Self-adaptive Parameter Control Mechanism in Evolutionary Computation

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Theory of Evolutionary Computation Group
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University of Birmingham
United Kingdom

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About Me

Hello!

I'm Xiaoyu QIN 秦霄羽.

I enrolled in Feb. 2020 as a PhD student
in University of Birmingham, supervised by
Per Kristian Lehre and Ata Kaban.

Research Interests:

- *Evolutionary Computation* $\sim$ *Optimisation & Search* $\sim$ *AI*
- *AI & ML and Applications*

Education:

- **PhD** in Computer Sci (*2020 - 2023*) *University of Birmingham*
- **MSc** in Advanced Computer Sci (*2019*) *University of Birmingham*
- **Visiting Student** in Computer Sci (*2017 - 2018*) *Loughborough University*
- **BEng** in Computer Sci & Tech (*2018*) *Shanxi University*

Teaching:

- **Teaching Assistance** (*2020 - 2022*) *University of Birmingham*

  Modules: *Neural Computation*, *AI & ML*, etc.

About Me

Outputs:

[1] *Self-adaptation Can Improve the Noise-tolerance of Evolutionary Algorithms.* (with P.K. Lehre)* In Proc. of FOGA '23.

[2] *Self-adaptation Can Help Evolutionary Algorithms Track Dynamic Optima.* (with P.K. Lehre)* In Proc. of GECCO '23.

[3] *Optimizing Chance-Constrained Submodular Problems with Variable Uncertainties.* (X. Yan, A.V. Do, F. Shi, X. Qin and F. Neumann) In Proc. of ECAI '23.

[4] *Self-adaptation via Multi-objectivisation: An Empirical Study.* (X. Qin and P.K. Lehre) In Proc. of PPSN '22.

[5] *More Precise Runtime Analyses of Non-elitist Evolutionary Algorithms in Uncertain Environments.* (with P.K. Lehre)* In *Algorithmica* (2022).

[6] *Self-adaptation via Multi-objectivisation: A Theoretical Study.* (with P.K. Lehre)* In Proc. of GECCO '22.

[7] *Fast Non-elitist Evolutionary Algorithms with Power-law Ranking Selection.* (with D.C. Dang, A. Eremeev and P.K. Lehre)* In Proc. of GECCO '22.

[8] *More Precise Runtime Analyses of Non-elitist EAs in Uncertain Environments.* (with P.K. Lehre)* In Proc. of GECCO '21.

[9] *Fast Non-elitist Evolutionary Algorithms with Power-law Ranking Selection.* (with D.C. Dang, A. Eremeev and P.K. Lehre)* Submitted to *Algorithmica* (Under review).

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*Following the convention of Theoretical Computer Science that the co-authors are listed in *Alphabetical Order*.

About Me

Professional Activities:

- **PC Member** of conferences GECCO[†], PPSN[‡], ECAI[§], etc.
- **Reviewer** of journals *IEEE Transactions on Evolutionary Computation* (TEVC), *Theoretical Computer Science* (TCS), *China Information Science*, etc.
- **Invited Talks** & **Presentations** in seminars and universities.

[†] GECCO: The Genetic and Evolutionary Computation Conference.

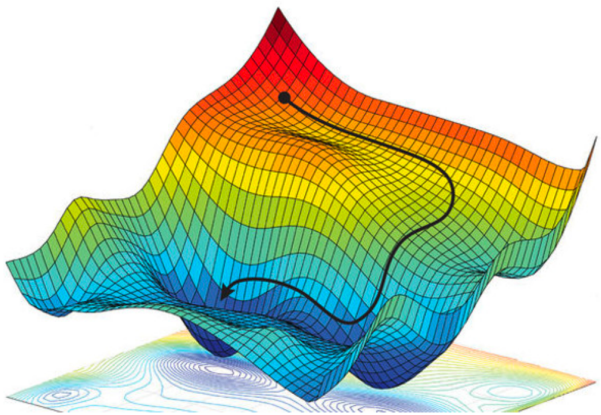
[‡] PPSN: International Conference on Parallel Problem Solving from Nature.

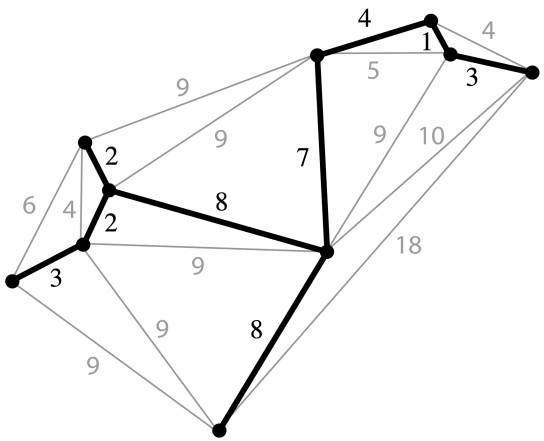
[§] ECAI: European Conference on Artificial Intelligence.

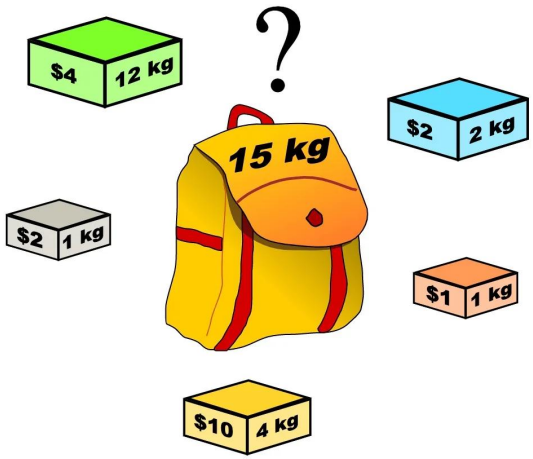
Self-adaptive Parameter Control Mechanism in Evolutionary Computation

$$\underline{Theory} \sim EvolutionaryComputation \sim Optimisation\&Search \sim AI$$
$$\underline{ParameterConfiguration} \sim EvolutionaryComputation \sim Optimisation\&Search \sim AI$$

Optimisation & Search ∼ AI







* Image sources: https://arxiv.org/pdf/1805.04829.pdf, https://www.51cto.com/article/641654.html, https://en.wikipedia.org/wiki/Combinatorial_optimization

Optimisation & Search in Real World

- **Machine Learning**
  - ML Models training & parameters tuning
  - Neural Architecture Search (NAS)
  - Neural network pruning
- **Software Engineering**
  - Project management
  - Testing
  - The next release problem
- **Intelligent Vehicles**
  - Route planning
  - Decision making
- **Industrial Field**
  - Robot design
  - Antenna design
  - Warehouse Management

‡ Image sources: https://arxiv.org/pdf/1506.02626.pdf, https://en.wikipedia.org/wiki/Evolved_antenna, https://www.nissan-global.com/EN/INNOVATION/TECHNOLOGY/ARCHIVE/IRP/, https://robot.ofweek.com/2022-10/ART-8321200-8100-30576584.html

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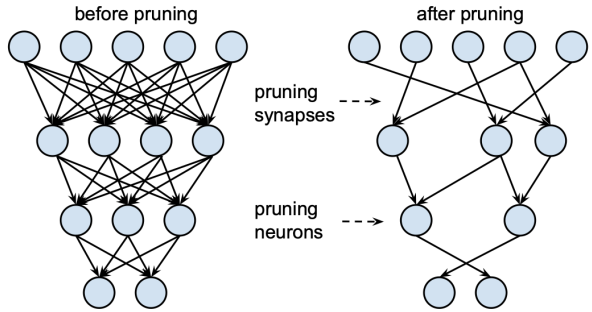

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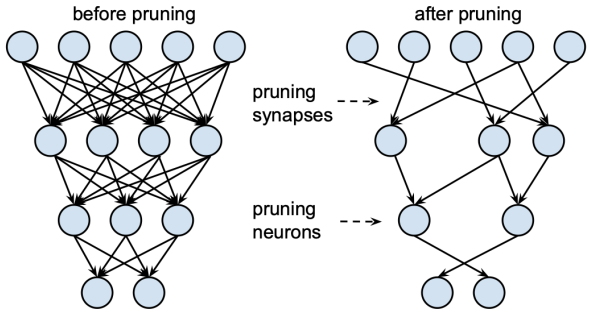

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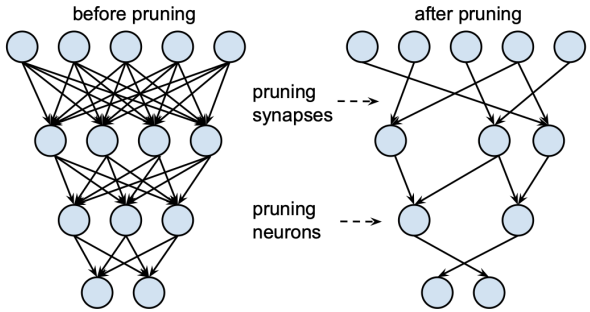

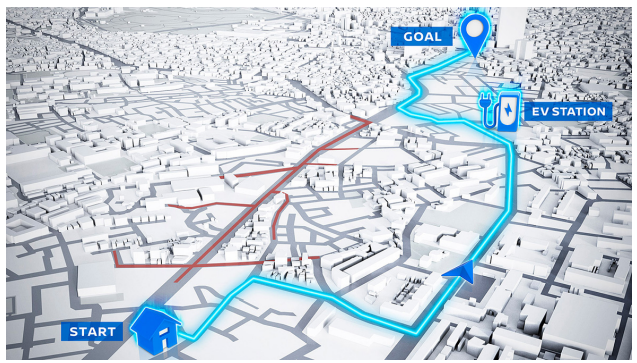

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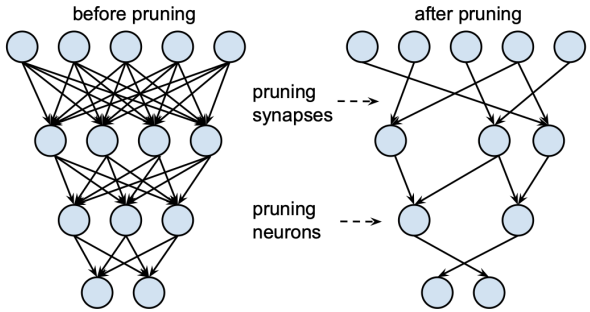

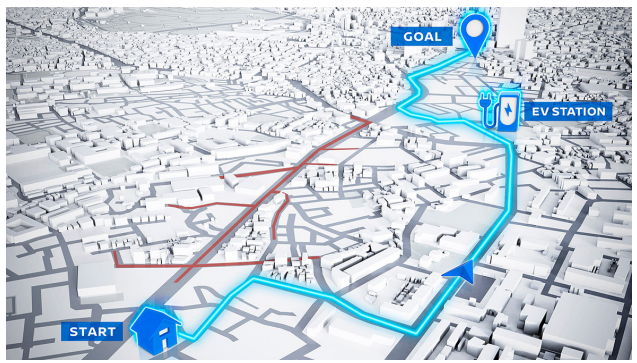

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Optimisation Problem

- **Aim:** To find the best solution from all feasible solutions.
- **Mathematical description:**
  - **Given:** a function $f : \mathcal{S} \to \mathbb{R}$
  - **Sought:** find an element $\hat{x} \in \mathcal{S}$ such that $f(\hat{x}) \geq f(x)$ for any $x \in \mathcal{S}$[¶]
- The *search space* $\mathcal{S}$ can be continuous, discrete, or mixed.
- The *objective (fitness) function* f is usually unknown for algorithms.
- The "property" of f is sometime unknown for algorithms.

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Black-box Optimisation Problem (BBOP)

[¶] Assume to maximise the function.

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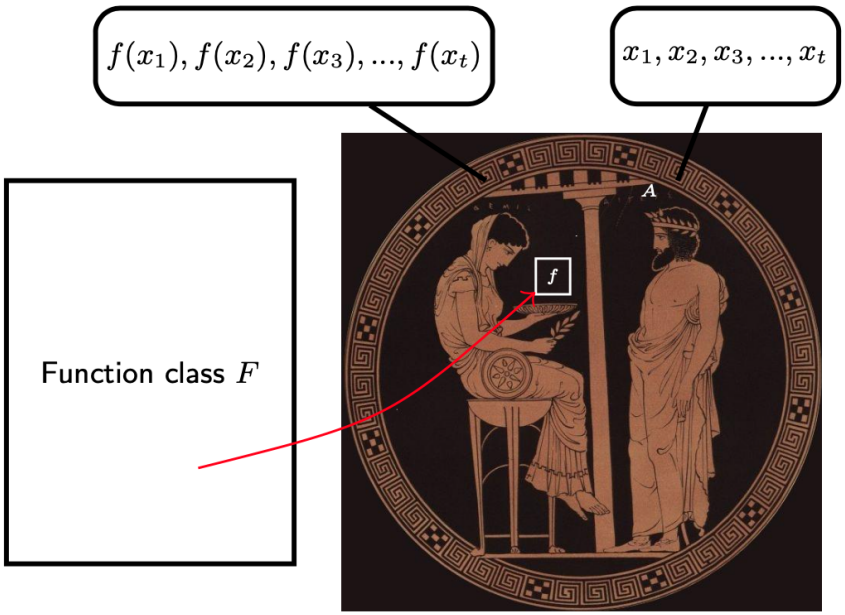

Photo: E. Gerhard (1846).

- Optimise function f by ONLY evaluating the fitness value of search points.
- **Formal description:**
  - An unknown optimisation problem f is chosen from a problem class $\mathcal{F} \subseteq \{f : S \to \mathbb{R}\}$ known to the algorithm A.
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Runtime*: $T_{A,f} := \min\{t \mid \forall y \in \{0,1\}^n,\ f(x_t) \geq f(y)\}$

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The number of evalutions until the optimal (desired) solution found.

*Reference: (Droste et al., 2006). (Assume that the objective is maximising f)

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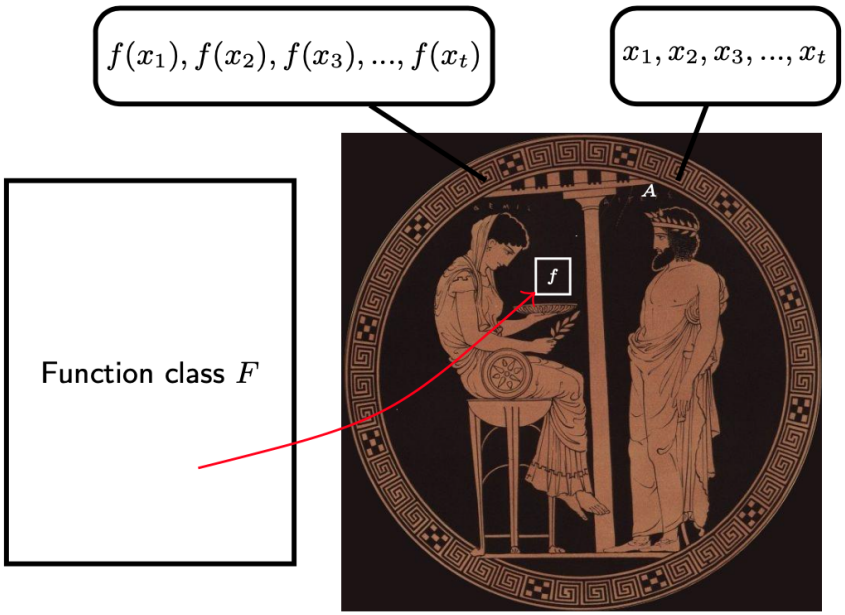

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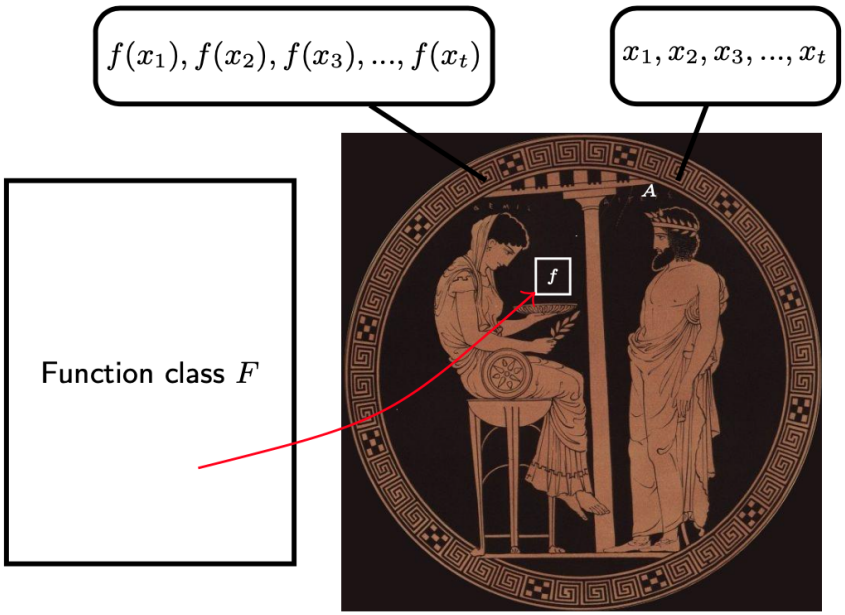

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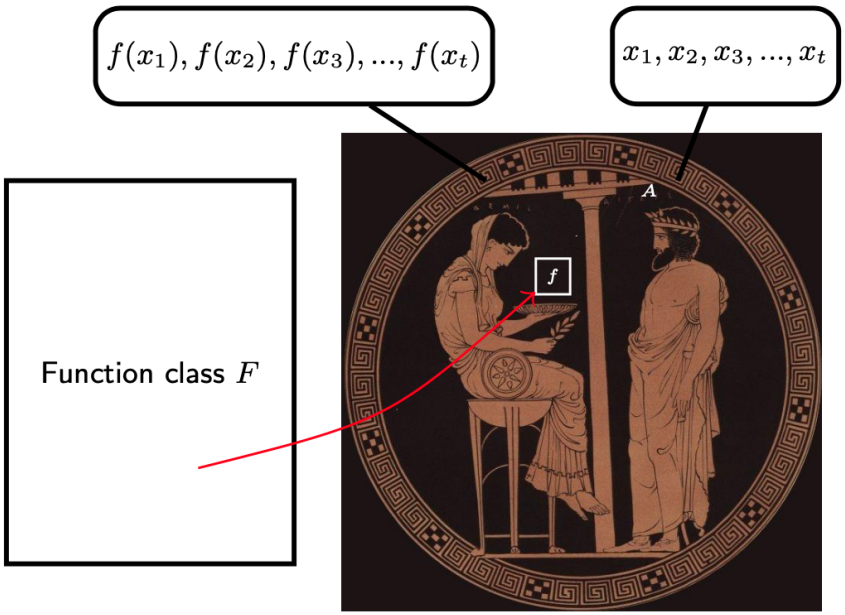

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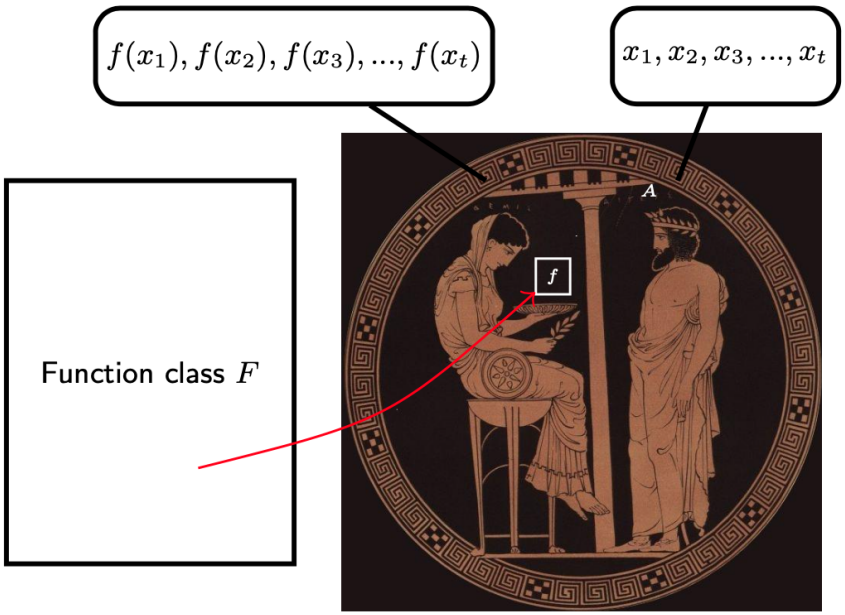

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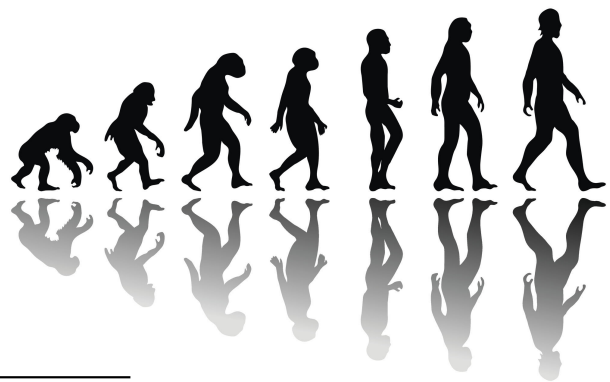
Evolutionary Computation (EC)

Evolutionary Computation is a family of *randomised heuristic algorithms* for optimisation inspired by biological evolution. *

- **Efficiency** $\Rightarrow$ Heuristic $\Rightarrow$ Achieve the (nearly) optimum in short time
  $\Rightarrow$ Generality $\Rightarrow$ Independent of problem 'property' (BBOP)
  $\Rightarrow$ Parallelism $\Rightarrow$ Population-based
- **Practicability** $\Rightarrow$ Multi-objectivisation
  $\Rightarrow$ Global optimisation $\Rightarrow$ Can escaping local optima
- **Robustness** $\Rightarrow$ Noisy/Dynamic Optimisation



*The generalised EC also includes other natural-inspired algorithms, e.g., Particle Swarm Optimisation (PSO), Ant Colony Optimisation (ACO), Differential Evolution (DE), Estimation of Distribution Algorithms (EDAs), Simulated Annealing (SA), etc.

