



**Systems and Software  
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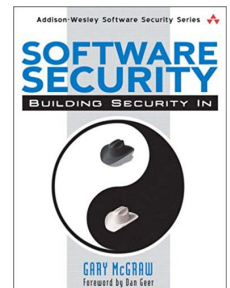
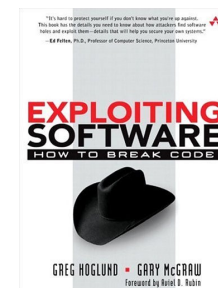
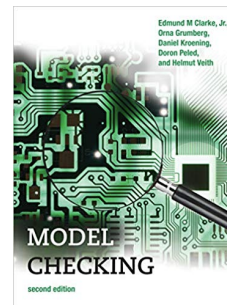
# Detection of Software Vulnerabilities: Static Analysis

**Lucas Cordeiro**  
**Department of Computer Science**  
[lucas.cordeiro@manchester.ac.uk](mailto:lucas.cordeiro@manchester.ac.uk)

# Detection of Software Vulnerabilities

- Lucas Cordeiro (Formal Methods Group)
  - [lucas.cordeiro@manchester.ac.uk](mailto:lucas.cordeiro@manchester.ac.uk)
  - Office: 2.28
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Exploiting Software: How to Break Code* (Chapter 7)
  - *C How to Program* (Chapter 1)

Rashid et al.: *The Cyber Security Body of Knowledge, CyBOK, v1.0*, 2019



# Intended learning outcomes

- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference between **static analysis** and **testing / simulation**
- Explain **Bounded** and **Unbounded Model Checking**
- Provide **practical examples** to detect software vulnerabilities statically

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# Motivating Example

- functionality demanded increased significantly
  - peer reviewing and testing
- multi-core processors with scalable shared memory / message passing
  - software model checking and testing

```
void *threadA(void *arg) {  
    lock(&mutex);  
    x++;  
    if (x == 1) lock(&lock);  
    unlock(&mutex); (CS1)  
    lock(&mutex); (CS3)  
    x--;  
    if (x == 0) unlock(&lock);  
    unlock(&mutex);  
}
```

**Deadlock**

```
void *threadB(void *arg) {  
    lock(&mutex);  
    y++;  
    if (y == 1) lock(&lock); (CS2)  
    lock(&mutex);  
    y--;  
    if (y == 0) unlock(&lock);  
    unlock(&mutex);  
}
```

# Detection of Vulnerabilities

- Detect the presence of vulnerabilities in the code during the development, testing and maintenance
- Techniques to detect vulnerabilities must make trade-offs between **soundness** and **completeness**
  - A detection technique is **sound** for a given category if it concludes that a given program has no vulnerabilities
    - An unsound detection technique may have **false negatives**, i.e., actual vulnerabilities that the detection technique fails to find
  - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
    - An incomplete detection technique may have **false positives**, i.e. it may detect issues that do not turn out to be actual vulnerabilities

# Detection of Vulnerabilities

- Achieving **soundness** requires reasoning about all executions of a program (usually an infinite number)
  - This is can done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, concrete executions of a program that are witnesses to any vulnerability reported
  - The analysis technique has to come up with concrete inputs for the program that trigger a vulnerability
    - A common dynamic approach is software testing: the tester writes test cases with concrete inputs, and specific checks for the outputs

