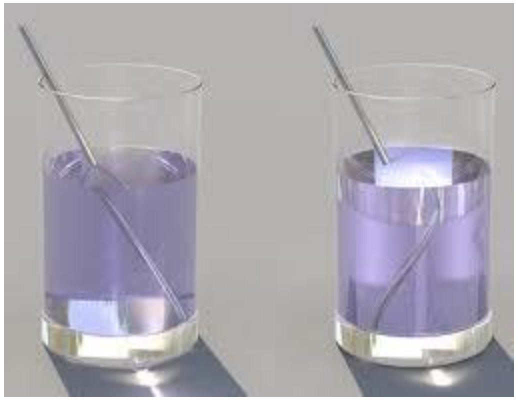
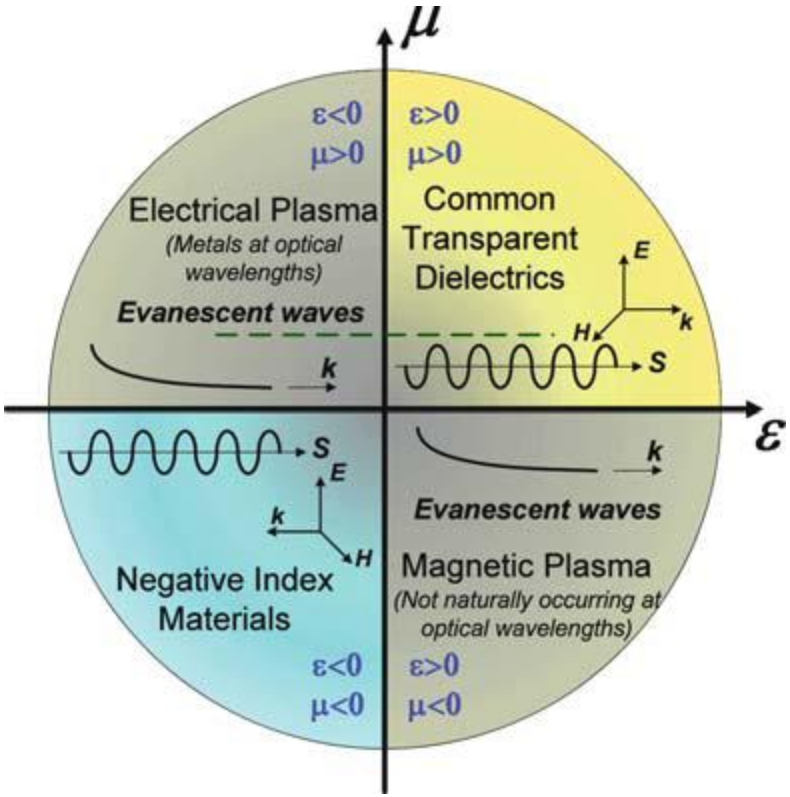
Light propagation in homogeneous media



Principles of Optics, fall semester, 2014

The four types of Isotropic media



1, dielectrics (*n*>0)
2, negative permittivity, *ε<0, μ>0*
3, negative permeability, *μ<0, ε>0*
4, negative refractive index (*n<0)*

5, near-zero *ε~=0*
6, near-zero *μ~=0*
7, near-zero *n~=0*

Wenshan Cai, *Vladimir Shalaev,* Optical Metamaterials: Fundamentals and Applications, Springer 2010

Propagation in metals (lossy materials)

Wave equation in metal:

$$\left(\nabla^2 + \omega^2 \mu \epsilon_c\right) \overline{E} = 0$$

Dispersion:

$$k^2 = \omega^2 \mu \epsilon_c = \omega^2 \mu \left(\epsilon + i\frac{\sigma}{\omega}\right)$$

Wave solutions in metal become:

$$\overline{E} = \hat{x} E_0 e^{-k_I z + i k_R z}$$

$$\overline{H} = \hat{y} \frac{(k_R + i k_I)}{\omega \mu} E_0 e^{-k_I z + i k_R z}$$

Here:

$$k^2 = k_R^2 - k_I^2 + i2k_R k_I = \omega^2 \mu \epsilon_c = \omega^2 \mu \left(\epsilon + i\frac{\sigma}{\omega}\right)$$

$$k_R = \omega\sqrt{\mu\epsilon}\left[\frac{1}{2}\left(\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}} + 1\right)\right]^{1/2}$$

$$k_I = \omega\sqrt{\mu\epsilon}\left[\frac{1}{2}\left(\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}} - 1\right)\right]^{1/2}$$

Propagation in plasma (negative permittivity materials)

Wave equation :

$$\left(\nabla^2 + \omega^2 \mu \epsilon_c\right) \overline{E} = 0$$

Dispersion:

$$k(\omega) = \frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

For $\omega > \omega_p$, the solution of a plane wave propagating in the $\hat{z}$ direction is

$$\begin{cases} k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \\ \overline{E} = \hat{x} E_0 e^{ikz} \\ \overline{H} = \hat{y} \frac{k}{\omega\mu} E_0 e^{ikz} \end{cases} \tag{3.2.30}$$

For $\omega < \omega_p$, k is imaginary. The solution (3.2.30) becomes

$$\begin{cases} k = ik_I = i\frac{\omega}{c}\sqrt{\frac{\omega_p^2}{\omega^2} - 1} \\ \overline{E} = \hat{x} E_0 e^{-k_I z} \\ \overline{H} = \hat{y} \frac{-k_I}{\omega\mu} E_0 e^{-k_I z} \end{cases} \tag{3.2.31}$$

//evanescent waves
// what's the difference between the lossy materials and negative permittivity materials?
//what will happen when ω ~= ωp (i.e., ε ~= 0)

Veselago's theory

Epsilon <0 & mu <0 → left-handed material

These relations are primarily the Maxwell equations and the constitutive relations

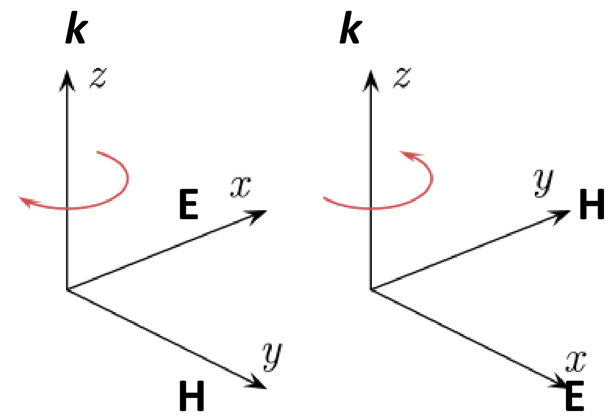
$$\left.\begin{aligned} \operatorname{rot}\mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ \operatorname{rot}\mathbf{H} &= \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \end{aligned}\right\} \qquad (4)^{*}$$

$$\left.\begin{aligned} \mathbf{B} &= \mu\mathbf{H}, \\ \mathbf{D} &= \varepsilon\mathbf{E}. \end{aligned}\right\} \qquad (4')$$

For a plane monochromatic wave, in which all quantities are proportional to $e^{i(kz-\omega t)}$, the expressions (4) and (4′) reduce to

$$\begin{aligned} [\mathbf{kE}] &= \frac{\omega}{c}\mu\mathbf{H}, \\ [\mathbf{kH}] &= -\frac{\omega}{c}\varepsilon\mathbf{E}. \end{aligned} \qquad (5)^{\dagger}$$

It can be seen at once from these equations that if eps > 0 and *mu > 0 then E, H, and k form a right - handed triplet of vectors, and if eps < 0 and mu < 0 they* are a left - handed set.



eps<0, mu<0 eps>0, mu>0

Definition of refractive index *n*

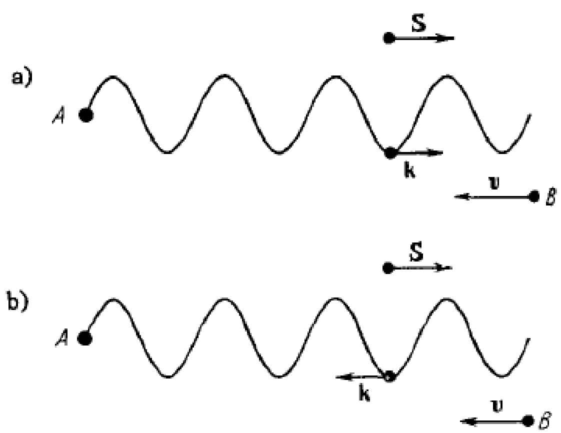
1) Convention: $n = \sqrt{\varepsilon\mu}$

2) Esp<0 & mu<0: ***k=nk₀*** **→ n < 0**

The sign of *n* is decided by the handedness of the material

V.G.Veselago, SOVIET PHYSICS USPEKHI, vol 10, 509 (1968)

S and k



normal. The energy flux carried by the wave is determined by the Poynting vector S, which is given by

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}]. \tag{8}$$

According to (8) *the vector* S *always forms a right-handed set with the vectors* E and H. Accordingly, for right-handed substances S and k are in the same direction, and for left-handed substances they are in opposite directions.[3] Since the vector k is in the the

Direction of energy flow and momentum:

between the Poynting vector S and the group velocity $\mathbf{v_g} = \partial\omega/\partial\mathbf{k}$,

$$\mathbf{S} = W \cdot \mathbf{v}_{gr}. \tag{26}$$

Combining the expressions (23)–(26), we get

$$\mathbf{p} = \frac{W}{\mathbf{v}_{ph}} = \frac{W}{\omega} \cdot \mathbf{k}. \tag{27}$$

It follows from this that in left-handed substances the field momentum p is directed opposite to the Poynting vector S.

V.G.Veselago, SOVIET PHYSICS USPEKHI, vol 10, 509 (1968)

Refraction

In the passage of a ray of light from one medium into another the boundary conditions

$$E_{t_1} = E_{t_2}, \qquad H_{t_1} = H_{t_2}, \qquad (12)$$

$$\varepsilon_1 E_{n_1} = \varepsilon_2 E_{n_2}, \qquad \mu_1 H_{n_1} = \mu_2 H_{n_2} \qquad (13)$$

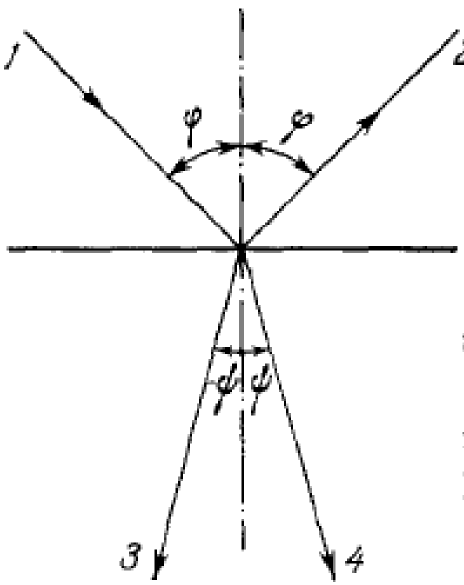


FIG. 3. Passage of a ray through the boundary between two media. 1 – incident ray; 2 – reflected ray; 3 – reflected ray if the second medium is left-handed; 4 – refracted ray if the second medium is right-handed.

Snell’s law is still correct.

