Systems and Software Verification Laboratory

The University of Manchester

Detection of Software Vulnerabilities: Static Analysis

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Static Analysis

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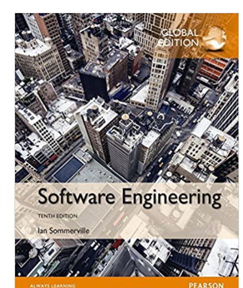
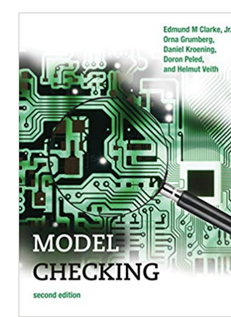
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Software model checking*. ACM Comput. Surv., 2009
  - *The Cyber Security Body of Knowledge*, 2019
  - *Software Engineering* (Chapters 8, 13)

Lecture notes "SAT/SMT-Based Bounded Model Checking of Software" by Bernd Fischer





Intended learning outcomes

- Introduce typical **BMC architectures** for verifying **software systems**
- Explain **bounded model checking of multi-threaded software**
- Explain **unbounded model checking of software**

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SAT/SMT-based BMC tools for C

- CBMC (C Bounded Model Checker)
  - http://www.cprover.org/
  - SAT-based (MiniSat) “workhorse”
  - also SystemC frontend

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  - branched off CBMC, also (rudimentary) C++ frontend

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  - branched off CBMC, also (rudimentary) C++ frontend
- LLBMC (Low-level Bounded Model Checker)
  - http://llbmc.org
  - SMT-based (Boolector or STP)
  - uses LLVM intermediate language

$\Rightarrow$ share common high-level architecture

SAT/SMT-based BMC tools for C

Typical features:

- full language support
  - bit-precise operations, structs, arrays, ...
  - heap-allocated memory
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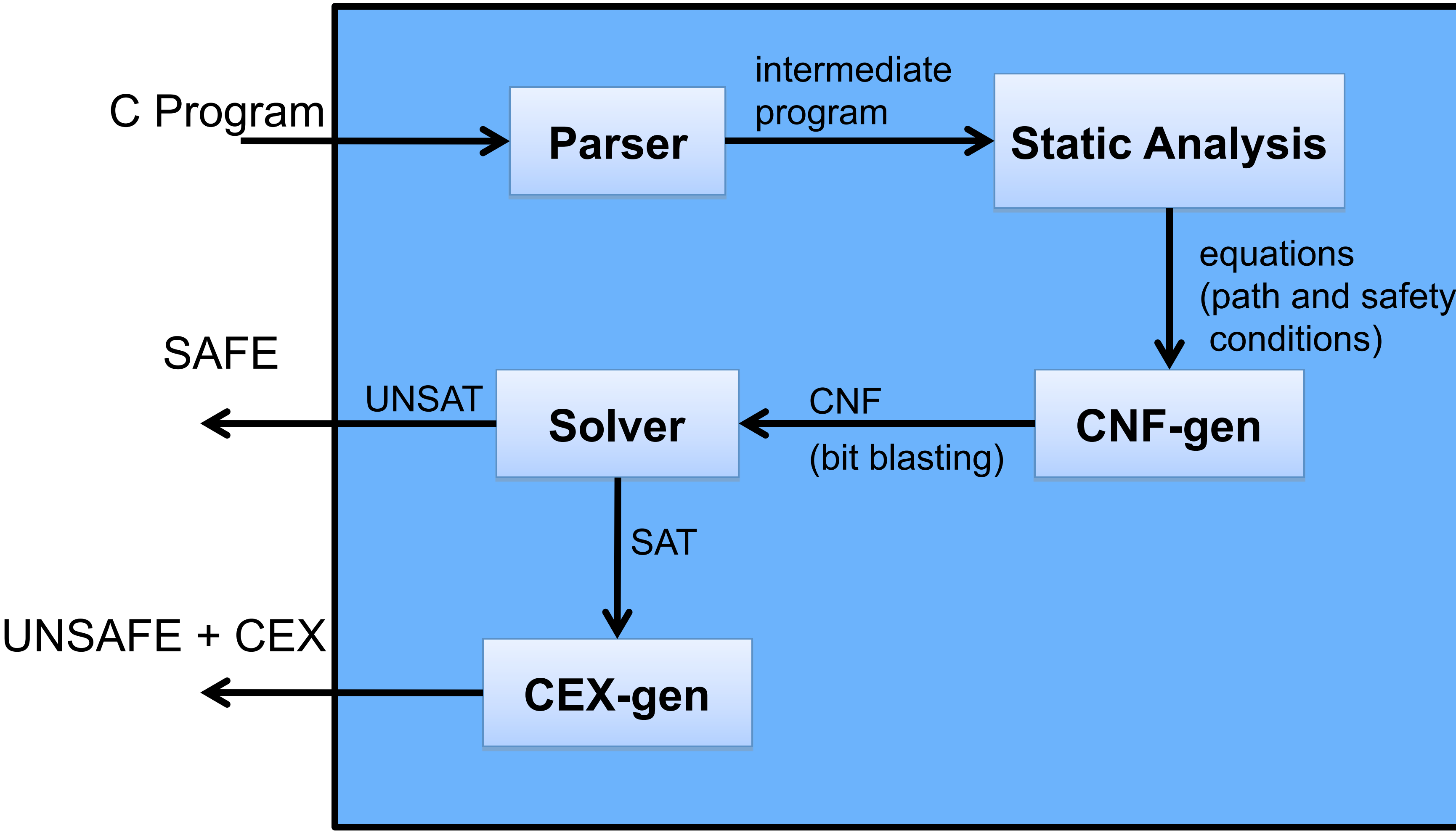
  - overflow, div-by-zero, array out-of-bounds indexing, ...
  - memory safety: nil pointer deref, memory leaks, ...
  - deadlocks, race conditions
- user-specified assertions and error labels
- non-deterministic modelling
  - nondeterministic assignments
  - assume-statements

SAT/SMT-based BMC tools for C

High-level architecture:



SAT/SMT-based BMC tools for C

General approach:

1. Simplify control flow
2. Unwind all of the loops
3. Convert into single static assignment (SSA) form
4. Convert into equations and simplify
5. (Bit-blast)
6. Solve with a SAT/SMT solver
7. Convert SAT assignment into a counterexample

Control flow simplifications

- remove all side effects
  - e.g., j = ++i; becomes i = i+1; j = i;

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  - e.g., replace for, do while by while
  - e.g., replace case by if

Control flow simplifications

- remove all side effects
  - e.g., j=++i; becomes i=i+1; j=i;
- simplify all control flow structures into core forms
  - e.g., replace for, do while by while
  - e.g., replace case by if
- make control flow explicit
  - e.g., replace continue, break by goto
  - e.g., replace if, while by goto

Control flow simplifications

Demo: esbmc --goto-functions-only example-1.c

```
int main() {
  int i,j;
  for(i=0; i<6; i++) {
   j=i;
  }
  assert(j==i);
  return j;
}
```

```
main (c::main):
         int i;
         int j;
         i = 0;
     1: IF !(i < 6) THEN GOTO 2
         j = i;
         i = i + 1;
        GOTO 1
     2:   ASSERT j == i
        RETURN: j
        END_FUNCTION
```

Loop unwinding

- all loops are “unwound”, i.e., replaced by several guarded copies of the loop body
  - same for backward **`gotos`** and recursive functions
  - can use different unwinding bounds for different loops

⇒ each statement is executed at most once

Loop unwinding

- all loops are "unwound", i.e., replaced by several guarded copies of the loop body
  - same for backward **`gotos`** and recursive functions
  - can use different unwinding bounds for different loops

⇒ each statement is executed at most once

- to check whether unwinding is sufficient special "unwinding assertion" claims are added

⇒ if a program satisfies all of its claims and all unwinding assertions then it is correct!