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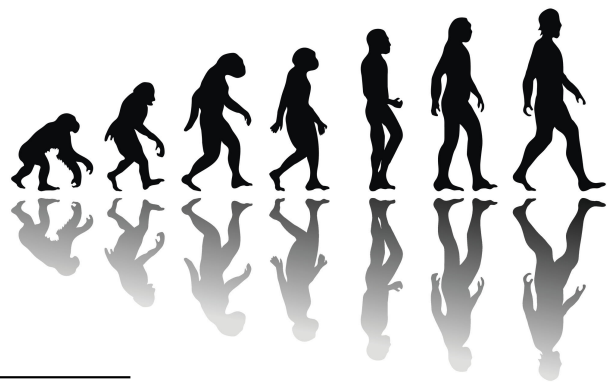

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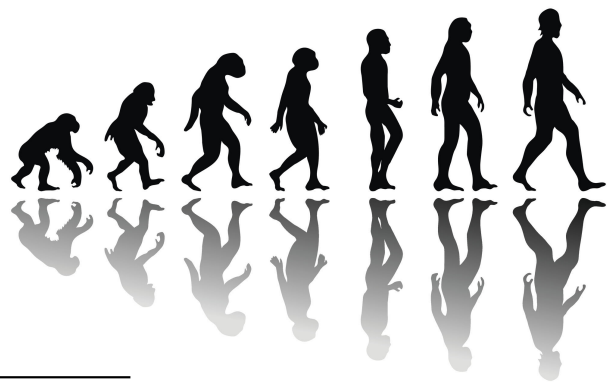

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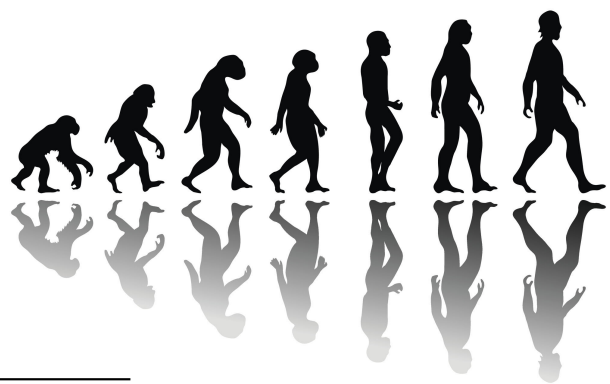
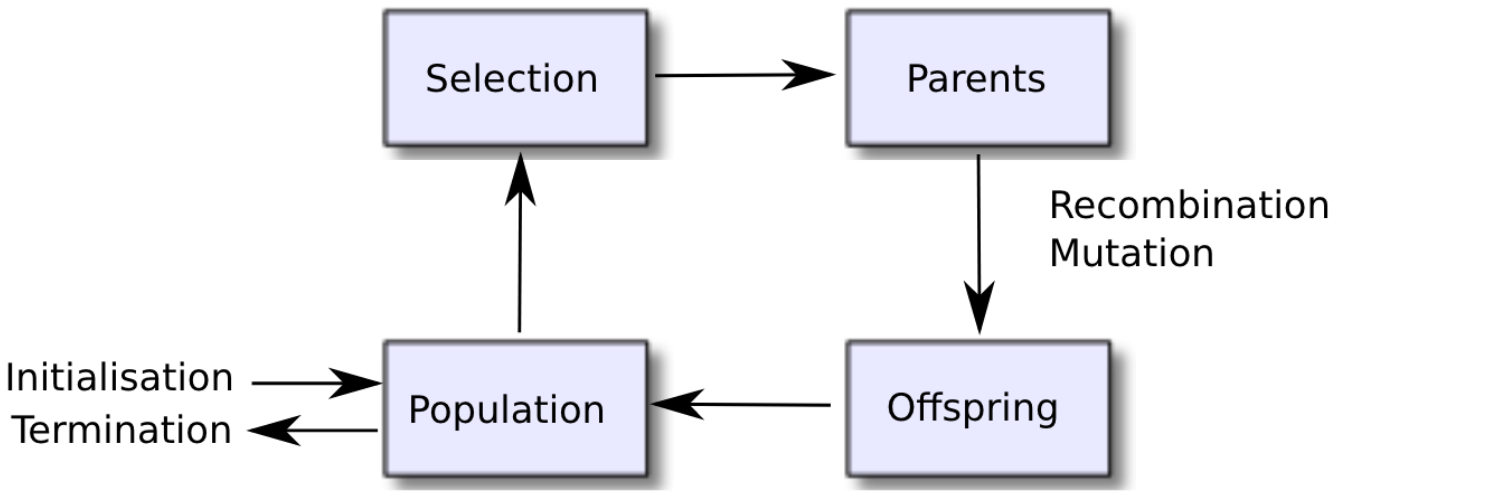


*The generalised EC also includes other natural-inspired algorithms, e.g., Particle Swarm Optimisation (PSO), Ant Colony Optimisation (ACO), Differential Evolution (DE), Estimation of Distribution Algorithms (EDAs), Simulated Annealing (SA), etc.

† Image source: https://www.baptistpress.com/resource-library/news/fossil-find-complicates-theory-of-human-evolution/

From Natural Evolution to Evolutionary Algorithms (EAs)



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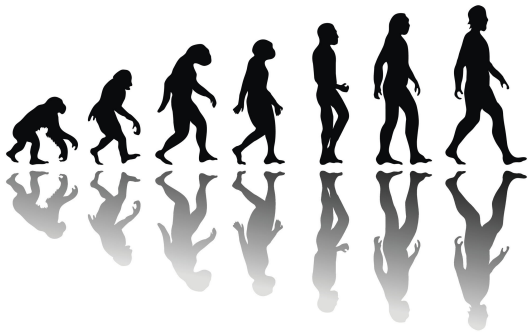

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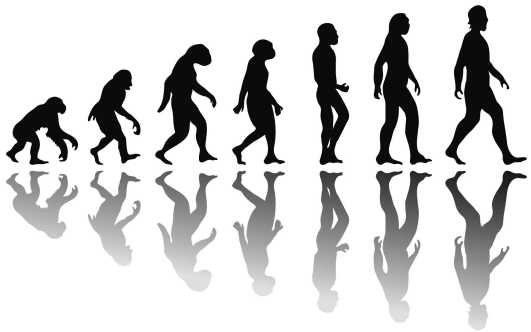

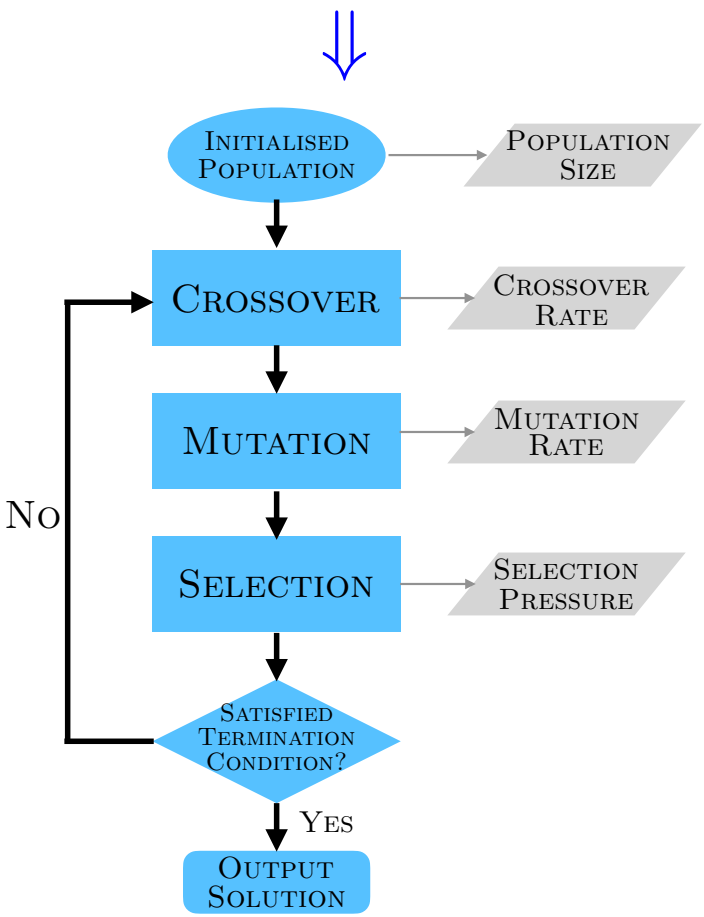

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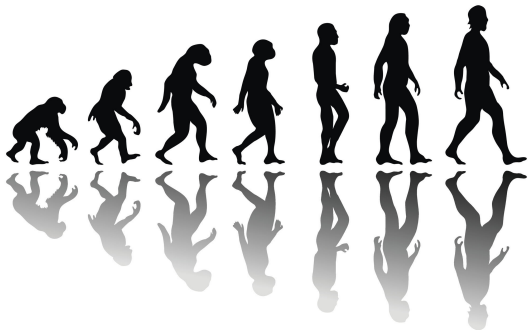

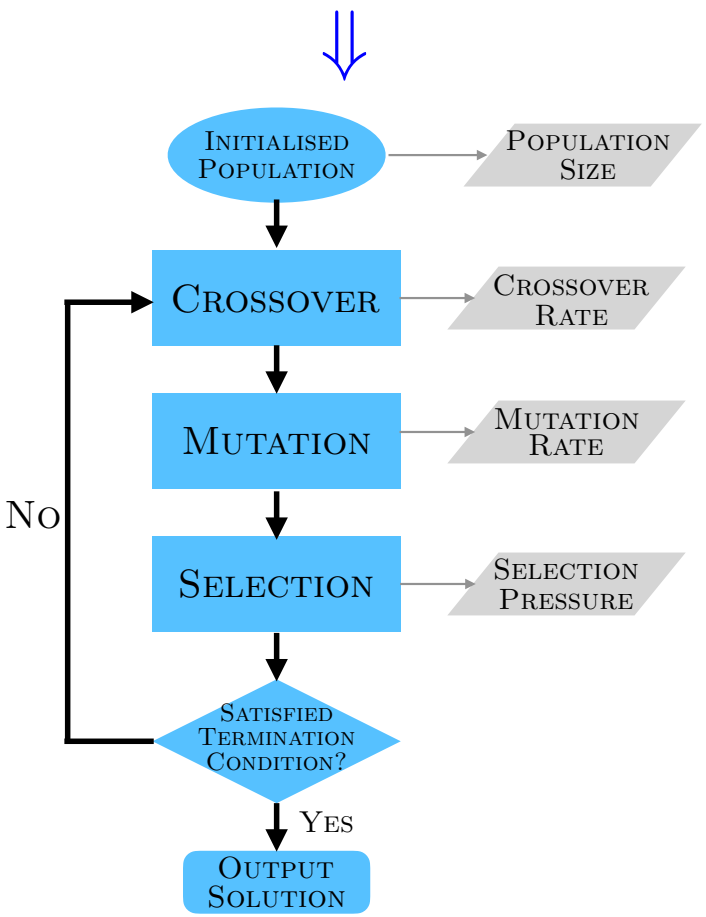

Classification of EAs:

- **Genetic Algorithms (GAs)** $\Rightarrow$ **Discrete/Binary**

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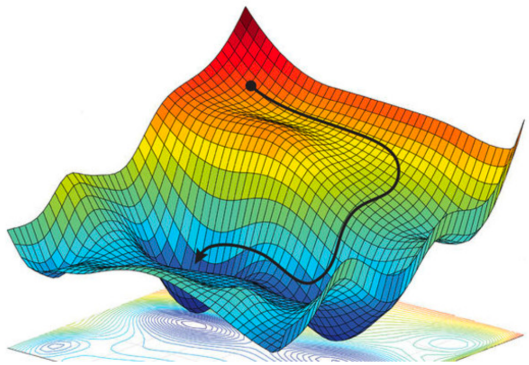
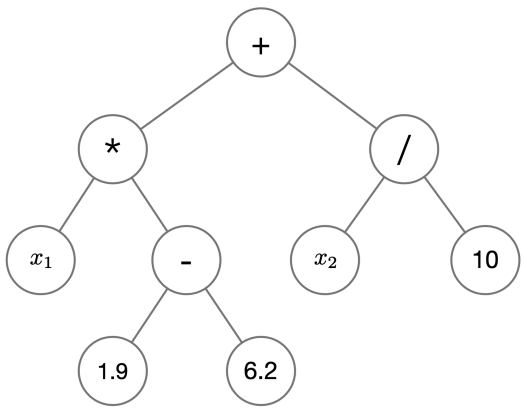
- **Evolutionary Strategy (ESs)** $\Rightarrow$ Continuous



- **Genetic Programming (GPs)** $\Rightarrow$ Tree



A Simple EA for Binary Search Space

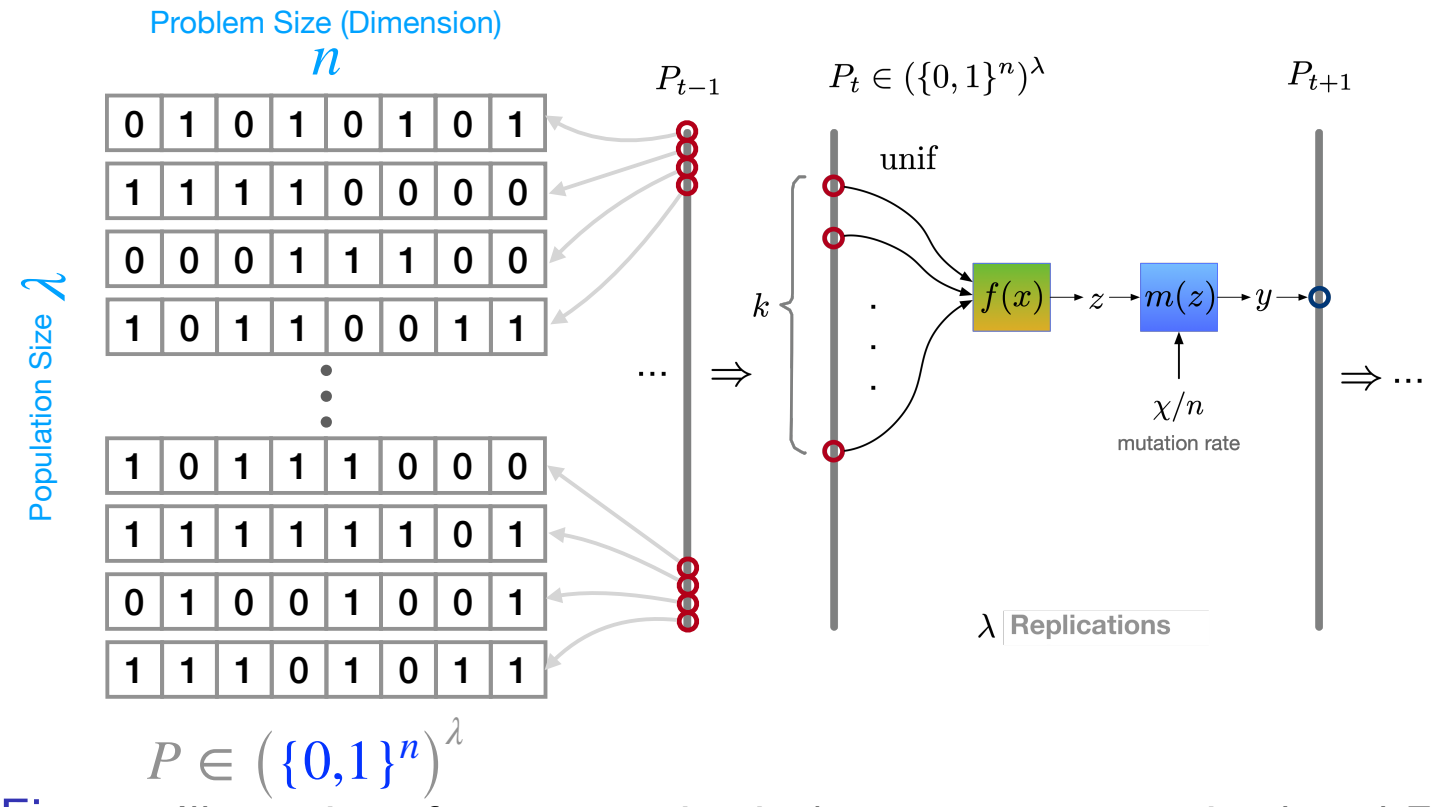


Figure: Illustration of one generation in k-tournament mutation-based EA

- **Selection Mechanism:** k-tournament (*here*), (μ,λ), Power-law, etc.
- **Mutation Operator:** $m : \{0,1\}^n \to \{0,1\}^n$
  Bit-wisely flip a solution z with probability χ/n, so-called **Mutation Rate**.

