Systems and Software Verification Laboratory

MANCHESTER 1824

The University of Manchester

Detection of Software Vulnerabilities: Static Analysis (Part I)

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Static Analysis

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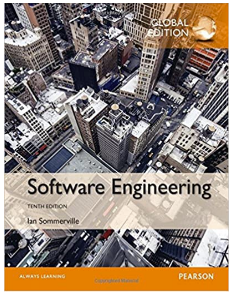
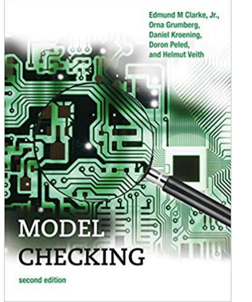
  - Office: 2.28
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Software model checking*. ACM Comput. Surv., 2009
  - *The Cyber Security Body of Knowledge,* 2019
  - *Software Engineering* (Chapters 8, 13)





Motivating Example

- **Functionality** demanded **increased significantly**
  - Peer reviewing and testing

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void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
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  lock(&mutex);
  x++;
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```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
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  unlock(&mutex);
}
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  lock(&mutex);   (CS3)
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```
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Deadlock

```
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  unlock(&mutex);
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  y--;
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}
```

Intended learning outcomes

- Introduce **software verification** and **validation**

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  - The software should **conform to its specification**
- **Validation**: *"Are we building the right product?"*
  - The software should do what the **user requires**
- Verification and validation must be applied at **each stage in the software process**
  - The **discovery of defects** in a system
  - The assessment of whether or not the system is **usable in an operational situation**

Static and Dynamic Verification

- **Software inspections** are concerned with the analysis of the static system representation to discover problems **(static verification)**
  - Supplement by **tool-based document** and **code analysis**
  - **Code analysis** can **prove the absence of errors** but might subject to **incorrect results**

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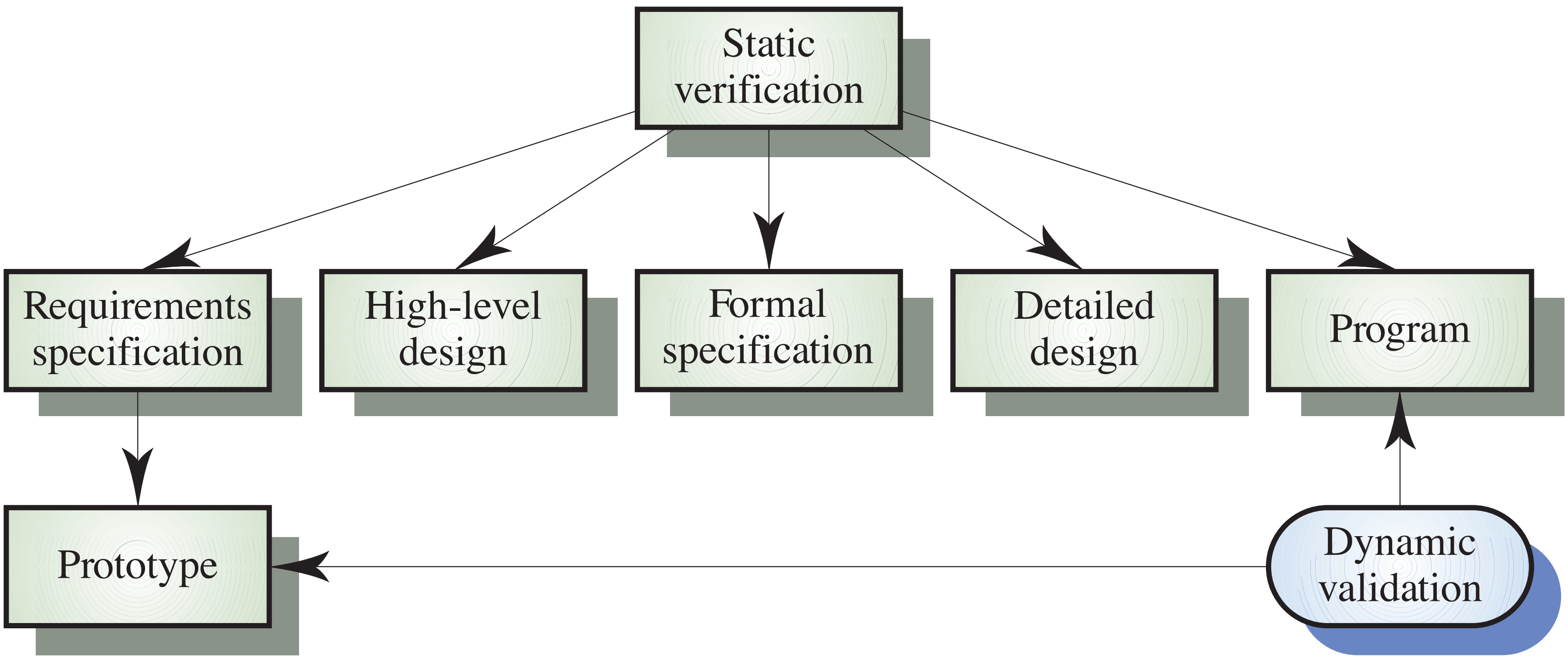
  - **Code analysis** can **prove the absence of errors** but might subject to **incorrect results**
- **Software testing** is concerned with exercising and observing product behaviour **(dynamic verification)**
  - The system is executed with **test data**
  - **Operational behaviour is observed**
  - Can reveal the presence of errors **NOT their absence**

Static and Dynamic Verification



Ian Sommerville. Software Engineering (6th,7th or 8th Edn) Addison Wesley

V & V planning

- **Careful planning** is required to get the most out of **dynamic and static verification**
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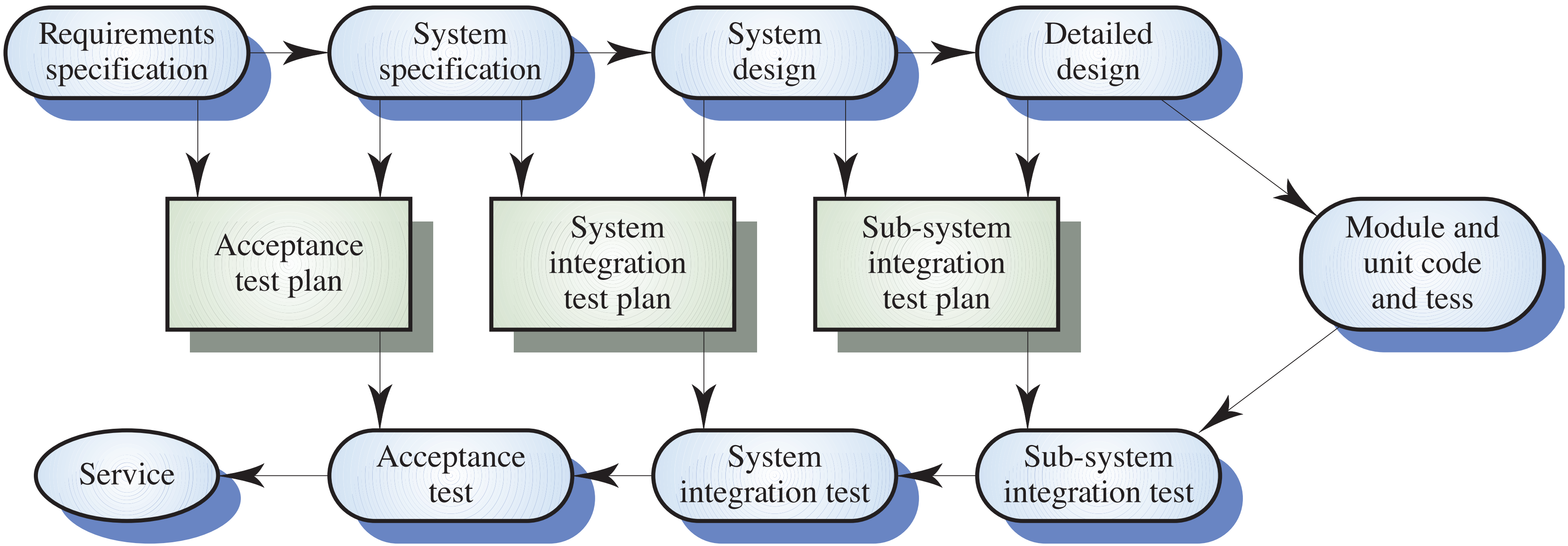
V & V planning depends **on system's purpose**, **user expectations** and **marketing environment**

The V-model of development



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  - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
    - An incomplete detection technique may have ***false positives***, i.e., it may detect issues that do not turn out to be actual vulnerabilities

Detection of Vulnerabilities

- Achieving **soundness** requires reasoning about **all executions** of a program (usually an infinite number)
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  - This can be done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, **concrete executions** of a program that are witnesses to any vulnerability reported
  - The analysis technique has to come up with concrete inputs for the program that triggers a vulnerability
  - A typical dynamic approach is software testing: the tester writes test cases with concrete inputs and specific checks for the outputs

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Detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**

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Dynamic verification should be used in conjunction with **static verification** to provide **full code coverage**

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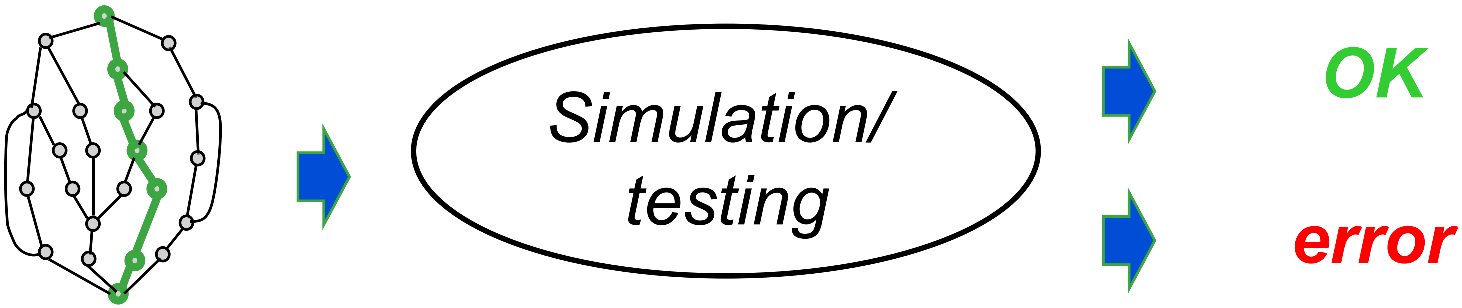
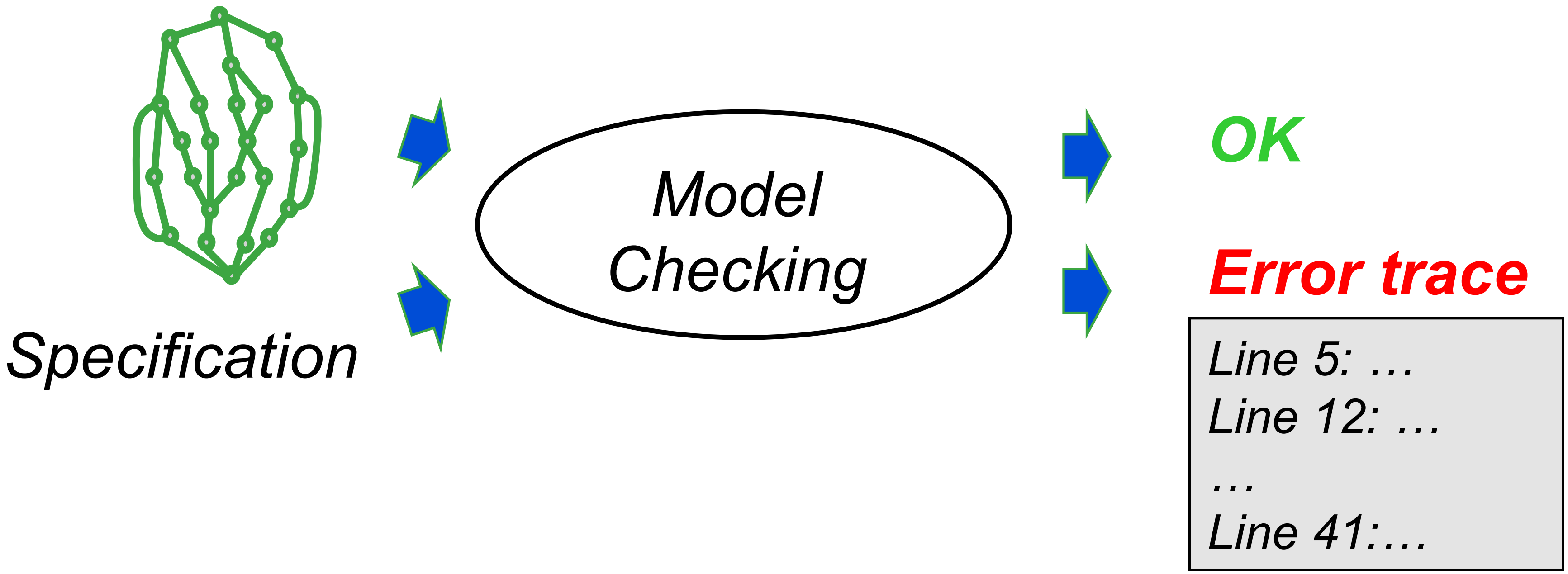
Static analysis vs Testing/ Simulation



- **Checks only some of the system executions**
  - May **miss errors**
- A **successful execution** is an execution that **discovers one** or **more errors**

