

算术 (上) - 课后练习册

1. Integer x and y which have the same tens digits. The sum of x and y is 87, and $x < y$ (x and y are positive integers). How many distinct pairs of x and y satisfy the above rule?

4

$$\begin{aligned} x &: \overline{AB} > 0 \\ y &: \overline{AB} > 0 \\ x+y &= 87 \\ x &< y \end{aligned}$$

$$\begin{aligned} x &= A \cdot 10 + B \\ y &= A \cdot 10 + C \\ x+y &= (10A+B) + (10A+C) \\ &= 20A+B+C \\ &= 87 \end{aligned}$$

$\therefore 2 \cdot A$
 $\therefore A$ must be 4 (to get 8)
 $\therefore 2 \cdot \text{any } B = \text{odd} \rightarrow \text{得不到 } 7$ (让 $B+C$ 进一位)
 $\therefore B \text{ or } C$ could be 1, 6, 2, 5, 3, 4
 $\therefore 40, 7$

2. In a store, price of small-sized coffee beans is \$ k per bag, while price of larger coffee beans is \$ m per bag. If someone buys four bags of coffee beans in the store at a price of \$30 in total, then k and m could be?

Indicate all such possible combinations.

- A. $k=7$ and $m=8$
 B. $k=5$ and $m=10$
 C. $k=5$ and $m=15$
 D. $k=8$ and $m=20$
 E. $k=10$ and $m=20$

if $m \text{ odd} = \text{even}$
 $\dots = \text{even}$ } $8 \cdot m = \text{odd}$

$$7 \cdot k + 8 \cdot m =$$

if $k \text{ odd} = \text{odd} + \text{odd} = \text{even}$ (could be)
 if $k \text{ even} = \text{even} + \text{even} = \text{even}$ (could be)

$$5 \cdot k + 15 \cdot m =$$

odd = odd
 even = even
 odd + odd = even (could be)

3. At most how many integers less than 25 could be the sum of the positive multiple of 4 and the positive multiple of 5?

- A. 7
 B. 8
 C. 9
 D. 10
 E. 11

$$\begin{aligned} x &< 25 \\ 4 \cdot x + 5 \cdot x & \\ 4 & 8 \quad 12 \quad 16 \quad 20 \quad 24 \\ 5 & 10 \quad 15 \quad 20 \quad 25 \end{aligned}$$

10 { 9, 14, 19, 24
 13, 18, 23
 17, 22
 21

4. When selecting four different integers from -5 to 4, inclusive, what is the least possible product of these four numbers?

-240

$$\begin{aligned} &[-5, 4] \\ &\downarrow \\ &\text{pick } 4 \\ &-5, 4, -4, -3 \\ &-20 \times -4 \times -3 \\ &= 80 \times -3 \\ &= -240 \end{aligned}$$

5. If the sum of 11 consecutive integers is 22, then what's the least of the list of numbers?

A. -6 $\frac{11(-6+4)}{2} = -11$

B. -5 $\frac{11(-5+3)}{2} = -11$

C. -3 $\frac{11(-3+7)}{2} = 22$

D. -2

E. -1

$\begin{matrix} 2 & 2 \\ \swarrow & \searrow \\ x & \cdots & y \\ \underbrace{\hspace{1cm}} \\ 11 \text{ consecutive} \end{matrix}$

$-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$

$\frac{n(a_1+a_n)}{2}$

6. The sum of n consecutive integers is 30

Quantity A: n

$1+2+3+4+5$

Quantity B: 4

其他可能是 9, 10, 11

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

7. The average of n consecutive integers is 0

分数得0 一定分子是0

Quantity A: The sum of the greatest and least value of the n consecutive integers

Quantity B: The sum of all the n consecutive integers

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

8. For positive integers X and Y, whose product is 24, which of following statements MUST be true?

Indicate all such statements.

