Systems and Software Verification Laboratory

The University of Manchester

Detection of Software Vulnerabilities: Static Analysis

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Detection of Software Vulnerabilities

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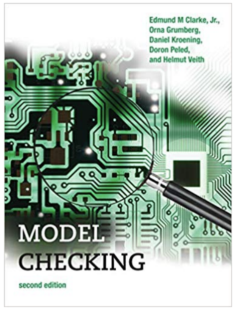
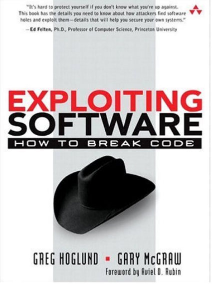
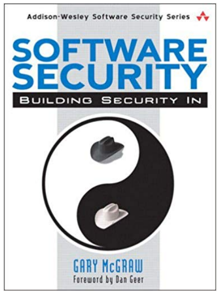
  - Office: 2.28
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Exploiting Software: How to Break Code* (Chapter 7)
  - *C How to Program* (Chapter 1)

Rashid et al.: *The Cyber Security Body of Knowledge*, CyBOK, v1.0, 2019







Intended learning outcomes

- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference between **static analysis** and **testing / simulation**
- Explain **Bounded** and **Unbounded Model Checking**
- Provide **practical examples** to detect software vulnerabilities statically

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Motivating Example

- functionality demanded increased significantly
  - peer reviewing and testing
- multi-core processors with scalable shared memory / message passing
  - software model checking and testing

```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);   (CS3)
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

Deadlock

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
```

Detection of Vulnerabilities

- Detect the presence of vulnerabilities in the code during the development, testing and maintenance
- Techniques to detect vulnerabilities must make trade-offs between **soundness** and **completeness**
  - A detection technique is **sound** for a given category if it concludes that a given program has no vulnerabilities
    - An unsound detection technique may have ***false negatives***, i.e., actual vulnerabilities that the detection technique fails to find
  - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
    - An incomplete detection technique may have ***false positives***, i.e. it may detect issues that do not turn out to be actual vulnerabilities

Detection of Vulnerabilities

- Achieving **soundness** requires reasoning about all executions of a program (usually an infinite number)
  - This is can done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, concrete executions of a program that are witnesses to any vulnerability reported
  - The analysis technique has to come up with concrete inputs for the program that trigger a vulnerability
    - A common dynamic approach is software testing: the tester writes test cases with concrete inputs, and specific checks for the outputs