Static analysis vs Testing/ Simulation



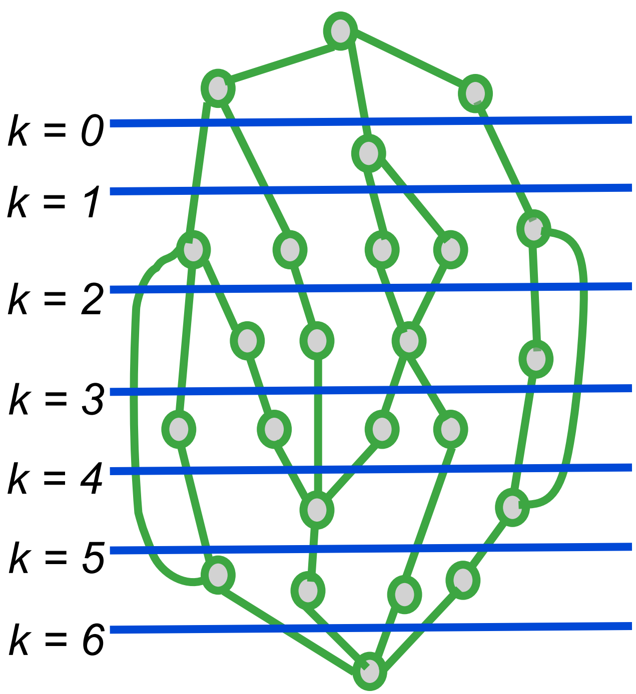
- **Exhaustively explores all executions**
- Report errors as **traces**
- May produce **incorrect results**

Avoiding state space explosion

- Bounded Model Checking (BMC)
  - **Breadth-first search** (BFS) approach
- Symbolic Execution
  - **Depth-first search** (DFS) approach

Bounded Model Checking

A graph G = (V, E) consists of:
- V: a set of vertices or nodes
- $E \subseteq V \times V$: set of edges connecting the nodes



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Breadth-First Search (BFS)

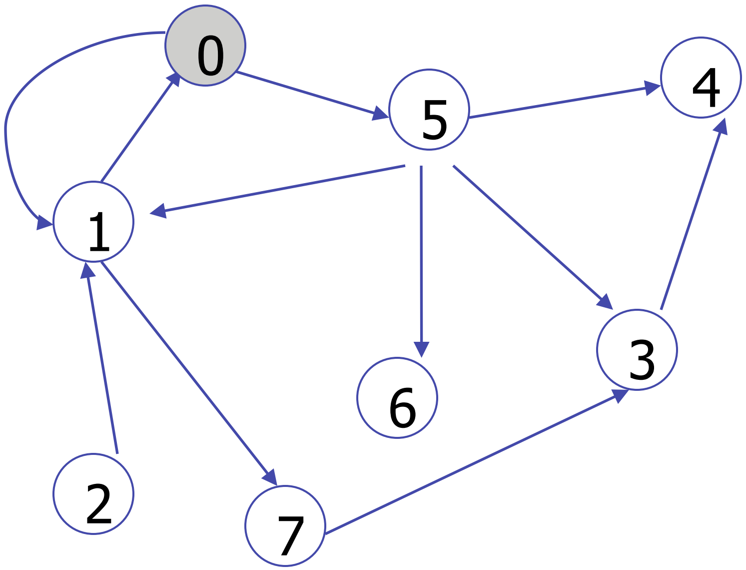
```
BFS(G,s)
for each vertex u ∈ V[G]-{s} // anchor (s)
   colour[u] ← white   // u colour
   d[u] ← ∞            // s distance
   π[u] ← NIL          // u predecessor
colour[s] ← grey
d[s] ← 0
π[s] ← NIL
enqueue(Q,s)
while Q ≠ ∅ do
   u ← dequeue(Q)
   for each v ∈ Adj[u] do
      If colour[v] = white then
         colour[v] ← grey
         d[v] ← d[u] + 1
         π[v] ← u
         enqueue(Q,v)
   colour[u] ← blue
```

Initialization of graph nodes

Initializes the anchor node (s)

Visit each adjacent node of *u*

BFS Example



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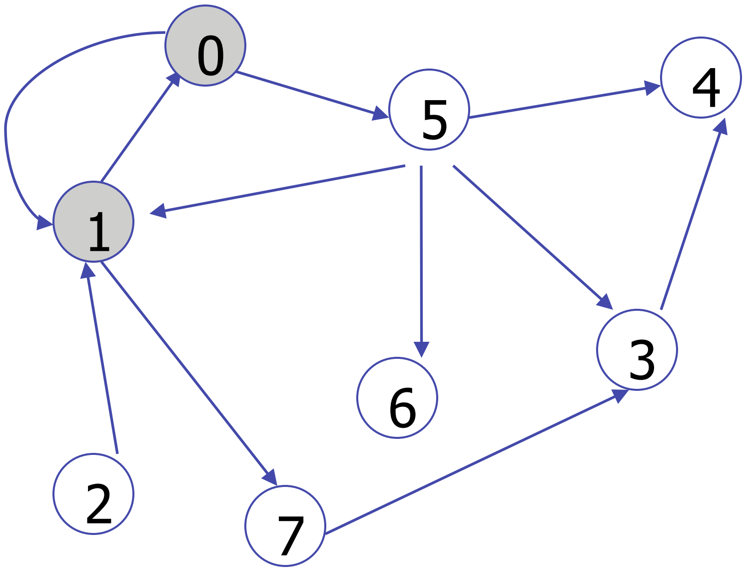

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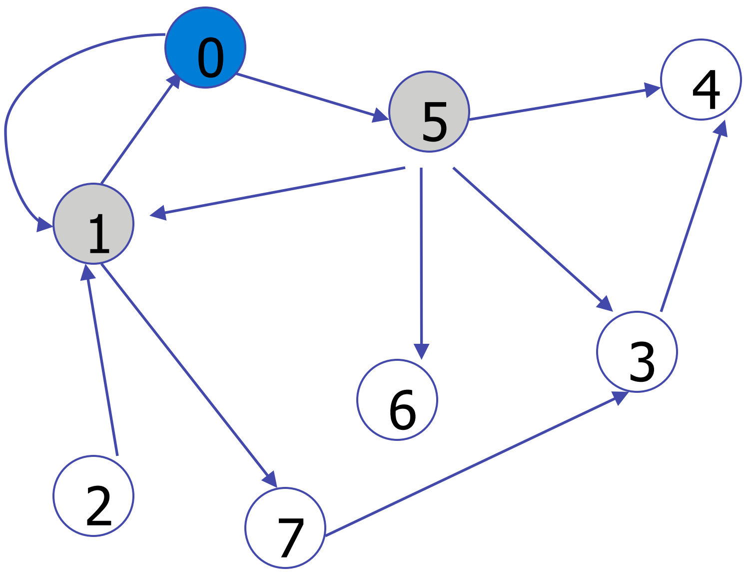

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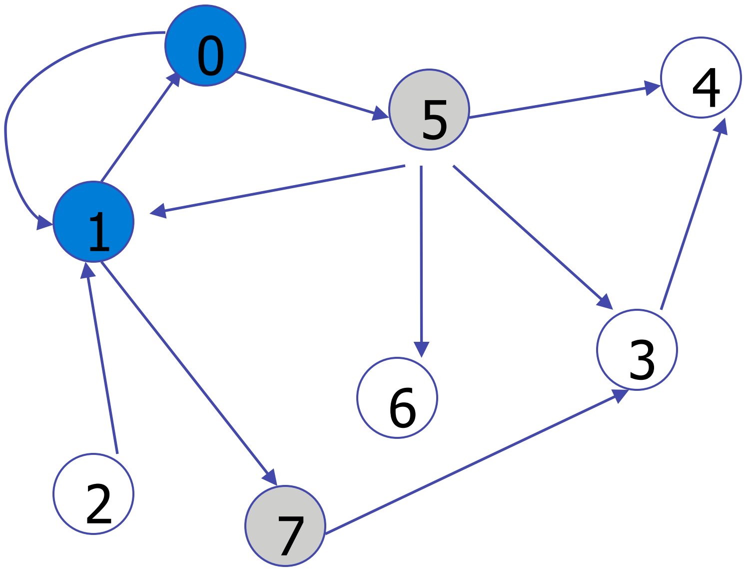

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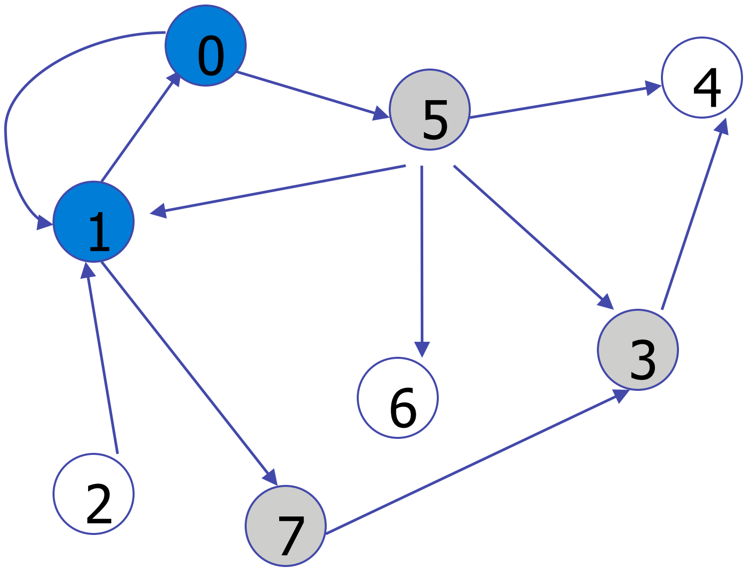

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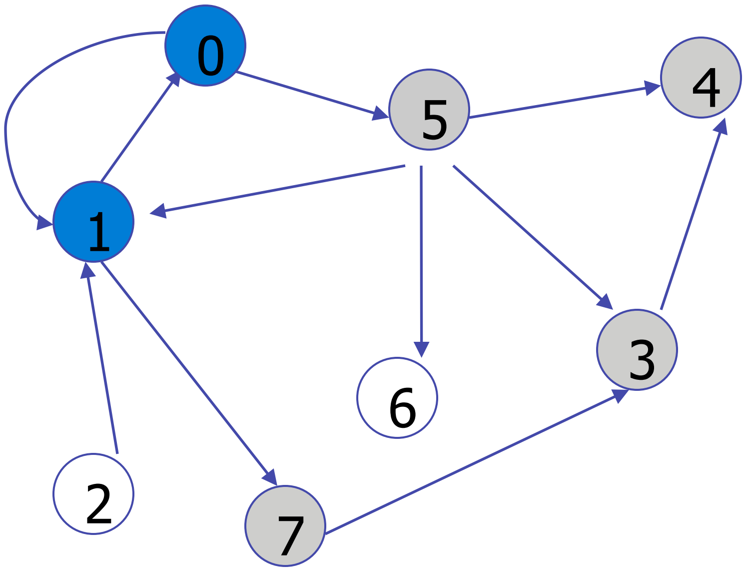

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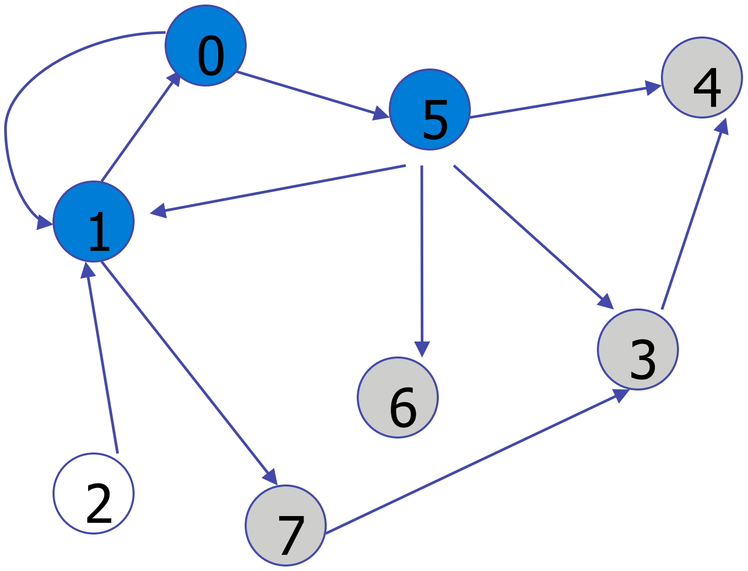

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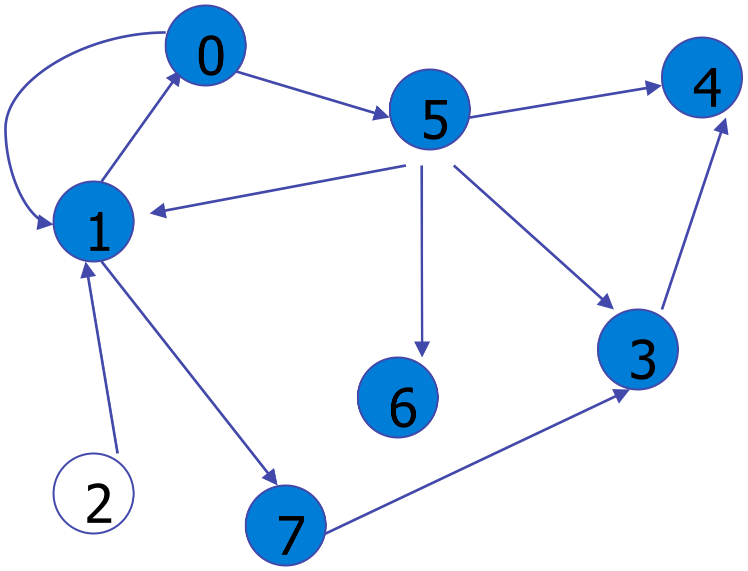
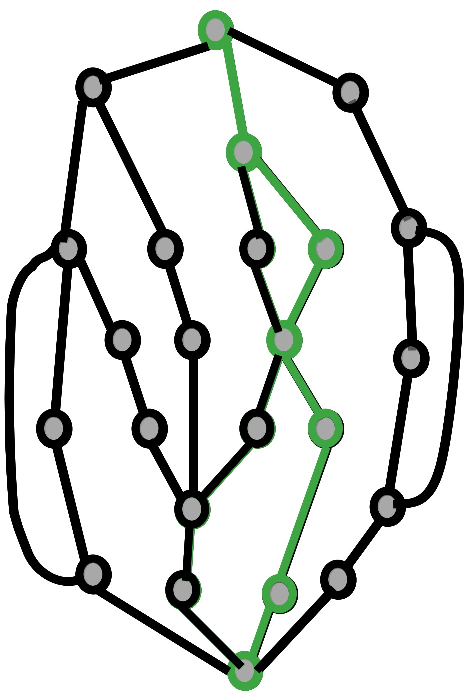