Loop unwinding

```
void f(...) {
  ...
  while(cond) {
    Body;
  }
  Remainder;
}
```

Loop unwinding

```
void f(...) {
  ...
  if(cond) {
    Body;
    while(cond) {
      Body;
    }
  }
  Remainder;
}
```

unwind one iteration

Loop unwinding

```
void f(...) {
  ...
  if(cond) {
    Body;
    if(cond) {
      Body;
      while(cond) {
        Body;
      }
    }
  }
  Remainder;
}
```

unwind one

unwind one iteration

Loop unwinding

```
void f(...) {
  ...
  if(cond) {
    Body;
    if(cond) {
      Body;
      if(cond) {
        Body;
        while(cond) {
          Body;
        }
      }
    }
  }
  Remainder;
}
```

unwind one

unwind one

unwind one iteration…

Loop unwinding

```
void f(...) {
  ...
  if(cond) {
    Body;
    if(cond) {
      Body;
      if(cond) {
        Body;
        assert(!cond);

      }
    }
  }
  Remainder;
}
```

unwind one

unwind one

unwind one iteration…

unwinding assertion

- unwinding assertion
  - inserted after last unwound iteration
  - violated if program runs longer than bound permits
  - ⇒ if not violated: (real) correctness result!

Loop unwinding

```
void f(...) {
  ...
  for(i=0; i<N; i++) {
    ...
    b[i]=a[i];
    ...
  };
  ...
  for(i=0; i<N; i++) {
    ...
    assert(b[i]-a[i]>0);
    ...
  };
  ...
  Remainder;
}
```

- unwinding assertion
  - inserted after last unwound iteration
  - violated if program runs longer than bound permits

  ⇒ if not violated: (real) correctness result!

⇒ what about multiple loops?
  - use --partial-loops to suppress insertion

  ⇒ unsound

Safety conditions

- Built-in safety checks converted into explicit assertions:

  e.g., array safety:

  ```
  a[i]=...;
   ⇒ assert(0 <= i && i <= N); a[i]=...;
  ```

Safety conditions

- Built-in safety checks converted into explicit assertions:

  e.g., array safety:

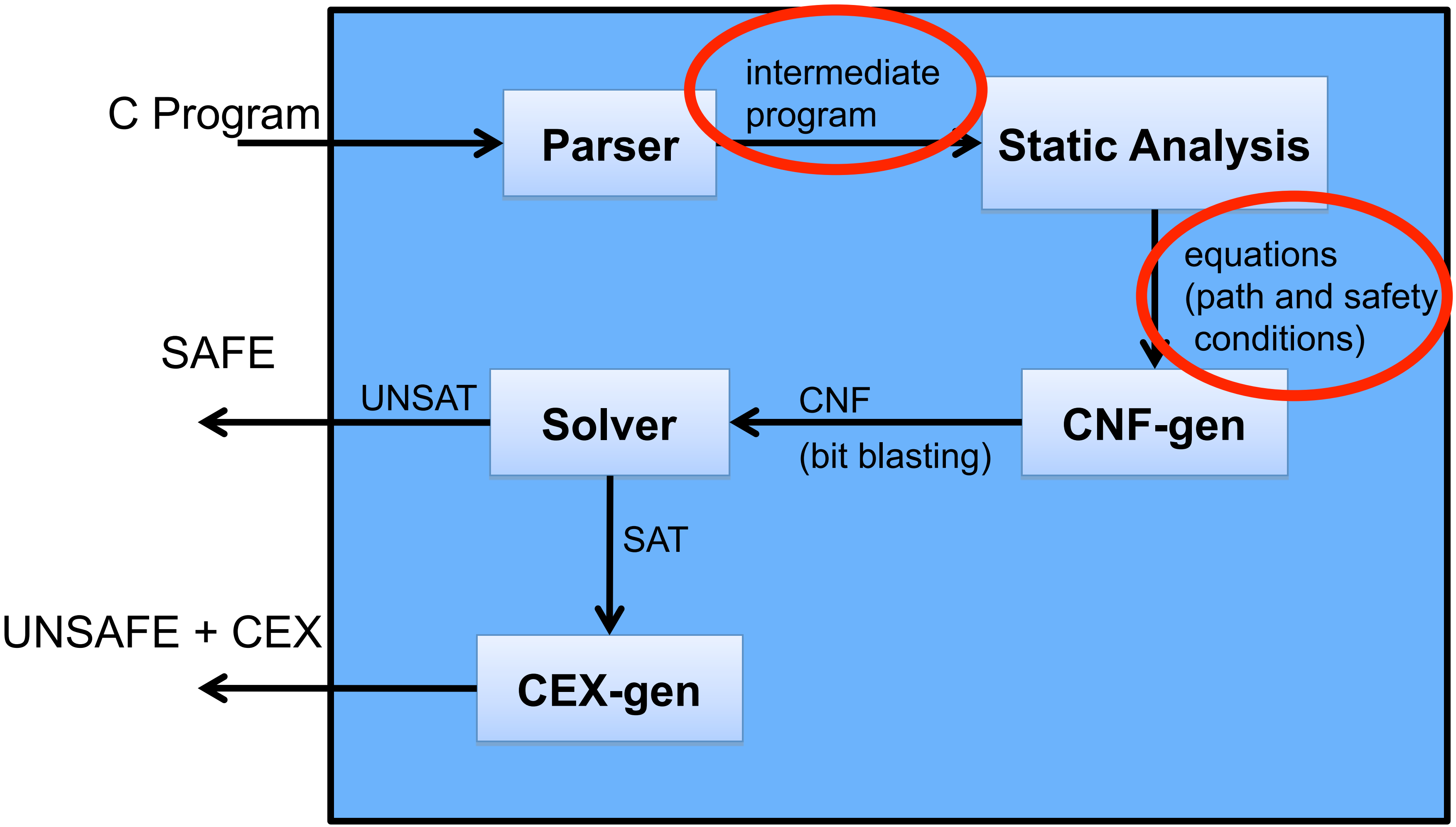
  ```
  a[i]=...;
   ⇒ assert(0 <= i && i <= N); a[i]=...;
  ```

⇒ sometimes easier at intermediate representation or formula level

  e.g., word-aligned pointer access, overflow, ...