Detection of Vulnerabilities

In practice, detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**

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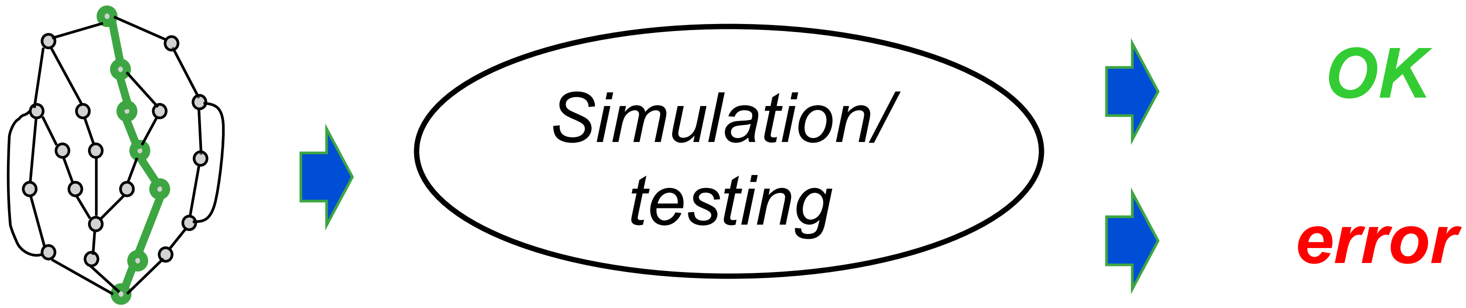
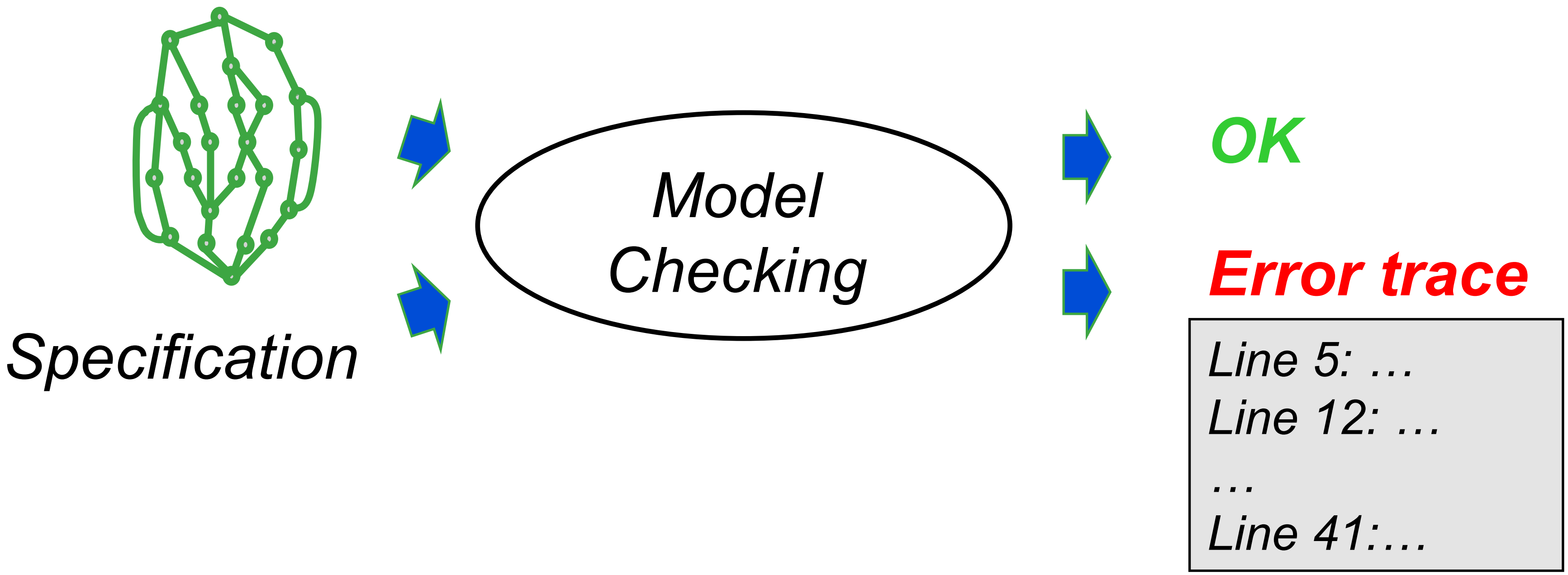
Static analysis vs Testing/ Simulation



