Systems and Software
Verification Laboratory

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Detection of Software Vulnerabilities: Static Analysis

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Detection of Software Vulnerabilities

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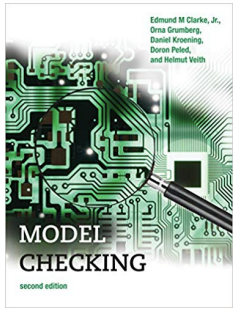
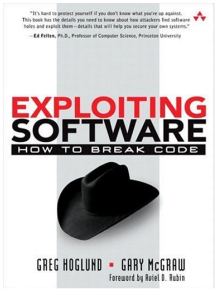
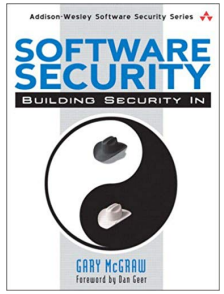
  - Office: 2.28
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Exploiting Software: How to Break Code* (Chapter 7)
  - *C How to Program* (Chapter 1)

Rashid et al.: *The Cyber Security Body of Knowledge*, CyBOK, v1.0, 2019







Intended learning outcomes

- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference between **static analysis** and **testing / simulation**
- Explain **bounded model checking of software**
- Explain **unbounded model checking**
- Provide **practical examples** to detect software vulnerabilities statically

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Motivating Example

- functionality demanded increased significantly
  - peer reviewing and testing
- multi-core processors with scalable shared memory / message passing
  - software model checking and testing

```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex);          (CS1)
  lock(&mutex);            (CS3)
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

Deadlock

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock);          (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
```

Detection of Vulnerabilities

- Detect the presence of vulnerabilities in the code during the development, testing and maintenance
- Techniques to detect vulnerabilities must make trade-offs between **soundness** and **completeness**
  - A detection technique is **sound** for a given category if it concludes that a given program has no vulnerabilities
    - An unsound detection technique may have ***false negatives***, i.e., actual vulnerabilities that the detection technique fails to find
  - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
    - An incomplete detection technique may have ***false positives***, i.e. it may detect issues that do not turn out to be actual vulnerabilities

Detection of Vulnerabilities

- Achieving **soundness** requires reasoning about all executions of a program (usually an infinite number)
  - This is can done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, concrete executions of a program that are witnesses to any vulnerability reported
  - The analysis technique has to come up with concrete inputs for the program that trigger a vulnerability
    - A common dynamic approach is software testing: the tester writes test cases with concrete inputs, and specific checks for the outputs

Detection of Vulnerabilities

In practice, detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**

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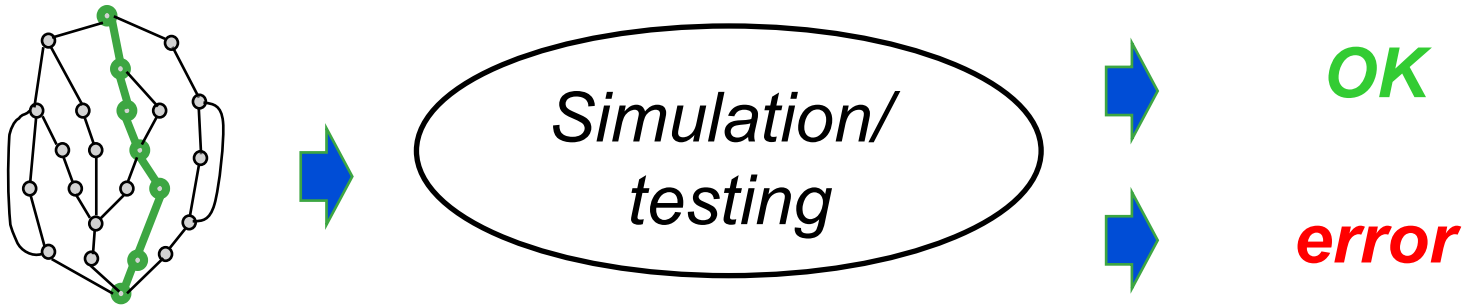
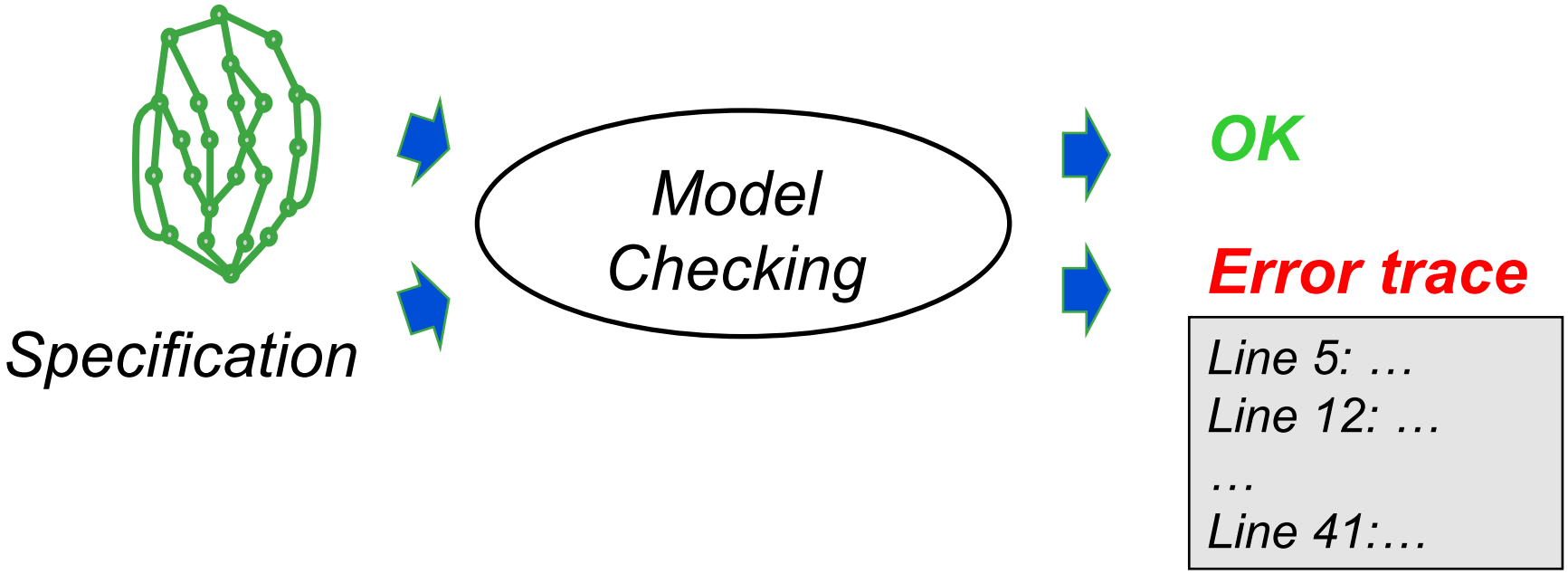
Static analysis vs Testing/ Simulation



- Checks only some of the system executions
- May miss errors

Static analysis vs Testing/ Simulation

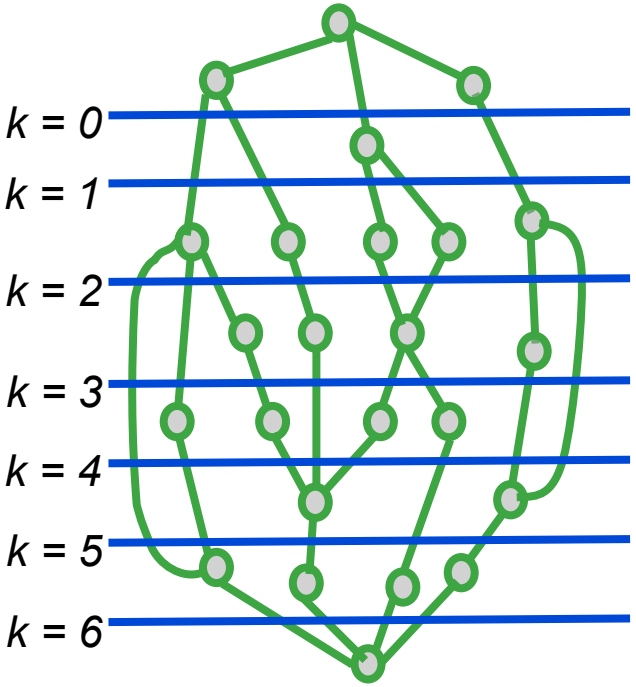


- Exhaustively explores all executions
- Report errors as traces

Avoiding state space explosion

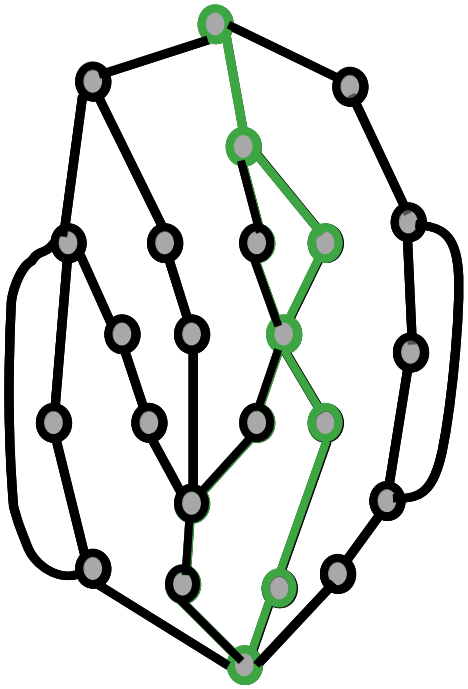
- Bounded Model Checking (BMC)
  - Breadth-first search (BFS) approach
- Symbolic Execution
  - Depth-first search (DFS) approach

Bounded Model Checking



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Symbolic Execution



- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

Intended learning outcomes

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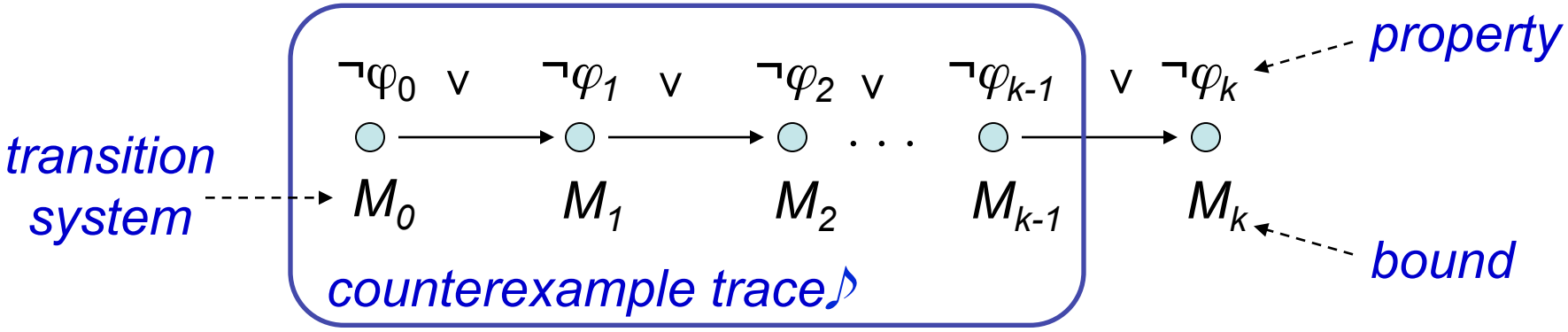
Bounded Model Checking

Basic Idea: check negation of given property up to given depth



- transition system M unrolled k times
  - for programs: unroll loops, unfold arrays, …
- translated into verification condition ψ such that

  ψ satisfiable iff φ has counterexample of max. depth k
- has been applied successfully to verify (sequential) software