# Detection of Vulnerabilities

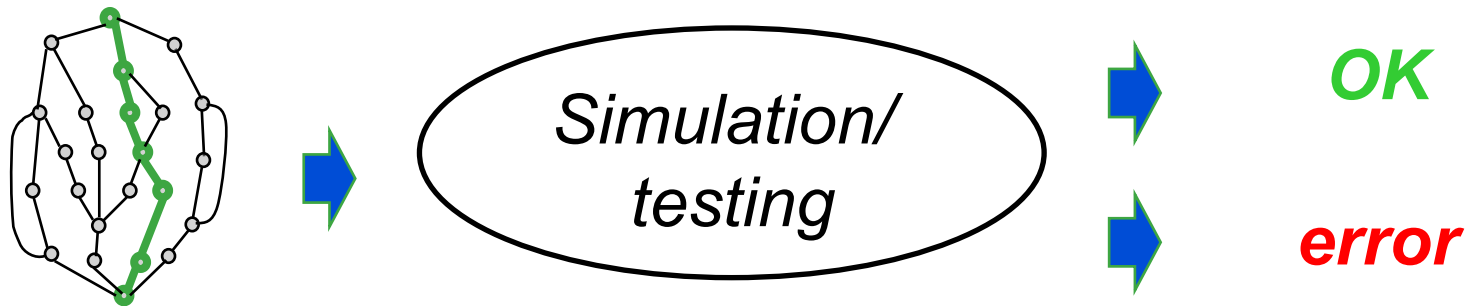
In practice, detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**



# Intended learning outcomes

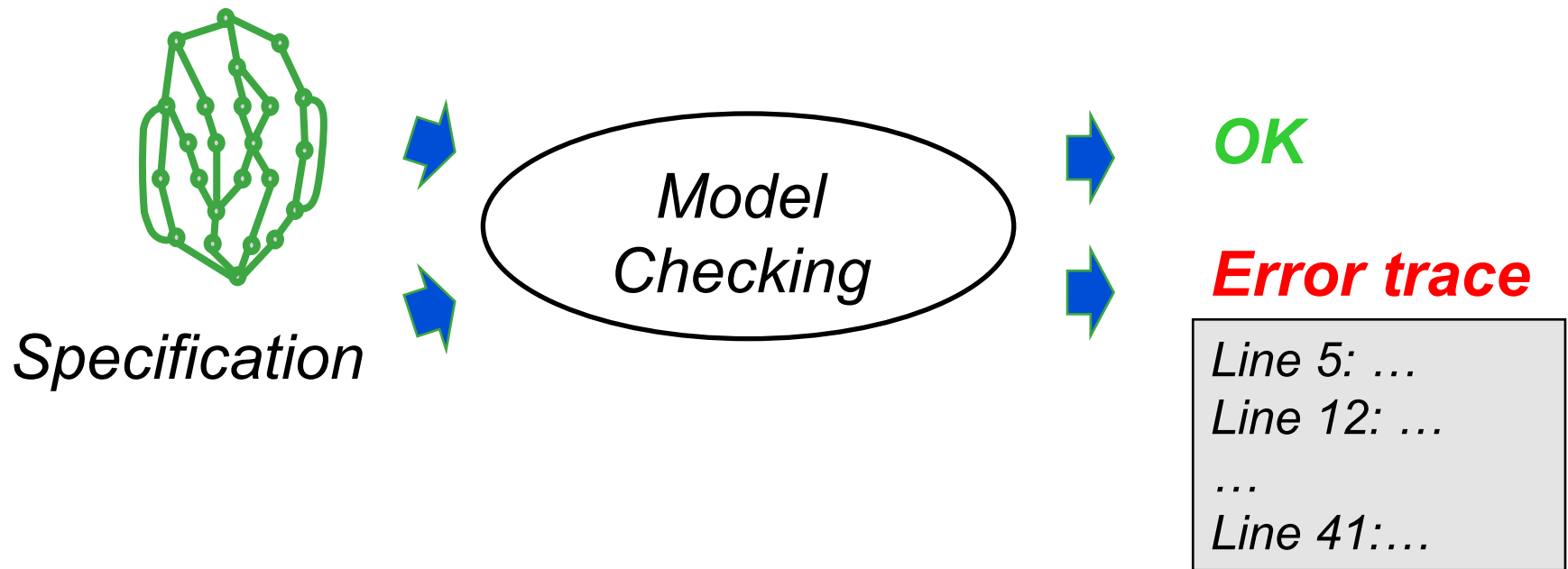
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# Static analysis vs Testing/ Simulation



