

GRE 数学讲义-直播二

三. 算术（下）-非整数相关

1. 分数和小数

（一）分数

相关术语：

fraction 分数

numerator 分子

denominator 分母

inverse 相反数

reciprocal 倒数

A over B B 分之 A

重要考点（1）：分数值的运算

1) 分数的乘除运算

分数乘法规则：分子乘以分子做分子，分母乘以分母做分母

分数除法规则：除以一个分数等于乘以这个分数的倒数

2) 分数的加减运算：通分与约分

通分：把异分母分数变成同分母的过程。（先找出几个分母的最小公倍数,再根据分数的基本性质,分子和分母同时乘上相同的数）

约分：把几个相乘的分式中相同的多项式或单项式约去

重要考点（2）：倒数与相反数

若 $a+b=0$ ，则 a 、 b 互为相反数

若 $a \times b=1$ ，则 a 、 b 互为倒数

补充：

分子不变,分母越大 (>0) ,分数值越小;

当 $0 < x < 1$, $1/x > 1$, x 越小, $1/x$ 越大;

当 $-1 < x < 0$, $1/x < -1$, x 越大, $1/x$ 越小

例 1: A certain brand of dishwashing liquid was sold in two different bottle sizes. The small bottle was sold with $2/5$ as many ounces of liquid as the large bottle and was sold at a price that was $1/2$ the price of the large bottle.

Quantity A: The price per ounce of the liquid in the small bottle

Quantity B: The price per ounce of the liquid in the large bottle

例 2: $1/a + 1/b = 1/c$, $a+b=20$, a, b, c are all positive. The value of c could be?

Indicate all such values.

- A.4
- B.6
- C.8
- D.10
- E.12

例 3: x and y are both integers

$$2 \leq x < y < 7$$

What is the maximum value of $\frac{x+y}{xy}$?

(二) 小数

相关术语:

tenths digit 十分位

hundredths digit 百分位

thousandths digit 千分位

decimal point 小数点

terminating decimal 有限小数

repeating decimal 循环小数

重要考点: (1) 四舍五入

be rounded to the nearest tenths 四舍五入到十分位

Give your answer to the nearest 0.1 保留到十分位

Give your answer to the nearest tenths 保留到十分位

重要考点: (2) 科学计数法

$$20130 = 2.013 \times 10^4$$

$$0.000191 = 1.91 \times 10^{-4}$$

关键点: 看小数点移动了多少位

运算方法:

第一步：确定 a 是整数位只有一位的数

第二步：确定 n ，当小数点向右移的时候， n 取正值；当小数点向左移的时候， n 取负值，小数点移动了几位， n 就是几

重要考点：(3) 如何判断有限小数

分子分母均为整数的分数，要么可以转换成有限小数，要么可以转换成无限循环小数

方法：

第一步：先把分数化简成最简分数

第二步：分母分解质因数，看是否只含有 2 或 5 两种质因数（或者看分母分解质因数之后是否符合三种形式的一种： $2^n, 5^m, 2^n \times 5^m$ ）

重要考点：(4) 无限循环小数与无限不循环小数

循环小数（repeating decimal）：一个数的小数部分从某一位起，一个或几个数字依次重复出现的无限小数叫循环小数

无理数（无限不循环小数） irrational number

例 4：The number $3.2\square\#6$, if \square and $\#$ each represent single digits in the decimal, when the decimal is rounded to the nearest tenths, 3.2 is the result, what digit does \square represent?

- A. 3
- B. 5
- C. 7
- D. 8
- E. 9

例 5：If k is an integer and $(0.0025)(0.025)(0.00025) \times 10^k$ is an integer, what is the least possible value of k ?

- A. -12
- B. -6
- C. 0
- D. 6
- E. 12

例 6：Which of the following fractions has a decimal equivalent that is a terminating decimal?

- A. $\frac{10}{189}$
- B. $\frac{15}{196}$
- C. $\frac{16}{225}$

- D. $\frac{25}{144}$
E. $\frac{39}{128}$

例 7: $0.\underline{abcde}$ is a repeating decimal (a, b, c, d, and e are all different)

$$0.\underline{abcde} = \frac{1}{X} \text{ (X is a positive integer)}$$

Quantity A: X

Quantity B: 83

例 8: n is an integer, and k is not an integer.

$$0 < k < n < k + 2$$

Quantity A: n

Quantity B: k+1

2. 实数

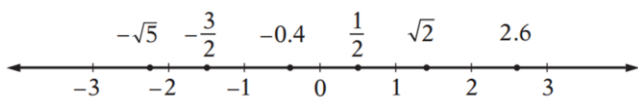
相关术语:

The set of real numbers consists of all rational numbers and all irrational numbers. The real numbers include all integers, fractions, and decimals.