- Checks only some of the system executions
- May miss errors

Static analysis vs Testing/ Simulation

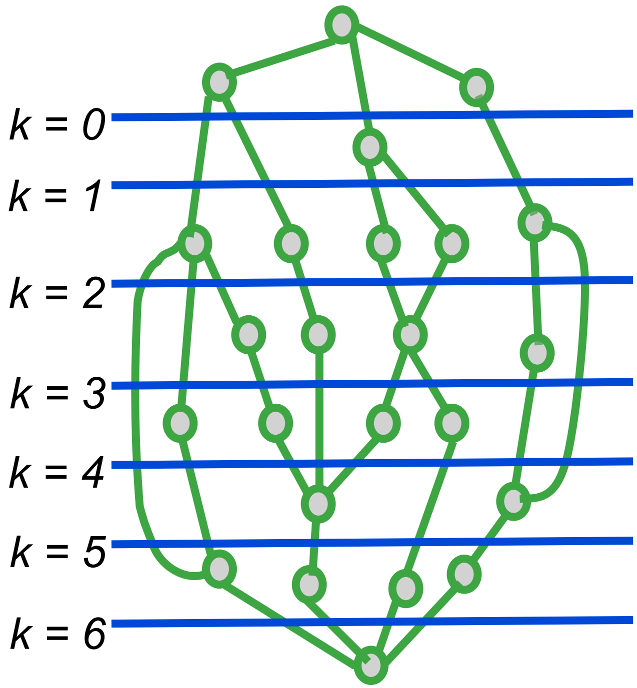


- Exhaustively explores all executions
- Report errors as traces

Avoiding state space explosion

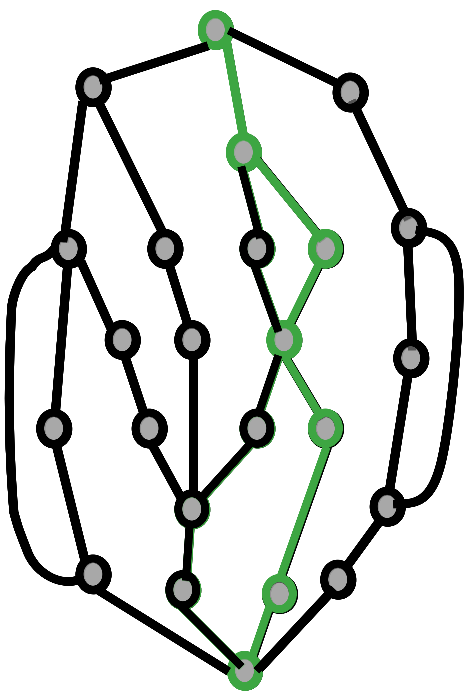
- Bounded Model Checking (BMC)
  - Breadth-first search (BFS) approach
- Symbolic Execution
  - Depth-first search (DFS) approach

Bounded Model Checking



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Symbolic Execution



- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

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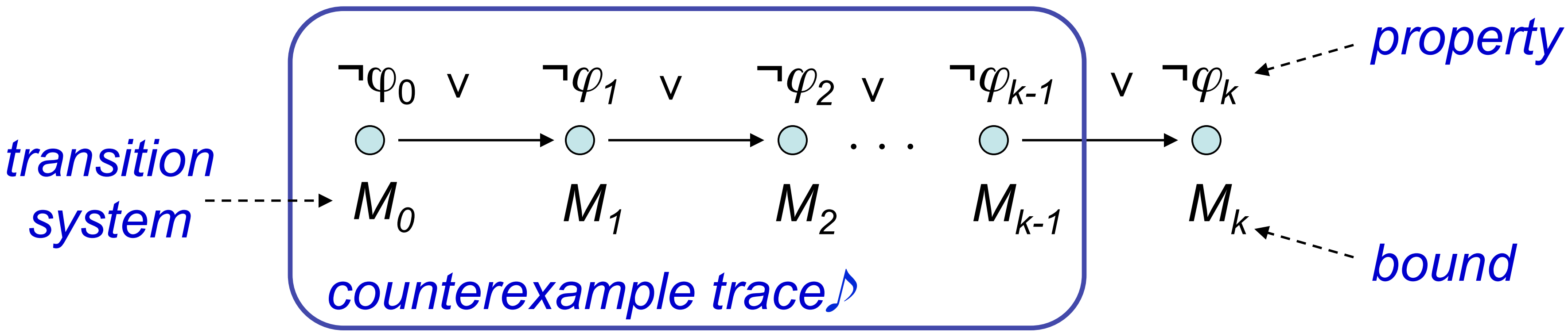
Bounded Model Checking

Basic Idea: check negation of given property up to given depth



- transition system M unrolled k times
  - for programs: unroll loops, unfold arrays, …
- translated into verification condition ψ such that

  ψ satisfiable iff φ has counterexample of max. depth k
- has been applied successfully to verify (sequential) software

Difficulties in proving the correctness of programs with loops in BMC

- BMC techniques can falsify properties up to a given depth k

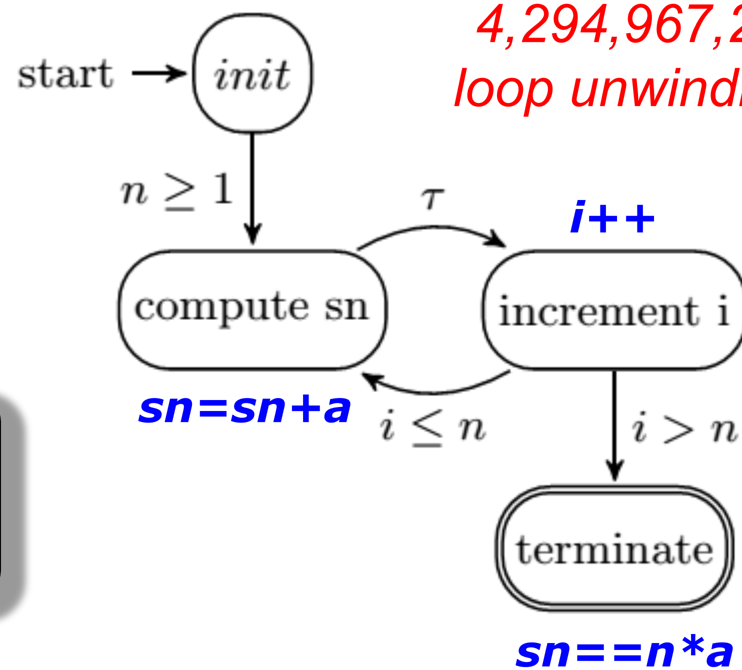
– they can prove correctness only if an upper bound of k is known (**unwinding assertion**)

  » BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

the loop will be unfolded 2^{n-1} times (in the worst case, 2^{32-1} times on 32 bits integer)

4,294,967,295 loop unwindings



BMC of Multi-threaded Software

- concurrency bugs are tricky to **reproduce/debug** because they usually occur under specific thread interleavings
    - most common errors: 67% related to atomicity and order violations, 30% related to deadlock [Lu et al.' 08]
- problem: the number of interleavings grows exponentially with the number of threads (n) and program statements (s)
    - number of executions: $O(n^s)$
    - context switches among threads increase the number of possible executions

BMC of single- and multi-threaded software

Bounded Model Checking of Software:

- symbolically executes programs into SSA, produces QF formulae
- unrolls loops and recursions up to a maximum bound *k*
- check whether corresponding formula is satisfiable
  - safety properties (array bounds, pointer dereferences, overflows,...)
  - user-specified properties

multi-threaded programs:

- combines explicit-state with symbolic model checking
- symbolic state hashing & monotonic POR
- context-bounded analysis (optional context bound)

Satisfiability Modulo Theories (1)

SMT decides the **satisfiability** of first-order logic formulae using the combination of different **background theories** (building-in operators)

Theory	Example
Equality	$x_1=x_2 \land \neg (x_1=x_3) \Rightarrow \neg(x_1=x_3)$
Bit-vectors	$(b >> i) \& 1 = 1$
Linear arithmetic	$(4y_1 + 3y_2 \geq 4) \lor (y_2 - 3y_3 \leq 3)$
Arrays	$(j = k \land a[k]=2) \Rightarrow a[j]=2$
Combined theories	$(j \leq k \land a[j]=2) \Rightarrow a[i] < 3$

Satisfiability Modulo Theories (2)

- Given
  - a decidable $\sum$-theory T
  - a quantifier-free formula φ

  φ is T-satisfiable iff $T \cup \{\varphi\}$ is satisfiable, i.e., there exists a structure that satisfies both formula and sentences of T
- Given
  - a set $\Gamma \cup \{\varphi\}$ of first-order formulae over T

  φ is a T-consequence of Γ ($\Gamma \models_T \varphi$) iff every model of $T \cup \Gamma$ is also a model of φ
- Checking $\Gamma \models_T \varphi$ can be reduced in the usual way to checking the T-satisfiability of $\Gamma \cup \{\neg\varphi\}$

Satisfiability Modulo Theories (3)

- let **a** be an array, **b**, **c** and **d** be signed bit-vectors of width 16, 32 and 32 respectively, and let **g** be an unary function.

$$g(select(store(a, c, 12)), SignExt(b, 16) + 3)$$
$$\neq g(SignExt(b, 16) - c + 4) \wedge SignExt(b, 16) = c - 3 \wedge c + 1 = d - 4$$

⇩ **b'** extends **b** to the signed equivalent bit-vector of size 32

$$step\,1 : g(select(store(a, c, 12), b' + 3)) \neq g(b' - c + 4) \wedge b' = c - 3 \wedge c + 1 = d - 4$$

⇩ replace b' by c−3 in the inequality

$$step\,2 : g(select(store(a, c, 12), c - 3 + 3)) \neq g(c - 3 - c + 4) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$

⇩ using facts about bit-vector arithmetic

$$step\,3 : g(select(store(a, c, 12), c)) \neq g(1) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$

Satisfiability Modulo Theories (4)

$$step\,3: g(select(store(a,c,12),c)) \neq g(1) \wedge c-3 = c-3 \wedge c+1 = d-4$$

applying the theory of arrays

$$step\,4: g(12) \neq g(1) \wedge c-3 \wedge c+1 = d-4$$

The function g implies that for all x and y,
if x = y, then g (x) = g (y) (*congruence rule*).