$x \cdot y = 24$

A. $x+y < 24$ $1 \cdot 24$

B. If one is odd, the other one must be even \checkmark $3 \cdot 8$

C. $(12/y)$ is an integer \checkmark

9. $y=4x^2+5x+7$ where y is an odd, which of the following is true about x ?

A. x is odd

$$y = \text{odd}$$

☒ B. x is even

$$\begin{array}{l} \text{if } x \text{ odd,} \\ x^2 \text{ is odd } \} 4 \cdot \text{odd} = \text{even} \\ 5 \cdot x \text{ is odd } \} \text{odd} \end{array} \quad \begin{array}{l} \text{even} \\ + \\ \text{odd} \end{array} \} \text{odd} + \text{odd} = \text{even}$$

C. x is positive odd

D. x is positive even

E. Cannot be determined

$$\begin{array}{l} x \text{ even} \\ x^2 \text{ even } \} \text{even} \\ 5 \cdot x \text{ even } \} \text{odd} \end{array} \quad \begin{array}{l} \text{even} \\ + \\ \text{odd} \end{array} = \text{odd}$$

☒ 10 Which of the following CANNOT be the value of $k+k^2$ where k is a positive integer?

A. A multiple of 10 \rightarrow even $\} \text{Could be } k(1+k)$

B. A multiple of 12 \rightarrow even $\} \text{odd even}$

☒ C. A number with the units digit as 5 \rightarrow odd

D. A number such that the sum of units digit and tens digit is 9 Could be

E. A number with tens digit as 4

11. m is a positive integer and $5m+9$ is divisible by 4, which of the following must be an odd?

Indicate all such items.

☒ A. $(m+2)(m+4) = \text{odd}$

B. $3-m = \text{even}$ or even

C. $3+m = \text{odd} + \text{odd} = \text{even}$

D. $m^3+9 = \text{even}$

$$\frac{5m+9}{4} = \text{int}$$

$$5m = \text{odd}$$

$$m = \text{odd}$$

$$15+9 = \frac{24}{4}$$

☒ 12. x and y are both integers from 1 to 10, inclusive, and Set S consists of all the possible products of $x \cdot y$.

Quantity A: The number of odd integers in Set S

Quantity B: The number of even integers in Set S

A. Quantity A is greater.

☒ B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

$$[1-10] \quad S [\text{Possible}]$$

$$P(x \text{ is odd}) = 0.5 \quad \text{even} = 0.5$$

$$y \text{ is odd} = 0.5 \quad \text{even} = 0.5$$

$$\therefore \text{only odd times odd give odd}$$

$$0.5 \cdot 0.5 = 0.25$$

13. **Quantity A:** Among all the consecutive integers from 1 to 50, the difference between the sum of all even numbers and the sum of all odd numbers *what meaning?*

$$(1 - 50)$$

Quantity B: 25

A. Quantity A is greater.

B. Quantity B is greater.

☒ C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

14. Which of the following CANNOT be the sum of six consecutive odd integers?

A. -24

☒ B. 42

C. 36

D. 96

E. 120

$$\begin{aligned} \text{Sum} &= na_1 + \frac{n(n-1)}{2}d \\ &= 6a_1 + \frac{6(5)}{2} \cdot 2 \\ &= 6a_1 + 30 \end{aligned}$$

15. a and b are positive integers and $ab=24$

$$\begin{aligned} &4 \cdot 6 \\ &\rightarrow 2 \cdot 12 \\ &\rightarrow 1 \cdot 24 \end{aligned}$$

Quantity A: a^2b

Quantity B: 192

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

☒ D. The relationship cannot be determined from the information given.



16. n, s and t are all positive integers

If n is a multiple of 7 and $n=s^2t$, which of the following must be a multiple of 49?

A. s

B. t

C. st^2

D. s^2t

☒ E. s^2t^2

$$\begin{aligned} &7 \mid n \cdot 1 \\ &n \in 1 \text{ or } 7 \end{aligned}$$

$$n = s^2t$$

$$n = 7m$$

$$7m = s^2t$$

$$1 = s = t$$

$$7 = s^2t$$