Trade and Growth with Indirect Additivity*

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Abstract

The workhorse CES model has limited our understandings of how trade liberalization can affect growth, especially via markups and near autarky, which is absent in most literature; the exceptions include Bykadorov et al. (2016) and Mrázová and Neary (2014) based on direct additive utilities. We propose a variety-expanding endogenous growth model with indirect additivity (IA hereafter), firm homogeneity, and symmetric countries. The IA utility used in this paper represents a general class of preferences that can not satisfy the direct additivity in Dixit and Stiglitz (1977) and also nests the CES preference which is the only hinge between IA and direct additivity. This enables us to draw general results irrespective of the specifications and to compare our results with the ones under the CES assumption. Our work follows a strand of papers that have developed IA preferences in many aspects (Boucekkine et al., 2017; Bertoletti and Etro, 2017; Bertoletti et al., 2018) and extends those models by combining the trade model with IA utility and endogenous growth.

Our model exhibits not only the direct cost effect but also the indirect income effect (or the Linder effect) absent in the CES trade model, which underlies the deviations in our results. Also, our VES utility provides a new channel where trade liberalization can affect growth via the average markups. More precisely, our model shows that the reduction in trade cost can foster growth and improve welfare near free trade while can slow down growth and harm welfare near autarky; the simulation further implies that there exists a U-shaped relation between growth or welfare and trade cost. The predictions of our model significantly deviate from the results in models with CES preferences (Baldwin and Robert-Nicoud, 2008; Ourens, 2016; Naito, 2017; Ourens, 2020) and attach importance to the introduction of endogenized variable markups as far as the impact of trade is concerned. In addition, the results of our model echo the recent evidence that trade liberalization can ultimately increase the markups faced by global producers and hence leads to anti-competitive effects.

Keywords: Indirect Additivity, Endogenous Growth, Trade Liberalization, Variable Markups

*This version serves as the writing sample of Jipeng Cheng, who is the main contributor and manuscript writer for this work. The work is still in progress and all errors are Cheng's own.

1 Introduction

A recent strand of papers has investigated how trade liberalization influences growth with heterogenous firm trade model based on CES utility (Baldwin and Robert-Nicoud, 2008; Ourens, 2016; Naito, 2017). However, they have apparently ignored the variation of markups caused by international trade and its impact on growth: a large number of literature has provided empirical evidence for the former (see Levinsohn, 1993; Harrison, 1994; Krishna and Mitra, 1998; Konings et al., 2001; Chen et al., 2009; De Loecker and Warzynski, 2012; Fan et al., 2018; Arkolakis et al., 2019; Hsu et al., 2020) and both empirical and theoretical evidence for the latter (see Aghion et al., 2005, 2009; Goldberg et al., 2010; Peters, 2012; De Loecker et al., 2016). Hence, the workhorse CES model has limited our further comprehension about the channels where trade liberalization can affect growth, especially via markups, which is even absent in the review about the connection between trade and growth of Grossman and Helpman (2015).

We here propose a model with homogenous firms, symmetric countries and indirectly additive preferences allowing us to incorporate endogenous variable markups so that we can tackle the aforementioned problem. According to Bertoletti and Etro (2017), the IA utility that we use in our paper represents a general class of preferences that can not satisfy the direct additivity in Dixit and Stiglitz (1977) and also nests the CES preference which is the only hinge between indirect additivity and direct additivity. These features of IA preferences enable us to draw a general result irrespective of the further specifications for preferences and to compare our results with the ones under CES assumption. Our work follows another strand of papers that have developed IA preferences in many aspects including the basic framework (Bertoletti and Etro, 2017), the closed model with endogenous growth (Boucekkine et al., 2017), and the introduction of asymmetric firms and countries (Bertoletti et al., 2018). Hence, this paper contributes to the further extension of the model with IA preferences by blending the trade model with IA utility and endogenous growth simultaneously.

In the seminal work of Baldwin and Robert-Nicoud (2008, BRN henceforth), they postulate a variety-expanding endogenous growth model with five different specifications of the mechanisms of international knowledge spillovers. Similarly, we also work with a model featuring monopolistic competition, variety-expanding endogenous growth while modify the specifications of spillovers due to the changes of fundamental preferences. Inasmuch as the sunk cost faced by firms are measured in units of knowledge produced in R&D sector, BRN has illustrated that trade liberalization can influence growth via the price of knowledge, i.e. the p_K channel, and via the amounts of knowledge required to invent a new variety, i.e. the κ channel. Besides those two classical channels, however, our model comes up with a new channel where trade liberalization affect growth through the average markups owing to the endogenized variable markups. This new channel together with the altered spillover mechanism distinguishes the predictions for the impact of trade on growth in our model from the ones in BRN model and its derivative models, which highlights the role of markup variability in this kind of estimation. In addition, some of the results are indistinct without further specifications of utility functions, which also emphasizes the importance of choosing a demand structure fitting the reality once again.

Our work also speaks to some state-of-art empirical literature. Inspection of how trade liberalization affects the three kinds of markups in our model—the markups for selling at home or home-sale markups, the markups for selling abroad or export markups, and the average markups across the two business of all firms above—implies that lowering trade cost, or intuitively the reduction in tariffs, can lead to lower home-sale markups, higher export markups and ambiguous average markups. Hence, the result of our work echoes the elusive pro-competitive effect of trade documented by Arkolakis et al. (2019, ACDR henceforth). More specifically, ACDR points out that though trade directly pulls down the markups of domestic producers, it also indirectly raises the markups of foreign producers (the export markups in our model) due to the incomplete pass-through from trade cost to prices, which causes a tension affecting the average markups. The case is also supported by De Loecker et al. (2016, DGKP henceforth) and Fan et al. (2018) in which both papers find that reducing input tariffs can significantly raise the markups based on the evidence from India and China. Moreover, our model also realize the conjecture in DGKP that trade spurs growth by promoting the average markups.

The rest of the paper is structured as follows. In Section 2 we present the fundamental model and the corresponding dynamics; Section 3 examines the growth and welfare effect of trade liberalization by comparative statics along BGP. Section 4 presents simulations with specific utilities including addilog, exponent and CES preferences. Section 5 concludes.

2 Framework

Our model assumes that time is continuous and the world economy is composed by two symmetric countries, Home and Foreign, each endowed with L workers who inelastically supply one unit of labor at every moment in time. Labor can be devoted to the production of final consumption goods or intermediate knowledge goods. Different varieties of the consumption good are produced monopolistically by homogeneous firms in the manufacture sector. The innovation sector produces knowledge enabling the emergence of new consumption varieties over time as in the standard model of endogenous growth with expanding product varieties.

2.1 Consumers

We consider a wider class of IA preferences which are identical across consumers. With the assumption of symmetric firms and countries, the instantaneous indirect utility function shared by all consumers is given by

$$V(t) = \int_0^{N(t)} v\Big(\frac{p_{iD}(t)}{e(t)}\Big)di + \int_0^{N(t)} v\Big(\frac{p_{iX}(t)}{e(t)}\Big)di = N(t)\Big[v\Big(\frac{p_D(t)}{e(t)}\Big) + v\Big(\frac{p_X(t)}{e(t)}\Big)\Big]$$

where N is the mass of varieties in one country, $e = Np_Dx_D + Np_Xx_X$ is the overall individual expenditure with p_D, x_D and p_X, x_X denoting the price and consumed quantity of each domestic variety and imported variety respectively. To make the characterization fit the law of demand, we impose a restriction to the sub-utility function $v(\cdot)$.

Assumption 1 $v(\cdot)$ *is thrice differentiable, decreasing and convex (as will be seen below, the convexity of v is necessary for the demand to be a decreasing function of the price).*

2.1.1 Dynamic Decision

In the long run, consumers would maximize their utility by choosing their optimal expenditure subject to their budget constraint. The intertemporal objective function shared by consumers is given by

$$\mathcal{V}(s_0) = \int_{s_0}^{\infty} e^{-\rho(t-s_0)} \log V(t) \mathrm{d}t \tag{1}$$

where $\rho > 0$ is the rate of pure time preference and s_0 denotes the time when consumers are born, and it is maximized with respect to (w.r.t. hereafter) $e(t)$ under the budget constraint

$$\dot{a}(t) = w(t) + r(t)a(t) - e(t),$$

with a denoting the individual stock of assets, w the wage rate, and r the interest rate.

The resulting Hamiltonian takes the form of

$$\mathcal{H}(t) = \exp\{-\rho(t-s_0)\} \log V(t) + \lambda(t)[w(t) + r(t)a(t) - e(t)],$$

where $\lambda(t)$ is the shadow price of the state variable $a(t)$. The first-order condition (FOC hereafter) w.r.t. the control variable $e(t)$ leads to

$$\frac{\exp\{-\rho(t-s_0)\}}{e(t)} \left[- \frac{v'\Big(\frac{p_D(t)}{e(t)}\Big)\frac{p_D(t)}{e(t)} + v'\Big(\frac{p_X(t)}{e(t)}\Big)\frac{p_X(t)}{e(t)}}{v\Big(\frac{p_D(t)}{e(t)}\Big) + v\Big(\frac{p_X(t)}{e(t)}\Big)} \right] = \lambda(t). \tag{2}$$

By introducing an auxiliary function ν:

$$\nu(t) = -\frac{v\Big(\frac{p_D(t)}{e(t)}\Big) + v\Big(\frac{p_X(t)}{e(t)}\Big)}{v'(\frac{p_D(t)}{e(t)}\Big)\frac{p_D(t)}{e(t)} + v'\Big(\frac{p_X(t)}{e(t)}\Big)\frac{p_X(t)}{e(t)}}, \tag{3}$$

we can rewrite (2) as

$$\lambda(t) = \frac{\exp\{-\rho(t-s_0)}{\}}\nu(t)e(t). \tag{4}$$

The FOC w.r.t. the state variable $a(t)$ leads to

$$r(t) = -\frac{\dot{\lambda}(t)}{\lambda(t)}, \tag{5}$$

and finally the transversality condition reads as

$$\lim_{t\to\infty} \lambda(t)a(t) = 0.$$

Log-differentiating (4) and using (5) yields the following enriched Euler equation as in Boucekkine et al. (2017):

$$\frac{\dot{e}(t)}{e(t)} + \rho + \frac{\dot{\nu(t)}}{\nu(t)} = r(t).$$

2.1.2 Static Decision

Having determined the optimal trajectory of the individual expenditure $e(t)$, we can further confirm the individual demand for a domestic variety x_D and for an imported variety x_X by applying Roy's identity respectively:

$$x_D = -\frac{\frac{\partial V(t)}{\partial p_D(t)}}{\frac{\partial V(t)}{\partial e(t)}} = \frac{v'\left(\frac{p_D(t)}{e(t)}\right)}{\mu(t)}, \quad x_X = -\frac{\frac{\partial V(t)}{\partial p_X(t)}}{\frac{\partial V(t)}{\partial e(t)}} = \frac{v'\left(\frac{p_X(t)}{e(t)}\right)}{\mu(t)}, \tag{6}$$

where $\mu(t)$ is the marginal utility of income and defined by

$$\mu(t) = -e(t)\frac{\partial V(t)}{\partial e(t)} = N(t)\Big[\frac{p_D(t)}{e(t)}v'(\frac{p_D(t)}{e(t)}) + \frac{p_X(t)}{e(t)}v'(\frac{p_X(t)}{e(t)})\Big]. \tag{7}$$

The market demand for a domestic variety q_D or for a imported variety q_X is in the form of

$$q_D(t) = x_D(t)L, \quad q_X(t) = x_X(t)L.$$

Also notice that we can rewrite (3) with (7) as

$$\nu(t) = -\frac{N(t)[v(\frac{p_D(t)}{e(t)}) + v(\frac{p_X(t)}{e(t)})]}{\mu(t)} = -\frac{V(t)}{\mu(t)} = -\frac{\mathrm{d}\ln e(t)}{\mathrm{d}\ln V(t)}. \tag{8}$$

Holding $v(\frac{p_D(t)}{e(t)})+v(\frac{p_X(t)}{e(t)})$ constant, we can express the elasticity of individual expenditure w.r.t. the mass of varieties with (8) as

$$\varepsilon_{e,N} \equiv \frac{\mathrm{d}\ln e(t)}{\mathrm{d}\ln N(t)} = \frac{\mathrm{d}\ln e(t)}{\mathrm{d}\ln N(t) + \mathrm{d}\ln\left[v\left(\frac{p_D(t)}{e(t)}\right) + v\left(\frac{p_X(t)}{e(t)}\right)\right]} = -\nu(t) < 0.$$

$\nu(t)$ measures the extent of the consumers' love for variety,[1] because the consumers faced with a decrease in $N(t)$ by 1% would increase their total expenditure $e(t)$ by $\nu(t)$% to keep the utility obtained from one pair of varieties (including one domestic and one imported variety) constant.

