An Approximate True Damping Solution of the Flutter Equation by Determinant Iteration

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The difference between a true damping, or rate-of-decay, solution of the flutter equation and the structural-damping-type of solution is highlighted. True damping solutions are possible if unsteady aerodynamics can be expressed in terms of the complex variable *p*. If the aerodynamics are given at discrete values of the reduced frequency *k*, an approximate determination of the true damping is possible by assuming that the aerodynamic forces for harmonic motion are a good approximation for the cases of slowly increasing or decreasing amplitude. A determinant iteration method for obtaining the solution is presented. Results obtained by different methods of solving the flutter equation are compared.

Nomenclature

$[A_1],[A_0]$	= quasi-steady aerodynamic matrices
$[A_2],[A_1],[A_0]$, $[B_2],[B_1],[B_0]$	= aerodynamic matrices defined by Eq. (8)
$[A(ik)],[A(k)]$, $[A_\alpha(k)]$	= complex aerodynamic matrix numerically available at any discrete value of k
$[A(p)]$	= aerodynamic matrix that is an explicit function of p
$[D]$	= viscous damping matrix
$[D_A],[D_B],[D_C]$	= matrices relating the control system to the degrees of freedom
$[D_\theta],[D_z]$	= matrices expressing local slope and local vertical displacement in terms of $\{q\}$
$\lvert[F(p,k)]\rvert$	= flutter determinant
$[K]$	= stiffness matrix
$[M]$	= inertia matrix
$[\bar{M}]$	= inertia matrix including effective apparent inertia of air
$\{Q\}$	= column of forces
$\{q\}$	= column of degrees of freedom
$\{\alpha\}$	= column of angles of attack
a_1,b_1,a_2,b_2	= constants defining the lag function
a_n,a_{n+1}	= successive amplitudes of oscillatory motion
c	= reference chord
F,G	= scalars
f	= frequency in cps
f_L	= scalar
g	= structural damping
g_0	= gravitational acceleration
H_A,H_B,H_C	= transfer functions for hydraulic controls and automatic control systems; explicit functions of s
k	= reduced frequency $\omega c/V$
p	= differential operator $(c/V)(d/dt)$
$p = \delta + ik$	= iterated values for the root of the flutter equation
s	= nondimensional time Vt/c
V	= true airspeed
γ	= true damping coefficient
$\lambda = \lambda_r + i\lambda_i$	= complex root of flutter equation
ρ,ρ_0	= air density, air density at sea level
σ	= air density ratio ρ/ρ_0
ω	= circular frequency

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The *p* and *k* Methods of Solution of the Flutter Equation

The flutter equation can be written in the general form

$$[V^2/c^2[M]p^2 + [K] - \tfrac{1}{2}\rho V^2[A(p)]]\{q\} = 0 \quad (1)$$

$[K]$ defines the elastic characteristics by relating generalized forces $\{Q\}$ to generalized displacements $\{q\}$

$$[K]\{q\} = \{Q\} \quad (2)$$

$[M]$ defines the inertia characteristics by relating the inertia forces $\{Q_{\text{in}}\}$ to the generalized accelerations

$$\{Q_{\text{in}}\} = -[M]\{\ddot{q}\} \quad (3)$$

$[A(p)]$ defines the unsteady aerodynamic forces through the equation

$$\{Q_{\text{aero}}\} = \tfrac{1}{2}\rho V^2[A(p)]\{q\} \quad (4)$$

In Eqs. (1) and (4) p is the nondimensional differential operator $(c/V)(d/dt)$. If the aerodynamic forces can be expressed as a sufficiently simple function of p, Eq. (1) defines a polynomial in p with real coefficients. For nonzero solutions for q the determinant formed by the matrix coefficients in Eq. (1) must be equal to zero. For a given value of the speed V, the determinant can then be solved directly for p. This leads to conjugate complex roots

$$p = \gamma k \pm ik \quad (5)$$

k defining a nondimensional reduced frequency $\omega c/V$ and γ defining a rate of decay

$$\gamma = (1/2\pi)\ln(a_{n+1}/a_n) \quad (6)$$

where a_n and a_{n+1} are the amplitudes of successive cycles.