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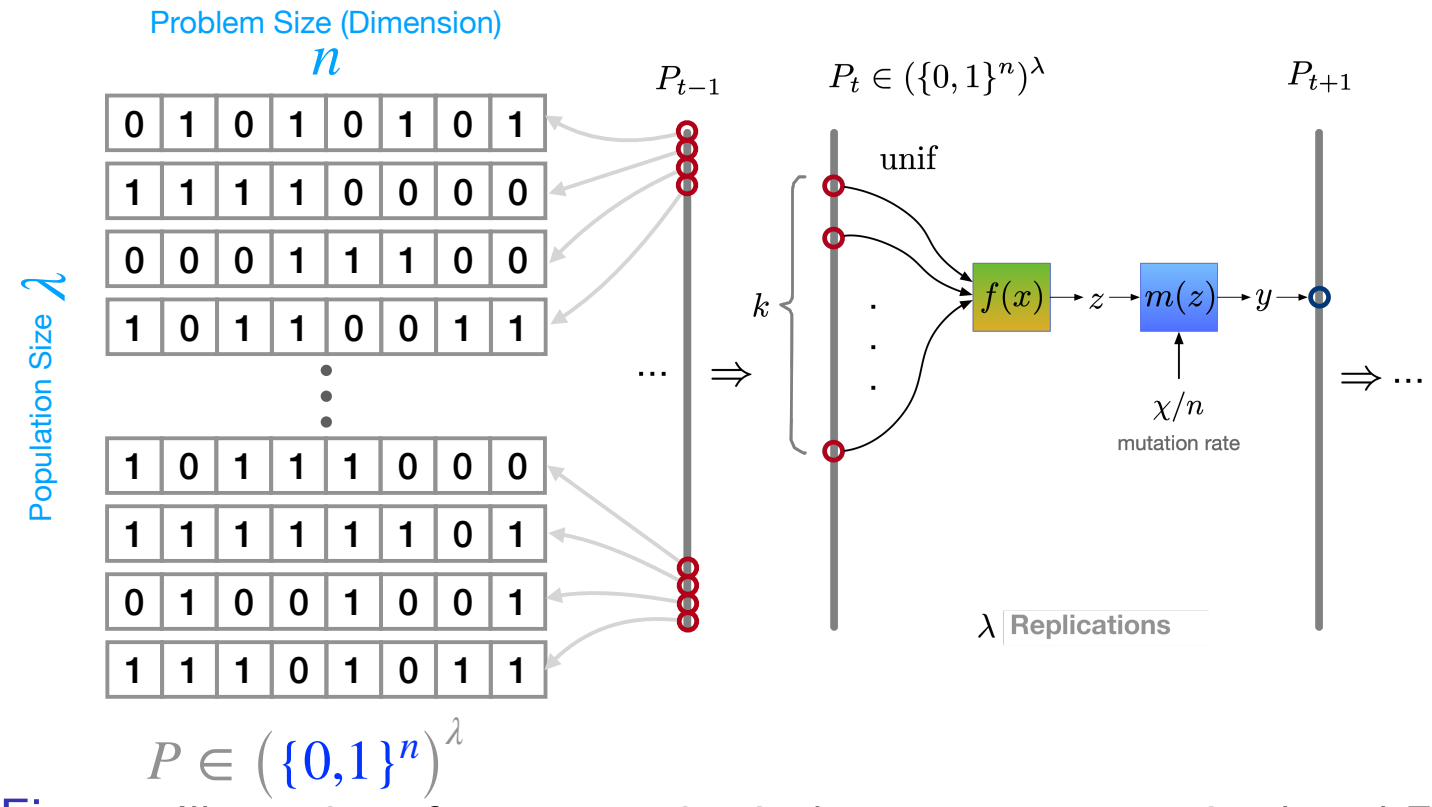

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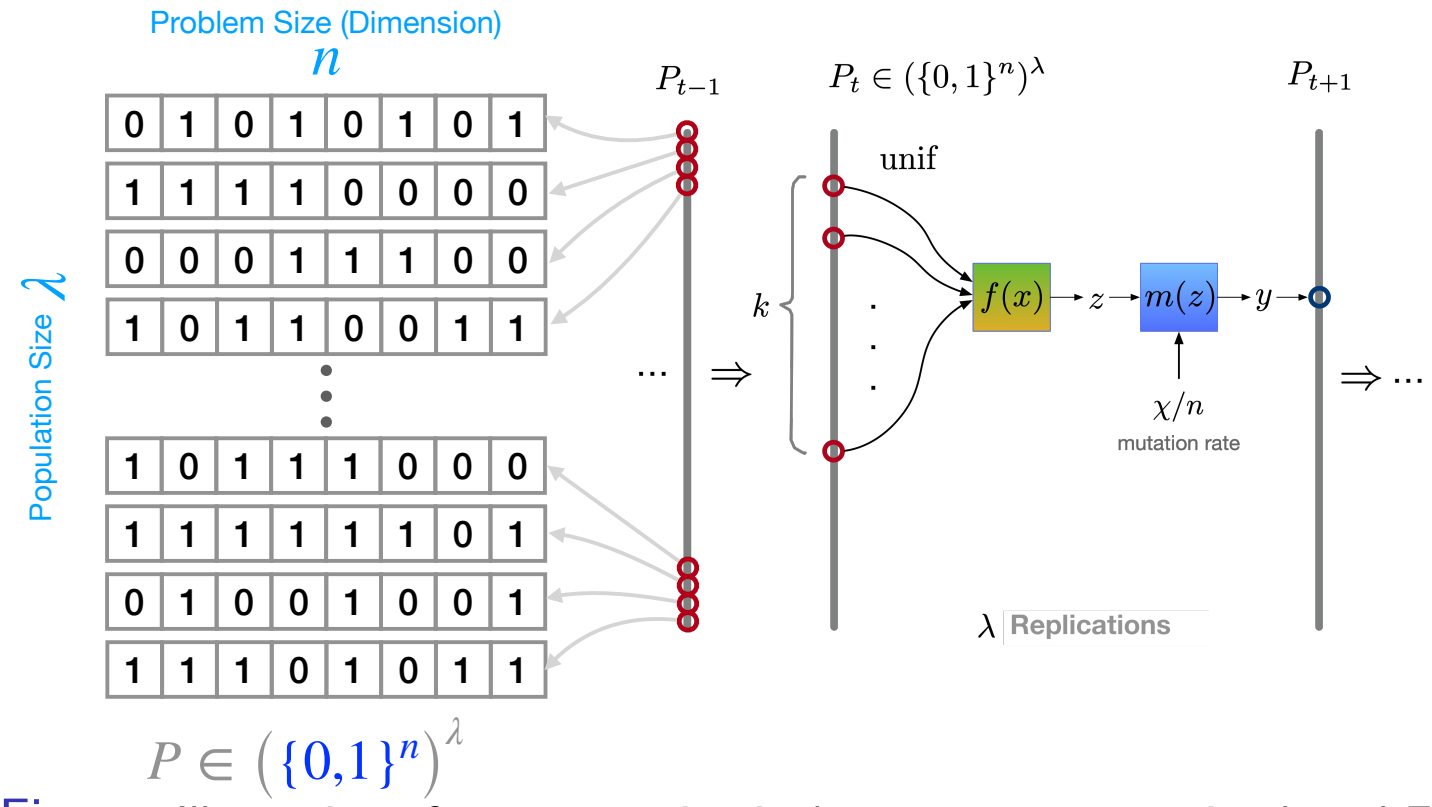

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Corollary 3.1. *The expected runtime of the 2-tournament EA using mutation rate $\chi = a/n$ for any constant $a > 0$ on LeadingOnes is $\Omega(n^2 \log(n))$.*

Theorem 5.2. *For any constant $\chi_{\text{high}} > 0$, $\chi_{\text{low}} = a/n$ where $a > 0$ is any constant, any constant $q \in [0, 1/2)$ and $p_c \in o(1) \cap \Omega(1/n)$, the expected runtime of the 2-tournament EA using SA-2mr from $\{\chi_{\text{high}}/n, \chi_{\text{low}}/n\}$ with adaptation parameter p_c and population size $\lambda > c \log(n)$ for a sufficiently large constant c on optimising LeadingOnes in the symmetric noise model (C, q) is $O\left(n\lambda \log(n) + n^3\right)$.*

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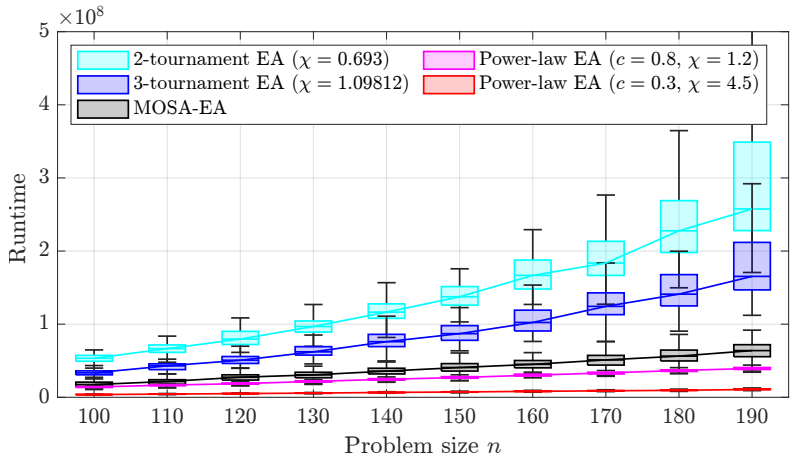

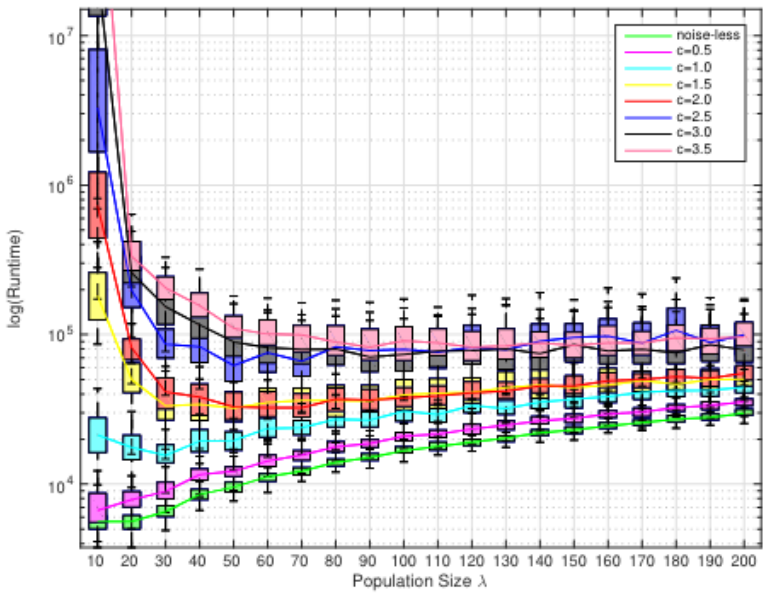

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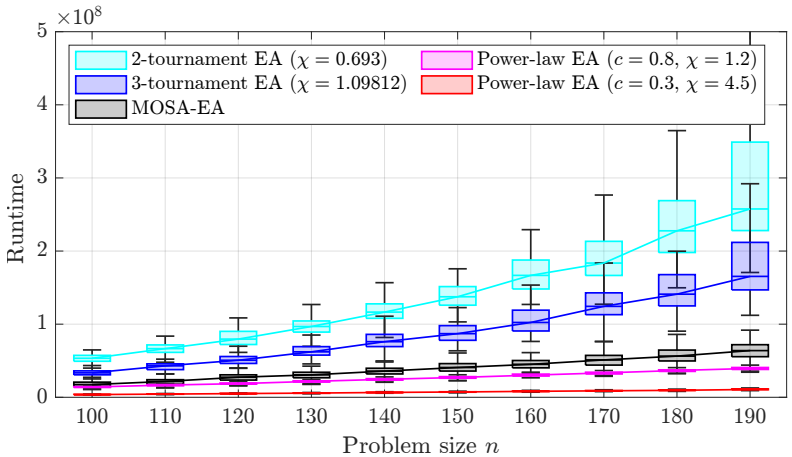

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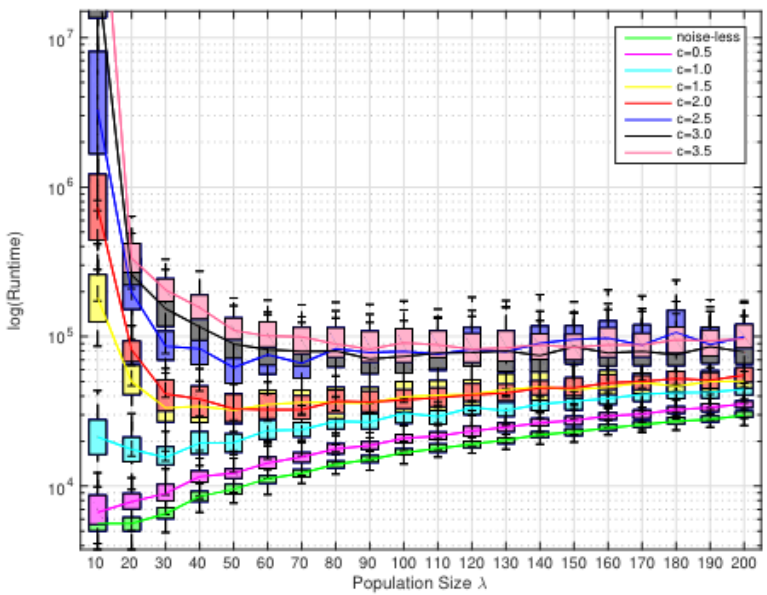

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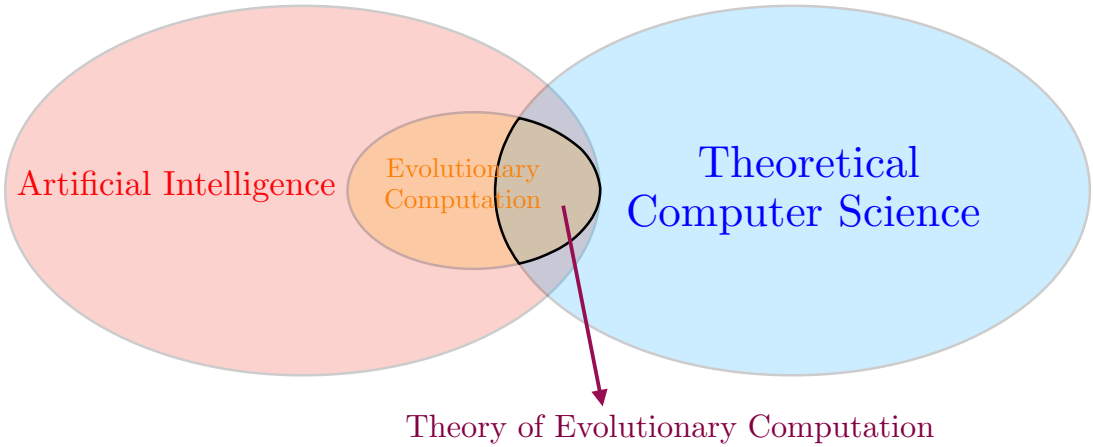

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After about 30 years of development,
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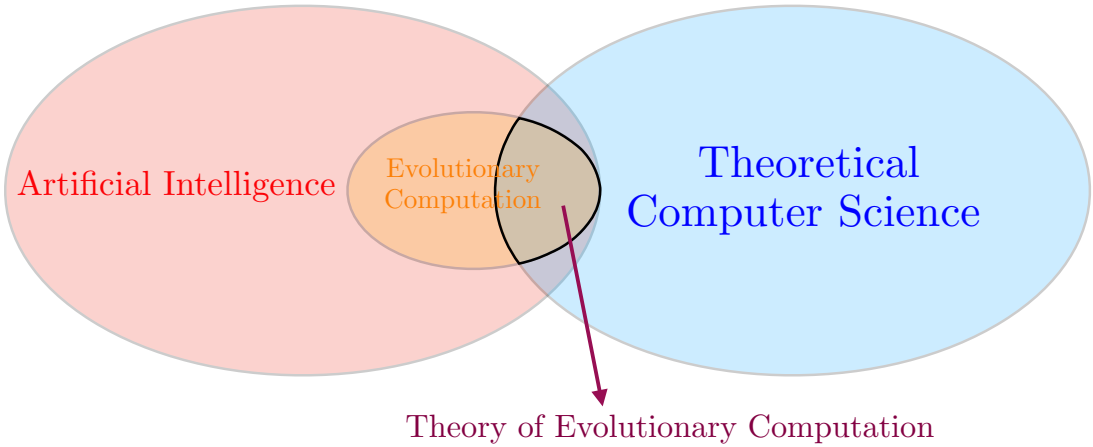

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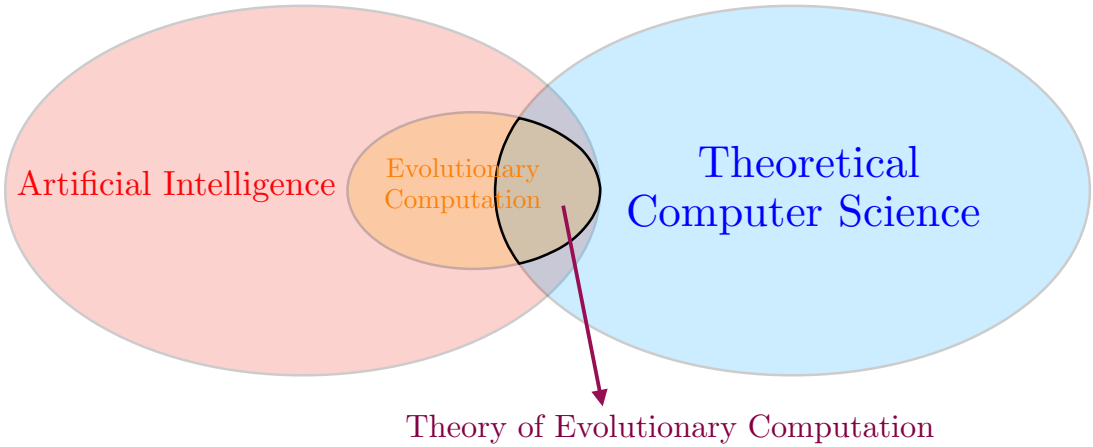

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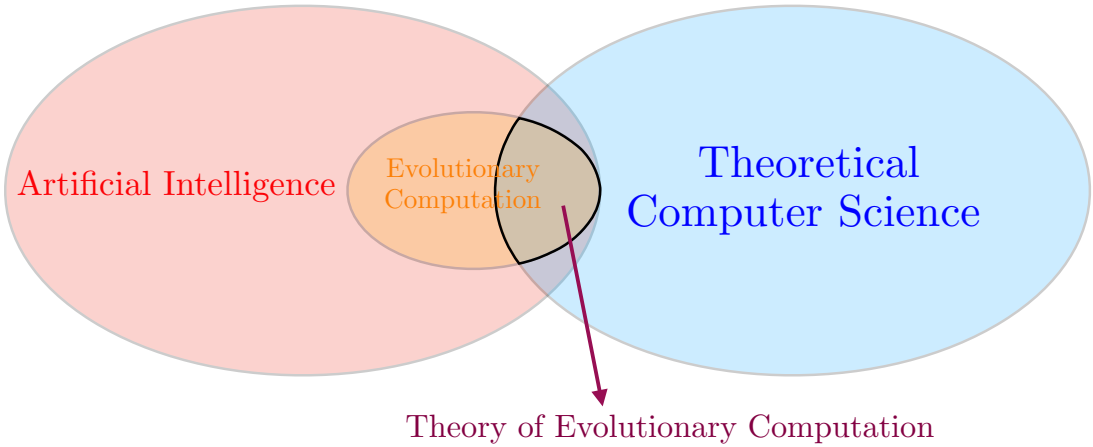