SAT/SMT-based BMC tools for C

High-level architecture:



Transforming straight-line programs into equations

- simple if each variable is assigned only once:

$x = a;$
$y = x + 1;$
$z = y - 1;$

program

→

$x = a \quad \&\&$
$y = x + 1 \quad \&\&$
$z = y - 1$

constraints

- still simple if variables are assigned multiple times:

$x = a;$
$x = x + 1;$
$x = x - 1;$

program

→

$x_0 = a;$
$x_1 = x_0 + 1;$
$x_2 = x_1 - 1;$

program in SSA-form

introduce fresh copy for each occurrence
(*static single assignment (SSA)* form)

Transforming loop-free programs into equations

But what about control flow branches (if-statements)?

```
if(v)
  x = y;
else
  x = z;

w = x;
```

```
if(v_0)
  x_0 = y_0;
else
  x_1 = z_0;

w_1 = ?
```

introduce & use new variable

- for each control flow join point, add a new variable with guarded assignment as definition
  - also called ϕ-function

Transforming loop-free programs into equations

But what about control flow branches (if-statements)?

```
if(v)
  x = y;
else
  x = z;

w = x;
```

$$
\begin{array}{l}
\texttt{if}(v_0) \\
\quad x_0 = y_0; \\
\texttt{else} \\
\quad x_1 = z_0; \\
x_2 = v_0 \; ? \; x_0 : x_1; \\
w_1 = x_2;
\end{array}
$$

introduce & use
new variable

- for each control flow join point, add a new variable with guarded assignment as definition
  - also called ϕ-function

Bit-blasting

Conversion of equations into SAT problem:

- simple assignments:

  $|[\ x = y\ ]| \triangleq \bigwedge_i x_i \Leftrightarrow y_i$

  effective bitwidth

  $\Rightarrow$ static analysis must approximate effective bitwidth well

- ϕ-functions:

  $|[\ x = v\ ?\ y : z\ ]| \triangleq (v \Rightarrow |[\ x = y\ ]|) \wedge (\neg\ v \Rightarrow |[\ x = z\ ]|)$

- Boolean operations:

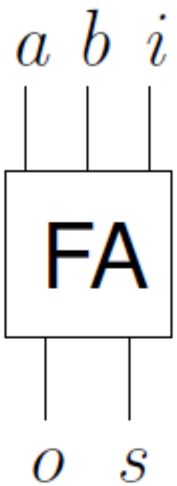
  $|[\ x = y \mid z\ ]| \triangleq \bigwedge_i x_i \Leftrightarrow (y_i \vee z_i)$

Exercise: relational operations

Bit-blasting arithmetic operations

Build **circuits** that implement the operations!

1-bit addition:



Full Adder				
s	$\equiv$	$(a+b+i)\bmod 2$	$\equiv$	$a \oplus b \oplus i$
o	$\equiv$	$(a+b+i)\operatorname{div} 2$	$\equiv$	$a \cdot b + a \cdot i + b \cdot i$

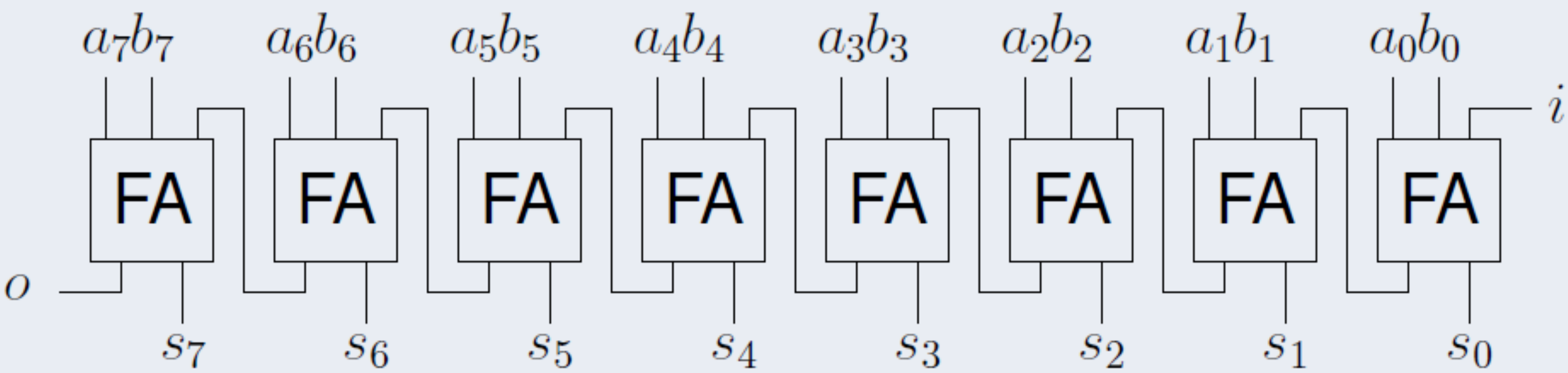
Full adder as CNF:

$$(a \vee b \vee \neg o) \wedge (a \vee \neg b \vee i \vee \neg o) \wedge (a \vee \neg b \vee \neg i \vee o) \wedge$$
$$(\neg a \vee b \vee i \vee \neg o) \wedge (\neg a \vee b \vee \neg i \vee o) \wedge (\neg a \vee \neg b \vee o)$$

Bit-blasting arithmetic operations

Build **circuits** that implement the operations!

8-Bit ripple carry adder (RCA)