2.2 Production

2.2.1 Manufacture Sector

Potential entrants at time t to the manufacture factor must incur in a fixed cost or sunk cost $F(t)$ to start up their production and sales. Moreover, once in production firms no longer pay fixed cost and continue to produce until they receives a negative shock that pushes them out of the business. The exogenous rate at which firms are hit by this shock is denoted by $\delta > 0$.

There are no cost for a firm to differentiate the variety they produce from those of other firms. This, together with the fact that all varieties enter the demand functions symmetrically by (6) provides incentives for every firm to produce a distinct variety of

[1] See Boucekkine et al. (2017) for more details about love for variety and $\nu(t)$'s role in enriched Euler equation.

the final good. Technology in the manufacture sector for each firm is represented by the following linear cost function

$$y(t) = l(t), \tag{9}$$

with labor being the sole input and a total of $L_E(t) = N(t)l(t)$ units of labor being devoted to the final goods production. Recall that we assume firm homogeneity in the model and therefore all the firms are exporters facing an exogenous iceberg cost $\tau \geq 1$ to sell abroad, which means that a manufacturing producer needs to send τ units of the good for one unit to reach the final destination. Thus, the marginal cost of each firm for selling domestically and selling abroad are

$$m_D(t) = w(t), \quad m_X(t) = \tau w(t).$$

Each firm has its domestic operating profits π_D and export operating profits π_X as

$$\pi_D = [p_D(t) - m_D(t)]q_D(t) = \frac{p_D(t) - w(t)}{\mu(t)} v'\Big(\frac{p_D(t)}{e(t)}\Big)L, \tag{10}$$

$$\pi_X = [p_X(t) - m_X(t)]q_X(t) = \frac{p_X(t) - \tau w(t)}{\mu(t)} v'\Big(\frac{p_X(t)}{e(t)}\Big)L. \tag{11}$$

Notice that monopolistic competition in the manufacture sector implies that μ is neutral with the price changes of a single variety and thus treaded as a constant in the following profit maximization problem.

Let $\xi_D = p_D/e$ and $\xi_X = p_X/e$. We define the price elasticity and the convexity of demand for one variety respectively by

$$\varepsilon(\xi) = -\frac{\xi v''(\xi)}{v'(\xi)} \text{ and } \zeta(\xi) = -\frac{\xi v'''(\xi)}{v''(\xi)}, \tag{12}$$

where ξ can be either ξ_D or ξ_X corresponding to ε_D and ζ_D for domestic goods or ε_X and ζ_X for imported goods, respectively. In the following, we require two extra conditions to guarantee the existence and uniqueness of a solution to the firm problem as in Bertoletti and Etro (2017) and Boucekkine et al. (2017).

Assumption 2

$$\varepsilon(\xi) > \max\{1, \zeta(\xi) - 1\}, \quad \forall \xi \geq 0.$$

Assumption 2 directly leads to two properties. On the one hand,

$$2\varepsilon(\xi) > \zeta(\xi), \quad \forall \xi \geq 0 \tag{13}$$

holds according to the summation of two components in the RHS. On the other hand, $\varepsilon(\cdot)$ is increasing according to

$$\varepsilon'(\xi) = [\varepsilon(\xi) - (\zeta(\xi) - 1)]\frac{v''((\xi)}{-v'(\xi)} > 0 \tag{14}$$

by Assumptions 1 and 2.

Each firm in Home maximizes its profits (10) and (11), giving the FOC w.r.t. p_D and p_X separately:

$$v'\Big(\frac{p_D(t)}{e(t)}\Big) + \frac{p_D(t) - w(t)}{e(t)} v''\Big(\frac{p_D(t)}{e(t)}\Big) = 0,$$

$$v'\Big(\frac{p_X(t)}{e(t)}\Big)+\frac{p_X(t)-\tau w(t)}{e(t)}v''\Big(\frac{p_X(t)}{e(t)}\Big)=0.$$

The two FOCs immediately lead to the two corresponding Lerner rules in the form of

$$\frac{p_D(t)-w(t)}{p_D(t)}=-\frac{v'\left(\frac{p_D(t)}{e(t)}\right)}{v''\left(\frac{p_D(t)}{e(t)}\right)\frac{p_D(t)}{e(t)}}=\frac{1}{\varepsilon\left(\frac{p_D(t)}{e(t)}\right)},\tag{15}$$

$$\frac{p_X(t)-\tau w(t)}{p_X(t)}=-\frac{v'\left(\frac{p_X(t)}{e(t)}\right)}{v''\left(\frac{p_X(t)}{e(t)}\right)\frac{p_X(t)}{e(t)}}=\frac{1}{\varepsilon\left(\frac{p_X(t)}{e(t)}\right)}.\tag{16}$$

Namely, $1/\varepsilon_D$ and $1/\varepsilon_X$ are the markups in the domestic and foreign markets, respectively. Meanwhile, (15) and (16) imply

$$p_X \geq p_D \text{ and } \varepsilon_X \geq \varepsilon_D \tag{17}$$

due to the increasing property of $\varepsilon(\cdot)$.

To keep in track of the indirect approach of BRN that connecting profits with expenditure, mass of varieties, and the price elasticity of demand, we can rearrange (10) and (11) with (15) and (16) as

$$\pi_D(t)=(1-\varpi(t))\frac{e(t)L}{N(t)\varepsilon_D(t)},\tag{18}$$

$$\pi_X(t)=\varpi(t)\frac{e(t)L}{N(t)\varepsilon_X(t)}\tag{19}$$

where we denote the ratio of expenditure on imported goods to total expenditure, i.e. the export share of GDP, by

$$\varpi(t)\equiv\frac{N(t)p_X(t)q_X(t)}{N(t)[p_D(t)q_D(t)+p_X(t)q_X(t)]}.$$

2.2.2 R&D (Innovation) Sector

One potential entrant for the manufacture sector has to buy one unit of blueprint from the R&D sector so that it can create its unique variety and sells domestically and abroad. Noticing that the blueprints are de facto the fixed cost sunk by firms, F, we can then model the cost for firms to buy blueprints of production process at t as

$$F(t)=P_K(t)\kappa,$$

where κ represents the amounts of knowledge required to reach a new blueprint and P_K is the price of knowledge. Free-entry conditions in our model are such that the cost of buying a blue print equals the accumulated total profits of a firm and are thus given by

$$F(t)=\frac{\pi_D(t)+\pi_X(t)}{\gamma(t)}$$

where $\gamma(t)$ is the discount rate of firms at the current moment given by

$$\gamma(t)=\frac{\pi(t)}{\displaystyle\int_t^\infty e^{-(\rho+\delta)(s-t)}\pi(s)\mathrm{d}s}.\tag{20}$$

Reusing (18) and (19) together with the previous equations we can get

$$\gamma(t)P_K(t)\kappa = \pi(t) = \pi_D(t) + \pi_X(t) = \frac{e(t)L}{N(t)\bar{\varepsilon}(t)}, \tag{21}$$

where $\pi(t)$ is the total profit gained by a firm at time t, and

$$\bar{\varepsilon} = [(1-\varpi)\varepsilon_D^{-1} + \varpi\varepsilon_X^{-1}]^{-1} \tag{22}$$

is the harmonic average of demand elasticities weighted by the market shares as in Bertoletti and Etro (2017). Therefore the discount rate $\gamma(t)$ of (20) can be written as

$$\gamma(t) = \frac{\dfrac{e(t)L}{N(t)\bar{\varepsilon}(t)}}{\displaystyle\int_t^{\infty} e^{-(\rho+\delta)(s-t)}\frac{e(s)L}{N(s)\bar{\varepsilon}(s)}\mathrm{d}s}.$$

We denote by $L_K(t)$ the amount of labor devoted to the production of knowledge, the process of which follows

$$Q_K(t) = \frac{L_K(t)w(t)}{c(\cdot)},$$

where Q_K is the amounts of created knowledge and $c(\cdot) = c[w(t), N(t)]$ is the marginal cost of this activity. The marginal cost of innovating is determined by labor price $w(t)$ and the amount of existing blueprints in the economy $N(t)$. We choose labor as the numeraire, then $w = 1$ holds in our model. The only assumption imposed on $c(\cdot)$ is to be homogeneous of degree minus one in N, i.e. $c[w,1] = N(t)c[w, N(t)]$, which captures the knowledge spillovers within the economy. We then assume perfect competition in the market of knowledge, which sets the price of knowledge equal to the corresponding marginal cost, i.e. $P_K(t) = c(\cdot)$. Hence, the property of $c(\cdot)$ allows us to express the price of knowledge in intensive terms $NP_K = c(w,1) = p_K$ for later use. Meanwhile, new varieties enter the economy following

$$\dot{N}(t) = \frac{Q_K(t)}{\kappa(t)} - \delta N(t), \tag{23}$$

Dividing (23) by $N(t)$ yields

$$\frac{\dot{N}(t)}{N(t)} = \frac{L_K(t)w}{p_K\kappa(t)} - \delta.$$

Namely, the rate at which the mass of firms grows over time equals the creation of new varieties net of firms' destruction by the exogenous shock.

