ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

TITLE CENTURY GOTHIC BOLD 18 PUNTO

Presenter or METU EEE Century Gothic Regular 14 Punto

April 29, 2014
Place

DESIGN OF KALMAN FILTER BASED ATTITUDE DETERMINATION AND CONTROL ALGORITHMS BY USING SOME ACTUATORS FOR A LEO SATELLITE

Gamze EFENDİOĞLU
Advisor: Prof. Dr. M. Kemal LEBLEBİCİOĞLU

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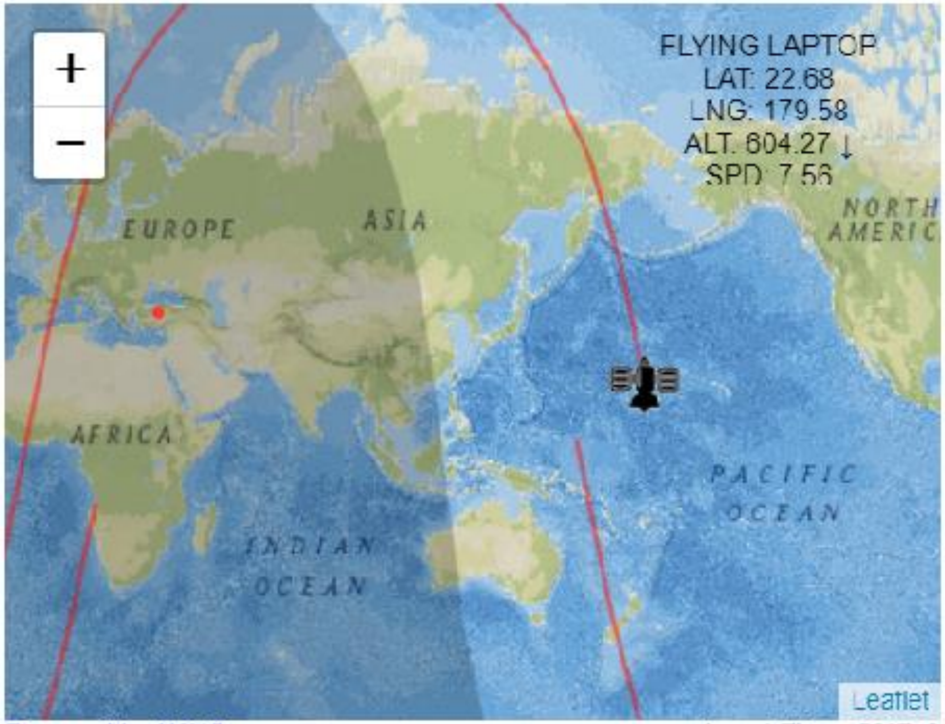
The Selected Satellite Specifications

FLYING LAPTOP

Track FLYING LAPTOP now!

10-day predictions

NORAD ID: 42831
Int'l Code: 2017-042G
Perigee: 591.7 km
Apogee: 612.0 km
Inclination: 97.6 °
Period: 96.6 minutes
Semi major axis: 6972 km
RCS: Unknown
Launch date: July 14, 2017
Source: Germany (GER)
Launch site: TYURATAM MISSILE AND SPACE COMPLEX (TTMTR)



Powered by N2YO.com   Local Time: GMT+3

NEXT PASS OF FLYING LAPTOP OVER YOUR CURRENT LOCATION						
START AZIMUTH		MAX ELEVATION		END AZIMUTH		TOTAL DURATION
Sep 5 09:08	113° ESE	09:13	11°	09:18	13° N	09m 35s

The Satellite Specifications

Line	TLE Data Set (FLP)							
1	42831U	17042G	19164.90037843	+.00000129	+00000-0	+18434-4	0	9993
2	**42831**	**097.5659**	**058.0490**	**0015745**	**077.4852**	**282.8127**	**14.91002723104220**	

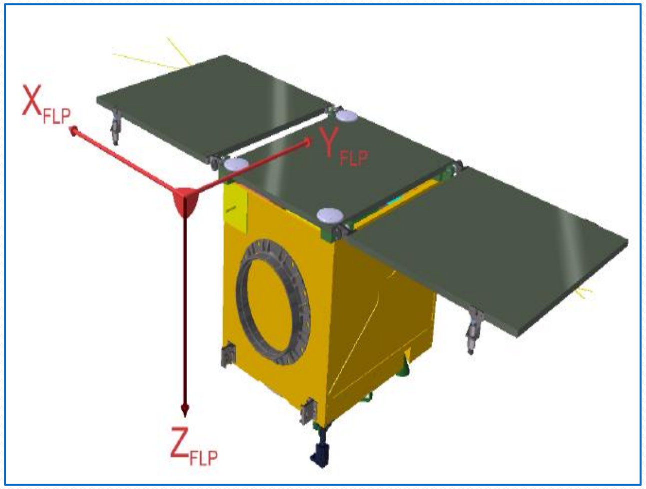
Orbital Parameters	Abb.	Value	Value
Inclination	i	097.5659 (deg)	1.7028462 rad
Right Ascension of The Ascending Node	Ω	612.9 (deg)	10.697123 rad
Eccentricity	e	0.015745	--
Argument of Perigee	ω	077.4852 (deg)	1.3523718 rad
Mean Anomaly	M	282.8127 (deg)	4.936013 rad
Mean Motion	n	14.910027 (rev/day)	0.00108 rad/s

Orbital Parameters	Abb.	Value	Dimension
Perigee	r_p	591.0	km
Apogee	r_a	612.9	km
Period	T	96.6	minutes
Semi Major Axis	a	6991.4	km

$$\frac{T^2}{a^3} = Cte$$

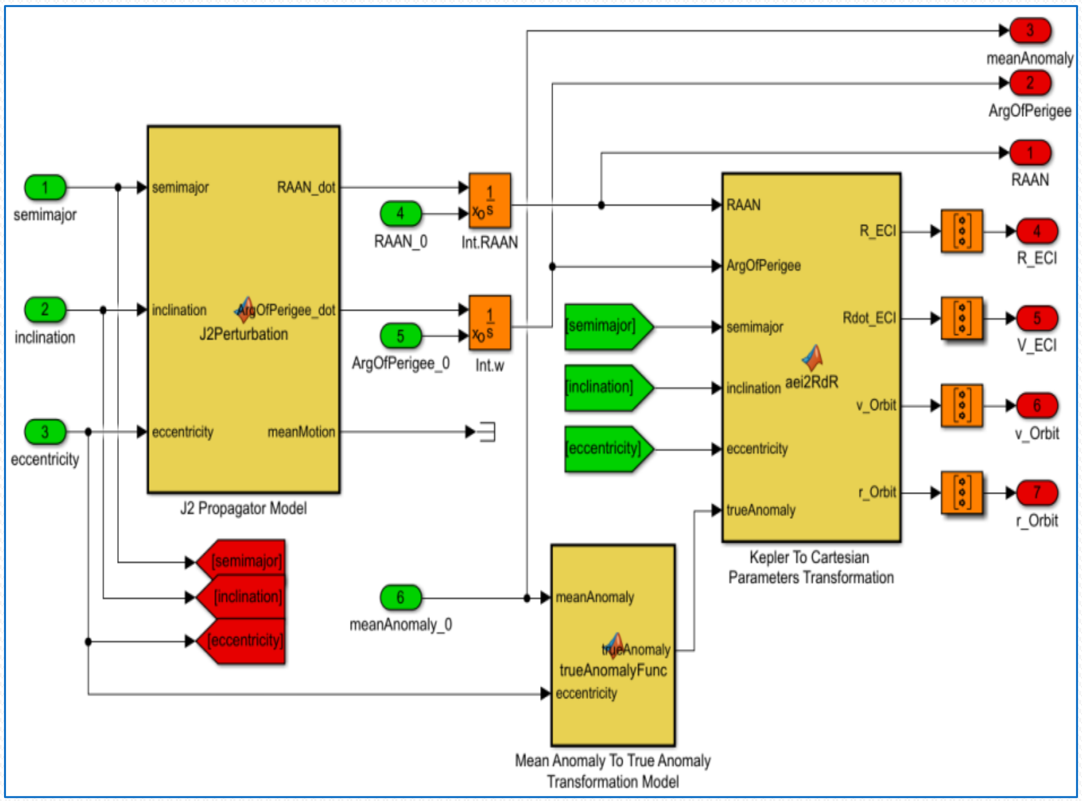
The Satellite Specifications

FLP Microsatellite Characteristics	
Dimensions	60 × 70 × 80 cm
Mass	117 kg
Orbit Type	Circular and Polar Orbit
Orbit Altitude	~ 700 km
Attitude Control	Three Axis Stabilized
Solar Panels	3 Solar Panels (2 deployable)



Orbit Propagator Model

- In order to determine the satellite attitude from the reference sensors, it is needed to know the satellite's orbit and its position in orbit.



$$E(t) = M(t) + e.sin(E(t))$$

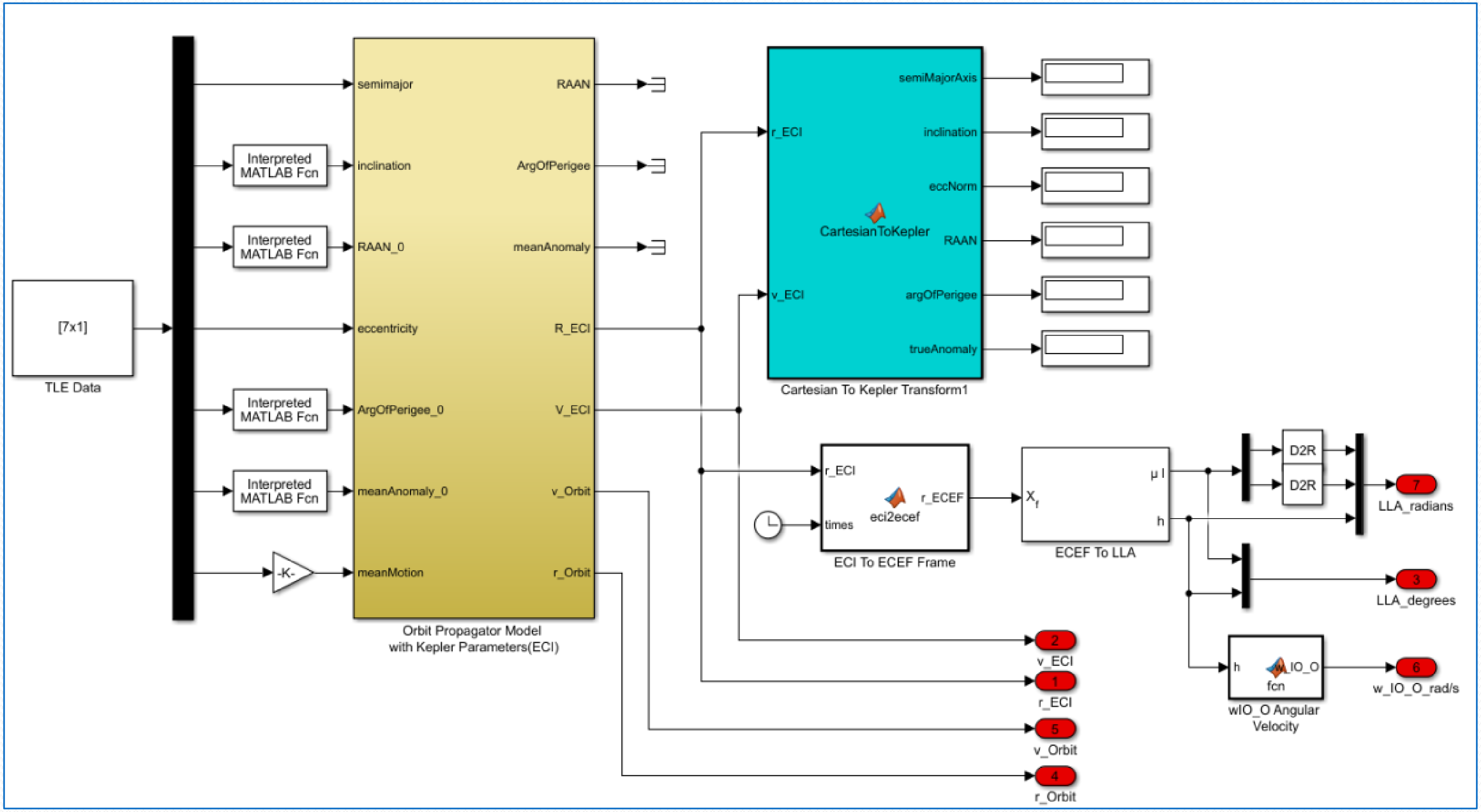
$$M(t_0 + t) = M(t_0) + n.t$$

$$M_{i+1} = M_i + n.t$$

$$E_{i+1} = M_i + e.sin(E_i)$$

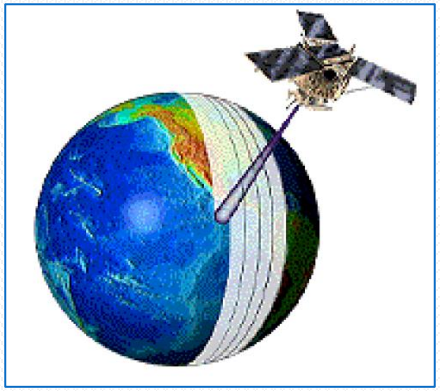
$$E_{i+1} = E_i + \frac{M_i + e.sin(E_i) - E_i}{1 - e.cos(E_i)}$$

Orbit Propagator Model



The Problem Definition

- Controlling a microsatellite attitude and its orientation
- Pointing towards a specific direction
- Maintaining a desired attitude



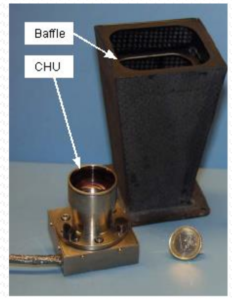
To provide these requirements:

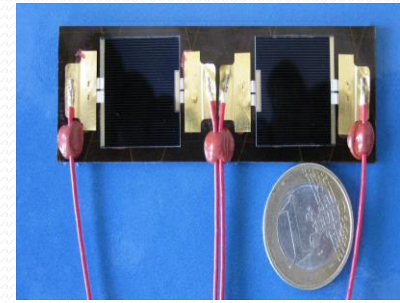
- Modelling satellite dynamics/kinematics
- Modelling attitude actuators (RW, MTR) and sensors
- Simulating space environment and external effects
- Linearizing the system models for LQR design
- Designing control methods (PID, LQR, SMC)
- Designing Desaturation and Detumbling Control
- Estimating sensor measurements with EKF

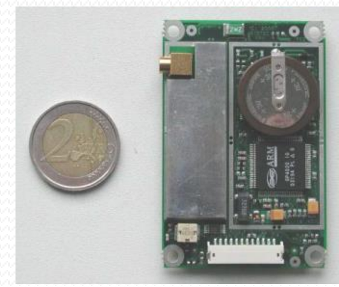
AOCS Sensors - Actuators

Sensors:

- Star Tracker
- Gyroscope
- Sun Sensor
- Magnetometer
- GPS





Actuators:

- Reaction Wheels
- Magnetorquer


AOCS Sensors

	MGM	STR	GPS	SS	FOG
Output	Magnetic Field Vector	Quaternion Vector	Position Velocity	Sun Position	Angular Rate
Dimension	(3×1)	(4×1)	(3×1)	(3×1)	(3×1)
Quantity	2 MGM	2 STR	3 GPS	8 SS	4 FOG
Accuracy	5 nT	5 arc sec	10 m 0.1 m/s	50 mA	2×10^{-6}deg/s
Control Rate	1.5, 3, 6 Hz	5 Hz	1 Hz	10 Hz	10 Hz

Satellite Dynamic Equations

$$M_{TOT} = M_C^{RW} + M_C^{MTR} + M_D$$

$$M_C^{RW} = \dot{H}_{RW}^B + (w_{IB}^B \times H_{RW}^B)$$

$$M_D = M_{GG} + M_{AERO} + M_{SR} + M_{MAG}$$

$$M_{TOT} = \frac{d\overrightarrow{H_I}}{dt} = \frac{d\overrightarrow{H_B}}{dt} + \overrightarrow{w_{IB}^B} \times \overrightarrow{H_B}$$

$$M_{TOT} = I_S \, . \, \dot{w}_{IB}^B + (w_{IB}^B \times I_S \, . \, w_{IB}^B)$$

$$H_B = H_S^B + H_{RW}^B$$

$$M_D = \frac{d}{dt}\left(H_S^B + H_{RW}^B \right) + w_{IB}^B \, x \, (H_S^B + H_{RW}^B)$$

$$M_D = I_S \, . \, \dot{w}_{IB}^B + \Omega(w_{IB}^B). I_S \, . \, w_{IB}^B + \dot{H}_{RW}^B + \Omega(w_{IB}^B) \, . \, H_{RW}^B$$

$$\dot{w}_{IB}^B = I_S^{-1}. \left[M_D - \dot{H}_{RW}^B - \Omega(w_{IB}^B). I_S . w_{IB}^B - \Omega(w_{IB}^B) \, . \, H_{RW}^B \right]$$

