

Circuit representation of Fredkin gate

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Further reading

Definition

The Fredkin gate is the reversible three-bit gate that swaps the last two bits if, and only if, the first bit is 1.

Permutation matrix form

INPUT			OUTPUT		
C	I_1	I_2	C	O_1	O_2
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It has the useful property that the numbers of 0s and 1s are conserved throughout, which in the billiard ball model means the same number of balls are output as input. This corresponds nicely to the conservation of mass in physics, and helps to show that the model is not wasteful.

Truth functions with AND, OR, XOR, and NOT

The Fredkin gate can be defined using truth functions with AND, OR, XOR, and NOT, as follows:

$$\begin{aligned}O_1 &= I_1 \text{ XOR } S \\O_2 &= I_2 \text{ XOR } S \\C_{\text{out}} &= C_{\text{in}}\end{aligned}$$

where $S = (I_1 \text{ XOR } I_2) \text{ AND } C$

Alternatively:

$$\begin{aligned}O_1 &= (\text{NOT } C \text{ AND } I_1) \text{ OR } (C \text{ AND } I_2) \\O_2 &= (C \text{ AND } I_1) \text{ OR } (\text{NOT } C \text{ AND } I_2) \\C_{\text{out}} &= C_{\text{in}}\end{aligned}$$

Completeness

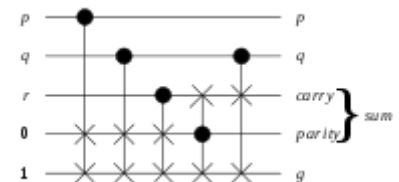
One way to see that the Fredkin gate is universal is to observe that it can be used to implement AND, NOT and OR:

If $I_2 = 0$, then $O_2 = C \text{ AND } I_1$.
 If $I_1 = 0$ and $I_2 = 1$, then $O_2 = \text{NOT } C$.
 If $I_2 = 1$, then $O_1 = C \text{ OR } I_1$.

Example

Three-bit full adder (add with carry) using five Fredkin gates. The "g" garbage output bit is $(p \text{ NOR } q)$ if $r=0$, and $(p \text{ NAND } q)$ if $r=1$.

Inputs on the left, including two constants, go through three gates to quickly determine the parity. The 0 and 1 bits swap places for each input bit that is set, resulting in parity bit on the 4th row and inverse of parity on 5th row



Then the carry row and the inverse parity row swap if the parity bit is set and swap again if one of the p or q input bits are set (it doesn't matter which is used) and the resulting carry output appears on the 3rd row

The p and q inputs are only used as gate controls so they appear unchanged in the output.

Quantum Fredkin gate

On March 25, 2016, researchers from Griffith University and the University of Queensland announced they had built a quantum Fredkin gate that uses the quantum entanglement of particles of light to swap qubits. The availability of quantum Fredkin gates may facilitate the construction of quantum computers^{[2][3]}

See also

- Quantum computing
- Quantum gate

- Quantum programming
- Toffoli gate, which is a *controlled-controlled-NOT gate*

References

1. Brown, Julian, The Quest for the Quantum Computer(<https://books.google.com/books?id=ECWm59h2pLAC&pg=RS8>), New York : Touchstone, 2000.
2. <http://www.pcworld.com/article/3048763/hardware/quantum-computing-is-now-a-big-step-closer-thanks-to-this-new-breakthrough.html>
3. A quantum Fredkin gate(<http://advances.sciencemag.org/content/2/3/e1501531>) Raj B. Patel, Joseph Ho, Franck Ferreyrol, Timothy C. Ralph and Geof J. Pryde, Science Advances, 25 Mar 2016, *vol.* 2, no. 3, e1501531, DOI: 10.1126/sciadv.1501531

Further reading

- Fredkin, Edward, Toffoli, Tommaso (1982). "Conservative Logic"(PDF). *International Journal of Theoretical Physics* **21** (3-4): 219–253. doi:10.1007/BF01857727. Archived from the original (PDF) on October 17, 2006.
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This page was last edited on 3 November 2017, at 22:38.

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