Difficulties in proving the correctness of programs with loops in BMC

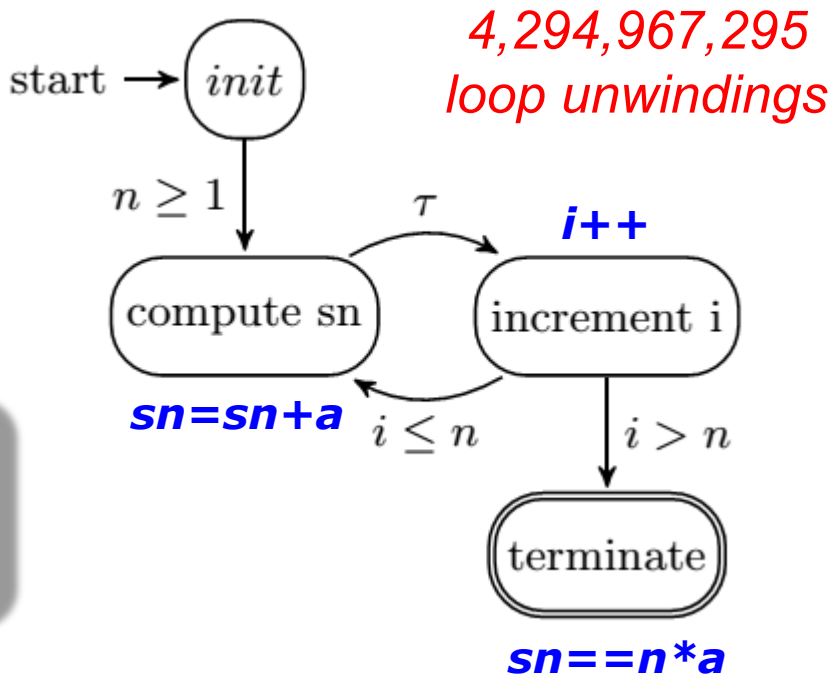
- BMC techniques can falsify properties up to a given depth k

– they can prove correctness only if an upper bound of k is known (**unwinding assertion**)

» BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

the loop will be unfolded 2^{n-1} times (in the worst case, 2^{32-1} times on 32 bits integer)



BMC of Multi-threaded Software

- concurrency bugs are tricky to **reproduce/debug** because they usually occur under specific thread interleavings
  - most common errors: 67% related to atomicity and order violations, 30% related to deadlock [Lu et al.' 08]
- problem: the number of interleavings grows exponentially with the number of threads (n) and program statements (s)
  - number of executions: $O(n^s)$
  - context switches among threads increase the number of possible executions

BMC of single- and multi-threaded software

Bounded Model Checking of Software:

- symbolically executes programs into SSA, produces QF formulae
- unrolls loops and recursions up to a maximum bound *k*
- check whether corresponding formula is satisfiable
  - safety properties (array bounds, pointer dereferences, overflows,...)
  - user-specified properties

multi-threaded programs:

- combines explicit-state with symbolic model checking
- symbolic state hashing & monotonic POR
- context-bounded analysis (optional context bound)

Satisfiability Modulo Theories (1)

SMT decides the **satisfiability** of first-order logic formulae using the combination of different **background theories** (building-in operators)

Theory	**Example**
Equality	$x_1=x_2 \wedge \neg (x_1=x_3) \Rightarrow \neg(x_1=x_3)$
Bit-vectors	(b >> i) & 1 = 1
Linear arithmetic	$(4y_1 + 3y_2 \geq 4) \vee (y_2 - 3y_3 \leq 3)$
Arrays	$(j = k \wedge a[k]=2) \Rightarrow a[j]=2$
Combined theories	$(j \leq k \wedge a[j]=2) \Rightarrow a[i] < 3$

Satisfiability Modulo Theories (2)

- Given
  - a decidable $\sum$-theory T
  - a quantifier-free formula φ

  φ is T-satisfiable iff $T \cup \{\varphi\}$ is satisfiable, i.e., there exists a structure that satisfies both formula and sentences of T
- Given
  - a set $\Gamma \cup \{\varphi\}$ of first-order formulae over T

  φ is a T-consequence of Γ ($\Gamma \models_T \varphi$) iff every model of $T \cup \Gamma$ is also a model of φ
- Checking $\Gamma \models_T \varphi$ can be reduced in the usual way to checking the T-satisfiability of $\Gamma \cup \{\neg\varphi\}$

Satisfiability Modulo Theories (3)

- let **a** be an array, **b**, **c** and **d** be signed bit-vectors of width 16, 32 and 32 respectively, and let **g** be an unary function.

$$g(select(store(a,c,12)), SignExt(b,16)+3)$$
$$\neq g(SignExt(b,16)-c+4) \wedge SignExt(b,16)=c-3 \wedge c+1=d-4$$

⇩ **b'** extends **b** to the signed equivalent bit-vector of size 32

$$step\,1: g(select(store(a,c,12),b'+3)) \neq g(b'-c+4) \wedge b'=c-3 \wedge c+1=d-4$$

⇩ replace b' by c−3 in the inequality

$$step\,2: g(select(store(a,c,12),c-3+3)) \neq g(c-3-c+4) \wedge c-3=c-3 \wedge c+1=d-4$$

⇩ using facts about bit-vector arithmetic

$$step\,3: g(select(store(a,c,12),c)) \neq g(1) \wedge c-3=c-3 \wedge c+1=d-4$$

Satisfiability Modulo Theories (4)

$$step\,3 : g(select(store(a,c,12),c)) \neq g(1) \wedge c-3 = c-3 \wedge c+1 = d-4$$

⇩ applying the theory of arrays

$$step\,4 : g(12) \neq g(1) \wedge c-3 \wedge c+1 = d-4$$

⇩ The function g implies that for all x and y, if x = y, then g (x) = g (y) (*congruence rule*).

$$step\,5 : SAT\,(c = 5, d = 10)$$

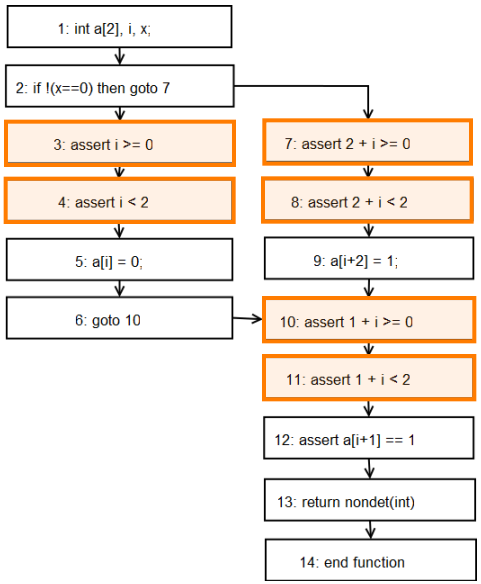
- SMT solvers also apply:
  - standard algebraic reduction rules $\boxed{r \wedge false \mapsto false}$
  - contextual simplification $\boxed{a = 7 \wedge p(a) \mapsto a = 7 \wedge p(7)}$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
  - derived from control-flow graph
  - checked safety properties give extra nodes
- program unfolded up to given bounds
  - loop iterations
  - context switches
- unfolded program optimized to reduce blow-up
  - constant propagation
  - forward substitutions

} crucial

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```



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  } crucial
- front-end converts unrolled and optimized program into SSA

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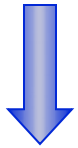

$g_1 = x_1 == 0$
$a_1 = a_0$ WITH $[i_0:=0]$
$a_2 = a_0$
$a_3 = a_2$ WITH $[2+i_0:=1]$
$a_4 = g_1 \; ? \; a_1 : a_3$
$t_1 = a_4[1+i_0] == 1$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
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  } crucial
- front-end converts unrolled and optimized program into SSA
- extraction of constraints C and properties P
  - specific to selected SMT solver, uses theories
- satisfiability check of C ∧ ¬P

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
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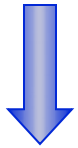


$$C := \begin{bmatrix} g_1 := (x_1 = 0) \\ \wedge\ a_1 := store(a_0, i_0, 0) \\ \wedge\ a_2 := a_0 \\ \wedge\ a_3 := store(a_2, 2 + i_0, 1) \\ \wedge\ a_4 := ite(g_1, a_1, a_3) \end{bmatrix}$$

$$P := \begin{bmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\ 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge\ 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\ select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

Encoding of Numeric Types

- SMT solvers typically provide different encodings for numbers:
  - abstract domains ($\mathbb{Z}$, $\mathbb{R}$)
  - fixed-width bit vectors (`unsigned int`, ...)
    - ▷ "internalized bit-blasting"
- verification results can depend on encodings

$$(a > 0) \wedge (b > 0) \Rightarrow (a + b > 0)$$

valid in abstract domains such as $\mathbb{Z}$ or $\mathbb{R}$

doesn't hold for bitvectors, due to possible overflows

  - majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
  - ESBMC supports both types of encoding and also combines them to improve scalability and precision

Encoding Numeric Types as Bitvectors

Bitvector encodings need to handle

- type casts and implicit conversions
  - arithmetic conversions implemented using word-level functions (part of the bitvector theory: Extract, SignExt, …)
    - o different conversions for every pair of types
    - o uses type information provided by front-end
  - conversion to / from bool via if-then-else operator

    t = ite(v ≠ k, true, false) //conversion to bool
    v = ite(t, 1, 0) //conversion from bool
- arithmetic over- / underflow
  - standard requires modulo-arithmetic for unsigned integer

    unsigned_overflow ⇔ $(r - (r \bmod 2^w)) < 2^w$
  - define error literals to detect over- / underflow for other types

    res_op ⇔ ¬ overflow(x, y) ∧ ¬ underflow(x, y)
    - o similar to conversions

Floating-Point Numbers

- Over-approximate floating-point by fixed-point numbers
  - encode the integral (i) and fractional (f) parts
- **Binary encoding:** get a new bit-vector b = i @ f with the same bitwidth before and after the radix point of a.

$$i = \begin{cases} Extract(b, n_b + m_a - 1, n_b) & : \quad m_a \le m_b \\ SignExt(Extract(b, t_b - 1, n_b), m_a - m_b) & : \quad otherwise \end{cases}$$

// m = number of bits of i

$$f = \begin{cases} Extract(b, n_b - 1, n_b - n_b) & : \quad n_a \le n_b \\ Extract(b, n_b, 0) \ @ \ SignExt(b, n_a - n_b) & : \quad otherwise \end{cases}$$

// n = number of bits of f

- **Rational encoding:** convert a to a rational number

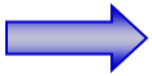
$$a = \begin{cases} \dfrac{\left(i * p + \left(\dfrac{f * p}{2^n} + 1\right)\right)}{p} & : \quad f \neq 0 \\ i & : \quad otherwise \end{cases}$$

// p = number of decimal places

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
  - p.o ≙ representation of underlying object
  - p.i ≙ index (if pointer used as array base)

```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```



C

$p_1 := \text{store}(p_0, 0, \&a[0])$
$\wedge\ p_2 := \text{store}(p_1, 1, 0)$
$\wedge\ g_2 := (x_2 == 0)$
$\wedge\ a_1 := \text{store}(a_0, i$
$\wedge\ a_3 := \text{store}(a_2, 1+ i_0, 1)$
$\wedge\ a_4 := \text{ite}(g_1, a_1, a_3)$
$\wedge\ p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1)+2)$

Store object at position 0

Store index at position 1

Update index

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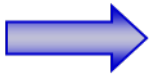

$$P := \begin{pmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\ 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\ \text{select}(p_3, 0) = \&a[0] \\ \wedge\ \text{select}(\text{select}(p_3, 0), \\ \qquad \text{select}(p_3, 1)) == 1 \end{pmatrix}$$

negation satisfiable (a[2] unconstrained) ⇒ assert fails

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
  - read and write operations to the memory array on the logic level
- each dynamic object d_o consists of
  - m ≙ memory array
  - s ≙ size in bytes of m
  - ρ ≙ unique identifier
  - υ ≙ indicate whether the object is still alive
  - l ≙ the location in the execution where m is allocated
- to detect invalid reads/writes, we check whether
  - d_o is a dynamic object
  - i is within the bounds of the memory array

$$l_{is\_dynamic\_object} \Leftrightarrow \left(\bigvee_{j=1}^{k} d_o.\rho = j \right) \wedge \left(0 \le i < n \right)$$

Encoding of Memory Allocation

- to check for invalid objects, we
  - set υ to true when the function malloc is called (d_o is alive)
  - set υ to false when the function free is called (d_o is not longer alive)

$$l_{valid\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow d_o.\upsilon)$$

- to detect forgotten memory, at the end of the (unrolled) program we check
  - whether the d_o has been deallocated by the function free

$$l_{deallocated\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow \neg\, d_o.\upsilon)$$

Example of Memory Allocation