Satellite Kinematic Equations

$$\dot{q} = \frac{1}{2}.\left[\Omega\left(w_{OB}^{B}\right)\right].q$$

$$\Omega\left(w_{OB}^{B}\right) = \begin{bmatrix} 0 & w_{OB_3}^{B} & -w_{OB_2}^{B} & w_{OB_1}^{B} \\ -w_{OB_3}^{B} & 0 & w_{OB_1}^{B} & w_{OB_2}^{B} \\ w_{OB_2}^{B} & -w_{OB_1}^{B} & 0 & w_{OB_3}^{B} \\ -w_{OB_1}^{B} & -w_{OB_2}^{B} & -w_{OB_3}^{B} & 0 \end{bmatrix}$$

$$w_{OB}^{B} = w_{IB}^{B} - [C(q)]_{O}^{B}.w_{IO}^{O}$$

$$w_{OB}^{B} = \begin{bmatrix} w_{IB_1}^{B} \\ w_{IB_2}^{B} \\ w_{IB_3}^{B} \end{bmatrix} + \begin{bmatrix} 2(q_1.q_2+q_3.q_4) \\ (q_4^2 - q_1^2 + q_2^2 - q_3^2) \\ 2(q_2.q_3-q_1.q_4) \end{bmatrix}.w_0$$

$$w_{IO}^{O} = [0 \quad -w_0 \quad 0]^T$$

$$w_0 \cong 0.0069\ \frac{rad}{s}$$

Disturbance Torques

$$M_D = M_{GG} + M_{SR} + M_{MAG} + M_{AERO}$$

Gravity Gradient Torque

Solar Radiation Torque

Magnetic Field Torque

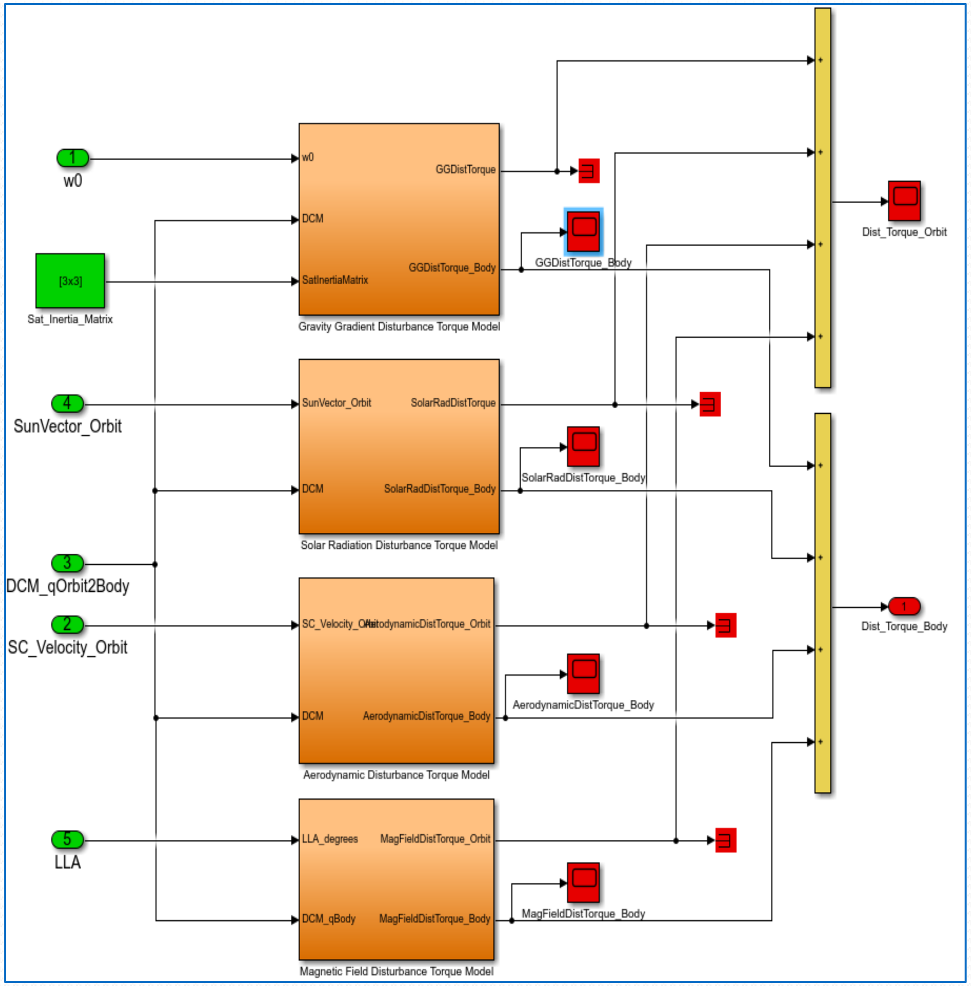
Aerodynamics Torque

→ $M_{GG}^{B} = 3.w_0^2.\Omega(R_3).I_S.R_3$

→ $M_{SR}^{B} = C_r\,\dfrac{k.I_S.A_S}{c}.\left(\dfrac{A_U}{R}\right)^2.\left(\dfrac{R_{sat} - R_{sun}}{R}\right)$

→ $M_{MAG}^{B} = m\ x\ B^B = \Omega(-B^B).m$

→ $M_{AERO}^{B} = \dfrac{1}{2}.A_s.\rho.C_D.V^2$



The magnitude of total disturbances torques ≅ 10^{-5} Nm

State Space Definition

- Nonlinear System State Equations – State and Input Vectors

$$\dot{x}_k = x_{k+1} = f(\, x_k \,, u_k \,, w_k \,, k) = A_k . x_k + B_k . u_k + G_k . w_k \;; w_k \sim N(0, Q_k)$$

$$\dot{x}_k = \begin{bmatrix} \dot{w}_{IB}^{B} \\ \dot{q} \\ \dot{H}_{RW}^{B} \end{bmatrix} = A_k . \begin{bmatrix} w_{IB}^{B} \\ q \\ H_{RW}^{B} \end{bmatrix} + B_k . \begin{bmatrix} M_C^{RW} \\ M_C^{MTR} \\ M_D \end{bmatrix} + Q_k . \begin{bmatrix} M_C^{RW} \\ M_C^{MTR} \\ M_D \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{w}_{IB}^{B} \\ \dot{q} \\ \dot{H}_{RW}^{B} \end{bmatrix} = \begin{bmatrix} I_S^{-1} . [\, M_D + M_C^{RW} + M_C^{MTR} - \Omega(w_{IB}^{B}) . I_S . w_{IB}^{B} - \Omega(w_{IB}^{B}) \, . H_{RW}^{B} \,] \\ \frac{1}{2} \, . \Omega\left(w_{OB}^{B}\right) . \, q \\ -M_C^{RW} \end{bmatrix}$$

State Space Definition

- Nonlinear System Measurement Equations - I

$$y_k = h\,(x_k\,, v_k\,, k) = H_k . x_k + D_k . u_k + v_k\,;\, v_k \sim N(0, R_k)$$

$$y_k = \begin{bmatrix} w_{meas} \\ q_{meas} \\ B_{meas} \\ SV_{meas} \\ r_{meas} \\ v_{meas} \end{bmatrix} = H_k . \begin{bmatrix} w_{IB}^{B} \\ q \\ H_{RW}^{B} \end{bmatrix} + D_k . \begin{bmatrix} M_C^{RW} \\ M_C^{MTR} \\ M_D \end{bmatrix} + v_k$$

$$y_k = \begin{bmatrix} w_{meas} \\ q_{meas} \\ B_{meas} \\ SV_{meas} \\ r_{meas} \\ v_{meas} \end{bmatrix} = \begin{bmatrix} w_{IB}^{B} + v_{GYRO} \\ q^B + v_{STR} \\ B_B + v_{MGM} \\ SV_B + v_{SuS} \\ r_B + v_{GPS} \\ v_B + v_{GPS} \end{bmatrix} = \begin{bmatrix} [C_O^B].w_O + v_{GYRO} \\ [C_O^B].q_O + v_{STR} \\ [C_O^B].B_O + v_{MGM} \\ [C_O^B].SV_O + v_{SuS} \\ [C_O^B].r_O + v_{GPS} \\ [C_O^B].v_O + v_{GPS} \end{bmatrix}$$

- Nonlinear System Measurement Equations - II

$$H_k.\Delta x_k = \begin{bmatrix} [I]_{3x3} & 0_{3x4} & 0_{3x3} \\ 0_{4x3} & [I]_{4x4} & 0_{4x3} \\ 0_{3x3} & [H_{MGM}^{non}]_{3x4} & 0_{3x3} \\ 0_{3x3} & [H_{SuS}^{non}]_{3x4} & 0_{3x3} \\ 0_{3x3} & [H_{GPS1}^{non}]_{3x4} & 0_{3x3} \\ 0_{3x3} & [H_{GPS2}^{non}]_{3x4} & 0_{3x3} \end{bmatrix} . \begin{bmatrix} \Delta w_{IB}^{B} \\ \Delta q \\ \Delta H_{RW}^{B} \end{bmatrix}$$

$$R_k = \begin{bmatrix} I_{3x3}.R_{GYRO} & 0_{3x4} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{4x3} & I_{4x4}.R_{STR} & 0_{4x3} & 0_{4x3} & 0_{4x3} & 0_{4x3} \\ 0_{3x3} & 0_{3x4} & I_{3x3}.R_{MGM} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x4} & 0_{3x3} & I_{3x3}.R_{SuS} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x3} & I_{3x3}.R_{GPS1} & 0_{3x3} \\ 0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3}.R_{GPS2} \end{bmatrix} \begin{bmatrix} w_B \\ q_B \\ B_B \\ SV_B \\ r_B \\ v_B \end{bmatrix}$$

$$R_{GYRO} = R_{STR} = 1x10^{-12} \; ; \; R_{MGM} = R_{SuS} = R_{GPS1} = R_{GPS2} = 1x10^{-7}$$

Linearization of System Model

- Linearization wrt. the first order of Taylor series expansion:

$$\dot{x}_k = x_{k+1} = f(x_k, u_k, k) = A_k . x_k + B_k . u_k$$

$$y_k = h(x_k, u_k, k) = H_k . x_k + D_k . u_k$$

$$A_k . x_k = \begin{bmatrix} \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{w}_{IB}^B} & \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{q}} & \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{H}_{RW}^B} \\ \frac{\partial \dot{\bar{q}}}{\partial \bar{w}_{IB}^B} & \frac{\partial \dot{\bar{q}}}{\partial \bar{q}} & \frac{\partial \dot{\bar{q}}}{\partial \bar{H}_{RW}^B} \\ \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{w}_{IB}^B} & \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{q}} & \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{H}_{RW}^B} \end{bmatrix} . \begin{bmatrix} w_{IB}^B \\ q \\ H_{RW}^B \end{bmatrix}$$

$$B_k . u_k = \begin{bmatrix} \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{M}_C^{RW}} & \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{M}_C^{MTR}} & \frac{\partial \dot{\bar{w}}_{IB}^B}{\partial \bar{M}_D} \\ \frac{\partial \dot{\bar{q}}}{\partial \bar{M}_C^{RW}} & \frac{\partial \dot{\bar{q}}}{\partial \bar{M}_C^{MTR}} & \frac{\partial \dot{\bar{q}}}{\partial \bar{M}_D} \\ \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{M}_C^{RW}} & \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{M}_C^{MTR}} & \frac{\partial \dot{\bar{H}}_{RW}^B}{\partial \bar{M}_D} \end{bmatrix} . \begin{bmatrix} M_C^{RW} \\ M_C^{MTR} \\ M_D \end{bmatrix}$$

Linearization of Measurement Model

	The Equations of Measurements
SS	$SV_{meas}^{BODY} = \left(\left[C_{ORBIT}^{BODY}\right].SV_{ORBIT}\right) + v_{SuS}$
MGM	$B_{meas}^{BODY} = \left(\left[C_{ORBIT}^{BODY}\right].B_{ORBIT}\right) + v_{MGM}$
GPS	$r_{meas}^{BODY} = \left(\left[C_{ORBIT}^{BODY}\right].r_{ORBIT}\right) + v_{GPS}$ $vel_{meas}^{BODY} = \left(\left[C_{ORBIT}^{BODY}\right].vel_{ORBIT}\right) + v_{GPS}$
STR	$q_{meas}^{BODY} = [H_{STR}].x_k + v_{STR}$
FOG	$w_{meas}^{GYRO} = [H_{GYRO}].x_k + v_{GYRO}$

$$\left.\frac{\partial\left[C_{ORBIT}^{BODY}\right]}{\partial x_k}\right|_{x_k=\bar{q}_1} = 2.\begin{bmatrix} q_1 & q_2 & q_3 \\ q_2 & -q_1 & q_4 \\ q_3 & -q_4 & -q_1 \end{bmatrix}$$

$$\left.\frac{\partial\left[C_{ORBIT}^{BODY}\right]}{\partial x_k}\right|_{x_k=\bar{q}_2} = 2.\begin{bmatrix} -q_2 & q_1 & -q_4 \\ q_1 & q_2 & q_3 \\ q_4 & q_3 & -q_2 \end{bmatrix}$$

$$\left.\frac{\partial\left[C_{ORBIT}^{BODY}\right]}{\partial x_k}\right|_{x_k=\bar{q}_3} = 2.\begin{bmatrix} -q_3 & q_4 & q_1 \\ -q_4 & -q_3 & q_2 \\ q_1 & q_2 & q_3 \end{bmatrix}$$

$$\left.\frac{\partial\left[C_{ORBIT}^{BODY}\right]}{\partial x_k}\right|_{x_k=\bar{q}_4} = 2.\begin{bmatrix} q_4 & q_3 & -q_2 \\ -q_3 & q_4 & q_1 \\ q_2 & -q_1 & q_0 \end{bmatrix}$$

Linear State Space Model - I

- Operating Points:

$\longrightarrow$ $\overline{w}_{IB}^{B} = \left[\, \overline{w}_{IB_1}^{B} \quad \overline{w}_{IB_2}^{B} \quad \overline{w}_{IB_3}^{B} \,\right]^T = [\,0 \quad 0 \quad 0\,]^T$

$\longrightarrow$ $\bar{q} = [\,\bar{q}_1 \quad \bar{q}_2 \quad \bar{q}_3 \quad \bar{q}_4\,]^T = [\,0 \quad 0 \quad 0 \quad 1\,]^T$

$\longrightarrow$ $\overline{H}_{RW}^{B} = \left[\, \overline{H}_{RW_1}^{B} \quad \overline{H}_{RW_2}^{B} \quad \overline{H}_{RW_3}^{B} \,\right]^T = [\,0 \quad 0 \quad 0\,]^T$

$$\bar{A}_k = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0.0033 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.010 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.0033 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0033 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{B}_k = \begin{bmatrix} 0.1415 & 0 & 0 & 0.1415 & 0 & 0 & 0.1415 & 0 & 0 \\ 0 & 0.1439 & 0 & 0 & 0.1439 & 0 & 0 & 0.1439 & 0 \\ 0 & 0 & 0.1169 & 0 & 0 & 0.1169 & 0 & 0 & 0.1169 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Linear State Space Model - II

$$
\overline{H}_k = \left[\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & 2\begin{bmatrix}0\\B_{Oz}\\-B_{Oy}\end{bmatrix} & 2\begin{bmatrix}-B_{Oz}\\0\\B_{Ox}\end{bmatrix} & 2\begin{bmatrix}B_{Oy}\\-B_{Ox}\\0\end{bmatrix} & 2\begin{bmatrix}B_{Ox}\\B_{Oy}\\B_{Oz}\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\
\begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & 2\begin{bmatrix}0\\SV_{Oz}\\-SV_{Oy}\end{bmatrix} & 2\begin{bmatrix}-SV_{Oz}\\0\\SV_{Ox}\end{bmatrix} & 2\begin{bmatrix}SV_{Oy}\\-SV_{Ox}\\0\end{bmatrix} & 2\begin{bmatrix}SV_{Ox}\\SV_{Oy}\\SV_{Oz}\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\
\begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & 2\begin{bmatrix}0\\r_{Oz}\\-r_{Oy}\end{bmatrix} & 2\begin{bmatrix}-r_{Oz}\\0\\r_{Ox}\end{bmatrix} & 2\begin{bmatrix}r_{Oy}\\-r_{Ox}\\0\end{bmatrix} & 2\begin{bmatrix}r_{Ox}\\r_{Oy}\\r_{Oz}\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\
\begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & 2\begin{bmatrix}0\\v_{Oz}\\-v_{Oy}\end{bmatrix} & 2\begin{bmatrix}-v_{Oz}\\0\\v_{Ox}\end{bmatrix} & 2\begin{bmatrix}v_{Oy}\\-v_{Ox}\\0\end{bmatrix} & 2\begin{bmatrix}v_{Ox}\\v_{Oy}\\v_{Oz}\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix}
\end{array}\right]
$$