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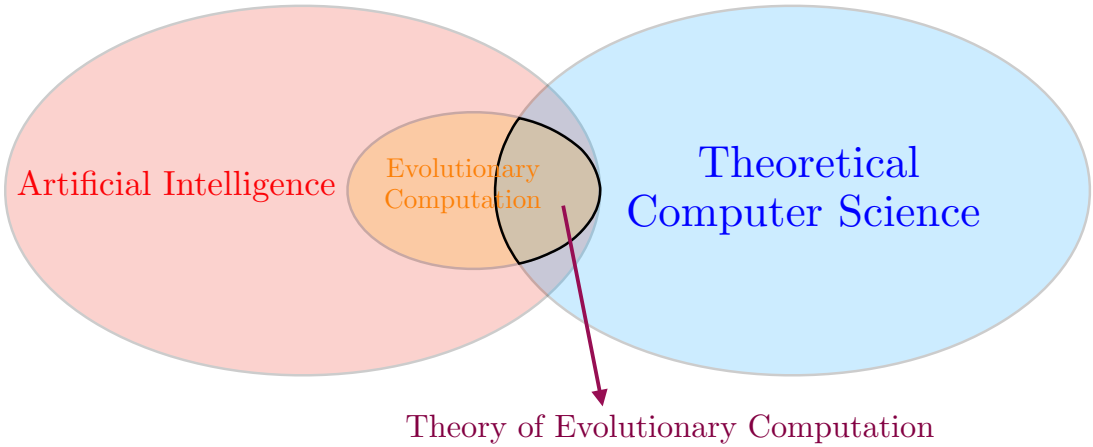

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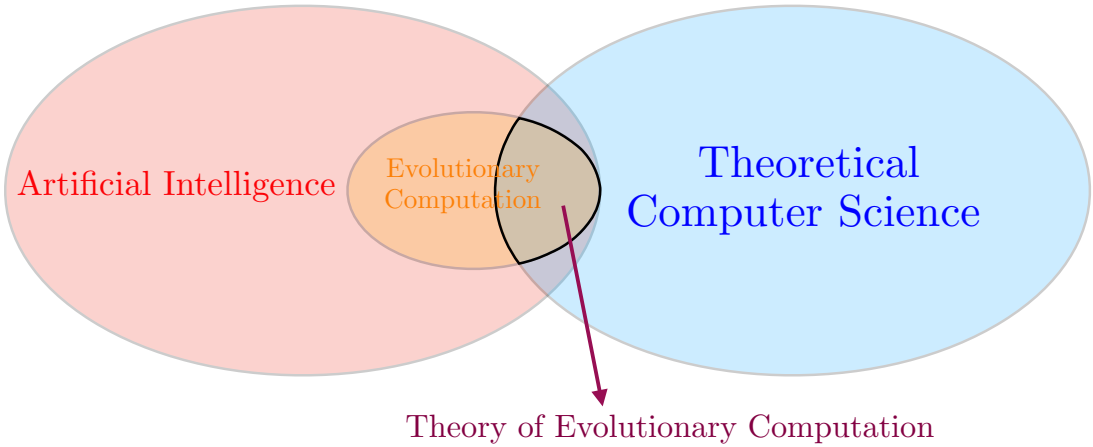

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      Problem size n
      TSP instances on n vertices
  - **A proof (automatically) give insight in**
    - Working principles of EC
    - Why the result is as it is
  - **Guide to design new algorithms**
    - Can see what exactly is the bottleneck for a result
    - How improve algorithms

Limitations of Theoretical Research (Doerr, 2020)

- **Limitations:**
  - **Restricted scope:**
    - So far, only known-structure optimisation problems, e.g., OneMax*, LeadingOnes†, Linear function‡, SparseLocalOpt Minimum Spanning Tree, Shortest Path, Knapsack, etc.
  - **Less precise results:** Constants are not tight, e.g.,
    - "$O(n^2)$" = "less than cn^2 for some unspecified constant c"
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Usually supplemented with experimental results.

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Recall: A Simple EA for Binary Search Space

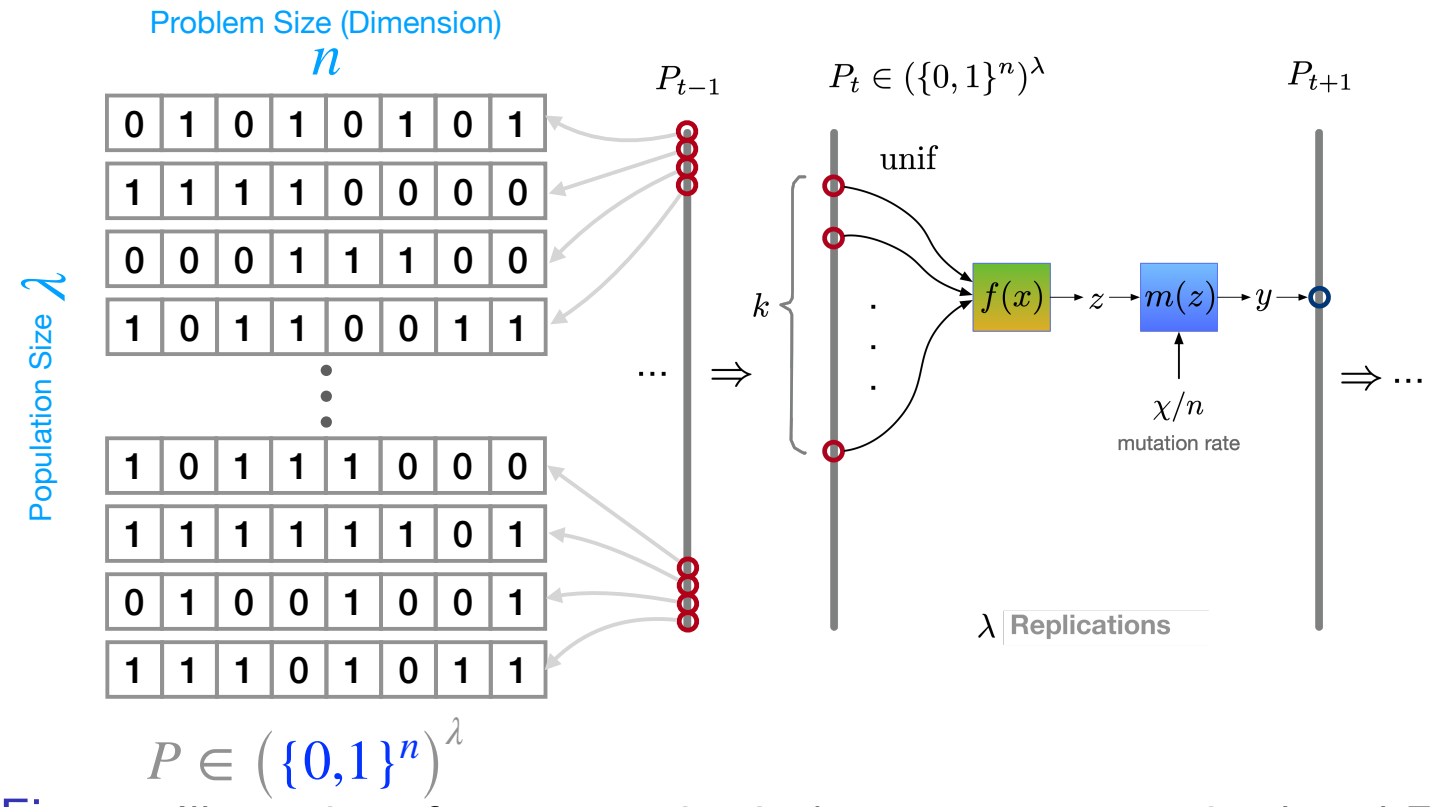

Figure: Illustration of one generation in k-tournament mutation-based EA

- **Selection Mechanism:** k-tournament (*This*), (μ,λ), Power-law, etc.
- **Mutation Operator:** $m : \{0,1\}^n \to \{0,1\}^n$
  Bit-wisely flip a solution z with probability χ/n, so-called **Mutation Rate**.

Parameter Settings in EAs

- EAs are parameterised algorithms.
- Parameter setting can dramatically impact the performance of EAs (Doerr and Doerr, 2020). $\Longrightarrow$ **IMPORTANCE**
- Parameter setting is instance- and state-dependent (Doerr and Doerr, 2020). $\Longrightarrow$ **DIFFICULTY**
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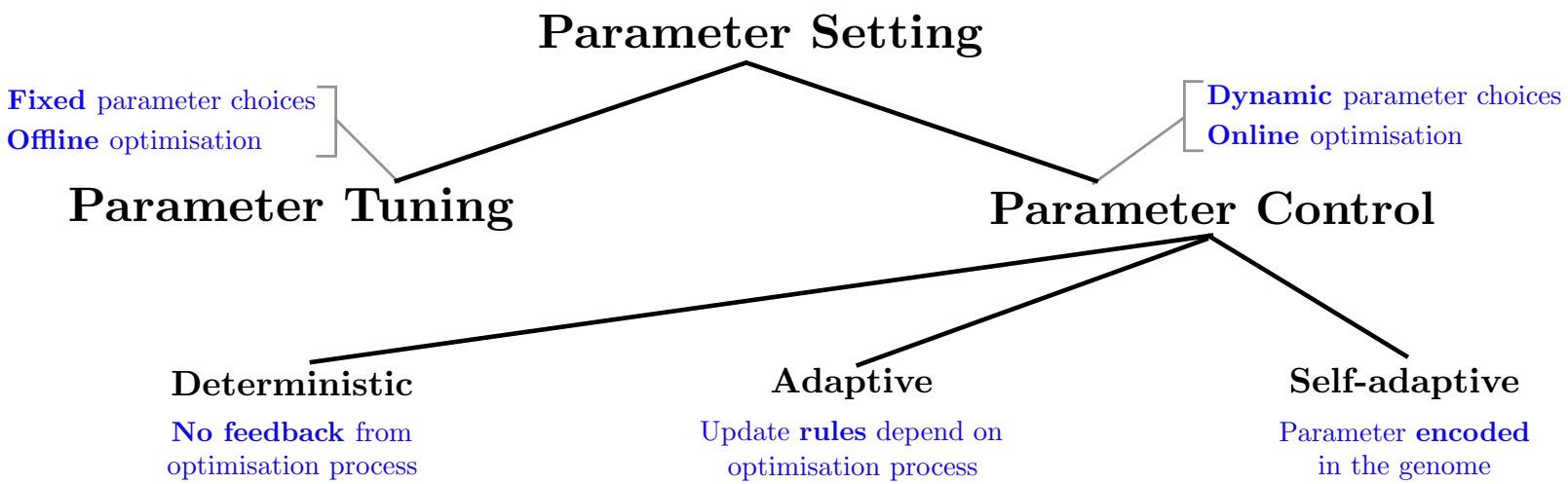

Self-adaptive Parameter Control Mechanism

The individual with "right" parameter setting is promising to improve.

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Self-adaptation is a more **natural** way to control parameters

$\Rightarrow$ **Encoded** parameters into genome and

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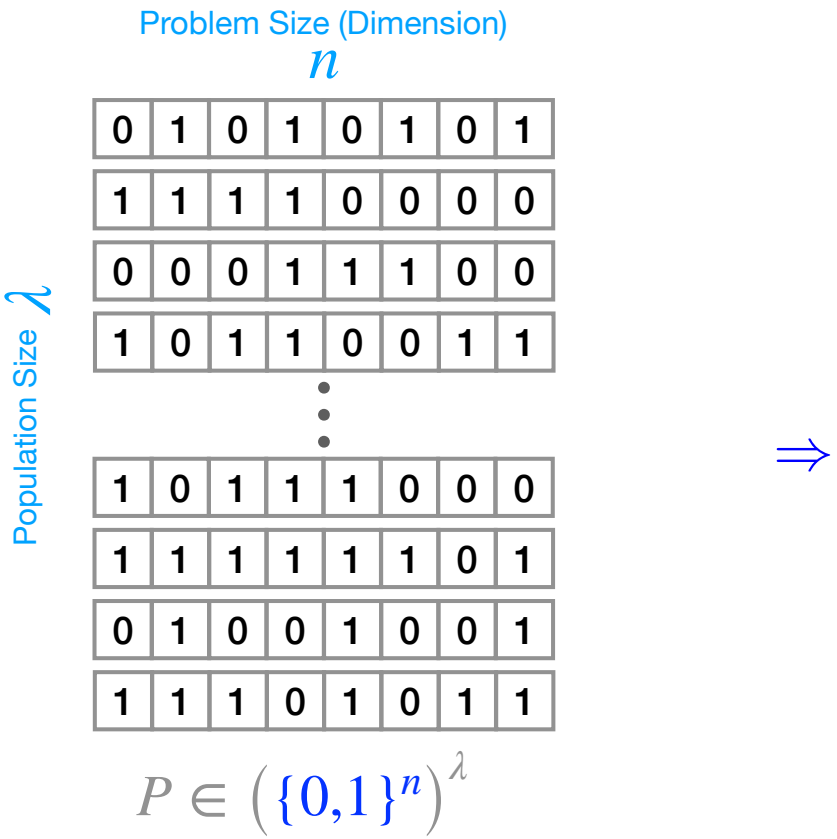

Figure: Static Population

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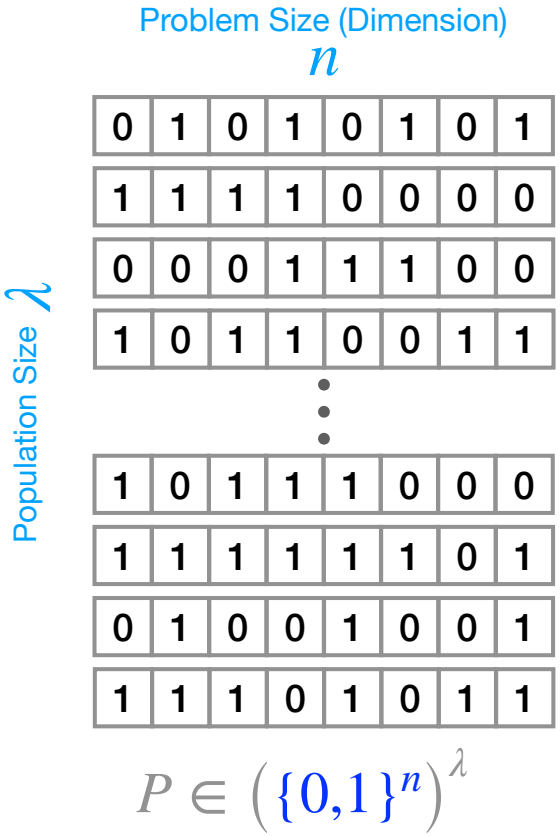

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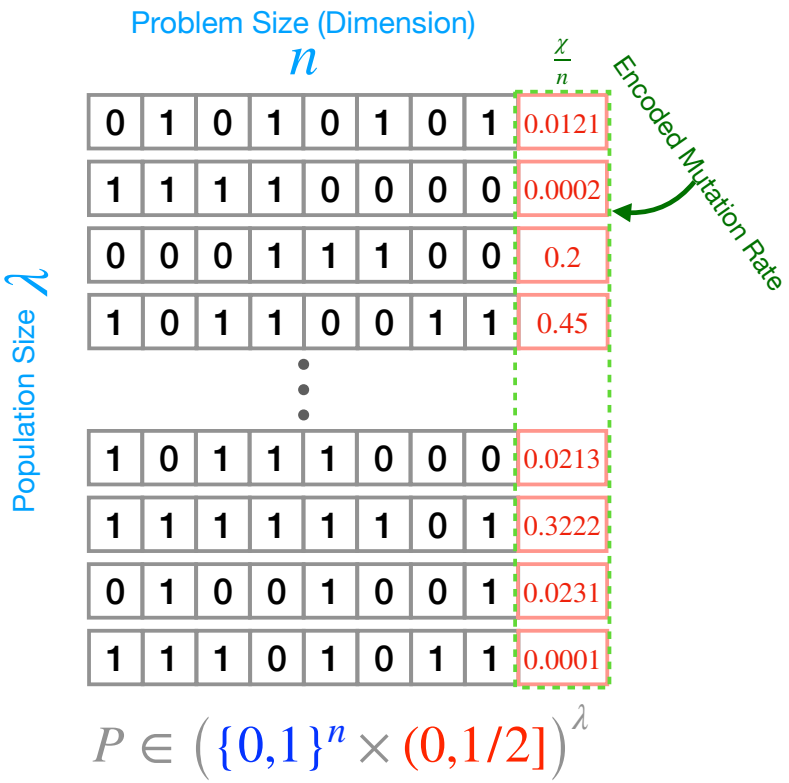

Figure: Self-adaptive Population

An example EA with Self-adapting Mutation Rate

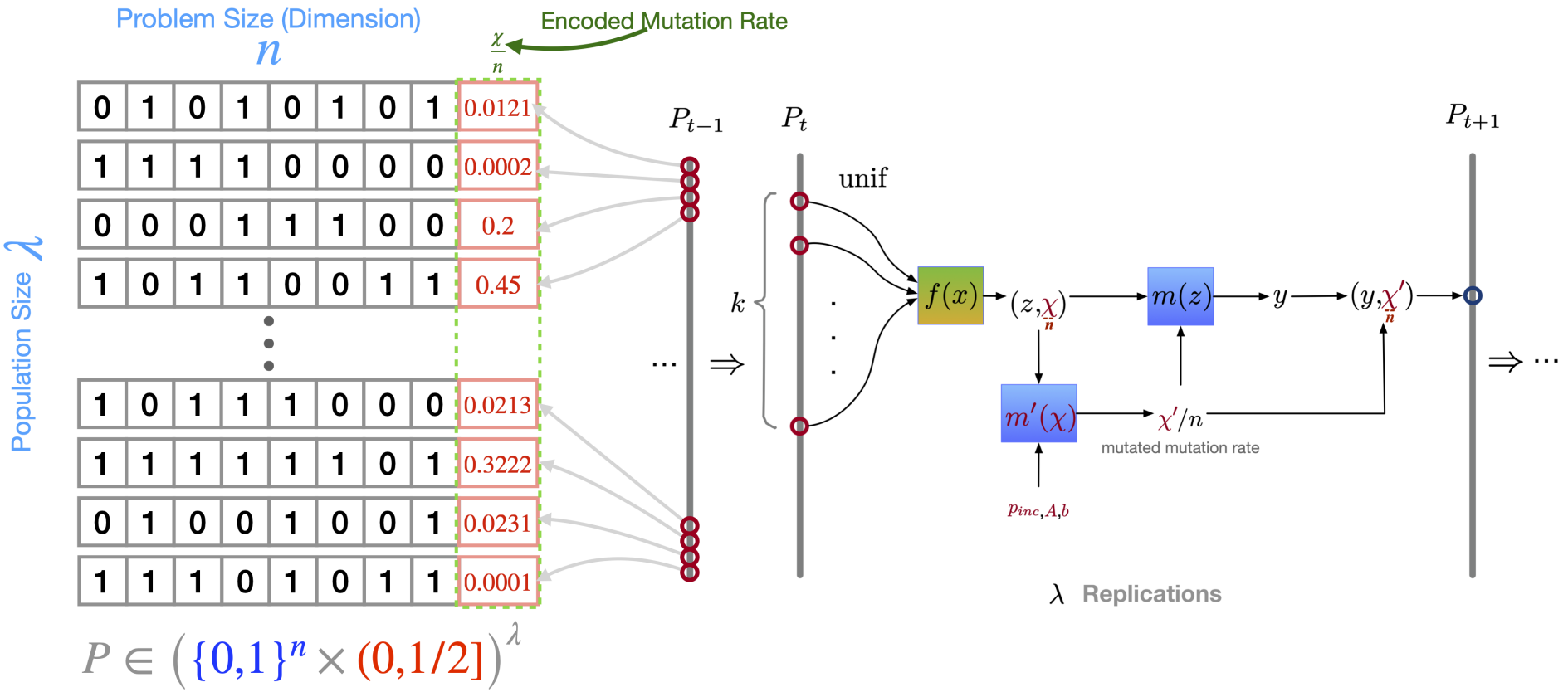

Figure: Illustration of one generation in k-tournament EA with self-adapting mutation rate

- **Comparison (sorting) rule:** Prefer to higher fitness than higher mutation rate.
- **Selection Mechanism:** k-tournament
- **Mutation Rate Adaptation Strategy:** $m'(\chi) : \mathbb{R} \to \mathbb{R}$
  $\Rightarrow$ Increase by $\times A$ with prob. p_{inc}, otherwise decrease by $\div A$.
- **Bitwise Mutation Operator:** Mutate the solution z with adapted (new) mutation rate.