```
#include <std
void main() {
  char *p =
  char *q = malloc(5);  // p
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

memory leak: pointer reassignment makes $d_{o1}.\upsilon$ to become an orphan

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

$$P := \left(\neg d_{o1}.\upsilon \wedge \neg d_{o2}.\upsilon \; \neg d_{o3}.\upsilon \right)$$

$$C := \left(\begin{array}{l} d_{o1}.\rho=1 \wedge d_{o1}.s=5 \wedge d_{o1}.\upsilon=\mathit{true} \wedge p=d_{o1} \\ \wedge \; d_{o2}.\rho=2 \wedge d_{o2}.s=5 \wedge d_{o2}.\upsilon=\mathit{true} \wedge q=d_{o2} \\ \wedge \; p=d_{o2} \wedge d_{o2}.\upsilon=\mathit{false} \\ \wedge \; d_{o3}.\rho=3 \wedge d_{o3}.s=5 \wedge d_{o3}.\upsilon=\mathit{true} \wedge p=d_{o3} \\ \wedge \; d_{o3}.\upsilon=\mathit{false} \end{array} \right)$$

Example of Memory Allocation

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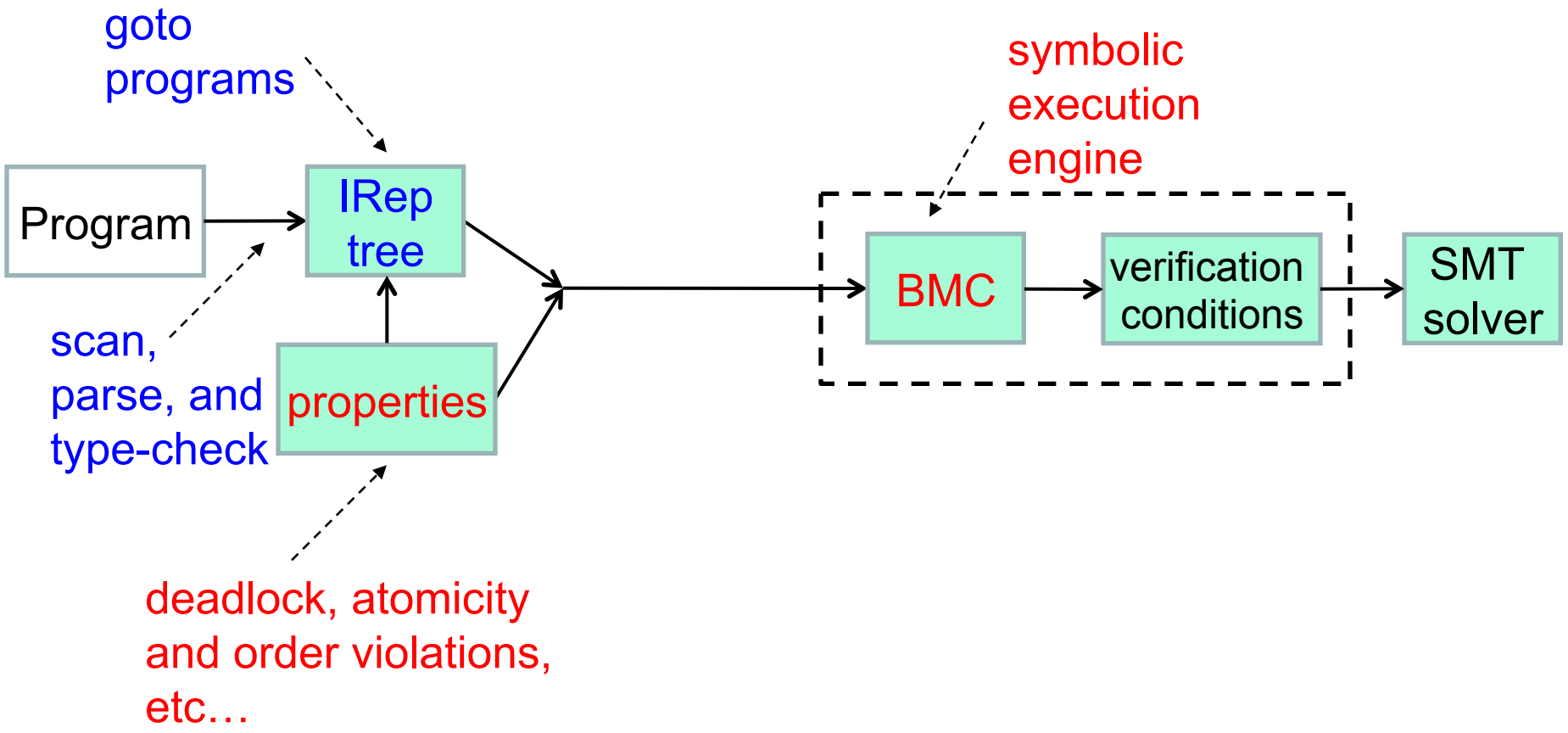
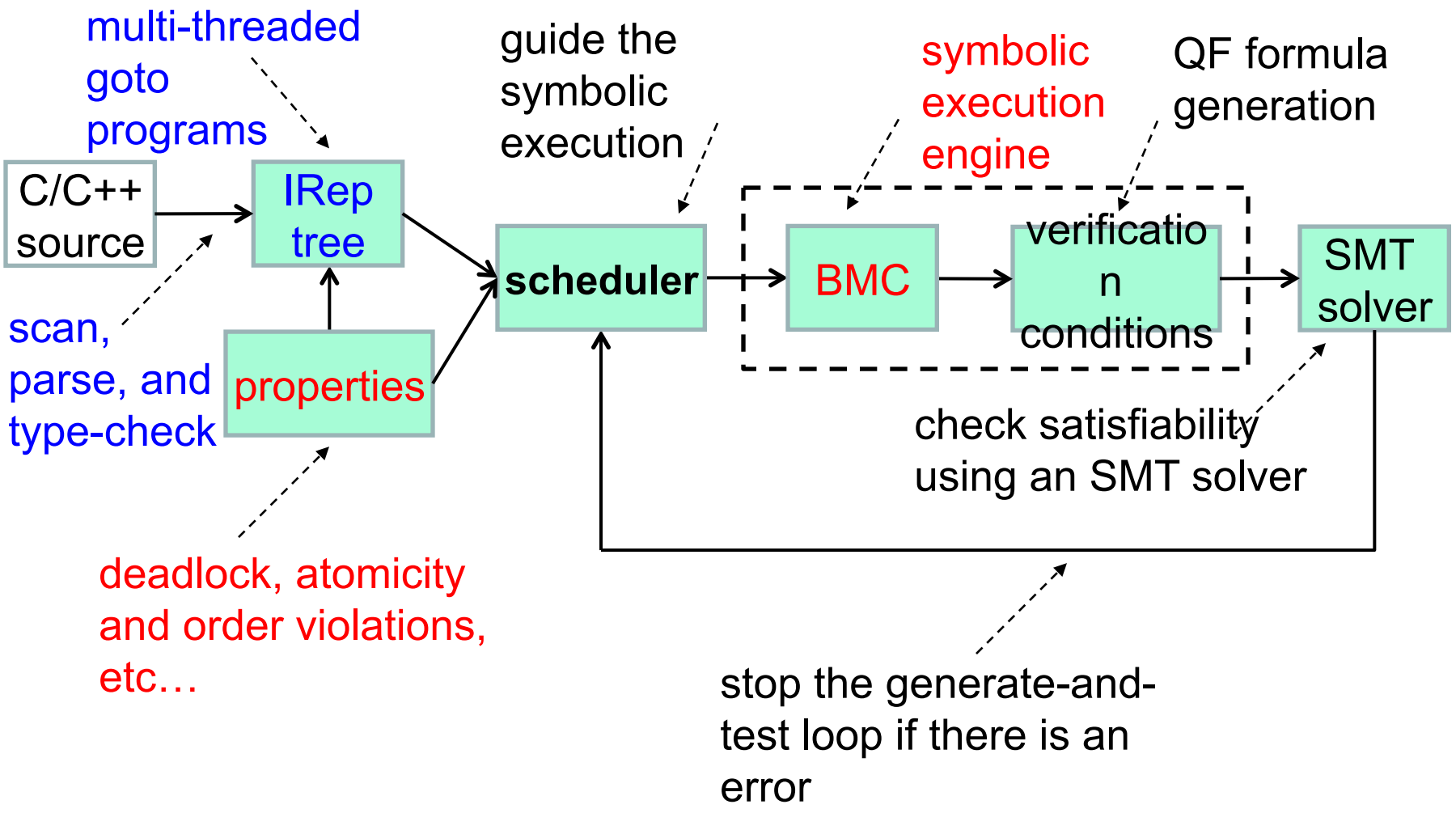
BMC Architecture

- A typical BMC architecture for verifying programs



BMC of Multi-threaded Software

Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving



Running Example

- the program has sequences of operations that need to be protected together to avoid atomicity violation
  - requirement: the region of code (val1 and val2) should execute atomically

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

A state $s \in S$ consists of the value of the program counter pc and the values of all program variables

```
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