For simplicity → $\overline{H}_k = [I_{10x10}]$ $\overline{D}_k = [0_{10x9}]$

Controllability - Stability

→ Q_C matrix has full row rank → $rank(Q_C) = 10 = n$

$$Q_C = [\, \bar{B}_k \mid \bar{A}_k \bar{B}_k \mid (\bar{A}_k)^2 \bar{B}_k \mid (\bar{A}_k)^3 \bar{B}_k \mid (\bar{A}_k)^4 \bar{B}_k \mid (\bar{A}_k)^5 \bar{B}_k \mid (\bar{A}_k)^6 \bar{B}_k \,]$$

→ Q_o matrix has full row rank → $rank(Q_o) = 10 = n$

$$Q_o = \left[\bar{H}_k{}^T \mid \bar{A}_k{}^T \bar{H}_k{}^T \mid (\bar{A}_k{}^2)^T \bar{H}_k{}^T \mid (\bar{A}_k{}^3)^T \bar{H}_k{}^T \mid (\bar{A}_k{}^4)^T \bar{H}_k H h^T \mid (\bar{A}_k{}^5)^T \bar{H}_k{}^T \mid (\bar{A}_k{}^6)^T \bar{H}_k{}^T \right]$$

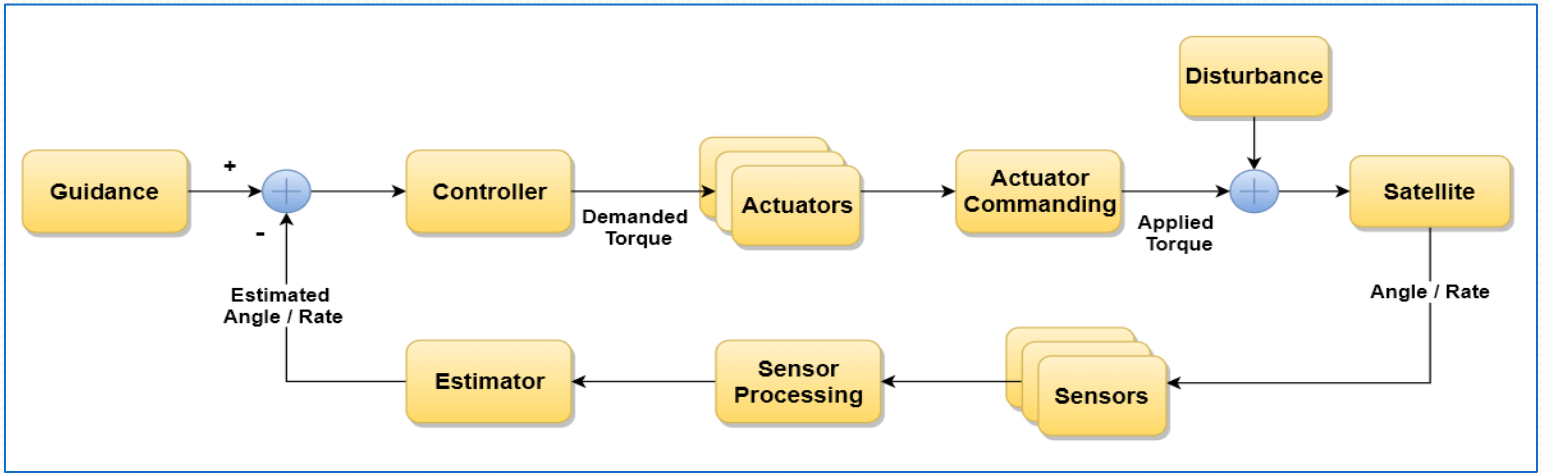
→ Lyapunov candidate function for system stability:

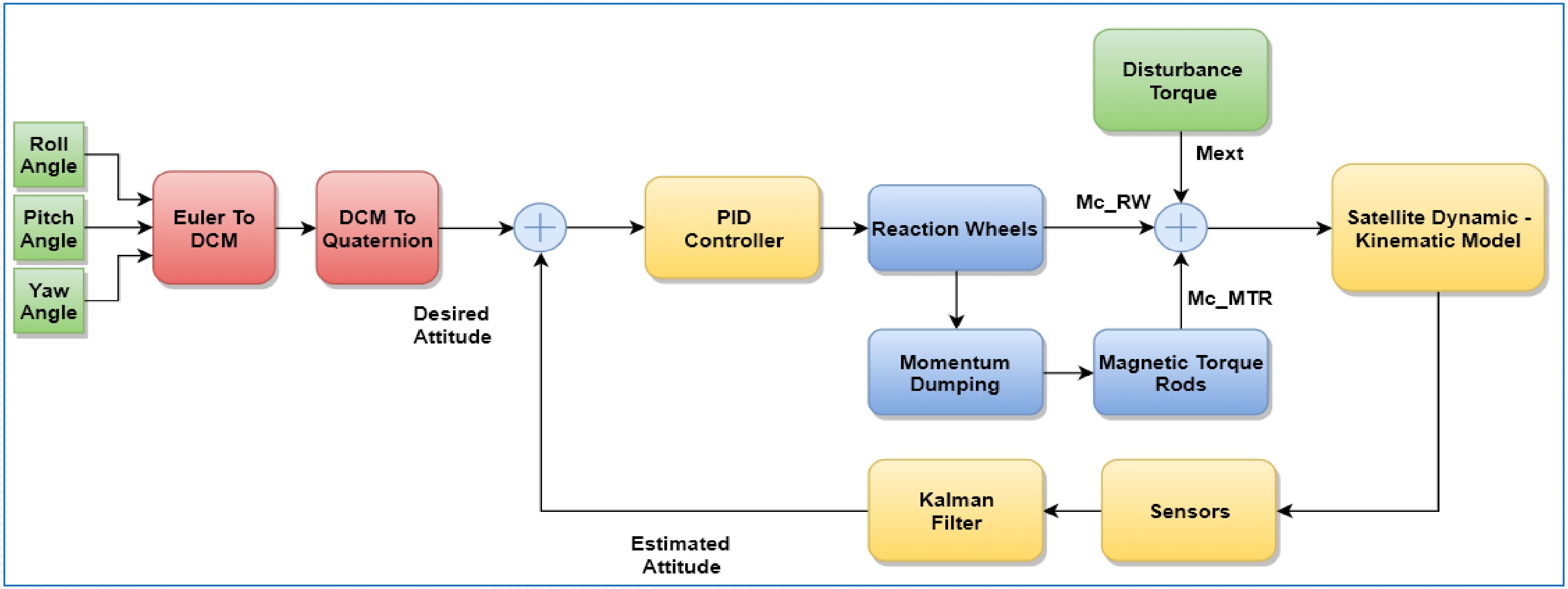
$$\boxed{V(x) = E_{TOT} = E_{KIN} + E_{POT} = E_{KIN} + E_{GG} + E_{GYRO}}$$

$$\boxed{V(q) = (q_v)^T . q_v + (1 - q_4)^2}$$

$$\boxed{\dot{V}(x) = \left(w_{OB}^B\right)^T . M_{cmd}}$$

Satellite Modelling and Control



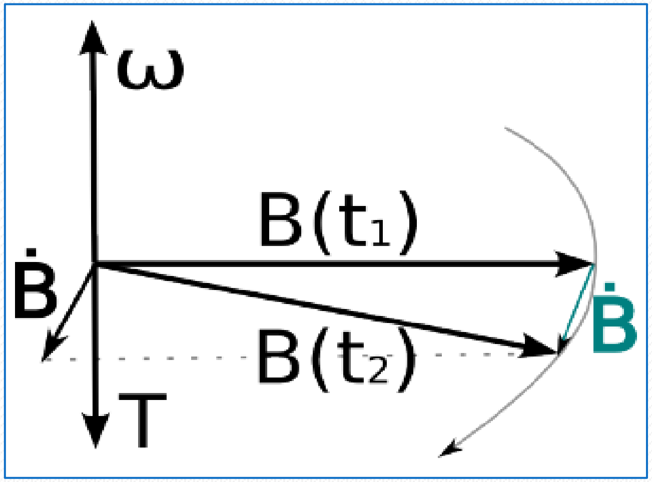


Detumbling Control

B-dot controller is to slow down satellite initial rotational motion.

$$\dot{V} = \dot{E}_{KIN} = -(m^B)^T . (w_{IB}^B \times B^B) < 0$$

$$\dot{B}^B = \left.\frac{dB^B}{dt}\right|_{B^B} = \left(\frac{dB^B}{dt}\right) - w_{IB}^B \times B^B \approx -w_{IB}^B \times B$$



$$\boxed{m^B = K_{Bdot} . (w_{IB}^B \times B^B) = \frac{K_{Bdot} . (w_{IB}^B \times B^B)}{\|B^B\|}}$$

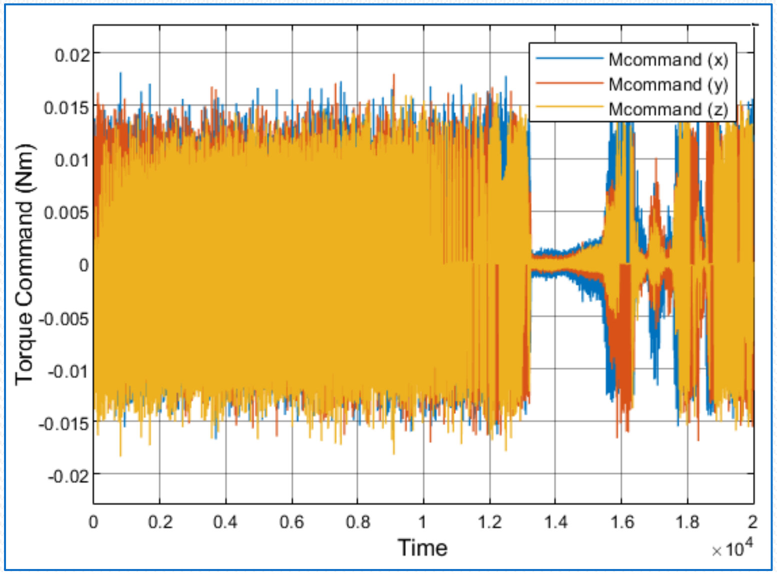
$\rightarrow$

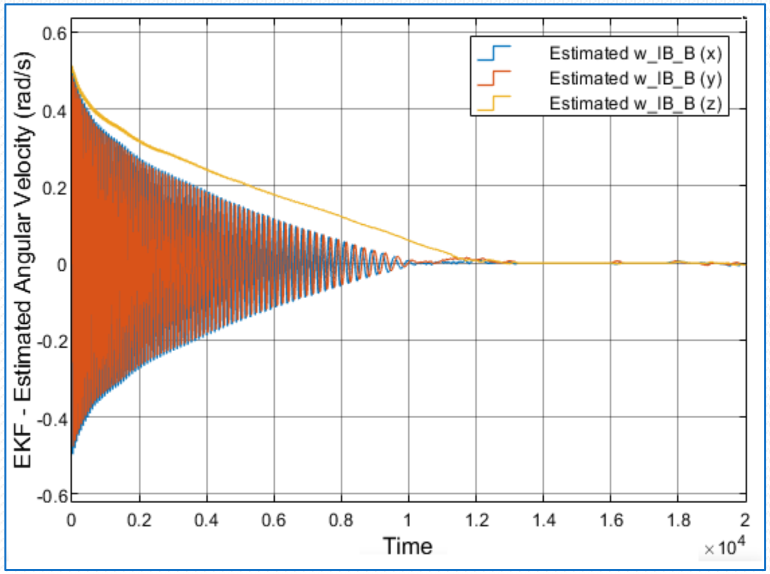
$$\boxed{m^B = \frac{-K_{Bdot} . \dot{B}^B}{\|B^B\|}}$$

$$\boxed{M_c^{MTR} = m^B \times B^B = \frac{-K_{Bdot} . \dot{B}^B}{\|B^B\|} \times B^B}$$

Detumbling Control - Test I

Parameters	Values
Initial Satellite Velocity	$w_0 = [0.5, 0.0, 0.5]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0,0]$; $[\psi_d, \theta_d, \Phi_d] = [-15, -5,5]$
Constant Controller Gain $K_{Bdot} = [\ I_{S_x} x\ 10^4 \quad I_{S_y} x\ 10^4 \quad I_{S_z} x\ 10^4\ ]$	$K_{Bdot,x} = 7.066197 \text{ x } 10^4$; $K_{Bdot,y} = 6.950219 \text{ x } 10^4$; $K_{Bdot,y} = 8.555828\ \ x\ 10^4$

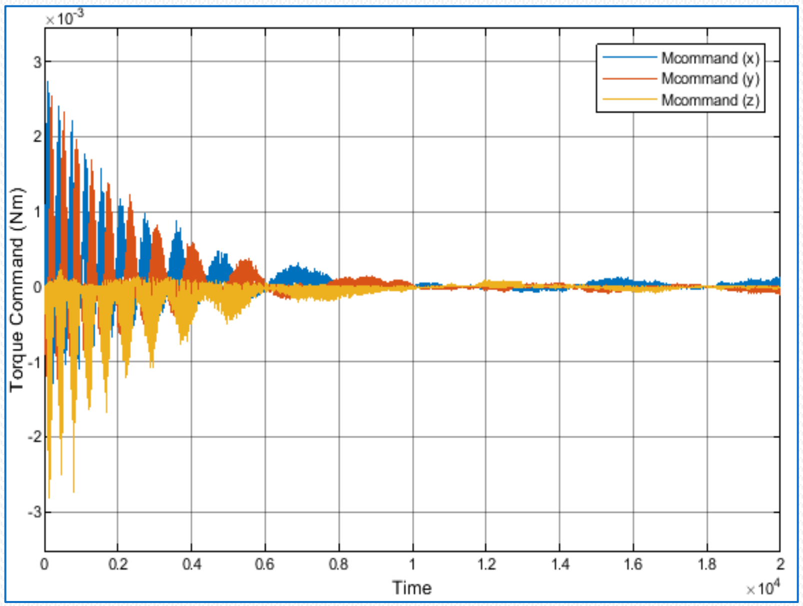


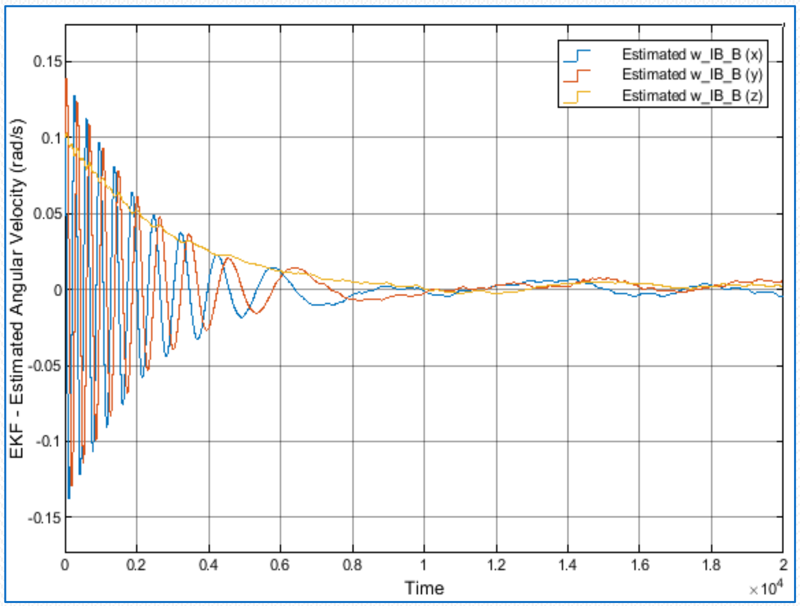


Settling time is about ~13250 sec. and angular velocity is between +/-0.01

Detumbling Control – Test II

Parameters	Values
Initial Satellite Velocity	$w_0 = [0.1, 0.1, 0.1]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0,0]$; $[\psi_d, \theta_d, \Phi_d] = [-15, -5,5]$
Constant Controller Gain	$K_{Bdot,x} = 10;\ K_{Bdot,y} = 10;\ K_{Bdot,y} = 10$





Settling time is about ~10000 sec. and angular velocity is in the range of ±0.007.

Desaturation Control

The unwanted angular momentum on reaction wheels must be desaturated by torque rods interacting with Earth magnetic field.