⇒ adds w variables, 6*w clauses

⇒ multiplication / division much more complicated

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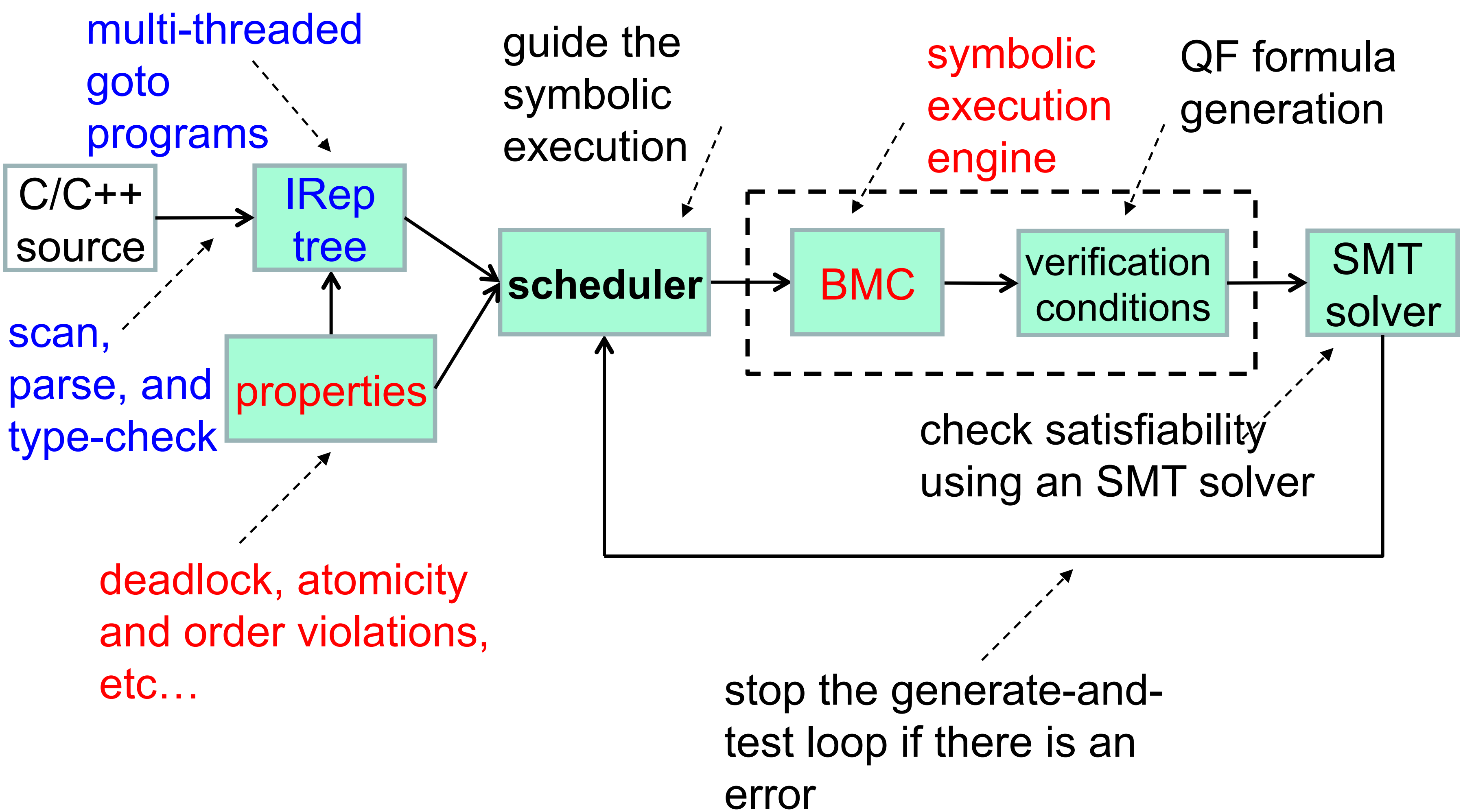
BMC of Multi-threaded Software

Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving



Running Example

- the program has sequences of operations that need to be protected together to avoid atomicity violation
  - requirement: the region of code (val1 and val2) should execute atomically

Thread twoStage
```
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

A state $s \in S$ consists of the value of the program counter pc and the values of all program variables

```
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
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program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
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Lazy exploration: interleaving I_s

statements: 1

val1-access:

val2-access:

Thread twoStage
```
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

Thread reader
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11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 1
mutexes: ***m1=1****; m2=0;*
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2

val1-access: $W_{twoStage,2}$

val2-access:

write access to the shared variable val1 in statement 2 of the thread twoStage

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 2

mutexes: m1=1; m2=0;

global variables: ***val1=1****; val2=0;*

local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3

val1-access: $W_{twoStage,_2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
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9:     unlock(m1);
10:   return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3
*mutexes: **m1=0**; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7

val1-access: $W_{twoStage,_2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7
mutexes: ***m1=1****; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I

read access to the shared variable val1 in statement 8 of the thread reader

statements: 1-2-3-7-8

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

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Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 8
mutexes: m1=1; m2=0;
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 11
mutexes: m1=1; m2=0;
global variables: val1=1; val2=0;
*local variabes: **t1= 1**; t2= -1;*

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 12

mutexes: ***m1=0****; m2=0;*

global variables: val1=1; val2=0;

local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 4

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4

val1-access: $W_{\text{twoStage},2}$ - $R_{\text{reader},8}$ - $R_{\text{reader},11}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 4
mutexes: m1=0; ***m2=1****;*
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:   return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS1

CS2

program counter: 5
mutexes: m1=0; m2=1;
global variables: val1=1; ***val2=2****;*
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 6
mutexes: m1=0; ***m2=0****;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

CS3

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 13

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

CS3

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 13
mutexes: m1=0; ***m2=1****;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS1

CS2

CS3

program counter: 14

mutexes: m1=0; m2=1;

global variables: val1=1; val2=2;

local variabes: t1= 1; ***t2= 2****;*

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

CS3

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 15

mutexes: m1=0; ***m2=0****;*

global variables: val1=1; val2=2;

local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

CS3

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
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10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 16

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

CS2

CS3

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

QF formula is unsatisfiable, i.e., assertion holds

Lazy exploration: interleaving I_f

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_f

statements: 1-2-3

val1-access: $W_{twoStage,_2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3
mutexes: m1=0; m2=0;
*global variables: **val1=1**; val2=0;*
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_f

statements: 1-2-3

val1-access: $W_{twoStage,_2}$

val2-access:

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access: $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 16
mutexes: m1=0; m2=0;
global variables: val1=1; val2=0;
local variabes: ***t1= 1***; ***t2= 0***;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access: $R_{reader,14}$

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 4
mutexes: m1=0; m2=0;
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$ - $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 6
mutexes: m1=0; m2=0;
global variables: val1=1; ***val2=2****;*
local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$ - $W_{twoStage,5}$

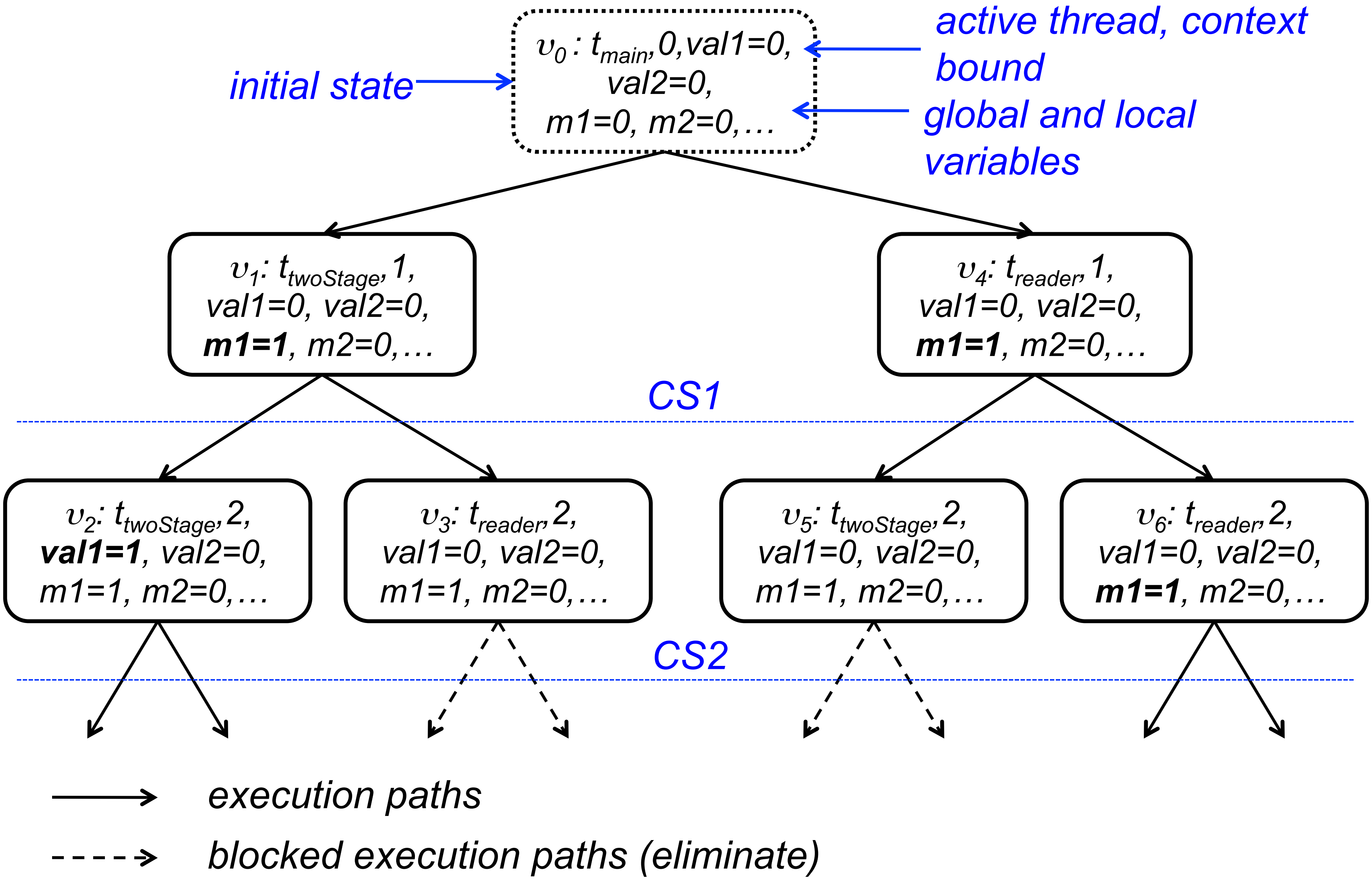
```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS2