2.2.3 Market Clearing

This section investigates the equilibrium conditions of labor market, product market and financial market. The total labor L in each country is an exogenous variable in our model. It is allocated into the manufacture sector and the R&D sector, giving the following labor market clearing condition:

$$L = L_E(t) + L_K(t).$$

In the equilibrium in the product market, each firm precisely supplies its demand, yielding

$$y(t) = q_D(t) + \tau q_X(t) = [x_D(t) + \tau x_X(t)]L. \tag{24}$$

Combining (24) with the linear cost function for firms (9) and the expression of the labor being devoted to the final goods' consumption, we get the product market clearing condition:

$$\frac{L_E(t)}{N(t)} = [x_D(t) + \tau x_X(t)]L.$$

Also, according to the definition of aggregate operating profits, we have

$$\Pi(t) = R(t) - wL_E(t) \tag{25}$$

with $\Pi(t) \equiv N(t)\pi(t)$ and $R(t)$ denoting the total revenue of firms in each country. Because the total expenditure on final goods of consumers equals the total revenue of firms, together with (25) and the assumption of perfect financial market, i.e. $\Pi(t) = A(t)r(t)$ with $A(t) = a(t)L$, we reach

$$E(t) = R(t) = wL_E(t) + \Pi(t), \tag{26}$$

with $E(t) \equiv e(t)L$. Using (21) and (26), we find the familiar formula appearing in BRN and Ourens (2016):

$$[1 - 1/\bar{\varepsilon}(t)]E(t) = w[L - L_K(t)], \tag{27}$$

which implies that not only the aggregate expenditure in final good but also the variable average price-elasticity of demand is linked to the allocation of labor resources in our model.

2.3 General Equilibrium

2.3.1 Specifications of p_K

Closing our model requires further specifications for p_K. BRN proposes five different specifications for p_K reflecting the diverse ways in which externalities in the R&D process could be introduced. But we are limited to consider only three of the five specifications in the following sections; literally, the specifications of knowledge spillovers in light of efficiency-linked and reverse engineering are excluded due to the firm homogeneity in our model.

The baseline specification is the well-known Grossman-Helpman product-innovation model with knowledge spillovers (Grossman and Helpman, 1991) where productivity in the creation of knowledge increases with knowledge accumulation, i.e. the increasing varieties (N) in our model. With $N = N^*$ where N^* is the mass of the varieties produced abroad, P_K and p_K are thus given by

$$[P_K]^{GH} = \frac{1}{N + N^*\psi} = \frac{1}{N + N\psi}; \quad [p_K]^{GH} = \frac{1}{1+\psi}, \tag{28}$$

where $\psi \in [0, 1]$ is the exogenous international knowledge spillover coefficient.

Coe and Helpman (1995) and Fracasso and Marzetti (2015) have provided evidence confirming the positive connection between trade flows and knowledge spillovers, which inspires us to set ψ as the import share of GDP ϖ and to derive p_K as

$$[p_K]^{CH} = \frac{1}{1+\varpi}.$$

Finally, a composition of the final goods can be regarded as the inputs in R&D sectors according to the lab-equipment model in Rivera-Batiz and Romer (1991). To measure the price of such a composition, we come up with a price index where the prices of domestic and imported goods are weighted by the corresponding expenditure share and set it as p_K:

$$[p_K]^{LE} = \varpi\xi_X + (1-\varpi)\xi_D.$$

2.3.2 Equilibrium Conditions

We define the normalized prices of domestic goods and imported goods as $\xi_D = p_D/e$ and $\xi_X = p_X/e$. With algebraic manipulation, the main equilibrium conditions are summarized as follows:

$$r(t) = \frac{\dot{e}(t)}{e(t)} + \rho + \frac{\dot{\nu(t)}}{\nu(t)}, \tag{29}$$

$$e(t) = \frac{1}{\xi_D(t)} \frac{w}{1 - \dfrac{1}{\varepsilon(\xi_D(t))}}, \tag{30}$$

$$e(t) = \frac{1}{\xi_X(t)} \frac{\tau w}{1 - \dfrac{1}{\varepsilon(\xi_X(t))}}, \tag{31}$$

$$\pi(t) = \frac{e(t)L}{N(t)\bar{\varepsilon}(t)}, \tag{32}$$

$$\pi(t) = \gamma(t)\frac{p_K\kappa(t)}{N(t)}, \tag{33}$$

$$e(t)L = N(t)\pi(t) + wL_E(t), \tag{34}$$

$$\frac{1}{\bar{\varepsilon}(t)} = (1-\varpi(t))\frac{1}{\varepsilon(\xi_D(t))} + \varpi(t)\frac{1}{\varepsilon(\xi_X(t))}, \tag{35}$$

$$\frac{\dot{N}(t)}{N(t)} = \frac{wL_K(t)}{p_K\kappa} - \delta, \tag{36}$$

$$L = L_K(t) + L_E(t), \tag{37}$$

$$\frac{L_E(t)}{N(t)L} = \frac{v'(\xi_D(t)) + \tau v'(\xi_X(t))}{\mu(t)}, \tag{38}$$

$$\nu(t) = -\frac{v(\xi_D(t)) + v(\xi_X(t))}{v'(\xi_D(t))\xi_D(t) + v'(\xi_X(t))\xi_X(t)}, \tag{39}$$

$$\varpi(t) = \frac{\xi_X(t)v'(\xi_X(t))}{\xi_D(t)v'(\xi_D(t)) + \xi_X(t)v'(\xi_X(t))}, \tag{40}$$

$$\mu(t) = N(t)[\xi_D(t)v'(\xi_D(t)) + \xi_X(t)v'(\xi_X(t))], \tag{41}$$

$$p_K(t) = \text{Three kinds of specifications.} \tag{42}$$

The equilibrium constitutes a system of 14 equations in 14 endogenous variables:

$$r(t), \xi_D(t), \xi_X(t), \pi(t), \gamma(t), e(t), \bar{\varepsilon}(t), N(t), L_K(t), L_E(t), \nu(t), \varpi(t), \mu(t), p_K(t).$$

Additionally, the model features 5 exogenous variables: the amounts of knowledge required for production κ, the rate of pure time preference ρ, the labor amount L, the export iceberg cost τ, and the negative shock to firms δ.

2.4 Dynamics and BGP

A balanced growth path (BGP hereafter) is defined as an equilibrium path along which every variable grows at a constant rate, either null or positive. We start from setting ξ_D (or ξ_X, the choice won't alter our result) as the state variable, which implies $\dot{\xi}_D = 0$ at BGP. Rearranging (30) with (31) yields the equation directly connecting ξ_D and ξ_X:

$$\xi_X\Big(1 - \frac{1}{\varepsilon(\xi_X)}\Big) - \tau\xi_D\Big(1 - \frac{1}{\varepsilon(\xi_D)}\Big) = 0, \tag{43}$$

which can be regarded as an implicit function $F(\xi_D, \xi_X) = 0$ to pin down ξ_X. The existence and uniqueness of this ξ_X is ensured by

$$F_{\xi_X} = 1 - \frac{1}{\varepsilon(\xi_X)} + \xi_X \frac{\varepsilon'(\xi_X)}{\varepsilon(\xi_X)^2} = 2 - \frac{\zeta_X}{\varepsilon_X} \neq 0,$$

where the equality is from (12) and the inequality is from Assumption 2. Therefore, constant ξ_D at BGP immediately makes ξ_X also constant at BGP.

Next, equations (30) and (39) imply that e and ν are also constant at BGP. Furthermore, equation (29) together with the facts of $\dot{e} = 0$ and $\dot{\nu} = 0$ gives $r = \rho$, which means that r keeps invariant with time at BGP. Also, using (22), (32)–(38), and (40)–(42) yields

$$\begin{aligned} \frac{1}{\bar{\varepsilon}}\Big[1 - \frac{\rho(\bar{\varepsilon}-1)p_K\kappa}{wL}\Big] &= 1 - \frac{v'(\xi_D) + \tau v'(\xi_X)}{\xi_D v'(\xi_D) + \xi_X v'(\xi_X)}, \\ \frac{1}{\bar{\varepsilon}} &= -\frac{(v'_D)^2}{(\xi_D v'_D + \xi_X v'_X)v''_D} - \frac{(v'_X)^2}{(\xi_D v'_D + \xi_X v'_X)v''_X}, \end{aligned} \tag{44}$$

where p_K equals to three different specifications related to ξ_D, ξ_X and ϖ. (44) defines the dynamics for the rest of our model. With (36) and (44) we can easily confirm that N grows at a constant rate over time, namely that $g = \dot{N}/N$ is constant at BGP.

The previous result illustrates that the BGP of this economy is characterized by a constant flow of varieties into the economy following

$$N(s) = N(t)\exp\{g(s-t)\},$$

which enables us to pin down γ at BGP. We furthermore normalize the mass of varieties at the beginning to 1, i.e., $N(s_0) = 1$. Then we have

$$N(t) = e^{g(t-s_0)}. \tag{45}$$

The value of a firm at time t is given by

$$V_f(t) = \int_t^\infty \exp\{-(\rho+\delta)(s-t)\}\pi(s)\mathrm{d}s$$

$$
\begin{aligned}
&= \int_t^\infty \exp\{-(\rho+\delta)(s-t)\}\frac{E(s)}{N(s)\bar{\varepsilon}(s)}\mathrm{d}s \\
&= \frac{E(t)}{\bar{\varepsilon}(t)}\int_t^\infty \exp\{-(\rho+\delta)(s-t)\}\frac{1}{N(t)\exp\{g(s-t)\}}\mathrm{d}s \\
&= \frac{E(t)}{\bar{\varepsilon}(t)N(t)}\int_t^\infty \exp\{-(\rho+\delta+g)(s-t)\}\mathrm{d}s = \frac{\pi(t)}{\rho+\delta+g},
\end{aligned}
$$

which shows that $\gamma(t)$ is constant at BGP:

$$
\gamma = \rho + \delta + g. \tag{46}
$$

Furthermore, notice that the continuous entrance of new firms into the economy pushes the marginal utility of income up at a constant rate according to the fact that log-differentiating (41) w.r.t. t at BGP yields

$$
g_\mu = \frac{\dot{\mu}}{\mu} = \frac{\dot{N}}{N} = g. \tag{47}
$$

The intuition behind the result is that the consumers can enjoy more varieties and can gain more utility given expenditure at BGP, which improves the marginal utility of income and re-highlights the "love for variety" featuring in our setting of preferences. This leads to another common conclusion under IA preferences that increasing varieties bring about a drop in the consumption for each variety while keep the overall consumption constant. Log-differentiating (6) w.r.t. t at BGP gives the rates at which the consumption for domestic goods and imported goods evolves with time:

$$
g_{x_D} = \frac{\dot{x}_D}{x_D} = -\frac{\dot{\mu}}{\mu} = -g_\mu \text{ and } g_{x_X} = \frac{\dot{x}_X}{x_X} = -\frac{\dot{\mu}}{\mu} = -g_\mu. \tag{48}
$$

On the other hand, the BGP in this economy imposes a constant level of individual overall expenditure, e, as well as any other untouched endogenous variable.