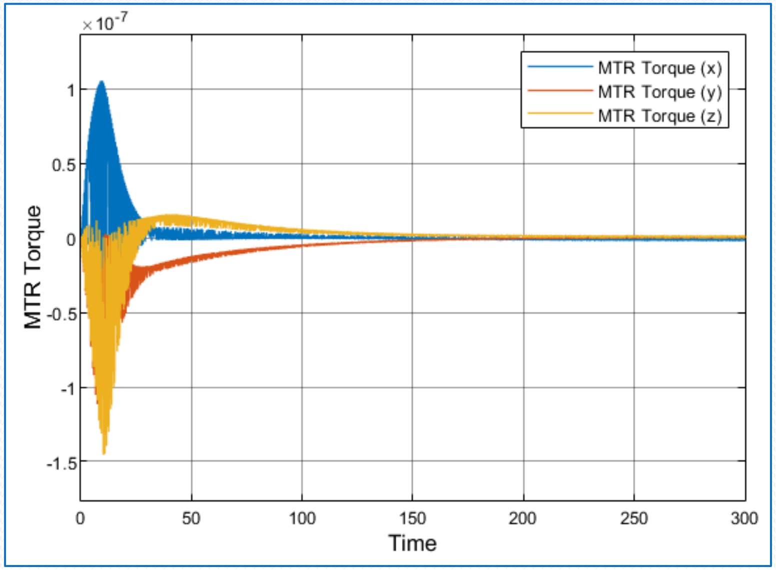
$$m^B = -\frac{K_{MD}}{\|B^B\|}.(B^B \times \Delta H_{RW}^B)$$

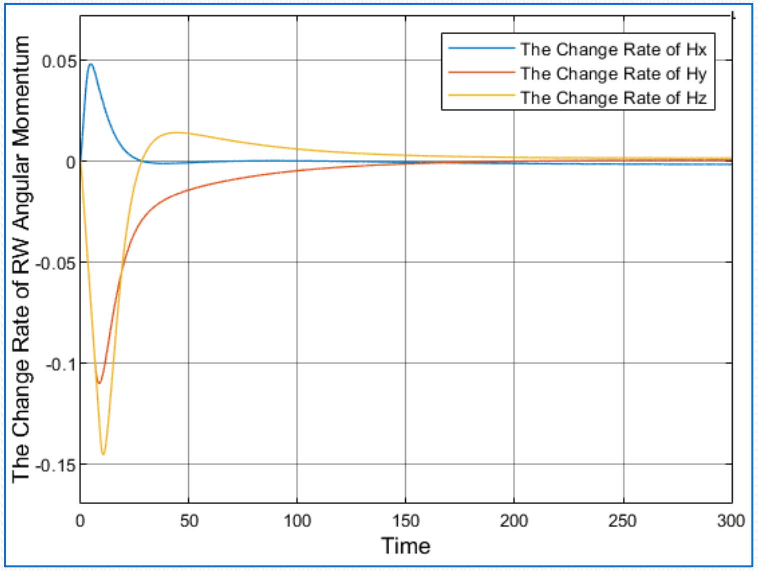
$$\Delta H_{RW}^B = H_{RW,nom}^B - H_{RW,sim}^B$$

$$M_C^{MTR} = m^B \times B^B = K_{MD}.\frac{(\Delta H_{RW}^B \times B^B)}{\|B^B\|^2}$$

Desaturation Control with PID Controller - Test I

Parameters	Values
Initial Satellite Velocity	$w_0 = [\ 0.0, 0.0, 0.0\ ]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0,0];\ [\psi_d, \theta_d, \Phi_d] = [-15, -5,5]$
Constant Controller Gain	$K_{dump} = 10^{-6}$
The Command Torque	$M_{cmd} = -K_P . q_{v,err} . q_{err,4} - K_{PD} . \dot{q}_{err}$

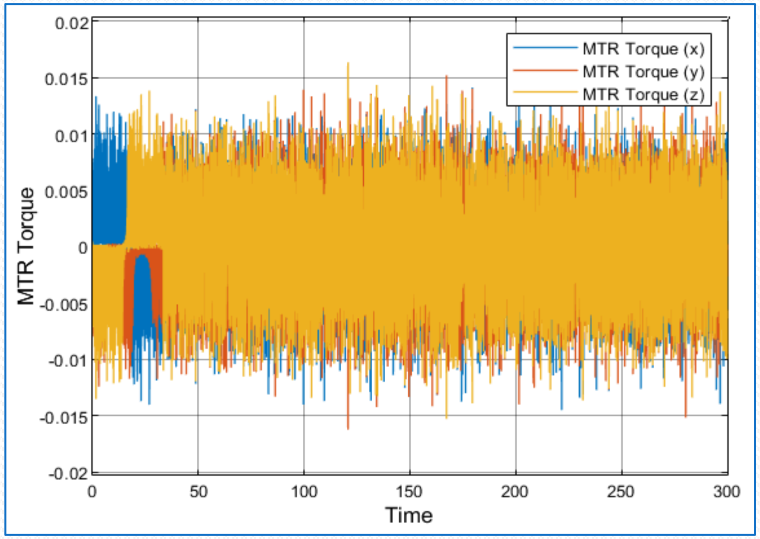


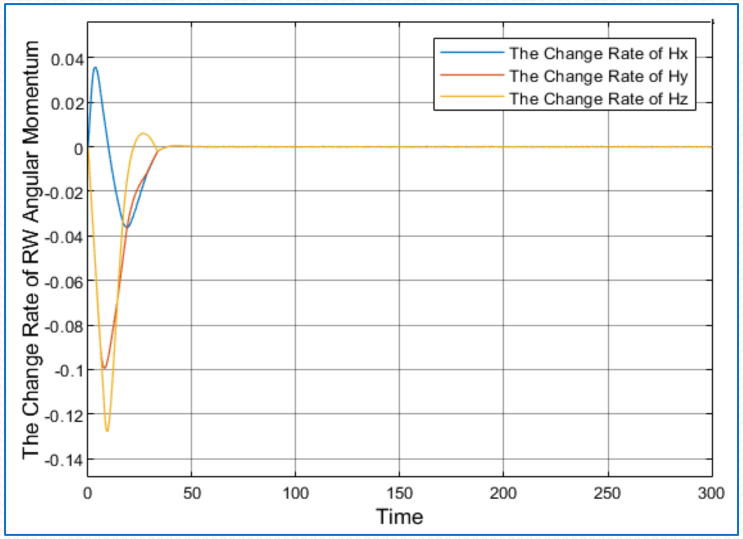


Settling time is about ~200 sec., angular momentum is around $\pm 0.005\ \mathrm{kgm}^2/\mathrm{sec}$.

Desaturation Control with PID Controller – Test II

Parameters	Values
Initial Satellite Velocity	$w_0 = [\,0.0, 0.0, 0.0\,]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0{,}0]$; $[\psi_d, \theta_d, \Phi_d] = [-15, -5{,}5]$
Constant Controller Gain	$K_{dump} = 10^9$
The Command Torque	$M_{cmd} = -K_P . q_{v,err} . q_{err,4} - K_{PD} . \dot{q}_{err}$

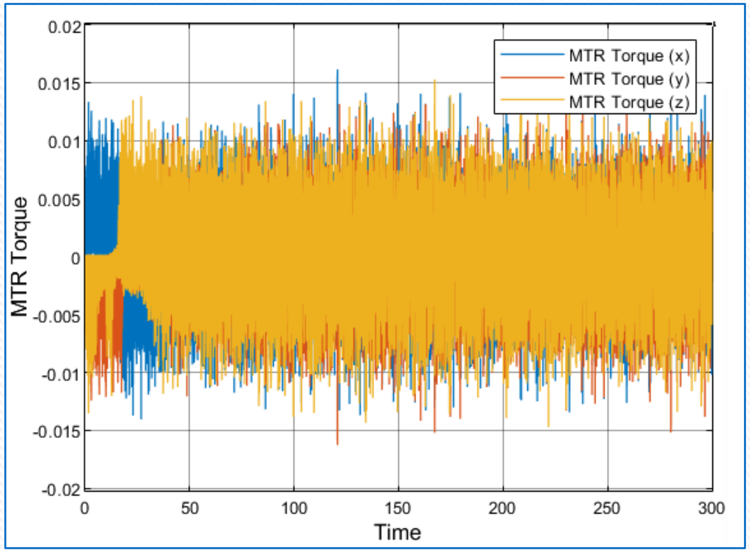


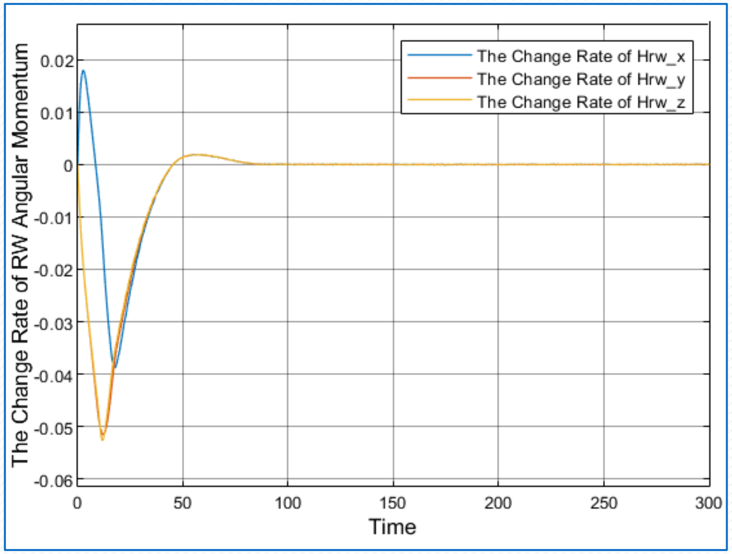


Settling time is about ~40 sec., angular momentum is around $\pm 0.0001\ \mathrm{kgm^2/sec}$.

Desaturation Control with PID Controller – Test III

Parameters	Values
Initial Satellite Velocity	$w_0 = [\,0.0, 0.0, 0.0\,]$ and RW1 Failure
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0,0];\ [\psi_d, \theta_d, \Phi_d] = [-15, -5,5]$
Constant Controller Gain	$K_{dump} = 10^9$
The Command Torque	$M_{cmd} = -K_P . q_{v,err} . q_{err,4} - K_{PD} . w_{OB}^B$





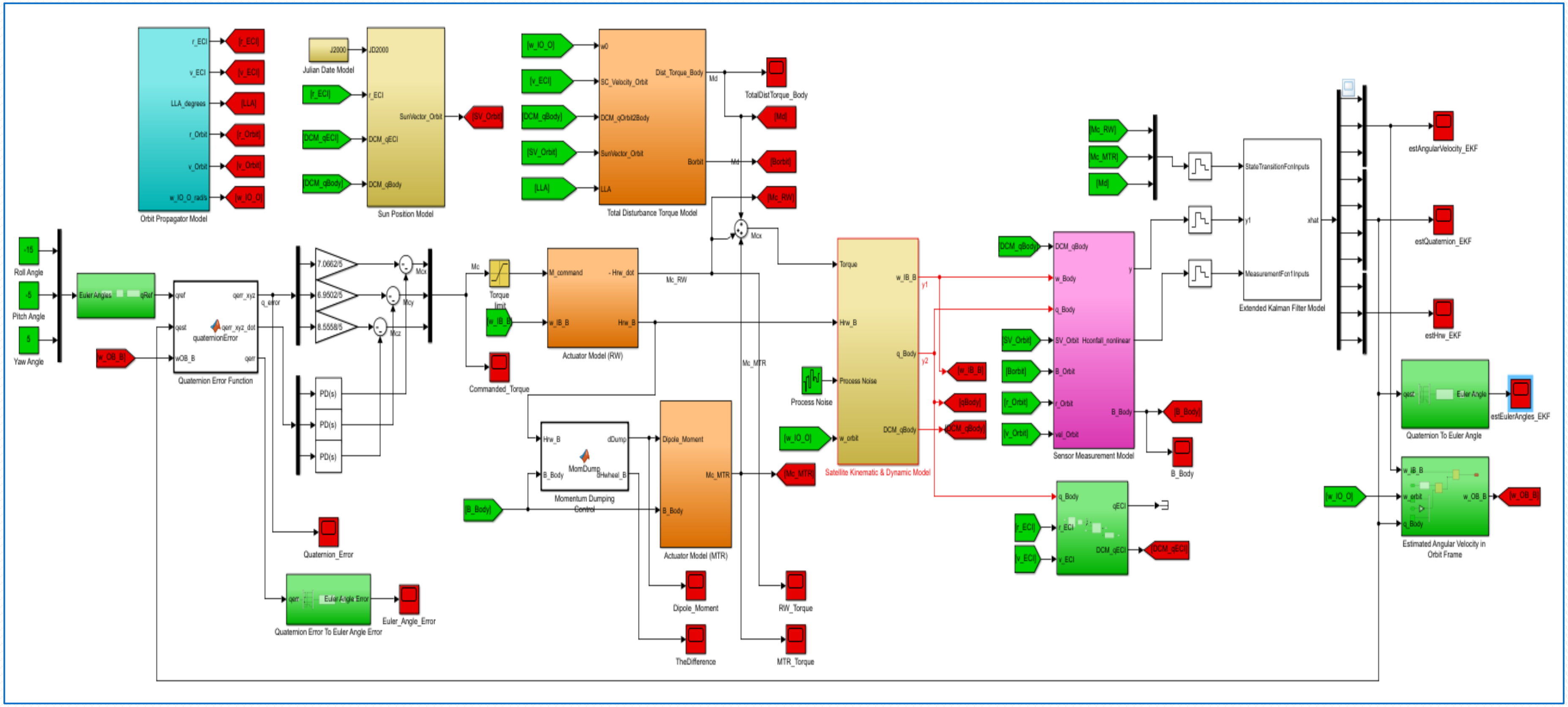
Settling time is about ~90 sec., angular momentum is around $\pm 0.0001\ \mathrm{kgm^2/sec.}$

PID Control

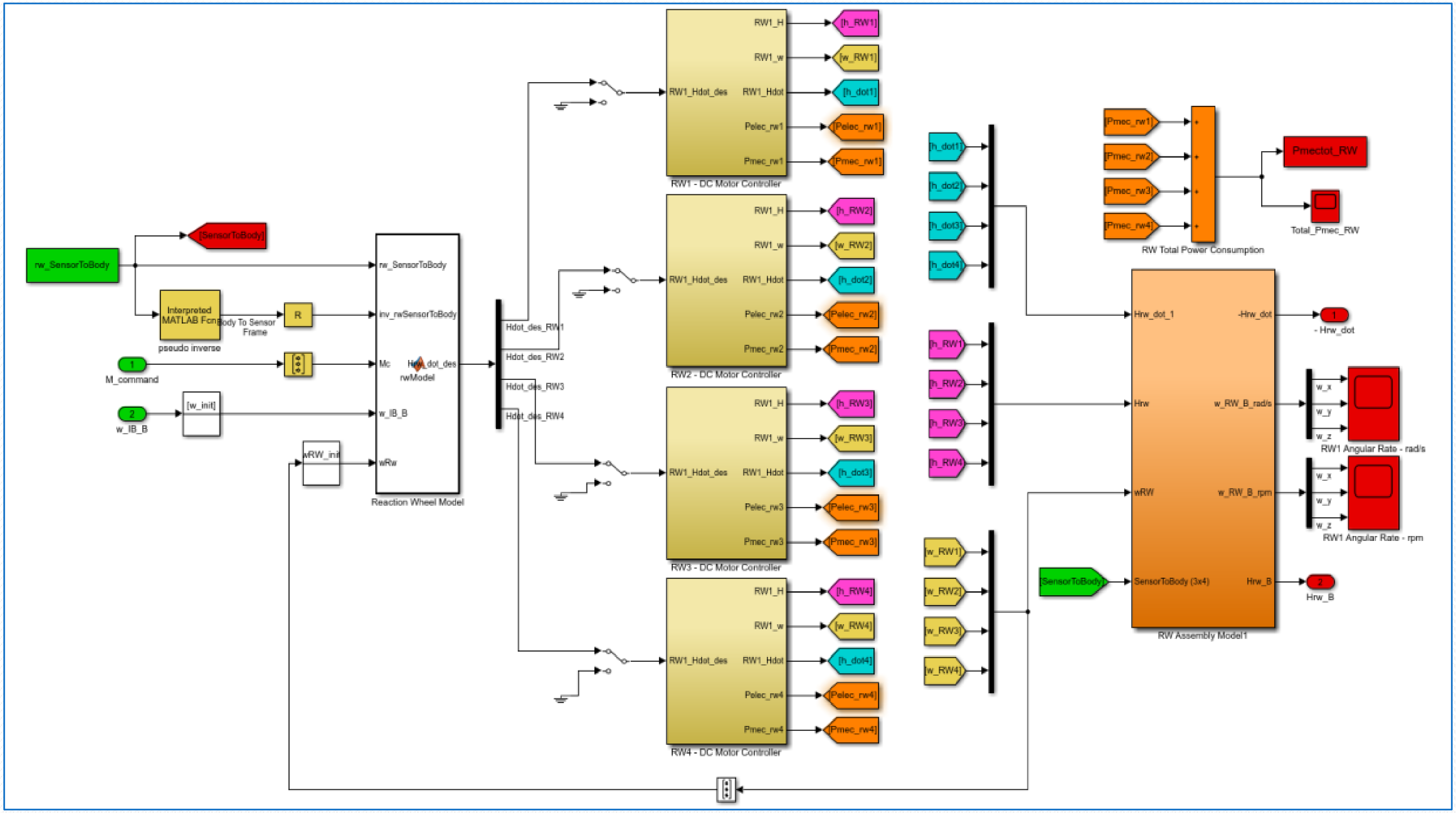
- The controller receives quaternion error, and propagates control torque command (M_{cmd}) to system actuators.