- Checks only some of the system executions
- May miss errors

# Static analysis vs Testing/ Simulation

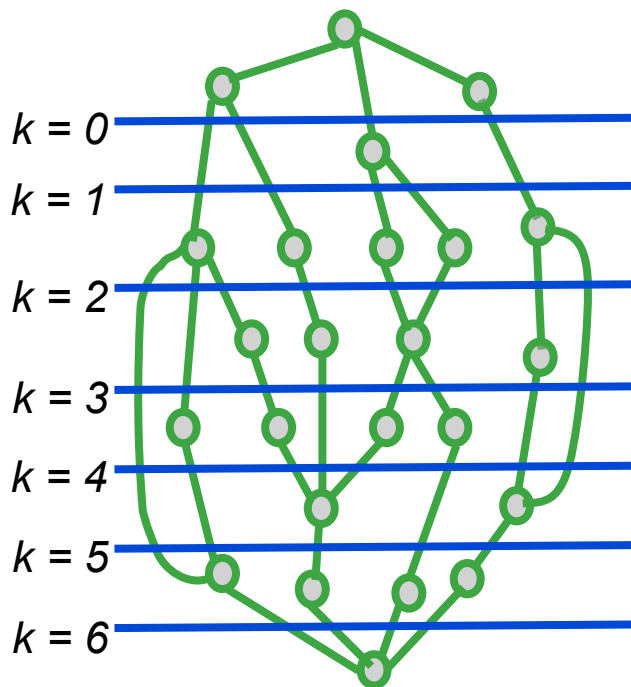


- Exhaustively explores all executions
- Report errors as traces

# Avoiding state space explosion

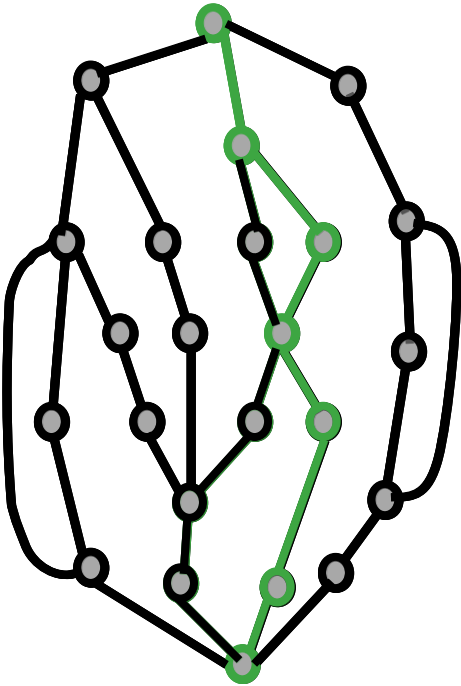
- Bounded Model Checking (BMC)
  - Breadth-first search (BFS) approach
- Symbolic Execution
  - Depth-first search (DFS) approach

# Bounded Model Checking



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

# Symbolic Execution



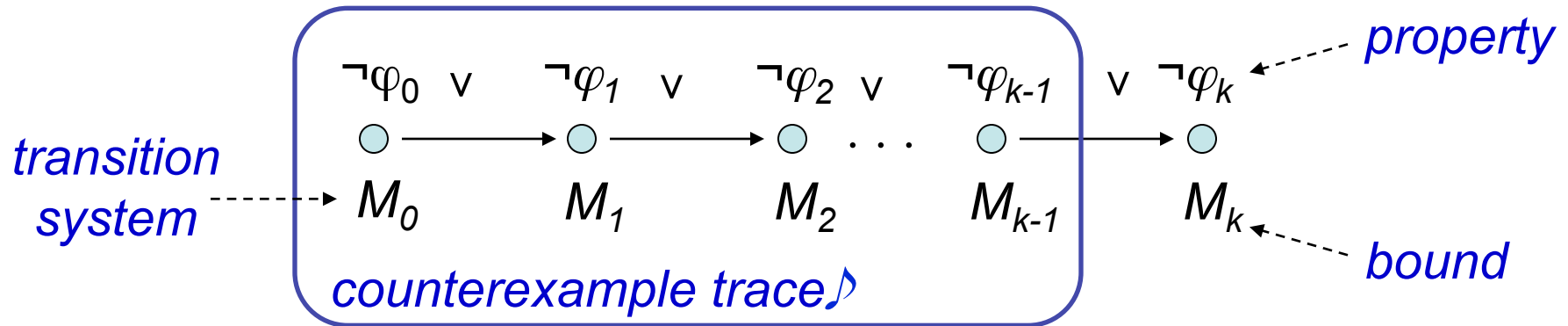
- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