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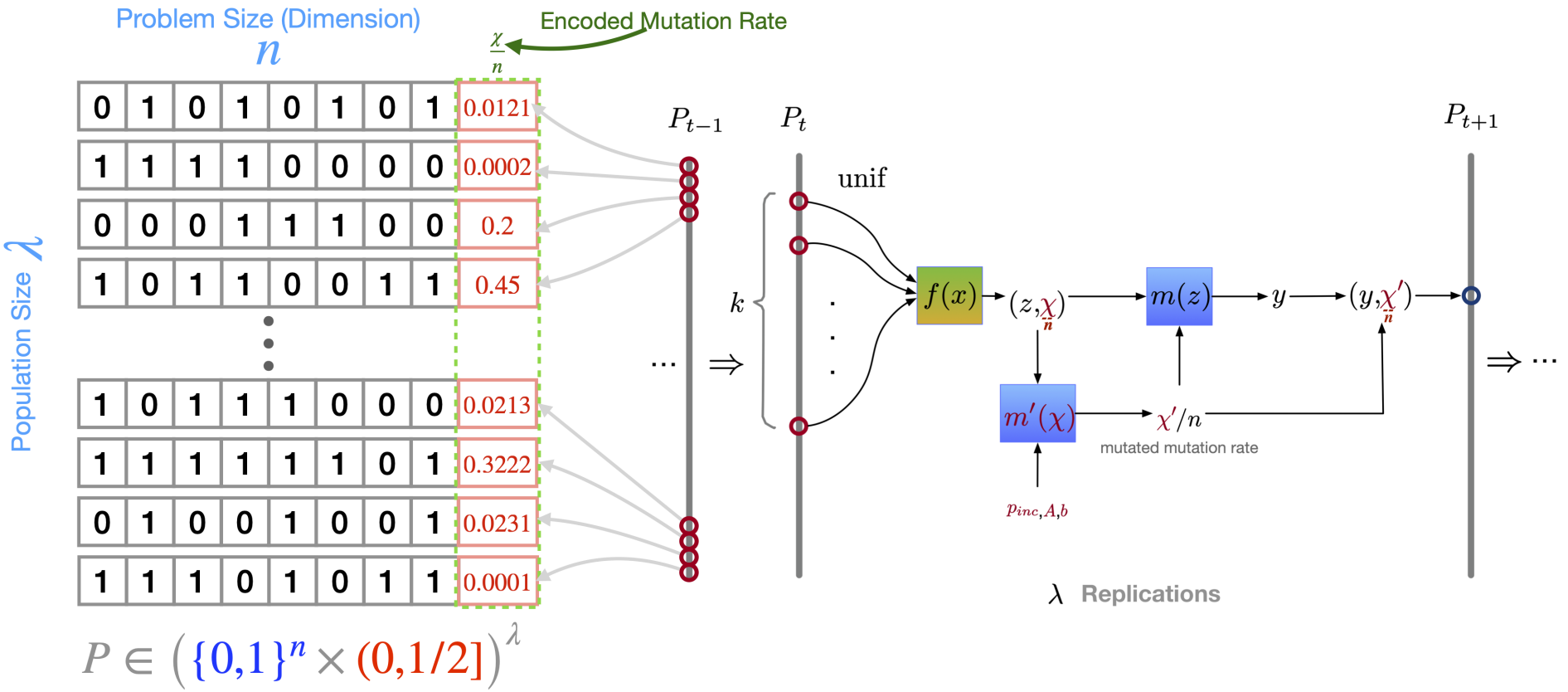

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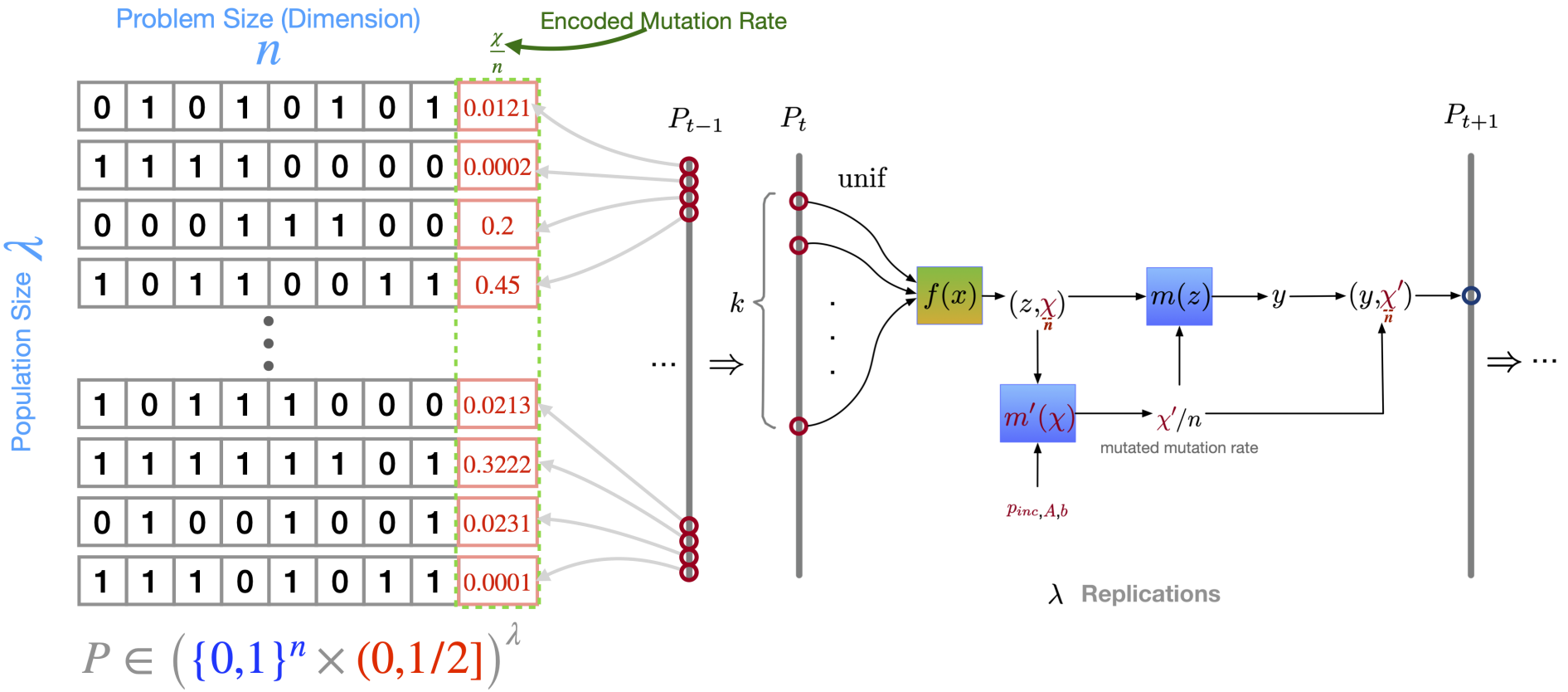

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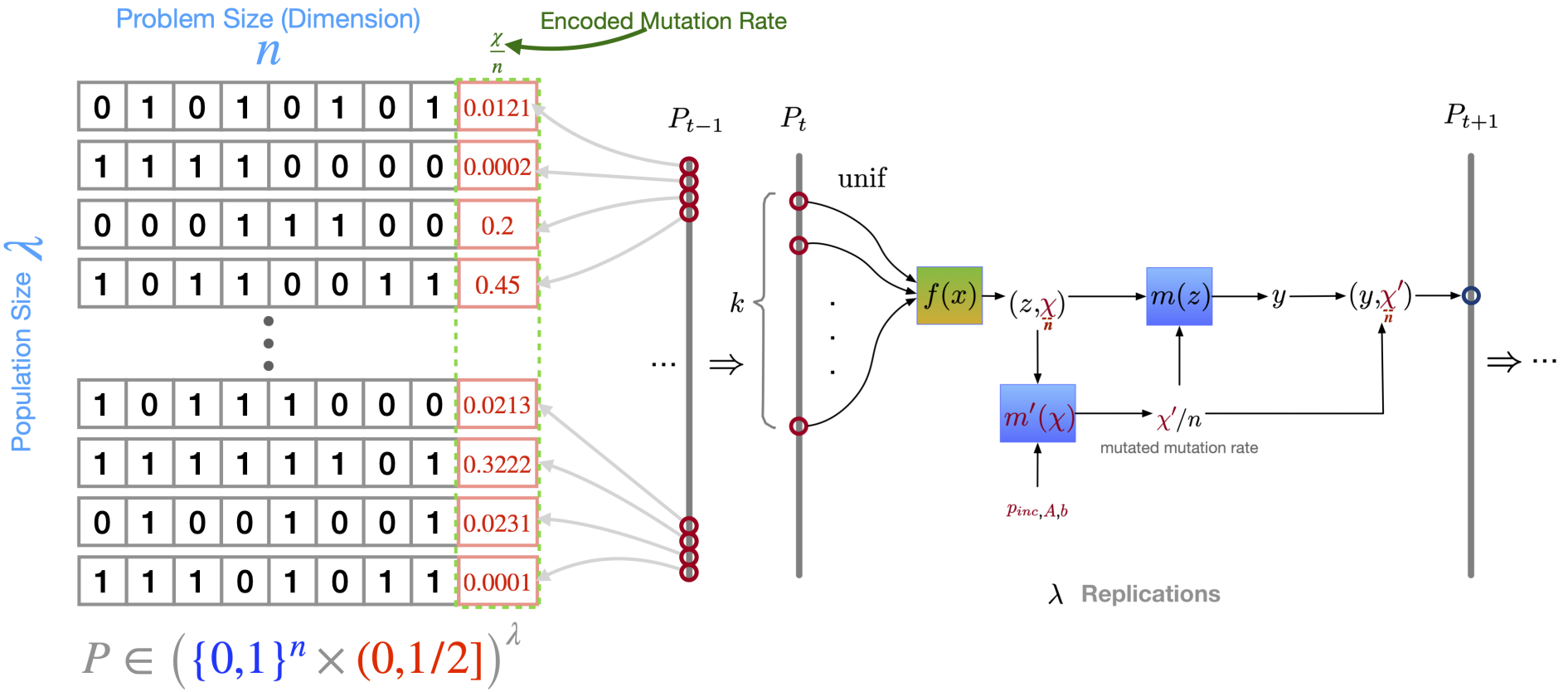

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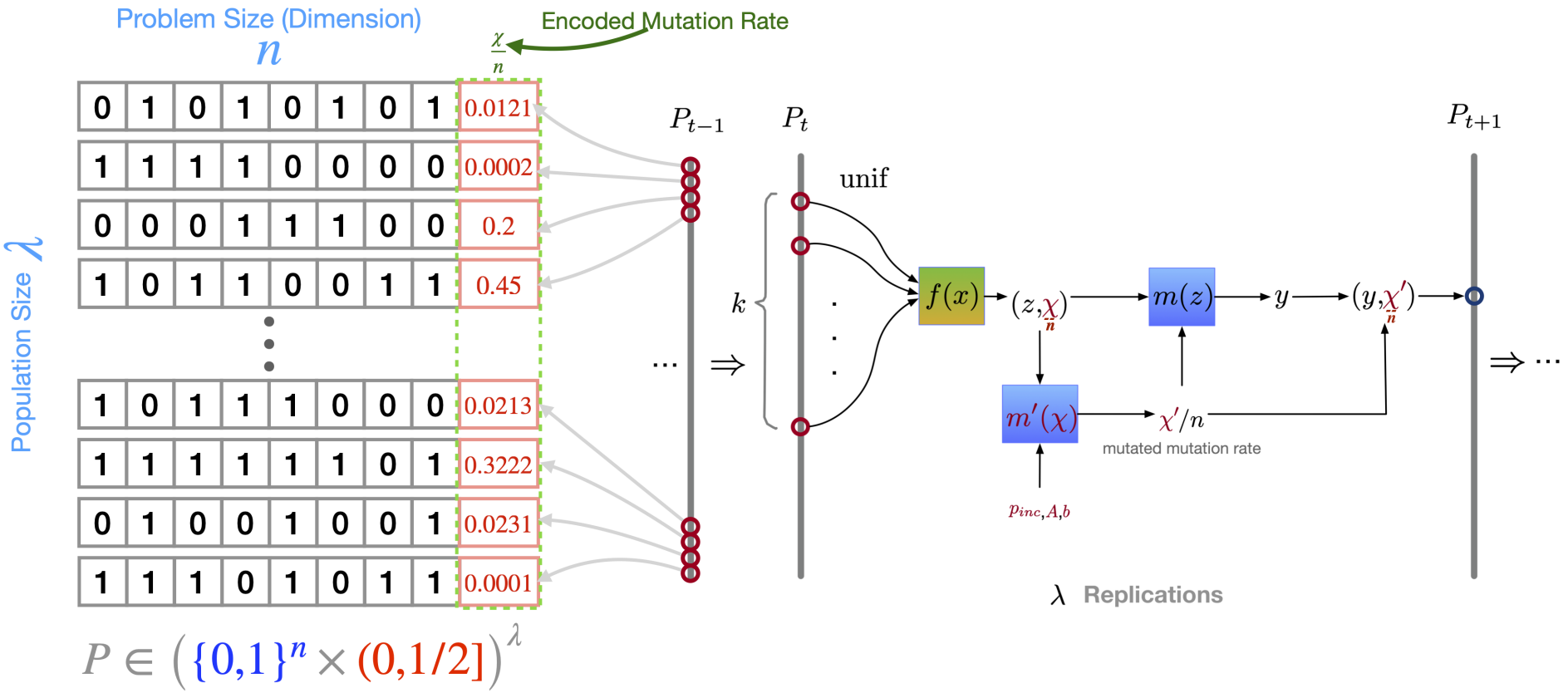

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Existing Results on Self-adaptation in EC

Existing Studies on Self-adaptive EAs (SAEAs)

Several theoretical and empirical studies have been conducted on SAEAs, demonstrating their advantages in some scenarios:

- **Toy Problems:**
  - (Doerr et al., 2021)
- **Uncertain Optimisation:**
  - **Unknown-length Functions**
    - (Case and Lehre, 2020)
  - **Noisy Optimisation**
    - (Lehre and Qin, 2021, 2022a, 2023a)
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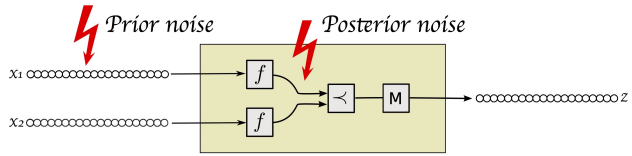

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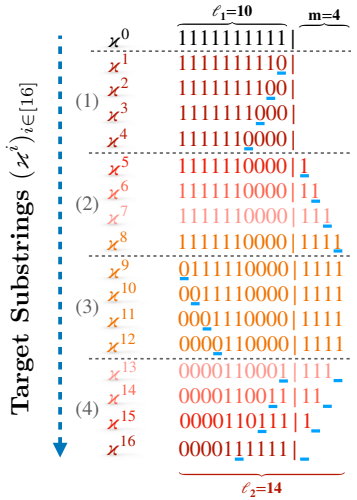

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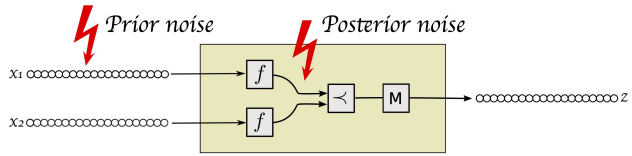

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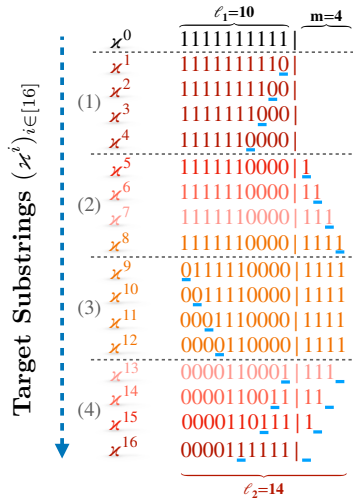

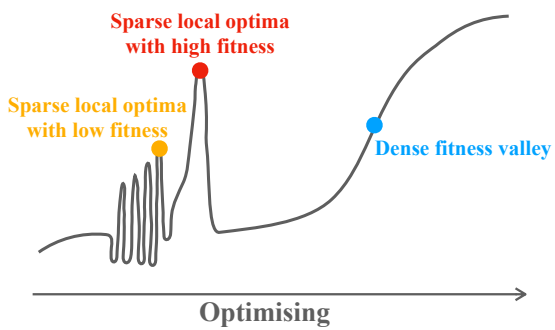

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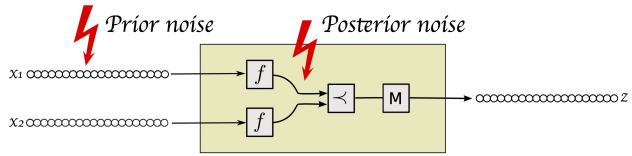

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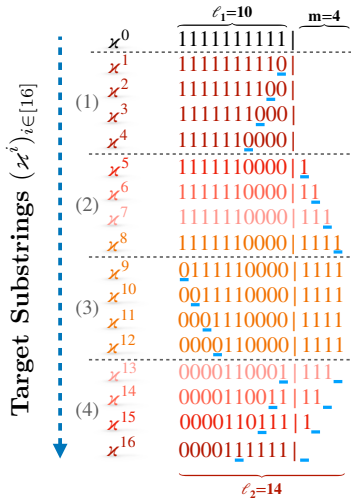
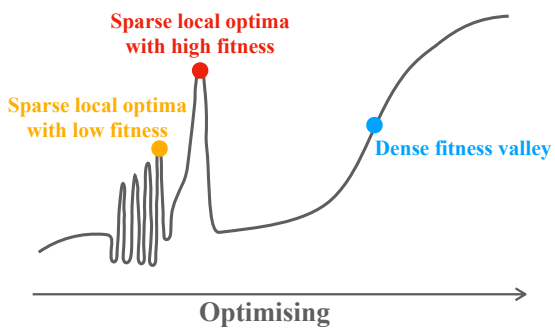
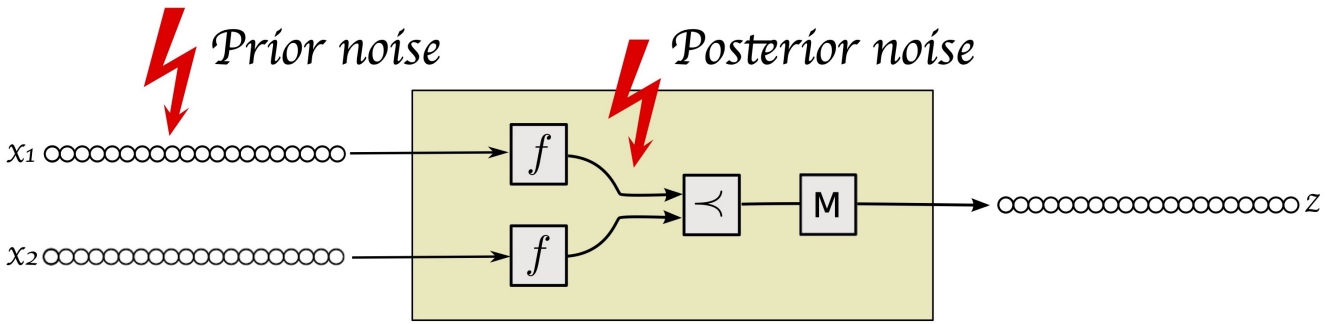




