STAT 576 HOMEWORK 2

DUE OCT. 4, 2024 (FRIDAY), 11:59 PM PST

NOTES. **NO** late submission will be accepted except for approval from the instructor. Answers should be either scanned or typed. For the coding part (if any), you may either prepare an R notebook or put the code in a separate file.

1 (Total Variation Distance)**.** Let F and G be two distributions over $\mathbb{R}$ with probability density functions f and g correspondingly. Define the total variation distance between F and G by

$$\|f - g\|_{TV} := \sup_{A \subset \mathbb{R}} |F(A) - G(A)|.$$

We ignore the measurability issues for this problem.

(a) Show that
$$\|f - g\|_{TV} = F(A) - G(A)$$
for the event $A = \{x : f(x) > g(x)\}$.

(b) Show that
$$\|f - g\|_{TV} = \int (f(x) - g(x))_+ d\mu(x),$$
where
$$(y)_+ = \begin{cases} y & \text{if } y \geq 0 \\ 0 & \text{if } y < 0. \end{cases}$$

(c) Show that
$$\|f - g\|_{TV} = \frac{1}{2} \int |f(x) - g(x)| d\mu(x)$$

2 (Textbook Problems)**.** Finish Problem 1 in Chapter 4 of the textbook. (You may skip the ploting step in part (c).)

3 (Hierarchical Poisson)**.** Suppose a datasets contains the numbers of traffic accidents in J districts for the last year. Denote the number of accidents in district j by y_j.

(a) The number of accidents y_j can be modeled by a Poisson distribution with intensity λ_j. Write down the probability mass function for y_j given λ_j.
(b) Build a hierarchical model for $(y_1, \ldots, y_J)$, where the λ_j follows a Gamma(α, β) distribution with the hyperprior $p(\alpha, \beta)$.
(c) Compute the marginal distribution of y_1.
(d) Compute the joint posterior distribution density (in proportional form) for $(\alpha, \beta, \lambda_1, \ldots, \lambda_J)$.
(e) Compute the exact conditional posterior distribution density for $\lambda_1, \ldots, \lambda_J \mid \alpha, \beta, y_1, \ldots, y_J$.
(f) Compute the marginal posterior distribution for (α, β) in proportional form.

4 (Textbook Problems)**.** Finish Problems 4 and 5 in Chapter 5 of the textbook.