V.G.Veselago, SOVIET PHYSICS USPEKHI, vol 10, 509 (1968)

Experiment

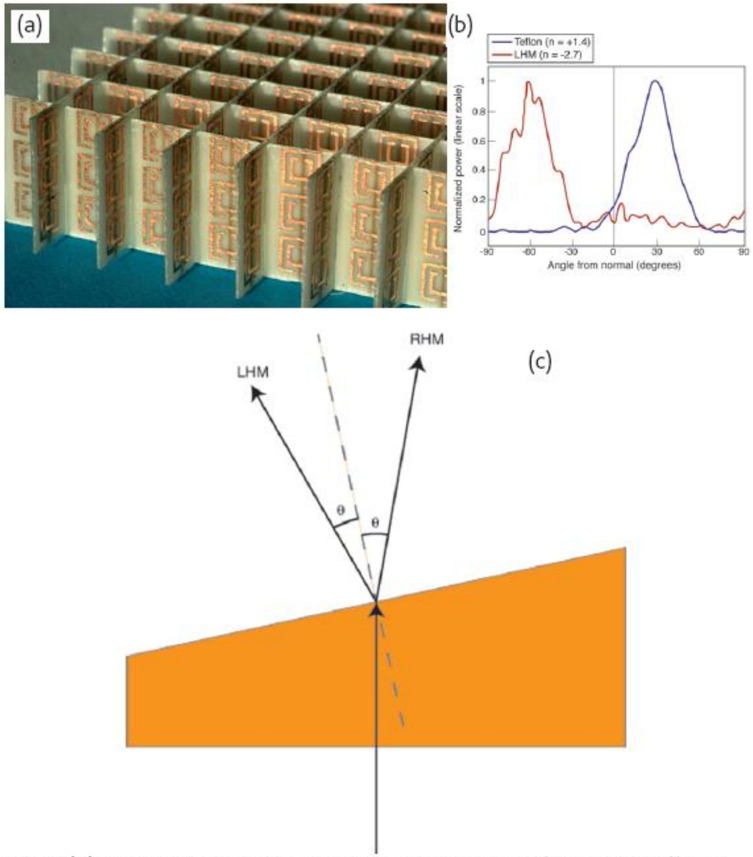


Fig. 3 (a) An NI MM formed by SRRs and wires deposited lithographically on opposite sides of a standard circuit board. The height of the structure is 1 cm. (b) The power detected as a function of angle in a Snell's law experiment performed on a Teflon sample (blue curve) and an NI sample (red curve). (c) A schematic showing the geometry used to experimentally verify the NI of refraction.

Shelby, R. A., et al., Science (2001) 292, 77

NIM slab can function as a lens

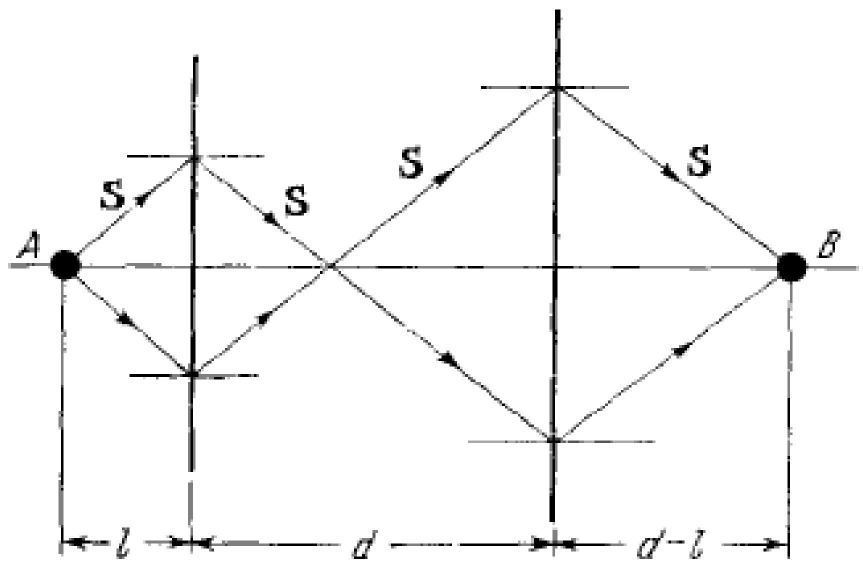
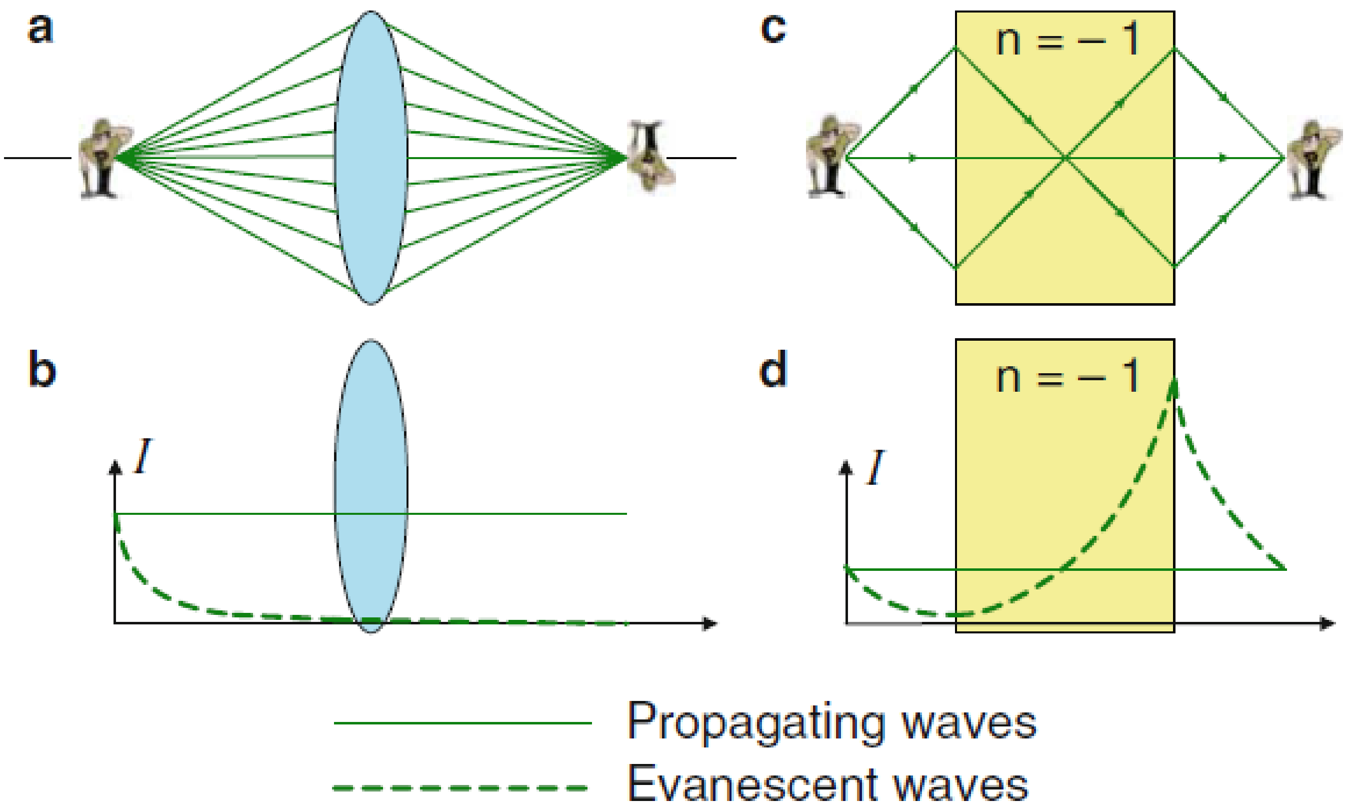


FIG. 4. Passage of rays of light through a plate of thickness d made of a left-handed substance. A – source of radiation; B – detector of radiation.

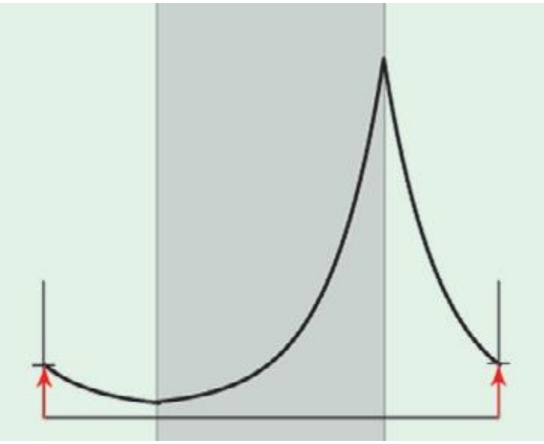
V.G.Veselago, SOVIET PHYSICS USPEKHI, vol 10, 509 (1968)

Perfect lens



W. Cai, & V. Shalaev, Optical Metamaterials, Springer (2010)

Perfect lens



In free space, features smaller than half wavelength cannot be resolved optically, because the detailed information of these features is carried by waves with a very large kx and ky. This large kx and ky results in an imaginary kz, and thus, the light cannot propagate into the free space.

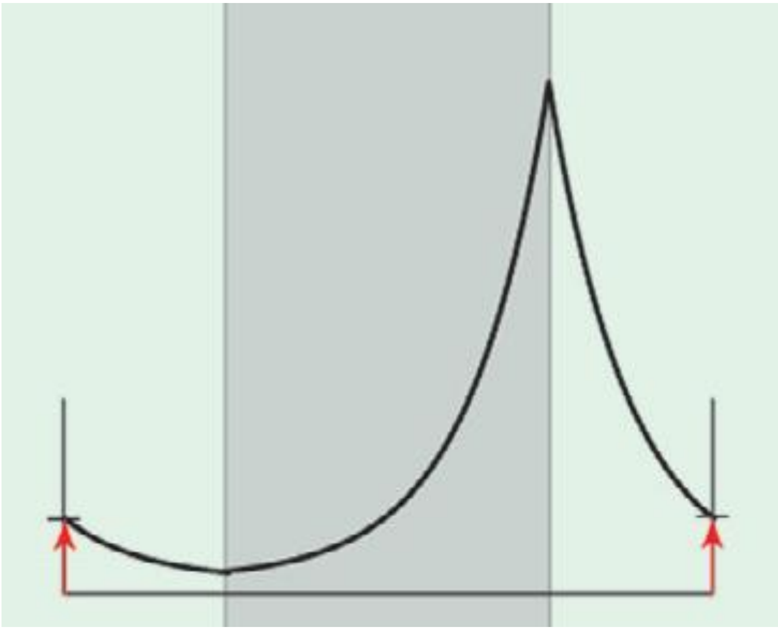
In NIM:

$$k_z = -\sqrt{\frac{\varepsilon\mu\omega^2}{c^2} - k_x^2 - k_y^2} \qquad k_z = -i\kappa_z = -i\sqrt{k_x^2 + k_y^2 - \frac{\varepsilon\mu\omega^2}{c^2}}.$$

The evanescent waves which carry the information with high spatial frequencies will be amplified. Therefore, this slab images objects perfects, without losing any details.

J. Pendry, PRL, 85, 3966 (2000)

Paradoxes



1. What will happen if the slab is infinitively thick?

   There must be material losses, and perfect lens cannot exist in reality.