```
program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;
```

Lazy exploration: interleaving l_s

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 1
mutexes: ***m1=1****; m2=0;*
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2

val1-access: $W_{twoStage,2}$

val2-access:

write access to the shared variable val1 in statement 2 of the thread twoStage

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 2
mutexes: m1=1; m2=0;
*global variables: **val1=1**; val2=0;*
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3
mutexes: ***m1=0****; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7
mutexes: ***m1=1****; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l

statements: 1-2-3-7-8

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$

val2-access:

read access to the shared variable val1 in statement 8 of the thread reader

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 8

mutexes: m1=1; m2=0;

global variables: val1=1; val2=0;

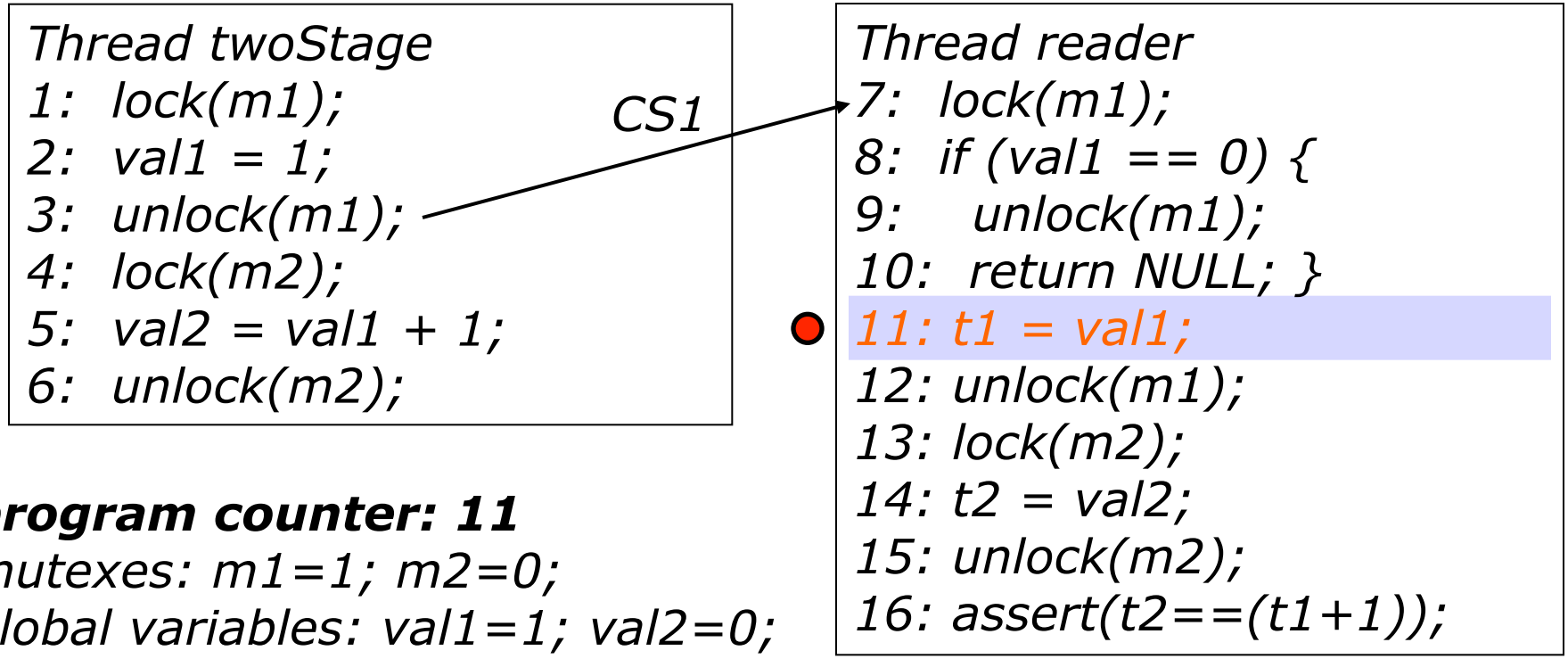
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



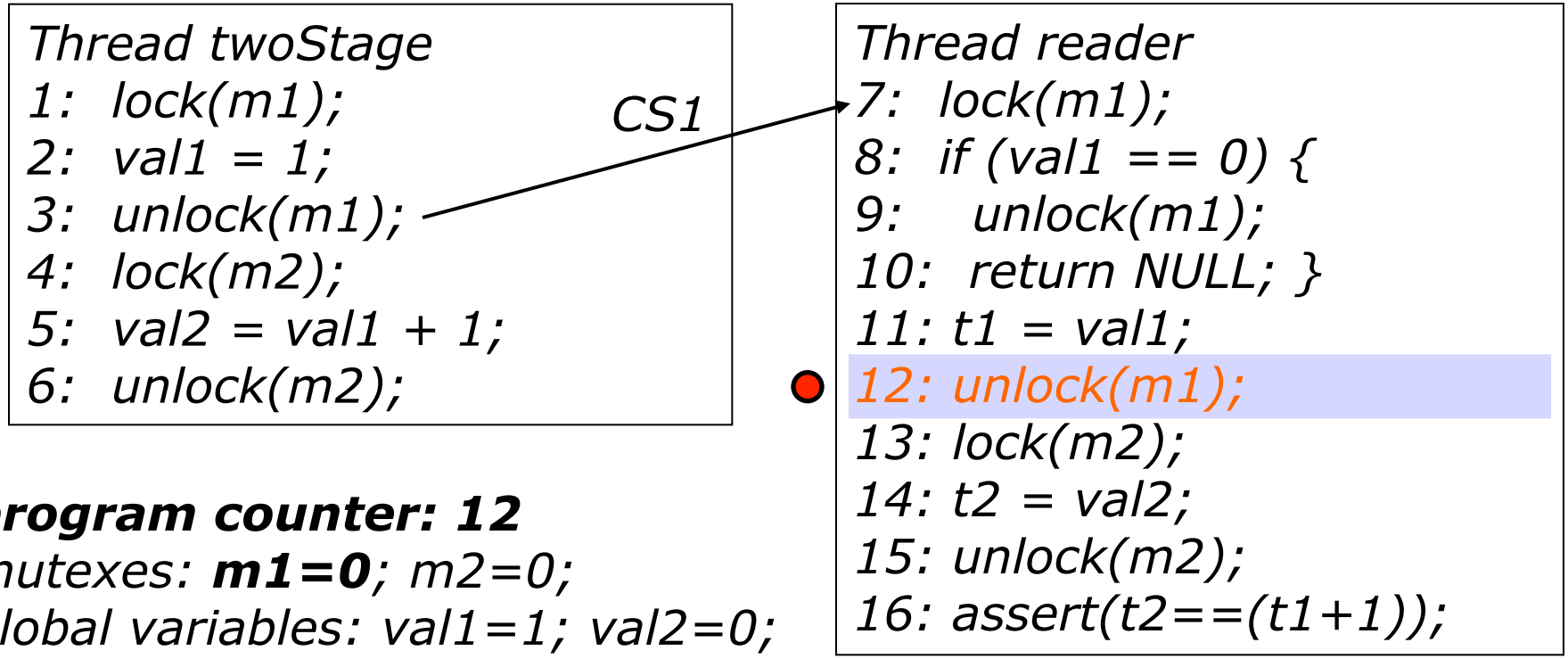
program counter: 11
mutexes: m1=1; m2=0;
global variables: val1=1; val2=0;
local variabes: ***t1= 1****; t2= -1;*

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



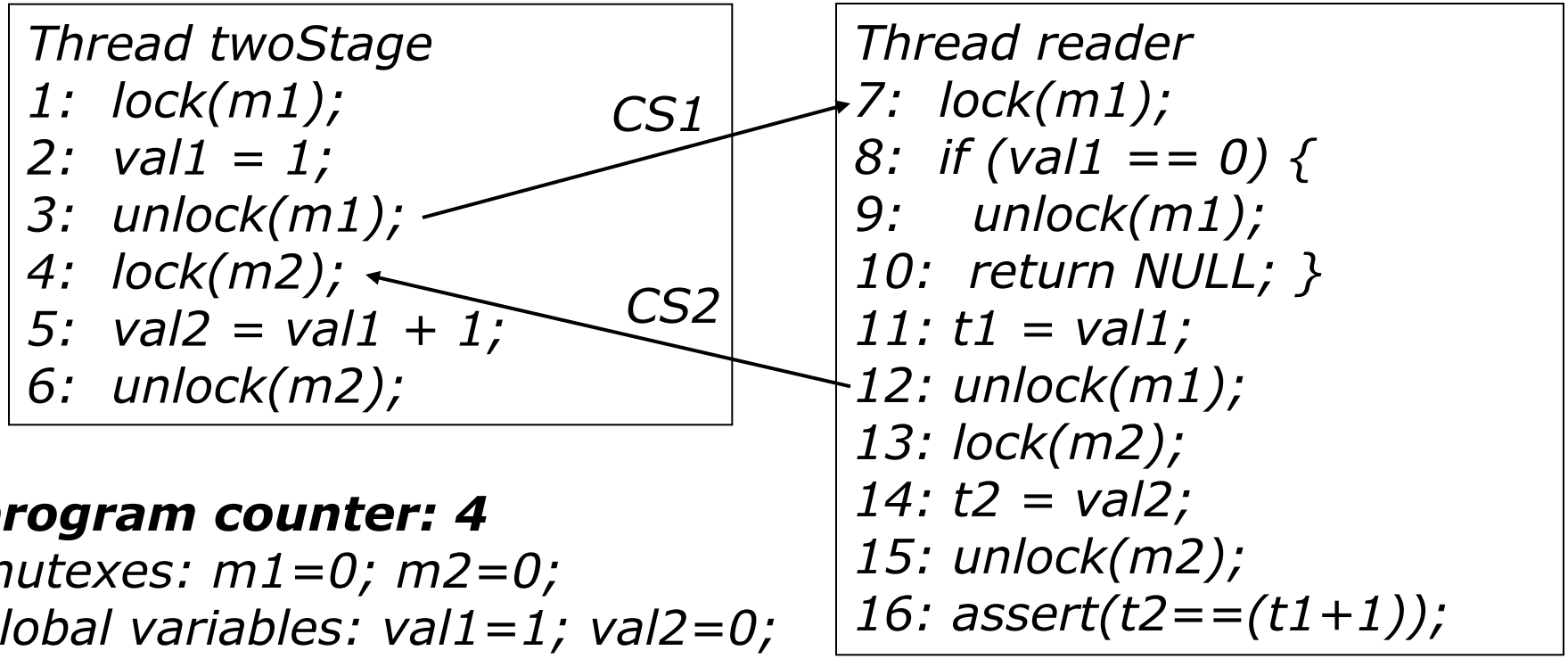
program counter: 12
*mutexes: **m1=0**; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



program counter: 4

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

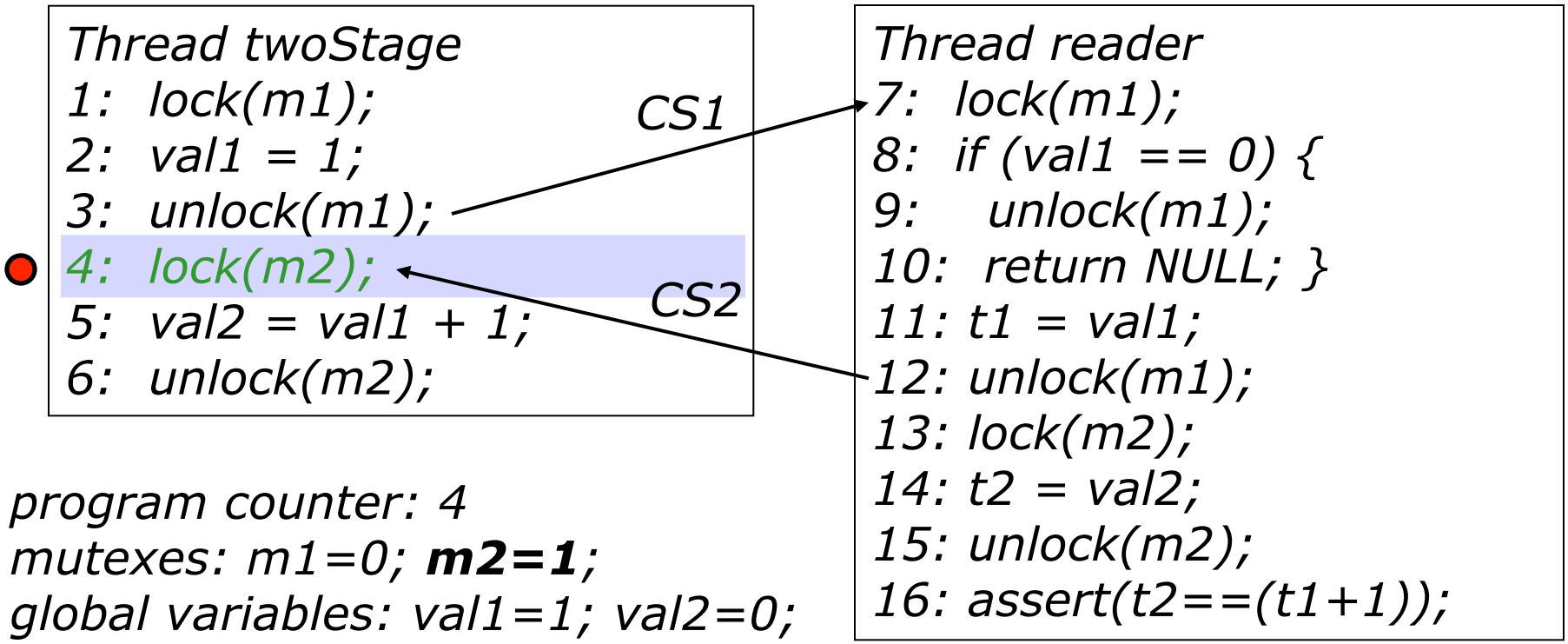
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



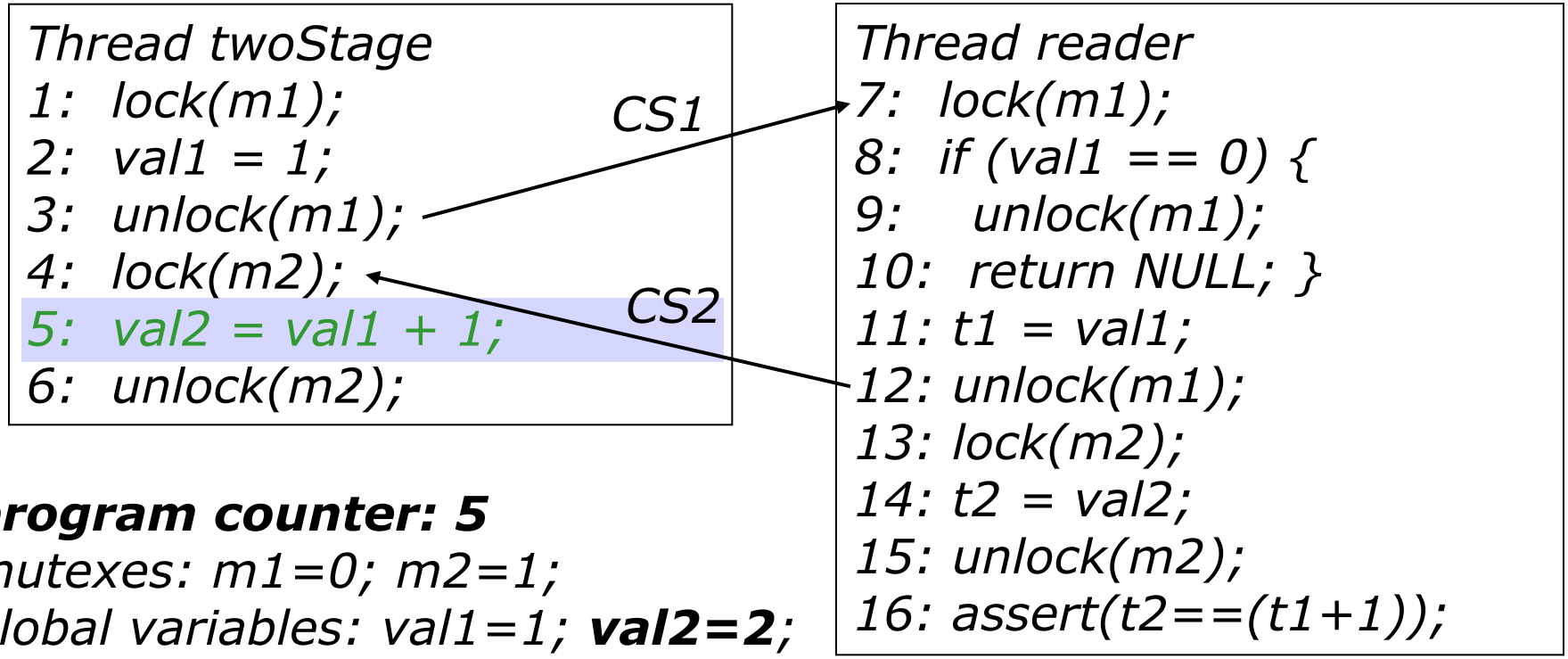
program counter: 4
*mutexes: m1=0; **m2=1**;*
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



program counter: 5

mutexes: m1=0; m2=1;

global variables: val1=1; ***val2=2****;*

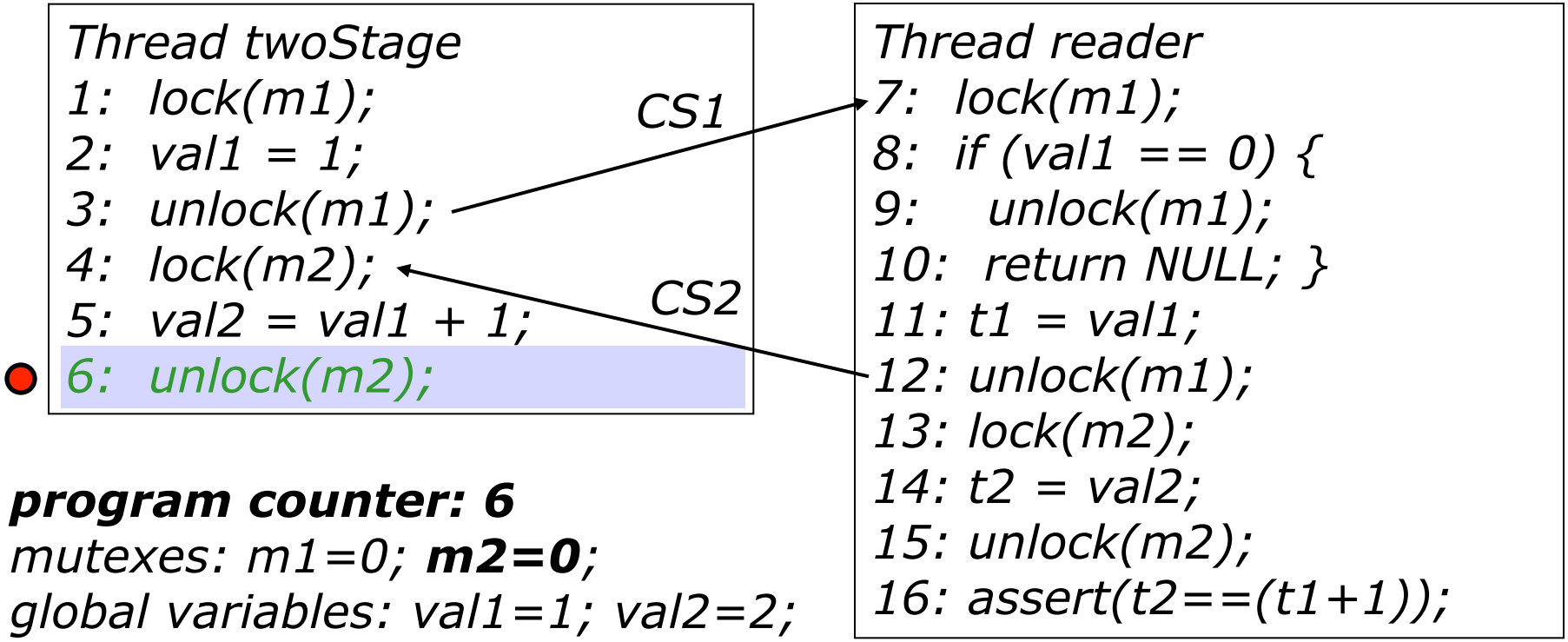
