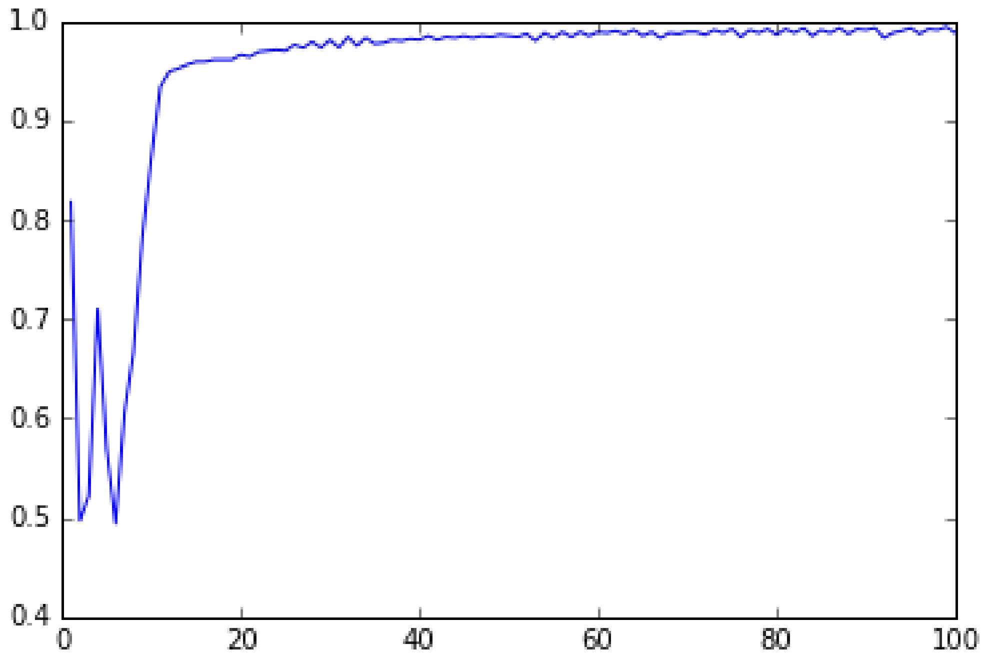
Homework 3

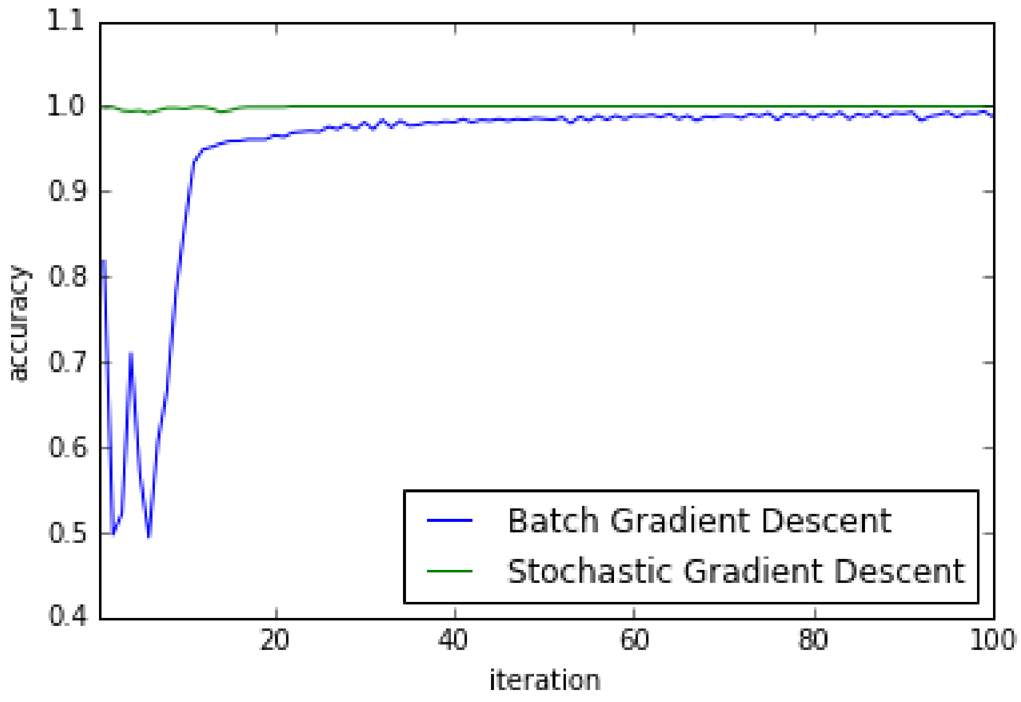
1(d)