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:     unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

QF formula is satisfiable,
i.e., assertion does not hold

Lazy Approach: State Transitions



Exploring the Reachability Tree

- Use a reachability tree (RT) to describe reachable states of a multi-threaded program

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- Each node in the RT is a tuple $\upsilon = \left(A_i, C_i, s_i, \left\langle l_i^j, G_i^j \right\rangle_{j=1}^{n} \right)_i$ for a given time step i, where:

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  - A_i represents the currently active thread
  - C_i represents the context switch number

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  - C_i represents the context switch number
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  - A_i represents the currently active thread
  - C_i represents the context switch number
  - s_i represents the current state
  - l_i^j represents the current location of thread j

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  - A_i represents the currently active thread
  - C_i represents the context switch number
  - s_i represents the current state
  - l_i^j represents the current location of thread *j*
  - G_i^j represents the control flow guards accumulated in thread *j* along the path from l_0^j to l_i^j

Expansion Rules of the RT

R1 (assign): If *l* is an assignment, we execute *l*, which generates s_{i+1}. We add as child to υ a new node υ'

$$l_{i+1}^{A_i} = l_i^{A_i} + 1$$

$$\upsilon' = \left(A_i, C_i, \underline{s_{i+1}}, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1}$$

- we have fully expanded υ if
  - I within an atomic block; or
  - *l* contains no global variable; or
  - the upper bound of context switches ($C_i = C$) is reached
- if υ is not fully expanded, for each thread $j \neq A_i$ where G_i^j is enabled in s_{i+1}, we thus create a new child node

$$\upsilon_j' = \left(\underline{j}, \underline{C_i + 1}, \underline{s_{i+1}}, \left\langle l_i^j, G_i^j \right\rangle\right)_{i+1}$$

Expansion Rules of the RT

R2 (skip): If I is a *skip*-statement with target l, we increment the location of the current thread and continue with it. We explore no context switches:

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

Expansion Rules of the RT

R2 (skip): If *l* is a *skip*-statement with target *l*, we increment the location of the current thread and continue with it. We explore no context switches:

$$l_{i+1}^{j} = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^{j}}, G_i^j \right\rangle\right)_{i+1}$$

R3 (unconditional goto): If *l* is an unconditional *goto*-statement with target *l*, we set the location of the current thread and continue with it. We explore no context switches:

$$l_{i+1}^{j} = \begin{cases} l & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^{j}}, G_i^j \right\rangle\right)_{i+1}$$

Expansion Rules of the RT

R4 (conditional goto): If *I* is a conditional *goto*-statement with test *c and* target *l*, we create two child nodes υ' and υ''.

- for υ', we assume that *c* is *true* and proceed with the target instruction of the jump:

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle \underline{l^j_{i+1}}, c \underline{\wedge\, G^j_i} \right\rangle\right)_{i+1} \qquad l^j_{i+1} = \begin{cases} l & : \quad j = A_i \\ l^j_i & : \quad otherwise \end{cases}$$

- for υ'', we add ¬*c* to the guards and continue with the next instruction in the current thread

$$\upsilon'' = \left(A_i, C_i, s_i, \left\langle \underline{l^j_{i+1}}, \neg c \underline{\wedge\, G^j_i} \right\rangle\right)_{i+1} \qquad l^j_{i+1} = \begin{cases} l^j_i + 1 & : \quad j = A_i \\ l^j_i & : \quad otherwise \end{cases}$$

- prune one of the nodes if the condition is determined statically

Expansion Rules of the RT

R5 (assume): If *I* is an *assume*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

Expansion Rules of the RT

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- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

R6 (assert): If *I* is an *assert*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- we generate a verification condition to check the validity of *c*

Expansion Rules of the RT

R5 (start_thread): If *I* is a *start_thread* instruction, we add the indicated thread to the set of active threads:

$$v' = \left(A_i, C_i, s_i, \underline{\left\langle l_{i+1}^{j}, G_{i+1}^{j} \right\rangle}_{j=1}^{n+1} \right)_{i+1}$$

- where l_{i+1}^{n+1} is the initial location of the thread and $G_{i+1}^{n+1} = G_i^{A_i}$
- the thread starts with the guards of the currently active thread

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- the thread starts with the guards of the currently active thread

R6 (join_thread): If *I* is a *join_thread* instruction with argument *Id*, we add a child node:

$$v' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle \right)_{i+1}$$

- where $l_{i+1}^j = l_i^{A_i} + 1$ only if the joining thread Id has exited

Lazy exploration of interleavings

- Main steps of the algorithm:
  1. Initialize the stack with the initial node ν_0 and the initial path $\pi_0 = \langle \upsilon_0 \rangle$
  2. If the stack is empty, terminate with "no error".
  3. Pop the current node υ and current path π off the stack and compute the set υ' of successors of υ using rules R1-R8.
  4. If υ' is empty, derive the VC φ_k^π for π and call the SMT solver on it. If φ_k^π is satisfiable, terminate with "error"; otherwise, goto step 2.
  5. If υ' is not empty, then for each node $\upsilon \in \upsilon'$, add ν to π, and push node and extended path on the stack. goto step 3.

