $P_0\left[b\omega - \gamma\omega\right] = \frac{e^2}{m}\mathrm{Im}(E_{refl})$. $\Rightarrow$

$$\Rightarrow \quad b = \gamma + \frac{e^2}{m\omega P_0}\mathrm{Im}(E_0)$$

$$\left.\begin{aligned} &\omega^2 - \omega_0^2 \simeq \frac{b^2}{4} - \frac{\gamma b}{2} - \frac{e^2}{mP_0}\mathrm{Re}(E_{refl}) \\ &\omega \simeq \omega_0 \end{aligned}\right\} \Rightarrow$$

$$\Delta\omega = \frac{b^2}{8\omega_0} - \frac{\gamma b}{2\omega_0} - \frac{e}{2m\omega_0 P_0}\mathrm{Re}(E_{refl})$$



$$E_{refl} = \frac{1}{4\pi\varepsilon_0} \frac{-p_0}{(2z)^3} \left[1 - ik(2z) - k^2(2z)^2\right] e^{ik2z}$$

$$b = \gamma + \frac{e^2}{m\omega p_0} \mathrm{Im}\,(E_{refl})$$

$$= \gamma - \frac{e^2\omega^2}{4\pi\varepsilon_0 m c^3} \left[\frac{\sin 2kz}{(2kz)^3} - \frac{\cos 2kz}{(2kz)^2} - \frac{\sin 2kz}{2kz} \right]$$