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



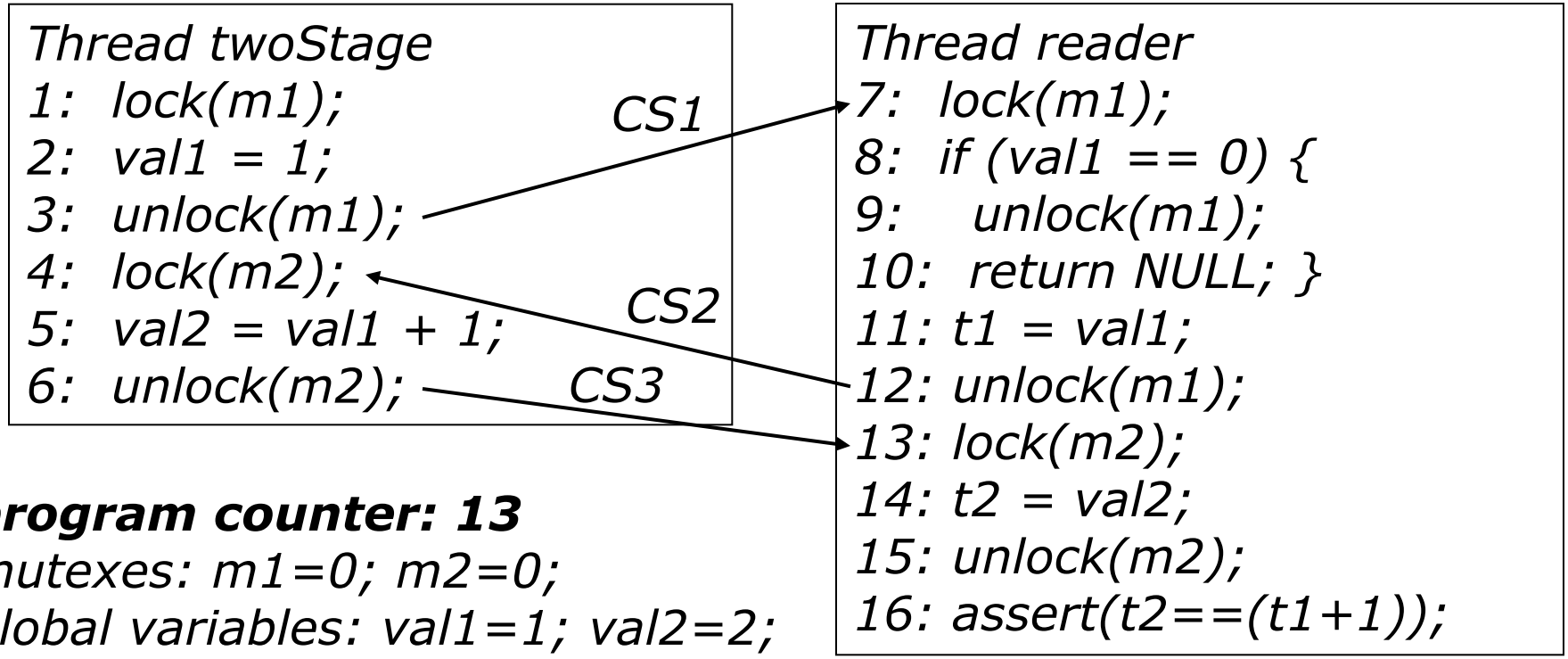
program counter: 6
*mutexes: m1=0; **m2=0**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



program counter: 13

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

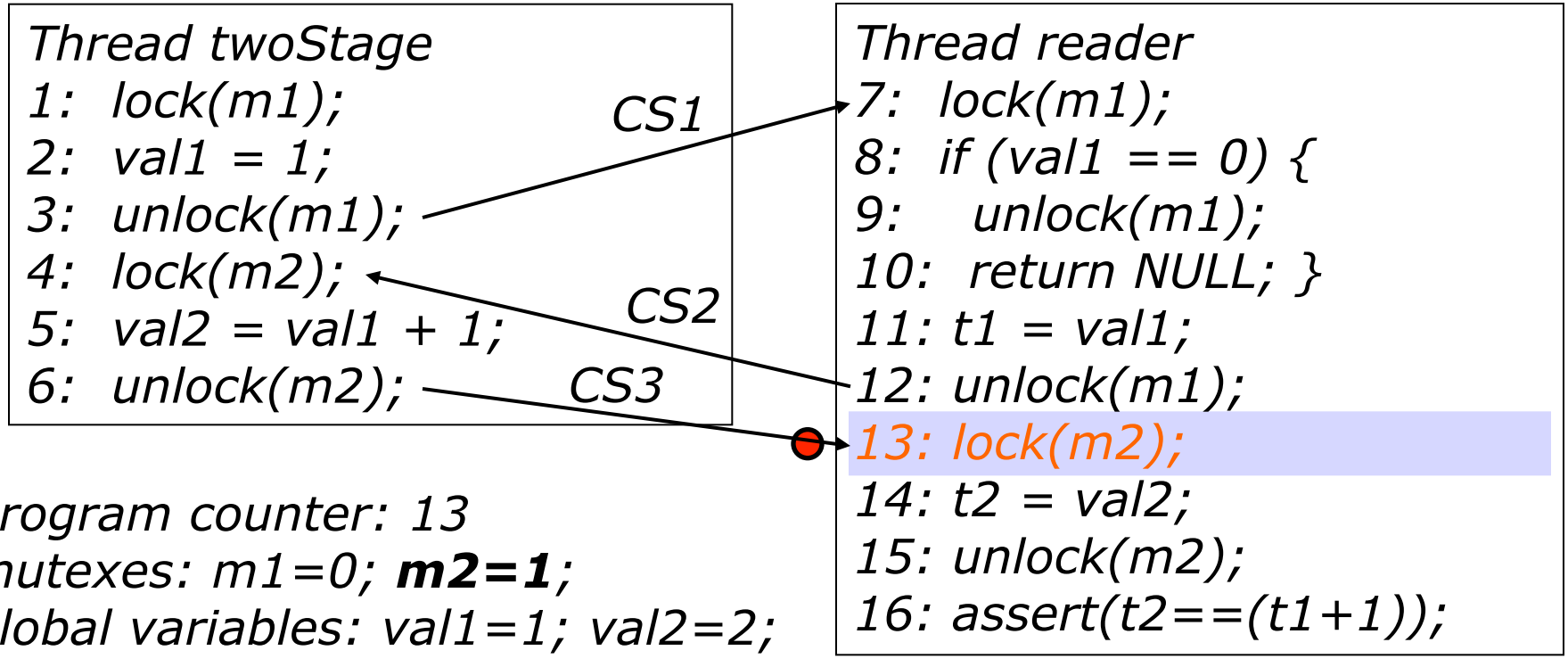
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



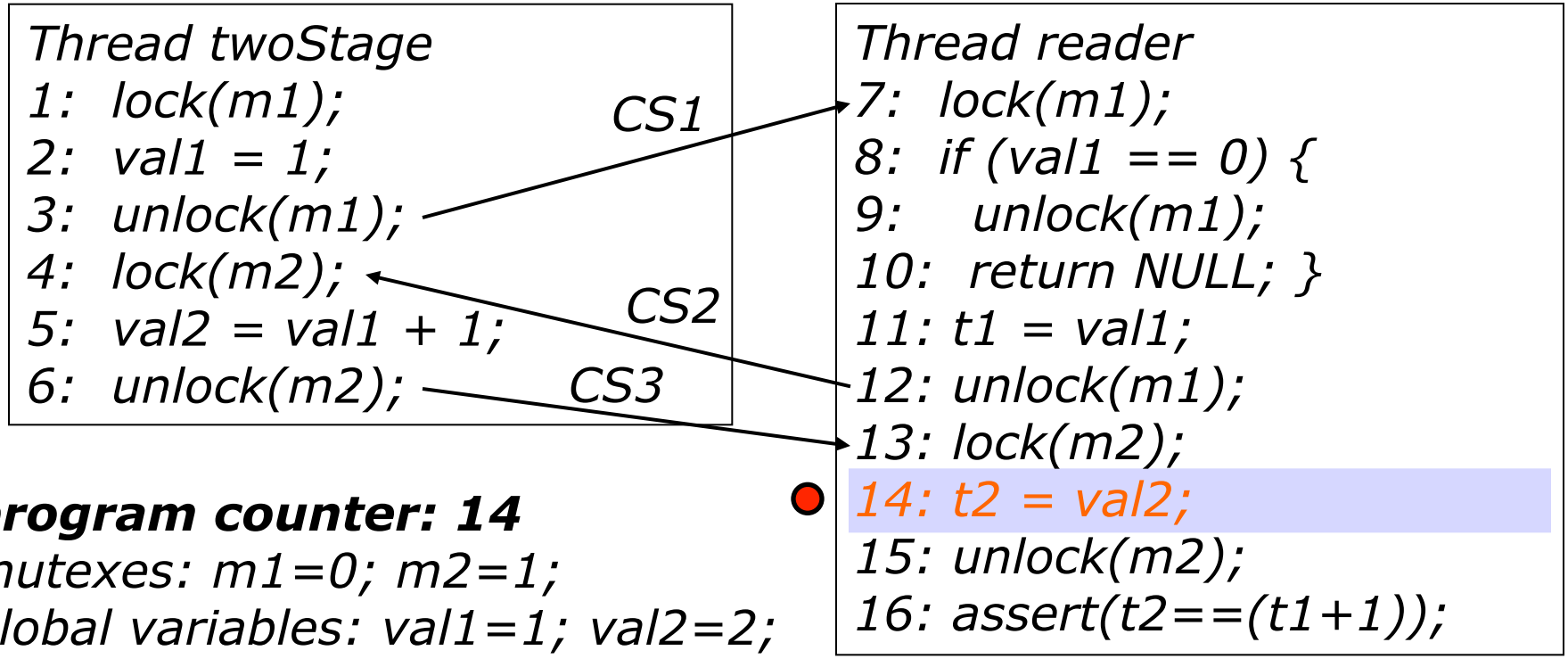
program counter: 13
*mutexes: m1=0; **m2=1**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



program counter: 14

mutexes: m1=0; m2=1;

global variables: val1=1; val2=2;

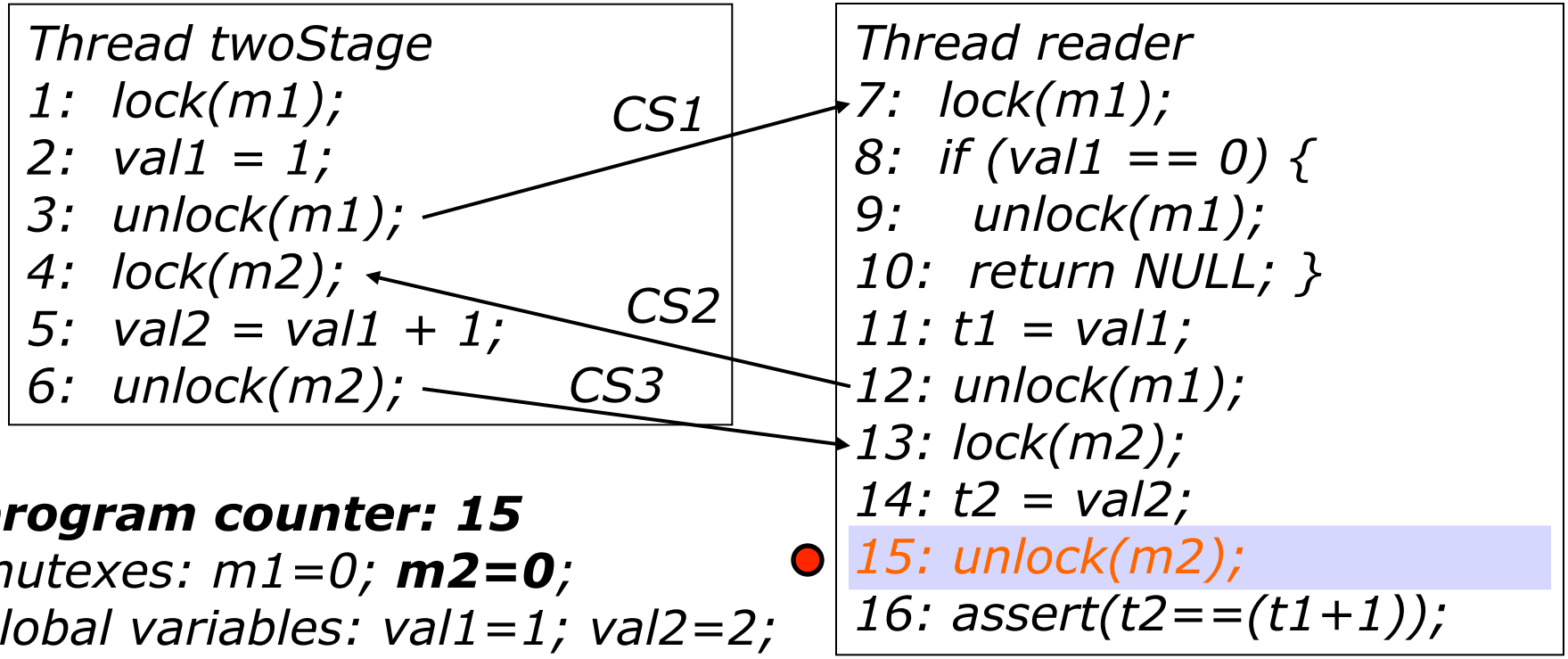
local variabes: t1= 1; ***t2= 2;***

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



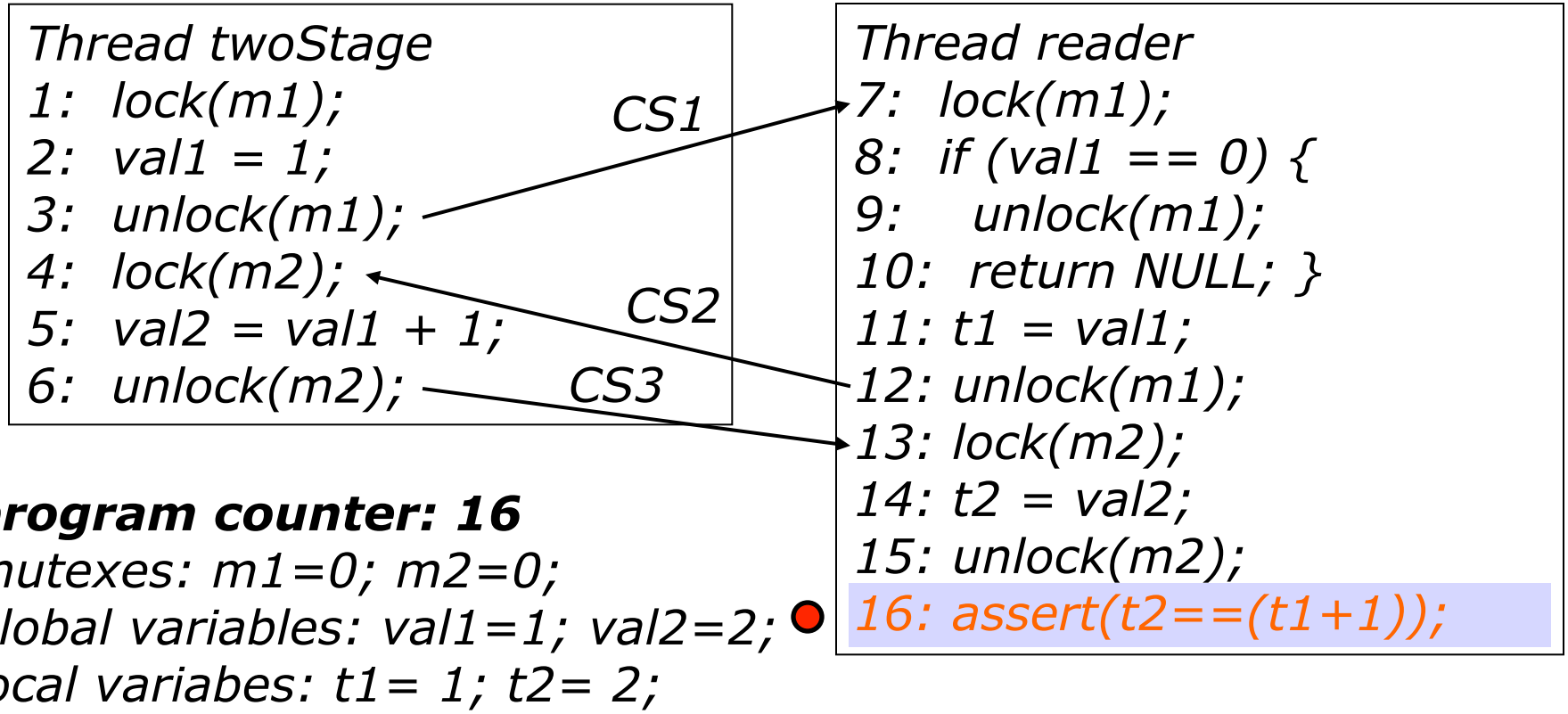
program counter: 15
*mutexes: m1=0; **m2=0**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



program counter: 16

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

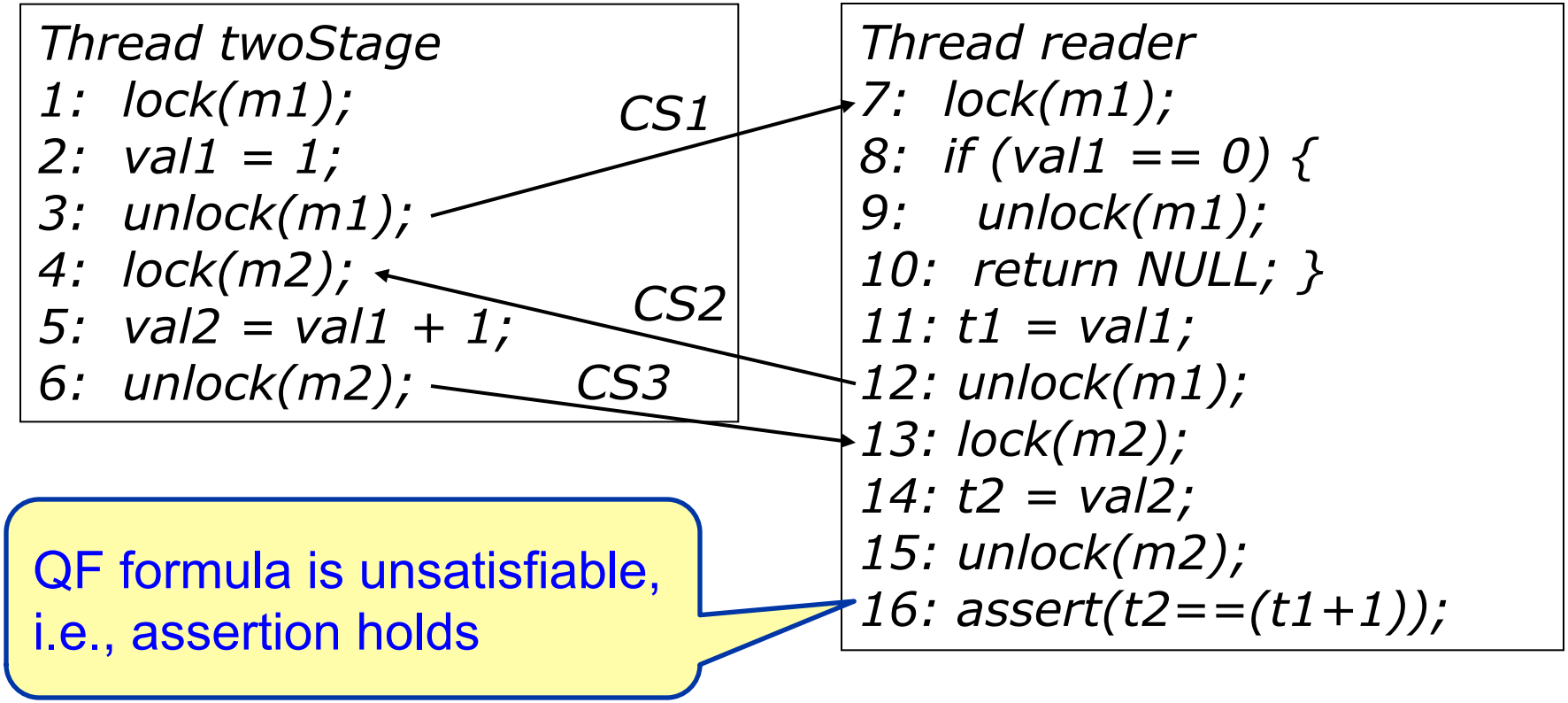
local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



Lazy exploration: interleaving l_f

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_f

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

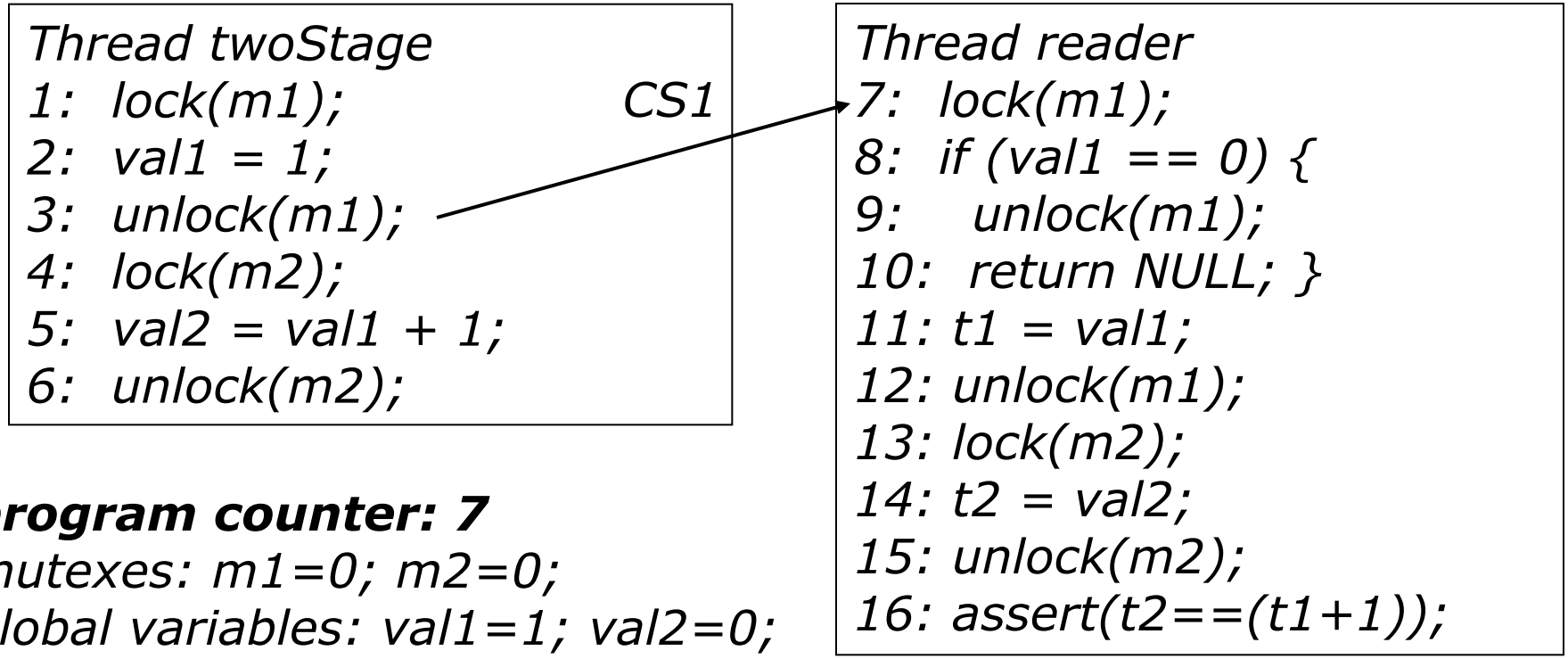
program counter: 3
mutexes: m1=0; m2=0;
*global variables: **val1=1**; val2=0;*
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_f

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:



program counter: 7

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

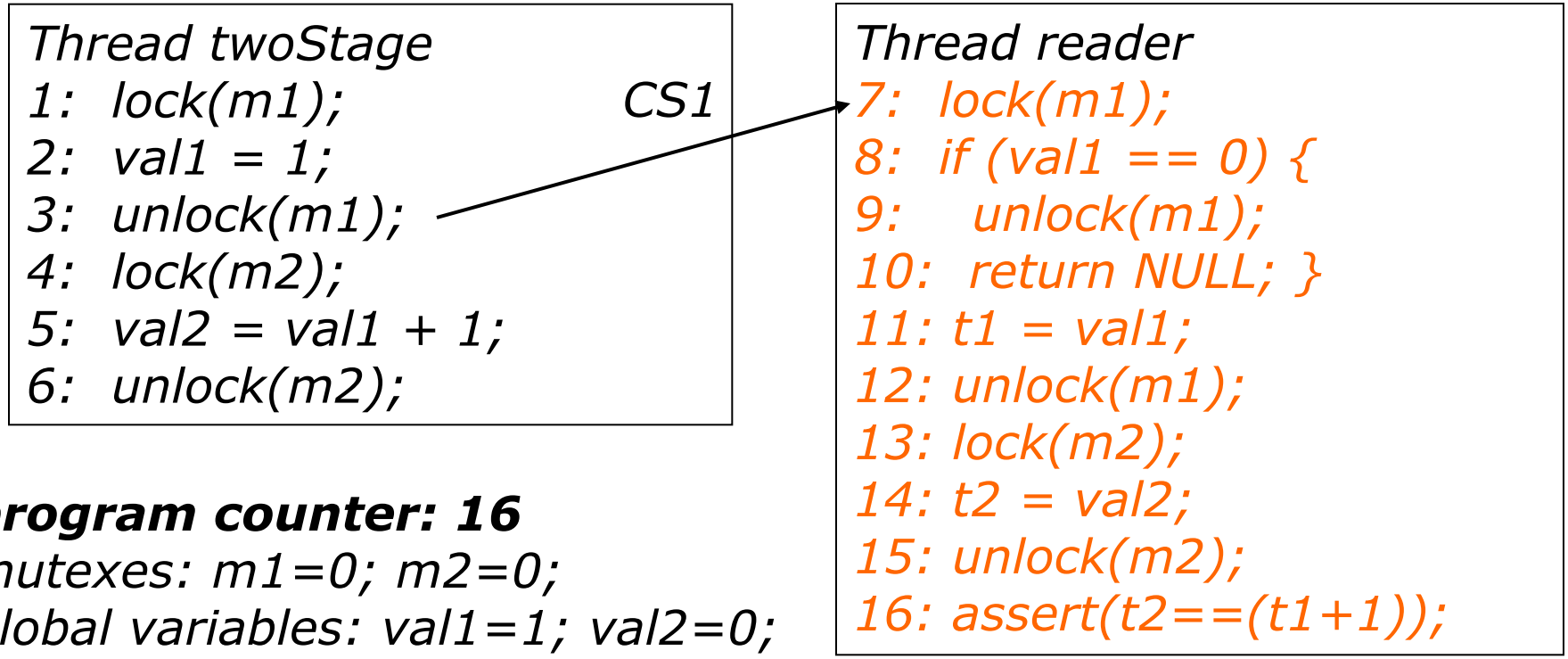
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$

val2-access: $R_{reader,14}$



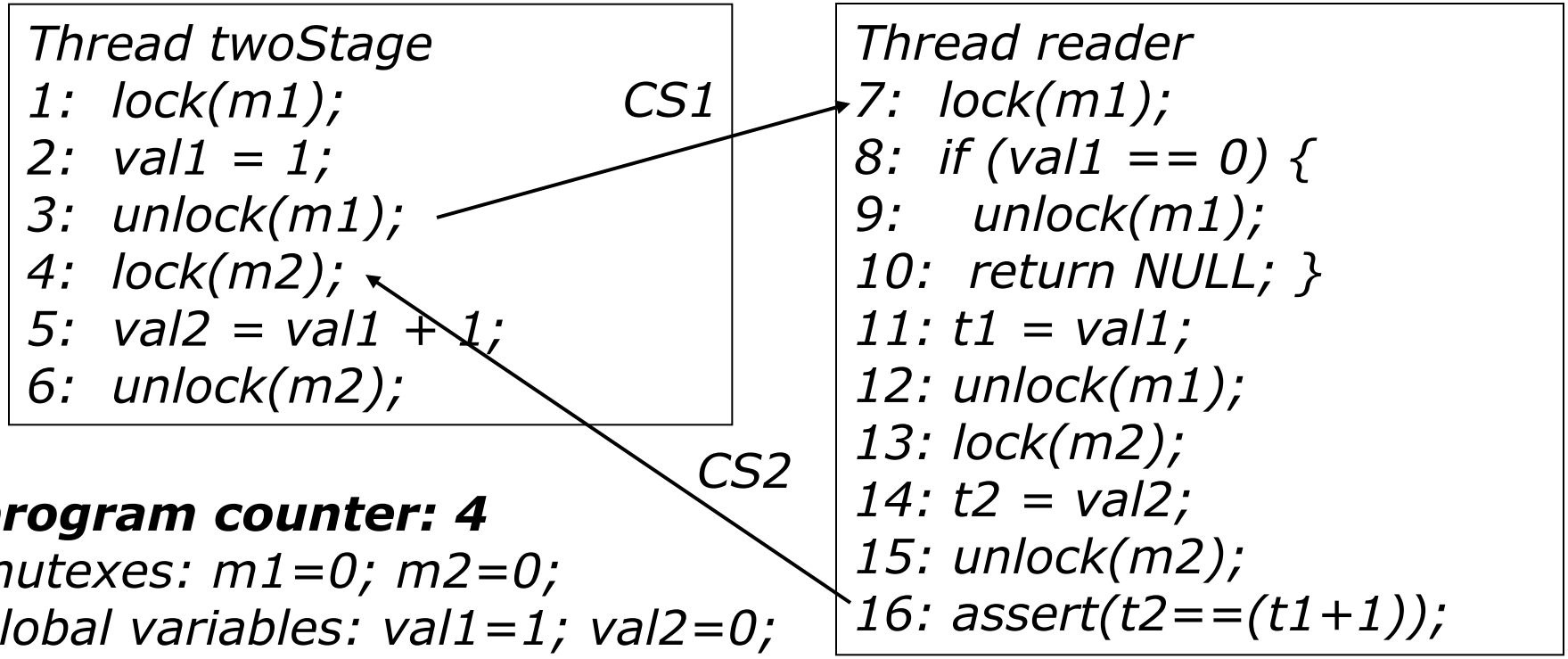
program counter: 16
mutexes: m1=0; m2=0;
global variables: val1=1; val2=0;
local variabes: ***t1= 1; t2= 0;***

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$