2. Causality

   Phase speed, energy flow, and the case of a pulse input

Magnetic response

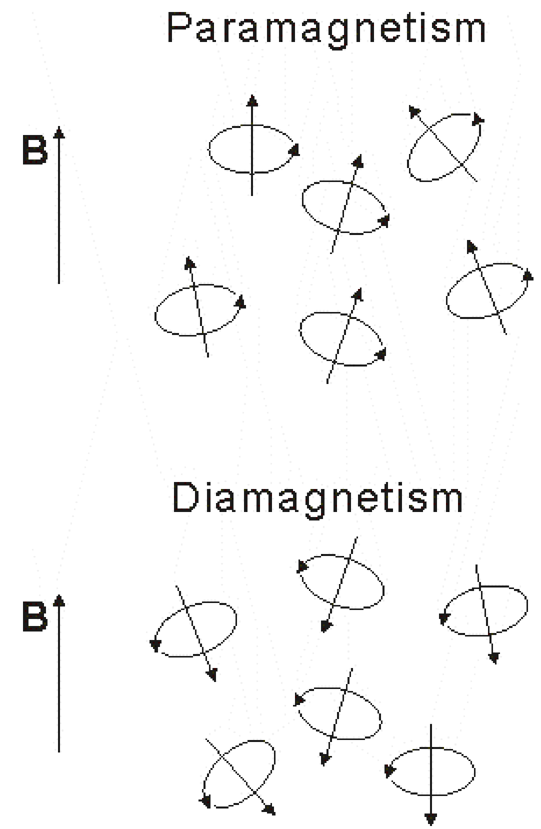
Maxwell equations in vacuum

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{c^2 \partial t} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$



J. D. Jackson, Classical Electrodynamics, 34d ed.

Magnetic response

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{c^2 \partial t} = \mu_0 \mathbf{J}$$

$$\mathbf{J} = \mathbf{J}_{ex} + \mathbf{J}_{mag} + \mathbf{J}_{pol}$$

$$\mathbf{J}_{mag} = \nabla \times \mathbf{M}$$

$$\mathbf{J}_{pol} = \partial \mathbf{P} / \partial t$$

$$\nabla \times (\mathbf{B} / \mu_0 - \mathbf{M}) - \partial(\varepsilon_0 \mathbf{E} + \mathbf{P}) / \partial t = \mathbf{J}_{ex}$$

$$\nabla \times \mathbf{H} - \partial \mathbf{D} / \partial t = \mathbf{J}_{ex}$$

here,

$$\mathbf{H} = \mathbf{B} / \mu_0 - \mathbf{M}$$

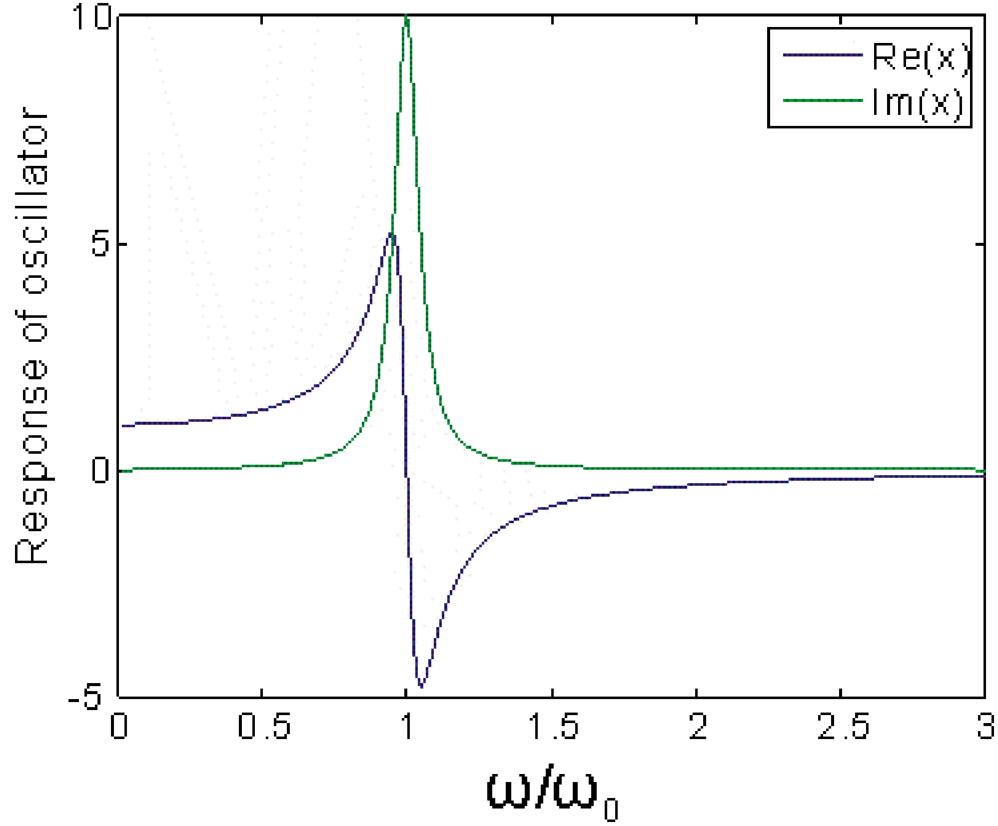
$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

uation. The magnetic coupling to an atom is proportional to the Bohr magneton $\mu_B = e\hbar/2m_e c = \alpha e a_0/2$, while the electric coupling is ea_0. The induced magnetic dipole also contains the fine structure constant $\alpha \approx 1/137$, so the effect of light on the magnetic permeability is α^2 times weaker than light's effect on the electric permittivity. This means that of the two field components of light – electric and magnetic – only the electric "hand" efficiently probes the atoms of a material, while the magnetic component remains relatively unused. Consequently, in all conven-

W. Cai, & V. Shalaev, Optical Metamaterials, Springer (2010)

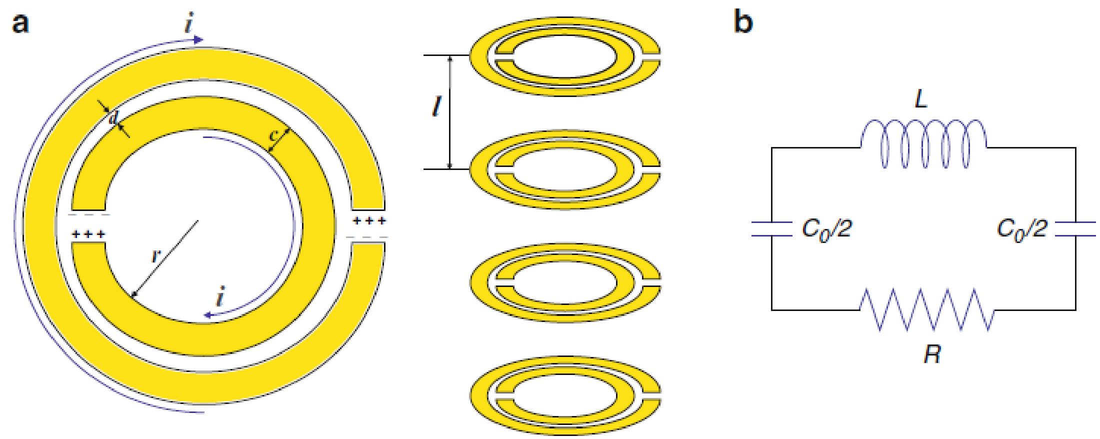
In 1970s, Veselago's theory attracted a lot interests, but the impact died out soon because people failed to find a material with negative mu.

Negative response and resonance



The negative response can be achieved using the resonant phenomenon. Consider a oscillator driven by external forces. The negative response occurs when frequency is higher than the resonance frequency.

Magnetic metamaterials – split ring resonator



$$\omega_0 = \sqrt{1/(L + R/j\omega_0)C}$$

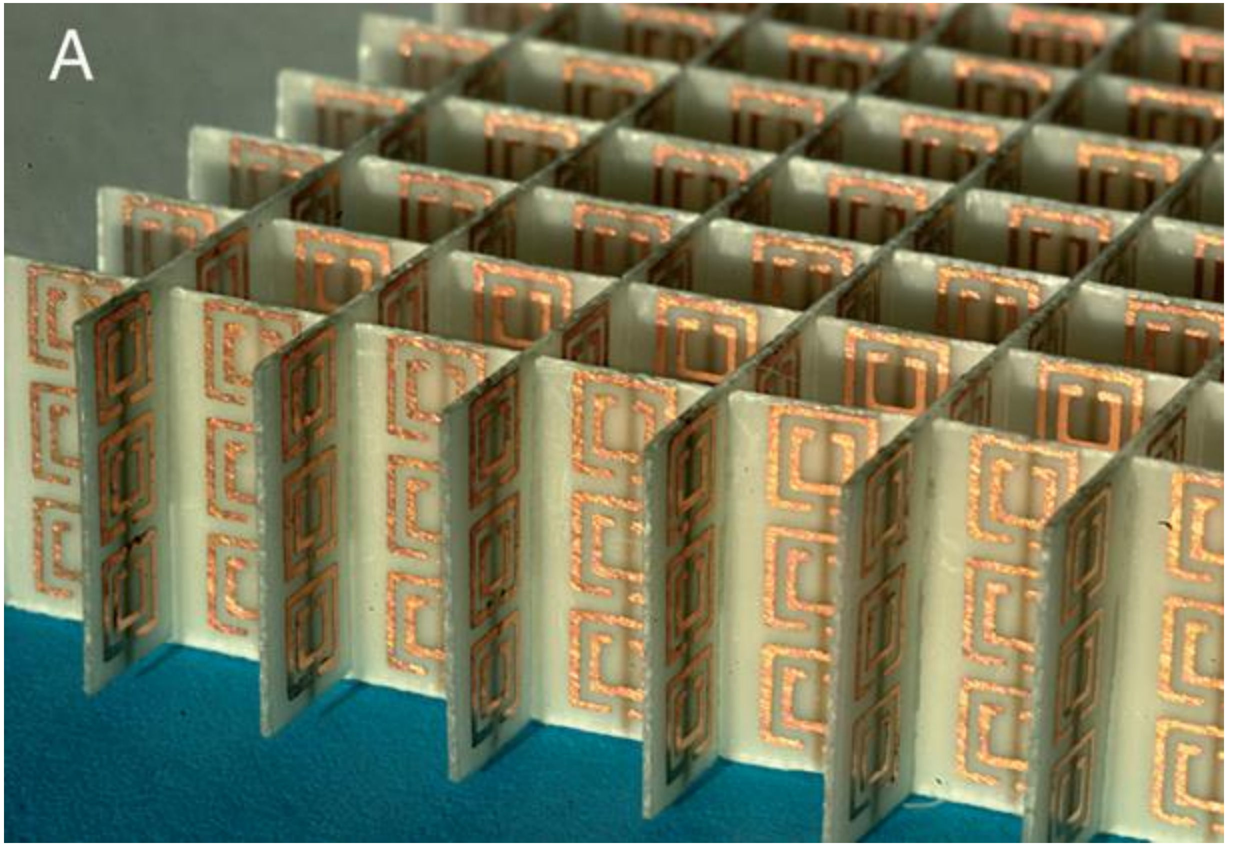
$$L \approx 2\mu_0 r$$
$$C_0 \approx 2\pi r \varepsilon_0 (c + t)/d$$
$$R \approx \pi r / c\delta\sigma$$

The magnetic resonator was invented by J. Pendry in 1990's and realized by D. Smith in the microwave spectral range.

1. Why did they choose split-ring instead of a closed ring?
2. The LCR circuit model is basically a toy model. Simulation is needed for engineering the resonance.

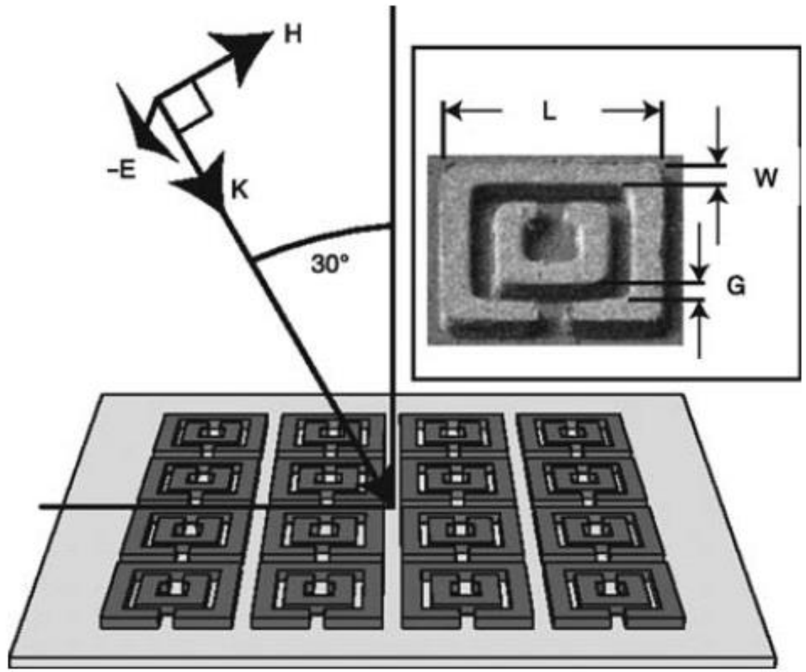
W. Cai, & V. Shalaev, Optical Metamaterials, Springer (2010)
IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 47, 2075 (1999)

The 1st NIM



Smith, D. R., et al., Phys. Rev. Lett. (2000) 84, 4184

Ring resonator in the THz spectral range



Using scaling techniques along with inclined incidence, the resonance frequency of double SRRs has been pushed up to 1 THz. A sample with terahertz SRRs and the associated polarization condition is illustrated in the above. The size of each SRR in this case is about 40 um.

