

GRE 数学讲义-直播一

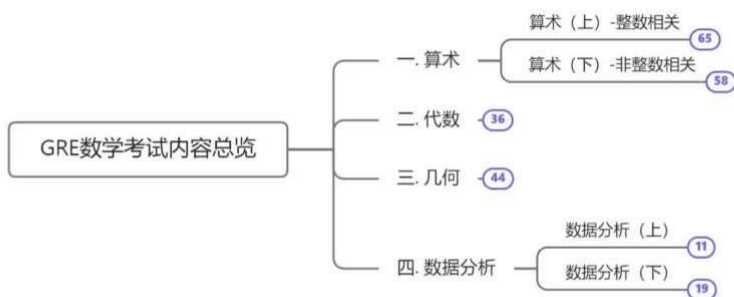
一.前言导学

1. 考试形式:

2 个 section → section I : 12 道题 (共 21 分钟); section II : 15 道题 (共 26 分钟) → 1 分 45 秒/题 → 分数: 130-170

考试特点: 自适应考试

2. 考试内容:



3. 考试题型:

1) 比较大小题【样题如下】

Quantity A	Quantity B
The least prime number greater than 24	The greatest prime number less than 28
(A) Quantity A is greater. (B) Quantity B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.	

2) 单选题【样题如下】

If $5x + 32 = 4 - 2x$, what is the value of x ?

- (A) -4
- (B) -3
- (C) 4
- (D) 7
- (E) 12

3) 不定项选择题【样题如下】



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Which of the following integers are multiples of both 2 and 3 ?

Indicate all such integers.

- ☐ A 8
☐ B 9
☐ C 12
☐ D 18
☐ E 21
☐ F 36

4) 数字填入题【样题如下】

One pen costs \$0.25 and one marker costs \$0.35. At those prices, what is the total cost of 18 pens and 100 markers?

\$

Rectangle R has length 30 and width 10, and square S has length 5. The perimeter of S is what fraction of the perimeter of R ?

注意：分数可以不约分

4. 考试注意事项：

- 1) GRE 数学中有计算器；
- 2) 椭圆是单选题，方框是不定项选择题
- 3) 所有的数字都是实数；
- 4) 所有的图表都假设位于一个平面，除非另外有提到
- 5) 几何图形（线，圆，三角形，四边形）不一定是按照比例画的；
- 6) 直角坐标系是按照比例画的
- 7) 图表（柱状图，饼状图，线形图）是按照比例画的

二. 算术（上）-整数相关

1. 整数与数位

(1) 整数 (Integer): 类似于 -2, -1, 0, 1, 2, 这样的数叫整数。注意 0 是整数

(2) 数位相关的英文术语：

ones digit / units digit 个位

tens digit 十位

hundreds digit 百位

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thousands digit 千位

double-digit integer/ two-digit integer 两位数

three-digit integer 三位数

(3) 重要考点 ① 数位与数值的转化

② 凑整

例 1: A and B are two-digit integers. The tens digit of A is 3, and the units digit of B is 5. If the product of A and (B+1) is 2016, what is the value of $3A+4B$?

- A. 216
- B. 328
- C. 290
- D. 306
- E. 296

例 2: If the value of a double-digit number is twice the sum of its tens digit and units digit, then double-digit number must be?

例 3: The sum of three different positive integers is 11. Select two of the following statements that can together identify the three numbers.

- A. None of those three can be 1
- B. None of those three can be 4
- C. None of those three can be 7
- D. None of those three can be 8

2. 正负整数

- positive integer 正整数
- negative integer 负整数

注意：① 0 是整数，但不具备正负性

② 当题目告诉一个数是整数的时候，不能惯性思维只考虑正整数

重要考点：正负整数的相关运算

➤ 1) 正整数 \times 正整数 = 正整数

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➤ 2) 负整数 \times 负整数 = 正整数

➤ 3) 正整数 \times 负整数 = 负整数

➤ 4) 正整数 + 正整数 = 正整数

➤ 5) 负整数 + 负整数 = 负整数

例 4:

x and y are both integers

$$x^2 - y^2 = 27$$

Quantity A: x

Quantity B: 10

例 5: If $a < b < 0$, which of the following numbers must be positive?

Indicate all such numbers.

