STAT 576 HOMEWORK 3

DUE NOV. 1, 2024 (FRIDAY), 11:59 PM PST

NOTES. **NO** late submission will be accepted except for approval from the instructor. Answers should be either scanned or typed. For the coding part (if any), you may either prepare an R notebook or put the code in a separate file.

In this homework, we consider the following Bayesian inference problem. Let $X_1, \ldots, X_5$ be five independent Poisson random variables with commom parameter λ. The prior distribution of λ is Uniform$(0, 10)$. The observed values are

$$(X_1, \ldots, X_5) = (3, 4, 2, 3, 2)$$

1 (Poseterior)**.** Derive the posterior distribution of λ (in proportional form).

2 (Inverse C.D.F.)**.**

(a) Relate the quantile function of the posterior to that of a Gamma distribution.
(b) Use inverse c.d.f. method to generate 4000 random samples from the poseterior. (You may need the `qgamma` function in R).
(c) Plot the histogram of the samples, and estimate the posterior mean and 95% credible interval.

3 (Rejection Algorithm)**.**

(a) Draw 4000 random samples from the posterior using the rejection algorithm with a uniform proposal distribution.
(b) Draw 4000 random samples from the posterior using the rejection algorithm with a Gamma distribution.
(c) Compare the acceptance rates of the two algorithms.
(d) For samples from part (b), plot the histogram of the samples, and estimate the posterior mean and 95% credible interval.

4 (Metropolis-Hastings Algorithm)**.**

(a) Draw 4000 random samples from the posterior using the Metropolis-Hastings algorithm with a normal proposal distribution and one Markov chain.
(b) Check the autocorrelation of the samples.
(c) Draw 4000 random samples from the posterior using the Metropolis-Hastings algorithm with a normal proposal distribution and four Markov chains (i.e. 1000 samples in each chain).
(d) For samples from part (c), plot the histogram of the samples, and estimate the posterior mean and 95% credible interval.

5 (Stan)**.**

(a) Write a Stan program to sample from the posterior. Use 4 chains with 1000 samples from each chain.
(b) Plot the histogram of the samples, and estimate the posterior mean and 95% credible interval.