The accuracy plot is the following:



1(f)

The accuracy plot is the following:

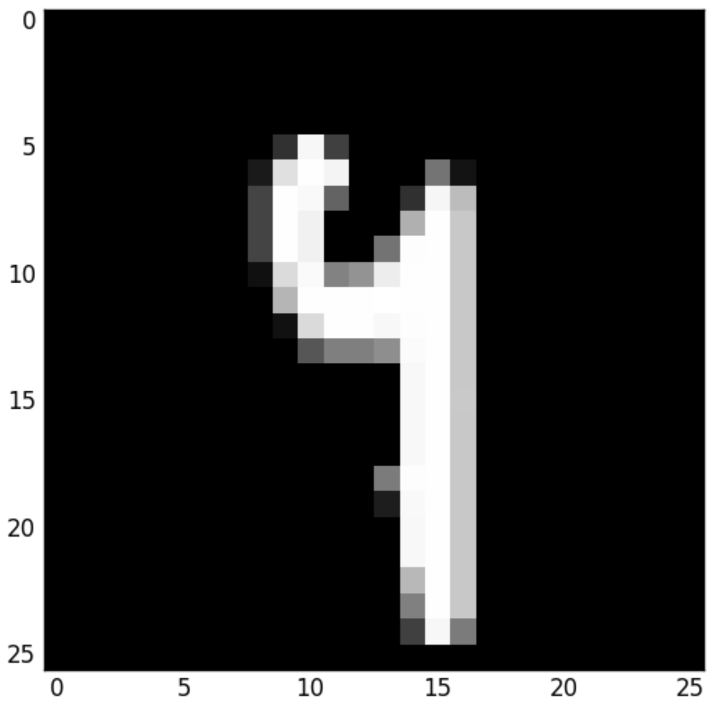


1(g)

The stochastic gradient descent converges faster than batch gradient decent. From the plot in 1(f), we can see the accuracy of stochastic gradient descent converges to 1 after approximate 30 iterations, while the batch gradient decent still has a little fluctuation. It is mainly because in the stochastic gradient descent, in each iteration, w & b are updated N times in 1 iteration. In batch gradient decent, w & b remain the same in 1 iteration.

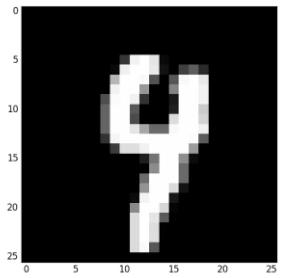
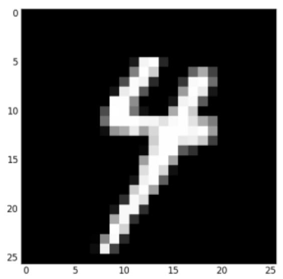
1(i)

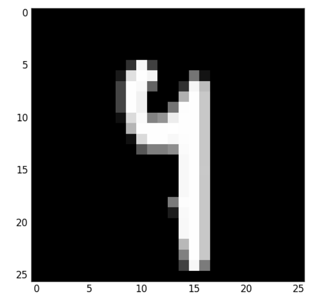
I choose Gamma=10^(-7), C=10 in this problem. The accuracy of training data and test data is 99.9% and 99.8%. Therefore, there is only 1 misclassified test image. It is the 164th of the test labels, it is "9" initially, but the SVM labels it "4". The picture is shown as follows:

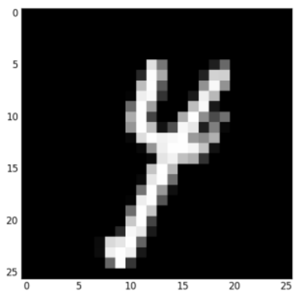


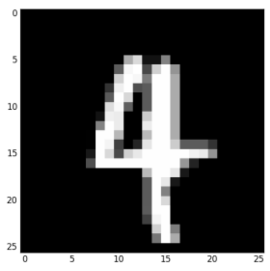
1(j)

Based on the separate final accuracy, there is no significant difference between LDA and SVM.
The The accuracy of training data and test data is 99.7% and 90.4%. 5 of the misclassified test images are :







The first and the third are misclassified as “4” and the others are misclassified “9”.

4b

The RMSE of the models are as follows:

	1	2	3	4
RMSE	7.412	4.831	2.962	13.199

Codes:

(1d)

```
import numpy as np
trainx = np.loadtxt("digits_training_data.csv",delimiter=",")
trainy=np.loadtxt("digits_training_labels.csv",delimiter=",")
w=np.zeros(676);b=0;
iterN=np.linspace(1,100,100);
for i in range(1000):
    if trainy[i]==4:
        trainy[i]=1;
    else:
        trainy[i]=-1;
from __future__ import division
def Ewb_grad(w,b):
    Ew=np.zeros(676)
    Eb=np.zeros((1000,1))
    for i in range(1000):
        if trainy[i]*(trainx[i,:].dot(w)+b)<1:
            Ew = Ew - trainy[i]*(trainx[i,:].T)
            Eb[i] = -3*trainy[i]
        else:
            Ew=Ew
            Eb[i]=0
    Ew_grad=Ew+w
    Eb_grad=np.sum(Eb)
    return Ew_grad, Eb_grad;
w0=w;b0=b;goal1=np.zeros(100)
for j in range(100):
    yita=0.001
    wgrad,bgrad=Ewb_grad(w0,b0)
    alpha=yita/(1+(j+1)*yita)
    w0=w0-alpha*wgrad
    b0=b0-alpha*bgrad
    yfit=trainy*(trainx.dot(w0)+b0)
    cali=np.zeros(1000)
    goal1[j]=np.sum(yfit>cali)/1000
from matplotlib import pyplot as plt
plt.plot(iterN,goal1)
```

(1f)

```
import numpy as np
trainx = np.loadtxt("digits_training_data.csv",delimiter=",")
trainy=np.loadtxt("digits_training_labels.csv",delimiter=",")
w=np.zeros(676);b=0;
```

```
iterN=np.linspace(1,100,100);
for i in range(1000):
        if trainy[i]==4:
                trainy[i]=1;
        else:
                trainy[i]=-1;
from __future__ import division
def wb_grad(w,b,j):
        Ew=np.zeros(676);
        w1=w;b1=b; yita=0.001;
        alpha=yita/(1+(j+1)*yita);
        r=np.random.permutation(1000)
        for i in r:
                if trainy[i]*(trainx[i,:].dot(w)+b)<1:
                        Ew = 0.001*w1 - 3*trainy[i]*(trainx[i,:].T)
                        Eb = -3*trainy[i]
                        w1=w1-alpha*Ew
                        b1=b1-alpha*Eb
                else:
                        Ew=0.001*w1
                        Eb=0
                        w1=w1-alpha*Ew
                        b1=b1-alpha*Eb
        return w1, b1;
w0=w;b0=b;goal=np.zeros(100)
for j in range(100):
        w0,b0=wb_grad(w0,b0,j)
        yfit=trainy*(trainx.dot(w0)+b0)
        cali=np.zeros(1000)
        goal[j]=np.sum(yfit>cali)/1000
from matplotlib import pyplot as plt
plt.plot(iterN,goal)
SVM1, =plt.plot(iterN,goal1,label='Batch Gradient Descent')
SVM2, =plt.plot(iterN,goal,label='Stochastic Gradient Descent')
plt.legend(handles=[SVM1,SVM2],loc='lower right')
plt.xlabel('iteration')
plt.ylabel('accuracy')
plt.axis([0.5,100,0.5,1.1])
```