val2-access: $R_{reader,14}$



program counter: 4

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$- $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS2

program counter: 6

mutexes: m1=0; m2=0;

global variables: val1=1; ***val2=2;***

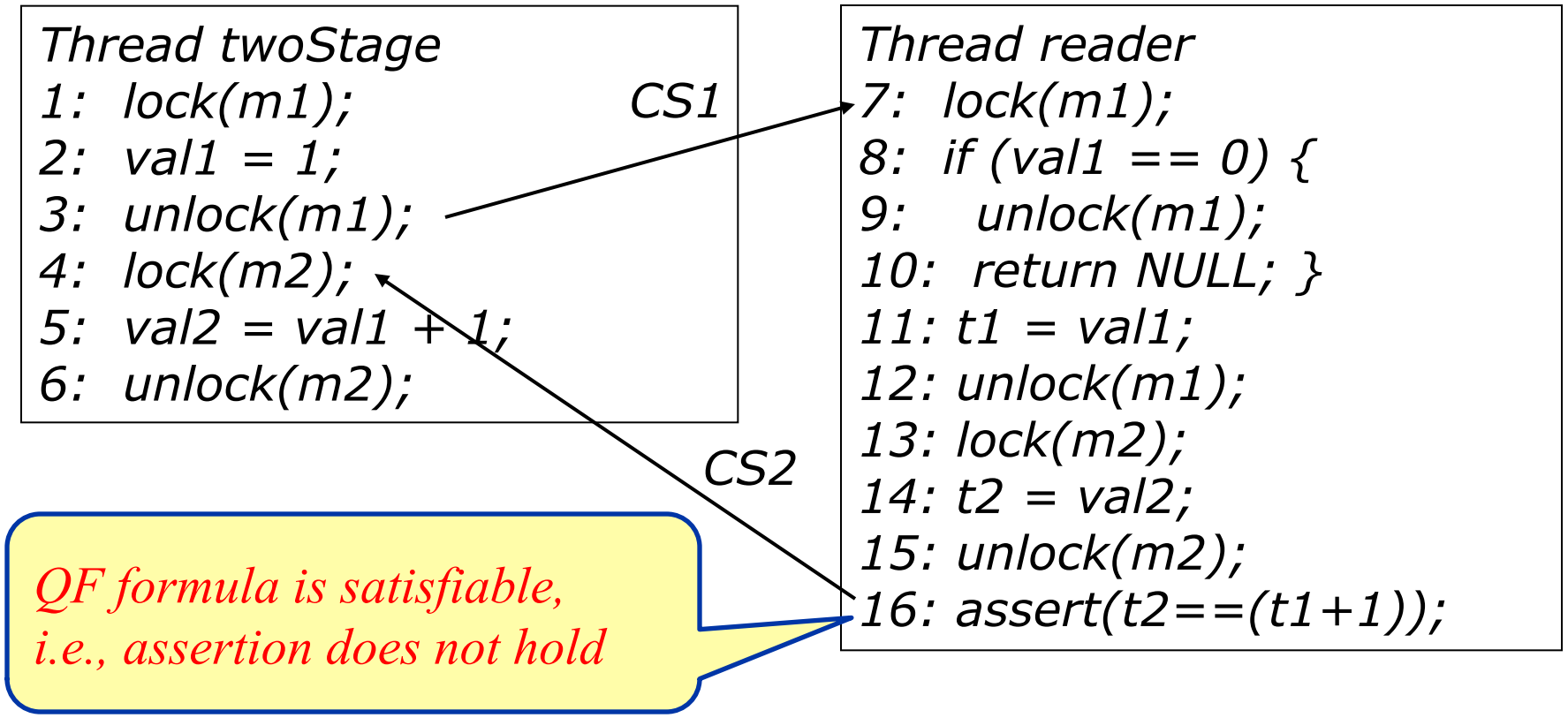
local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

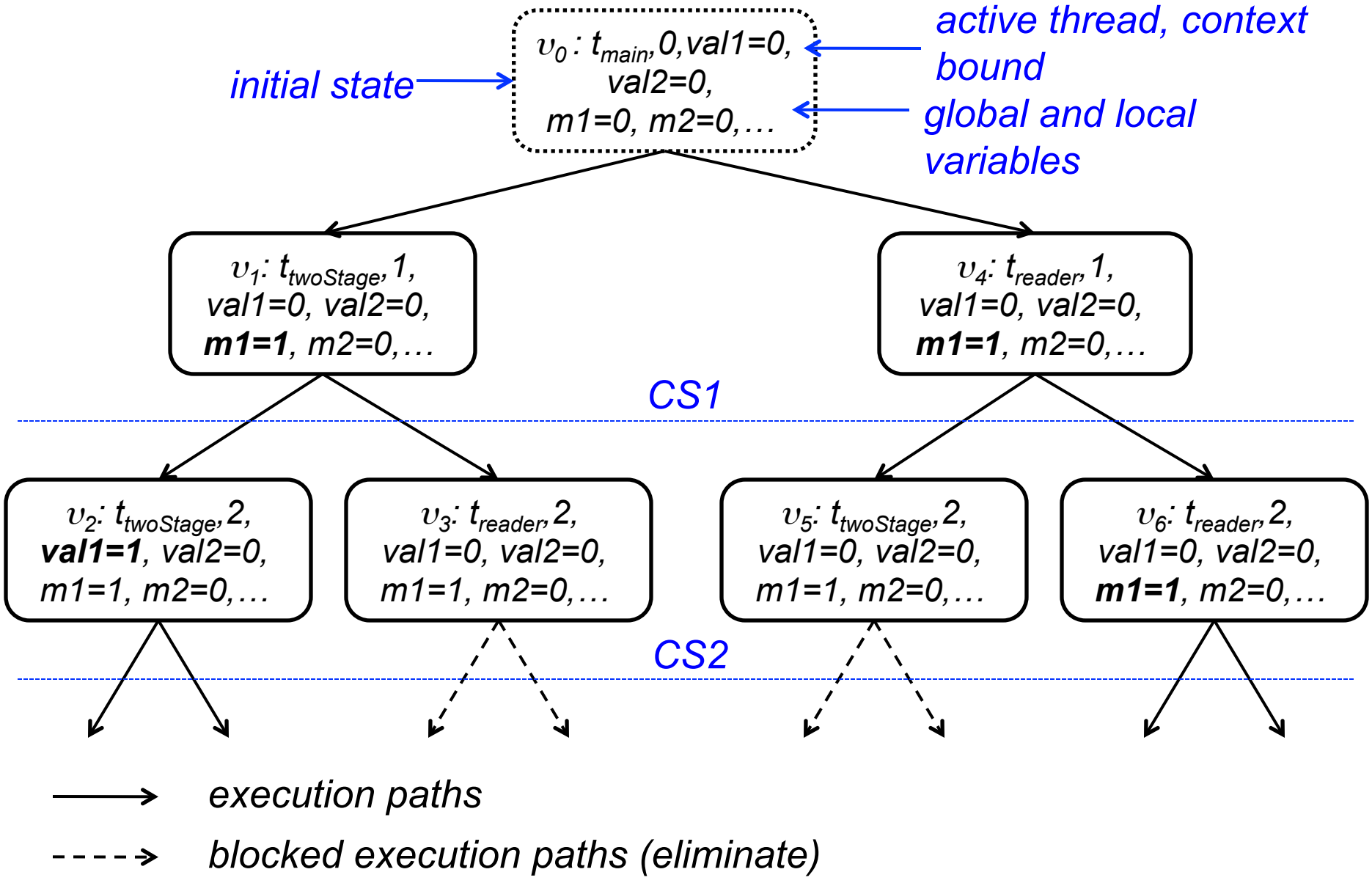
statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$- $W_{twoStage,5}$



Lazy Approach: State Transitions



Lazy exploration of interleavings

- Main steps of the algorithm:

1. Initialize the stack with the initial node ν_0 and the initial path $\pi_0 = \langle \upsilon_0 \rangle$
2. If the stack is empty, terminate with "no error".
3. Pop the current node υ and current path π off the stack and compute the set υ' of successors of υ using rules R1-R8.
4. If υ' is empty, derive the VC φ_k^π for π and call the SMT solver on it. If φ_k^π is satisfiable, terminate with "error"; otherwise, goto step 2.
5. If υ' is not empty, then for each node $\upsilon \in \upsilon'$, add ν to π, and push node and extended path on the stack. goto step 3.

computation path

$\pi = \{\upsilon_1, \ldots \upsilon_n\}$

$$\varphi_k^\pi = \overbrace{I(s_0) \wedge R(s_0, s_1) \wedge \ldots \wedge R(s_{k-1}, s_k)}^{\text{constraints}} \wedge \overbrace{\neg \phi_k}^{\text{property}}$$

bound

Exploring the Reachability Tree

- use a reachability tree (RT) to describe reachable states of a multi-threaded program
- each node in the RT is a tuple $\upsilon = \left(A_i, C_i, s_i, \left\langle l_i^j, G_i^j \right\rangle_{j=1}^{n} \right)_i$ for a given time step i, where:
  - A_i represents the currently active thread
  - C_i represents the context switch number
  - s_i represents the current state
  - l_i^j represents the current location of thread *j*