Symbolic Execution



- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

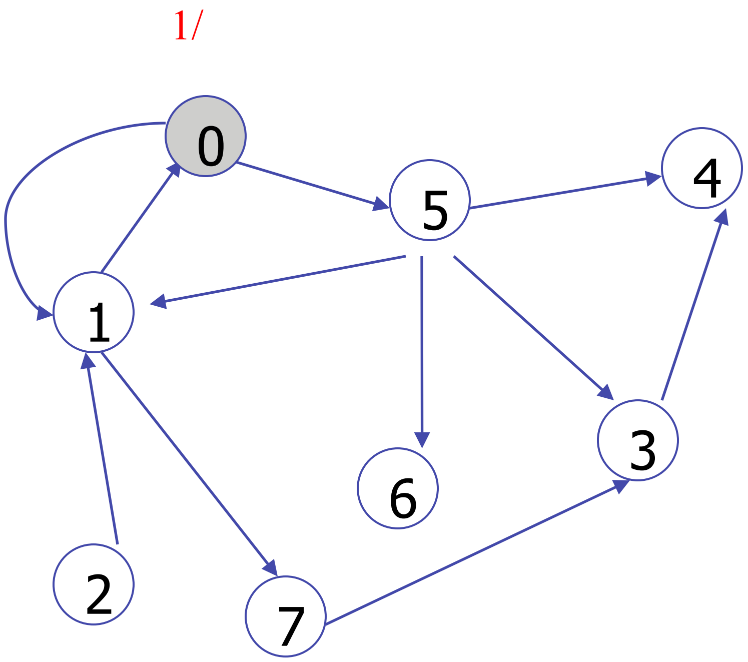
Depth-first search (DFS)

```
DFS(G)
for each vertex u ∈ V[G]
      do color[u] ← WHITE
         π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
      do if color[u] = WHITE
            then DFS-VISIT(u)
```

Paint all vertices white and initialize the fields π with NIL where π [u] represents the predecessor of u

```
DFS-VISIT(u)
color[u] ← GRAY              ▷ White vertex u has just been discovered.
time ← time + 1
d[u] ← time
for each v ∈ Adj[u]          ▷ Explore edge (u, v).
      do if color[v] = WHITE
            then π[v] ← u
                 DFS-VISIT(v)
color[u] ← BLACK             ▷ Blacken u; it is finished.
f[u] ← time ← time + 1
```

DFS Example



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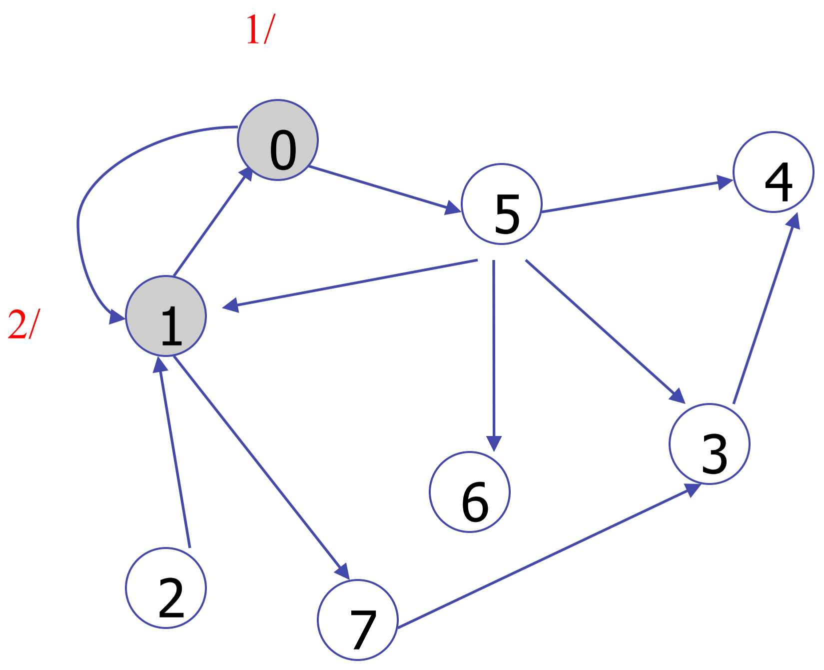

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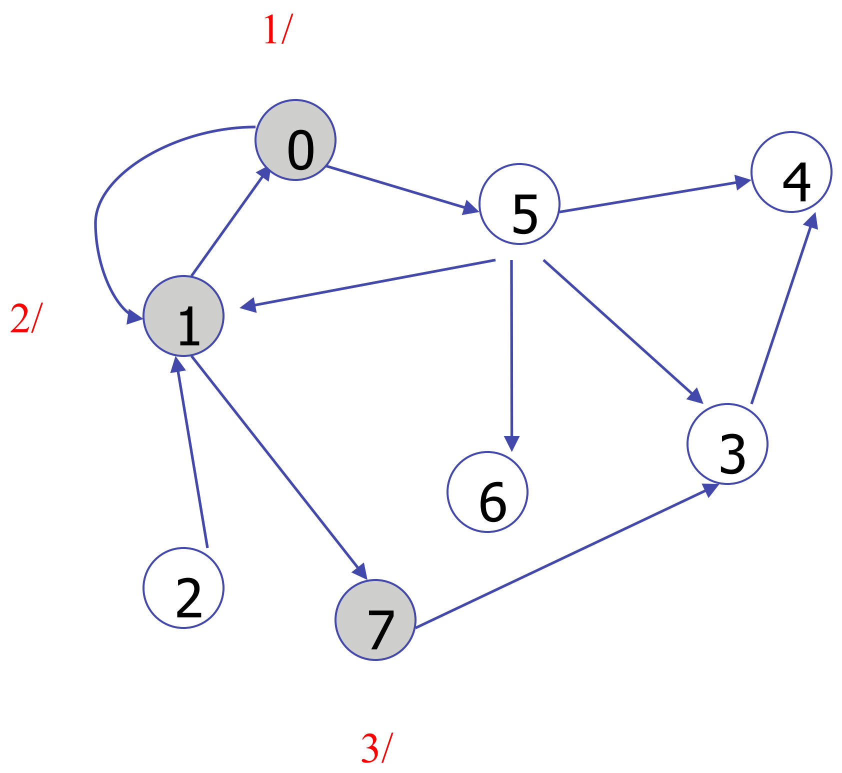

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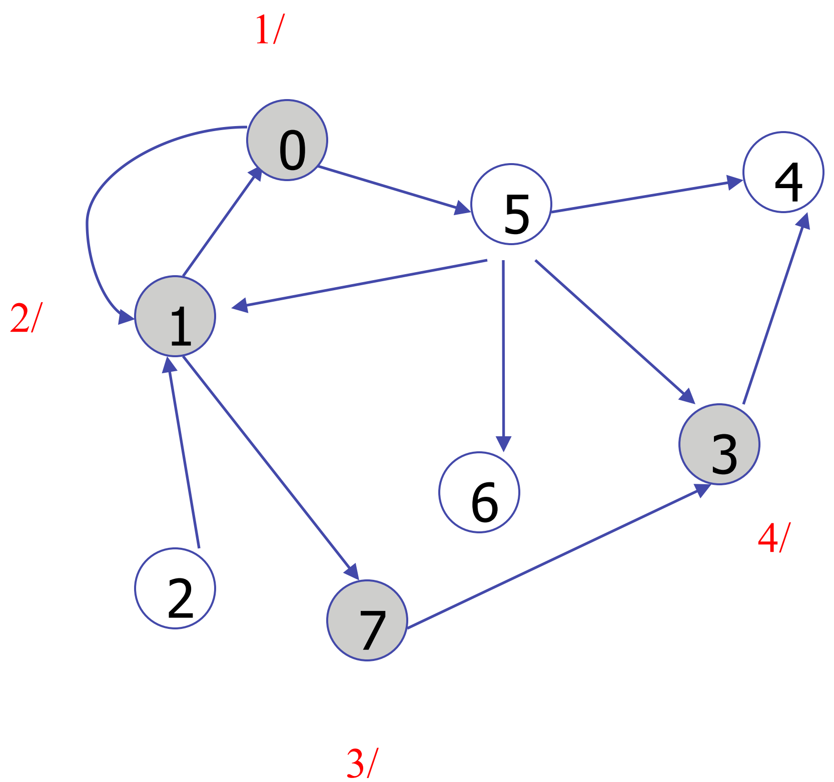

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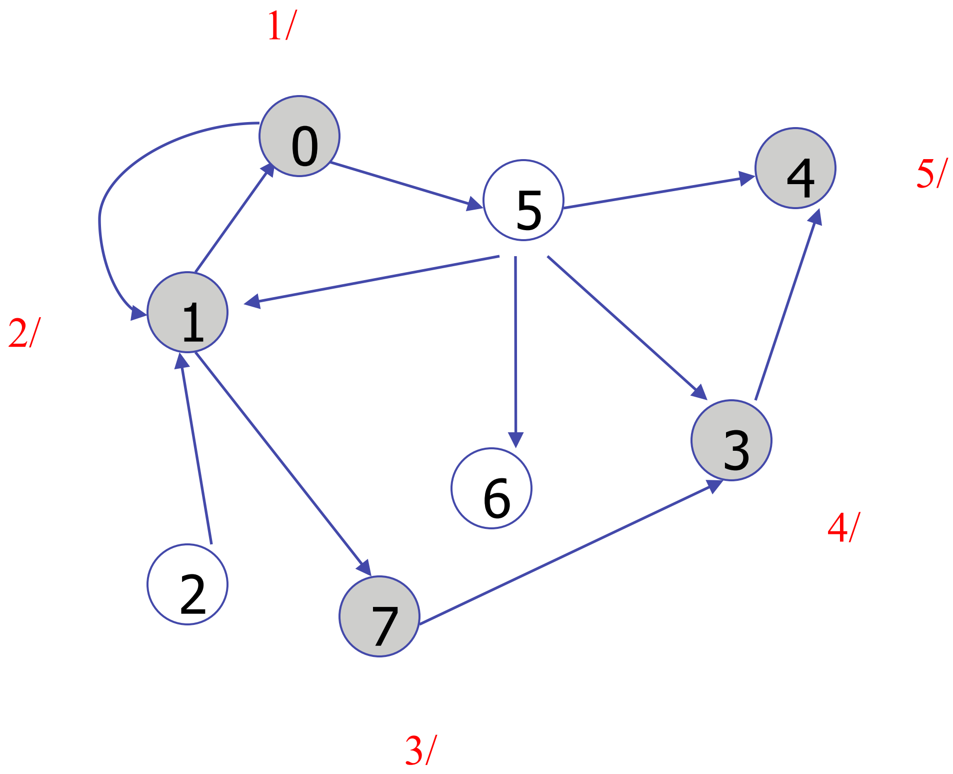

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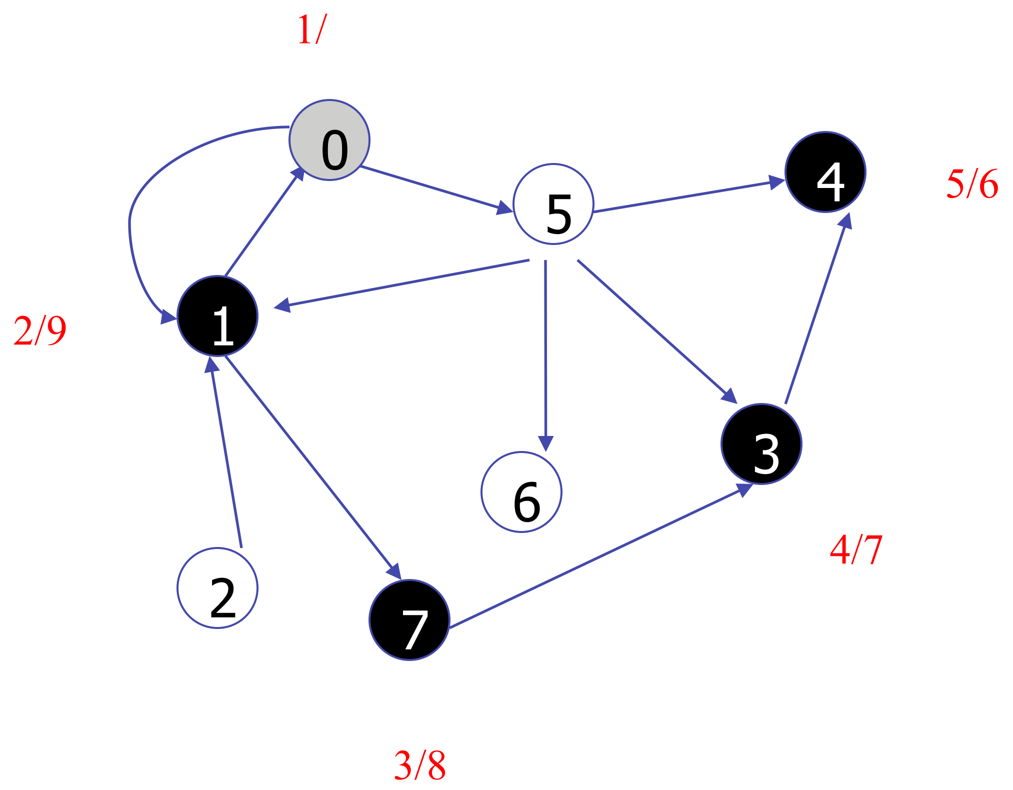