Finally, we reformulate the two variables of interest, i.e. the growth rate g and the total individual utility over time $\mathcal{V}$, to facilitate the analysis of the comparative statics along the BGP in our model. Inserting (46) into (32) and (33) yields

$$
\frac{E}{\bar{\varepsilon}} = p_K\kappa(g+\rho+\delta). \tag{49}
$$

Merging (27) and (36) gives

$$
E(1-\frac{1}{\bar{\varepsilon}}) = wL - (g+\delta)p_K\kappa. \tag{50}
$$

Joining (49) and (50) gets

$$
E = wL + \rho p_K\kappa. \tag{51}
$$

Plugging (51) back into (49) delivers the familiar expression (in the CES model) for the growth rate at BGP:

$$
g = \frac{wL}{\bar{\varepsilon}p_K\kappa} - \frac{\rho(\bar{\varepsilon}-1)}{\bar{\varepsilon}} - \delta. \tag{52}
$$

Using (1), (45), and (52), we derive the individual welfare in the BGP as

$$\begin{aligned}\mathcal{V}(t) =& \frac{1}{\rho}\Big\{ \log(v_D + v_X) + \frac{g}{\rho}\Big\} \\ =& \frac{1}{\rho}\Big\{ \log(v_D + v_X) + \frac{1}{\rho}\Big[\frac{wL}{\bar{\varepsilon} p_K \kappa} - \frac{\rho(\bar{\varepsilon}-1)}{\bar{\varepsilon}} - \delta\Big]\Big\},\end{aligned} \tag{53}$$

where $v_D = v(\xi_D)$ and $v_X = v(\xi_X)$.

3 Growth Effect of Openness

Following BRN and Ourens (2016), we check the impact of trade on growth by comparing results in equilibrium before and after the shock and therefore explicitly ignoring the adjustments that each economy needs to undertake in order to achieve the new equilibrium. We here list the expressions for $p_K\kappa$ and g in each version of models as below for the convenience of further analysis[2]:

$$[p_K\kappa]^{GH} = \frac{\kappa}{1+\psi}, \quad g^{GH} = \frac{wL(1+\psi)}{\bar{\varepsilon}\kappa} - \frac{\rho(\bar{\varepsilon}-1)}{\bar{\varepsilon}} - \delta; \tag{54}$$

$$[p_K\kappa]^{CH} = \frac{\kappa}{1+\varpi}, \quad g^{CH} = \frac{wL(1+\varpi)}{\bar{\varepsilon}\kappa} - \frac{\rho(\bar{\varepsilon}-1)}{\bar{\varepsilon}} - \delta; \tag{55}$$

$$\begin{aligned}[p_K\kappa]^{LE} &= (\varpi\xi_X + (1-\varpi)\xi_D)\kappa, \\ g^{LE} &= \frac{wL}{(\varpi\xi_X + (1-\varpi)\xi_D)\kappa} - \frac{\rho(\bar{\varepsilon}-1)}{\bar{\varepsilon}} - \delta.\end{aligned}$$

To keep our work comparable to the literature, where trade liberalization is usually modeled in 3 ways (lowering trade cost, τ, increasing spillovers, ψ, and lowering technical trade barriers which is related to κ), we will not normalize κ and ψ that are in fact of little importance in our model; however, much of the focus next would be on τ. Also, We divide the process of trade liberalization into radical approach, which means the economy opens up from autarky directly to free trade, and progressive approach including 3 stages (near-autarky, transition and near-free-trade).

In addition, to discuss the extreme cases at/near free trade and autarky, we have to rely on IA preferences enabling autarky in the following work; however, it is worth noting that the following analysis for the general case and the case at/near free trade, can also apply to the IA preferences without permitting autarky. Here, we focus on the IA preferences featuring a choke price, which requires a threshold trade cost to make zero imports. One case is the addilog function $v(\xi) = (b-\xi)^{1+\iota}/(1+\iota)$ with choke price $b > 0$ and $\iota > 0$, and the simulation part would be based on the addilog specification.

3.1 Radical Opening-up: From Autarky to Free Trade

In our model, autarky means that there exists a $\tau = \tau^{aut}$ such that $\varpi = 0$, while perfect integrations means $\tau = \tau^{fre} = 1$ and $\varpi = 1/2$. At free trade, (30) together with (31) yields $\xi_D^{fre} = \xi_X^{fre}$ and thus (35) yields $\varepsilon_D^{fre} = \varepsilon_X^{fre} = \bar{\varepsilon}^{fre}$. At autarky, it is immediate that $\bar{\varepsilon}^{aut} = \varepsilon_D^{aut}$ due to (35).

[2]For expositional convenience, we do not examine the Lab-equipment model analytically. Numerical examples in Section 4 show that results are similar.

According to the related CES model, trade can affect growth via p_K channel and κ channel; though only p_K channel would take effect in our model due to firm homogeneity, $p_K\kappa$ is still the pivot in the following analysis. The values of $p_K\kappa$ at extremes are summarized in Table 1.

Table 1: Values of $p_K\kappa$ for each specification

Specifications	Autarky	Free Trade
Grossman-Helpman	$\frac{\kappa}{1+\psi}$	$\frac{\kappa}{1+\psi}$
Coe-Helpman	κ	$\frac{2\kappa}{3}$

However, inspection of (52) apparently shows that our model presents a new $\bar{\varepsilon}$ channel stemming from IA preferences. Notice that, unlike $p_K\kappa$, $\bar{\varepsilon}$ cannot be given clearly without the specification of a utility function; but it is still possible to compare $\bar{\varepsilon}^{aut}$ and $\bar{\varepsilon}^{fre}$. Using Appendix A and Appendix B we have that a decrease in τ (from τ^{aut} to τ^{fre}) leads to a decrease in xi_D and thus a decrease in ε_D. Then we have that

$$\bar{\varepsilon}^{aut} = \bar{\varepsilon}_D^{aut} > \bar{\varepsilon}_D^{fre} = \bar{\varepsilon}^{fre}$$

in the Grossman-Helpman model and the Coe-Helpman model.

The growth rates at extremes for the two specifications are provided in Table 2. It is clear that the $\bar{\varepsilon}$ channel is always pro-growth because the higher average markups at free trade can boost firm entry. Moreover, the pro-growth (or neutral) p_K channel together with the $\bar{\varepsilon}$ channel generates pro-growth radical trade liberalization with certainty, which is a result different from the related CES model due to the absent κ channel and the extra $\bar{\varepsilon}$ channel in our model.

Table 2: Growth effect of radical trade liberalization

Specifications	g at Autarky $\propto$	g at Free Trade $\propto$	Growth Effect
Grossman-Helpman	$\frac{1}{\varepsilon_D^{aut}}(\frac{(1+\psi)wL}{\kappa} + \rho)$	$\frac{1}{\varepsilon_D^{fre}}(\frac{(1+\psi)wL}{\kappa} + \rho)$	pro-growth
Coe-Helpman	$\frac{1}{\varepsilon_D^{aut}}(\frac{wL}{\kappa} + \rho)$	$\frac{1}{\varepsilon_D^{fre}}(\frac{3wL}{2\kappa} + \rho)$	vague

3.2 Progressive Opening-up

3.2.1 General Case: Transition Period

Let us start with the most general case: how the variables of interest respond to trade shocks when the economy is still on the way from autarky to free trade. Putting this at first also lays the foundation for our further analysis near extremes.

Prices, quantities, and income. As is the case with Bertoletti and Etro (2017), the Linder effect also appears in our model. Using hat algebra, i.e. defining $\hat{x} = \mathrm{d}\ln x$, we can rewrite (see Appendix B) the FOCs with the conclusion of Appendix A as

$$\hat{\xi}_X = \frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}(\hat{\tau} - \hat{E}) \text{ and } \hat{\xi}_D = -\frac{\varepsilon_D - 1}{2\varepsilon_D - \zeta_D}\hat{E}. \tag{56}$$

Note that a change in τ has a direct effect on the economy and an indirect effect via income E. Equivalently, we have

$$\hat{p}_X = \frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}\hat{\tau} + \frac{\varepsilon_X + 1 - \zeta_X}{2\varepsilon_X - \zeta_X}\hat{E} \text{ and } \hat{p}_D = \frac{\varepsilon_D + 1 - \zeta_D}{2\varepsilon_D - \zeta_D}\hat{E}, \tag{57}$$

which clearly shows that the prices for both domestic and imported goods would increase in the marginal production cost and the income of the consumers. In fact, (56) can further give

$$\mathrm{d}\ln\frac{p_X}{p_D} = \hat{\xi}_X - \hat{\xi}_D = \Big(\frac{\varepsilon_D - 1}{2\varepsilon_D - \zeta_D} - \frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}\Big)\hat{E} + \frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}\hat{\tau}, \tag{58}$$

showing how p_X changes relatively to p_D. Using the fact that $\widehat{v'} = -\varepsilon\hat{\xi}$ from (12) and (56), we get

$$\mathrm{d}\ln\frac{q_X}{q_D} = \varepsilon_D\hat{\xi}_D - \varepsilon_X\hat{\xi}_X = \Big[\frac{\varepsilon_X(\varepsilon_X - 1)}{2\varepsilon_X - \zeta_X} - \frac{\varepsilon_D(\varepsilon_D - 1)}{2\varepsilon_D - \zeta_D}\Big]\hat{E} - \frac{\varepsilon_X(\varepsilon_X - 1)}{2\varepsilon_X - \zeta_X}\hat{\tau}, \tag{59}$$

showing how consumer demands respond to the changing τ and E.

Meanwhile, the income of consumers is directly affected by trade liberalization in our model because log-differentiating (51) leads to

$$\hat{E} = \frac{\rho p_K\kappa}{E}\widehat{p_K\kappa}, \tag{60}$$

Equation (60) highlights that in our model trade liberalization would take effect by affecting both the cost of producers and the income of consumers, which differs many of our conclusions away from those in traditional trade models.

In addition, equation (57) leads to

$$\frac{\partial\ln p_X}{\partial\ln\tau} = \frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X} < 1,$$

which embodies the feature of "incomplete pass-thorough" in our model. More specifically, a decrease in τ amounted to a decrease in m_X results in a less-proportional decrease in p_X, as recorded in De Loecker and Warzynski (2012) and DGKP.

The export share of GDP. We will move to ϖ since it plays an important role in the $\bar{\varepsilon}$ channel and the p_K channel. With algebraic reformulation, log-differentiating (40) reaches

$$\hat{\varpi} = (1 - \varpi)[(\hat{p}_X - \hat{p}_D) + (\hat{q}_X - \hat{q}_D)], \tag{61}$$

which straight demonstrates that the export share of GDP would increase with price and consumption of imported goods while decrease with price and consumption of domestic goods. Combined with the fact that $\widehat{v'} = -\varepsilon\hat{\xi}$ (61) can be rewritten as

$$\begin{aligned}\hat{\varpi} =& (1 - \varpi)(\hat{\xi}_X - \hat{\xi}_D + \widehat{v'_X} - \widehat{v'_D}) = (1 - \varpi)[(\varepsilon_D - 1)\hat{\xi}_D + (1 - \varepsilon_X)\hat{\xi}_X] \\ =& (1 - \varpi)\Big\{ - \frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X}\hat{\tau} + \Big[\frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X} - \frac{(\varepsilon_D - 1)^2}{2\varepsilon_D - \zeta_D}\Big]\hat{E}\Big\} \qquad (62) \\ =& (1 - \varpi)\Big\{ - \frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X}\hat{\tau} + \Big[\frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X} - \frac{(\varepsilon_D - 1)^2}{2\varepsilon_D - \zeta_D}\Big]\frac{\rho p_K\kappa}{E}\widehat{p_K\kappa}\Big\}, \qquad (63)\end{aligned}$$

where (62) is from (56), and (63) is from (60).