  - G_i^j represents the control flow guards accumulated in thread *j* along the path from l_0^j to l_i^j
- expand the RT by executing symbolically each instruction of the multi-threaded program

Expansion Rules of the RT

R1 (assign): If *l* is an assignment, we execute *l*, which generates s_{i+1}. We add as child to υ a new node υ'

$$\upsilon' = \left(A_i, C_i, s_{i+1}, \left\langle l_{i+1}^{j}, G_i^{j} \right\rangle\right)_{i+1} \qquad l_{i+1}^{A_i} = l_i^{A_i} + 1$$

- we have fully expanded υ if
  - I within an atomic block; or
  - *l* contains no global variable; or
  - the upper bound of context switches ($C_i = C$) is reached
- if υ is not fully expanded, for each thread $j \neq A_i$ where G_i^j is enabled in s_{i+1}, we thus create a new child node

$$\upsilon_j' = \left(j, C_i + 1, s_{i+1}, \left\langle l_i^j, G_i^j \right\rangle \right)_{i+1}$$

Expansion Rules of the RT

R2 (skip): If *l* is a *skip*-statement with target *l*, we increment the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

R3 (unconditional goto): If *l* is an unconditional *goto*-statement with target *l*, we set the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

Expansion Rules of the RT

R4 (conditional goto): If *l* is a conditional *goto*-statement with test *c and* target *l*, we create two child nodes υ' and υ''.

- for υ', we assume that *c* is *true* and proceed with the target instruction of the jump:

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^j, c \wedge G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

- for υ'', we add $\neg c$ to the guards and continue with the next instruction in the current thread

$$\upsilon'' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^j, \neg c \wedge G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

- prune one of the nodes if the condition is determined statically