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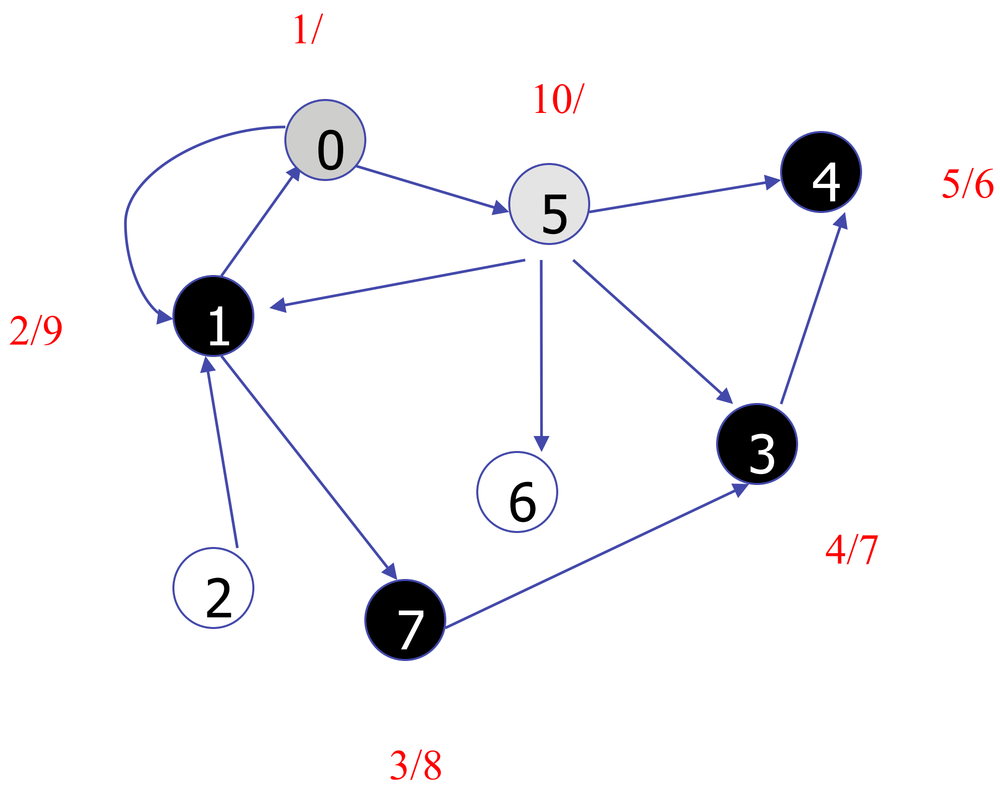

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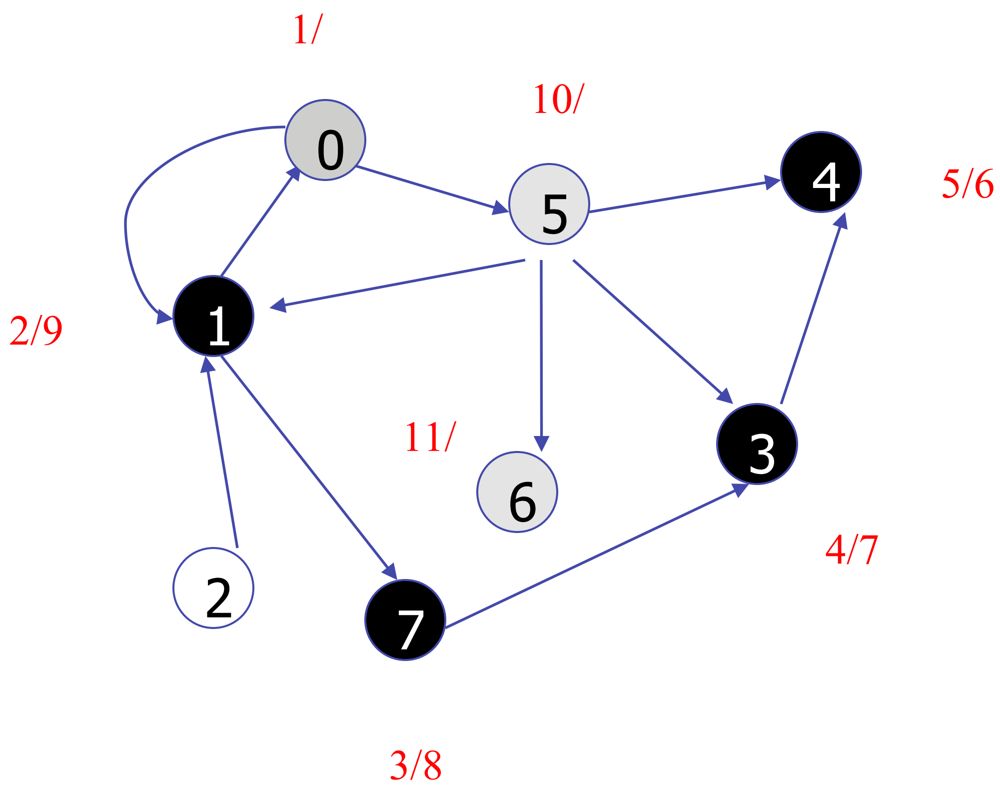

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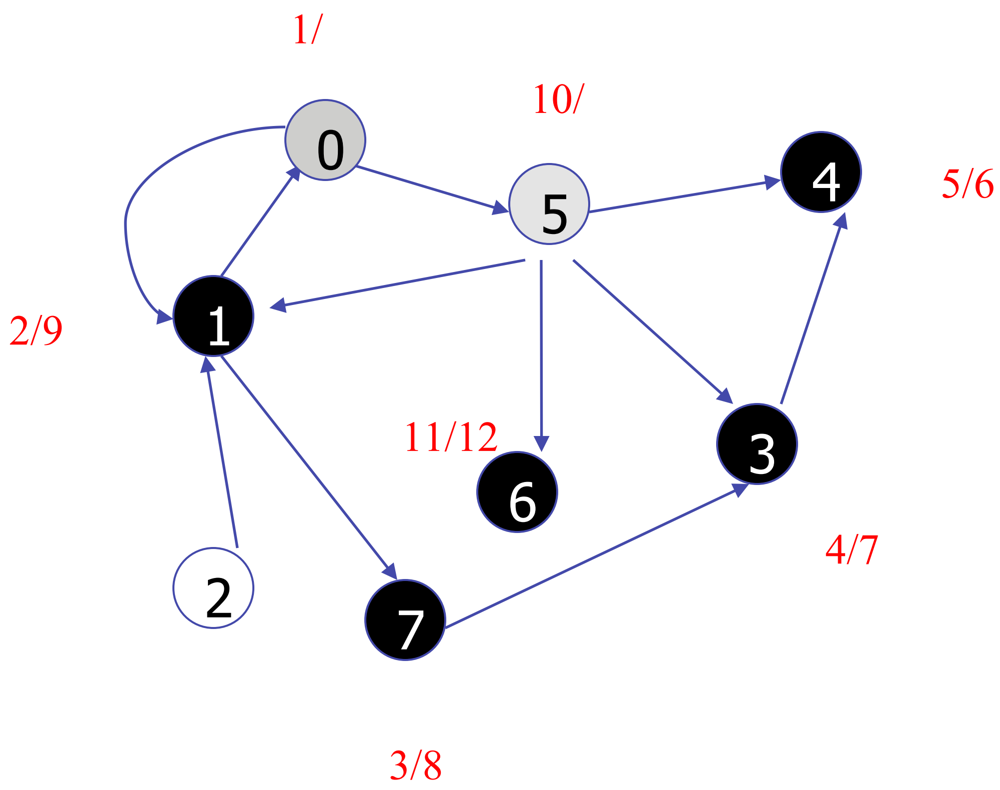

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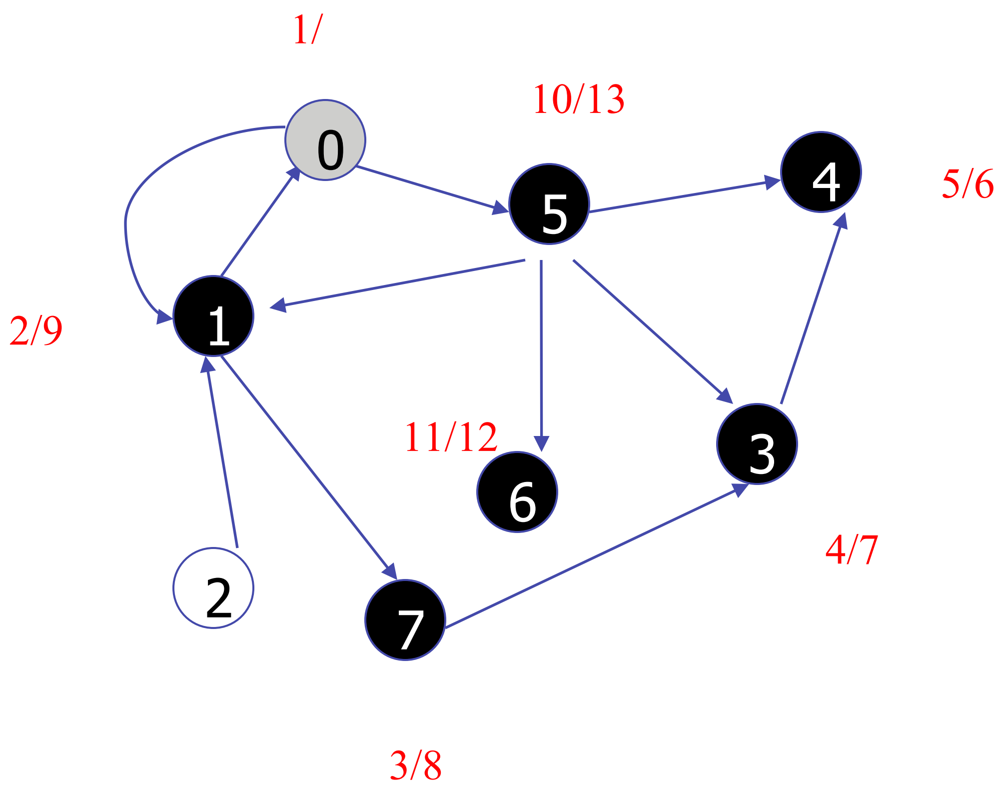

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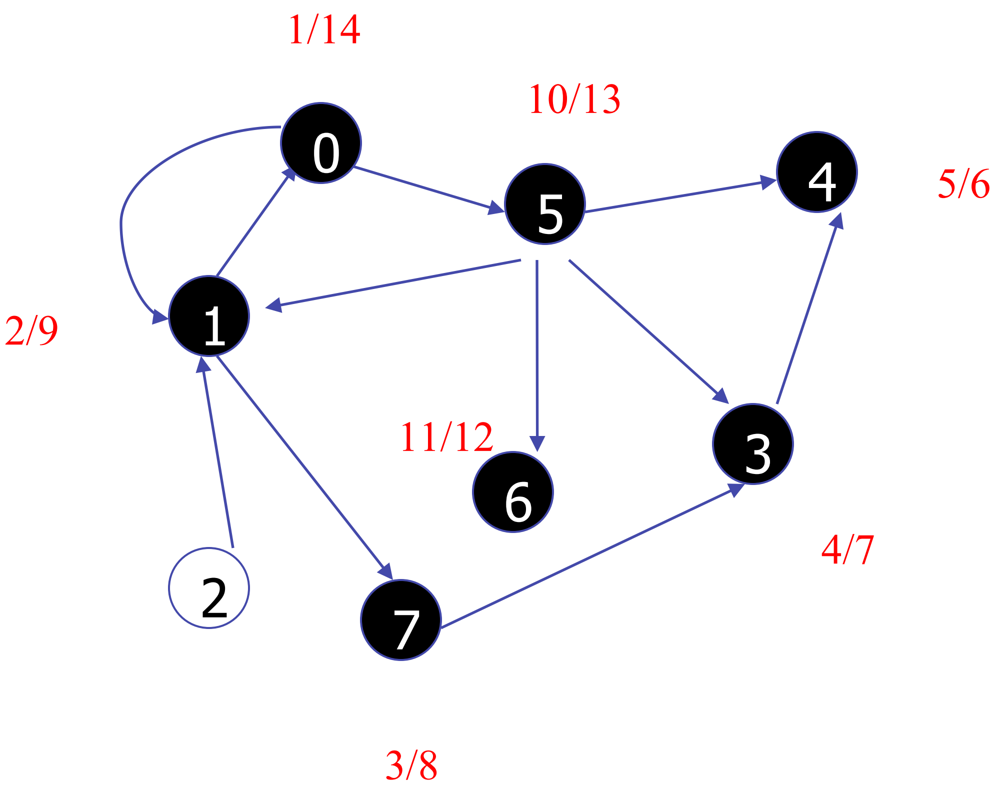

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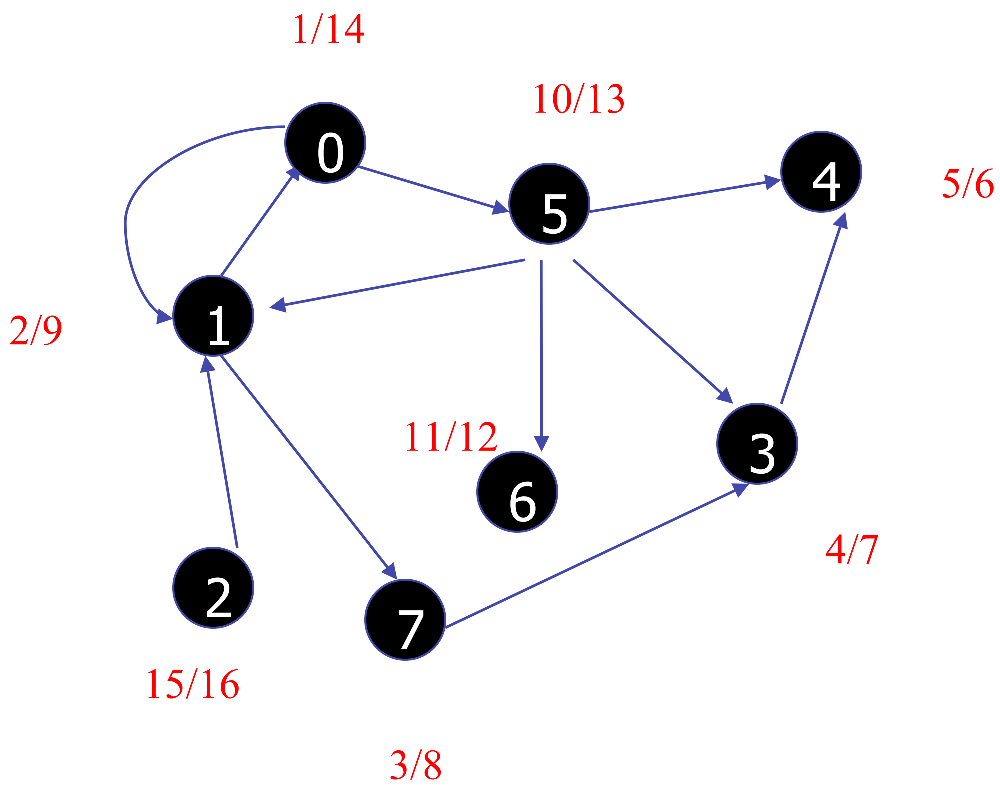