(一) 数轴

相关术语:

number line 数轴 (原点, 正方向, 单位长度)



interval 间隔

注意: 在数轴右边的数字总是大于数轴上它左边的数字

重要考点: 数轴三要素

例 9: On a number line, Point R is between 0 and 1, Point S is between 3 and 4, while Point Q is between -1 and -2

Quantity A: the distance between Q and R

Quantity B: the distance between R and S

(二) 绝对值

相关术语:

absolute value 绝对值

重要考点: (1) 绝对值的性质

- 绝对值一定非负
- $|a+b| \leq |a|+|b|$
- $|a*b| = |a|*|b|$

重要考点: (2) 绝对值相关的方程式

例 10: $s=|t-2|$

Quantity A: $s+2$

Quantity B: $|t|$

例 11: $x \neq 0, y \neq 0$

$$|x|+|y|=|x+y|$$

Which of the following statements must be true?

Indicate all such statements.

- A. $xy < 0$
- B. $xy > 0$
- C. $x+y > 0$
- D. $x+y < 0$

例 12: $|1-|x-250||=1$

What's the number of all the possible values of x ?

例 13: If $x+|x|+y=7$ and $x+|y|-y=6$, then $x+y=$

- A. -1
- B. 1
- C. 3
- D. 5
- E. 13

例 14: What is the sum of all possible solutions of the equation $|x+4|^2-10|x+4|+24=0$?

A. -16

B.-14

C.-12

D.-8

E.-6

3.比例和百分比

(一) 比例

相关术语:

the ratio of s to $t = s$ to $t = s:t = s/t$

重要考点: 比例运算&比例相关的应用题

- 1) 交叉相乘: 若 $a/b = c/d$, 则 $ad=bc$
- 2) 交叉相乘法解等比关系未知数

例 15: A certain bag contains red balls, green balls, and blue balls and no other balls. The ratio of the number of red balls to the number of blue balls is 2:3, and the ratio of the number of blue balls to the number of green balls is 4:3. The number of blue balls in the bag is what fraction of the total number of balls in the bag?

- A.3/8
- B.12/29
- C.7/13
- D.15/23
- E.12/17

(二) 百分比

相关术语:

The term **percent** means per hundred, or hundredths. Percents are ratios that are often used to represent parts of a whole, where the whole is considered as having 100 parts.

【例如】

- 1 percent means 1 part out of 100 parts, or $1/100$
- 32 percent means 32 parts out of 100 parts, or $32/100$
- 50 percent means 50 parts out of 100 parts, or $1/2$

相关术语:

single interest 单利

compound interest 复利

principal 本金

interest rate 利率

Interest 利息

Compounding annually 按年计息 Compounding quarterly 按季度计息

Compounding monthly 按月计息 Compounding daily 按日计息

重要考点:

1) 百分比所属关系的描述 (of)

A is m percent of B: A 是 B 的百分之 m: $A = B \times m\%$

A is half of B : A 是 B 的 1/2: $A = B \times 1/2$

A is m times as many as B: A 是 B 的 m 倍: $A = B \times m$

2) 百分比多少关系的描述

A is m percent greater than B: A 比 B 多百分之 m: $A = B + B \times m\% = B (1 + m\%)$

A is m percent less than B: A 比 B 少百分之 m: $A = B - B \times m\% = B (1 - m\%)$

3) 百分比增减关系的描述

A increased by m percent: A 增长了百分之 m, 增到了 $A + A \times m\% = A (1 + m\%)$

A decreased by m percent: A 减少了百分之 m, 降到了 $A - A \times m\% = A (1 - m\%)$

4) 变化关系的描述

... increase by m percent from A to B: 从 A 到 B, 以 A 为基础, 涨幅百分比 = $(B - A) / A$

... decrease by m percent from A to B: 从 A 到 B, 以 A 为基础, 降幅百分比 = $(A - B) / A$

5) 单利计算公式:

单利利息 = $P \cdot R \cdot T$ (本金 \times 利率 \times 时间)

6) 复利计算公式:

总金额 = 本金 $\times (1 + \text{年利率} / \text{复利频率})^{\text{复利频率} \times \text{时间 (年)}}$

$$P' = P \left(1 + \frac{r}{n} \right)^{nt}$$

复利利息 = 本金 $\times (1 + \text{年利率} / \text{复利频率})^{\text{复利频率} \times \text{时间}} - \text{本金}$

$$I = P \left(1 + \frac{r}{n} \right)^{nt} - P$$

例 16: A pianist agreed to perform one concert at a fee 12.5 percent less than her usual fee and a second concert at a fee 20 percent greater than the first fee. The fee for the second concert was what percent greater than her usual fee?

A.5%

B.7.5%

- C.15%
- D.16.25%
- E.32.5%

例 17: At a certain elementary school, 10 percent of the fifth-grade students are members of the school band. If 12 percent of the fifth-grade boys and 8 percent of the fifth-grade girls are members of the band, what percent of the fifth-grade students at the school are boys?

- A.10%
- B.12%
- C.20%
- D.30%
- E.50%

例 18: At a certain company, employees who earn \$20.00 per hour will be given an increase of \$1.00 per hour. For each of the other employees, either the employee will be given an increase of \$1.00 per hour or the employee will be given a percent increase equal to the percent increase that will be given to the employees who earn \$20.00 per hour, whichever results in a larger increase for that employee. Which of the following statements are true?

