

Kripke Structures

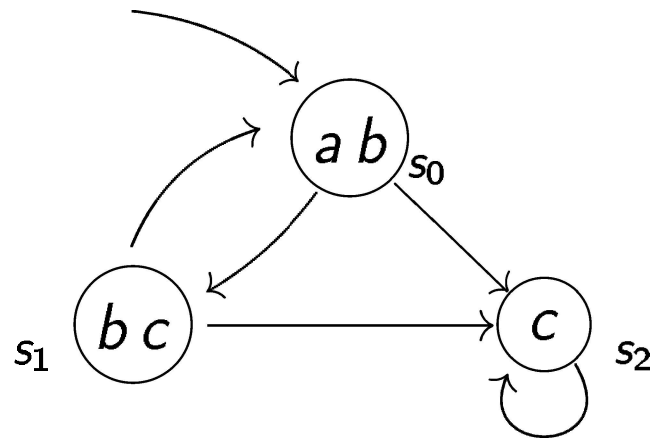
A **Kripke structure** M over a set $Prop$ of *atomic propositions* is given by:

- ▶ a *finite* set S of **states**
- ▶ a subset $S_0 \subseteq S$ of **initial states**
- ▶ a **transition relation** $R \subseteq S \times S$ between states
- ▶ a **valuation** V giving, for each state s , the atomic propositions which are true in that state, $V(s) \subseteq Prop$

Think of S as modelling the states of a system, of R as modelling the computation steps, and of V as describing basic properties of states.

Paths through the Kripke structure then correspond to possible system executions !

Kripke Structures - Example



► set of states: $S = \{ s_0, s_1, s_2 \}$

► set of initial states: $\{ s_0 \}$

► transition relation:

$\{ (s_0, s_1), (s_0, s_2), (s_1, s_0), (s_1, s_2), (s_2, s_2) \}$

$s_0 \longrightarrow s_2 \quad s_2 \not\longrightarrow s_0$

► valuation V gives labelling of states with atomic propositions:

$V(s_0) = \{a, b\}$

$V(s_1) = \{b, c\}$

$V(s_2) = \{c\}$

Example: Mutual Exclusion Protocol

```
bool turn;
P = m : cobegin  $P_0 \parallel P_1$  coend m'
 $P_0 = n_0$  : while True do
     $t_0$  : wait (turn = 0);
     $c_0$  : use resource; turn := 1;
endwhile ;  $n'_0$ 
 $P_1 = n_1$  : while True do
     $t_1$  : wait (turn = 1);
     $c_1$  : use resource; turn := 0;
endwhile ;  $n'_1$ 
```

Note: **wait** (*c*) repeatedly tests *c* until it becomes true.

Extract a Kripke structure which models $P_0 \parallel P_1$:

- ▶ states are given by pairs of states of P_0 and P_1 , together with the value of the shared variable *turn*
- ▶ transitions correspond to execution steps in either P_0 or P_1
- ▶ *Prop* and *V* will depend on the properties we want to verify ...

Mutual Exclusion: the Model

bool *turn*;

$P = m : \text{cobegin } P_0 \parallel P_1 \text{ coend } m'$

$P_0 = n_0 : \text{while True do}$

$t_0 : \text{wait } (\text{turn} = 0);$

$c_0 : \text{use resource; } \text{turn} := 1;$

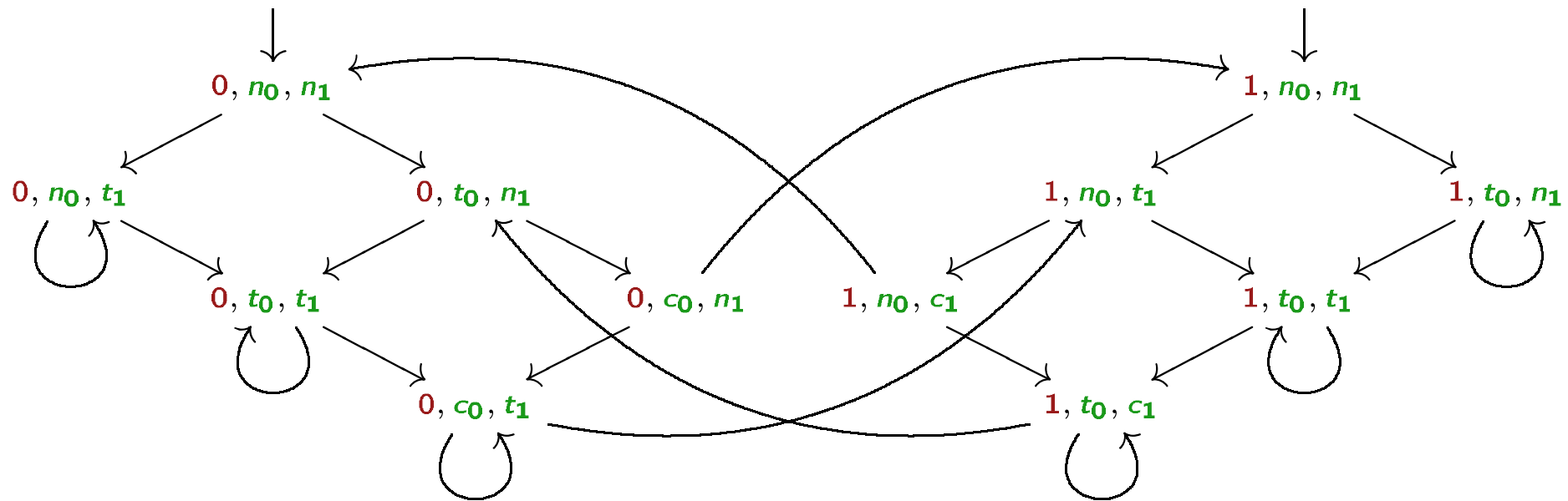
endwhile; n'_0

$P_1 = n_1 : \text{while True do}$

$t_1 : \text{wait } (\text{turn} = 1);$

$c_1 : \text{use resource; } \text{turn} := 0;$

endwhile; n'_1



(More on extracting Kripke structures from concurrent programs in Chapter 2 of Clarke, Grumberg and Peled.)