Science 303:1494–1496

Single split-ring resonator

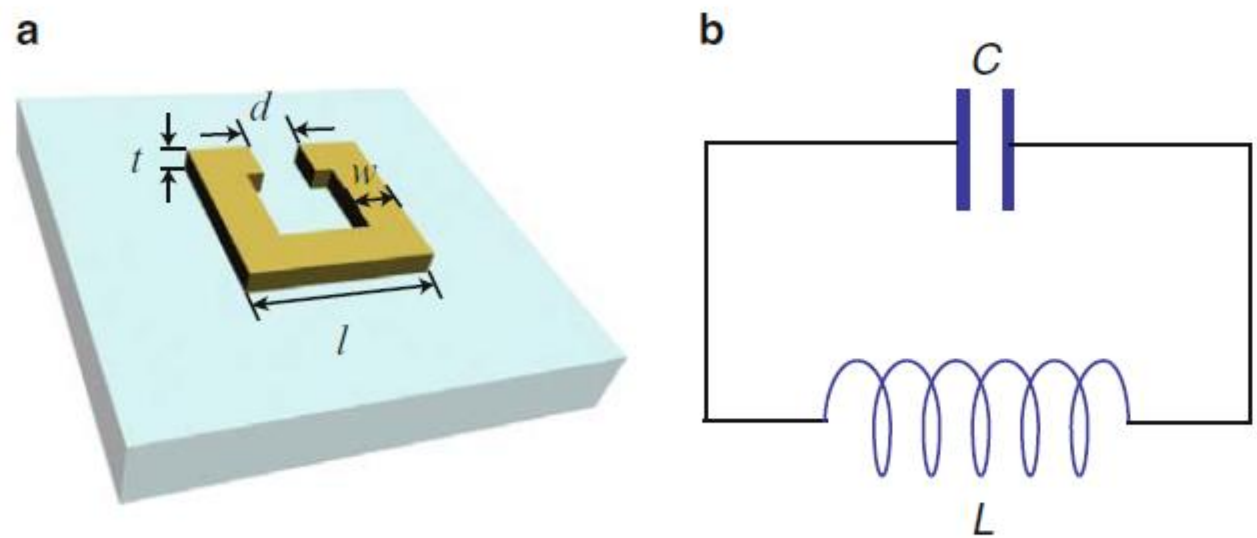
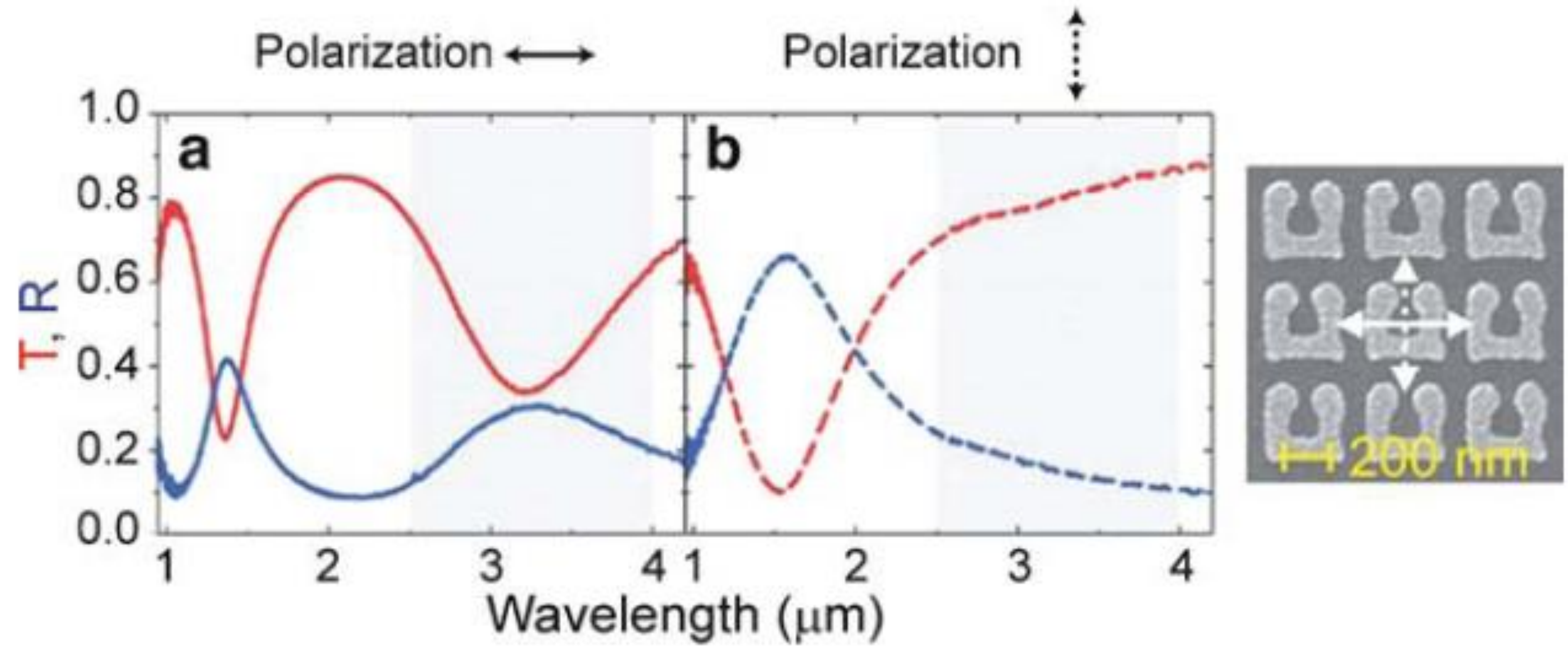
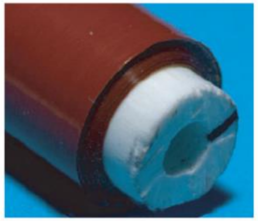
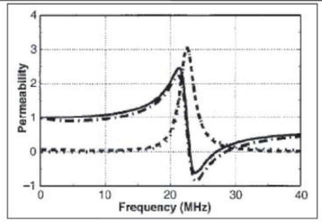
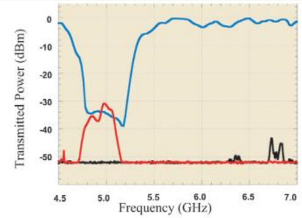
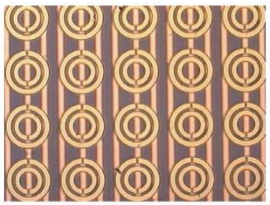
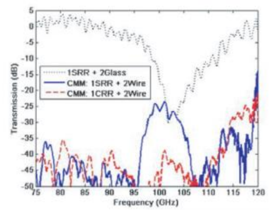
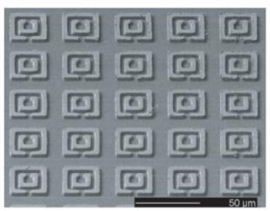
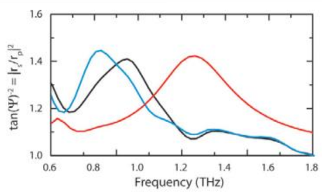
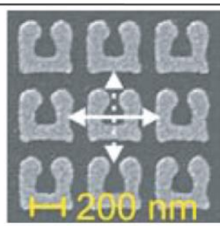
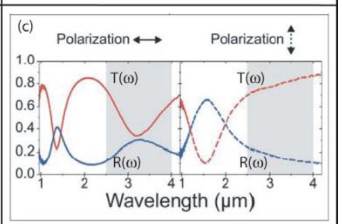
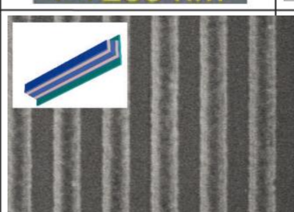
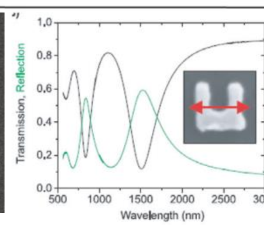


Fig. 5.4 (**a**) A single-SRR as a magnetic element and (**b**) its equivalent *LC* circuit