$$M_{cmd} = -K_P . q_{v,err} . q_{err,4} - K_{PD} . w_{IB}^{B}$$

$$M_{cmd} = -K_P . q_{v,err} . q_{err,4} - K_{PD} . \dot{q}_{err}$$

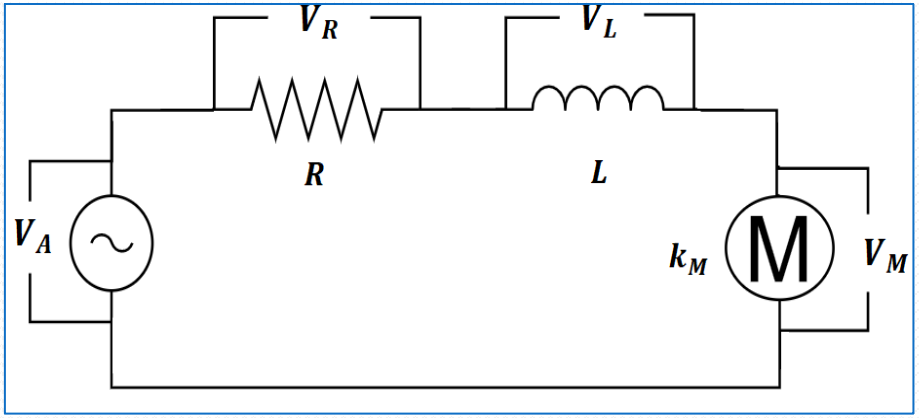


Reaction Wheel Model - I



$$M_C^{RW} = \dot{H}_{RW}^{B} + (w_{IB}^{B} \times H_{RW}^{B})$$

Reaction Wheel Model - II



Electrical part

$$V_A(t) = V_R(t) + V_L(t) + V_M(t)$$

$$V_A(t) = V_R(t) + V_L(t) + V_M(t)$$

$$V_A(t) = R.i(t) + L.\frac{di(t)}{dt} + k_M.w_{RW}(t)$$

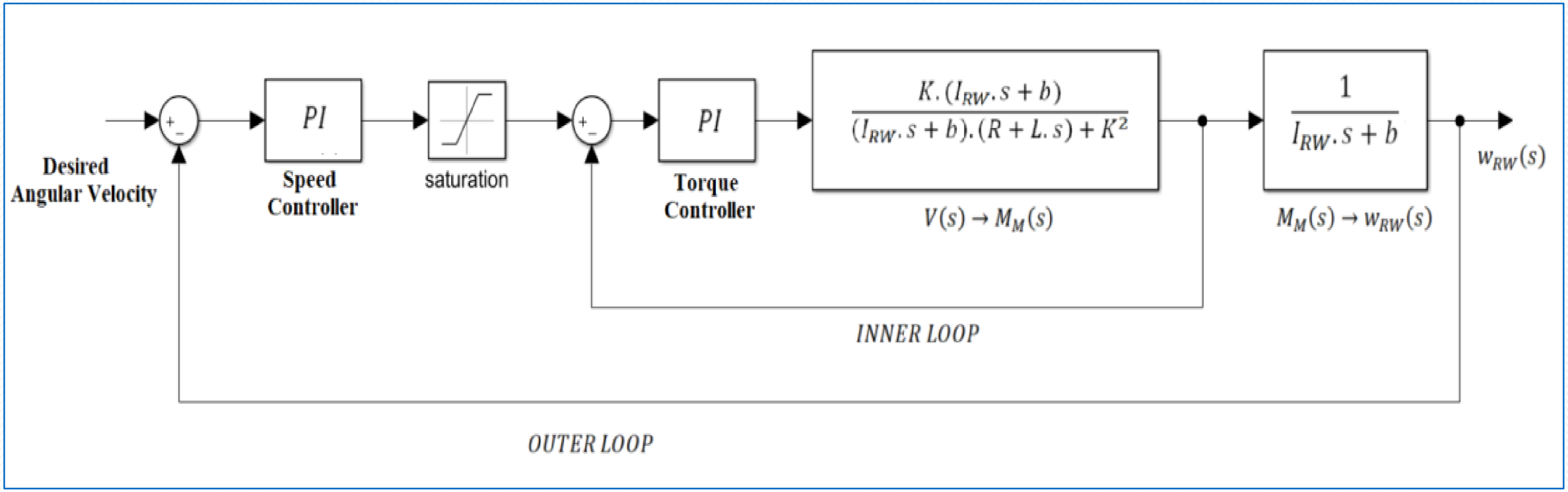
Mechanical part

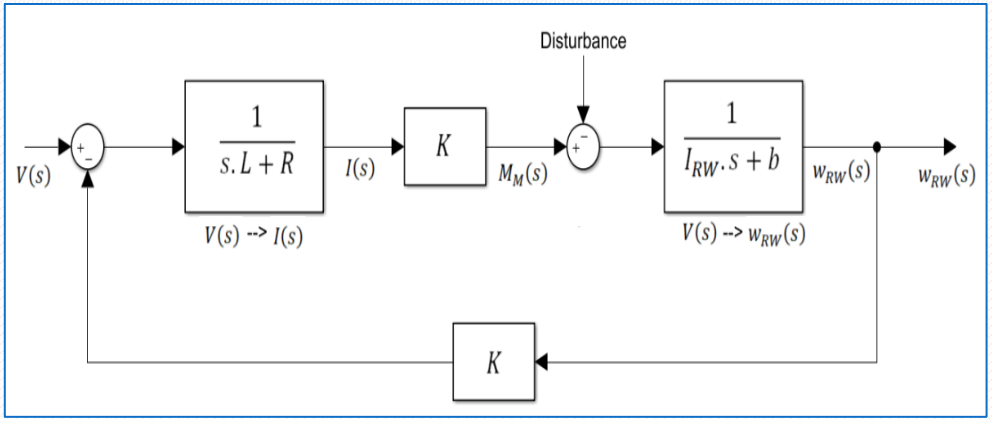
$$M_M = k_t.i = I_{RW}.\dot{w}_{RW} + b.w_{RW}$$

$$H_{RW} = I_{RW}.w_{RW}\ ;\ \ I_{RW} = I_{RW_x} = I_{RW_y} = I_{RW_z}$$

$$H_{RW}^{B} = C_{RW}^{BODY}.H_{RW}$$

RW PI Control – I

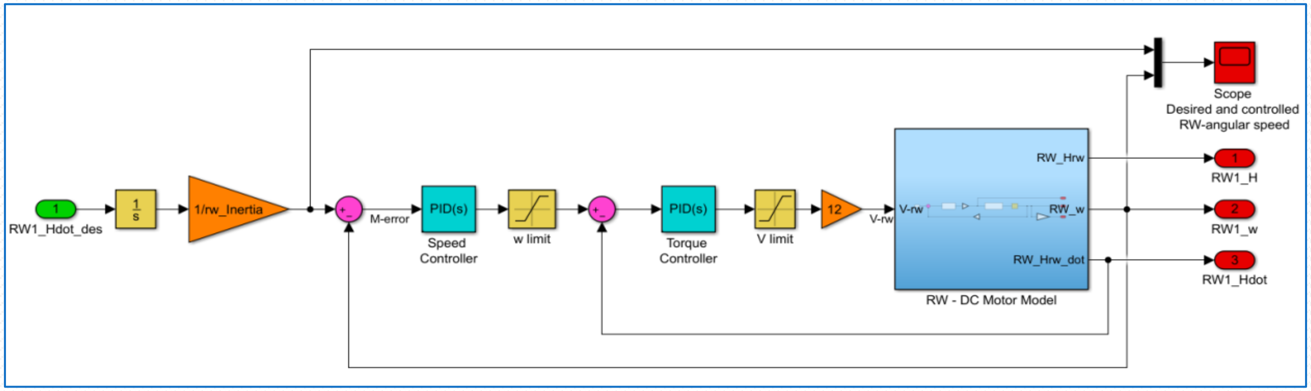




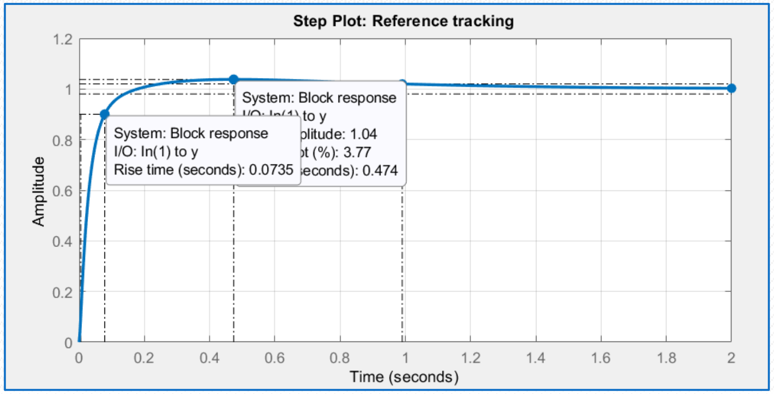
$$\frac{M_M(s)}{V(s)} = \frac{K.(I_{RW}.s+b)}{(I_{RW}.s+b).(R+L.s)+K^2}$$

$$\frac{w_{RW}(s)}{V(s)} = \frac{K}{(I_{RW}.s+b).(R+L.s)+K^2}$$

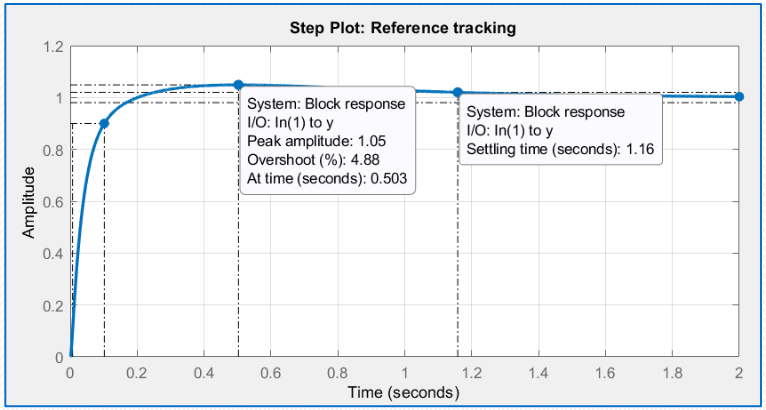
RW PI Control – II



K_P	K_I	K_D	N	Settling Time	Rise Time	Ovrsht
2.503e-5	0.022215	0	100	0.99 s	0.0735 s	3.77 %

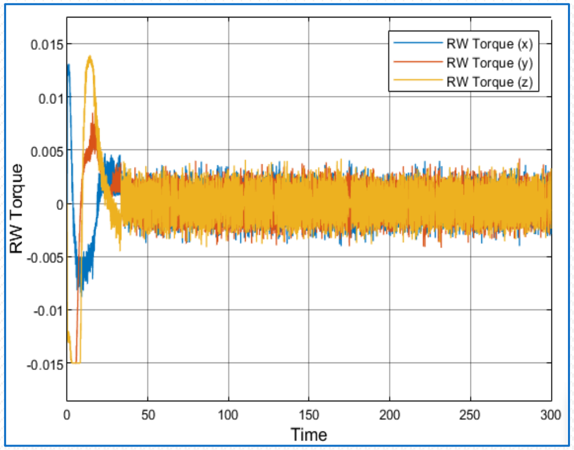


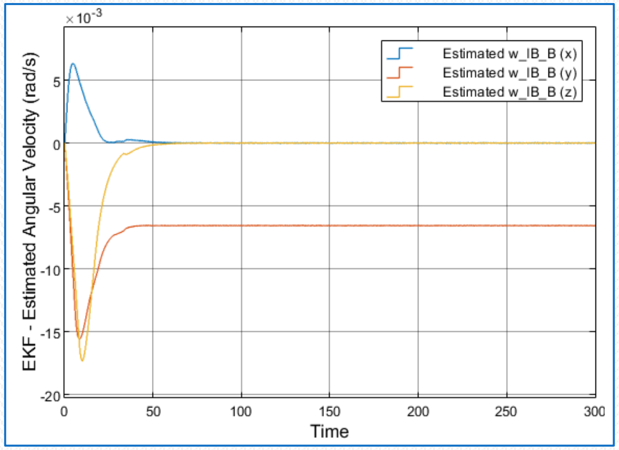
K_P	K_I	K_D	N	Settling Time	Rise Time	Ovrsht.
9.5605	13.4366	1.1625	2668.9531	1.16 s	0.0948 s	4.88 %

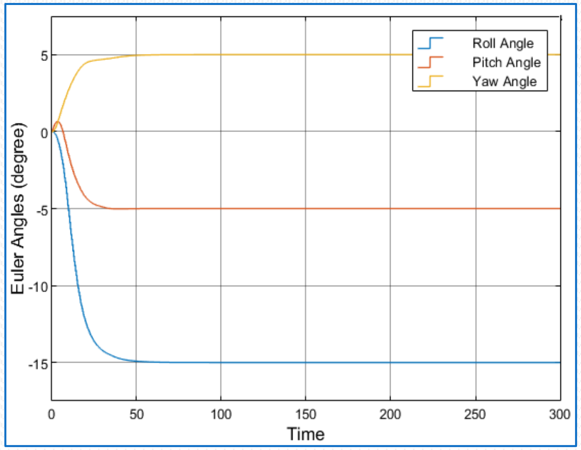


PID Control – Test I

Parameters	Values
Initial Satellite Velocity	$w_0 = [\ 0.0, 0.0, 0.0\ ]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0{,}0]; [\psi_d, \theta_d, \Phi_d] = [-15, -5{,}5]$
Constant Controller Gains $K_{P_{x,y,z}} = \left[I_{S_x}/5,\ \ I_{S_y}/5,\ \ I_{S_z}/5\right]; K_{PD_{x,y,z}} = 10 * K_{P_{x,y,z}}$	$K_{P_{x,y,z}} = [7.0662/5, \quad 6.9502/5, \quad 8.5558/5]$ $K_{PD_{x,y,z}} = [14.1324, \quad 13.9004, \quad 17.1117]$



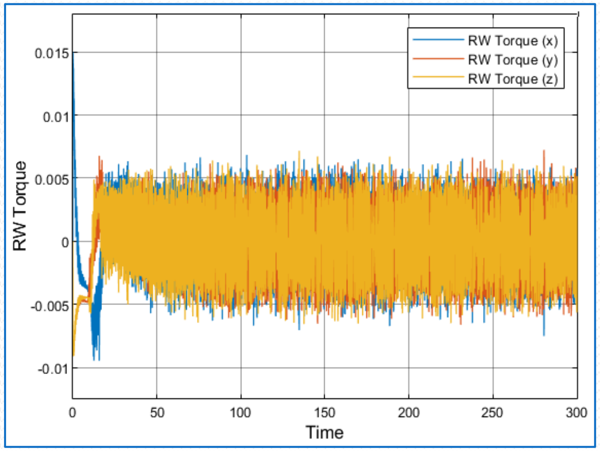


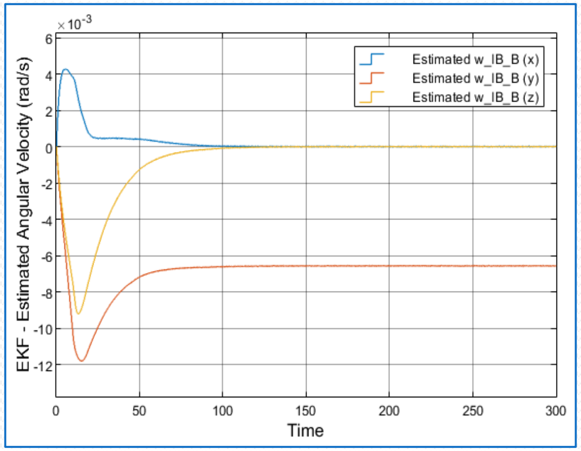


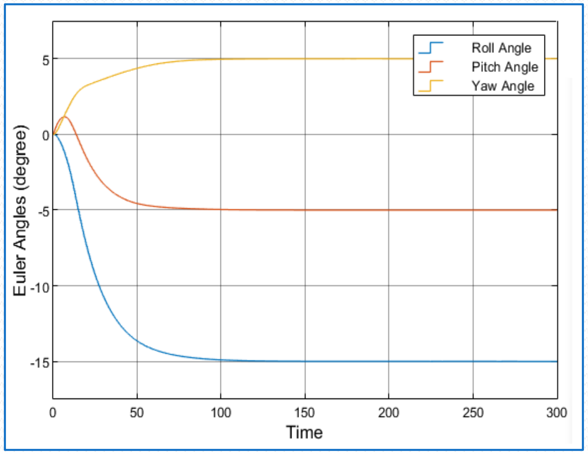
	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~50 s	~18.205 s	2.009 %
Pitch Angle	~40 s	~14.901 s	1.994 %
Yaw Angle	~50 s	~17.302 s	0.497 %