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- **Debugging** is concerned with two main tasks
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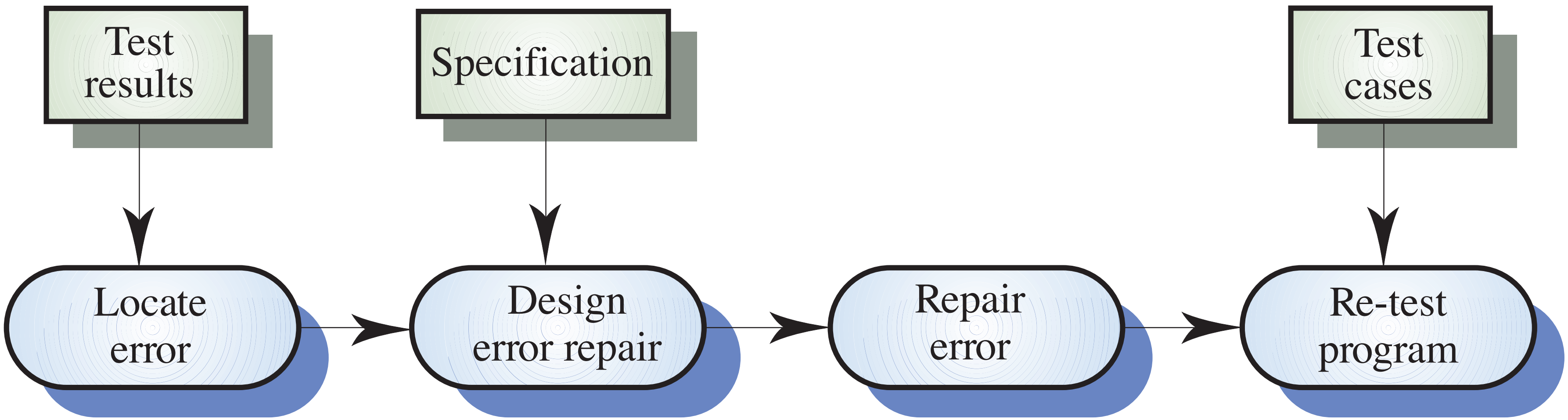
- Debugging involves
  - Formulating a hypothesis about program behaviour
  - Test these hypotheses to find the system error

The debugging process



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- A Boolean formula contains
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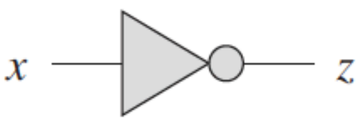
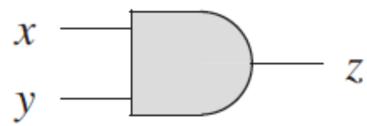
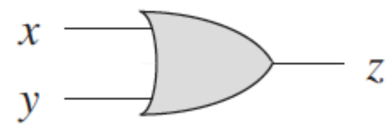
  - **Connectives**: ∧ **(AND)**, ∨ **(OR)**, and ¬ **(NOT)**







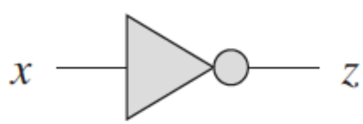
x	$\neg x$
0	1
1	0

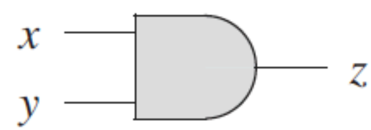
x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

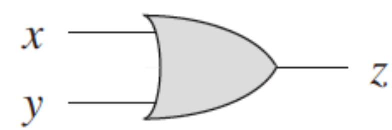
x	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

Circuit Satisfiability

- A Boolean formula contains
  - **Variables** whose values are **0** or **1**
  - **Connectives**: ∧ **(AND)**, ∨ **(OR)**, and ¬ **(NOT)**







x	$\neg x$
0	1
1	0

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

x	y	$x \vee y$
0	0	0
0	1	1
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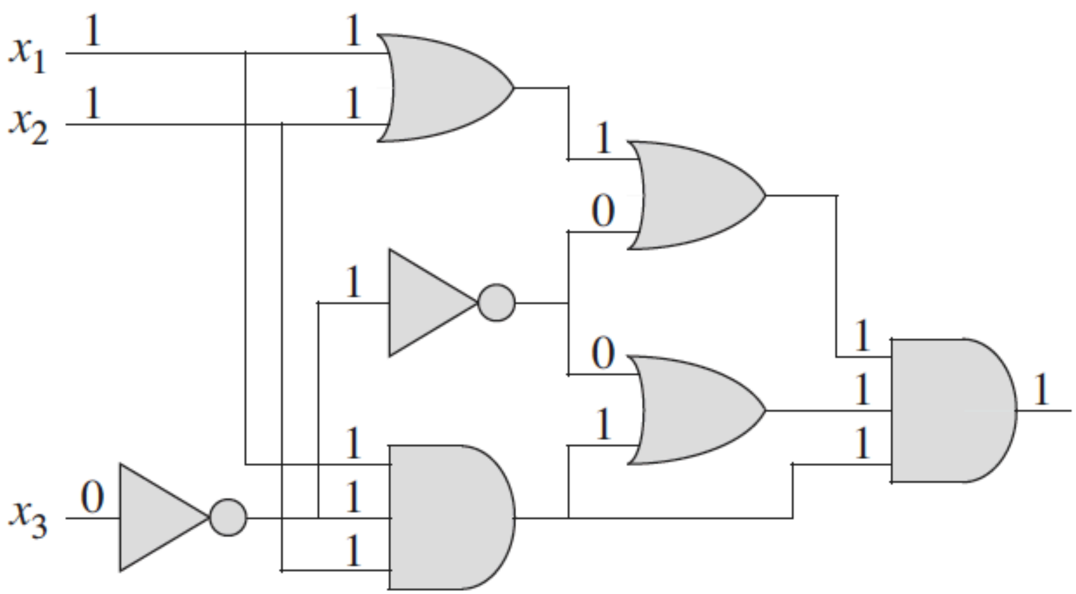
- A Boolean formula is **SAT** if there exists some assignment to its variables that **evaluates it to 1**

Circuit Satisfiability

- A **Boolean combinational circuit** consists of one or more **Boolean combinational elements** interconnected by **wires**



SAT: <x_1 = 1, x_2 = 1, x_3 = 0>

Circuit-Satisfiability Problem

- Given a **Boolean combinational circuit** of AND, OR, and NOT gates, is it **satisfiable**?

CIRCUIT-SAT = {<C> : C is a satisfiable Boolean combinational circuit}

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- When the **size of *C* is polynomial in *k***, checking each one takes ***$\Omega(2^k)$***
  - Super-polynomial in the size of *k*

Formula Satisfiability (SAT)

- The SAT problem asks whether a given Boolean formula is satisfiable

$$\boxed{\text{SAT} = \{\langle\Phi\rangle : \Phi \text{ is a satisfiable Boolean formula}\}}$$

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    - o $\Phi = 1$

DPLL satisfiability solving

Given a Boolean formula φ in *clausal form* **(an AND of ORs)**

$$\{\{a, b\}, \{\neg a, b\}, \{a, \neg b\}, \{\neg a, \neg b\}\}$$

determine whether a *satisfying assignment* of variables to truth values exists.

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Solvers based on Davis-Putnam-Logemann-Loveland algorithm:

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2. If $\square \in \varphi$ then UNSAT
3. If $\varphi = \varphi' \cup \{x\}$ then DPLL($\varphi'[x \mapsto \text{true}]$)
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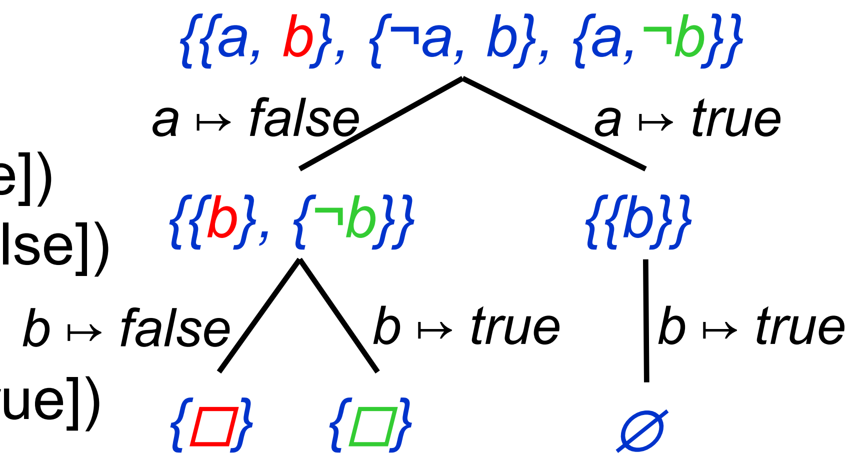

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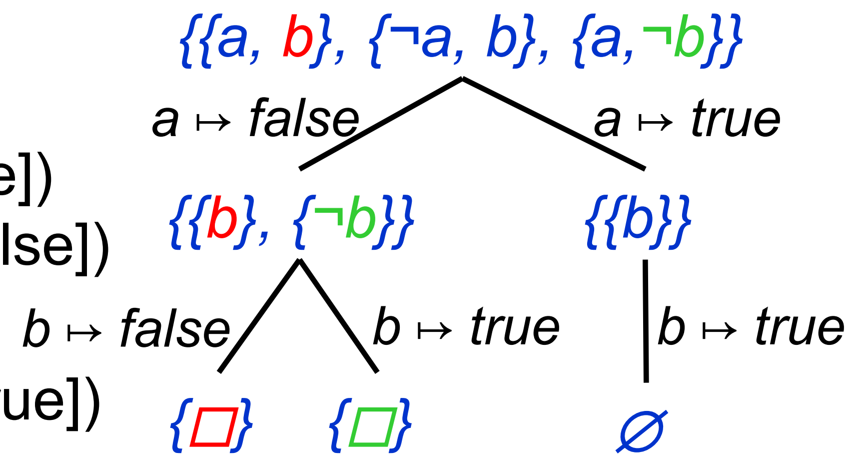
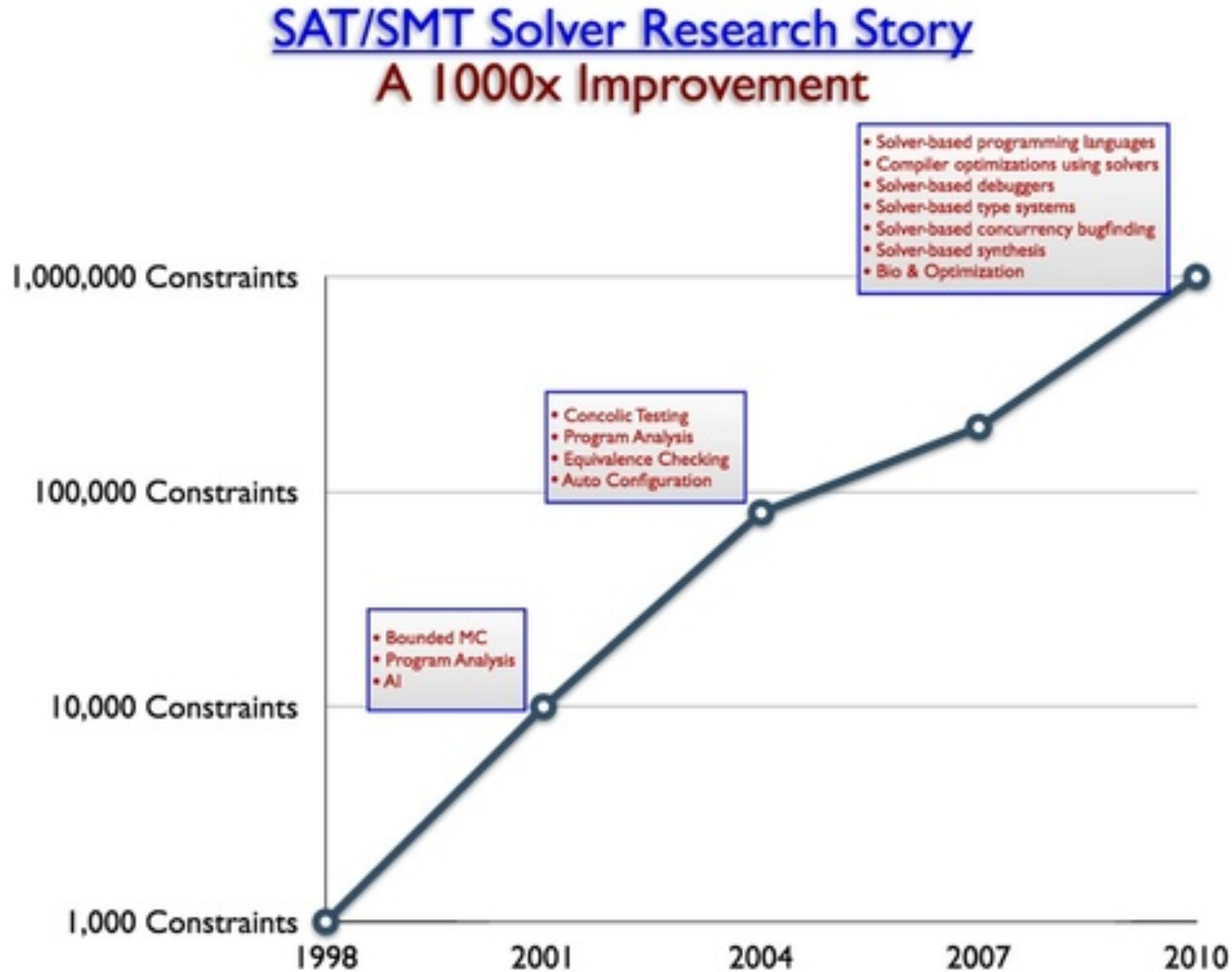