This method of solving the flutter equation will be called the p method. It can be used in the case of quasisteady aerodynamics when $[A(p)]$ assumes the simple form

$$[A(p)] = [A_1]p + [A_0] \quad (7)$$

but also in the case of simple forms of unsteady aerodynamics.

Mazelsky and O'Connell[1] formulated approximate unsteady aerodynamics that accounted for apparent mass, aero-

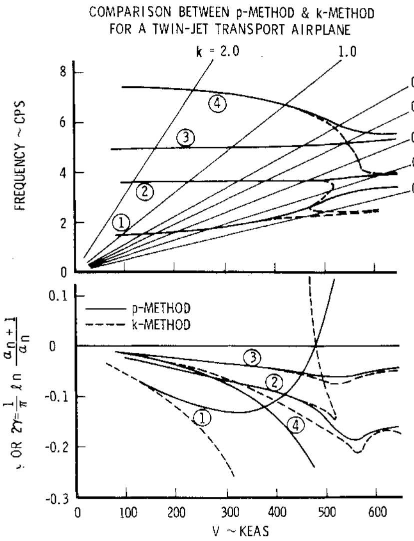

Fig. 1 Comparison between p method and k method for a twin-jet transport airplane.

dynamic lag, and aerodynamic coupling between different strips on the wing and which were a relatively simple function of p. With their formulation, the flutter equation takes the form

$$\left[p^2[\bar{M}] + (1/V^2)[K] + \sigma/(p + b_1)[p^2[A_2] + p[A_1] + [A_0]] + \sigma/(p + b_2)[p^2[B_2] + p[B_1] + [B_0]]\right]\{q\} = 0 \quad (8)$$

The quantities b_1 and b_2 are from the two-exponential lag function

$$1 - a_1e^{-b_1s} + a_2e^{-b_2s} \quad (9)$$

The matrices A_0,A_1,A_2 and B_0,B_1,B_2 contain the lift curve slope, aerodynamic influence coefficients, the aerodynamic center and rotation center (corresponding to the $\frac{3}{4}$ chord point in two-dimensional theory) for each strip, and the damping in pitch. Equation (8) has been used at the Lockheed-California Company since the middle fifties and has been routinely solved by determinant iteration since 1960 for roots $p = \gamma k + ik$ of interest.†

More sophisticated formulations of the aerodynamics, such as follow from the kernel function or doublet lattice approach or the supersonic Mach box, lead to aerodynamic matrices valid only for harmonic motion, $p = ik$. In that case, Eq. (1) takes the form

$$\left[-\frac{1}{c^2}[M]k^2 + \frac{1}{V^2}[K] - \frac{1}{2}\rho[A(ik)]\right]\{q\} = 0 \quad (10)$$

This is the traditional American form of the flutter equation. At chosen values of k, complex roots for $1/V^2$, $\lambda_r + i\lambda_i$, are found and interpreted as

$$\lambda_r + i\lambda_i = (1/V^2)(1 + ig) \quad (11)$$

where g is the structural damping required for harmonic motion. This method of solving the flutter equation is here called the k method.

Several authors have discussed the differences between the p method and the k method and variations thereof.[2-5] The significance of the difference between the rate-of-decay damping, γ, found with the p method and the structural damping, g, found with the k method is, in general, well understood. The numerical differences have been demonstrated by direct calculation by Richardson,[4] Rodden and Stahl.[5] A result obtained by this author is shown in Fig. 1. It is found by formulating the flutter equation for a twin-jet transport according to Eq. (8) and solving it according to the p method and the k method.

The upper part of Fig. 1 shows the frequency-vs-speed relation obtained with the two methods. Each of the curves numbered 1–4 corresponds to a root of the flutter equation and an associated flutter mode. The lower part of Fig. 1 shows how the damping obtained with the two methods varies with speed.

For the k method g is plotted; for the p method the comparable quantity

$$2\gamma = (1/\pi)\ln(a_{n+1}/a_n) \quad \text{(Ref. 6)}$$

The numbering of the curves corresponds to the numbering in the upper part of the figure. One can see that the p method solution shows flutter in the first mode; the k method solution shows flutter in the second mode. Even the modal coupling seems different. Yet at the flutter speed, both methods of solution give identical results.