IR

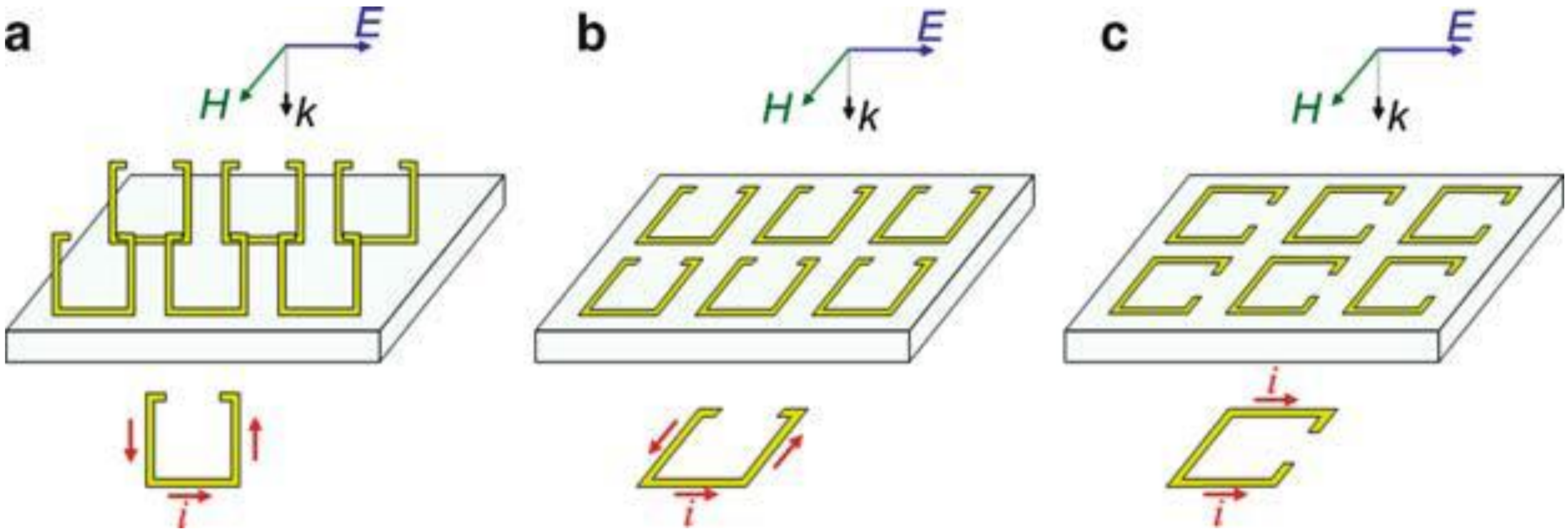


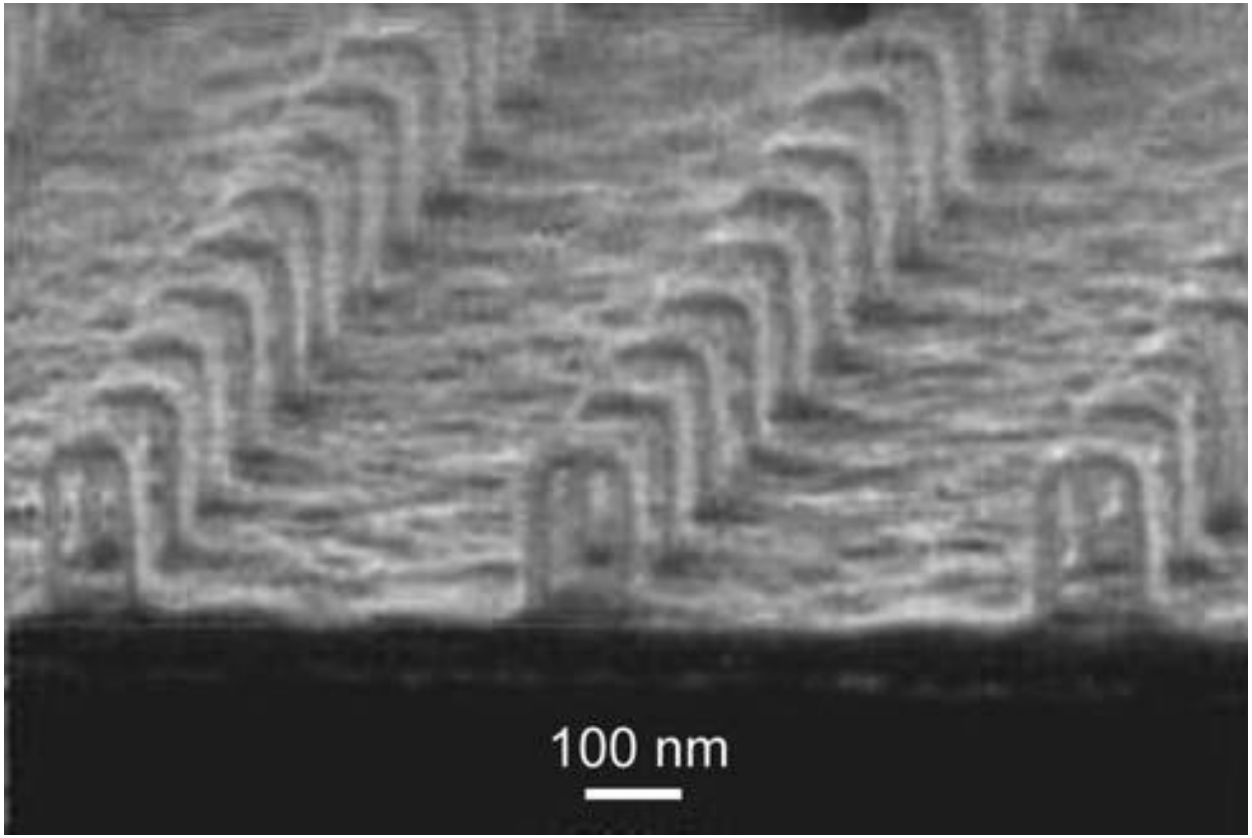
Linden S, Enkrich C, Wegener M, Zhou JF, Koschny T, Soukoulis CM (2004) Magnetic response of metamaterials at 100 terahertz. Science 306:1351–1353

Radio Frequency 21 MHz Ref. [5]			Wiltshire, M. C. K., et al., Science (2001) 291, 849
Microwave Frequency 5 GHz Ref. [1]			Smith, D. R., et al., Phys. Rev. Lett. (2000) 84, 4184
mm-Wave 100 GHz Ref. [6]			www.nanotechnology.bilkent.edu.tr/research%20areas/documents/mm-waveleft-handed.htm
Terahertz 1 THz Ref. [7]			Yen, T. J., et al., Science (2004) 303, 1494
Mid Infrared 100 THz Ref. [8]			Linden, S., et al., Science (2004) 306, 1351
λ ≈ 725 nm			Yuan, H.-K. *et al. A negative permeability material at red light. Preprint at <http://arxiv.org/abs/* physics/0610118> (2006)

Mat. Today, 9, 28 (2006)

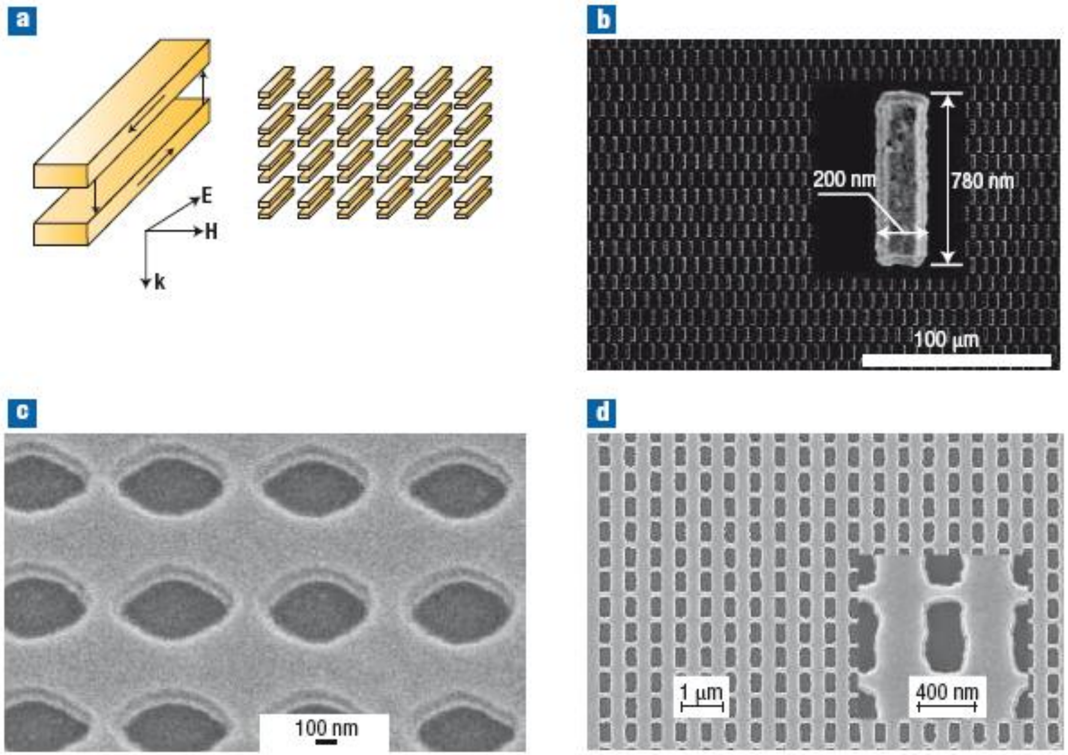
Orientation matters





PRL 94, 037402 (2005)

Visible spectral range – beyond split ring resonator



These structures work for normal incident light
Issue: the artificial magnetic "atoms" are still large.

Opt. Lett. **30,** 3356 (2005); *Opt.Lett.* **31,** 1800(2006); Nature Photonics, v.1, 41-48 (2007).

Wave propagation in anisotropic and bianisotropic media, & kDB system

Maxwell equations:

$$\nabla \times \overline{E} = i\omega \overline{B}$$
$$\nabla \times \overline{H} = -i\omega \overline{D}$$
$$\nabla \cdot \overline{B} = 0$$
$$\nabla \cdot \overline{D} = 0$$

In the case of plane waves:

$$\overline{k} \times \overline{E} = \omega \overline{B} \tag{3.3.6}$$
$$\overline{k} \times \overline{H} = -\omega \overline{D} \tag{3.3.7}$$
$$\overline{k} \cdot \overline{B} = 0 \tag{3.3.8}$$
$$\overline{k} \cdot \overline{D} = 0 \tag{3.3.9}$$

We see from (3.3.8) and (3.3.9) that the complex vectors $\overline{D}$ and $\overline{B}$ are always perpendicular to the wave vector $\overline{k}$. Let me call this plane, which is perpendicular to $\overline{k}$ and contains both $\overline{D}$ and $\overline{B}$, the DB plane, and establish a coordinate system formed by the three vectors $\overline{k}$, $\overline{D}$, and $\overline{B}$, the kDB system, noticing that $\overline{D}$ and $\overline{B}$ may not be perpendicular to each other. Since $\overline{E}$ and $\overline{H}$ may not lie on the DB plane, the Poynting vector is in the direction of $\overline{E} \times \overline{H}$ is not necessarily in the same direction of $\overline{k}$ inside an anisotropic medium. Thus the direction of power flow of a plane wave may not always be in the direction of the wave vector $\overline{k}$. To facilitate the study

kDB system

To facilitate the discussion on the propagation of light, people establish a convenient coordinate system called kDB system, which consists of k vector and the DB plane.
Here, we let **e3** be in the direction o**f k.** form perspective of the xyz coordinate system , we have:

$$\hat{e}_3 = \hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

Assume e2 is in the theta direction, we have

$$\hat{e}_2 = \hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$$

Since e1 is perpendicular to both e2 and e3,

$$\hat{e}_1 = \hat{e}_2 \times \hat{e}_3 = \hat{x}\sin\phi - \hat{y}\cos\phi$$

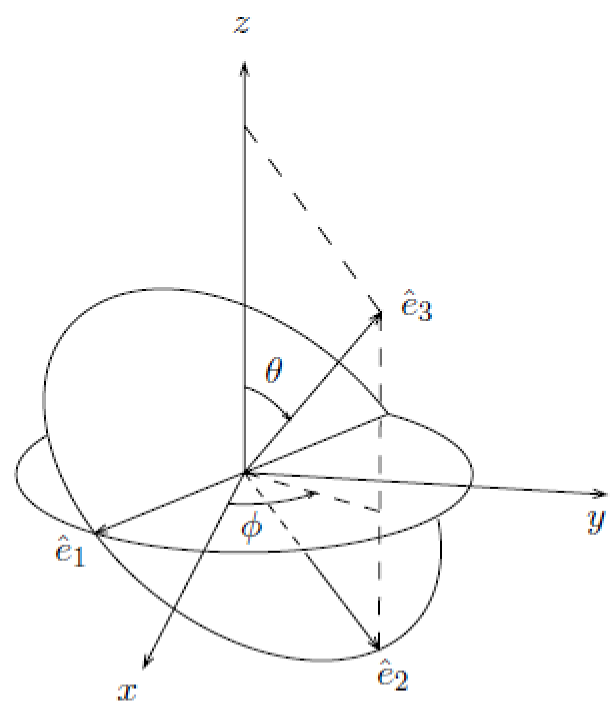


Figure 3.3.1 The kDB system.