A. $a - b$

B. $a^2 - b^2$

C. ab

D. a^2b

E. $a^2b + ab^2$

3. 奇数与偶数

基础知识:

偶数 (even number): 能够被 2 所整除的整数, 如: $-4, -2, 0, 2, 8, \dots$, 通用表达式: $2k$ (k 为整数)

奇数 (odd number): 不能够被 2 所整除的整数, 如: $-3, -1, 1, 3, 5, \dots$, 通用表达式: $2k+1$ (k 为整数)

注意:

➤ ① 0 是偶数

- ② 所有整数要么是奇数，要么是偶数，与其正负性无关

相关术语：

- even integer 偶数
- odd integer 奇数
- sum 和
- difference 差
- product 乘积

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重要考点：（1）加减乘除判定规律

- •1) 偶数+偶数=偶数
- •2) 奇数+奇数=偶数
- •3) 偶数+奇数=奇数
- •4) 偶数×偶数=偶数
- •5) 奇数×奇数=奇数
- •6) 偶数×奇数=偶数

重要考点：（2）奇偶数的性质

- 1) a^n 的奇偶性由底数(base number) a 来决定 ($n \neq 0$)
 $odd^n = odd$
 $even^{n \neq 0} = even, even^0 = 1 (odd)$
- 2) 连续的两个整数相乘，结果为偶数

例 6: Integer a is 125 more than integer b . Which of the following statement(s) must be true?

Indicate all such statements.

- A. b is an even integer
- B. a is an odd integer
- C. $a+b$ is an odd integer
- D. ab is an even integer

例 7: If x is an odd negative integer and y is an even integer, which of the following statements must be true?

- I. $(3x - 2y)$ is odd
- II. xy^2 is an even negative integer
- III. $(y^2 - x)$ is an odd negative integer

... of ... negative integer

- A. I only
- B. II only
- C. I and II
- D. I and III
- E. II and III

例 8:

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r and t are consecutive integers and $p=r^2+t$.

Quantity A: $(-1)^p$

Quantity B: -1

例 9:

If c and d are odd positive integers, which of the following could be odd?

Indicate all such expressions.

- A. c^d
- B. c^{d+1}
- C. $(c+1)^{d+1}$
- D. $(c+d)^{c+d}$
- E. c^d/d^c

4. 质合数

基础知识:

质数 (Prime Numbers): 除了 1 和它本身之外, 不能被其他正整数所整除的自然数, 如: 2,3,5,7,11,13...

合数 (Composite Numbers): 除了 1 和它本身之外, 还有其他因数的自然数, 如: 4,6,8,9,10...

质因数 (prime factor): 因数中的质数

质因数与因数的关系是?

注意:

- ① 1 既不是质数也不是合数

➤ ② 最小的质数是 2，最小的合数是 4

➤ ③ 熟记 30 以内的质数

相关术语：

➤ prime number 质数

➤ composite number 合数

➤ prime divisor/factor 质因数

➤ prime factorization 分解质因数

➤ greatest prime divisor/factor 最大质因数

重要考点：(1) 分解质因数

分解质因数方法有两种：

1) 常规法，即短除法，适用于较小的数字

2) 快速分解法，适用于含有“一个 0 或者多个 0”结尾的数据

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重要考点：(2) 质数与合数的判断

质合数主要依据定义来判断：

质数：有且只有两个因数，即 1 和本身

合数：至少有 3 个因数，即 1、本身、其它

重要考点：(3) 质因数的个数

质因数的个数：

第一步：分解质因数

第二步：直接看底数有几个质数即可

重要考点：(4) 其他相关重要性质

1) 如果一个正整数 n 有且只有三个因数，那么 n =质数的平方

2) 质数中 2 是唯一的偶数，其他均为奇数

例 10: X is the positive difference between 3^{100} and 3^{97} . What is the greatest prime divisor of X ?

例 11:

If p and n are prime numbers, $p-n=4$, and $\frac{3}{2} < \frac{p}{n} < 2$, what is the value of p ?

例 12:

Quantity A: The number of different prime factors of 500

Quantity B: The number of different prime factors of 360.

例 13:

n is an even integer.

Quantity A: The number of prime factors of n

Quantity B: The number of prime factors of $n/2$

例 14: From 1 to 900, how many integers have exactly three factors?