Expansion Rules of the RT

R5 (assume): If *l* is an *assume*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

R6 (assert): If *l* is an *assert*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- we generate a verification condition to check the validity of *c*

Expansion Rules of the RT

R5 (start_thread): If *l* is a *start_thread* instruction, we add the indicated thread to the set of active threads:

$$v' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^{j}, G_{i+1}^{j} \right\rangle_{j=1}^{n+1} \right]_{i+1}$$

- where l_{i+1}^{n+1} is the initial location of the thread and $G_{i+1}^{n+1} = G_i^{A_i}$
- the thread starts with the guards of the currently active thread

R6 (join_thread): If *l* is a *join_thread* instruction with argument *ld*, we add a child node:

$$v' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^{j}, G_i^{j} \right\rangle \right)_{i+1}$$

- where $l_{i+1}^{j} = l_i^{A_i} + 1$ only if the joining thread Id has exited

Observations about the lazy approach

- naïve but useful:
  - bugs usually manifest with few context switches [Qadeer&Rehof'05]
  - keep in memory the parent nodes of all unexplored paths only
  - exploit which transitions are enabled in a given state
  - bound the number of preemptions (C) allowed per threads
    - ▷ *number of executions: $O(n^c)$*
  - as each formula corresponds to one possible path only, its size is relatively small
- can suffer performance degradation:
  - in particular for correct programs where we need to invoke the SMT solver once for each possible execution path

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- Explain **bounded model checking of software**
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- Provide **practical examples** to detect software vulnerabilities statically

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