It is generally conceded that it is desirable to formulate and solve the flutter equation such that the solution leads to a value for the rate of decay. Ideally, this requires the formulation of the unsteady aerodynamics matrix as a function of the complex variable $p = \gamma k + ik$. Some of the approximate formulations of the aerodynamic matrices have made this possible.[1,4,5] An attempt to develop Theodorsen-type aerodynamics as a function of $p = \gamma k + ik$ led to a dispute as to whether the formulation was valid for a motion with decaying amplitude.[7-9] The only published application of generalized Theodorsen aerodynamics is by Stümke.[10] No such attempt for kernel function aerodynamics or supersonic Mach box aerodynamics has come to the attention of the author.

In general, then, when one wants to work with exact theoretical aerodynamics one must work with a formulation for harmonic motion and devise approximate methods to determine the rate of decay.

Zisfein, Frueh, and Miller[11,12] have shown that under simplifying assumptions, the rate of decay as a function of speed, assuming zero structural damping, can be obtained from the traditional k-method solution. If g, ω, and V are found with the k method, then, according to Ref. 12

$$2\gamma = \frac{1}{\pi}\ln\frac{a_{n+1}}{a_n} = g\left(1 - \frac{V}{\omega}\frac{d\omega}{dV}\right) \quad (12)$$

Landahl[13] plots the real and imaginary part of the flutter determinant as a function of ω and obtains approximations for γ and ω. Such a plot must be made for each speed to obtain complete frequency damping-velocity diagrams. Natke[2] also describes a method of obtaining rate-of-decay from what is in principle a k-method type solution. These methods are approximate and indirect. Indirect in the sense that they require the availability of the k-method solution.

The p-k Method

Equation (1) suggests an approximate method of finding a rate-of-decay type solution directly. Writing Eq. (1) in a form indicating that the aerodynamic matrix is only available for harmonic motion, and computing $[A(ik)]$ for an estimated value of k:

$$\left[(V^2/c^2)[M]p^2 + [K] - \tfrac{1}{2}\rho V^2[A(ik)]\right]\{q\} = 0 \quad (13)$$

one can solve for $p = \gamma k_1 + ik_1$, compute $[A(ik_1)]$, solve Eq. (13) again which leads to $p = \gamma k_2 + ik_2$, etc., until the imaginary part of the solution equals the k value of the aerodynamics. This method of solution is here called the p-k method. The rationale for this approach is that for sinusoidal motions with slowly increasing or decreasing amplitude, aerodynamics based on constant amplitude are a good approximation.

† Credit for the original determinant iteration program goes to R. F. O'Connell and G. E. Smith of the Lockheed-California Company.

The method is shown in an actual flutter analysis by Irwin and Guyett[14]‡ and is mentioned by Natke[2] and Dat and Muerzec.[3] Recently, Jocelyn Lawrence and Jackson[15] devoted an ARC report to a comparison of methods of solving the flutter equation and they presented the method of Ref. 14 in detail, calling it the "British Method" in contrast to the "American Method" (k method).§

In Refs. 14 and 15, a graphical method is presented to match the imaginary part of p with the k value of the aerodynamics. This author has shown the feasibility of finding roots directly. Figure 2 compares p-k method results and p method results obtained by applying Eq. (8) to the same case of a twin-jet transport as in Fig. 1. Corresponding curves in the upper and lower halves of the figure have like symbols. At all rates-of-decay of significance the p-k method gives a very good approximation of the true rate-of-decay as found with the p method.

In judging the significance of Fig. 2 it must be considered that the aerodynamics implied by Eq. (8), although containing most important aspects of unsteady aerodynamics, is in a rather simple form. It does include aerodynamic lag and steady-state aerodynamic coupling between strips, but not the signal delay between strips. However, Fig. 2 represents the most general comparison known to this author at the moment.

In view of the good agreement shown in Fig. 2, a computer program was developed that makes it possible to find roots according to the p-k method by determinant iteration.