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# Bounded Model Checking

Basic Idea: check negation of given property up to given depth



- transition system  $M$  unrolled  $k$  times
  - for programs: unroll loops, unfold arrays, ...
- translated into verification condition  $\psi$  such that
  - $\psi$  **satisfiable iff  $\varphi$  has counterexample of max. depth  $k$**
- has been applied successfully to verify (sequential) software

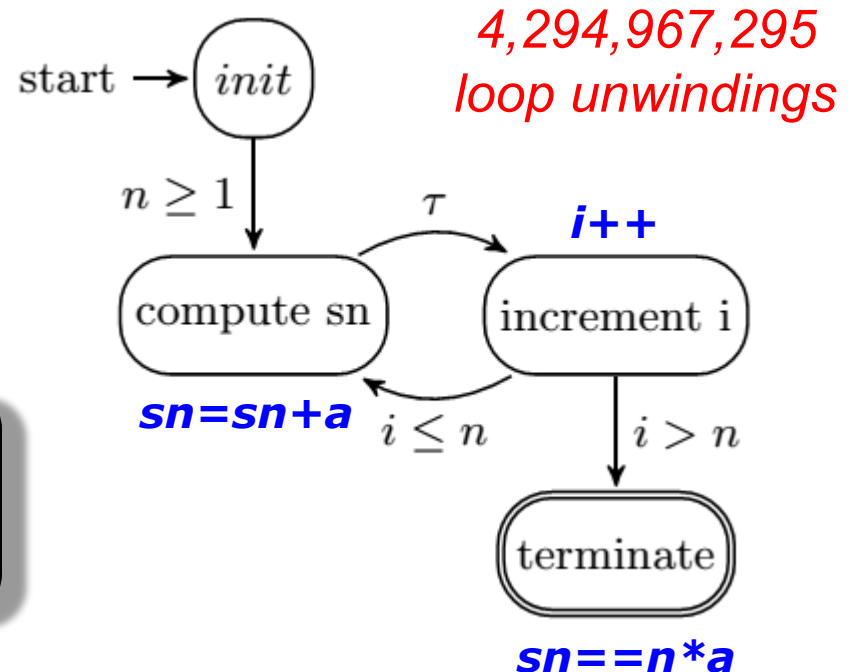


# Difficulties in proving the correctness of programs with loops in BMC

- BMC techniques can falsify properties up to a given depth  $k$ 
  - they can prove correctness only if an upper bound of  $k$  is known (**unwinding assertion**)
    - » BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^n a = na, n \geq 1$$

the loop will be unfolded  $2^{n-1}$  times  
(in the worst case,  $2^{32-1}$  times on 32  
bits integer)



# BMC of Multi-threaded Software

- concurrency bugs are tricky to **reproduce/debug** because they usually occur under specific thread interleavings
  - most common errors: 67% related to atomicity and order violations, 30% related to deadlock [Lu et al.' 08]
- problem: the number of interleavings grows exponentially with the number of threads ( $n$ ) and program statements ( $s$ )
  - number of executions:  $O(n^s)$
  - context switches among threads increase the number of possible executions