(1i)

```
import numpy as np
from sklearn import svm
trainx = np.loadtxt("digits_training_data.csv",delimiter=",")
trainy=np.loadtxt("digits_training_labels.csv",delimiter=",")
```

```
testx= np.loadtxt("digits_test_data.csv",delimiter=",")
testy=np.loadtxt("digits_test_labels.csv",delimiter=",")
for i in range(500):
        if testy[i]==4:
                testy[i]=1;
        else:
                testy[i]=-1;
for i in range(1000):
        if trainy[i]==4:
                trainy[i]=1;
        else:
                trainy[i]=-1;
from __future__ import division
clf = svm.SVC(kernel='rbf',gamma=10**(-7), C=10)
clf.fit(trainx, trainy)
z=clf.predict(trainx)
goal=np.sum(z==trainy)/1000
clf = svm.SVC(kernel='rbf',gamma=10**(-7), C=10)
clf.fit(testx, testy)
z=clf.predict(testx)
goal1=np.sum(z==testy)/500
for i in range(500):
        if z[i]*testy[i]<0:
                k=i
import matplotlib.cm as cm
from matplotlib import pyplot as plt
plt.imshow(testx[k,:].reshape((26,26)), interpolation="nearest", cmap=cm.Greys_r)
```

(1j)

```
import numpy as np
trainx = np.loadtxt("digits_training_data.csv",delimiter=",")
trainy=np.loadtxt("digits_training_labels.csv",delimiter=",")
testx = np.loadtxt("digits_test_data.csv",delimiter=",")
testy=np.loadtxt("digits_test_labels.csv",delimiter=",")
for i in range(500):
        if testy[i]==4:
                testy[i]=1;
        else:
                testy[i]=-1;
for i in range(1000):
        if trainy[i]==4:
                trainy[i]=1;
        else:
                trainy[i]=-1;
```

```
from __future__ import division
trainx1=trainx[(trainy==-1),]
trainx2=trainx[(trainy==1),]
mean1=np.reshape(np.mean(trainx1,0),(1,len(trainx1[0])))
mean2=np.reshape(np.mean(trainx2,0),(1,len(trainx2[0])))
trainx1norm=trainx1-mean1;
trainx2norm=trainx2-mean2;
trainxnorm=np.concatenate((trainx1norm,trainx2norm))
sigma=(trainxnorm.T.dot(trainxnorm))/1000
gamma1                                    =                                    -
0.5*mean1.dot(np.linalg.pinv(sigma)).dot(mean1.T)+np.log(1.0*len(trainx1)/len(trainx))
beta1 = np.linalg.pinv(sigma).dot(mean1.T)
gamma2                                    =                                    -
0.5*mean2.dot(np.linalg.pinv(sigma)).dot(mean2.T)+np.log(1.0*len(trainx2)/len(trainx))
beta2 = np.linalg.pinv(sigma).dot(mean2.T)
trainp1                                                                        =
np.exp((trainx.dot(beta1)+gamma1))/(np.exp((trainx.dot(beta1)+gamma1))+np.exp((trainx.dot(be
ta2)+gamma2)))
trainp2                                                                        =
np.exp((trainx.dot(beta2)+gamma2))/(np.exp((trainx.dot(beta1)+gamma1))+np.exp((trainx.dot(be
ta2)+gamma2)))
trainyfit=np.zeros((1000))
for i in range(1000):
        if trainp1[i]>=trainp2[i]:
                trainyfit[i]=-1
        else:
                trainyfit[i]=1
goal=np.sum(trainyfit==trainy)/1000

testp1                                                                         =
np.exp((testx.dot(beta1)+gamma1))/(np.exp((testx.dot(beta1)+gamma1))+np.exp((testx.dot(beta
2)+gamma2)))
testp2                                                                         =
np.exp((testx.dot(beta2)+gamma2))/(np.exp((testx.dot(beta1)+gamma1))+np.exp((testx.dot(beta
2)+gamma2)))
testyfit=np.zeros((500))
for i in range(500):
        if testp1[i]>=testp2[i]:
                testyfit[i]=-1
        else:
                testyfit[i]=1
goal1=np.sum(testyfit==testy)/500
m=np.zeros((500))
for i in range(500):
        if testyfit[i]*testy[i]<0:
                m[i]=i
k=m[m>0]
import matplotlib.cm as cm
from matplotlib import pyplot as plt
for i in range(5):
        plt.figure()
        plt.imshow(testx[k[i],:].reshape((26,26)), interpolation="nearest", cmap=cm.Greys_r)
```