Matrix Iteration Applied to the p-k Method

When the p-k method symbolized by Eq. (13) is programed for the digital computer, significant advantages are gained by generalizing the equation. The equation used in the Lockheed-California Company program is

$$\left[\frac{1}{g_0}[M]\frac{V^2}{c^2}p^2 + [D]\frac{V}{c}p + (1+ig)[K] - \sigma V^2 \frac{1}{2}\rho_0 f_L [A(k)] - H_A\left(\frac{V}{c}p\right)[D_A] - H_B\left(\frac{V}{c}p\right)[D_B] - H_C\left(\frac{V}{c}p\right)[D_C]\right]\{q\} = 0 \quad (14)$$

or

$$[F(p,K)]\{q\} = 0 \quad (15)$$

$[D]$ represents viscous damping; e.g., due to flutter dampers. H_A, H_B, and H_C are transfer functions for hydraulic controls and automatic control systems; they are related to the degrees of freedom by $[D_A]$, $[D_B]$, and $[D_C]$. $1/g_0$ and f_L are scalars, introduced for convenience.

All matrices in Eq. (14) are real and uniquely defined, except $[A(k)]$, which is complex and must be given for a sufficient number of k values. Equation (14) is solved at several values of V and σ, or combinations thereof, for complex roots p associated with modes of interest. Modes of interest are determined from vibration analysis or from previous flutter analyses.

The process of determinant iteration is completed mode by mode for one speed and then at successive preselected speeds. For one mode at one particular speed, the process is started by initial trials for p:

$$p_1 = \delta_1 + ik_1 \qquad p_2 = \delta_2 + ik_2 \quad (16)$$

$[A(k_1)]$ and $[A(k_2)]$ are computed by interpolation. Using

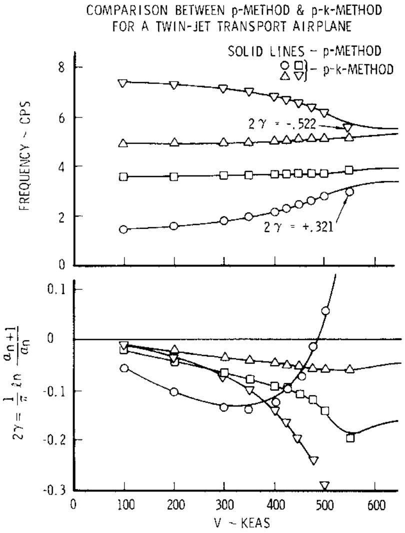

Fig. 2 Comparison between p method and p-k method for a twin-jet transport airplane.

Eq. (14) the values

$$F_1 = |[F(p_1,k_1)]| \qquad F_2 = |[F(p_2,k_2)]| \quad (17)$$

are determined. The Reglua Falsi method gives a first iterated value for p:

$$p_3 = (p_2F_1 - p_1F_2)/(F_1 - F_2) \quad (18)$$

The process is repeated according to the recurrence formula

$$p_{i+2} = (p_{i+1}F_i - p_iF_{i+1})/(F_i - F_{i+1}) \quad (19)$$

until a specified degree of convergence is attained. From the converged root $p_c = \delta_c + ik_c$, the frequency and damping can be computed

$$f = \frac{Vk_c}{2\pi c} \qquad 2\gamma = \frac{1}{\pi}\ln\frac{a_{n+1}}{a_n} = 2\frac{\delta_c}{k_c} \quad (20)$$

To complete one frequency-damping-velocity diagram for several modes, one initial trial for each mode, at the first speed only, must be input. This trial, p_2, may be given as $p_2 = \delta_2 + ik_2$ if available from an earlier computation, or as

$$p_2 = 0 + (2\pi fc/V)i \quad (21)$$

where f is the natural vibration frequency of the mode sought. When using Eq. (21), the first speed, V, should be sufficiently small such that Eq. (21) is a reasonably good trial for p_2. The program computes p_1 as

$$p_1 = -Fk_2 + iGk_2 \quad (22)$$

where usually $F = 0.01$ and $G = 1.00$.