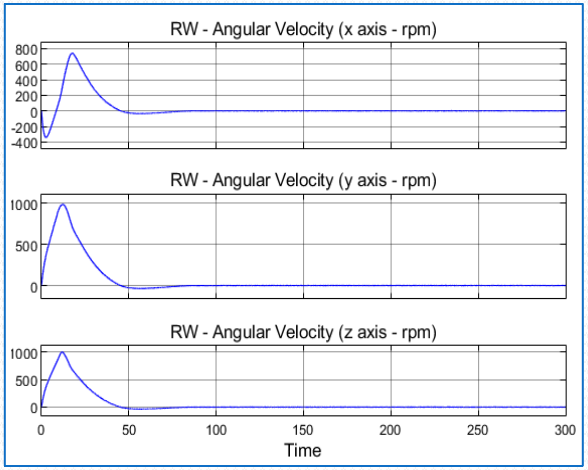
PID Control - Test II

Parameters	Values
Same Test Conditions	RW1 Failure









	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~110 s	~39.503 s	2.000 %
Pitch Angle	~70 s	~32.602 s	2.001 %
Yaw Angle	~100 s	~50.405 s	0.493 %

LQR Control

Linear Quadratic Regulator method is based on linear attitude model.

$$\boxed{J(x,u) = \frac{1}{2}\int_{0}^{\infty} [\, x^T . Q . x + u^T . R . u]\, dt}$$

$$\boxed{u(t) = -K . x(t)}$$

K is the optimal gain and computed from the solution to Riccati Equation:

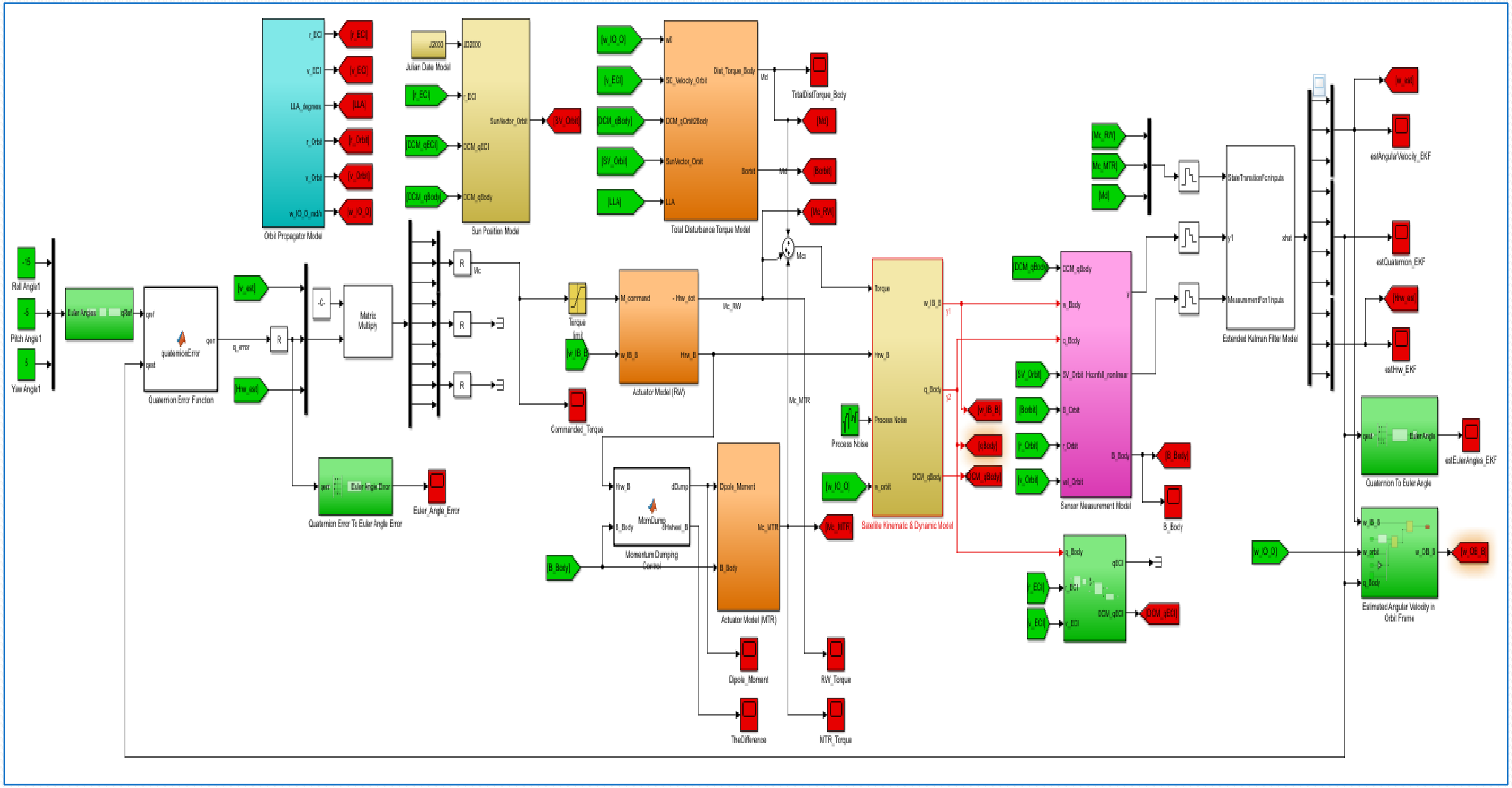
$$A^T . S + S . A - S . B . R^{-1} . B^T . S + Q = 0$$

$$K = R^{-1} . B^T . S$$

$$u = -(R^{-1} . B^T . S) . x$$

$$\boxed{[K_{LQR}, S, E] = lqr(A, B, Q, R)}$$

LQR Control Model



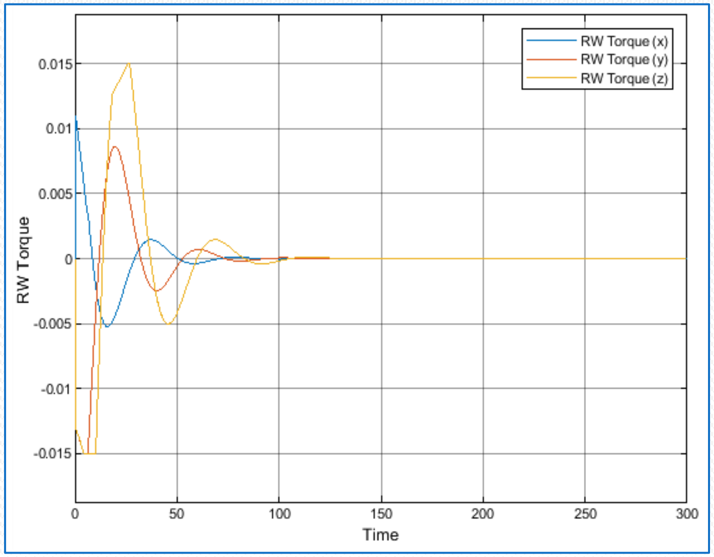
LQR Control – Test I

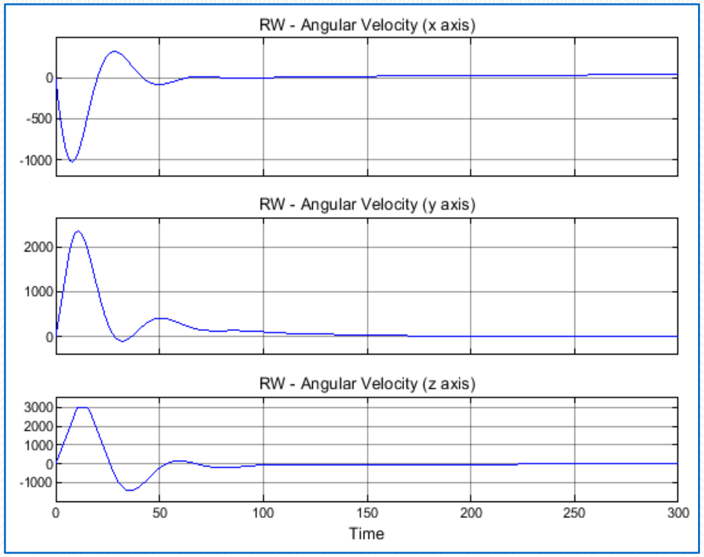
Parameters	Values
Initial Satellite Velocity	$w_0 = [\ 0.0, 0.0, 0.0\ ]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0{,}0]; \quad [\psi_d, \theta_d, \Phi_d] = [-15, -5, 5]$
Constant Weight State Matrix	$Q_{RW} = \begin{bmatrix} [I_{3x3}] & 0 & 0 \\ 0 & [I_{3x3}] * (1000) & 0 \\ 0 & 0 & [I_{4x4}] \end{bmatrix}$
Constant Weight Input Matrix	$R_{RW} = [I_{6x6}] * (2500)$
Controller Gain Matrix	$K_{LQR} = \left[Kw_{LQR,}\, Kq_{LQR,}\, Khrw_{LQR,}\right]$

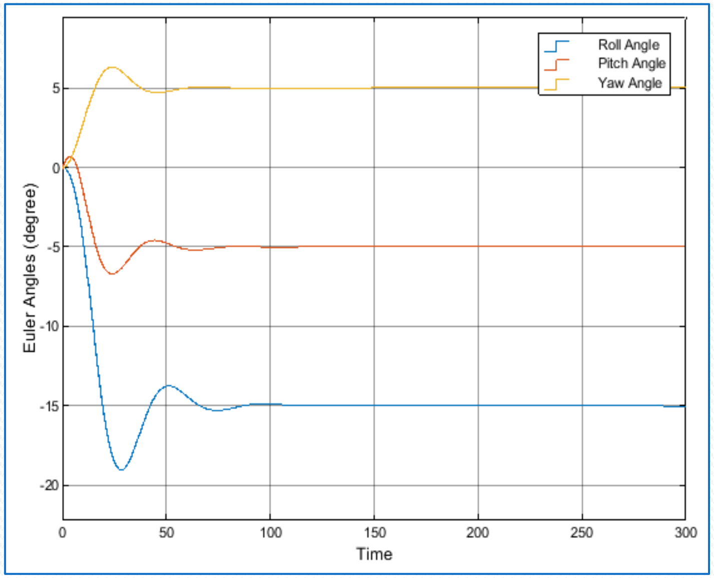
$$K_{LQR} = \begin{bmatrix} 0.8895 & 0.0000 & -0.0009 & 0.3640 & -0.0000 & 0.0028 & 0.0000 & -0.0164 & -0.0000 & -0.0000 \\ -0.0000 & 0.9046 & -0.0000 & 0.0000 & 0.3819 & -0.0000 & 0.0066 & -0.0000 & -0.0164 & -0.0000 \\ -0.0007 & 0.0000 & 0.9745 & -0.0035 & -0.0000 & 0.3637 & -0.0000 & 0.0000 & -0.0000 & -0.0164 \\ 0.9470 & -0.0000 & -0.0009 & 0.3657 & -0.0000 & 0.0029 & 0.0000 & 0.0081 & 0.0000 & -0.0000 \\ -0.0000 & 0.9612 & -0.0000 & 0.0000 & 0.3836 & -0.0000 & 0.0077 & -0.0000 & 0.0081 & -0.0000 \\ -0.0007 & -0.0000 & 1.0441 & -0.0035 & -0.0000 & 0.3658 & 0.0000 & 0.0000 & 0.0000 & 0.0081 \\ 0.9470 & -0.0000 & -0.0009 & 0.3657 & -0.0000 & 0.0029 & 0.0000 & 0.0081 & 0.0000 & -0.0000 \\ -0.0000 & 0.9612 & -0.0000 & 0.0000 & 0.3836 & -0.0000 & 0.0077 & -0.0000 & 0.0081 & -0.0000 \\ -0.0007 & -0.0000 & 1.0441 & -0.0035 & -0.0000 & 0.3658 & 0.0000 & 0.0000 & 0.0000 & 0.0081 \end{bmatrix}$$

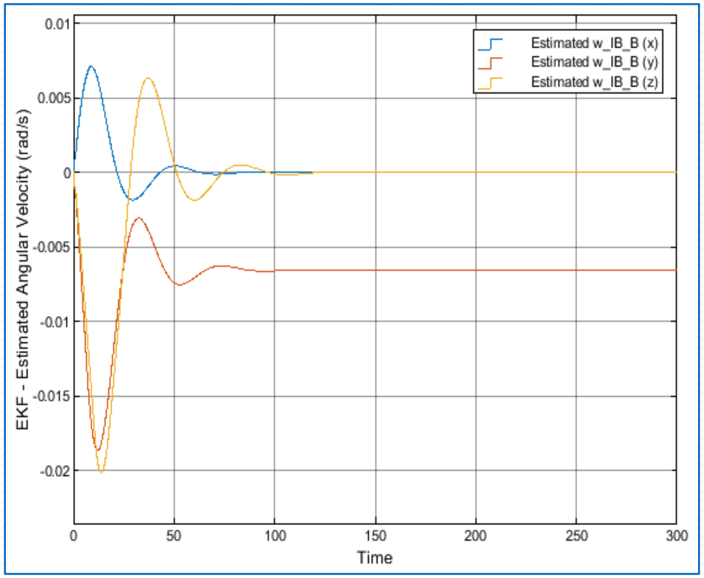
	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~100 s	- 11.606 s	8.028 %
Pitch Angle	~75 s	- 8.005 s	4.072 %
Yaw Angle	~60 s	+10.302 s	25.949 %

LQR Control – Test I



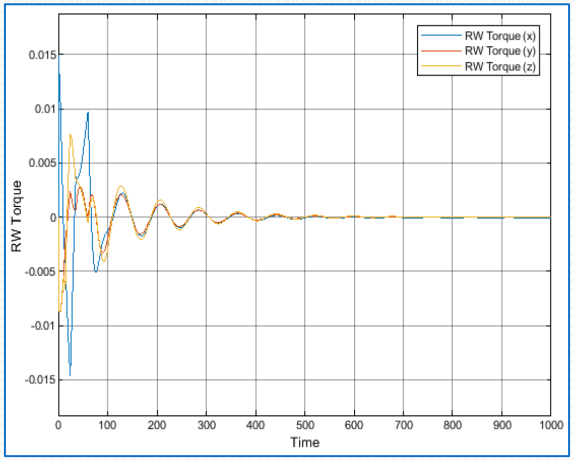


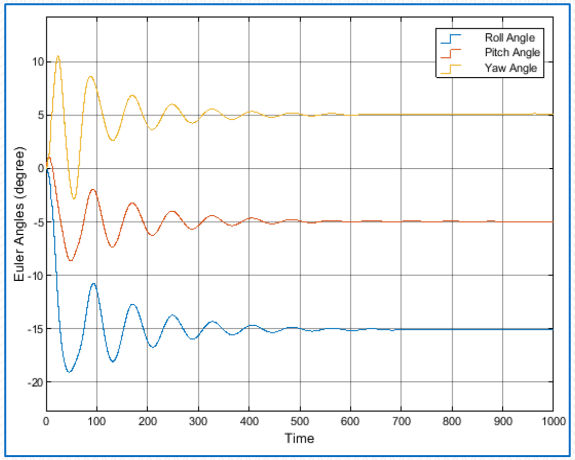


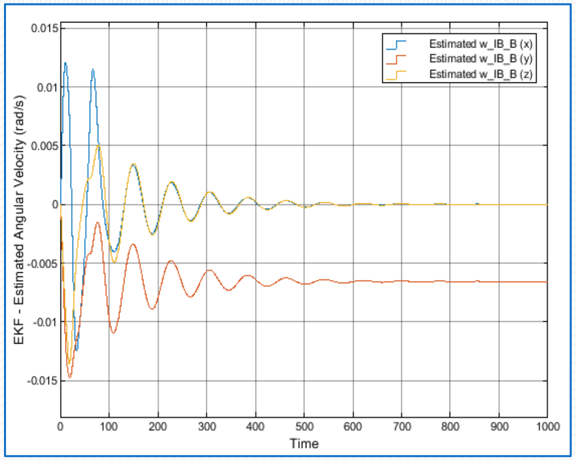


LQR Control – Test II

Parameters	Values
Same Test Conditions	RW1 Failure







	Settling Time (s)	Rise/Fall Time (s)	Overshoot (%)
Roll Angle	~ 600	- 16.903	- 7.133
Pitch Angle	~ 600	8.654 / 8.337	38.072 / 6.877
Yaw Angle	~ 600	6.305 / 6.803	137.676 / 3.450

Sliding Mode Control

There is a predefined sliding line or surface to force state trajectories to lie on it.