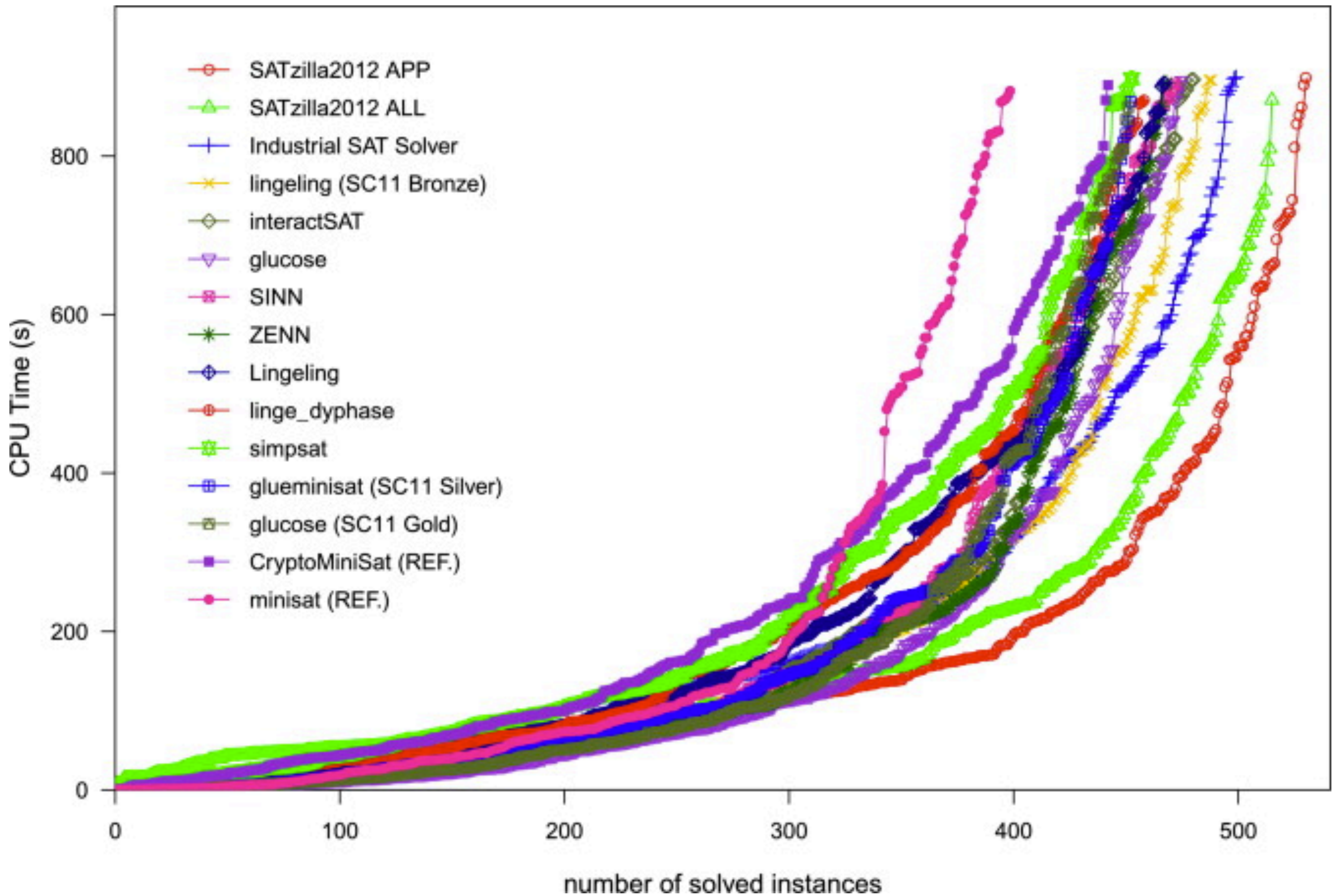
+ NP-complete but many heuristics and optimizations

⇒ can handle problems with 100,000's of variables

SAT solving as enabling technology

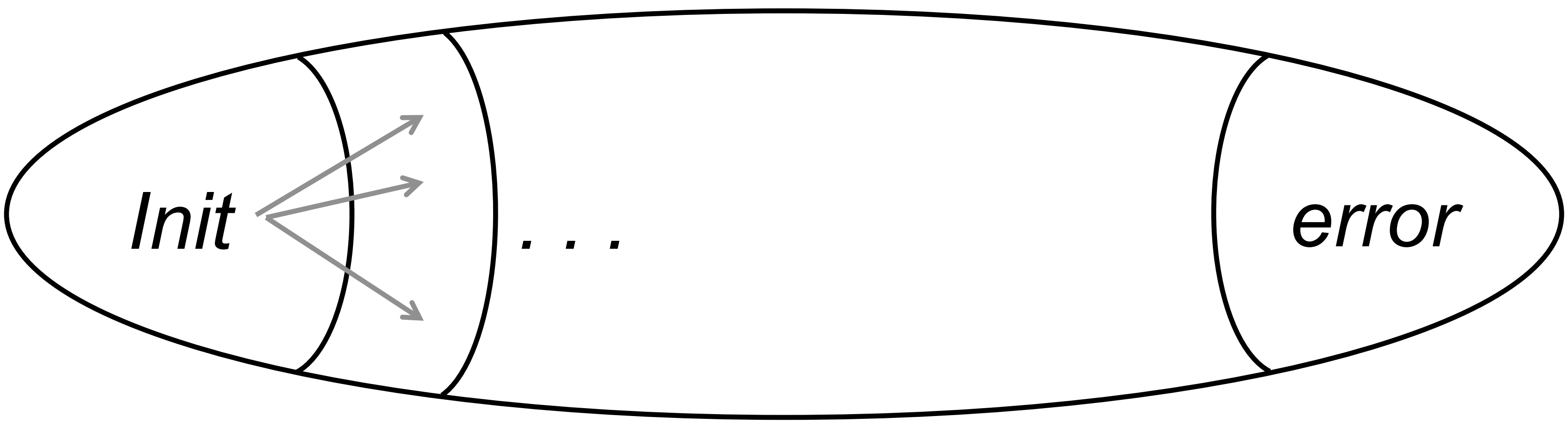


SAT Competition



Bounded Model Checking (BMC)

MC: check if a property holds for all states



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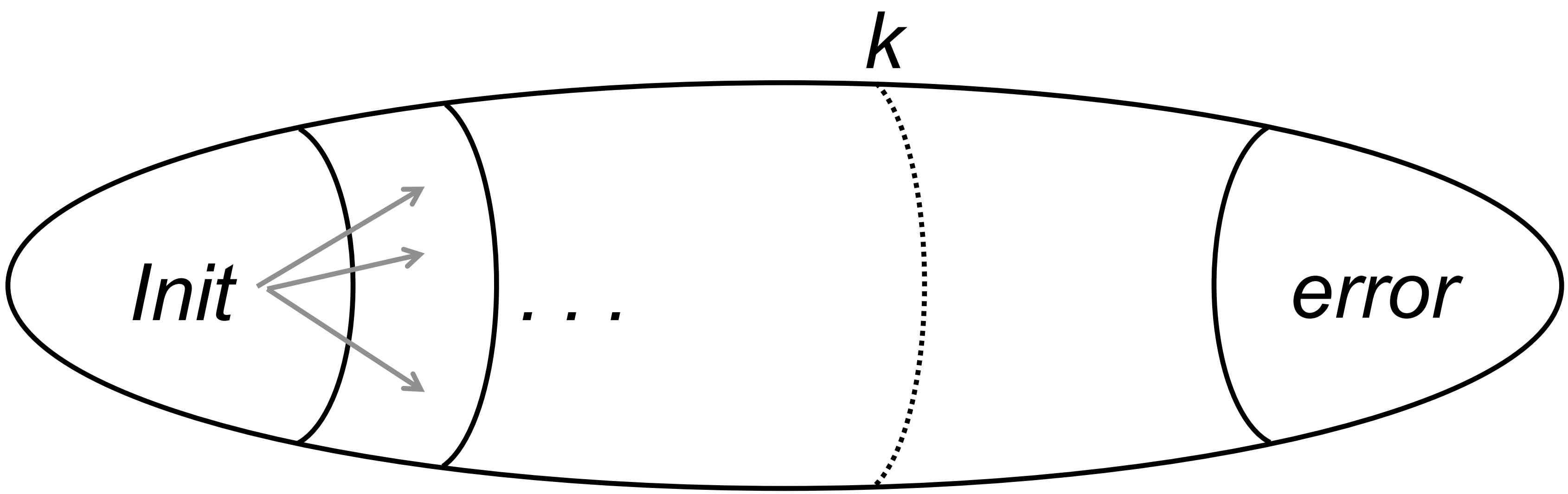
BMC: check if a property holds for a subset of states



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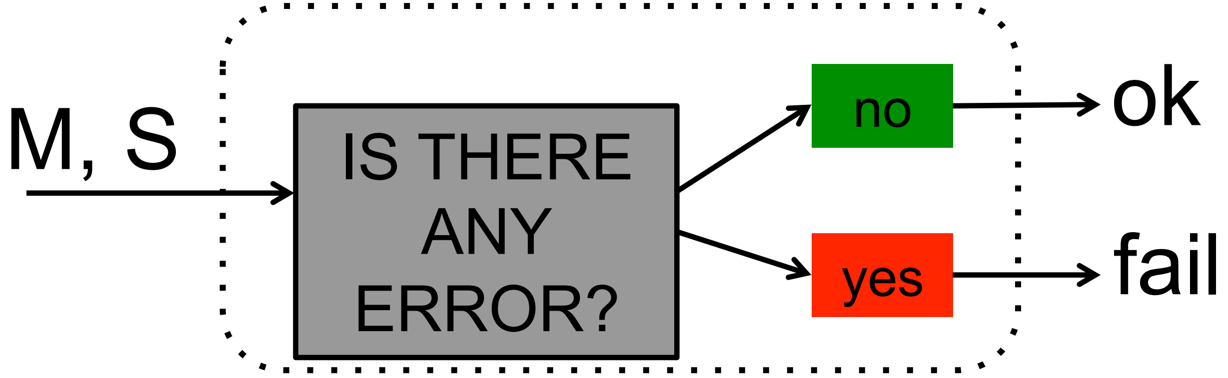

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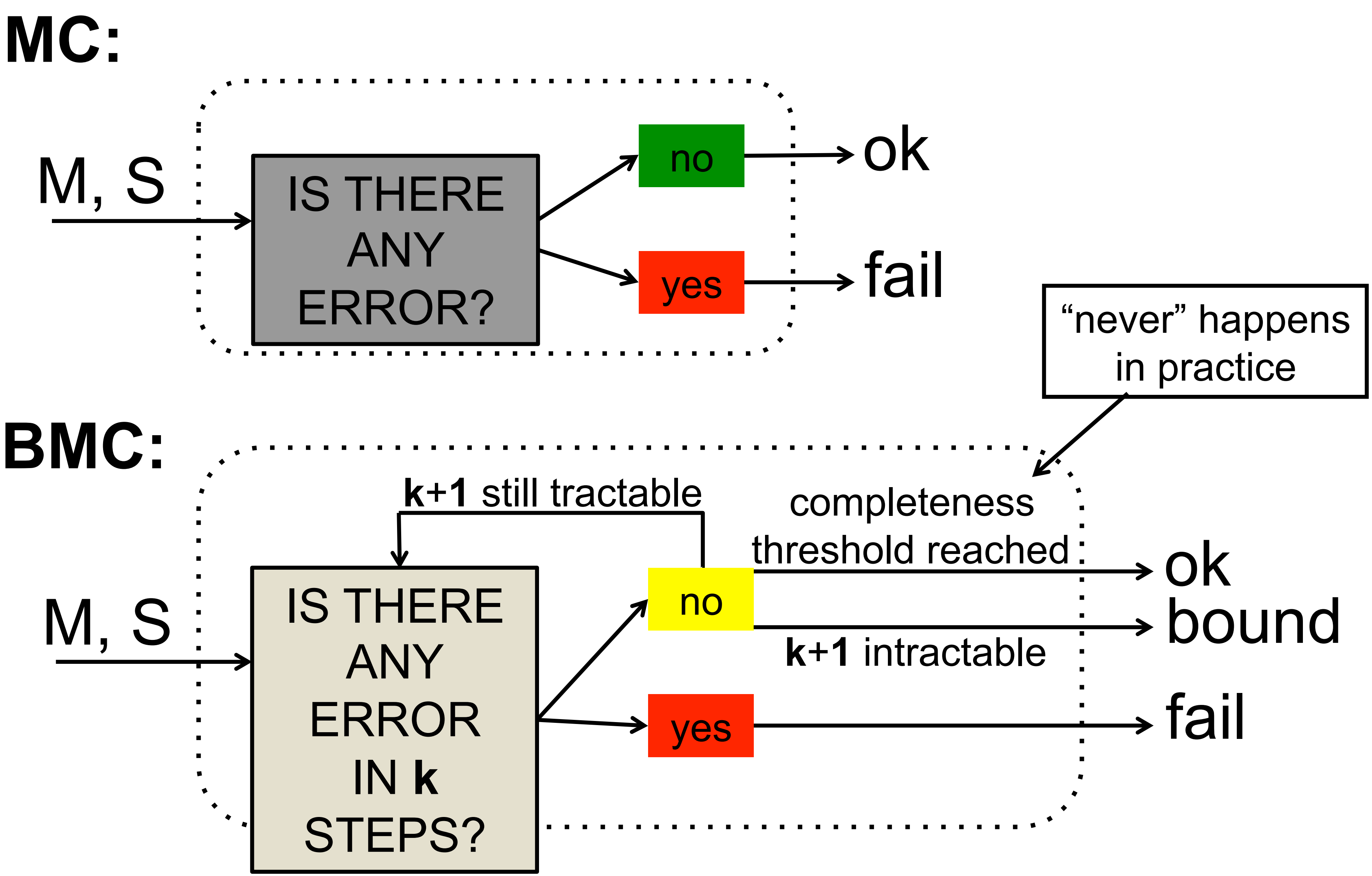
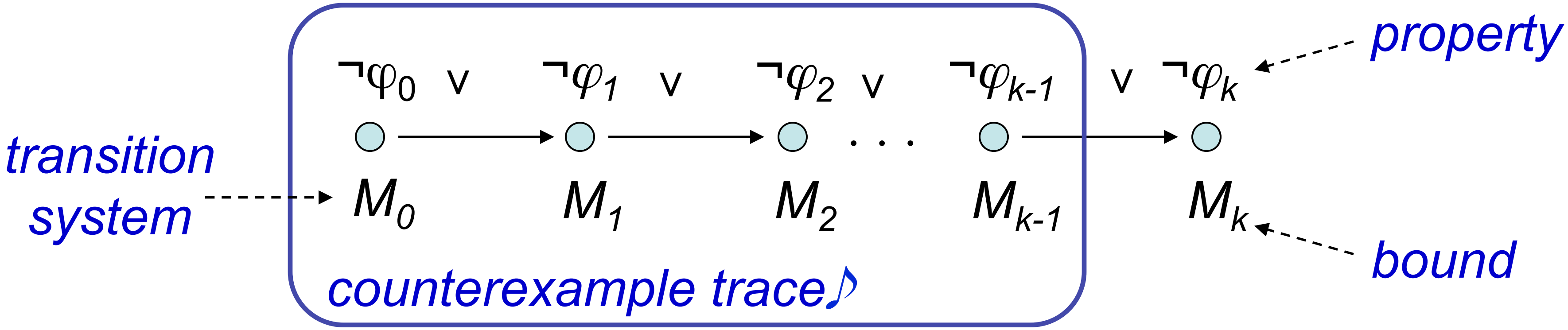