Let

$$\mathcal{A} = \frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X}, \quad \mathcal{B} = \frac{(\varepsilon_D - 1)^2}{2\varepsilon_D - \zeta_D}, \quad \mathcal{C} = \frac{\rho p_K \kappa}{E} = \frac{\rho}{\bar{\varepsilon}(\rho + g + \delta)}.$$

All of them are positive according to (13). Then equation (63) can be simplified into

$$\hat{\varpi} = (1 - \varpi)\Big[- \mathcal{A}\hat{\tau} + (\mathcal{A} - \mathcal{B})\mathcal{C}\widehat{p_K\kappa}\Big].$$

Notice that we have

$$[\widehat{p_K\kappa}]^{GH} = 0 \tag{64}$$

from (54) and

$$[\widehat{p_K\kappa}]^{CH} = \frac{(1-\varpi)\varpi\mathcal{A}}{1 + \varpi + (1-\varpi)\varpi(\mathcal{A} - \mathcal{B})\mathcal{C}}\hat{\tau} \tag{65}$$

from (55). According to Appendix C, we always have the results in Table 3.

Table 3: Values of the income effect of progressive trade liberalization

Specifications	$\widehat{p_K\kappa}/\hat{\tau}$	$\hat{E}/\hat{\tau}$
Grossman-Helpman	0	0
Coe-Helpman	> 0	$(0, 1)$

Note that in the Grossman-Helpman model we have

$$[\hat{\varpi}]^{GH} = -(1-\varpi)\frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X}\hat{\tau} \tag{66}$$

and in the Coe-Helpman model we have

$$[\hat{\varpi}]^{CH} = -(1-\varpi)\Big[\frac{(\varepsilon_X - 1)^2}{2\varepsilon_X - \zeta_X}(\hat{\tau} - \hat{E}) + \frac{(\varepsilon_D - 1)^2}{2\varepsilon_D - \zeta_D}\hat{E}\Big]. \tag{67}$$

Thus, trade liberalization would always reduce the export share of GDP no matter whether there exists income effect or not.

Table 3 clearly shows that trade liberalization can curb income by spillovers. Since free entry condition has connected firms' fixed cost with their revenue, which is exactly amounted to the income of consumers (and profits), trade liberalization would reduce the fixed cost by reducing p_K via stronger spillovers and hence lower the average profits that would be paid back to consumers as their investment returns (in the Coe-Helpman model).

To give a full description on how ϖ responds to trade liberalization, we here start with the Coe-Helpman model to highlight the income effect from trade liberalization. In the Coe-Helpman model, a decrease in τ would lead to decrease in both p_X and p_D by (57); however, the decrease in p_D is totally owing to the income effect while the decrease in p_X is the result of both cost effect and income effect from trade liberalization. In other words, reducing trade costs lowers the consumer income, which tends to pull down

p_X as the described income effect or the Linder effect; meanwhile, it directly cuts p_X as their production costs. And by (58) p_X would decrease by more than p_D because of $[\mathrm{d}\ln(p_X/p_D)/\mathrm{d}\ln\tau] > 0$, which shows that the cost effect together with the income effect on p_X prevails over the single income effect on p_D of trade liberalization. As for the demand, there would be a bigger change in q_X than q_D in response to trade liberalization due to $[\mathrm{d}\ln(q_X/q_D)/\mathrm{d}\ln\tau] < 0$ by (59). The result is natural because of the law of demand.

Though the impacts of prices and demand on the export share of GDP are opposite, the fact that $\hat{v}' = -\varepsilon\hat{\xi}$ where $\varepsilon > 1$ gurantees that the relative change in demand q_X/q_D would dominate the change in the relative sales $p_X q_X/p_D q_D$. Hence, decreasing τ yields a higher export share of GDP with a higher q_X/q_D and thus boost the bilateral trade in the Coe-Helpman model.

In the Grossman-Helpman model, the income of consumers is indifferent to the trade cost, which makes the cost effect of τ work alone. Hence, decreasing τ leads to lower p_X and higher q_X/q_D while keeps p_D invariable. The consequences in this model mirrors those in the Coe-Helpman model except for the results of p_D. All the results in this part are summarized in Table 4.

Table 4: Sign of the impacts of progressive trade liberalization

Specifications	$\hat{\tau} < 0$				
	$\hat{p}_D$	$\hat{p}_X$	$(\widehat{\frac{p_X}{p_D}})$	$(\widehat{\frac{q_X}{q_D}})$	$\hat{\varpi}$
Grossman-Helpman	N*	−	−	+	+
Coe-Helpman	−	−	−	+	+

* N refers to no impacts.

The average markup. Rewrite (35) in hat algebra as

$$\hat{\bar{\varepsilon}} = \bar{\varepsilon}\Big[\varpi\Big(\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_X}\Big)\hat{\varpi} + \frac{\varpi}{\varepsilon_X}\hat{\varepsilon}_X + \frac{1-\varpi}{\varepsilon_D}\hat{\varepsilon}_D\Big], \tag{68}$$

which implies that the average markup across domestic and foreign producers is not only increasing in domestic and export markups, but also connected to the relative size of them. The property that the home-sale markups are always higher than export markups endows enlarged trade flows with anti-competitive effect because the fact that $\varepsilon_D - \varepsilon_X \leq 0$ gives rise to the negative correlation between ϖ and $\frac{1}{\bar{\varepsilon}}$ as revealed in (68).

Using (14) and (56), we can reformulate (68) as

$$\begin{aligned}\hat{\bar{\varepsilon}} =&\bar{\varepsilon}\Big[\varpi\Big(\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_X}\Big)\hat{\varpi} + \frac{(\varepsilon_X + 1 - \zeta_X)(\varepsilon_X - 1)\varpi}{(2\varepsilon_X - \zeta_X)\varepsilon_X}\hat{\tau} \\ &- \Big[\frac{(\varepsilon_X + 1 - \zeta_X)(\varepsilon_X - 1)\varpi}{(2\varepsilon_X - \zeta_X)\varepsilon_X} + \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)(1-\varpi)}{(2\varepsilon_D - \zeta_D)\varepsilon_D}\Big]\hat{E}\Big\},\end{aligned} \tag{69}$$

which attaches importance to income again. It will not be weird that higher income generates higher markups according to (69) because of the significant Linder effect modeled by us. Also, $1/\bar{\varepsilon}$ is negatively related to ϖ because enlarged trade flows would reallocate more business to the exporters who charge lower markups and would eventually lead to lower average markups consisting of more export markups than before. Using (63), we obtain another version of (69) (the proof is deferred to Appendix D):

$$\hat{\bar{\varepsilon}} = \bar{\varepsilon}\{(\mathcal{D} - (1-\varpi)\mathcal{A}\mathcal{F})\hat{\tau} + [(1-\varpi)(\mathcal{A}-\mathcal{B})\mathcal{F} - (\mathcal{D}+\mathcal{E})]\hat{E}\}, \tag{70}$$

where

$$\begin{aligned}
\mathcal{D} &= \frac{(\varepsilon_X + 1 - \zeta_X)(\varepsilon_X - 1)\varpi}{(2\varepsilon_X - \zeta_X)\varepsilon_X}, \\
\mathcal{E} &= \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)(1-\varpi)}{(2\varepsilon_D - \zeta_D)\varepsilon_D}, \\
\mathcal{F} &= \varpi\Big(\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_X}\Big).
\end{aligned}$$

Hence, (70) shows that the effect on the average markups can also be decomposed into cost effect and income effect. Moreover, we have

$$[\hat{\bar{\varepsilon}}]^{GH} = \bar{\varepsilon}(\mathcal{D} - (1-\varpi)\mathcal{A}\mathcal{F})\hat{\tau}, \tag{71}$$

$$[\hat{\bar{\varepsilon}}]^{CH} = \bar{\varepsilon}[(\mathcal{D} - (1-\varpi)\mathcal{A}\mathcal{F}) + \frac{(1-\varpi)\mathcal{A}\varpi((1-\varpi)(\mathcal{A}-\mathcal{B})\mathcal{F} - (\mathcal{D}+\mathcal{E}))\mathcal{C}}{1+\varpi+(1-\varpi)(\mathcal{A}-\mathcal{B})\mathcal{C}\varpi}]\hat{\tau}. \tag{72}$$

Since the signs of the coefficients before $\hat{\tau}$ are undetermined, the ultimate impacts of changing τ would be ambiguous, which will be further discussed in extreme cases and suggests U-shaped effect according to simulations.

Notice that (14) and (56) give

$$\hat{\varepsilon}_D = -(\varepsilon_D + 1 - \zeta_D)\frac{\varepsilon_D - 1}{2\varepsilon_D - \zeta_D}\hat{E} \text{ and } \hat{\varepsilon}_X = (\varepsilon_X + 1 - \zeta_X)\frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}(\hat{\tau} - \hat{E}), \tag{73}$$

which was applied in obtaining (69). (73) demonstrates that a higher trade cost level would result in a divergence of markups. Though increasing τ will elevate home-sale markups due to the increased income, it would ultimately decrease export markups (recall that $\hat{\tau} - \hat{E} > 0$ under this circumstance according to Appendix C). This is coherently in line with the effect on p_X/p_D of changing τ as mentioned before, which is related to the presence of cost effect and incomplete pass-through for export business. More precisely, an increase in τ can only lead to a less-proportional increase in p_X, and charging relatively lower prices compared to production cost finally delivers lower export markups. This is consistent with the anti-competitive effect of trade liberalization on export business as documented by ACDR, DGKP, and Fan et al. (2018).

Consequently, decreasing τ yields higher export markups due to incomplete pass-through (where the cost effect dominates) and lower home-sale markups due to the Linder effect. Furthermore, it relegates the home-sale markups in the composition of the average markups by expanding trade flows and hence pulls down $1/\bar{\varepsilon}$ in this way. The final effect of changing τ on the average markups, as we have mathematically proved, are undetermined. The results of this part are summarized in Table 5.

Table 5: Sign of the impacts of progressive trade liberalization on markups

Specifications	$\hat{\tau} < 0$			
	$\hat{\varpi}$	$-\hat{\varepsilon}_D{}^*$	$-\hat{\varepsilon}_X{}^*$	$-\hat{\bar{\varepsilon}}^*$
Grossman-Helpman	+	N	+	V**
Coe-Helpman	+	−	+	V**

* The sign of $-\hat{\varepsilon}$ is exactly the sign of its corresponding markups since $\hat{\frac{1}{\varepsilon}} = -\hat{\varepsilon}$.
** V refers to vague signs.

The vague impacts of cutting trade cost on the average markups echoes the empirical finding of ACDR, which presents that the increase in the markups of foreign producers is shown to partly antagonize the decrease in the markups of domestic producers; or rather, the former effect could balance out the latter one, or at best slightly win over the latter one. Therefore, (69) decomposes the "elusive pro-competitive effect of trade" into three parts: the incomplete pass-through, the pro-competitive trade flows, and the Linder effect. The ambiguity can be traced back to the tension between the first part and the second part ultimately for they are the two sides of cutting trade cost.

Growth and welfare. According to (49), the growth effect of trade openness is represented by

$$\begin{aligned} \mathrm{d}g =& -\frac{wL + \rho p_K \kappa}{\bar{\varepsilon}^2 p_K \kappa}\mathrm{d}\bar{\varepsilon} - \frac{wL}{\bar{\varepsilon}(p_K\kappa)^2}\mathrm{d}(p_K\kappa), \\ =& -(g+\rho+\delta)\hat{\bar{\varepsilon}} - [g + (1-\frac{1}{\bar{\varepsilon}})\rho + \delta]\widehat{p_K\kappa}, \end{aligned} \tag{74}$$

where the second equality is from (52). The above result demonstrates that either higher markups or lower fixed costs can boost growth. When trade cost τ are reduced, the decreased fixed cost tends to increase the growth while the change in markups is ambiguous.