computation path

$\pi = \{\upsilon_1, \ldots \upsilon_n\}$

$$\varphi_k^\pi = \overbrace{I(s_0) \wedge R(s_0, s_1) \wedge \ldots \wedge R(s_{k-1}, s_k)}^{\text{constraints}} \wedge \overbrace{\neg \phi_k}^{\text{property}}$$

bound

Observations about the lazy approach

- naïve but useful:
  - bugs usually manifest with few context switches [Qadeer&Rehof'05]

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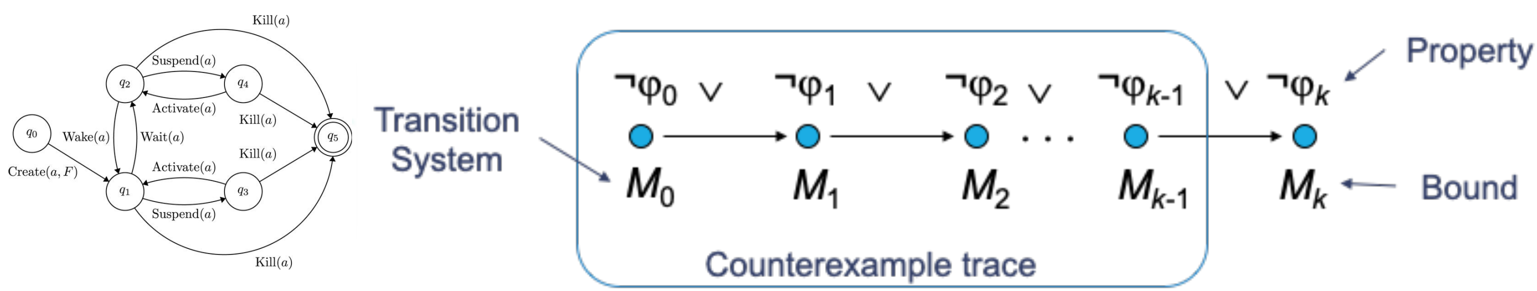
  - bound the number of preemptions (C) allowed per threads
    - ▹ *number of executions: $O(n^c)$*
  - as each formula corresponds to one possible path only, its size is relatively small
- can suffer performance degradation:
  - in particular for correct programs where we need to invoke the SMT solver once for each possible execution path

Intended learning outcomes

- Introduce typical **BMC architectures** for verifying **software systems**
- Explain **bounded model checking of multi-threaded software**
- Explain **unbounded model checking of software**

Revisiting BMC

- Basic Idea: given a transition system *M*, check negation of a given property φ up to given depth *k:*



- Translated into a VC ψ such that: ψ *satisfiable iff* φ *has counterexample of max. depth k*

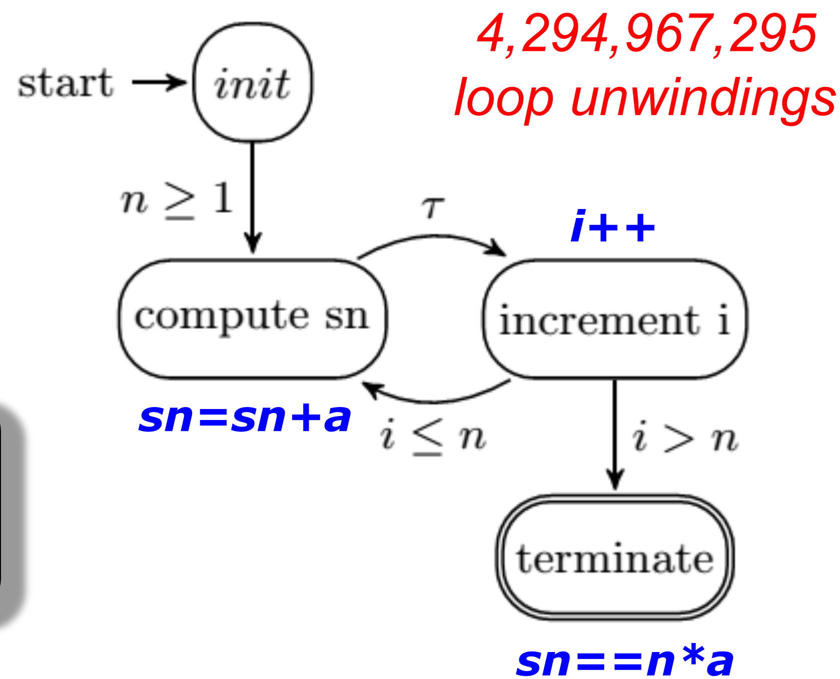
BMC is aimed at finding bugs; it cannot prove correctness, unless the bound *k* safely reaches all program states

Difficulties in proving the correctness of programs with loops in BMC

- BMC techniques can falsify properties up to a given depth k
  - they can prove correctness only if an upper bound of k is known (**unwinding assertion**)
    - » BMC tools typically fail to verify programs that contain bounded and unbounded loops

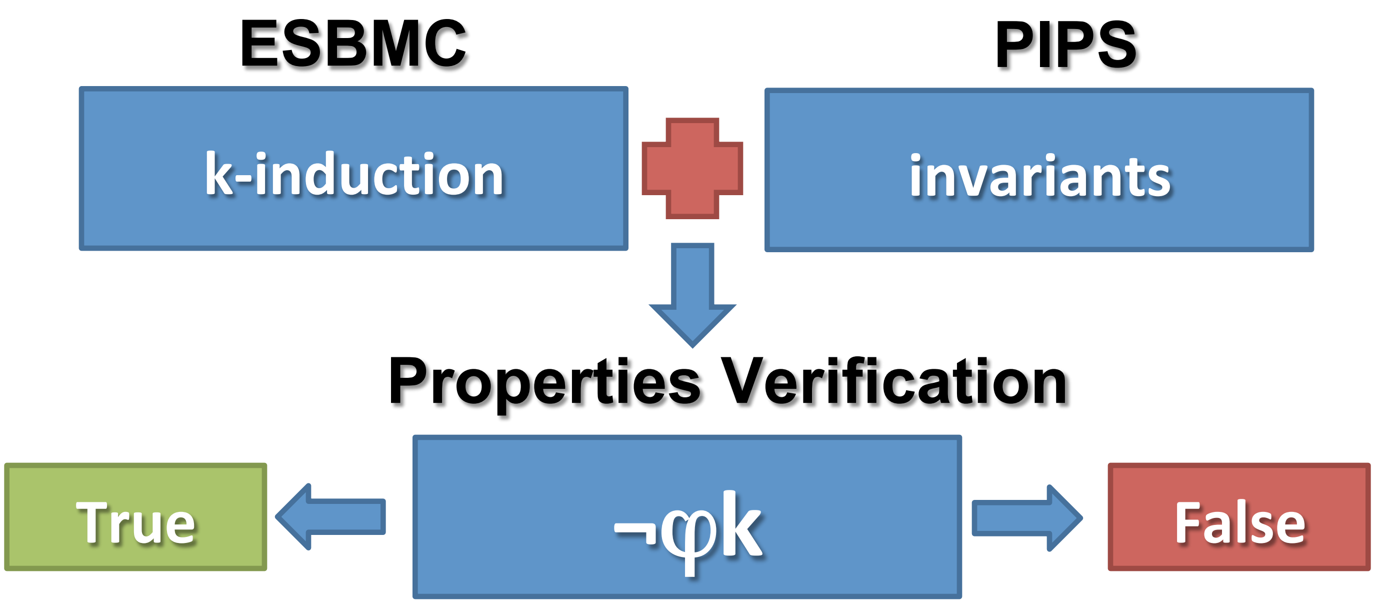
$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