A. 9

B. 10

C. 11

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D. 12

E. 13

例 15: For every positive even integer n , the function $h(n)$ is defined to be the product of all the even integers from

2 to n , inclusive. If p is the smallest prime factor of $h(100)+1$, then p is

A. Between 2 and 10

B. Between 10 and 20

C. Between 20 and 30

D. Between 30 and 40

E. Greater than 50

5. 因数和倍数

基础知识:

- 如果 $\frac{A}{B} = \text{integer}$ (A 和 B 都是整数且 $B \neq 0$), 则 A 是 B 的倍数 (multiple), B 是 A 的因数 (factor/divisor)
- 最大公约数 (Greatest Common Factor or Divisor): 如果一个数同时是几个正整数的约数, 则称这个

数为它们的公约数或公因数；公约数中最大的被称为最大公约数

- 最小公倍数 (Least Common Multiple): 如果一个数同时是几个正整数的倍数, 则称这个数为它们的公倍数; 公倍数中最小的被称为最小公倍数

注意:

- ① 1 是任何整数的因数; 0 是任何非零自然数的倍数
- ② 一个数字本身也是自己的一个因数

重要考点: (1) 因数倍数间的包含关系

如果 $\frac{A}{B} = \text{integer}$ (A 和 B 都是整数且 $B \neq 0$)

则: A is a multiple of B=B is a factors of A=A is divisible by B 【此时 A 能够完全包含 B】

重要考点: (2) 因数的个数

1. 分解质因数

$$n = a^x \times b^y \times c^z$$

a, b and c are different prime factors

2. 次方数+1 的乘积

$$\text{The number of positive factors} = (x+1)(y+1)(z+1)$$

重要考点: (3) 倍数的个数

重要考点: (4) 最大公约数和最小公倍数

1) 最大公约数&最小公倍数的求值方法:

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方法①: 短除法 (适用于两个数之间的求值)

- 从最小质数 2 开始验证, 寻找公共质因数, 不断短除
- 短除左侧值全部乘起来即为最大公约数
- 全部短除结果乘起来即为最小公倍数

方法②: 分解质因数的方法 (适用于两个及以上的数之间的求值)

- 将各个数字先分解质因数
- 取共同的质因数指数较小的数字相乘得到最大公约数
- 取所有的质因数指数较大的数字相乘得到最小公倍数

Eg: 求 84 和 90 的最小公倍数和最大公约数?

2) 推论/性质

两个数的乘积=这两个数最大公约数和最小公倍数的乘积

例 16: What is the least positive integer that is not a factor of 25! and is not a prime number?

- A. 26
- B. 28
- C. 36
- D. 56
- E. 58

例 17: If 2^n is the highest power of 2 that is a divisor of the product $(10^2)(12^5)(18^6)$, then $n=?$

- A. 12
- B. 13
- C. 18
- D. 45
- E. 60

例 18: p is a positive odd number, while 5 is a factor of $p+p^2$

Quantity A: The remainder when p is divided by 5

Quantity B: 0

例 19: How many positive factors does 147,000 have?

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例 20: If $T=2^2 \cdot 5 \cdot 7^2 \cdot 9^2$, how many positive factors does T have?

例 21: How many integers from 100 to 200 are both multiples of 3 and odd numbers?

例 22: How many integers are multiples of 2 or 3 from 1 to 603, inclusive?

例 23: Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisors of every integer n in S ?

Indicate all such integers.

A.12

B.24

C.36

D.72

例 24: In a certain medical group, Dr. Schwartz schedules appointments to begin 30 minutes apart, Dr. Ramirez schedules appointments to begin 25 minutes apart, and Dr. Wu schedules appointments to begin 50 minutes apart. All three doctors schedule their first appointments to begin at 8:00 in the morning, which are followed by their successive appointments throughout the day without breaks. Other than at 8:00 in the morning, at what times before 1:30 in the afternoon do all three doctors schedule their appointments to begin at the same time?

Indicate all such times

A.9:30 in the morning

B.10:30 in the morning

C.11:30 in the morning

D.12:00 noon

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E.1:00 in the afternoon

6. 连续正整数

基础知识:

连续整数 (consecutive integers): 连续且递增的整数

consecutive positive integers 连续正整数

consecutive negative integers 连续负整数

three consecutive integers 三个连续整数

consecutive odd integers 连续奇数

consecutive even integers 连续偶数

重要考点：（1）三条性质

1) 2 个连续正整数相乘必然为偶数（2 的倍数）。

2) 两个连续偶数相乘为 8 的倍数。

3) 3 个连续正整数相乘必然为 6 的倍数。

重要考点：（2）连续奇数/连续偶数的个数

连续偶数与连续奇数当中，每相邻两项的差值为 2

三个连续偶数，可设未知数为 $2k$, $2k+2$, $2k+4$

三个连续奇数，可设未知数为 $2k+1$, $2k+3$, $2k+5$

从奇数 a 到奇数 b ，有 $(b-a)/2 + 1$ 个奇数

从偶数 a 到偶数 b ，有 $(b-a)/2 + 1$ 个偶数

例 25: If n is a positive integer greater than 1, then $n(n^2-1)$ must be a multiple of which of the following integers?