For a given t, we now examine how welfare is affected by a trade shock. The welfare effect of trade openness is represented by the total derivatives of (53):

$$\mathrm{d}\mathcal{V} = \frac{1}{\rho}\Big(\frac{v'_D\xi_D}{v_D+v_X}\hat{p}_D + \frac{v'_X\xi_X}{v_D+v_X}\hat{p}_X - \frac{v'_D\xi_D + v'_X\xi_X}{v_D+v_X}\hat{E} + \frac{1}{\rho}\mathrm{d}g\Big). \tag{75}$$

Note that we applied $dN = 0$ in (75) according to (45). This is convenient for us to compare our results with Ourens (2016), who also ignores the transition before and after the trade shock and hence approximates the welfare in the long run. The expansion of (75) for reducing trade cost is given by

$$\begin{aligned} \mathrm{d}\mathcal{V} =& \frac{1}{\rho}\Big(\frac{v'_D\xi_D}{v_D+v_X}\underbrace{\mathrm{d}(p_K\kappa)\frac{\varepsilon_D + 1 - \zeta_D}{2\varepsilon_D - \zeta_D}\frac{\rho}{E}}_{\text{static effect on } p_D} \\ &+ \frac{v'_X\xi_X}{v_D+v_X}\underbrace{[\mathrm{d}(\tau)\frac{\varepsilon_X - 1}{\tau(2\varepsilon_X - \zeta_X)} + \mathrm{d}(p_K\kappa)\frac{\varepsilon_X + 1 - \zeta_X}{2\varepsilon_X - \zeta_X}\frac{\rho}{E}]}_{\text{static effect on } p_X} \end{aligned}$$

$$-\left(\frac{v'_D\xi_D + v'_X\xi_X}{v_D+v_X}\right)\underbrace{\mathrm{d}(p_K\kappa)\frac{\rho}{E}}_{\text{static effect on } E}+\frac{1}{\rho}\underbrace{\left\{-\frac{g+\rho+\delta}{\bar{\varepsilon}}\mathrm{d}\bar{\varepsilon}-\frac{g+\delta+(1-\frac{1}{\bar{\varepsilon}})\rho}{p_K\kappa}\mathrm{d}(p_K\kappa)\right\}}_{\text{dynamic effect}}\Bigg),$$

Scrutiny of the equation reveals that the impact of trade liberalization on welfare can be decomposed into two parts: the static effect owing to the changes of p_D, p_X, and E; the dynamic effect owing to the change of g. Trade liberalization increase consumer welfare by pulling down the consumer prices but also decrease welfare by lowering consumer incomes (except in the Grossman-Helpman model) in terms of static effect, and the sign of dynamic effect is just as with the previous description about growth. In fact, we can further prove that the static effect of income can dominate the static effect of prices when there exists no cost effect of τ[3]; otherwise, the sign of static effect would be ambiguous. Hence, the welfare effect of trade liberalization is totally vague and the relevant results heavily depend on parameterization. The results of this part are summarized in Table 6.

Table 6: Sign of the impacts of progressive trade liberalization on welfare

Specifications	static effect of			dynamic effect	total effect
	prices	income	total		
Grossman-Helpman	+	N	+	V	V
Coe-Helpman	+	−	V	V	V

Note that the signs presented here refers to how the variables of interest respond to the decrease in τ ultimately.

3.2.2 Near Free Trade

In the general case, we find that many results of progressive trade liberalization are inexplicit, especially for those variables of interests such as the reciprocal of average markups $\bar{\varepsilon}$, growth rate g, welfare $\mathcal{V}$. To find out what will happen in the process of trade liberalization, we here shed light on the marginal changes of those variables near extreme cases, which is also the common practice in the literature.

For the convenience of reader, we here reclarify that τ, $\varpi = \frac{1}{2}$, $\varepsilon_D = \varepsilon_X = \bar{\varepsilon} = \varepsilon$, $\zeta_D = \zeta_X = \zeta$ at free trade. Using these properties, we can rewrite red the inverse average markups (70) as:

$$\hat{\bar{\varepsilon}}^{fre} = \frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2(2\varepsilon-\zeta)}\hat{\tau} - \frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2\varepsilon-\zeta}\frac{\rho}{\varepsilon(g+\rho+\delta)}\widehat{p_K\kappa}. \tag{76}$$

Compared to the general case, the relationship between markups and trade liberalization's two effects is unequivocal in the neighborhood of free trade: the average markups would absolutely be decreased by cost effect owing to increased trade cost while increased by income effect owing to increased knowledge requirement.

For further analysis, we present the free trade versions of (64), (65) as:

$$[\widehat{p_K\kappa}^{fre}]^{GH} = 0,$$

[3]By way of checking the sign of $(\frac{v'_D\xi_D}{v_D+v_X}\frac{\varepsilon_D+1-\zeta_D}{2\varepsilon_D-\zeta_D}+\frac{v'_X\xi_X}{v_D+v_X}\frac{\varepsilon_X+1-\zeta_X}{2\varepsilon_X-\zeta_X}-\frac{v'_D\xi_D+v'_X\xi_X}{v_D+v_X})\frac{\rho}{E}\mathrm{d}(p_K\kappa)$ or further $\left[\frac{v'_D\xi_D}{v_D+v_X}(\frac{\varepsilon_D+1-\zeta_D}{2\varepsilon_D-\zeta_D}-1)+\frac{v'_X\xi_X}{v_D+v_X}(\frac{\varepsilon_X+1-\zeta_X}{2\varepsilon_X-\zeta_X}-1)\right]\frac{\rho}{E}\mathrm{d}(p_K\kappa)$.

$$[\widehat{p_K\kappa}^{fre}]^{CH} = \frac{(\varepsilon-1)^2}{6(2\varepsilon-\zeta)}\hat{\tau},$$

Then we write down $\hat{\bar{\varepsilon}}^{fre}$ using the two specifications:

$$\begin{aligned}
[\hat{\bar{\varepsilon}}^{fre}]^{GH} =& \frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2(2\varepsilon-\zeta)}\hat{\tau} \\
[\hat{\bar{\varepsilon}}^{fre}]^{CH} =& \frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2(2\varepsilon-\zeta)}\left[1-\frac{(\varepsilon-1)^2\rho}{3\varepsilon(2\varepsilon-\zeta)(g+\rho+\delta)}\right]\hat{\tau},
\end{aligned}$$

Increased trade cost reduces the average markups solely via the cost effect in the Grossman-Helpman model while via the cost effect partly antagonized by the income effect in the the Coe-Helpman model where the cost effect dominates the income effect.

As for the growth rate near free trade, combining (74) with (76) yields

$$\mathrm{d}g^{fre} = -(g+\rho+\delta)\frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2(2\varepsilon-\zeta)}\hat{\tau} - [g+\delta+\frac{(\varepsilon-1)^2}{\varepsilon(2\varepsilon-\zeta)}\rho]\widehat{p_K\kappa}. \tag{77}$$

Rewriting (77) with the two specifications leads to:

$$\begin{aligned}
[\mathrm{d}g^{fre}]^{GH} =& -(g+\rho+\delta)\frac{(\varepsilon+1-\zeta)(\varepsilon-1)}{2(2\varepsilon-\zeta)}\hat{\tau}, \\
[\mathrm{d}g^{fre}]^{CH} =& -\frac{\varepsilon-1}{2(2\varepsilon-\zeta)}\left\{(g+\rho+\delta)(\varepsilon+1-\zeta)+\left[g+\delta+\frac{(\varepsilon-1)^2}{\varepsilon(2\varepsilon-\zeta)}\rho\right]\frac{\varepsilon-1}{3}\right\}\hat{\tau}
\end{aligned}$$

The previous results have demonstrated that there exist linear relationships between the growth rate and different ways of trade liberalization near free trade.

From (75), the subtle change of welfare near free trade is represented by

$$\mathrm{d}\mathcal{V}^{fre} = \frac{1}{\rho}\Big[\frac{v'\xi}{v}\frac{1-\varepsilon}{2\varepsilon-\zeta}\frac{\rho}{\varepsilon(g+\rho+\delta)}\widehat{p_K\kappa} + \frac{v'\xi}{2v}\frac{\varepsilon-1}{2\varepsilon-\zeta}\hat{\tau} + \frac{1}{\rho}\mathrm{d}g\Big\}.$$

Since the the impact of progressive trade liberalization on the dynamic part of welfare is no longer uncertain near free trade, we here take a glance at the changes of the static part of welfare $\mathrm{d}\mathcal{V}^{fre}_{static}$ under the same circumstance, which are described by

$$\begin{aligned}
[\mathrm{d}\mathcal{V}^{fre}_{static}]^{GH} =& \frac{1}{\rho}\frac{v'\xi}{2v}\frac{\varepsilon-1}{2\varepsilon-\zeta}\hat{\tau} \\
[\mathrm{d}\mathcal{V}^{fre}_{static}]^{CH} =& \frac{1}{\rho}\frac{v'\xi}{v}\frac{\varepsilon-1}{2\varepsilon-\zeta}\Big[1-\frac{\rho(\varepsilon-1)^2}{3(g+\rho+\delta)\varepsilon(2\varepsilon-\zeta)}\Big]\hat{\tau}
\end{aligned}$$

The previous results are summarized in summarized in Table 7.

Table 7: Sign of the impacts of progressive trade liberalization near free trade

Specifications	$\hat{\bar{\varepsilon}}^{fre}$	$\mathrm{d}\mathcal{V}^{fre}_{static}$	g^{fre}	$\mathrm{d}\mathcal{V}^{fre}$
Grossman-Helpman	−	+	+	+
Coe-Helpman	−	+	+	+

Note that the signs presented here refers to how the variables of interest respond to the decrease in τ ultimately.

3.2.3 Near Autarky

Similarly, we first reclarify that $\tau = \tau^{aut}$, $v'_X = q_X = \varpi = 0$, $\xi_X = \xi_X^{aut}$, $\varepsilon_X = \varepsilon_X^{aut}$, $\bar{\varepsilon} = \varepsilon_D^{aut}$ when the economy is in autarky and pay attention to $\bar{\varepsilon}$, g and $\mathcal{V}$. Rewriting (68) from hat algebra to total derivatives yields[4]

$$\hat{\bar{\varepsilon}} = \bar{\varepsilon}\Big[\Big(\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_X}\Big)\mathrm{d}\varpi + \frac{\varpi}{\varepsilon_X^2}\mathrm{d}\varepsilon_X + \frac{1-\varpi}{\varepsilon_D}\hat{\varepsilon}_D\Big].$$

Recall $\varepsilon_X = -\xi_X v''_X / v'_X$, thus when $\tau \to \tau^{aut}$, $v'_X \to 0$, and $\varepsilon_X \to +\infty$. We can further rewrite $\hat{\bar{\varepsilon}}^{aut}$ as

$$\begin{aligned}\hat{\bar{\varepsilon}}^{aut} &= \mathrm{d}\varpi + \hat{\varepsilon}_D \\ &= \mathrm{d}\varpi - \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)}{(2\varepsilon_D - \zeta_D)\,\varepsilon_D}\frac{\rho}{\rho + g + \delta}\widehat{p_K \kappa}.\end{aligned} \tag{78}$$

It is easy to prove $(\hat{\varepsilon}_D/\hat{\tau}) < 0$ with Appendix A and Appendix B. With $(\partial\varpi/\partial\tau) < 0$, (78) shows that $(\partial\bar{\varepsilon}/\partial\tau) < 0$. Thus, the average markups would increase when economies start trade liberalization from autarky, which generates the anti-competitive effect. As a comparison, trade liberalization near free trade would decrease the average markups and cause the ideal pro-competitive effects that is emphasized by traditional trade theory. This shows that deviating from autarky can bring to heterodox results different from the canonical conclusions in international economics.