$$step\,5: SAT\,(\mathrm{c}=5, \mathrm{d}=10)$$

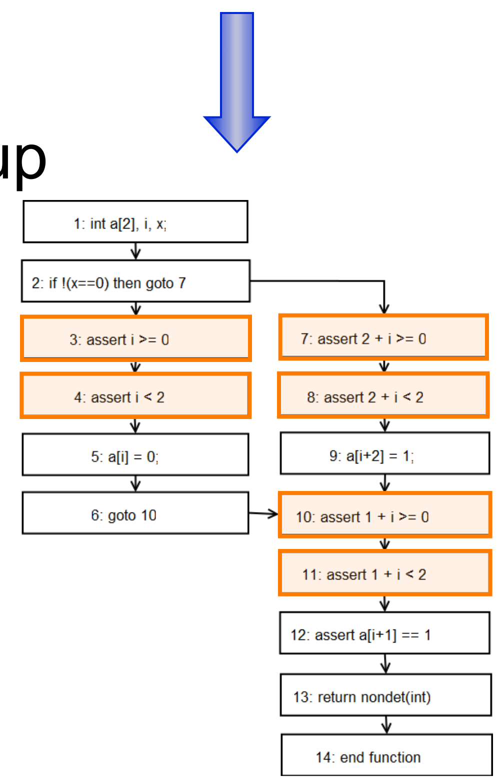
- SMT solvers also apply:
  - standard algebraic reduction rules $\boxed{r \wedge false \mapsto false}$
  - contextual simplification $\boxed{a = 7 \wedge p(a) \mapsto a = 7 \wedge p(7)}$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
  - derived from control-flow graph
  - checked safety properties give extra nodes
- program unfolded up to given bounds
  - loop iterations
  - context switches
- unfolded program optimized to reduce blow-up
  - constant propagation
  - forward substitutions

crucial

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```



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  crucial
- front-end converts unrolled and optimized program into SSA

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```

$$
\begin{aligned}
g_1 &= x_1 == 0\\
a_1 &= a_0 \text{ WITH } [i_0{:=}0]\\
a_2 &= a_0\\
a_3 &= a_2 \text{ WITH } [2+i_0{:=}1]\\
a_4 &= g_1 \,?\, a_1 : a_3\\
t_1 &= a_4[1+i_0] == 1
\end{aligned}
$$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
  - derived from control-flow graph
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- program unfolded up to given bounds
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- unfolded program optimized to reduce blow-up
  - constant propagation
  - forward substitutions

  crucial
- front-end converts unrolled and optimized program into SSA
- extraction of constraints C and properties P
  - specific to selected SMT solver, uses theories
- satisfiability check of C ∧ ¬P

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```

$$C := \begin{bmatrix} g_1 := (x_1 = 0) \\ \wedge\, a_1 := store(a_0, i_0, 0) \\ \wedge\, a_2 := a_0 \\ \wedge\, a_3 := store(a_2, 2 + i_0, 1) \\ \wedge\, a_4 := ite(g_1, a_1, a_3) \end{bmatrix}$$

$$P := \begin{bmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\, 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge\, 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\, select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

Encoding of Numeric Types

- SMT solvers typically provide different encodings for numbers:
  - abstract domains ($\mathbf{Z}$, $\mathbf{R}$)
  - fixed-width bit vectors (`unsigned int`, …)
    - ▹ "internalized bit-blasting"
- verification results can depend on encodings

$$(a > 0) \land (b > 0) \Rightarrow (a + b > 0)$$

valid in abstract domains such as $\mathbf{Z}$ or $\mathbf{R}$

doesn't hold for bitvectors, due to possible overflows

  - majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
  - ESBMC supports both types of encoding and also combines them to improve scalability and precision

Encoding Numeric Types as Bitvectors

Bitvector encodings need to handle

- type casts and implicit conversions
  - arithmetic conversions implemented using word-level functions (part of the bitvector theory: Extract, SignExt, …)
    - different conversions for every pair of types
    - uses type information provided by front-end
  - conversion to / from bool via if-then-else operator

    t = ite(v $\neq$ k, true, false) //conversion to bool
    v = ite(t, 1, 0) //conversion from bool
- arithmetic over- / underflow
  - standard requires modulo-arithmetic for unsigned integer

    $\text{unsigned\_overflow} \Leftrightarrow (r - (r \bmod 2^w)) < 2^w$
  - define error literals to detect over- / underflow for other types

    $\text{res\_op} \Leftrightarrow \neg\, \text{overflow}(x, y) \wedge \neg\, \text{underflow}(x, y)$
    - similar to conversions

Floating-Point Numbers

- Over-approximate floating-point by fixed-point numbers
  - encode the integral (i) and fractional (f) parts
- **Binary encoding:** get a new bit-vector b = i @ f with the same bitwidth before and after the radix point of a.