(2)

```
import numpy as np
from sklearn import svm
trainy = np.loadtxt('trainingLabels.gz', dtype=np.uint8, delimiter=',')
trainx = np.loadtxt('trainingData.gz', dtype=np.uint8, delimiter=',')
testx = np.loadtxt('testData.gz', dtype=np.uint8, delimiter=',')
from __future__ import division
clf = svm.SVC(kernel='rbf',gamma=10**(-8), C=10)
clf.fit(trainx, trainy)
z=clf.predict(testx)
```

(4b)

```
import numpy as np
train = np.loadtxt("steel_composition_train.csv", delimiter=",", skiprows=1,
usecols=(1,2,3,4,5,6,7,8,9))
x1=train[:,0:8]
y1=train[:,8]
x1 = (x1-np.mean(x1,0))/np.std(x1,0)
y1 = np.reshape(y1,((len(y1),1)))

K1 = (x1.dot(x1.T)+1)**2
a1 = np.linalg.inv(np.identity(len(x1))+K1).dot(y1)
Err1 = a1.T.dot(K1).dot(K1).dot(a1)-2*y1.T.dot(K1).dot(a1)+y1.T.dot(y1)+a1.T.dot(K1).dot(a1)
rmse1 = np.sqrt(Err1/len(x1))

K2 = (x1.dot(x1.T)+1)**3
a2 = np.linalg.inv(np.identity(len(x1))+K2).dot(y1)
Err2= a2.T.dot(K2).dot(K2).dot(a2)-2*y1.T.dot(K2).dot(a2)+y1.T.dot(y1)+a2.T.dot(K2).dot(a2)
rmse2 = np.sqrt(Err2/len(x1))

K3 = (x1.dot(x1.T)+1)**4
a3 = np.linalg.inv(np.identity(len(x1))+K3).dot(y1)
Err3= a3.T.dot(K3).dot(K3).dot(a3)-2*y1.T.dot(K3).dot(a3)+y1.T.dot(y1)+a3.T.dot(K3).dot(a3)
```

```
rmse3 = np.sqrt(Err3/len(x1))

from numpy.linalg import norm
K4 = []
for i in range(0,len(x1)):
        K4.append(np.exp(-0.5*norm(x1[i]-x1,axis=1)**2))
K4 = np.array(K4)
a4 = np.linalg.inv(np.identity(len(x1))+K4).dot(y1)
Err4 = a4.T.dot(K4).dot(K4).dot(a4)-2*y1.T.dot(K4).dot(a4)+y1.T.dot(y1)+a4.T.dot(K4).dot(a4)
rmse4 = np.sqrt(Err4/len(x1))
```