Self-adaptation in Noisy Optimisation

SAEAs on Noisy Optimisation



- From the previous theoretical study (Lehre and Qin, 2021, 2022a):
  - Non-elitist EAs can cope with the higher levels of noise by reducing the mutation rate.
  - Too small mutation rate slows down optimisation process.
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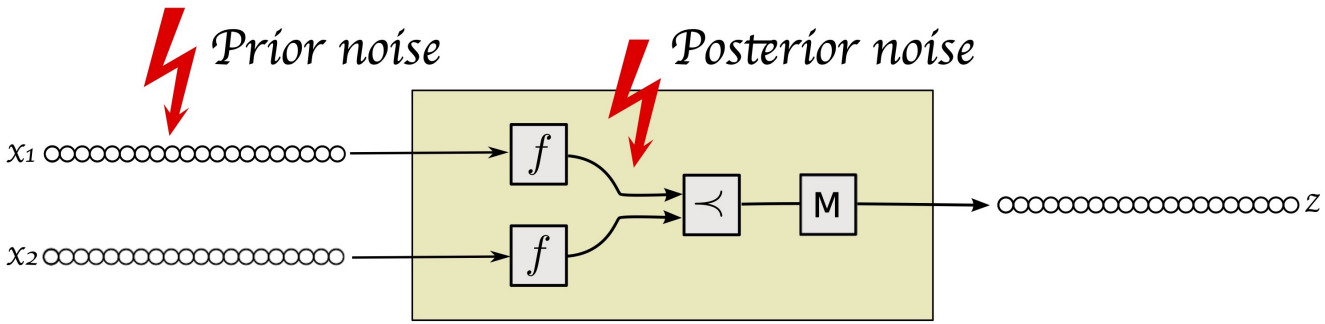

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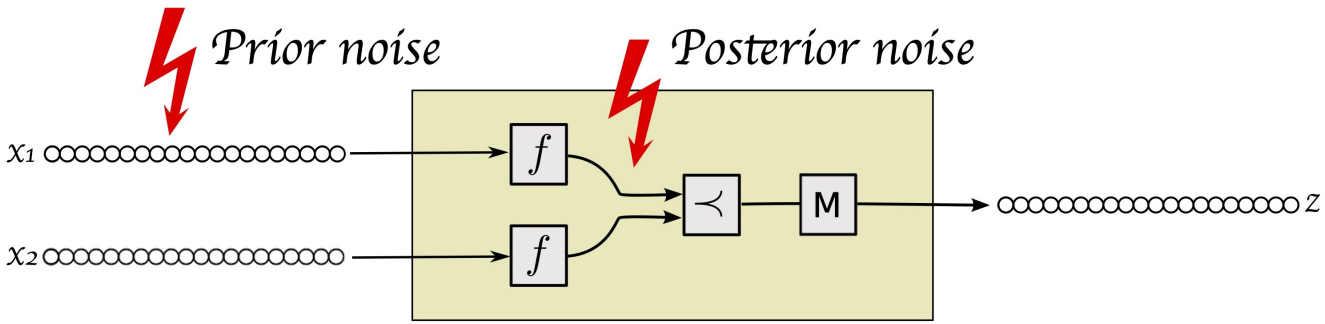

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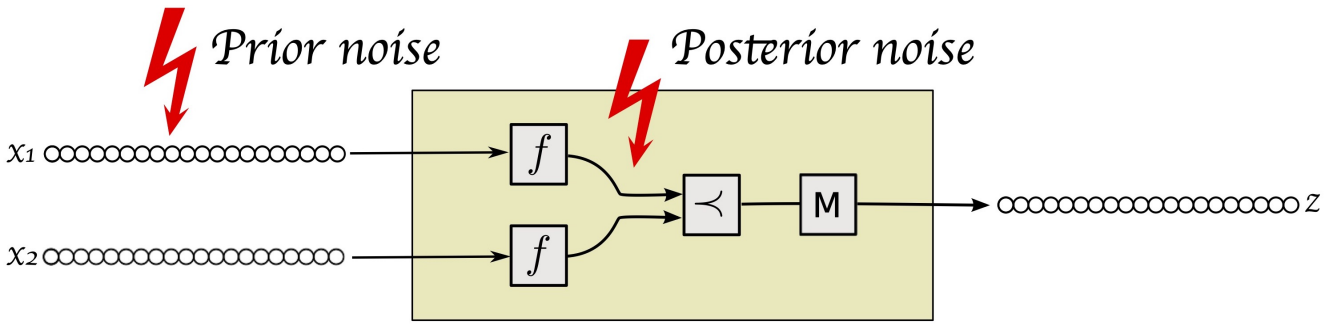

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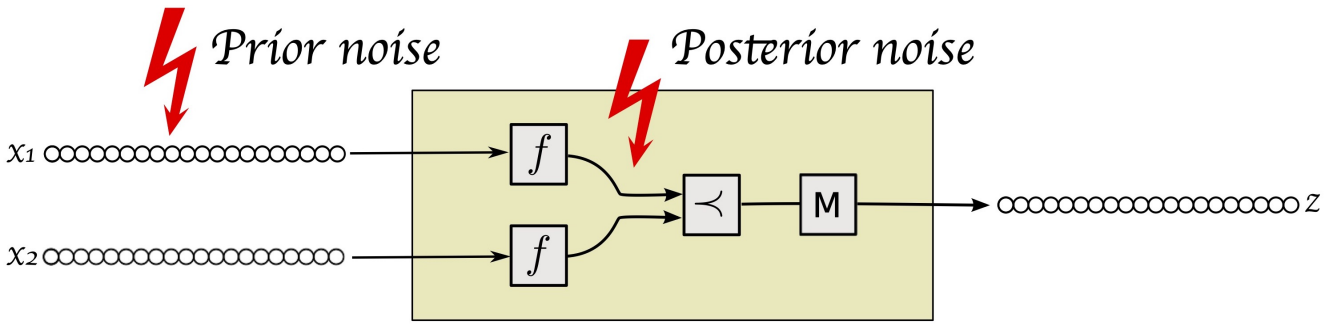

- From the previous theoretical study (Lehre and Qin, 2021, 2022a):
  - Non-elitist EAs can cope with the higher levels of noise by reducing the mutation rate.
  - Too small mutation rate slows down optimisation process.
  - However, we need to know the **exact** noise level to set a **proper** mutation rate.

Runtime Analysis for SAEAs under Noise

Lehre and Qin (2023) rigorously analysed runtimes of a EA with self-adapting from high/low mutation rates under symmetric noise:

Algorithm	Noise-free	Under Noise
2-tour' EA with χ_{high}	$O(n^2)$ (Well-known)	$e^{\Omega(n)}$ * (Lehre and Qin, 2022a)
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- **High mutation rate** fails EA under noise.
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Empirical study on Noise

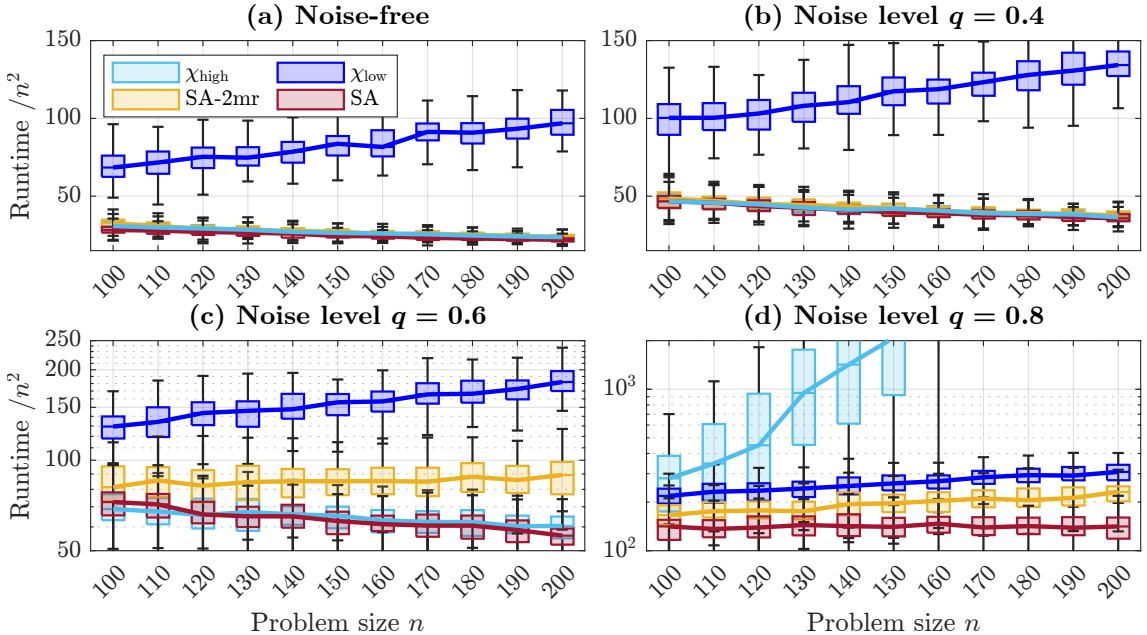


Figure: Runtimes on LeadingOnes under one-bit noise with noise levels q.

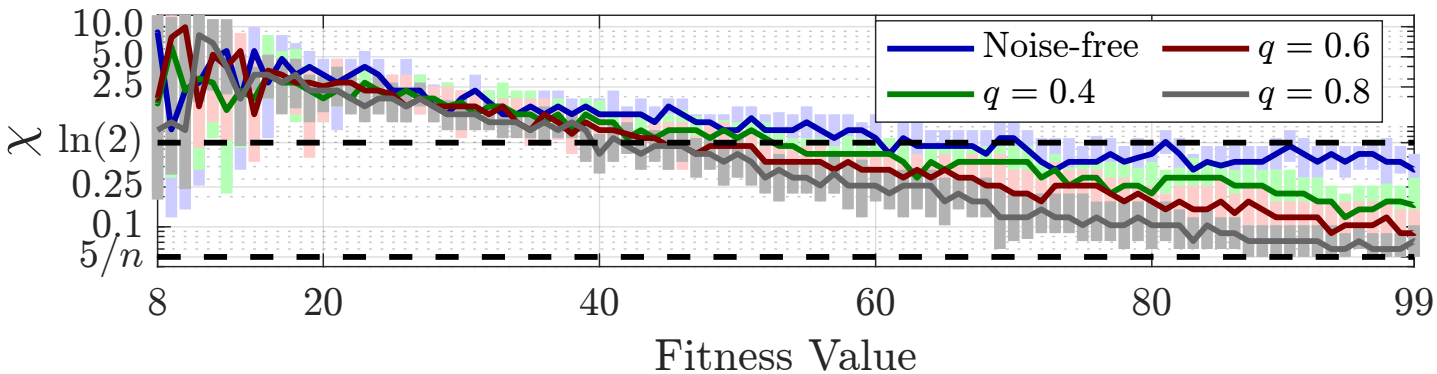
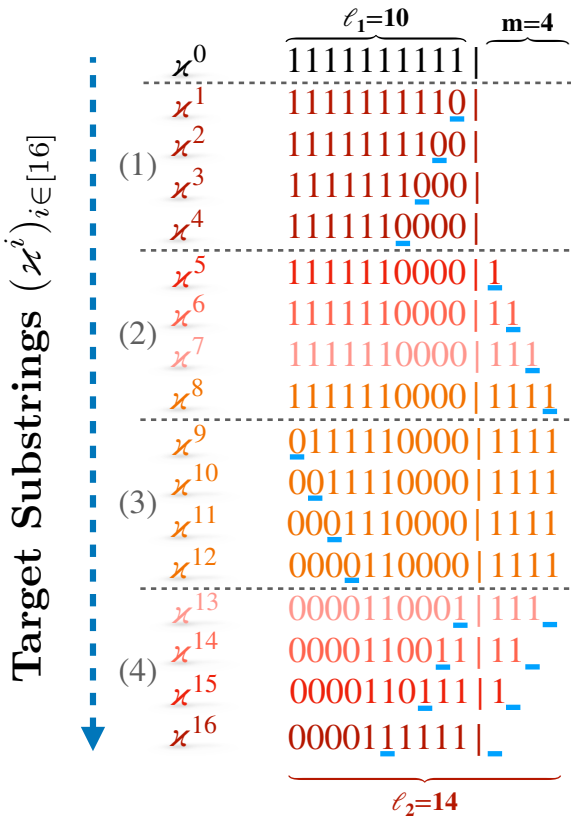


Figure: Mutation rate of SAEAs on LeadingOnes under one-bit noise with noise levels q.

Self-adaptation in Dynamic Optimisation

Dynamic Substring Matching ($\mathrm{DSM}^{\varkappa,m,\varepsilon,k}$) problem



Four phases:

$\Leftarrow$ Starting target substring $\varkappa \in \{0,1\}^{\ell_1}$

$\Leftarrow$ (1) $i \in [m]$, $\varkappa^{i-1}$ and $\varkappa^i$ are the same length but one bit different

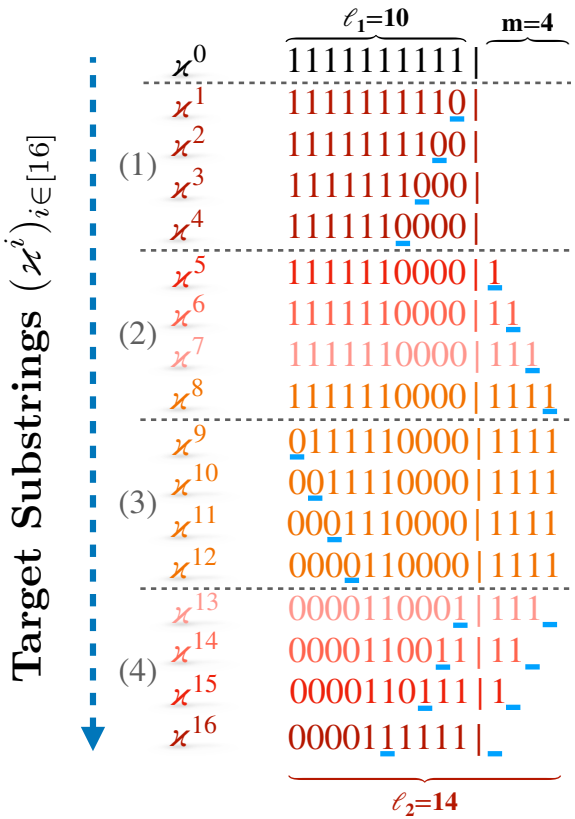
$\Leftarrow$ (2) $i \in [m+1..2m]$, the target substrings are becoming longer

$\Leftarrow$ (3) $i \in [2m+1..3m]$, similar to stage (1): $\varkappa^{i-1}$ and $\varkappa^i$ are the same length but one bit different

$\Leftarrow$ (4) $i \in [3m+1..4m]$, the target substrings are becoming shorter

Algorithms are required to find solutions that match the current target substring within the evaluation budget of $\boxed{kn^{\varepsilon}|\varkappa^i|}$.

Dynamic Substring Matching ($\mathrm{DSM}^{\varkappa,m,\varepsilon,k}$) problem



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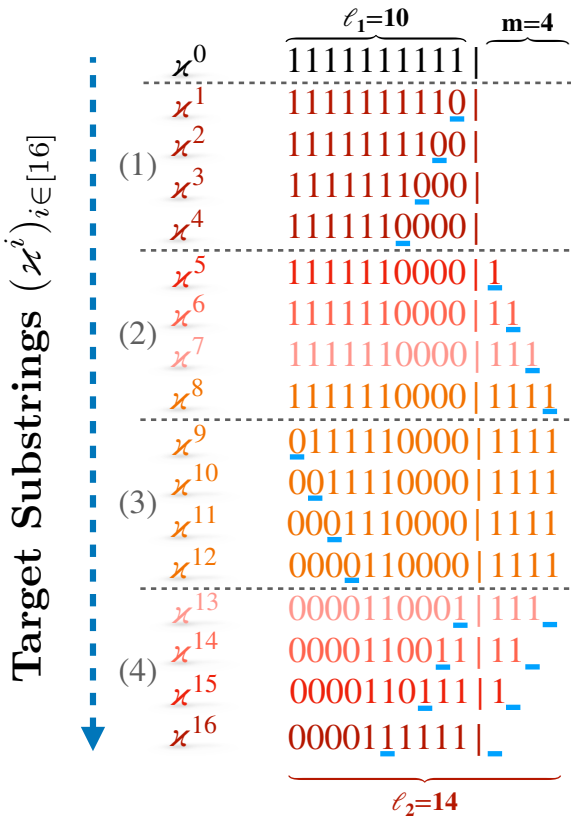

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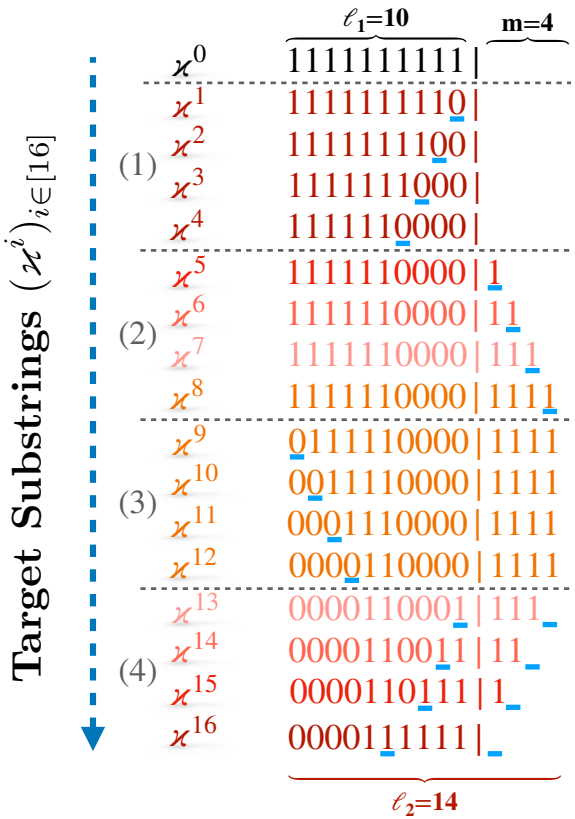

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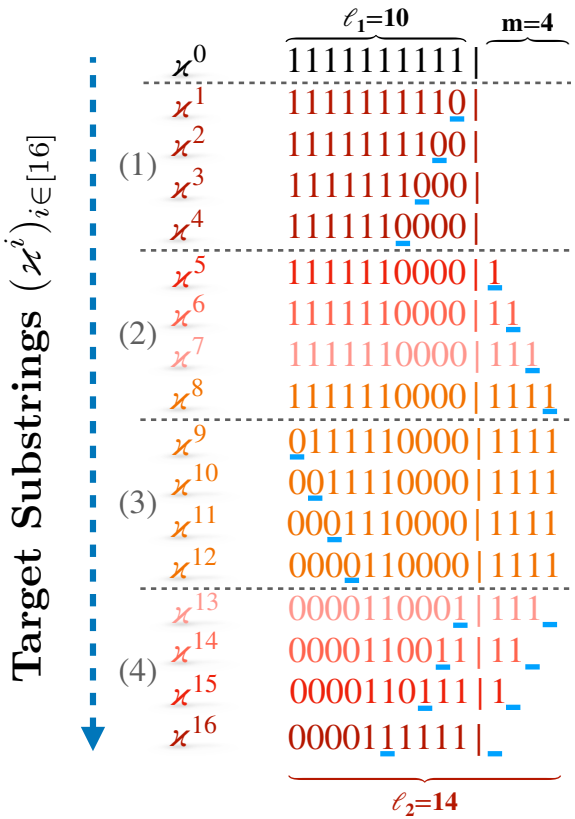