# BMC of single- and multi-threaded software

## Bounded Model Checking of Software:

- symbolically executes programs into SSA, produces QF formulae
- unrolls loops and recursions up to a maximum bound  $k$
- check whether corresponding formula is satisfiable
  - safety properties (array bounds, pointer dereferences, overflows,...)
  - user-specified properties

## multi-threaded programs:

- combines explicit-state with symbolic model checking
- symbolic state hashing & monotonic POR
- context-bounded analysis (optional context bound)

# Satisfiability Modulo Theories (1)

SMT decides the **satisfiability** of first-order logic formulae using the combination of different **background theories** (building-in operators)

Theory	Example
Equality	$x_1 = x_2 \wedge \neg (x_1 = x_3) \Rightarrow \neg (x_1 = x_3)$
Bit-vectors	$(b \gg i) \& 1 = 1$
Linear arithmetic	$(4y_1 + 3y_2 \geq 4) \vee (y_2 - 3y_3 \leq 3)$
Arrays	$(j = k \wedge a[k] = 2) \Rightarrow a[j] = 2$
Combined theories	$(j \leq k \wedge a[j] = 2) \Rightarrow a[i] < 3$

# Satisfiability Modulo Theories (2)

- Given

- a decidable  $\Sigma$ -theory  $T$
- a quantifier-free formula  $\varphi$

$\varphi$  is **T-satisfiable** iff  $T \cup \{\varphi\}$  is satisfiable, i.e., there exists a structure that satisfies both formula and sentences of  $T$

- Given

- a set  $\Gamma \cup \{\varphi\}$  of first-order formulae over  $T$

$\varphi$  is a **T-consequence of  $\Gamma$**  ( $\Gamma \models_T \varphi$ ) iff every model of  $T \cup \Gamma$  is also a model of  $\varphi$

- Checking  $\Gamma \models_T \varphi$  can be reduced in the usual way to checking the T-satisfiability of  $\Gamma \cup \{\neg\varphi\}$

# Satisfiability Modulo Theories (3)

- let **a** be an array, **b**, **c** and **d** be signed bit-vectors of width 16, 32 and 32 respectively, and let **g** be an unary function.

$$g(\text{select}(\text{store}(a, c, 12)), \text{SignExt}(b, 16) + 3) \\ \neq g(\text{SignExt}(b, 16) - c + 4) \wedge \text{SignExt}(b, 16) = c - 3 \wedge c + 1 = d - 4$$



**b'** extends **b** to the signed equivalent bit-vector of size 32

$$\text{step 1: } g(\text{select}(\text{store}(a, c, 12), b' + 3)) \neq g(b' - c + 4) \wedge b' = c - 3 \wedge c + 1 = d - 4$$



replace **b'** by **c-3** in the inequality

$$\text{step 2: } g(\text{select}(\text{store}(a, c, 12), c - 3 + 3)) \neq g(c - 3 - c + 4) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$



using facts about bit-vector arithmetic

$$\text{step 3: } g(\text{select}(\text{store}(a, c, 12), c)) \neq g(1) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$

# Satisfiability Modulo Theories (4)

*step 3*:  $g(\text{select}(\text{store}(a, c, 12), c)) \neq g(1) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$

↓ applying the theory of arrays

*step 4*:  $g(12) \neq g(1) \wedge c - 3 \wedge c + 1 = d - 4$

↓ The function  $g$  implies that for all  $x$  and  $y$ ,  
if  $x = y$ , then  $g(x) = g(y)$  (congruence rule).

*step 5*:  $\text{SAT}(c = 5, d = 10)$

- SMT solvers also apply:
  - standard algebraic reduction rules
  - contextual simplification

$$\boxed{r \wedge \text{false} \mapsto \text{false}}$$

$$\boxed{a = 7 \wedge p(a) \mapsto a = 7 \wedge p(7)}$$