the loop will be unfolded 2^{n-1} times (in the worst case, 2^{32-1} times on 32 bits integer)



Handling loops in BMC of C programs via *k*-induction and invariants

- Algorithmic method to prove correctness of C programs
  - combining *k*-induction with invariants
  - in a completely automatic way



Induction-Based Verification

***k*-induction** checks...

- **base case (*$base_k$*):** find a counter-example with up to *k* loop unwindings (plain BMC)
- **forward condition (*fwd_k*):** check that *P* holds in all states reachable within *k* unwindings
- **inductive step (*$step_k$*):** check that whenever *P* holds for *k* unwindings, it also holds after next unwinding
  – havoc state
  – run *k* iterations
  – assume invariant
  – run final iteration

⇒ iterative deepening if inconclusive

The *k*-induction algorithm

```
k=initial bound
while true do
    if  base_k then
        return trace s[0..k]
    else if fwd_k
        return true
    else if step_k then
        return true
    end if
    k=k+1
end
```

The *k*-induction algorithm

k=initial bound

while *true* **do**

if $base_k$ **then**

return *trace s[0..k]*

else if fwd_k

return *true*

else if $step_k$ **then**

return *true*

end if

k=k+1

end

inserts unwinding assumption after each loop

The *k*-induction algorithm

k=initial bound
while *true* **do**
    if $base_k$ **then**
        return *trace s[0..k]*
    else if fwd_k
        return *true*
    else if $step_k$ **then**
        return *true*
    end if
    k=k+1
end

inserts unwinding assumption after each loop

inserts unwinding assertion after each loop

The *k*-induction algorithm

k=initial bound
while *true* **do**
    if $base_k$ **then**
        return *trace s[0..k]*
    else if fwd_k
        return *true*
    else if $step_k$ **then**
        return *true*
    end if
    k=k+1
end

inserts unwinding assumption after each loop

inserts unwinding assertion after each loop

havoc variables that occur in the loop's termination condition

The *k*-induction algorithm

k=initial bound
while *true* **do**
  if *base*$_k$ **then**
    return *trace s[0..k]*
  else if *fwd*$_k$
    return *true*
  else if *step*$_k$ **then**
    return *true*
  end if
  k=k+1
end

inserts unwinding assumption after each loop

inserts unwinding assertion after each loop

havoc variables that occur in the loop's termination condition

unable to falsify or prove the property

Loop-free Programs (base$_k$ and fwd$_k$)

- A loop-free program is represented by a **straight-line program** (without loops) using *if*-statements

```
for(B; c; D) { E; }   ⇒   B while(c) { E; D;}
```

```
L1: while(c) {
      E; D;
    }
```

Loop Body

Condition

```
L1: if(!c) goto L2
      E; D;
      goto L1
L2: ASSUME or ASSERT
```

Loop-free Programs ($\mathbf{step_k}$)

In the inductive step, loops are converted into:

the code to remove redundant states

```
while(c) { E; }   ⇒   A while(c) { S; E; U; } R;
```

- **A:** assigns **non-deterministic values** to all loops variables (the state is havocked before the loop)
- **c:** is the **halt condition** of the loop
- **S: stores the current state** of the program variables before executing the statements of E
- **E:** is the actual **code inside the loop**
- **U: updates all program variables** with local values after executing E

Running example

Prove that $S_n = \sum_{i=1}^{n} a = na$ for $n \geq 1$

```
unsigned int nondet_uint();
int main() {
  unsigned int i, n=nondet_uint(), sn=0;
  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assert(sn==n*a);
}
```

Running example: *base case*

Insert an **unwinding assumption** consisting of the termination condition after the loop

– find a counter-example with *k* loop unwindings

```
unsigned int nondet_uint();
int main() {
  unsigned int i, n=nondet_uint(), sn=0;
  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assume(i>n);
  assert(sn==n*a);
}
```

Running example: *forward condition*

Insert an **unwinding assertion** consisting of the termination condition after the loop

- check that *P* holds in all states reachable with k unwindings

```
unsigned int nondet_uint();
int main() {
  unsigned int i, n=nondet_uint(), sn=0;
  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assert(i>n);
  assert(sn==n*a);
}
```

Running example: *inductive step*

Havoc (only) the variables that occur in the loop's termination and branch conditions

```
unsigned int nondet_uint();
typedef struct state {
    unsigned int i, n, sn;
} statet;
int main() {
    unsigned int i, n=nondet_uint(), sn=0, k;
    assume(n>=1);
    statet cs, s[n];
    cs.i=nondet_uint();
    cs.sn=nondet_uint();
    cs.n=n;
```

Running example: *inductive step*

Havoc (only) the variables that occur in the loop's termination and branch conditions

```
unsigned int nondet_uint();
typedef struct state {
    unsigned int i, n, sn;
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    assume(n>=1);
    statet cs, s[n];
    cs.i=nondet_uint();
    cs.sn=nondet_uint();
    cs.n=n;
```

define the type of the program state

Running example: *inductive step*

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```

define the type of the program state

state vector

Running example: *inductive step*

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unsigned int nondet_uint();
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int main() {
   unsigned int i, n=nondet_uint(), sn=0, k;
   assume(n>=1);
   statet cs, s[n];
   cs.i=nondet_uint();
   cs.sn=nondet_uint();
   cs.n=n;
```

define the type of the program state

state vector

explore all possible values implicitly

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
    s[i-1]=cs;
    sn = sn + a;
    cs.i=i;
    cs.sn=sn;
    cs.n=n;
    assume(s[i-1]!=cs);
  }
assume(i>n);
assert(sn ==  n*a);
}
```