$$i = \begin{cases} Extract(b, n_b + m_a - 1, n_b) & : \quad m_a \leq m_b \\ SignExt(Extract(b, t_b - 1, n_b), m_a - m_b) & : \quad otherwise \end{cases}$$

// m = number of bits of i

$$f = \begin{cases} Extract(b, n_b - 1, n_b - n_b) & : \quad n_a \leq n_b \\ Extract(b, n_b, 0) @ SignExt(b, n_a - n_b) & : \quad otherwise \end{cases}$$

// n = number of bits of f

- **Rational encoding:** convert a to a rational number

$$a = \begin{cases} \dfrac{\left(i * p + \left(\dfrac{f * p}{2^n} + 1\right)\right)}{p} & : \quad f \neq 0 \\ i \quad : \quad otherwise \end{cases}$$

// p = number of decimal places

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
  - p.o ≜ representation of underlying object
  - p.i ≜ index (if pointer used as array base)

Store object at position 0

```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

$$C := \left(\begin{array}{l} p_1 := \mathrm{store}(p_0, 0, \&a[0]) \\ \wedge\ p_2 := \mathrm{store}(p_1, 1, 0) \\ \wedge\ g_2 := (x_2 == 0) \\ \wedge\ a_1 := \mathrm{store}(a_0, i \ldots \\ \ldots \\ \wedge\ a_3 := \mathrm{store}(a_2, 1+ i_0, 1) \\ \wedge\ a_4 := \mathrm{ite}(g_1, a_1, a_3) \\ \wedge\ p_3 := \mathrm{store}(p_2, 1, \mathrm{select}(p_2, 1)+2) \end{array}\right)$$

Store index at position 1

Update index

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
  - p.o $\triangleq$ representation of underlying object
  - p.i $\triangleq$ index (if pointer used as array base)

negation satisfiable
(a[2] unconstrained)
$\Rightarrow$ assert fails

```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

$$
P := \left(
\begin{array}{l}
i_0 \geq 0 \wedge i_0 < 2 \\
\wedge\ 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\
\wedge\ \mathrm{select}(p_3, 0) == \&a[0] \\
\wedge\ \mathrm{select}(\mathrm{select}(p_3, 0), \\
\qquad\qquad \mathrm{select}(p_3, 1)) == 1
\end{array}
\right)
$$

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
  - read and write operations to the memory array on the logic level
- each dynamic object d_o consists of
  - m ≜ memory array
  - s ≜ size in bytes of m
  - ρ ≜ unique identifier
  - υ ≜ indicate whether the object is still alive
  - l ≜ the location in the execution where m is allocated
- to detect invalid reads/writes, we check whether
  - d_o is a dynamic object
  - i is within the bounds of the memory array

$$l_{is\_dynamic\_object} \Leftrightarrow \left(\bigvee_{j=1}^{k} d_o.\rho = j \right) \wedge \left(0 \le i < n\right)$$

Encoding of Memory Allocation

- to check for invalid objects, we
  - set υ to true when the function malloc is called (d_o is alive)
  - set υ to false when the function free is called (d_o is not longer alive)

$$l_{valid\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow d_o.\upsilon)$$

- to detect forgotten memory, at the end of the (unrolled) program we check
  - whether the d_o has been deallocated by the function free

$$l_{deallocated\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow \neg\, d_o.\upsilon)$$

Example of Memory Allocation

```
#include <std
void main() {
  char *p = 
  char *q = malloc(5); // ρ = 2
  p=q;
  free(p)
  p = malloc(5);          // ρ = 3
  free(p)
}
```

memory leak: pointer reassignment makes $d_{o1}.\upsilon$ to become an orphan

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

$$P := \left(\neg d_{o1}.\upsilon \land \neg d_{o2}.\upsilon \; \neg d_{o3}.\upsilon \right)$$

$$C := \left(\begin{array}{l} d_{o1}.\rho=1 \land d_{o1}.s=5 \land d_{o1}.\upsilon=\mathit{true} \land p=d_{o1} \\ \land\ d_{o2}.\rho=2 \land d_{o2}.s=5 \land d_{o2}.\upsilon=\mathit{true} \land q=d_{o2} \\ \land\ p=d_{o2} \land d_{o2}.\upsilon=\mathit{false} \\ \land\ d_{o3}.\rho=3 \land d_{o3}.s=5 \land d_{o3}.\upsilon=\mathit{true} \land p=d_{o3} \\ \land\ d_{o3}.\upsilon=\mathit{false} \end{array} \right)$$

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
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