Transformation between different coordinate systems

Consider a vector **A, in xyz system:** $\overline{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$ **and in kDB system:** $\overline{A}_k = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$

The two representations are linked by a transformation matrix T $\overline{A}_k = \overline{\overline{T}} \cdot \overline{A}$

Since $\overline{A}$ and $\overline{A}_k$ denote the same vector, we find

$$\begin{aligned} A_1 &= \hat{e}_1 \cdot \overline{A} = \hat{e}_1 \cdot \hat{x} A_x + \hat{e}_1 \cdot \hat{y} A_y + \hat{e}_1 \cdot \hat{z} A_z \\ &= \sin\phi A_x - \cos\phi A_y \\ A_2 &= \hat{e}_2 \cdot \overline{A} = \hat{e}_2 \cdot \hat{x} A_x + \hat{e}_2 \cdot \hat{y} A_y + \hat{e}_2 \cdot \hat{z} A_z \\ &= \cos\theta\cos\phi A_x + \cos\theta\sin\phi A_y - \sin\theta A_z \\ A_3 &= \hat{e}_3 \cdot \overline{A} = \hat{e}_3 \cdot \hat{x} A_x + \hat{e}_3 \cdot \hat{y} A_y + \hat{e}_3 \cdot \hat{z} A_z \\ &= \sin\theta\cos\phi A_x + \sin\theta\sin\phi A_y + \cos\theta A_z \end{aligned}$$

Transformation between different coordinate systems

Consider a vector **A, in xyz system:** $\overline{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$ **and in kDB system:** $\overline{A}_k = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$

The two representations are linked by a transformation matrix T $\overline{A}_k = \overline{\overline{T}} \cdot \overline{A}$

After rearrangement of the equations:

$$\overline{\overline{T}} = \begin{pmatrix} \sin\phi & -\cos\phi & 0 \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix}$$

$$\overline{\overline{T}}^{-1} = \begin{pmatrix} \sin\phi & \cos\theta\cos\phi & \sin\theta\cos\phi \\ -\cos\phi & \cos\theta\sin\phi & \sin\theta\sin\phi \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

Transformation of tensors

In xyz system, the constitutive relations are:

$$\overline{E} = \overline{\overline{\kappa}} \cdot \overline{D} + \overline{\overline{\chi}} \cdot \overline{B}$$
$$\overline{H} = \overline{\overline{\nu}} \cdot \overline{B} + \overline{\overline{\gamma}} \cdot \overline{D}$$

Utilizing relation $\overline{E} = \overline{\overline{T}}^{-1} \cdot \overline{E}_k$, and transformation relations for D, B, and H yields:

$$\overline{E}_k = (\overline{\overline{T}} \cdot \overline{\overline{\kappa}} \cdot \overline{\overline{T}}^{-1}) \cdot \overline{D}_k + (\overline{\overline{T}} \cdot \overline{\overline{\chi}} \cdot \overline{\overline{T}}^{-1}) \cdot \overline{B}_k$$
$$\overline{H}_k = (\overline{\overline{T}} \cdot \overline{\overline{\nu}} \cdot \overline{\overline{T}}^{-1}) \cdot \overline{B}_k + (\overline{\overline{T}} \cdot \overline{\overline{\gamma}} \cdot \overline{\overline{T}}^{-1}) \cdot \overline{D}_k$$

We thus obtain

$$\overline{\overline{\kappa}}_k = \overline{\overline{T}} \cdot \overline{\overline{\kappa}} \cdot \overline{\overline{T}}^{-1}$$
$$\overline{\overline{\chi}}_k = \overline{\overline{T}} \cdot \overline{\overline{\chi}} \cdot \overline{\overline{T}}^{-1}$$
$$\overline{\overline{\nu}}_k = \overline{\overline{T}} \cdot \overline{\overline{\nu}} \cdot \overline{\overline{T}}^{-1}$$
$$\overline{\overline{\gamma}}_k = \overline{\overline{T}} \cdot \overline{\overline{\gamma}} \cdot \overline{\overline{T}}^{-1}$$

such that in the kDB system

$$\overline{E}_k = \overline{\overline{\kappa}}_k \cdot \overline{D}_k + \overline{\overline{\chi}}_k \cdot \overline{B}_k$$
$$\overline{H}_k = \overline{\overline{\nu}}_k \cdot \overline{B}_k + \overline{\overline{\gamma}}_k \cdot \overline{D}_k$$

Maxwell Equations in kDB system

$$\overline{k} \times \overline{E}_k = \omega \overline{B}_k$$
$$\overline{k} \times \overline{H}_k = -\omega \overline{D}_k$$
$$\overline{k} \cdot \overline{B}_k = 0$$
$$\overline{k} \cdot \overline{D}_k = 0$$

$$\overline{k} = \hat{e}_3 k$$

$$\omega B_2 = kE_1 = k(\kappa_{11} D_1 + \kappa_{12} D_2 + \chi_{11} B_1 + \chi_{12} B_2)$$
$$\omega B_1 = -kE_2 = -k(\kappa_{21} D_1 + \kappa_{22} D_2 + \chi_{21} B_1 + \chi_{22} B_2)$$
$$\omega D_2 = -kH_1 = -k(\nu_{11} B_1 + \nu_{12} B_2 + \gamma_{11} D_1 + \gamma_{12} D_2)$$
$$\omega D_1 = kH_2 = k(\nu_{21} B_1 + \nu_{22} B_2 + \gamma_{21} D_1 + \gamma_{22} D_2)$$

Maxwell Equations in kDB system

$$\overline{k} \times \overline{E}_k = \omega \overline{B}_k$$
$$\overline{k} \times \overline{H}_k = -\omega \overline{D}_k$$
$$\overline{k} \cdot \overline{B}_k = 0$$
$$\overline{k} \cdot \overline{D}_k = 0$$

$$\overline{k} = \hat{e}_3 k$$

divide both sides by k and let $u = \omega / k$

$$\boxed{\begin{aligned} \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} &= - \begin{pmatrix} \chi_{11} & \chi_{12} - u \\ \chi_{21} + u & \chi_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \\ \begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} &= - \begin{pmatrix} \gamma_{11} & \gamma_{12} + u \\ \gamma_{21} - u & \gamma_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \end{aligned}}$$

Wave in isotropic media

The constitutive relations for isotropic media in the $\overline{D}\,\overline{B}$ representation can be written as

$$\overline{E} = \kappa\overline{D}$$
$$\overline{H} = \nu\overline{B}$$

where $\kappa = 1/\epsilon$ is called the impermittivity and $\nu = 1/\mu$ is called the impermeability.

In the kDB system, we find that

$$\overline{E}_k = \kappa\overline{D}_k$$
$$\overline{H}_k = \nu\overline{B}_k$$

Substitute the above equations in the dispersion relation:

$$\kappa \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$\nu \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 & -u \\ u & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

$$\boxed{\begin{aligned} \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} &= -\begin{pmatrix} \chi_{11} & \chi_{12} - u \\ \chi_{21} + u & \chi_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \\ \begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} &= -\begin{pmatrix} \gamma_{11} & \gamma_{12} + u \\ \gamma_{21} - u & \gamma_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \end{aligned}}$$

Eliminating $\overline{B}_k$ from the above two equations yields

$$\begin{pmatrix} u^2 - \kappa\nu & 0 \\ 0 & u^2 - \kappa\nu \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

Wave in isotropic media

Eliminating $\overline{B}_k$ from the above two equations yields

$$\begin{pmatrix} u^2-\kappa\nu & 0 \\ 0 & u^2-\kappa\nu \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

To satisfy the above equation, we have following cases:

(A) $D_1 = D_2 = 0$
(B) $D_1 \neq 0$ and $D_2 = 0\,,\quad u^2-\nu\kappa = 0$
(C) $D_1 = 0$ and $D_2 \neq 0\,,\quad u^2-\nu\kappa = 0$
(D) $D_1 \neq 0$ and $D_2 \neq 0\,,\quad u^2-\nu\kappa = 0$

In order to have non-zero $\overline{D}_k$, we must have the dispersion relation

$$u^2-\nu\kappa = 0$$

Case (A) is a trivial case with zero field. Case (B) is a plane wave linearly polarized in the $\hat{e}_1$ direction and Case (C) is a plane wave linearly polarized in the $\hat{e}_2$ direction. Case (D) represents a plane wave of any polarization.

Uniaxial case

In the principal, xyz, coordinate system of a uniaxial medium the constitutive relations under $\overline{D}\,\overline{B}$ representation are

$$\overline{E} = \overline{\overline{\kappa}} \cdot \overline{D} \tag{3.3.42}$$
$$\overline{H} = \nu \overline{B} \tag{3.3.43}$$

where

$$\overline{\overline{\kappa}} = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & \kappa_z \end{pmatrix} \tag{3.3.44}$$

is called the impermittivity tensor. The optic axis is in the $\hat{z}$ direction. In terms of the permittivity tensor

$$\overline{\overline{\epsilon}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix} \tag{3.3.45}$$

we find that $\kappa = 1/\epsilon$ and $\kappa_z = 1/\epsilon_z$, since $\overline{\overline{\kappa}} = \overline{\overline{\epsilon}}^{-1}$. The impermeability is related to the permeability μ by $\nu = 1/\mu$. Transforming to the kDB system, we find from (3.3.25), that

$$\overline{\overline{\kappa}}_k = \overline{\overline{T}} \cdot \overline{\overline{\kappa}} \cdot \overline{\overline{T}}^{-1} = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa \cos^2\theta + \kappa_z \sin^2\theta & (\kappa - \kappa_z)\sin\theta\cos\theta \\ 0 & (\kappa - \kappa_z)\sin\theta\cos\theta & \kappa\sin^2\theta + \kappa_z\cos^2\theta \end{pmatrix} \tag{3.3.46}$$

Since the uniaxial medium has cylindrical symmetry around the z-axis, the transformed relation is ϕ-independent as one should expect. Applying

Uniaxial case

Applying relation

$$\boxed{\begin{aligned} \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} &= -\begin{pmatrix} \chi_{11} & \chi_{12} - u \\ \chi_{21} + u & \chi_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \\ \begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} &= -\begin{pmatrix} \gamma_{11} & \gamma_{12} + u \\ \gamma_{21} - u & \gamma_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \end{aligned}}$$

we have:

$$\begin{pmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$\nu \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 & -u \\ u & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

Eliminating B, we obtain:

$$\begin{pmatrix} u^2 - \nu\kappa_{11} & 0 \\ 0 & u^2 - \nu\kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

To satisfy above equations, we need:

$$\begin{array}{ll} \text{(A)} \ D_1 = D_2 = 0 & \\ \text{(B)} \ D_1 \neq 0 \ \text{and} \ D_2 = 0\,, & u^2 - \nu\kappa_{11} = 0 \\ \text{(C)} \ D_1 = 0 \ \text{and} \ D_2 \neq 0\,, & u^2 - \nu\kappa_{22} = 0 \\ \text{(D)} \ D_1 \neq 0 \ \text{and} \ D_2 \neq 0\,, & u^2 - \nu\kappa_{11} = u^2 - \nu\kappa_{22} = 0 \end{array}$$

Uniaxial case

To satisfy above equations, we need:

$$
\begin{array}{ll}
\text{(A)}\ D_1 = D_2 = 0 & \\
\text{(B)}\ D_1 \neq 0 \ \text{and}\ D_2 = 0\,, & u^2 - \nu\kappa_{11} = 0 \\
\text{(C)}\ D_1 = 0 \ \text{and}\ D_2 \neq 0\,, & u^2 - \nu\kappa_{22} = 0 \\
\text{(D)}\ D_1 \neq 0 \ \text{and}\ D_2 \neq 0\,, & u^2 - \nu\kappa_{11} = u^2 - \nu\kappa_{22} = 0
\end{array}
$$

a. Ordinary and Extraordinary Waves

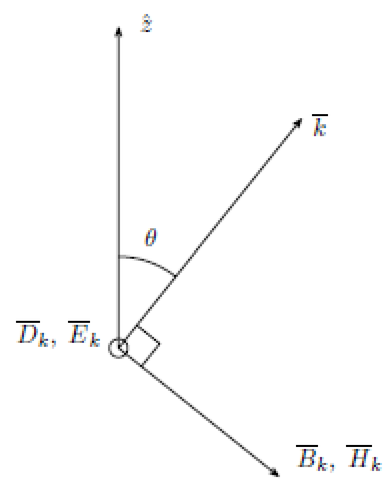
Case (B) corresponds to a wave which is linearly polarized in the $\hat{e}_1$ direction. Notice from Figure 3.3.2 that $\hat{e}_1$ is perpendicular to the plane formed by the optic axis and the $\overline{k}$ vector. This linearly polarized plane wave propagates with the phase velocity

$$u = \pm\sqrt{\nu\kappa_{11}}$$

$$\begin{pmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$\nu \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 & -u \\ u & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

$$\overline{D}_k = \hat{e}_1 D_1$$
$$\overline{B}_k = \hat{e}_2 \frac{u}{\nu} D_1$$
$$\overline{H}_k = \hat{e}_2\, u D_1$$
$$\overline{E}_k = \hat{e}_1\, \kappa D_1$$



Thus $\overline{D}_k$ and $\overline{E}_k$ are in the same direction, as are $\overline{B}_k$ and $\overline{H}_k$

We call this the ordinary wave in the uniaxial medium.