Indicate all such numbers.

A. 2

B. 3

C. 4

D. 5

E. 6

例 26: If a , b , c are three consecutive positive even integers, which of the following must be an integer?

Indicate all that's possible.

A. $(a+b+c)/2$

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B. $(a+b+c)/4$

C. $(a+b+c)/6$

D. $(a+b+c)/12$

E. $(a+b+c)/15$

例 27: If j and k are even integers and $j < k$, which of the following equals the number of even integers that are greater than j and less than k ?

- A. $(k-j-2)/2$
- B. $(k-j-1)/2$
- C. $((k-j)/2)$
- D. $k-j$
- E. $k-j-1$

7. 整除和余数

基础知识:

quotient / result 商

remainder 余数

注意: GRE 数学不研究被除数和除数是负数的情况

重要考点: (1) 余数的基础知识

- 1) 余数的取值范围: 余数必为非负整数, 且 $0 \leq \text{余数} < \text{除数}$
- 2) 余数与小数的转化关系:

如果 $A \div B = C.xx$, 则 A 除以 B 的余数为: $0.xx * B$

重要考点: (2) 公式表达被除数

标准问法: what is the remainder when xxx is divided by ### ?

- 1) 一个表达式时:

带有未知数的式子除以一个常数, 余数是否唯一固定? 就看未知数项除以常数能否整除

- 2) 两个表达式时--构造通式, 变成一个表达式

第一步: 找已知的两个除数的最小公倍数作为新的未知数项的系数

第二步: 通过试数, 找出满足两个式子的第一次等值, 把这个值作为新的常数项

重要考点: (3) 自然数高次幂求余数

方向一: 找个位的规律

- 1) 掌握个位数循环规律的万能做法
- 2) 常考的四种提问方式

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方向二: 找余数的规律

- 1) 注意常考的提问方式

重要考点: (4) 整数的整除特性

1)、个位除以 2, 5, 10 余数是多少就是整个数除以 2, 5, 10 余数是多少

2 的倍数的个位是: 2, 4, 6, 8, 0

5 的倍数的个位是: 0, 5

2)、末两位除以 4, 20, 25 的余数是多少就表示整个数除以 4, 20, 25 余数是多少

3)、末三位数除以 8, 125 的余数是多少就表示整个数除以 8, 125 的余数是多少

4)、各个数位数字之和除以 3 或 9 余数是多少就表示整个数除以 3 或 9 余数是多少

例 28: What is the remainder of $\frac{7^{23}-5}{7}$?

A. 1

B. 2

C. 4

D. 5

E. 6

例 29: How many integers from 1 to 1000, inclusive, have the same remainder when divided by

2, 3, 5, 7?

例 30: $X=123^4-123^3+123^2-123$

What is the remainder when X is divided by 122?

例 31: When the positive integer x is divided by 42, the remainder is 19. What is the remainder when x is divided by 7?

A.0

B.2

C.3

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D.4

E.5

例 32: When a positive integer (less than 100) is divided by 5 and 6, the remainder is 3 and 2, respectively.

Quantity A: The number of such integers

Quantity B: 4

例 33: The remainder is 30 when $6n$ is divided by 75, which of the following could be the remainder when $7n$ is divided by 75?

Indicate all such numbers.

A. 5

B. 10

C. 15

D. 25

E. 35

F. 45

G. 60

例 34: When the even integer n is divided by 7, the remainder is 3.

Quantity A: The remainder when n is divided by 14

Quantity B: 10

例 35: Quantity A: The units digit of 888^{888}

Quantity B: The units digit of 555^{555}

例 36: Quantity A: The remainder when 7^{13} is divided by 10

Quantity B: 7

例 37: n, k and r are all positive integers. If $n^k = 10r + 3$, then n could be?

Indicate all that are possible.

A. 11 B. 12 C. 13 D. 15 E. 17 F. 19

例 38: n, k and r are all positive integers. If $n^k = 10r + 3$, then k could be?

Indicate all that are possible.

A. 11 B. 12 C. 13 D. 15 E. 17 F. 19

例 39: What is the remainder when 7^{38} is divided by 9?

例 40: $m = 10^{32} + 2$, when m is divided by 11, the remainder is r .

Quantity A: r

Quantity B: 3

例 41: Quantity A: The remainder when $2 \cdot 10^{1000} + 1$ is divided by 3

Quantity B: 1

例 42: Quantity A: The remainder when 12345678910 is divided by 9

Quantity B: 1

例 43: What is the remainder when the square of 345,606 is divided by 20?