As for the growth rate, we need to analyze it specification by specification since in the Coe-Helpman model p_K is connected with ϖ. Combining (74) with (78) leads to

$$\mathrm{d}g^{aut} = -(g + \rho + \delta)\mathrm{d}\varpi - \left\{g + \delta + \left[1 - \frac{1}{\bar{\varepsilon}} - \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)}{(2\varepsilon_D - \zeta_D)\varepsilon_D}\right]\rho\right\}\widehat{p_K\kappa}, \tag{79}$$

where $0 < 1 - \frac{1}{\bar{\varepsilon}} - \frac{(\varepsilon_D+1-\zeta_D)(\varepsilon_D-1)}{(2\varepsilon_D-\zeta_D)\varepsilon_D} < 1$ since $1 - \frac{1}{\bar{\varepsilon}} - \frac{(\varepsilon_D+1-\zeta_D)(\varepsilon_D-1)}{(2\varepsilon_D-\zeta_D)\varepsilon_D} > 1 - \frac{1}{\bar{\varepsilon}} - \frac{(\varepsilon_D-1)}{\varepsilon_D} = \frac{1}{\varepsilon_D} - \frac{1}{\bar{\varepsilon}} \geq 0$. In the Grossman-Helpman model, we have

$$[\widehat{p_K\kappa}^{aut}]^{GH} = 0,$$

which leads to

$$[\mathrm{d}g^{aut}]^{GH} = -(g + \rho + \delta)\mathrm{d}\varpi,$$

In the Coe-Helpman model, we have

$$[\widehat{p_K\kappa}^{aut}]^{CH} = -\mathrm{d}\varpi,$$

which leads to

$$[\mathrm{d}g^{aut}]^{CH} = -\Big[\frac{1}{\bar{\varepsilon}} + \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)}{(2\varepsilon_D - \zeta_D)\,\varepsilon_D}\Big]\rho\mathrm{d}\varpi.$$

It is straight that near autarky $\partial g/\partial\tau < 0$ (i.e. $\partial g/\partial\varpi > 0$) in both two models.

The welfare change near autarky is represented by the total derivative of (53) as

$$\mathrm{d}\mathcal{V}^{aut} = -\frac{v'_D\xi_D}{v_D + v_X}\frac{\varepsilon_D - 1}{2\varepsilon_D - \zeta_D}\frac{1}{\bar{\varepsilon}(g + \delta + \rho)}\widehat{p_K\kappa} + \frac{\mathrm{d}g}{\rho^2}. \tag{80}$$

[4]Because many variables like ϖ equals 0 in autarky and the relevant hat algebra do not make sense, we should base the analysis on total derivatives instead of hat algebra in this subsection.

We know the first term on the RHS of (80) is the so-called static welfare and the second term is the dynamic part. It is easy to confirm that

$$\frac{\partial [p_K \kappa^{aut}]^{GH}}{\partial \varpi} = 0, \quad \frac{\partial [p_K \kappa^{aut}]^{CH}}{\partial \varpi} < 0;$$

together with $\partial g^{aut}/\partial \varpi < 0$, it is straight that $\partial \mathcal{V}/\partial \varpi < 0$ and thus $\partial \mathcal{V}/\partial \tau > 0$ holds near autarky.

4 Examples and Simulations

In the analysis of BGP, we have obtained equations (31), (35), (38), (40), (41), (42), (43), (44), (45, and (52). They can be applied to pin down 11 variables

$$\kappa, p_K, \xi_D, \xi_X, \bar{\varepsilon}, \bar{\varpi}, g, e, L_E, \mu(t), N(t).$$

Unfortunately, they are not analytically solvable. Therefore, we conduct simulations based on 3 kinds of IA utilities: addilog utilities, exponent utilities and CES utilities.

Addilog utilities are given by $(b - x)^{1+\phi}/(1 + \phi)$, where b is the choke prices which allows the existence of autarky. Exponent utilities are given by $\exp\{-ax\}$ which leads to economy without autarky. CES utilities are given by $x^{1-\sigma}$ where σ is the constant elasticity of substitution. Parameters and exogenous variables are set as:

$$\kappa = 7, \rho = 0.8, L = 50, \delta = 0.25, \psi = 0.7, \phi = 1, b = 2, a = 2, \sigma = 3.3, t = 1.$$

We show how important variables g, $\bar{\varpi}$, x_X, and welfare depend on τ in Figures 1–3. Three kinds of curves are plotted in each figure. The G-H model is the blue, the C-H is the red, the L-E model is the green.

In the ϖ panels, a non-positive value corresponds to an autarky state. The corresponding value of x_X is also non-positive.

Since v_D and v_X may be smaller than 1, the welfare level might become negative. Since the addilog utility features a choke price, curves in Figure 1 are plotted for τ smaller than the autarky trade cost. We can observe the U-shape of the welfare curves with respect to τ in the all models in Figure 1.

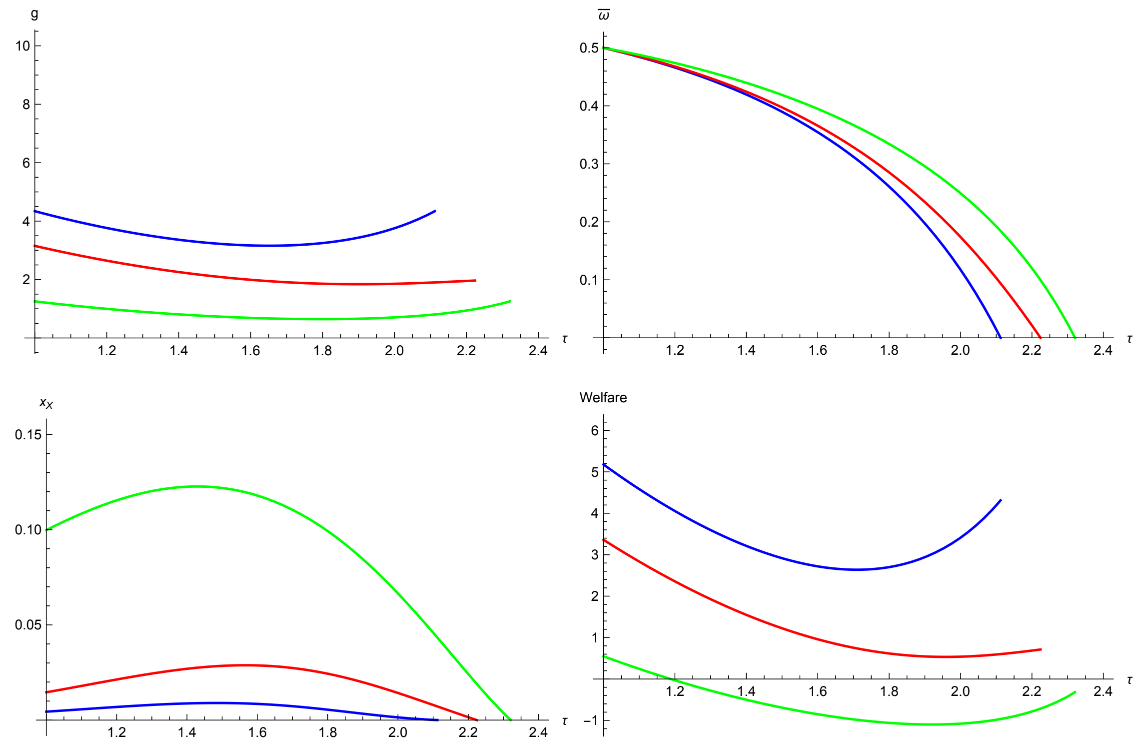

Figure 1: Variables of interests responding to τ shocks based on addilog utilities

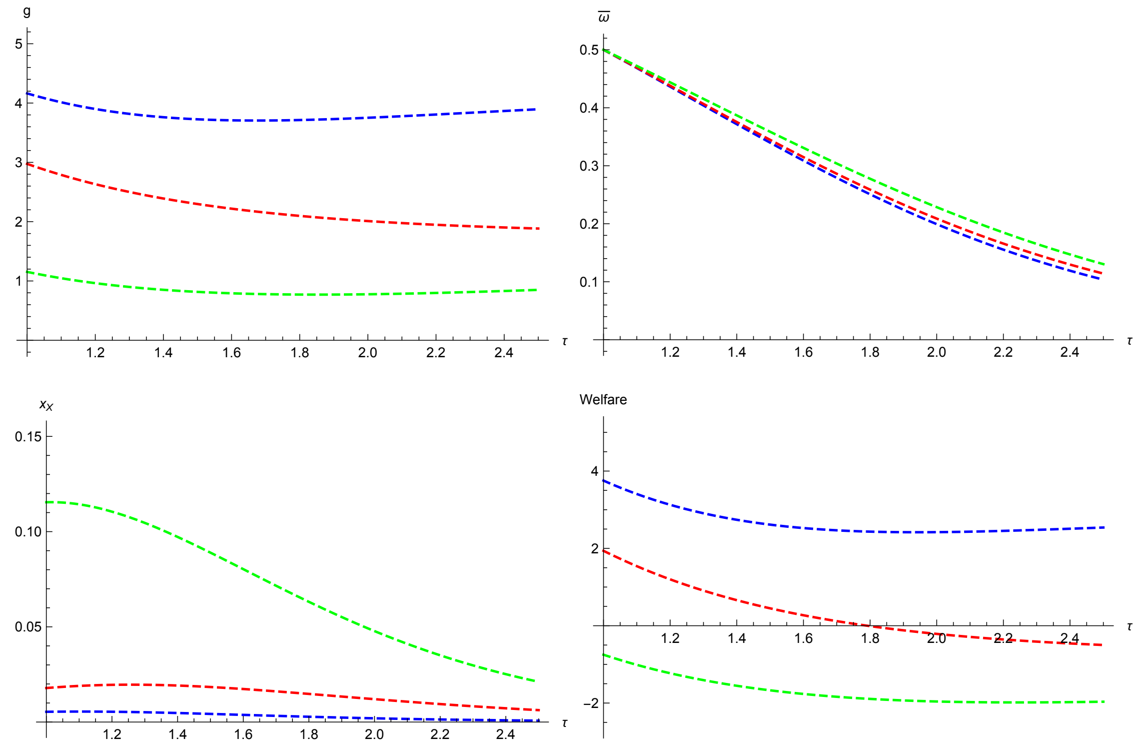

Figure 2: Variables of interests responding to τ shocks based on exponent utilities