Uniaxial case

To satisfy above equations, we need:

$$\begin{aligned}
&\text{(A)}\ \ D_1 = D_2 = 0 \\
&\text{(B)}\ \ D_1 \neq 0 \ \text{ and } \ D_2 = 0\,, \quad u^2 - \nu\kappa_{11} = 0 \\
&\text{(C)}\ \ D_1 = 0 \ \text{ and } \ D_2 \neq 0\,, \quad u^2 - \nu\kappa_{22} = 0 \\
&\text{(D)}\ \ D_1 \neq 0 \ \text{ and } \ D_2 \neq 0\,, \quad u^2 - \nu\kappa_{11} = u^2 - \nu\kappa_{22} = 0
\end{aligned}$$

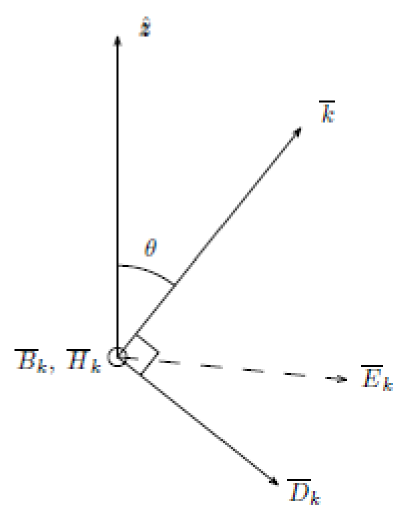
For case (C), the wave is linearly polarized in the $\hat{e}_2$ direction. Remember that $\hat{e}_2$ lies in the plane determined by the optic axis and the $\overline{k}$ vector and is perpendicular to the $\overline{k}$ vector. This linearly polarized wave propagates with the phase velocity

$$u = \pm\sqrt{\nu\kappa_{22}} = \pm[\nu(\kappa\cos^2\theta + \kappa_z\sin^2\theta)]^{1/2}$$

$$\begin{pmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$\nu \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 & -u \\ u & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

$$\begin{aligned}
\overline{D}_k &= \hat{e}_2\, D_2 \\
\overline{B}_k &= -\hat{e}_1\, \frac{u}{\nu} D_2 \\
\overline{H}_k &= -\hat{e}_1\, u D_2 \\
\overline{E}_k &= \hat{e}_2\, \kappa_{22} D_2 + \hat{e}_3\, (\kappa - \kappa_z) \sin\theta\cos\theta\, D_2
\end{aligned}$$



Extraordinary wave in a positive uniaxial medium.

Plane waves in Gyrotropic media

As another example of the application of the kDB system to the solution of characteristic waves inside homogeneous media, consider a gyrotropic medium possessing the following constitutive relations:

$$\overline{H} = \nu \overline{B}$$
$$\overline{E} = \overline{\overline{\kappa}} \cdot \overline{D}$$

where

$$\overline{\overline{\kappa}} = \begin{pmatrix} \kappa & i\kappa_g & 0 \\ -i\kappa_g & \kappa & 0 \\ 0 & 0 & \kappa_z \end{pmatrix}$$

We transform the constitutive matrices to the kDB system We find $\nu_k = \nu$ and

$$\begin{aligned} \overline{\overline{\kappa}}_k &= \overline{\overline{T}} \cdot \overline{\overline{\kappa}} \cdot \overline{\overline{T}}^{-1} \\ &= \begin{pmatrix} \kappa & i\kappa_g \cos\theta & i\kappa_g \sin\theta \\ -i\kappa_g \cos\theta & \kappa \cos^2\theta + \kappa_z \sin^2\theta & (\kappa - \kappa_z)\sin\theta\cos\theta \\ -i\kappa_g \sin\theta & (\kappa - \kappa_z)\sin\theta\cos\theta & \kappa\sin^2\theta + \kappa_z\cos^2\theta \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = -\begin{pmatrix} \chi_{11} & \chi_{12} - u \\ \chi_{21} + u & \chi_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$
$$\begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = -\begin{pmatrix} \gamma_{11} & \gamma_{12} + u \\ \gamma_{21} - u & \gamma_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

$$\begin{pmatrix} \kappa & i\kappa_g \cos\theta \\ -i\kappa_g \cos\theta & \kappa\cos^2\theta + \kappa_z \sin^2\theta \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$\nu \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 & -u \\ u & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$$

Eliminating $\overline{B}_k$ yields

$$\begin{pmatrix} u^2 - \nu\kappa & -i\nu\kappa_g \cos\theta \\ i\nu\kappa_g\cos\theta & u^2 - \nu(\kappa\cos^2\theta + \kappa_z \sin^2\theta) \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

Plane waves in Gyrotropic media

For nontrivial solutions for $\overline{D}_k$, we set the determinant of the 2×2 matrix equal to zero and obtain

$$u^2 = \frac{\nu}{2}\left[\kappa(1+\cos^2\theta) + \kappa_z \sin^2\theta \pm \sqrt{(\kappa-\kappa_z)^2 \sin^4\theta + 4\kappa_g^2 \cos^2\theta}\right]$$

In terms of the components of the $\overline{k}$ vector, we have

$$\omega^2 = \frac{\nu}{2}\left[\kappa(k^2+k_z^2) + \kappa_z k_s^2 \pm \sqrt{(\kappa-\kappa_z)^2 k_s^4 + 4\kappa_g^2 k_z^2 k^2}\right]$$

This is the dispersion relation relating ω and $\overline{k}$.

The two components of the field vector $\overline{D}_k$ are related by

$$\frac{D_2}{D_1} = \frac{-2i\kappa_g \cos\theta}{(\kappa-\kappa_z)\sin^2\theta \pm \sqrt{(\kappa-\kappa_z)^2 \sin^4\theta + 4\kappa_g^2\cos^2\theta}}$$

we define an angle ψ such that $\tan 2\psi = \dfrac{2\kappa_g \cos\theta}{(\kappa-\kappa_z)\sin^2\theta}$

for the characteristic wave with the phase velocity u having the plus sign $\dfrac{D_2}{D_1} = -i\tan\psi$ Type I wave

For the characteristic wave with the phase velocity u having the minus sign $\dfrac{D_2}{D_1} = i\cot\psi$ Type II wave

Plane waves in Gyrotropic media

When the wave propagation direction is along $\hat{z}$, we have $\theta = 0$ and $\begin{pmatrix} u^2 - \nu\kappa & -i\nu\kappa_g \\ i\nu\kappa_g & u^2 - \nu\kappa \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$

// $\begin{pmatrix} u^2 - \nu\kappa & -i\nu\kappa_g \cos\theta \\ i\nu\kappa_g \cos\theta & u^2 - \nu(\kappa\cos^2\theta + \kappa_z \sin^2\theta) \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$

The magnitudes of the phase velocity are seen to be

$$u = \omega/k = \sqrt{\nu(\kappa \pm \kappa_g)}$$

and the ratio of the components of $\overline{D}_k$ is

$$\frac{D_2}{D_1} = \mp i$$

Thus, both characteristic waves are circularly polarized. The Type I wave has a velocity $(\nu\kappa + \nu\kappa_g)^{1/2}$ with

$$k^I = \omega/\sqrt{\nu(\kappa + \kappa_g)}$$

and the Type II wave has a velocity $(\nu\kappa - \nu\kappa_g)^{1/2}$ with

$$k^{II} = \omega/\sqrt{\nu(\kappa - \kappa_g)}$$

Plane waves in Gyrotropic media ——Faraday rotation

Consider a linearly polarized plane wave entering a gyrotropic medium along the $\hat{z}$ direction, neglecting reflections at the boundaries. The incoming wave is decomposed into two circularly polarized waves propagating at different velocities. We let

$$\overline{D} = \hat{e}_1 D_o = \frac{D_o}{2}(\hat{e}_1 + \hat{e}_2 i) + \frac{D_o}{2}(\hat{e}_1 - \hat{e}_2 i)$$

where D_o is a real number. After traveling a distance z_0 inside the medium, the two waves are phase-shifted by different amounts,

$$\begin{aligned} \overline{D} &= \frac{D_o}{2}(\hat{e}_1 + \hat{e}_2 i)e^{i\phi_{\mathrm{II}}} + \frac{D_o}{2}(\hat{e}_1 - \hat{e}_2 i)e^{i\phi_{\mathrm{I}}} \\ &= \hat{e}_1 \frac{D_o}{2}(e^{i\phi_{\mathrm{II}}} + e^{i\phi_{\mathrm{I}}}) + \hat{e}_2 \frac{iD_o}{2}(e^{i\phi_{\mathrm{II}}} - e^{i\phi_{\mathrm{I}}}) \end{aligned}$$

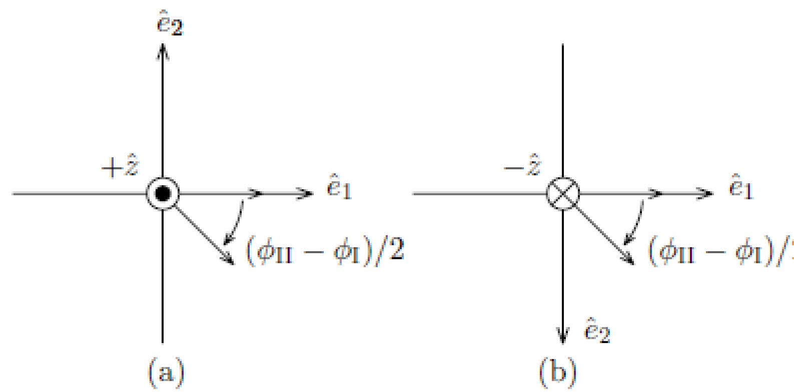
where

$$\phi_{\mathrm{I}} = \frac{\omega z_0}{\sqrt{\nu(\kappa + \kappa_g)}} = k^I z_0$$

$$\phi_{\mathrm{II}} = \frac{\omega z_0}{\sqrt{\nu(\kappa - \kappa_g)}} = k^I I z_0$$

For the ratio of the two components of $\overline{D}_k$, we find

$$\frac{D_2}{D_1} = i\frac{e^{i\phi_{\mathrm{II}}} - e^{i\phi_{\mathrm{I}}}}{e^{i\phi_{\mathrm{II}}} + e^{i\phi_{\mathrm{I}}}} = -\tan\frac{(\phi_{\mathrm{II}} - \phi_{\mathrm{I}})}{2}$$



Faraday rotation.