Bounded Model Checking

Basic Idea: check negation of given property up to given depth



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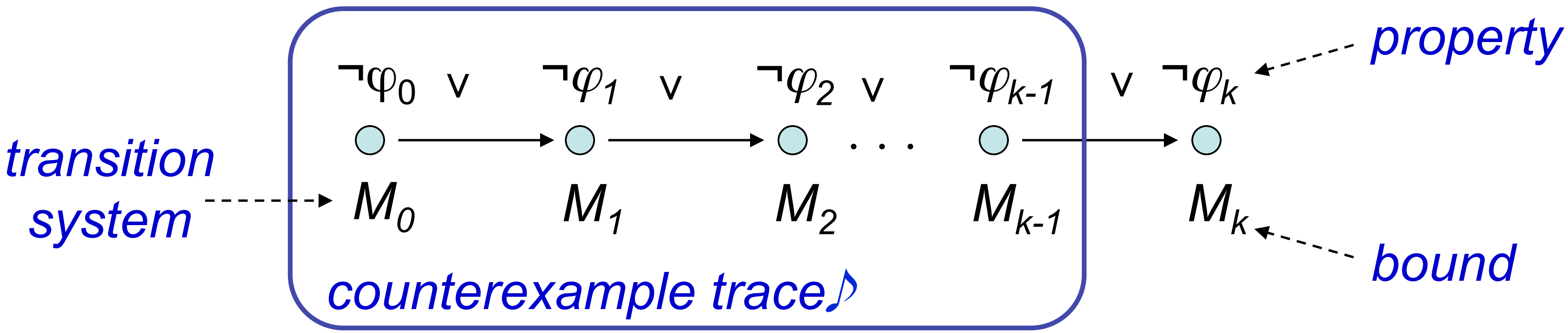

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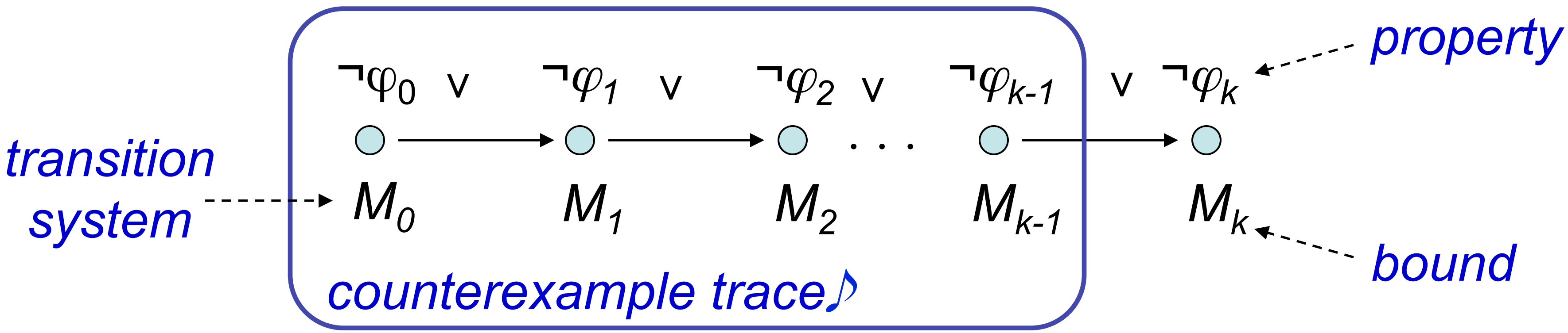

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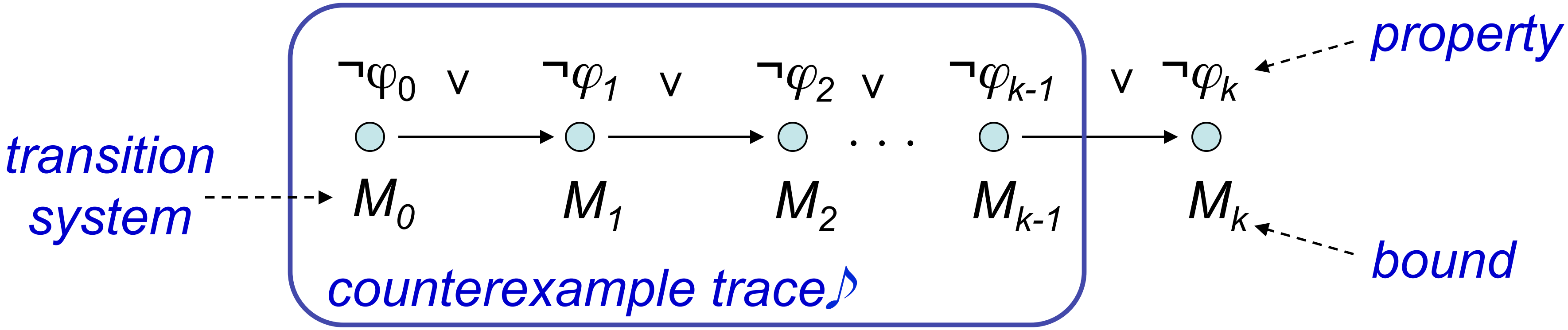

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- has been applied successfully to verify HW/SW systems

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Arrays	$(j = k \land a[k]=2) \Rightarrow a[j]=2$
Combined theories	$(j \leq k \land a[j]=2) \Rightarrow a[i] < 3$

Satisfiability Modulo Theories (2)

- Given
  - a decidable $\sum$-theory T
  - a quantifier-free formula φ

φ is T-satisfiable iff $T \cup \{\varphi\}$ is satisfiable, i.e., there exists a structure that satisfies both formula and sentences of T

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- Checking $\Gamma \models_T \varphi$ can be reduced in the usual way to checking the T-satisfiability of $\Gamma \cup \{\neg\varphi\}$

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& \neq g(SignExt(b,16)-c+4) \wedge SignExt(b,16)=c-3 \wedge c+1=d-4
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⇩ using facts about bit-vector arithmetic

$$step3: g(select(store(a,c,12), c)) \neq g(1) \wedge c-3=c-3 \wedge c+1=d-4$$

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The function g implies that for all x and y,
if x = y, then g (x) = g (y) (*congruence rule*).

$$step\,5: SAT\,(\mathrm{c} = 5, \mathrm{d} = 10)$$

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- SMT solvers also apply:
  - standard algebraic reduction rules $\boxed{r \wedge false \mapsto false}$
  - contextual simplification $\boxed{a=7 \wedge p(a) \mapsto a=7 \wedge p(7)}$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
  - derived from control-flow graph
  - checked safety properties give extra nodes
- program unfolded up to given bounds
  - loop iterations
  - context switches
- unfolded program optimized to reduce blow-up
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int main() {
  int a[2], i, x;
  if (x==0)
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    a[i+2]=1;
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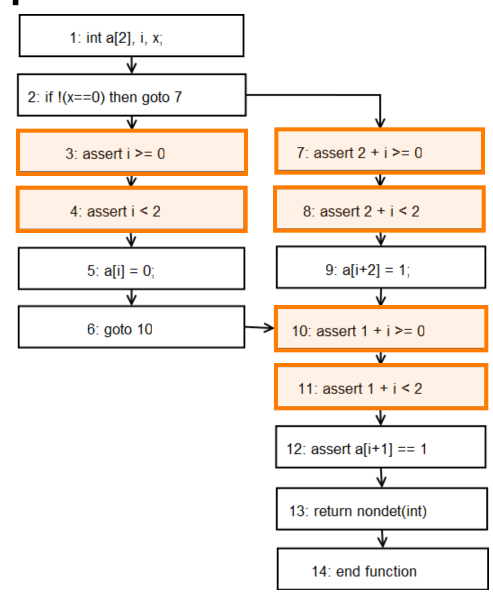

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$$
\begin{aligned}
g_1 &= x_1 == 0 \\
a_1 &= a_0 \text{ WITH } [i_0{:=}0] \\
a_2 &= a_0 \\
a_3 &= a_2 \text{ WITH } [2+i_0{:=}1] \\
a_4 &= g_1 \;?\; a_1 : a_3 \\
t_1 &= a_4[1+i_0] == 1
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  - specific to selected SMT solver, uses theories
- satisfiability check of C ∧ ¬P

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$$C := \begin{bmatrix} g_1 := (x_1 = 0) \\ \wedge\, a_1 := store(a_0, i_0, 0) \\ \wedge\, a_2 := a_0 \\ \wedge\, a_3 := store(a_2, 2 + i_0, 1) \\ \wedge\, a_4 := ite(g_1, a_1, a_3) \end{bmatrix}$$

$$P := \begin{bmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\, 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge\, 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\, select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

Encoding of Numeric Types

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  - abstract domains ($\mathbf{Z}$, $\mathbf{R}$)
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    - ▹ "internalized bit-blasting"

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- verification results can depend on encodings

$$(a > 0) \wedge (b > 0) \Rightarrow (a + b > 0)$$

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- verification results can depend on encodings

$$(a > 0) \land (b > 0) \Rightarrow (a + b > 0)$$

valid in abstract domains such as $\mathbf{Z}$ or $\mathbf{R}$

doesn't hold for bitvectors, due to possible overflows

Encoding of Numeric Types

- SMT solvers typically provide different encodings for numbers:
  - abstract domains ($\mathbb{Z}$, $\mathbb{R}$)
  - fixed-width bit vectors (`unsigned int`, …)
    - ▹ "internalized bit-blasting"
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  - majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
  - ESBMC supports both types of encoding and also combines them to improve scalability and precision

Encoding Numeric Types as Bitvectors

Bitvector encodings need to handle

- type casts and implicit conversions
  - arithmetic conversions implemented using word-level functions (part of the bitvector theory: Extract, SignExt, …)
    - different conversions for every pair of types
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    $\text{unsigned\_overflow} \Leftrightarrow (r - (r \bmod 2^w)) < 2^w$
  - define error literals to detect over- / underflow for other types

    $\text{res\_op} \Leftrightarrow \neg\, \text{overflow}(x, y) \wedge \neg\, \text{underflow}(x, y)$
    - similar to conversions

Floating-Point Numbers

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- **Binary encoding:** get a new bit-vector b = i @ f with the same bitwidth before and after the radix point of a.

$$i = \begin{cases} Extract(b, n_b + m_a - 1, n_b) & : \quad m_a \leq m_b \\ SignExt(Extract(b, t_b - 1, n_b), m_a - m_b) & : \quad otherwise \end{cases}$$

// m = number of bits of i

$$f = \begin{cases} Extract(b, n_b - 1, n_b - n_b) & : \quad n_a \leq n_b \\ Extract(b, n_b, 0) \ @ \ SignExt(b, n_a - n_b) & : \quad otherwise \end{cases}$$

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// n = number of bits of f

- **Rational encoding:** convert a to a rational number

$$a = \begin{cases} \dfrac{\left(i * p + \left(\dfrac{f * p}{2^n} + 1\right)\right)}{p} & : \quad f \neq 0 \\ i \quad : \quad \mathit{otherwise} \end{cases}$$

// p = number of decimal places

Floating-point SMT Encoding

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- The SMT floating-point theory is an addition to the SMT standard, proposed in 2010 and formalises:
  - Floating-point arithmetic
  - Positive and negative infinities and zeroes
  - NaNs
  - Comparison operators
  - Five rounding modes: round nearest with ties choosing the even value, round nearest with ties choosing away from zero, round towards zero, round towards positive infinity and round towards negative infinity

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- Two solvers currently support the standard:
  - Z3: implements all operators
  - MathSAT: implements all but two operators
    - *fp.rem:* remainder: x - y * n, where n in Z is nearest to x/y
    - *fp.fma:* fused multiplication and addition; (x * y) + z

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    - *fp.rem:* remainder: x - y * n, where n in Z is nearest to x/y
    - *fp.fma:* fused multiplication and addition; (x * y) + z
- Both solvers offer non-standard functions:
  - *fp_as_ieeebv:* converts floating-point to bitvectors
  - *fp_from_ieeeb*v: converts bitvectors to floating-point

How to encode Floating-point programs?