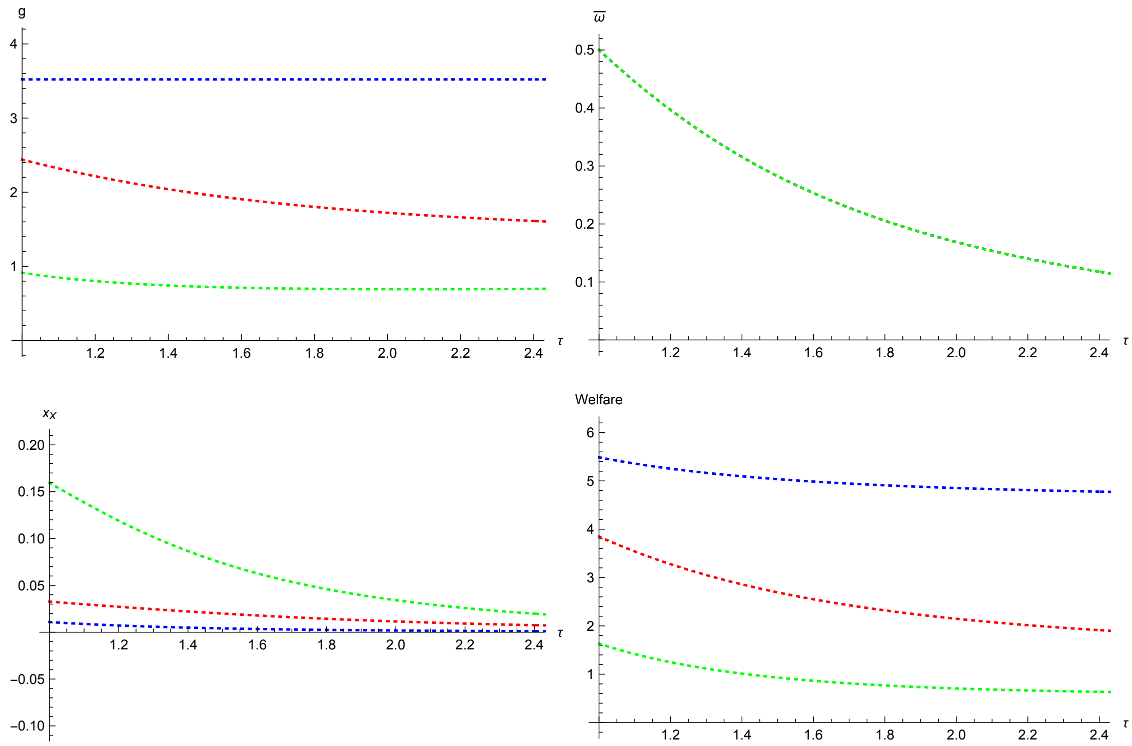

Figure 3: Variables of interests responding to τ shocks based on CES utilities

5 Concluding Remarks

Further extensions of our research can be expected. The introduction of firm heterogeneity á la Melitz (2003) can bring the selection effects of trade back in our model with homogenous firms. Considering the case with asymmetric countries can reveal the stylized fact of pricing-to-market that is a common conclusion under IA preferences. Better calibration for the parameters is ideally wanted. The model with the previous features can be more precise in evaluating the impact of trade on growth.

Appendix

Appendix A

Define $\theta_{\varepsilon,\xi}$ as the elasticity of ε w.r.t. ξ (for both home and foreign variables). Differentiating $\varepsilon(\xi) = -\xi v''(\xi)/v'(\xi)$ yields

$$\frac{\mathrm{d}\varepsilon(\xi)}{\mathrm{d}\xi} = \frac{(v''(\xi))^2\xi - v'''(\xi)v'(\xi)\xi - v''(\xi)v'(\xi)}{(v'(\xi))^2} = \frac{(v''(\xi))^2\xi}{(v'(\xi))^2} - \frac{v'''(\xi)\xi}{v'(\xi)} - \frac{v''(\xi)}{v'(\xi)}.$$

Express the elasticity as

$$\theta_{\varepsilon,\xi} \equiv \frac{\xi}{\varepsilon(\xi)}\frac{\mathrm{d}\varepsilon(\xi)}{\mathrm{d}\xi} = 1 + \frac{\xi v'''(\xi)}{v''(\xi)} - \frac{\xi v''(\xi)}{v'(\xi)},$$

which boils down to

$$\theta_{\varepsilon,\xi} = \frac{\mathrm{d}\ln\varepsilon(\xi)}{\mathrm{d}\ln\xi} = \varepsilon(\xi) + 1 - \zeta(\xi) > 0, \tag{A.1}$$

where the inequality is from Assumption 2.

Appendix B

Log-differentiating (15) and (16) yields

$$\mathrm{d}\ln\xi = \mathrm{d}\ln p - \mathrm{d}\ln E = \mathrm{d}\ln m + \mathrm{d}\ln\varepsilon - \mathrm{d}\ln(\varepsilon-1) - \mathrm{d}\ln E. \tag{A.2}$$

Using the fact that $\mathrm{d}\ln(\varepsilon-1) = (\varepsilon/(\varepsilon-1))\mathrm{d}\ln\varepsilon$, (A.2) leads to

$$\mathrm{d}\ln\xi = \mathrm{d}\ln m - \frac{1}{\varepsilon-1}\mathrm{d}\ln\varepsilon - \mathrm{d}\ln E. \tag{A.3}$$

Combining (A.1), (A.2) with (A.3) gives

$$\begin{aligned}\mathrm{d}\ln\xi &= (\mathrm{d}\ln m - \mathrm{d}\ln E)\frac{\varepsilon-1}{2\varepsilon-\zeta},\\ \mathrm{d}\ln p &= \frac{\varepsilon-1}{2\varepsilon-\zeta}\mathrm{d}\ln m + \frac{\varepsilon+1-\zeta}{2\varepsilon-\zeta}\mathrm{d}\ln E.\end{aligned} \tag{A.4}$$

Appendix C

The effect of lowering trade cost τ is complex in the Coe-Helpman model at first glance because there exist tensions between the income effect and the cost effect, i.e. $(\hat{\tau}-\hat{E})$ in (58) and (59). Nevertheless, we are able to show that the cost effect dominates the income effect when the contradictory effect exists. Combining (60) with (65) will give

$$\left[\frac{\hat{E}}{\hat{\tau}}\right]^{CH} = \frac{(1-\varpi)\mathcal{AC}\varpi}{1+\varpi-(1-\varpi)\mathcal{BC}\varpi+(1-\varpi)\mathcal{AC}\varpi},$$

Notice that

$$\bar{\varepsilon}-\varepsilon_D = \frac{\varpi\varepsilon_D(\varepsilon_X-\varepsilon_D)}{(1-\varpi)\varepsilon_X+\varpi\varepsilon_D} > 0 \Leftrightarrow \frac{\varepsilon_D-1}{\bar{\varepsilon}} < 1,$$

by which we have

$$1+\varpi-(1-\varpi)\mathcal{BC}\varpi = 1+\varpi\left[1-(1-\varpi)\frac{(\varepsilon_D-1)^2}{\bar{\varepsilon}(2\varepsilon_D-\zeta_D)}\frac{\rho}{(\rho+g+\delta)}\right] > 0$$

since $(1-\varpi)\frac{(\varepsilon_D-1)^2}{\bar{\varepsilon}(2\varepsilon_D-\zeta_D)}\frac{\rho}{(\rho+g+\delta)} < 1$. Together with $(1-\varpi)\mathcal{AC}\varpi > 0$, we have

$$0 < \frac{(1-\varpi)\mathcal{AC}\varpi}{1+\varpi-(1-\varpi)\mathcal{BC}\varpi+(1-\varpi)\mathcal{AC}\varpi} < 1.$$

Then we have $[\frac{\hat{E}}{\hat{\tau}}]^{CH} \in (0,1)$ and $[\frac{\widehat{p_K\kappa}}{\hat{\tau}}]^{CH} > 0$. Hence, E is less proportionally increasing in τ, which suggests $\hat{\tau}-\hat{E} > 0$ if $\hat{\tau} > 0$. Thenceforth it is easy to corroborate our full description on how ϖ changes with τ.

Appendix D

Log-differentiating (35) yields

$$\begin{aligned}-\hat{\bar{\varepsilon}} &= \bar{\varepsilon}[(1-\varpi)d\frac{1}{\varepsilon_D} + \frac{1}{\varepsilon_D}d(1-\varpi) + \varpi d\frac{1}{\varepsilon_X} + \frac{1}{\varepsilon_X}d\varpi]\\ -\hat{\bar{\varepsilon}} &= \bar{\varepsilon}[(\frac{1}{\varepsilon_X}-\frac{1}{\varepsilon_D})d\varpi - \frac{\varpi}{\varepsilon_X^2}d\varepsilon_X - \frac{1-\varpi}{\varepsilon_D^2}d\varepsilon_D]\\ \hat{\bar{\varepsilon}} &= \bar{\varepsilon}[\varpi(\frac{1}{\varepsilon_D}-\frac{1}{\varepsilon_X})\hat{\varpi} + \frac{\varpi}{\varepsilon_X}\hat{\varepsilon}_X + \frac{1-\varpi}{\varepsilon_D}\hat{\varepsilon}_D].\end{aligned} \tag{A.5}$$

Remember that $\hat{\varepsilon} = (\varepsilon + 1 - \zeta)\hat{\xi}$ and $\hat{\xi} = (\hat{m} - \hat{E})\frac{\varepsilon - 1}{2\varepsilon - \zeta}$, so we have

$$\hat{\varepsilon}_X = (\varepsilon_X + 1 - \zeta_X)\frac{\varepsilon_X - 1}{2\varepsilon_X - \zeta_X}(\hat{\tau} - \hat{E}) \text{ and } \hat{\varepsilon}_D = -(\varepsilon_D + 1 - \zeta_D)\frac{\varepsilon_D - 1}{2\varepsilon_D - \zeta_D}\hat{E}.. \tag{A.6}$$

Combing (A.5) with (A.6) gives

$$\begin{aligned}\hat{\bar{\varepsilon}} = \bar{\varepsilon}[\varpi(\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_X})\hat{\varpi} + \frac{(\varepsilon_X + 1 - \zeta_X)(\varepsilon_X - 1)\varpi}{(2\varepsilon_X - \zeta_X)\varepsilon_X}\hat{\tau} \\ - (\frac{(\varepsilon_X + 1 - \zeta_X)(\varepsilon_X - 1)\varpi}{(2\varepsilon_X - \zeta_X)\varepsilon_X} + \frac{(\varepsilon_D + 1 - \zeta_D)(\varepsilon_D - 1)(1 - \varpi)}{(2\varepsilon_D - \zeta_D)\varepsilon_D})\hat{E}].\end{aligned} \tag{A.7}$$

Mechanical rearrangement using (60), (63) and (A.7) finally delivers

$$\hat{\bar{\varepsilon}} = \bar{\varepsilon}[(\mathcal{D} - (1 - \varpi)\mathcal{A}\mathcal{F})\hat{\tau} + ((1 - \varpi)(\mathcal{A} - \mathcal{B})\mathcal{F} - (\mathcal{D} + \mathcal{E}))\mathcal{C}\hat{E}]. \tag{A.8}$$

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