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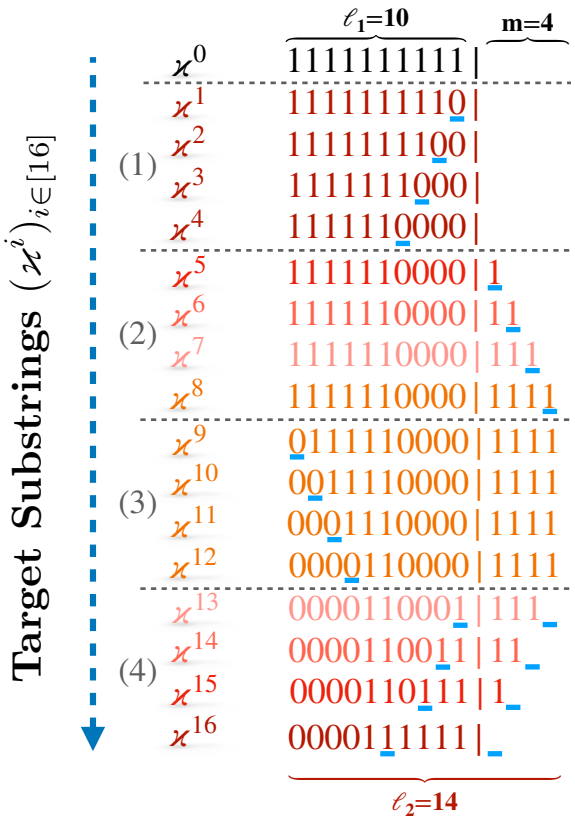

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Theoretical Analysis

Lehre and Qin (2023b) performed runtime analysis of EAs on the DSM problem:

Static mutation-based EAs	(μ, λ) **self-adaptive EA***
Constants $\varepsilon, a, \beta, \xi \in (0,1)$ and $k > 0$	
Starting target substring $\lvert\varkappa\rvert = \ell_1 \in \Theta(n^a)$ and $m \in \Theta(n^\beta)$	
$1/2 + \varepsilon < a + 2\varepsilon < \beta \leq 1 - \varepsilon$	$1/34 \leq a \leq \beta \leq 1$
Tracks with prob. $e^{-\Omega(n^\xi)}$	Tracks with prob. $1 - e^{-\Omega(n^\xi)}$

*Using parameters satisfying $\lambda, \mu = \Theta(n^\xi)$, $\lambda/\mu = \alpha_0 \geq 4$, $A > 1$, $0 < b < 1/(1 + \sqrt{1/\alpha_0(1 - p_{\mathsf{inc}})})$, $(1+\delta)/\alpha_0 < p_{\mathsf{inc}} < 2/5$, and $\epsilon := b'/n$ for any constant $b' > 0$, where A and b are constants.

Design new Self-adaptive EAs (SAEAs)

Moreover, the exploration of **designing self-adaptive algorithm** is currently ongoing in the community.

- **Sorting and selection mechanism**
  How to address the "order" of fitness and parameters.
- **Parameter adaptation mechanism**
  How to "mutate" parameters.
- **Parameter encoding method**
  How to encode parameters into the genome.
- **Self-adaptation of other parameters**
  How to self-adapt other parameters,
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A new EA for single-objective: MOSA-EA

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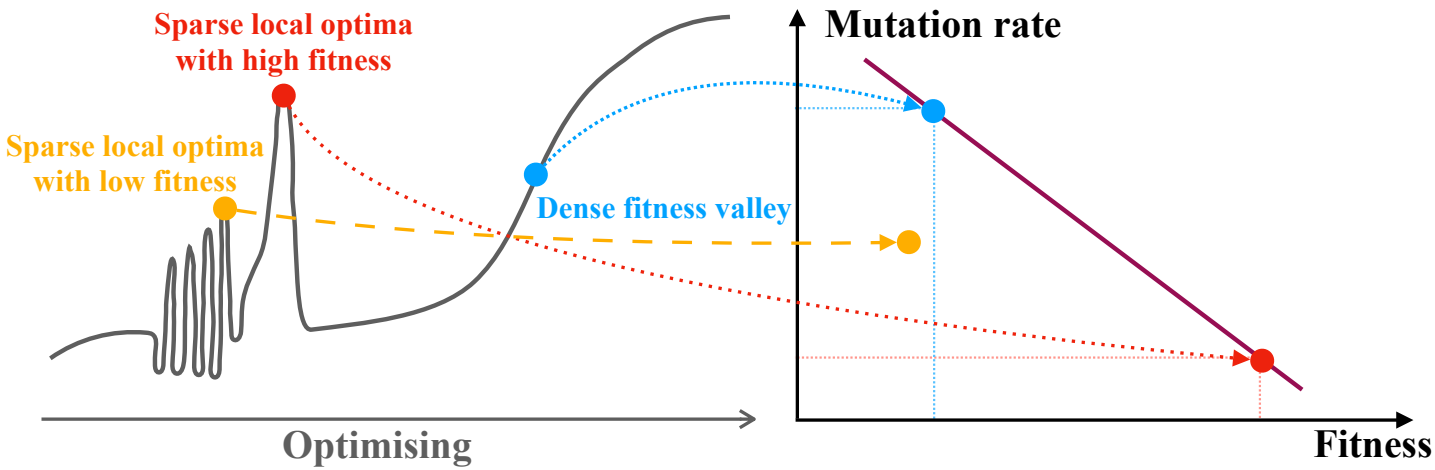
- The Multi-Objective Self-Adaptive Evolutionary Algorithms

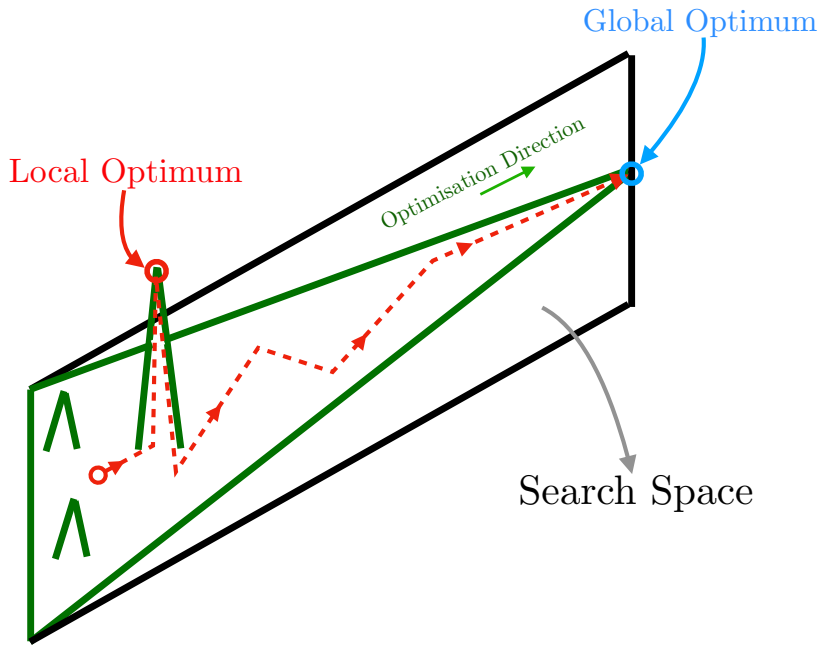
  In short, MOSA-EA[‖] (Lehre and Qin, 2022b).

  - **Non-elitism**
  - **Self-adaptation**
  - **Multi-objectivisation**



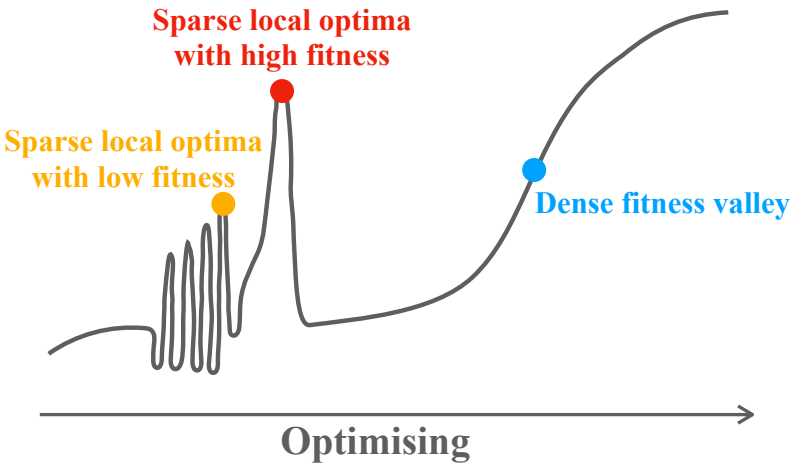
[‖] Implementation can be found in https://github.com/ChengCheng-Qin/mosa-ea.

Motivation: Sparse Local Optima



- Elitist EAs can get stuck on local optima (Jagerskupper and Storch, 2007; Dang et al., 2021a; Doerr, 2022; Dang et al., 2021b).
- *Sparse deceptive regions* (sparse local optima)

Motivation: Sparse Local Optima - Previous Works



- SparseLocalOpt $\Rightarrow$ a kind of fitness landscapes with *sparse deceptive regions* (local optima) and *dense fitness valleys* (Dang et al., 2021b).
- *Non-linear non-elitist selection* and *sufficiently high mutation rate* (Dang et al., 2021b) $\Rightarrow$
  - Sparse local optimal individuals $\Rightarrow$ higher chance to be selected but only survive a small percentage of such individuals after mutation;
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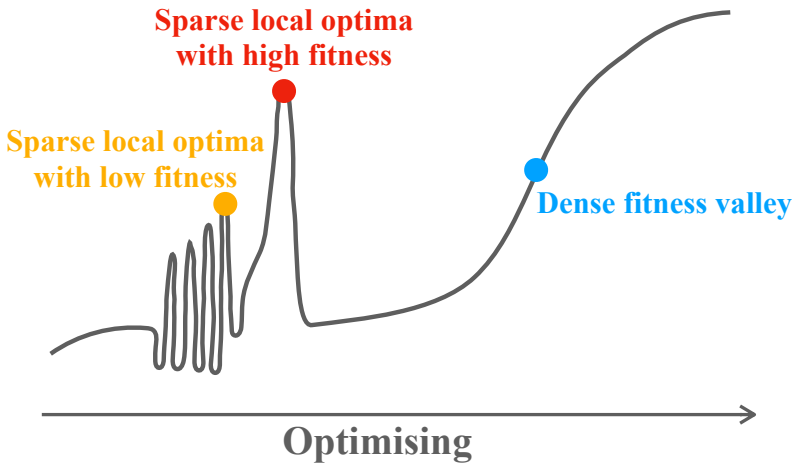

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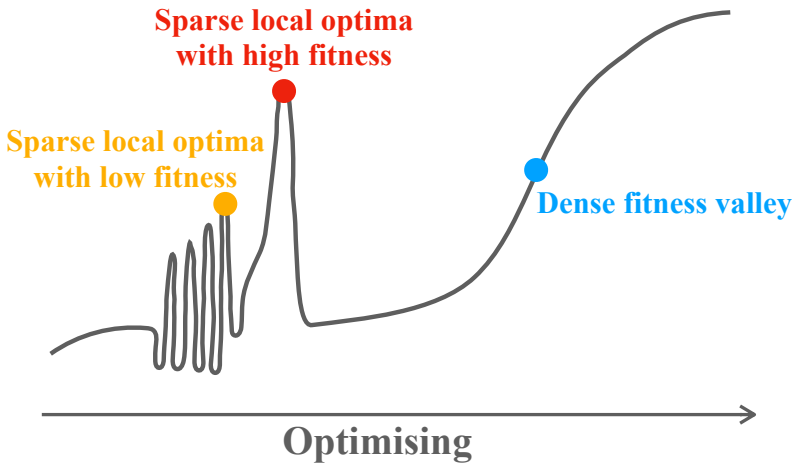

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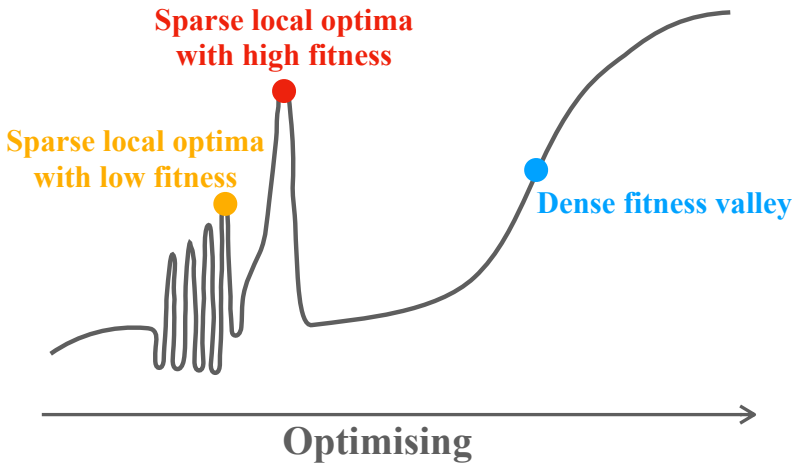

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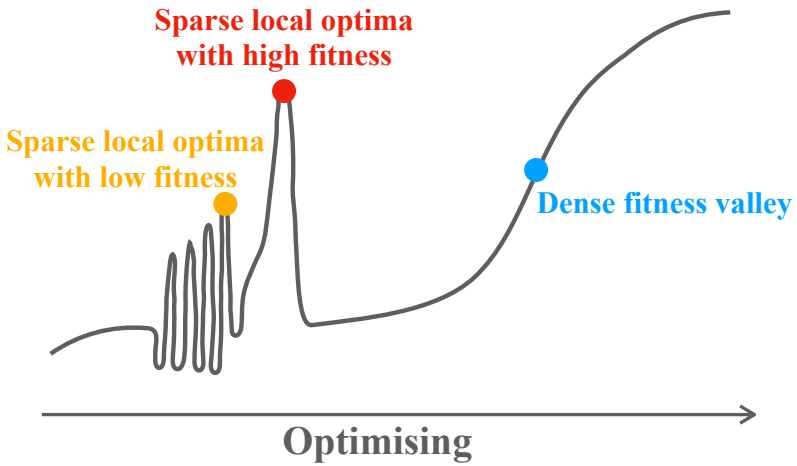

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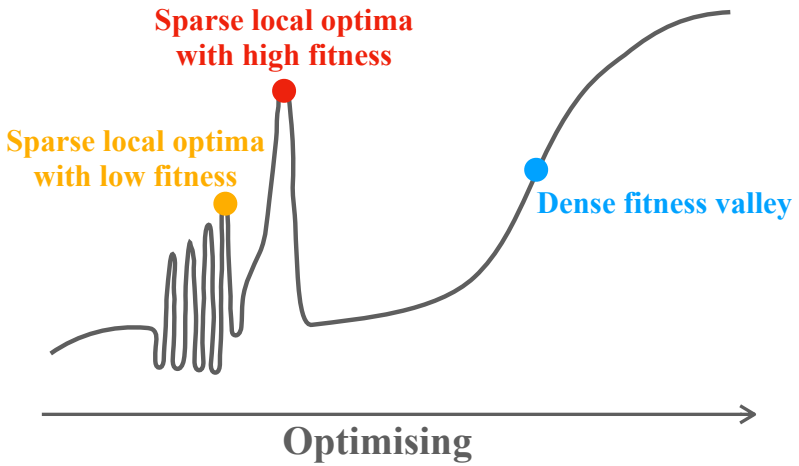
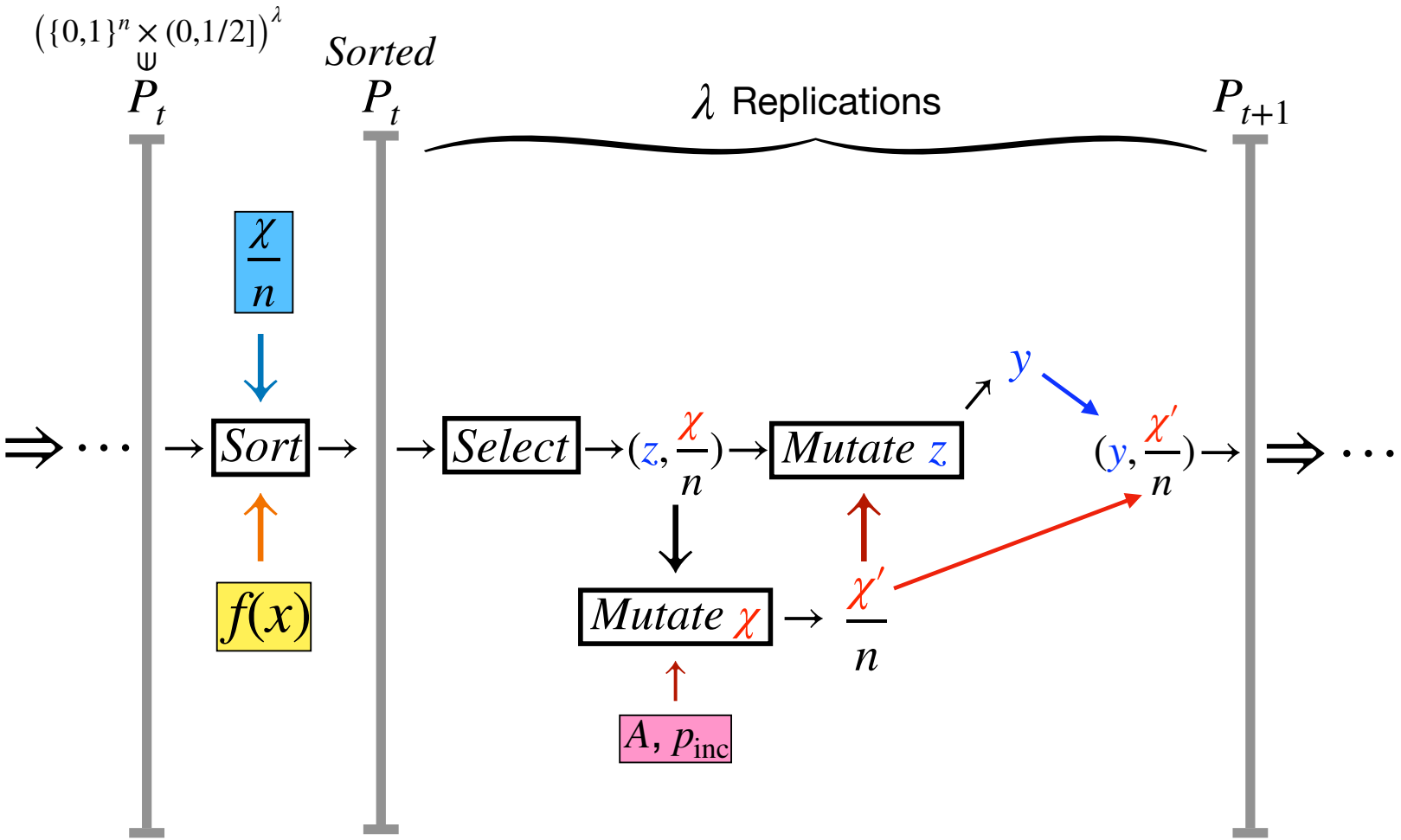
Motivation: Sparse Local Optima - Problems



- However,
  - We need to know the sparsity of local optima to set the mutation rate;
  - Fitness functions could contain several local optimums with different sparsities.