When all the desired roots at the first speed are found, they are used in determining the initial trials for the next speed, $\bar{V}$

$$\bar{p}_1 = (V/\bar{V})p_2 \qquad \bar{p}_2 = (V/\bar{V})p_c \quad (23)$$

and repeated application of Eq. (19) leads to converged roots at the second speed

$$\bar{p}_c = \bar{\delta}_c + i\bar{k}_c \quad (24)$$

The trials for the third speed, $\bar{\bar{V}}$, are

$$\bar{\bar{p}}_1 = (V/\bar{\bar{V}})p_c \qquad \bar{\bar{p}}_2 = (\bar{V}/\bar{\bar{V}})p_c \quad (25)$$

and similarly for the following speeds.

‡ This author's attention was directed to this method by P. R. Guyett when he was visiting the Lockheed-California Co.

§ This author favors identifying methods by names that say something about the method, if it can be done without using unduly long names.

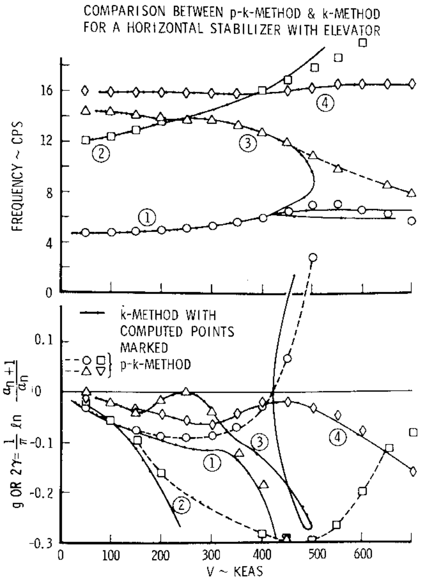

Fig. 3 Comparison between *p-k* method and *k* method for a horizontal stabilizer with elevator.

Features of the Lockheed-California Company Program

The program, Determinant Iteration *p-k* Method, was programed by R. B. Neveceral for the IBM 360 Model 91 as part of the FAMAS system.[16] The existing version is designed such that each iteration can be done largely within core. Therefore, the following restrictions have been imposed: a) maximum order: 50 × 50; b) use four $[A(k)]$ matrices for the interpolation of the aerodynamic matrices; c) the matrices $[D_A]$, $[D_B]$, and $[D_C]$ have a maximum of 20 nonzero elements; and d) H_A, H_B, and H_C each are the quotient of 8th degree polynomials in $(V/c)p$.

The four aerodynamic matrices used for each interpolation are determined by the value of the imaginary part of p used to evaluate $|[F(p,k)]|$. During the first three iteration steps, the aerodynamic matrices are chosen such that two of their k values are above and two are below the imaginary part of p. From then on, the same four aerodynamic matrices are used; even if during the iteration process the imaginary part of p wanders outside the range between the inner two k values, as long as it does not wander too far into the adjacent range. This feature was included after it was found that, when the final k value of the solution is close to one of the input values, "hunting" may occur. Namely, going from one iterated value to the next, the imaginary part may demand a different set of aerodynamic matrices and the next iterated value may call the earlier four aerodynamic matrices back.

For the determinant iteration to work satisfactorily, the initial trials that start the iteration must be reasonably good. Thus, when going from one preselected speed to the next, a root may vary so much that the trials provided by the program do not lead to convergence. In that case, for the mode concerned, the program cuts the interval to the previous speed in smaller subintervals.

As a result of the previous feature, speed intervals may be chosen rather large, such that when flutter occurs there are not enough points to determine the flutter speed accurately by fairing a curve. To get a better definition of the flutter speed, the program will solve Eq. (14) at additional speeds in the flutter region any time a mode becomes unstable.

There is an option to run the program at constant σ (a maximum of five σ values per case) or at σ varying with speed. If in the latter case σ varies as defined by a constant Mach number in the atmosphere, the program computes the σ values corresponding to the velocities chosen. The σ values for any other speed-vs-σ relationship must be listed on a control card. The output of the program is arranged such that f vs V and 2γ vs V plots can be obtained from the CALCOMP digital incremental plotter.