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
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}
```

capture the state *cs* before the iteration

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

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for(i=1; i<=n; i++) {
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   cs.sn=sn;
   cs.n=n;
   assume(s[i-1]!=cs);
 }
assume(i>n);
assert(sn ==  n*a);
}
```

capture the state *cs* before the iteration

capture the state *cs* after the iteration

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
    s[i-1]=cs;
    sn = sn + a;
    cs.i=i;
    cs.sn=sn;
    cs.n=n;
    assume(s[i-1]!=cs);
  }
assume(i>n);
assert(sn ==  n*a);
}
```

capture the state *cs* before the iteration

capture the state *cs* after the iteration

constraints are included by means of assumptions

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
   s[i-1]=cs;
   sn = sn + a;
   cs.i=i;
   cs.sn=sn;
   cs.n=n;
   assume(s[i-1]!=cs);
 }
assume(i>n);
assert(sn ==  n*a);
}
```

capture the state *cs* before the iteration

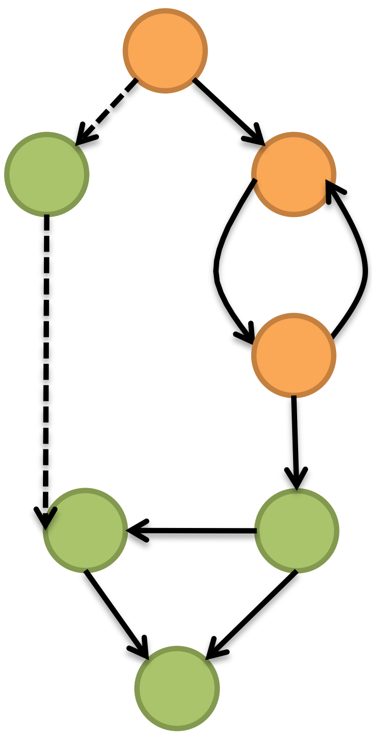
capture the state *cs* after the iteration

constraints are included by means of assumptions

insert unwinding assumption

Program Invariant

- Invariants are properties of program variables and relationships between these variables in a specific line of code (program point)



n = size(b)

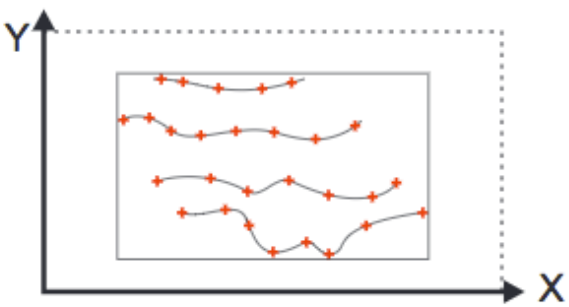
```
i  := 0;
s  := 0;
while i ≠ n{
   s  := s+b[i];
   i  := i+1;
}
```

s = sum(b[0..i-1])

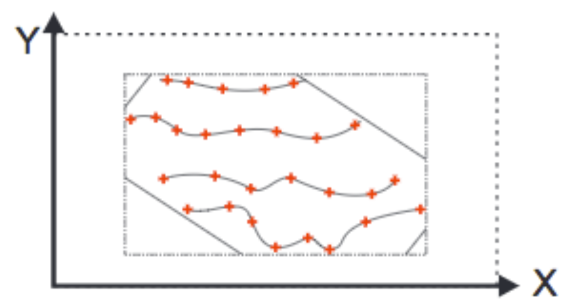
s = sum(b)

Automatic Invariant Generation

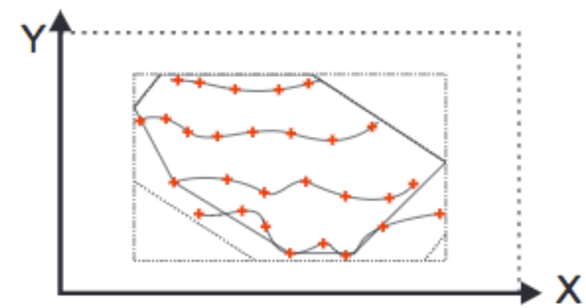
- Infer invariants using **intervals**, **octagons**, and **convex polyhedral** constraints for the inductive step
  - *e.g., $a \le x \le b$*; *$x \le a$, $x\text{-}y \le b$*; and *$ax + by \le c$*



intervals



octagons



convex polyhedral

- Use existing libraries to discover linear/polynomial relations among integer/real variables to infer **loop invariants**
  - compute **pre-** and **post-conditions**

Running Example: Plain BMC

- Plain BMC unrolls this *while*-loop 100 times…

```c
int main() {
  int x=0, t=0, phase=0;
  while(t<100) {
   if(phase==0) x=x+2;
   if(phase==1) x=x-1;
   phase=1-phase;
   t++;
  }
  assert(x<=100);
  return 0;
}
```

```
$esbmc example.c --clang-frontend
ESBMC version 4.2.0 64-bit x86_64 macos
file example.c: Parsing
Converting
Type-checking example
Generating GOTO Program
GOTO program creation time: 0.232s
GOTO program processing time: 0.001s
Starting Bounded Model Checking
Unwinding loop 1 iteration 1 file example.c line 5 function
main
Unwinding loop 1 iteration 2 file example.c line 5 function
main
…
Unwinding loop 1 iteration 100 file example.c line 5 function
main
Symex completed in: 0.340s (313 assignments)
Slicing time: 0.000s
Generated 1 VCC(s), 0 remaining after simplification
VERIFICATION SUCCESSFUL
BMC program time: 0.340s
```

Running Example: *k*-induction + invariants

- Inductive step proves correctness for *k*-step 2…

```
int main() {
  int x=0, t=0, phase=0;
  while(t<100) {
    assume(-2*x+t+3*phase == 0);
    assume(3-2*x+t >= 0);
    assume(-x+2*t >= 0);
    assume(147+x-2*t >= 0);
    assume(2*x-t >= 0);
    if(phase==0) x=x+2;
    if(phase==1) x=x-1;
    phase=1-phase;
    t++;
  }
  assert(x<=100);
  return 0;
}
```

```
$esbmc example.c --clang-frontend --k-induction
*** K-Induction Loop Iteration 2 ***
*** Checking inductive step
Starting Bounded Model Checking
Unwinding loop 1 iteration 1 file example_pagai.c line 6 function main
Unwinding loop 1 iteration 2 file example_pagai.c line 6 function main
Symex completed in: 0.002s (53 assignments)
Slicing time: 0.000s
Generated 1 VCC(s), 1 remaining after simplification
No solver specified; defaulting to Boolector
Encoding remaining VCC(s) using bit-vector arithmetic
Encoding to solver time: 0.001s
Solving with solver Boolector 2.4.0
Encoding to solver time: 0.001s
Runtime decision procedure: 0.144s
VERIFICATION SUCCESSFUL
BMC program time: 0.148s
Solution found by the inductive step (k = 2)
```

inductive invariants

reuse k-induction counterexamples to speed-up bug finding

reuse results of previous steps (caching SMT queries)

Summary

- Described the difference between **soundness** and **completeness** concerning **detection techniques**
  - **False positive** and **false negative**
- Pointed out the difference between **static analysis** and **testing / simulation**
  - **hybrid combination** of static and dynamic analysis techniques to achieve a good trade-off between **soundness** and **completeness**
- Explained **bounded** and **unbounded model checking of software**
  - they have been applied successfully to verify **single- and multi-threaded software**