Indicate all such statements.

- A. An employee who earns less than \$20.00 per hour will be given a percent increase that is greater than the percent increase that will be given to the employees who earn \$20.00 per hour.
- B. An employee who earns \$22.00 per hour will be given an increase of \$1.10 per hour.
- C. An employee who earns \$24.00 per hour will earn \$25.20 per hour after the increase.

例 19: Last year Leo bought two paintings. This year he sold them for \$2,000 each. On one, he made a 25% profit, and on the other he had a 25% loss. What was his net loss or profit?

- A. He broke even.
- B. He lost less than \$100.
- C. He lost more than \$100.
- D. He earned less than \$100.
- E. He earned more than \$100.

例 20: If A is the initial amount put into an account, R is the annual percentage of interest written as a decimal, and the interest compounds annually, then which of the following would be an expression, in terms of A and R , for the interest accrued in three years?

- A. $A(R)^3$
- B. $A(R+R^3)$
- C. $A(3R+3R^2+R^3)$
- D. $3A(R)^3$
- E. $3A(R+R^2+R^3)$

例 21: Diana invested \$61,293 in an account with a fixed annual percent of interest, compounding quarterly. At the end of five full years, she had \$76,662.25 in principal plus interest. Approximately what was the annual percent rate of interest for this account?

- A. 1.2%
- B. 4.5%
- C. 10%
- D. 18%
- E. 25.2%

例 22: If someone puts away \$92,000 in a bank with an annual interest rate of $r\%$, and earns \$920 in the first month, then what is the value of r ?

Give your answer as percent.

4. 文字应用题

(1) 工作问题:

工作总量 = 工作总时间 * 工作总效率

工作总效率 = 单个人的效率和

如果总工作量没有告知具体数值且总工作量不变的可以把总工作量设为 1 份

(2) 路程问题:

路程(或距离)=速度*时间

①反向运动: 反向运动有两种, 一是两个物体同时同地向相反的方向运动, 二是两个物体同时但相隔一定距离向相反的方向运动。

②同向运动: 亦称“追赶运动”, 两个物体同时同地以不同的速度向相同的方向运动。

(3) 浓度问题:

浓度计算公式:

溶质质量/总重量 $\times 100\%$

A makes up x percent of B: B 当中 A 的浓度为 x%

What percent of B is/are A? B 当中 A 的浓度为多少?

(4) 分段计费问题:

注意看清楚题目上的分段要求

(5) 最大可能值与最小可能值问题:

最大(可能)值 the greatest possible value

最小(可能)值 the least possible value

the greatest possible value = 总和- (其他数据的最小可能之和)

the least possible value = 总和- (其他数据的最大可能之和)

(6) 握手问题:

总净握手次数计算公式= $\frac{\text{总握手人数} \times \text{每人需握手次数}}{2}$

(7) 最少抽取数量问题:

解题思路:

为了确保, 所以需要假设在最坏的情况下, 至少经过多少次才一定能达到预期效果

(8) 人数取整应用题

例 23: Pumps A, B, and C operate at their respective constant rates. Pumps A and B, operating simultaneously, can fill a certain tank in $\frac{6}{5}$ hours; pumps A and C, operating simultaneously, can fill the tank in $\frac{3}{2}$ hours; and pumps B and C, operating simultaneously, can fill the tank in 2 hours. How many hours does it take pumps A, B, and C, operating simultaneously, to fill the tank?

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{5}{6}$

E.1

例 24: A-town and B-ville are connected by a straight, 420-mile road. At noon, Atu left A-town for B-ville, and Brek left B-ville for A-town. If Atu travels at 56 miles per hour and Brek travels at 49 miles per hour, how many miles apart will Atu and Brek be 1 hour before they meet?

例 25: How many ounces of 20% grape juice needs to be added to a bottle of 20-ounce 12% grape juice so that the mixture is 15% grape?

例 26: The taxi fee charged in a certain place is calculated as follows: the starting price is 11 RMB, no extra charge within 2 kilometers, and 3 RMB per kilometer over 2 kilometers. If the fee for a taxi ride is 41 RMB, how many kilometers is the distance travelled?

例 27: Seven pieces of rope have an average (arithmetic mean) length of 68 centimeters and a median length of 84 centimeters. If the length of the longest piece of rope is 14 centimeters more than 4 times the length of the shortest piece of rope, what is the maximum possible length, in centimeters, of the longest piece of rope?

- A.82
- B.118
- C.120
- D.134
- E.152

例 28: Quantity A: The number of different line segments that can be formed when connecting 6

different points on a circle

Quantity B: 15

例 29: Ben has 30 pencils in a box. Each of the pencils is one of 5 different colors, and there are 6 pencils of each color. If Ben selects pencils one at a time from the box without being able to see the pencils, what is the minimum number of pencils that he must select in order to ensure that he selects at least 2 pencils of each color?

A.24

B.25

C.26

D.27

E.28

例 30: There are n positive integers. The sum of the numbers is greater than 50, while the arithmetic average of the numbers is 2.5. What is the least value of n ?