Plane waves in bianisotropic media

Consider bianisotropic media with the following constitutive relations:

$$\overline{E} = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & \kappa_z \end{pmatrix} \cdot \overline{D} + \begin{pmatrix} \chi & 0 & 0 \\ 0 & \chi & 0 \\ 0 & 0 & \chi_z \end{pmatrix} \cdot \overline{B}$$

$$\overline{H} = \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma_z \end{pmatrix} \cdot \overline{D} + \begin{pmatrix} \nu & 0 & 0 \\ 0 & \nu & 0 \\ 0 & 0 & \nu_z \end{pmatrix} \cdot \overline{B}$$

In the kDB system, the constitutive matrix $\overline{\overline{\kappa}}_k$ becomes

$$\overline{\overline{\kappa}}_k = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa\cos^2\theta + \kappa_z\sin^2\theta & (\kappa-\kappa_z)\sin\theta\cos\theta \\ 0 & (\kappa-\kappa_z)\sin\theta\cos\theta & \kappa\sin^2\theta + \kappa_z\cos^2\theta \end{pmatrix}$$

A similar form holds for the other matrices, $\overline{\overline{\chi}}_k$, $\overline{\overline{\gamma}}_k$ and $\overline{\overline{\nu}}_k$.

$$\begin{pmatrix} \kappa_\theta(u^2\nu + \nu_\theta\chi\gamma - \kappa\nu\nu_\theta) & u\kappa_\theta(\nu_\theta\chi - \nu\gamma_\theta) \\ u\kappa(\nu_\theta\gamma - \nu\chi_\theta) & \kappa(u^2\nu_\theta + \nu\chi_\theta\gamma_\theta - \kappa_\theta\nu\nu_\theta) \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

where we use this short notation:

$$\kappa_\theta = \kappa\cos^2\theta + \kappa_z\sin^2\theta$$
$$\nu_\theta = \nu\cos^2\theta + \nu_z\sin^2\theta$$
$$\chi_\theta = \chi\cos^2\theta + \chi_z\sin^2\theta$$
$$\gamma_\theta = \gamma\cos^2\theta + \gamma_z\sin^2\theta$$

Plane waves in lossless magnetoelectric medium

Consider a lossless magnetoelectric medium in which $\bar{\bar{\gamma}} = \bar{\bar{\chi}}$ are both real.

$$\begin{pmatrix} \kappa_\theta(u^2\nu + \nu_\theta\chi\gamma - \kappa\nu\nu_\theta) & u\kappa_\theta(\nu_\theta\chi - \nu\gamma_\theta) \\ u\kappa(\nu_\theta\gamma - \nu\chi_\theta) & \kappa(u^2\nu_\theta + \nu\chi_\theta\gamma_\theta - \kappa_\theta\nu\nu_\theta) \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

$\rightarrow$ In the $\hat{z}$ direction $\theta = 0$

$$\begin{pmatrix} u^2 - \kappa\nu + \chi^2 & 0 \\ 0 & u^2 - \kappa\nu + \chi^2 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

This is a degenerate case. The characteristic waves can have any polarization, and the phase velocity is $u^2 = \kappa\nu - \chi^2$. Note that we must have $\kappa\nu > \chi^2$; otherwise the velocity becomes imaginary.

Optical activity

Chiral media possess the following constitutive relations

$$\overline{E} = \kappa \overline{D} + i\chi \overline{B}$$
$$\overline{H} = -i\chi \overline{D} + \nu \overline{B}$$

Letting $\kappa_z = \kappa$, $\nu_z = \nu$, and $\chi_z = \chi$

$$\boxed{\begin{array}{l} \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = -\begin{pmatrix} \chi_{11} & \chi_{12} - u \\ \chi_{21} + u & \chi_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \\ \begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = -\begin{pmatrix} \gamma_{11} & \gamma_{12} + u \\ \gamma_{21} - u & \gamma_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \end{array}}$$

$$\begin{pmatrix} u^2 - \kappa\nu + \chi^2 & i2\chi u \\ -i2\chi u & u^2 - \kappa\nu + \chi^2 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0$$

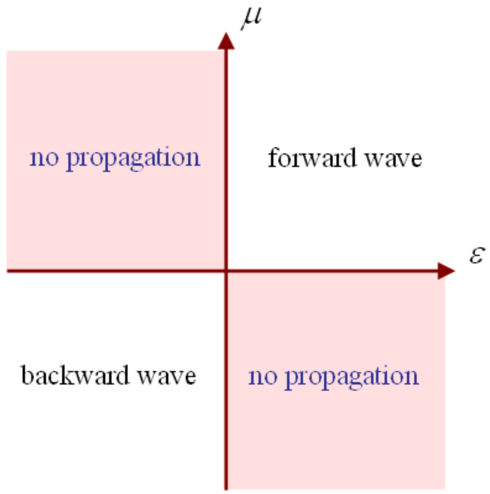
The velocity of the propagation is determined from $u^2 - \kappa\nu + \chi^2 = \pm 2\chi u$, and the corresponding polarization is $D_2/D_1 = \pm i$. Thus both characteristic waves are circularly polarized. The right-hand circularly polarized wave has a velocity $\sqrt{\kappa\nu} + \chi$, and the left-hand circularly polarized wave has a velocity $\sqrt{\kappa\nu} - \chi$.

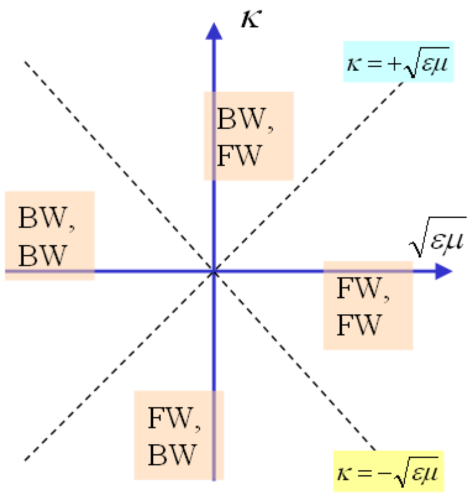
Chiral material

Unlike to the ordinary isotropic materials and double-negative Veselago media, bi-isotropic materials are birefringent: the two eigenwaves propagating in bi-isotropic media have different propagation factors:

$$k_{\pm} = \omega\left(\sqrt{\mu\varepsilon - \chi^2} \pm \kappa\right) \tag{3}$$

Both magnetoelectric parameters χ and κ affect the phase of the wave.[6]





The eigenwaves in homogeneous chiral media are the two circularly polarized (right- and left-handed) waves [10]. Let us assume that the waves are propagating into the direction of the positive z axis. Then their dependence upon the propagation distance is exp(-j$k_\pm$z), with $k_\pm$ being the propagation constant of the right-handed (+) and left-handed (-) eigenwave.

The electric field vectors rotate in the xy plane (unit vectors $\mathbf{u}_x$ and $\mathbf{u}_y$), and as complex vectors they can be written as

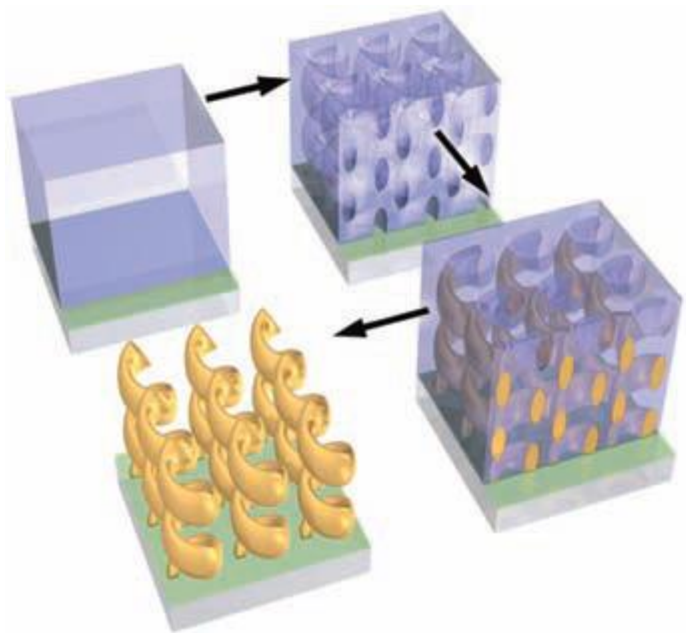
$$\begin{cases} \text{RCP}: \quad \mathbf{u}_\text{x} - \text{j}\mathbf{u}_\text{y}; \quad \exp(-\text{j}k_+ z), \quad k_+ = \omega(\sqrt{\mu\varepsilon} + \kappa) \\ \text{LCP}: \quad \mathbf{u}_\text{x} + \text{j}\mathbf{u}_\text{y}; \quad \exp(-\text{j}k_- z), \quad k_- = \omega(\sqrt{\mu\varepsilon} - \kappa) \end{cases} \tag{4}$$

If the wave is linearly x polarized at $z = 0$, it is the sum of RCP and LCP, both of equal amplitudes. For positive κ, the wave number of RCP is larger than that of LCP. Hence its phase changes faster. The result is that at a distance into positive z, the vector direction of the electric field is

$$\begin{aligned} &(\mathbf{u}_\text{x} - \text{j}\mathbf{u}_\text{y})\exp(-\text{j}k_+ z) + (\mathbf{u}_\text{x} - \text{j}\mathbf{u}_\text{y})\exp(-\text{j}k_- z) \\ &\quad = 2\left[\mathbf{u}_\text{x}\cos(\omega\kappa z) - \mathbf{u}_\text{y}\sin(\omega\kappa z)\right]\exp(-\text{j}\omega\sqrt{\mu\varepsilon}\, z) \end{aligned} \tag{5}$$

from which it can be seen that at position $z = 0$, the field vector is x-polarized as assumed.

Experiment



A positive-tone photoresist (blue) is spun onto a glass substrate covered with a 25-nm thin film of conductive indium-tin oxide (ITO) shown in green. After 3D DLW and development, an array of air helices in a block of polymer results. After plating with gold in an electrolyte, the polymer is removed by plasma etching, leading to a square array of freestanding 3D gold helices.

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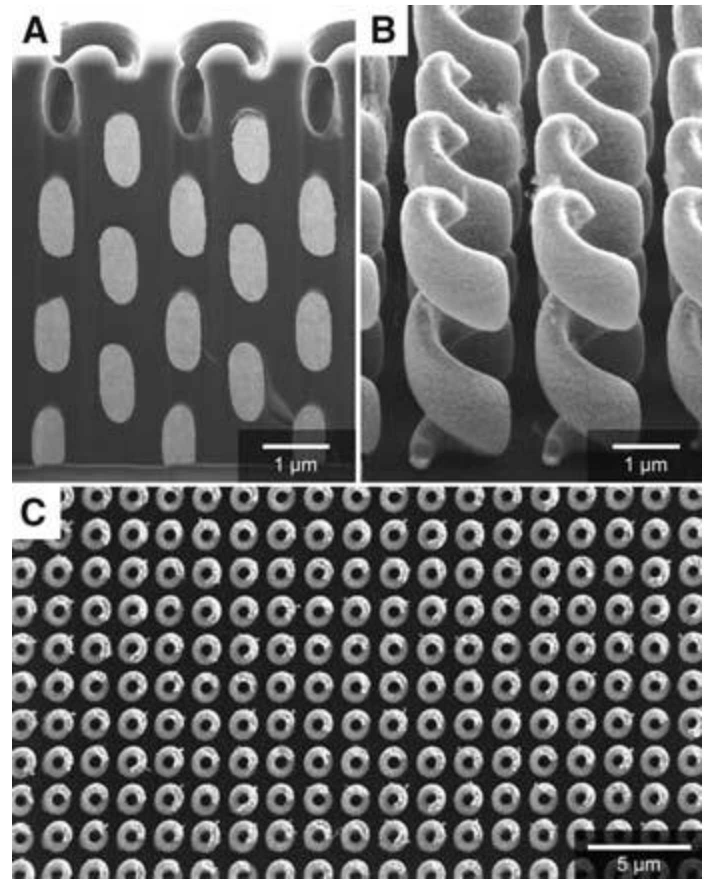

(A) Focused-ion-beam (FIB) cut of a polymer structure partially filled with gold by electroplating (compare lower right part of Fig. 2). (B) Oblique view of a left-handed helix structure after removal of the polymer by plasma etching. (C) Top-view image revealing the circular cross section of the helices and the homogeneity on a larger scale. The lattice constant of the square lattice is a = 2 mm.