When a system is out of a sliding surface, system dynamics reach this surface and the control torque is also needed to force the system states towards it.

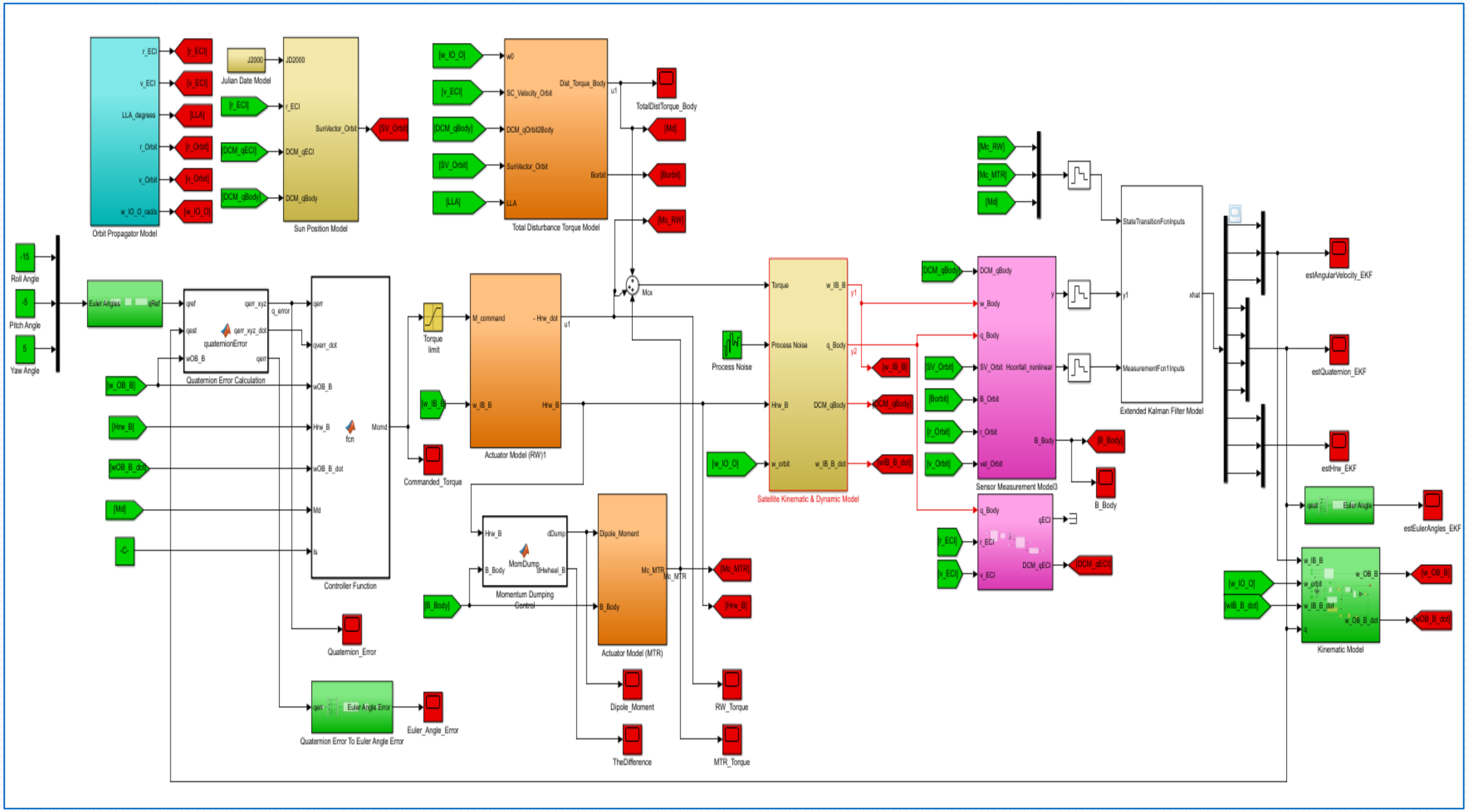
When a system is on the surface ($s = 0$), its states provide the system stability and its control torque is needed to keep the system at the surface.

$$\boxed{s = w_{OB}^{B} + K_{SMC}.\, q_{v,err} \; = 0}$$

$$\frac{1}{2}.\, S(q_{v,err}).\, w_{OB}^{B} + \frac{1}{2}.\, S(q_{v,err}).\, K_{SMC}.\, q_{v,err} = 0$$

$$\dot{q}_{v,err} + \frac{1}{2}.\, S(q_{v,err}).\, K_{SMC}.\, q_{v,err} = 0$$

Sliding Mode Control Model



Sliding Mode Control - Stability

$$V = \frac{1}{2}.s^T.s$$

$$\dot{V} = s^T.\dot{s} = s^T.\left(\dot{w}_{OB}^B + K_{SMC}.\dot{q}_{v,err}\right)$$

$$\dot{V} = s^T.I_S^{-1}\left(M_D + M_{cmd} - w_{OB}^B \times \left(I_S.w_{OB}^B + H_{RW}^B\right) + I_S.K_{SMC}.\dot{q}_{v,err}\right)$$

$$\boxed{M_{cmd} = w_{OB}^B \times \left(I_S.w_{OB}^B + H_{RW}^B\right) - M_D - I_S.\dot{w}_{OB}^B - I_S.K_{SMC}.\dot{q}_{v,err} - I_S.G_{SMC}.sign(s)}$$

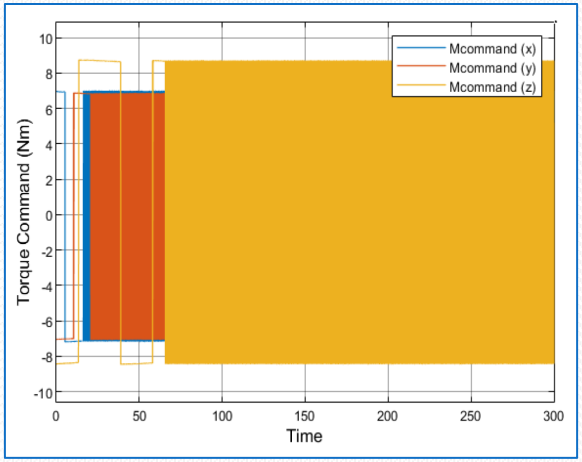
$$\dot{V} = -s^T.\left(\dot{w}_{OB}^B + G_{SMC}.sign(s)\right)$$

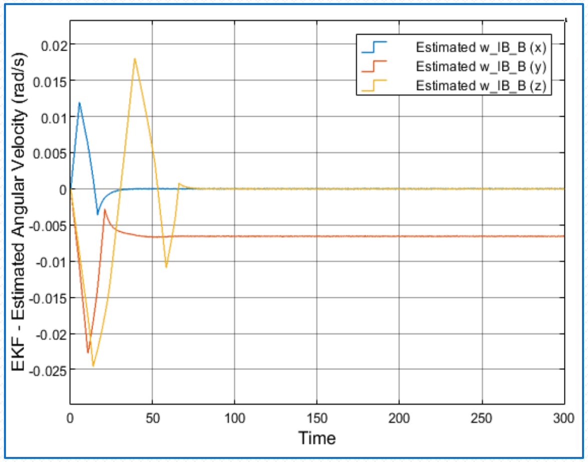
$$\boxed{M_{cmd} = w_{OB}^B \times \left(I_S.w_{OB}^B + H_{RW}^B\right) - M_D - I_S.\dot{w}_{OB}^B - I_S.K_{SMC}.\dot{q}_{v,err} - I_S.G_{SMC}.tanh\left(\frac{S}{\varepsilon}\right)}$$

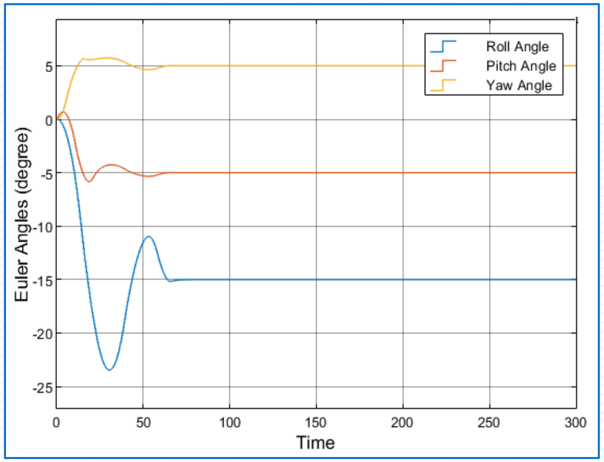
SMC Control – Test I

Parameters	Values
Initial Satellite Velocity	$w_0 = [\ 0.0, 0.0, 0.0\ ]$
Initial / Desired Euler Angels	$[\psi_0, \theta_0, \Phi_0] = [0, 0,0]\ ;\ [\psi_d, \theta_d, \Phi_d] = [-15, -5, 5]$
Constant Controller Gains	$K_{SMC} = 0.5 * [I]_{3x3}\ ;\ G_{SMC} = 1 * [I]_{3x3}$
Sliding Thickness	$\varepsilon = 0.02$

$$M_{cmd} = w_{OB}^{B} \times \left(I_S . w_{OB}^{B} + H_{RW}^{B}\right) - M_D - I_S . \dot{w}_{OB}^{B} - I_S . K_{SMC} . \dot{q}_{v,err} - I_S . G_{SMC} . sign(s)$$





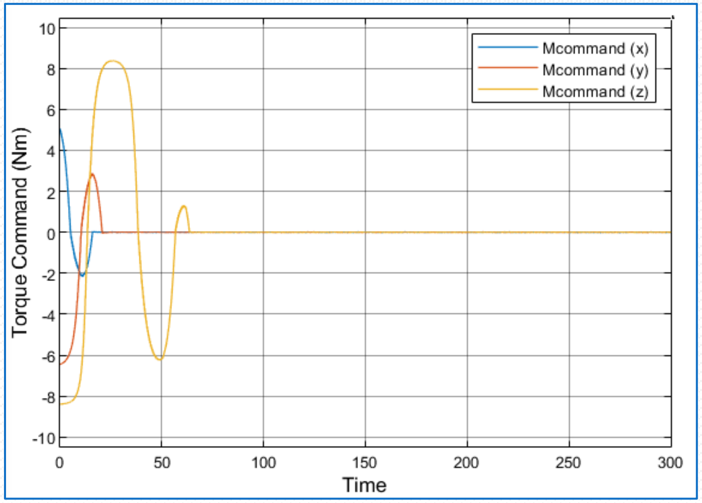


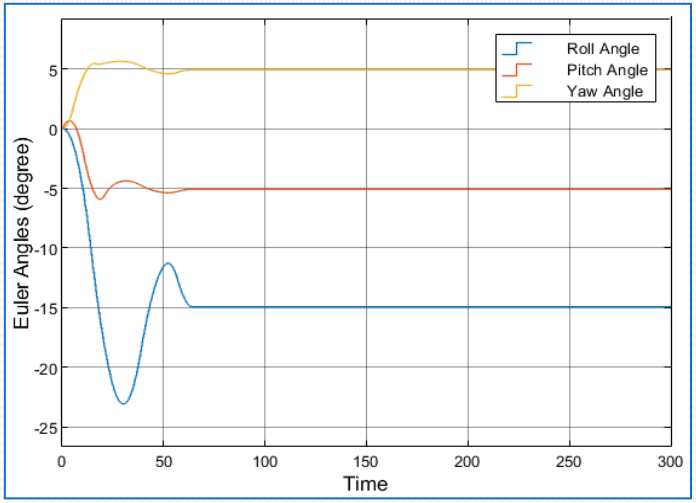
	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~75 s	5.703 s	4.479 %
Pitch Angle	~60 s	7.005 s	13.333 %
Yaw Angle	~60 s	7.407 s	14.368 %

SMC Control – Test II

Parameters	Values
Same Test Conditions	Without Chattering Problem

$$M_{cmd} = w_{OB}^{B} \times \left(I_S . w_{OB}^{B} + H_{RW}^{B}\right) - M_D - I_S . \dot{w}_{OB}^{B} - I_S . K_{SMC} . \dot{q}_{v,err} - I_S . G_{SMC} . tanh\left(\frac{S}{\varepsilon}\right)$$

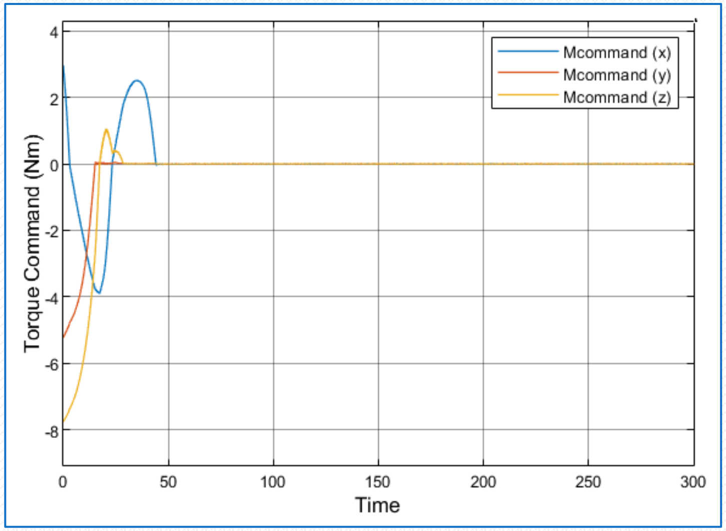


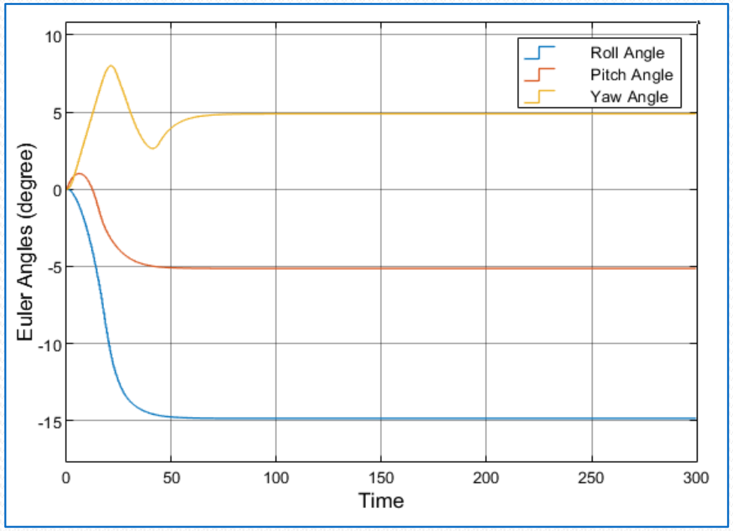


	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~75 s	5.502 s	4.217 %
Pitch Angle	~60 s	7.006 s	12.459 %
Yaw Angle	~60 s	7.602 s	14.368 %

SMC Control – Test III

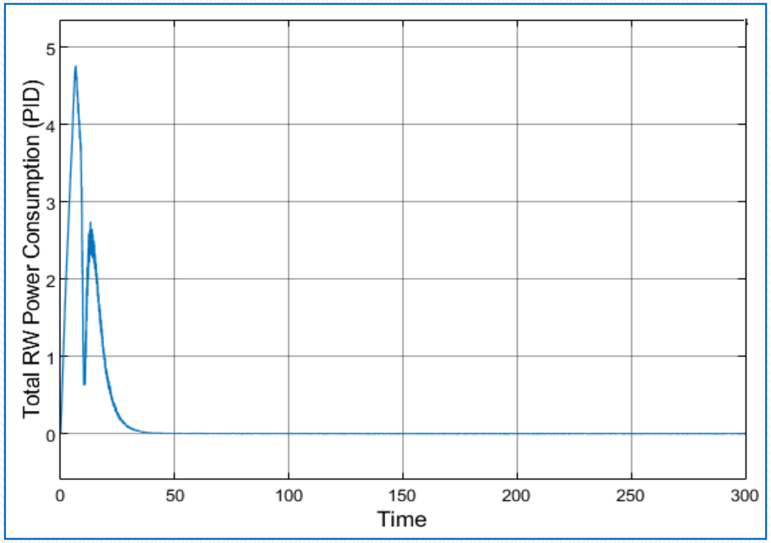
Parameters	Values
Same Test Conditions	RW1 Failure
Constant Controller Gains	$K_{SMC} = 0.25 * [I]_{3x3}$; $G_{SMC} = 15 * [I]_{3x3}$