Discussion

A good deal of experience with the Determinant Iteration *p-k* Method Program has been gathered at the Lockheed-California Company. Figure 3 shows one of the early check cases for which an earlier *k*-method solution was available. The *k*-method solution is represented by the solid lines with computed points marked. Corresponding curves in upper and lower half of the figure are numbered 1–4. The *p-k* method solution is represented by the symbols and where it is significantly different from the *k*-method solution, by dashed lines. A case of a horizontal stabilizer with elevator is chosen for which the original k solution can be easily interpreted incorrectly even though the k values at which the flutter equation is solved are reasonably close together. A sufficient number of check cases has been done to show the agreement at the flutter speed between k method and *p-k* method.

It is thought that the *p-k* method is economically competitive with the k method, although no precise comparisons have been made. Much depends on details of the computer system available. The *p-k* method has two distinct advantages, however, the frequency-damping-speed plots are more easily interpreted and frequency-damping-speed plots can be obtained in which the solution at each speed is for the density in the atmosphere corresponding to that speed and the Mach number. Thus, computer runs for several different constant densities are avoided.

In developing the program, elements of the $[A(k)]$ matrix were plotted vs k, and surprisingly irregular curves were found for the case in which a 64-degree-of-freedom system was reduced to a modalized 15-degree-of-freedom system, using natural vibration modes. This has given rise to the hunting mentioned earlier. When eliminating hunting, by retaining the same four aerodynamic matrices as long as the imaginary part of p does not wander away too far, the problem of nonuniqueness of the aerodynamics is introduced. A compromise can be found in which hunting is virtually avoided, and nonuniqueness of the aerodynamics is restricted to small ranges of k values (say, 5% of each interval at each end of the interval).

Possibly better behaved aerodynamic matrices are found if, instead of writing Eq. (4), the aerodynamic forces are expressed as

$$\{Q_{\text{aero}}\} = \tfrac{1}{2}\rho V^2[A_\alpha(k)]\{\alpha\} \tag{26}$$

from which

$$\{Q_{\text{aero}}\} = \tfrac{1}{2}\rho V^2[A_\alpha(k)]([D_\theta] + p[D_z])\{q\} \tag{27}$$

However, this increases the number of matrices that must be kept in store for the iteration; as a result, the maximum order that can be handled will be less than 50.

At low speed and high frequency, the k values are high and often lead to extrapolation for $[A(k)]$, which in turn may lead to nonconvergence or convergence on the wrong roots. To avoid problems, an aerodynamic matrix for an arbitrarily high k value is included (say, $k = 50$ or 100). The $[A(k)]$ for that value is derived from piston theory or made equal to the regularly computed $[A(k)]$ matrix at the highest k value.

The present program makes it possible to find control surface rotation modes that in the k method often escape attention because, in the frequency-vs-speed diagram, they fall between two constant k lines. Also, the stability modes, except the phugoid, can be found at the same time that the flutter modes are found.

The program can be expanded to include computing the effect of parameter variations on the frequency and the damping at one speed. Once the frequency and damping at that speed have been determined, say for $[M_1]$, $[D_1]$ or $[K_1]$ the roots can be used as first trials to start the iteration at the same speed for a slightly different matrix $[M_2]$, $[D_2]$ or $[K_2]$. This may have advantages in the case of the flutter analysis of airplanes with a large number of possible external wing store combinations, or for other parameter studies. The feasibility of this process was recently demonstrated for the variation of the viscous damping $[D]$.

Determinant iteration was used for finding the flutter roots because it was readily available. It can be used without first putting the flutter equation in the canonical form and the flutter determinant can be an intricate function of p. (Thus, determinant iteration can be used if aerodynamic approximations are complicated formulas of p, such as those proposed by Richardson.[4]) However, other methods of finding the roots may be modified to suit the needs of the p-k method, and it has already been suggested[17] that a modified power method (matrix iteration) may be used successfully.

Conclusions

1) An approximate but direct method of finding rate-of-decay type solutions of the flutter equation, even when the aerodynamic forces are available for harmonic motion only, is presented. 2) A determinant iteration method is used for solving the flutter equation which is generalized to include viscous damping, structural damping and several transfer functions representing automatic control systems. 3) The validity of the method is demonstrated by comparing solutions obtained by different methods. 4) Matrix iteration (power method) and other methods, may, after modification, provide alternate means of finding the flutter roots according to the p-k method.

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