- Most operations performed at program-level to encode FP numbers have a **one-to-one conversion to SMT**
- Special cases being casts to boolean types and the fp.eq operator
  - Usually, cast operations are encoded using **extend/extract operation**
  - Extending floating-point numbers is non-trivial because of the format

```
int main()
{
  _Bool c;

  double b = 0.0f;
  b = c;
  assert(b != 0.0f);

  c = b;
  assert(c != 0);
}
```

Cast to/from booleans

- Simpler solutions:
  - Casting **booleans** to **floating-point numbers** can be done using an ite operator

```
(assert (= (ite |main::c|
          (fp #b0 #b01111111111 #x0000000000000)
          (fp #b0 #b00000000000 #x0000000000000))
   |main::b|))
```

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If true, assign 1f to b

```
(assert (= (ite |main::c|
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(assert (= (ite |main::c|
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```

Otherwise, assign 0f to b

Cast to/from booleans

- Simpler solutions:
  - Casting **floating-point numbers** to **booleans** can be done using an equality and one not:

```
(assert (= (not (fp.eq |main::b|
         (fp #b0 #b00000000000 #x0000000000000)))
   |main::c|))
```

```
:note
"(fp.eq x y) evaluates to true if x evaluates to -zero and y
to +zero, or vice versa. fp.eq and all the other comparison op-
erators evaluate to false if one of their arguments is NaN."
```

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true when the floating is not 0.0

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otherwise, the result is false

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erators evaluate to false if one of their arguments is NaN."
```

Floating-point Encoding: Illustrative Example

```
int main()
{
  float x;
  float y = x;
  assert(x==y);
  return 0;
}
```

Floating-point Encoding: Illustrative Example

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))

; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))

; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))

; assert x == y
(assert (let ((a!1 (not (=> true
                    (=> |execution_statet::\\guard_exec|
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  (or a!1)))
```

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Variable declarations

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Nondeterministic symbol declaration (optional)

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Guard used to check satisfiability

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Assignment of nondeterministic *value to x*

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(assert (let ((a!1 (not (=> true
                    (=> |execution_statet::\\guard_exec|
                        (fp.eq |main::x| |main::y|))))))
  (or a!1)))
```

Assignment *x to y*

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                        (fp.eq |main::x| |main::y|))))))
  (or a!1)))
```

Check if the comparison *satisfies the guard*

Floating-point Encoding: Illustrative Example

- Z3 produces:

```
sat
(model
  (define-fun |main::x| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |main::y| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |execution_statet::\\\\guard_exec| () Bool
    true)
)
```

Floating-point Encoding: Illustrative Example

- MathSAT produces:

```
sat
( (|main::x| (_ NaN 8 24))
  (|main::y| (_ NaN 8 24))
  (|nondet_symex::nondet0| (_ NaN 8 24))
  (|execution_statet::\\guard_exec| true) )
```

Floating-point Encoding: Illustrative Example

```
Counterexample:

State 1 file main3.c line 3 function main thread 0
main
----------------------------------------------------
  main3::main::1::x=-NaN (11111111100000000000000000000001)

State 2 file main3.c line 4 function main thread 0
main
----------------------------------------------------
  main3::main::2::y=-NaN (11111111100000000000000000000001)

State 3 file main3.c line 5 function main thread 0
main
----------------------------------------------------
Violated property:
  file main3.c line 5 function main
  assertion
  (_Bool)(x == y)

VERIFICATION FAILED
```

Intended learning outcomes

- Introduce **software verification** and **validation**
- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference among **static analysis**, **testing / simulation**, and **debugging**
- Explain **bounded model checking of software**
- Explain **precise memory model for software verification**

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
  - p.o $\triangleq$ representation of underlying object
  - p.i $\triangleq$ index (if pointer used as array base)

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```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

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```

⇒

$$C := \left(\begin{array}{l} p_1 := \mathrm{store}(p_0, 0, \&a[0]) \\ \wedge\ p_2 := \mathrm{store}(p_1, 1, 0) \\ \wedge\ g_2 := (x_2 == 0) \\ \wedge\ a_1 := \mathrm{store}(a_0, i_0, 0) \\ \wedge\ a_2 := a_0 \\ \wedge\ a_3 := \mathrm{store}(a_2, 1+ i_0, 1) \\ \wedge\ a_4 := \mathrm{ite}(g_1, a_1, a_3) \\ \wedge\ p_3 := \mathrm{store}(p_2, 1, \mathrm{select}(p_2 , 1)+2) \end{array}\right)$$

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  - p.o $\triangleq$ representation of underlying object
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Store object at position 0

```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

$$
C := \left(\begin{array}{l}
p_1 := \mathrm{store}(p_0, 0, \&a[0]) \\
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\wedge\ g_2 := (x_2 == 0) \\
\wedge\ a_1 := \mathrm{store}(a_0, i_0, 0) \\
\wedge\ a_2 := a_0 \\
\wedge\ a_3 := \mathrm{store}(a_2, 1+ i_0, 1) \\
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  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

$$
C := \left(\begin{array}{l}
p_1 := \mathrm{store}(p_0, 0, \&a[0]) \\
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\wedge\ a_1 := \mathrm{store}(a_0, i \\
\wedge\ a_2 := a_0 \\
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\end{array}\right)
$$

Store index at position 1

Encoding of Pointers

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- pointers modelled as tuples
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```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

⇒ C

$$
\left(\begin{array}{l}
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\wedge\ p_2 := \mathrm{store}(p_1,\ 1,\ 0) \\
\wedge\ g_2 := (x_2 == 0) \\
\wedge\ a_1 := \mathrm{store}(a_0,\ i \ldots \\
\wedge\ a_3 := \mathrm{store}(a_2,\ 1+ i_0,\ 1) \\
\wedge\ a_4 := \mathrm{ite}(g_1,\ a_1,\ a_3) \\
\wedge\ p_3 := \mathrm{store}(p_2,\ 1,\ \mathrm{select}(p_2\ ,\ 1)+2)
\end{array}\right)
$$

Store object at position 0

Store index at position 1

Update index

Encoding of Pointers

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  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

$$P := \left(\begin{array}{l} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\ 1+ i_0 \geq 0 \wedge 1+ \quad < 2 \\ \wedge\ \mathrm{select}(p_3, 0) == \&a[0] \\ \wedge\ \mathrm{select}(\mathrm{select}(p_3, 0), \\ \qquad \mathrm{select}(p_3, 1)) == 1 \end{array} \right)$$

negation satisfiable (a[2] unconstrained)
⇒ assert fails

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- model memory just as an array of bytes (array theories)
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  - $m \triangleq$ memory array
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  - $\rho \triangleq$ unique identifier
  - $\upsilon \triangleq$ indicate whether the object is still alive
  - $l \triangleq$ the location in the execution where m is allocated

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- model memory just as an array of bytes (array theories)
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  - m ≜ memory array
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  - ρ ≜ unique identifier
  - υ ≜ indicate whether the object is still alive
  - l ≜ the location in the execution where m is allocated
- to detect invalid reads/writes, we check whether
  - d_o is a dynamic object
  - i is within the bounds of the memory array

$$l_{is\_dynamic\_object} \Leftrightarrow \left(\bigvee_{j=1}^{k} d_o.\rho = j \right) \wedge \left(0 \leq i < n\right)$$

Encoding of Memory Allocation

- to check for invalid objects, we
  - set υ to *true* if the function `malloc` can allocate memory (d_o is alive)
  - set υ to *false* if the function `free` is called (d_o is not longer alive)

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- to detect forgotten memory, at the end of the (unrolled) program we check
  - whether the d_o has been deallocated by the function free

$$l_{deallocated\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow \neg\, d_o.\upsilon)$$

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

Assume that the malloc call succeeds

Example of Memory Allocation

memory leak: pointer reassignment makes $d_{o1}.\upsilon$ to become an orphan

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$$C := \left(\begin{array}{l} d_{o1}.\rho=1 \wedge d_{o1}.s=5 \wedge d_{o1}.\upsilon=\mathit{true} \wedge p=d_{o1} \\ \wedge\ d_{o2}.\rho=2 \wedge d_{o2}.s=5 \wedge d_{o2}.\upsilon=\mathit{true} \wedge q=d_{o2} \\ \wedge\ p=d_{o2} \wedge d_{o2}.\upsilon=\mathit{false} \\ \wedge\ d_{o3}.\rho=3 \wedge d_{o3}.s=5 \wedge d_{o3}.\upsilon=\mathit{true} \wedge p=d_{o3} \\ \wedge\ d_{o3}.\upsilon=\mathit{false} \end{array} \right)$$

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- Alignment rules require that any pointer variable must be aligned to at least the alignment of the pointer type
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  - E.g., an integer pointer's value must be aligned to at least 4 bytes, for 32-bit integers
- Encode **property assertions** when dereferences occur during symbolic execution
  - To guard against executions where an unaligned pointer is dereferenced
  - This is not as strong as the C standard requirement, that a pointer variable may never hold an unaligned value
    - But it provides a guarantee that any pointer dereference will either be correctly aligned or result in a verification failure

ESBMC's memory model

- statically tracks possible pointer variable targets (objects)
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struct foo {
  uint16_t bar[2];
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pointer and object types do not match

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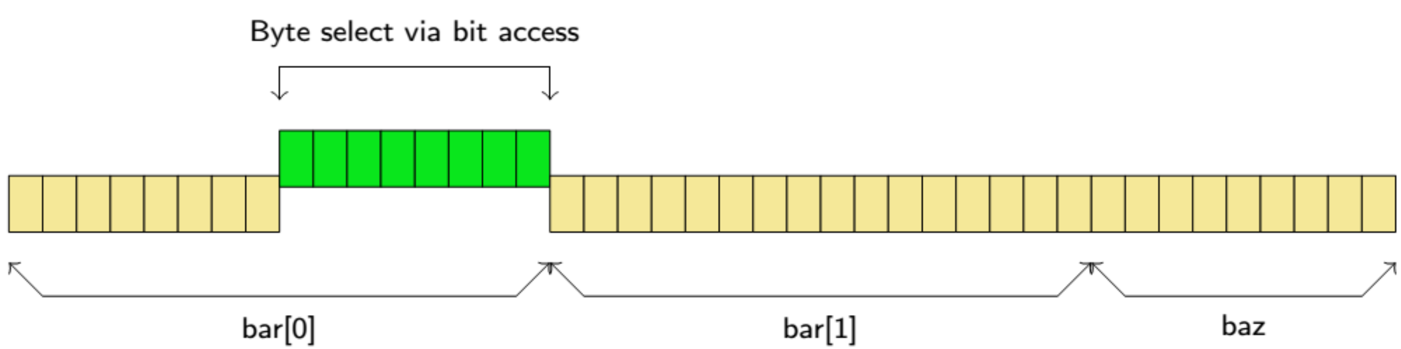
- **SAT: immediate access** to bit-level representation



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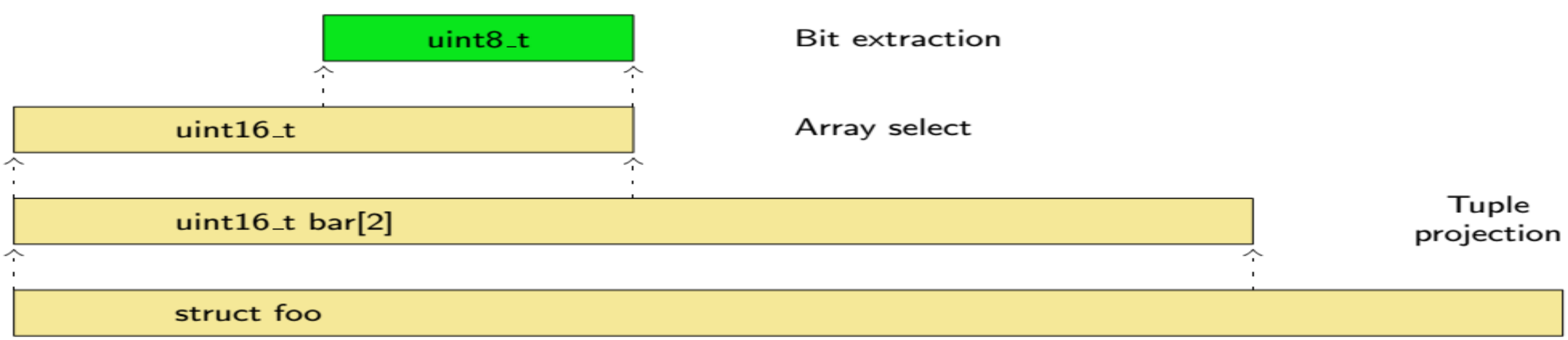
- **SMT:** sorts must be **repeatedly unwrapped**



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if (nondet_bool()) {
    fuzz = &qux.bar[0];
  } else {
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- supporting **all legal behaviors** at SMT layer **difficult**
  - extract (unaligned) 16bit integer from *fuzz
- experiments showed significantly increased **memory consumption**

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- **enforce C alignment rules**
  - static analysis of pointer alignment eliminates need to encode unaligned data accesses
    → reduces number of behaviors that must be modeled
  - add alignment assertions (if static analysis not conclusive)
  - extracting 16-bit integer from `*fuzz` if guard is true:
    - offset = 0: project `bar[0]` out of foo
    - offset = 1: **“unaligned memory access” failure**
    - offset = 2: project `bar[1]` out of foo
    - offset = 3: **“unaligned memory access” failure**
    - offset = 4: “access to object out of bounds” failure

Summary

- Described the difference between **soundness** and **completeness** concerning **detection techniques**
  - **False positive** and **false negative**
- Pointed out the difference between **static analysis** and **testing / simulation**
  - **hybrid combination** of static and dynamic analysis techniques to achieve a good trade-off between **soundness** and **completeness**
- Explained **bounded model checking of software**
  - they have been applied successfully to verify **single-threaded software using a precise memory model**