STAT 423/523 Statistical Methods for Engineers and Scientists

Lecture 4: Hypothesis Testing I

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- The average height of WSU students is 68 inches.
- v.s. The average height of WSU students is NOT 68 inches.

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The decision is based on the sample data. (So the decision is a random variable.)

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	H_0 is true	H_0 is false
Reject H_0	Type I error	Correct decision
Fail to reject H_0	Correct decision	Type II error

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- Usually, Type I error and Type II error are inversely related.
- Type I error can be controlled by the **significance level** α.

$$P(\text{reject} \mid H_0) = \alpha$$

- α is also called the **size** of the test.

Example

Suppose the heights of WSU students are normally distributed with mean μ and a standard deviation of 2 inches.
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The significance level of this test is

$$\alpha = P(\text{reject}|H_0) = P(N(68, 0.04) > 68.5) = 1 - \Phi(3).$$

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In our previous sample, our decision is based on the sample mean $\bar{X}$, which is called the test statistic.

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- The rejection region for the test of whether the average height of WSU students is greater than 68 inches is $(68.6, \infty)$.
- The rejection region for the test of whether the average height of WSU students is exactly 68 inches is $(-\infty, 67.6) \cup (68.4, \infty)$.

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The significance level of this test is

$$\alpha = P(\text{reject}|H_0) = P(\text{Binom}(100, 0.5) \leq 37) = 0.006$$