	Settling Time	Rise/Fall Time	Overshoot
Roll Angle	~100 s	19.907 s	2.008 %
Pitch Angle	~90 s	19.403 s	2.018 %
Yaw Angle	~120 s	10.602 s	70.560 / 2.267 %

RW Power Consumption (PID,SMC)



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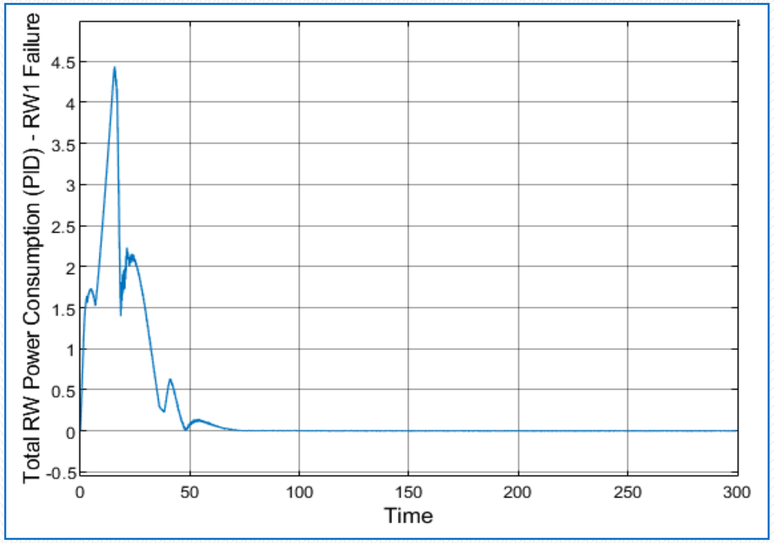

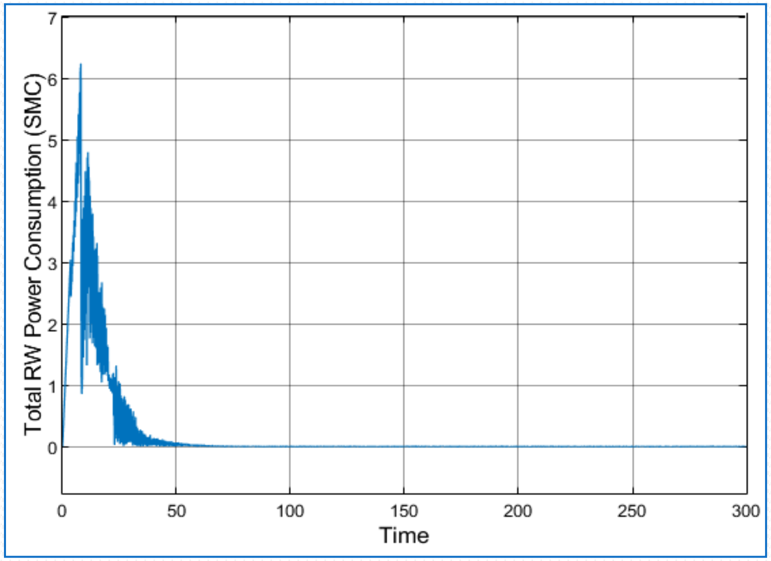

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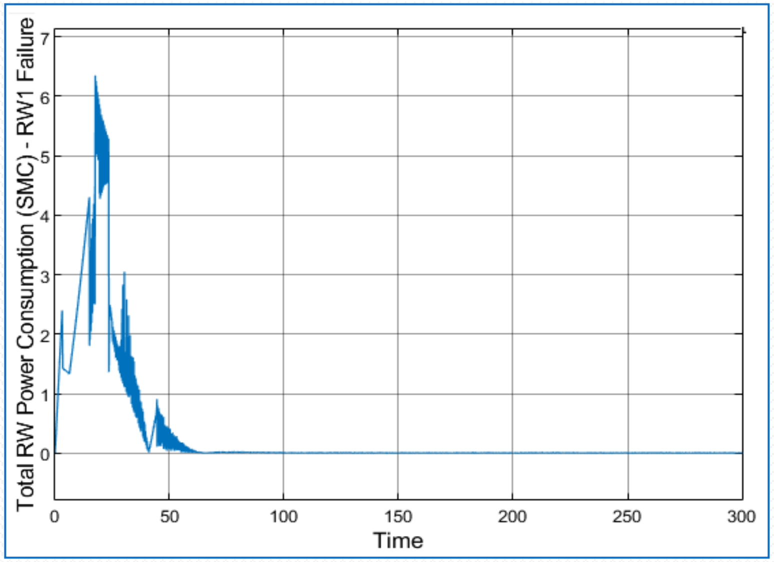
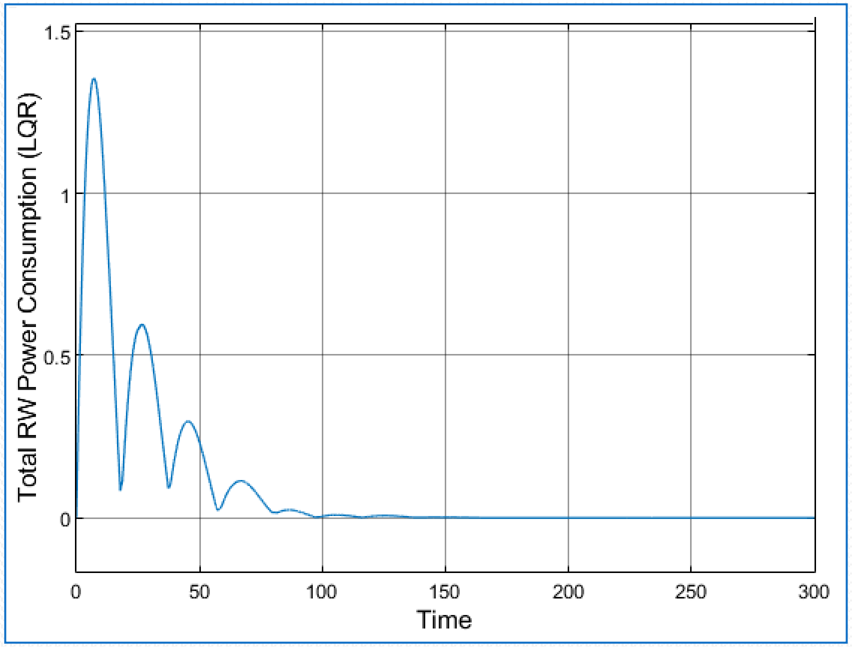


RW Power Consumption (LQR)



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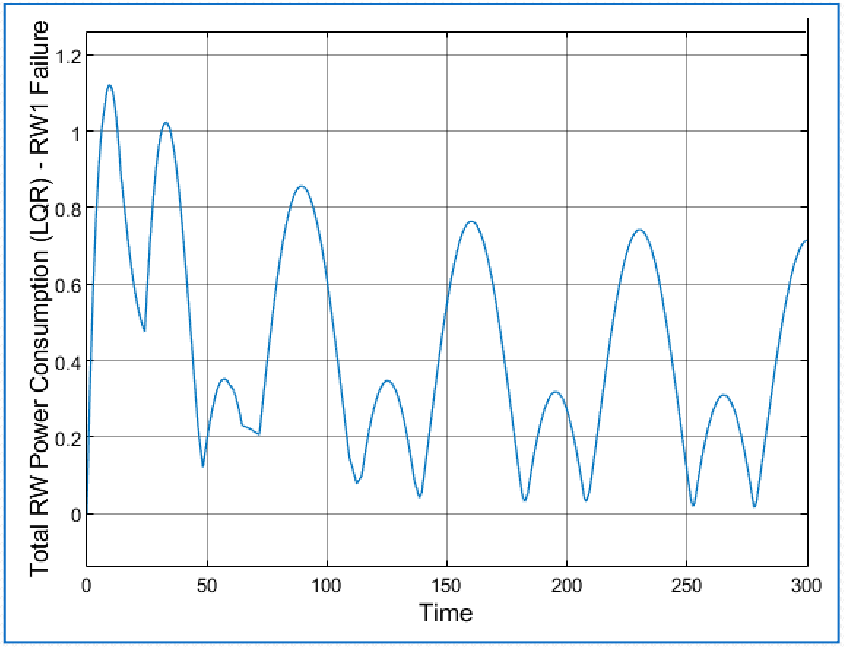
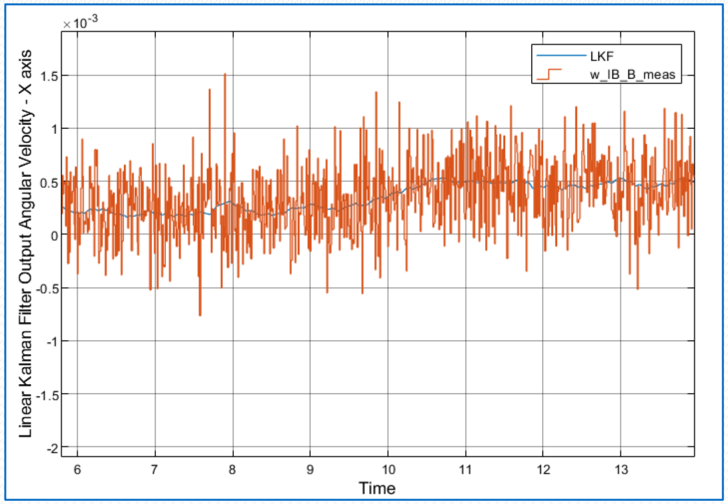
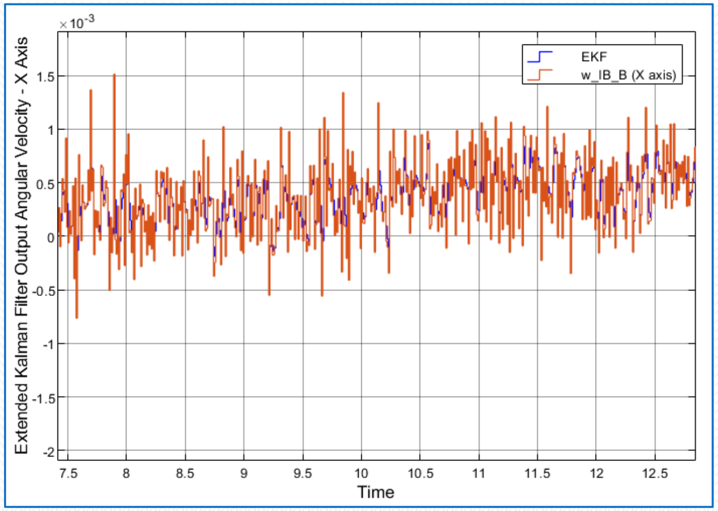
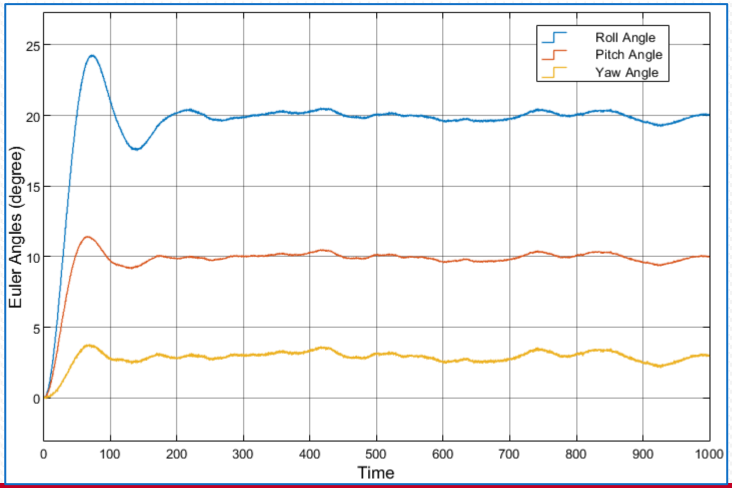


LKF and EKF Results - I

- $Q_k = 1x10^{-7}$, $R_{gyro} = 1x10^{-7}$, $R_{str} = 1x10^{-7}$
- Roll Angle = 20°, Pitch Angle = 10°, Yaw Angle = 3°

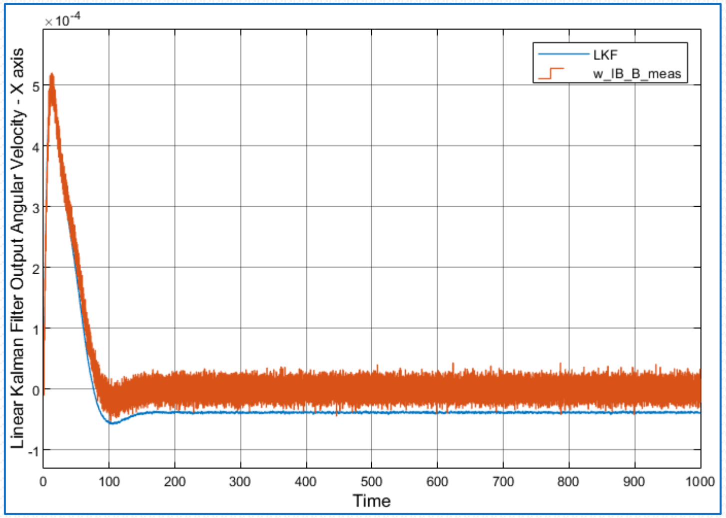


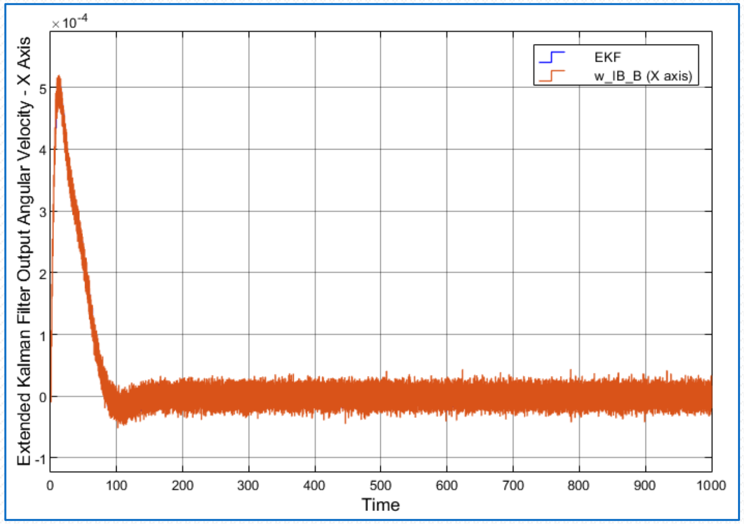


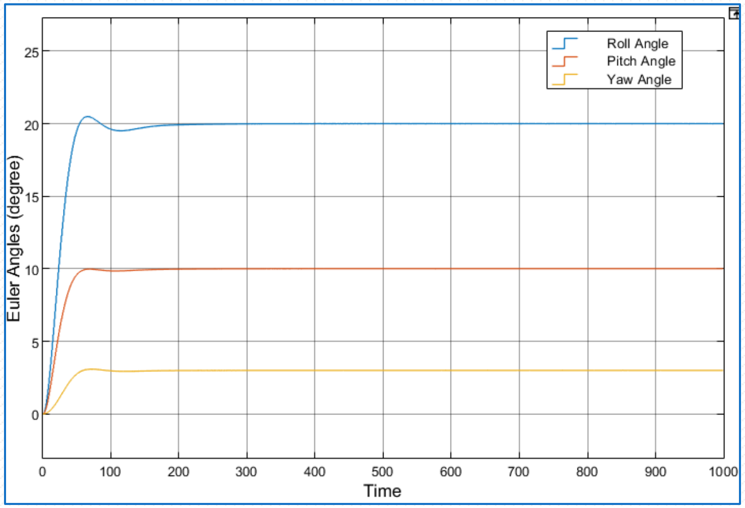


LKF and EKF Results - II

- $Q_k = 1x10^{-10}$, $R_{gyro} = 1x10^{-10}$, $R_{str} = 1x10^{-10}$







Results - Conclusions

- The Kalman filter provides better stability with low noise measurement and process covariance matrices with multi-sensor configuration for the roll, pitch and yaw axes.
- All controller types (PID,LQR,SMC) give stable results based on Lyapunov stability theorem in terms of the desired orientation.
- Detumbling and Desaturation are realized with satisfied results for each type of attitude controller.
- SMC is the best results among of all other controllers in terms of settling time and robustness.

Future Works

- Different sensor configurations can be used with UKF Filters

- Optimal path desicion based on minimum power consumption of reaction wheels

- The following controllers can be used:
  - H Infinity Controller
  - Fuzzy Logic Control
  - Neural Networks Control

Thank you for your attention.