A Framework of Self-adaptive EAs



Components of MOSA-EA

- MOSA-EA:
  - Multi-objective sorting mechanism $\Rightarrow$ Strict non-dominated Pareto fronts
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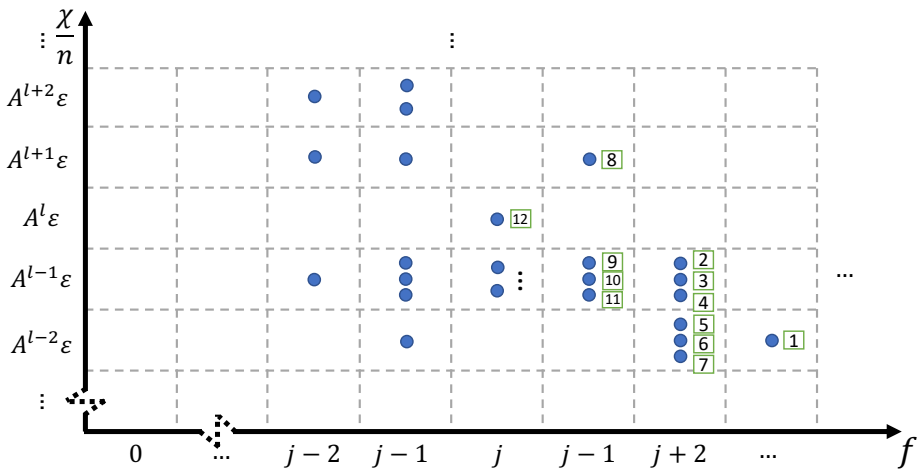

Figure: Fitness-first sorting (Case and Lehre, 2020)

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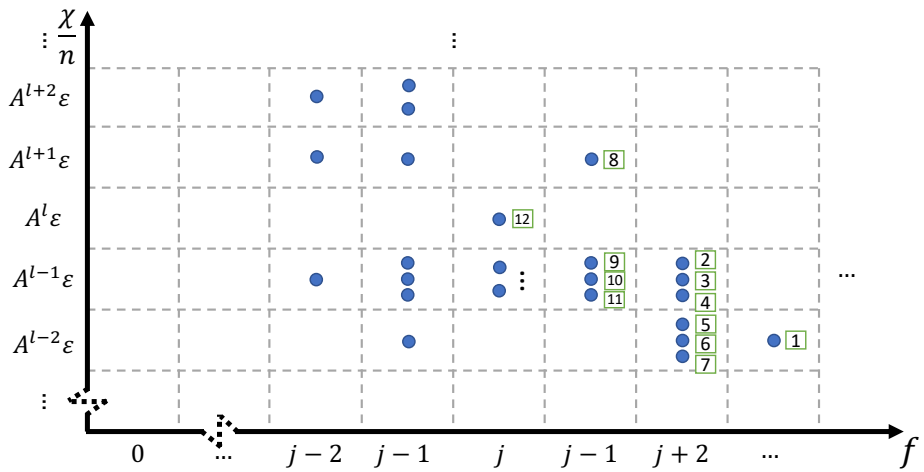

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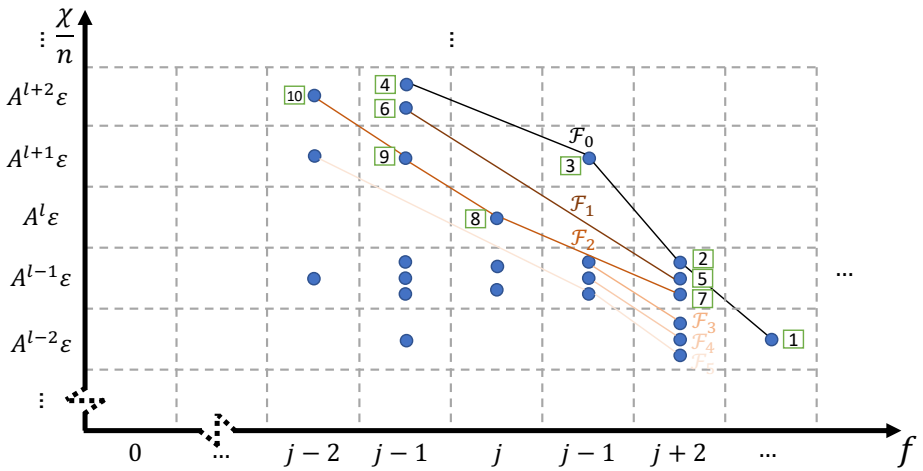

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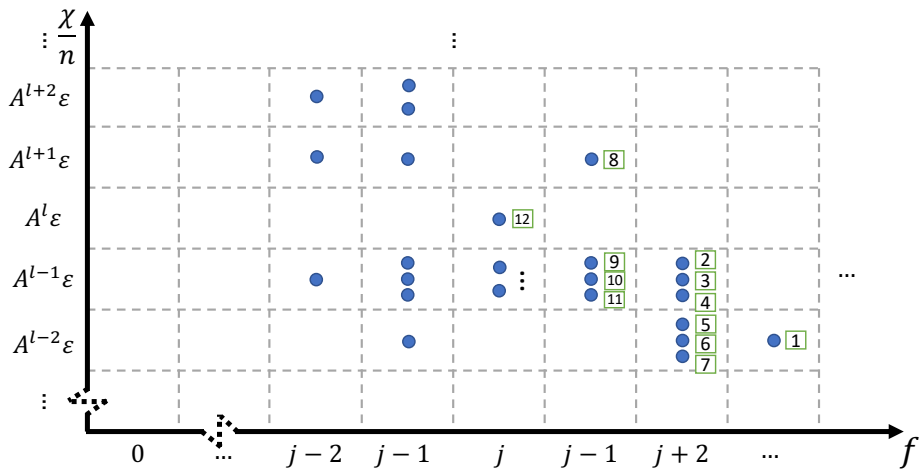

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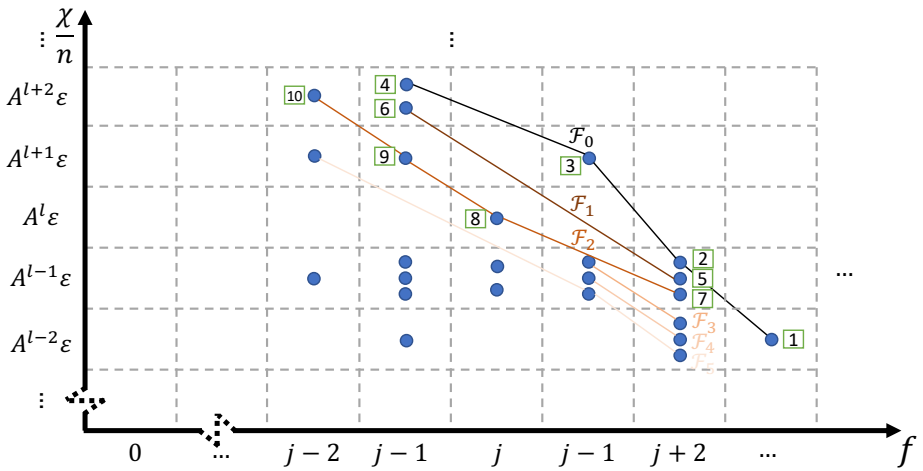

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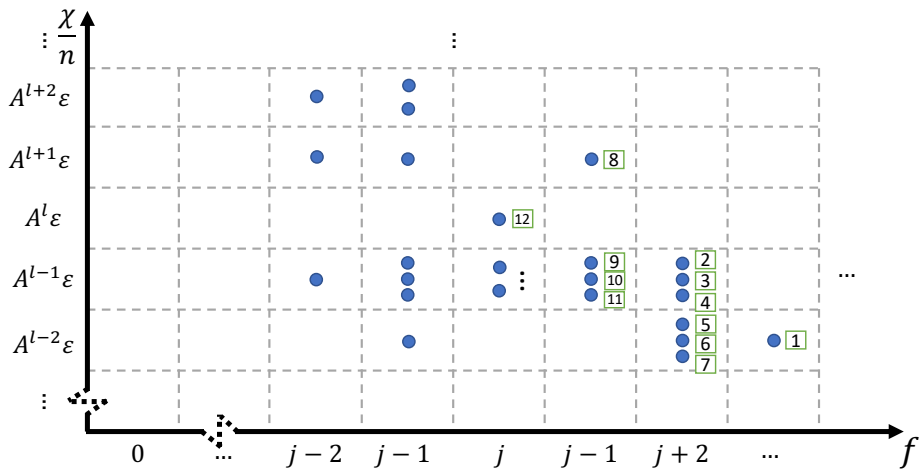

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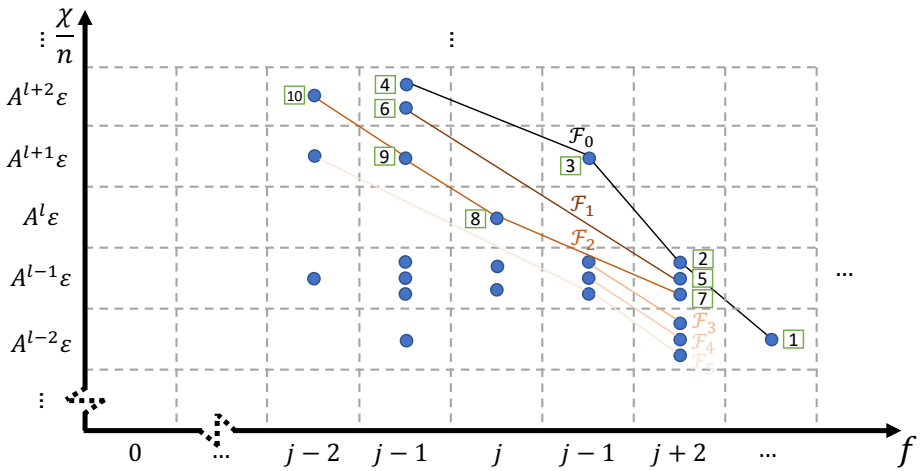

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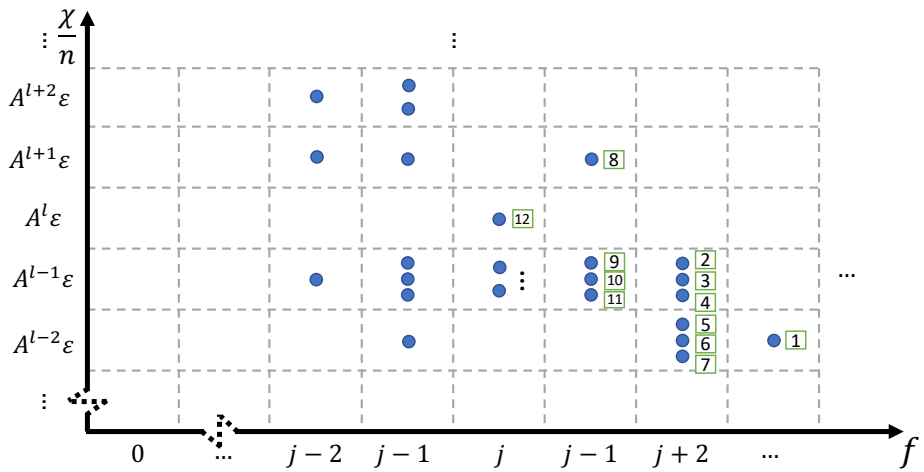

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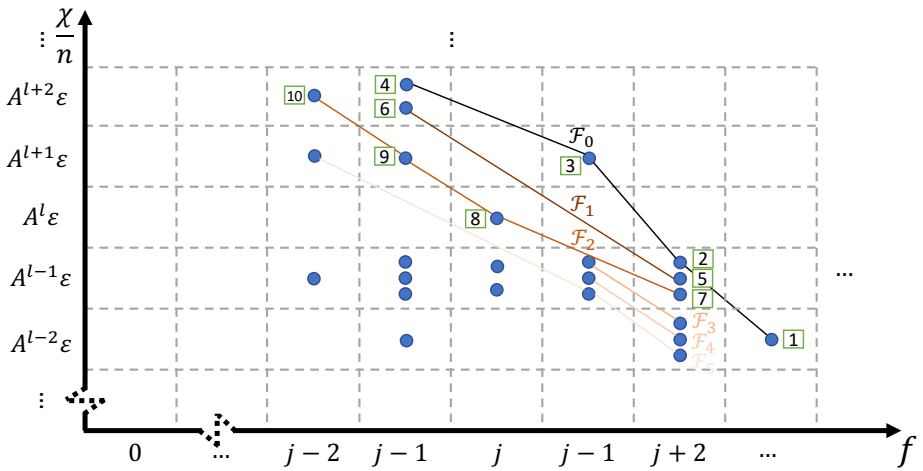

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Theoretical Study on MOSA-EA

Lehre and Qin (2022b) mathematical analysed MOSA-EA on an artificial two-peak function $\textsc{PeakedLO}_{m,k}$ with tunable sparsity:

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Algorithm	$\textsc{PeakedLO}_{m,k}$	Runtime T
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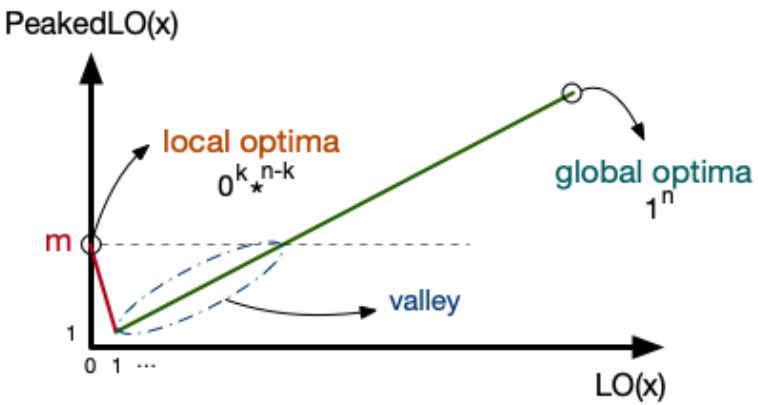

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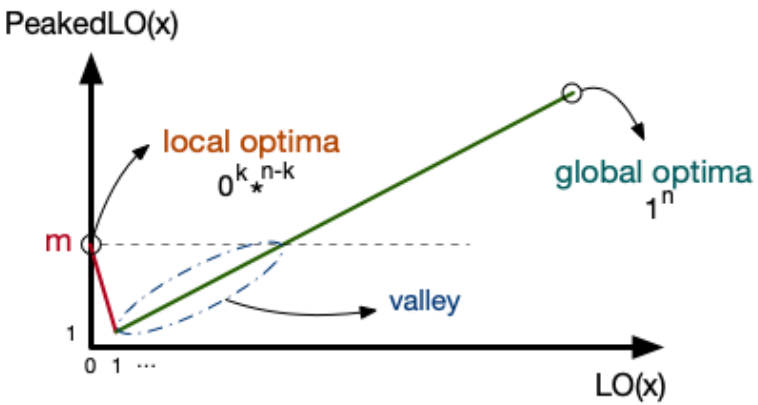

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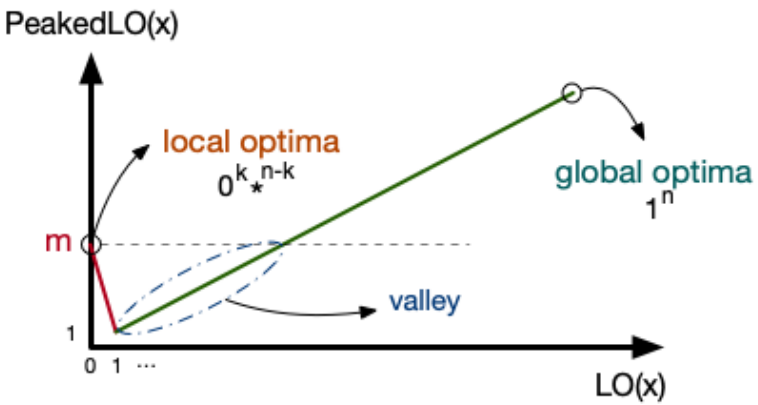

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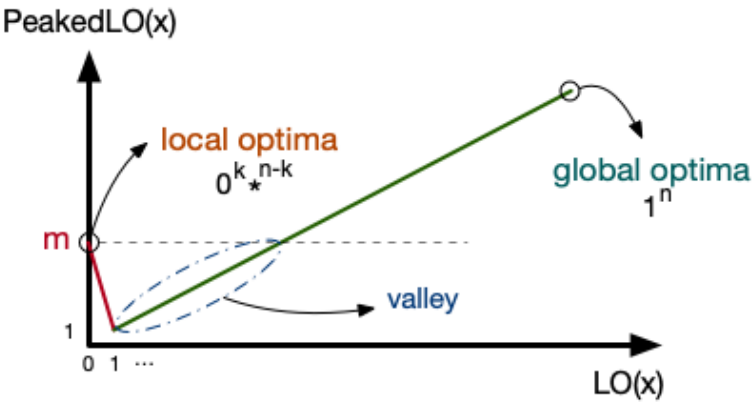

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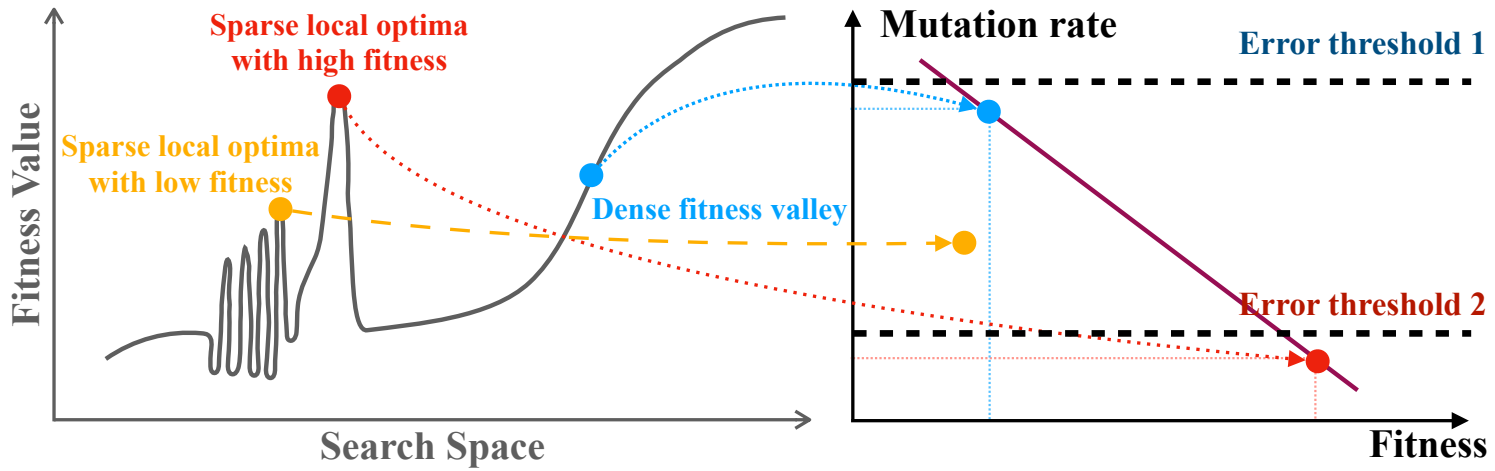

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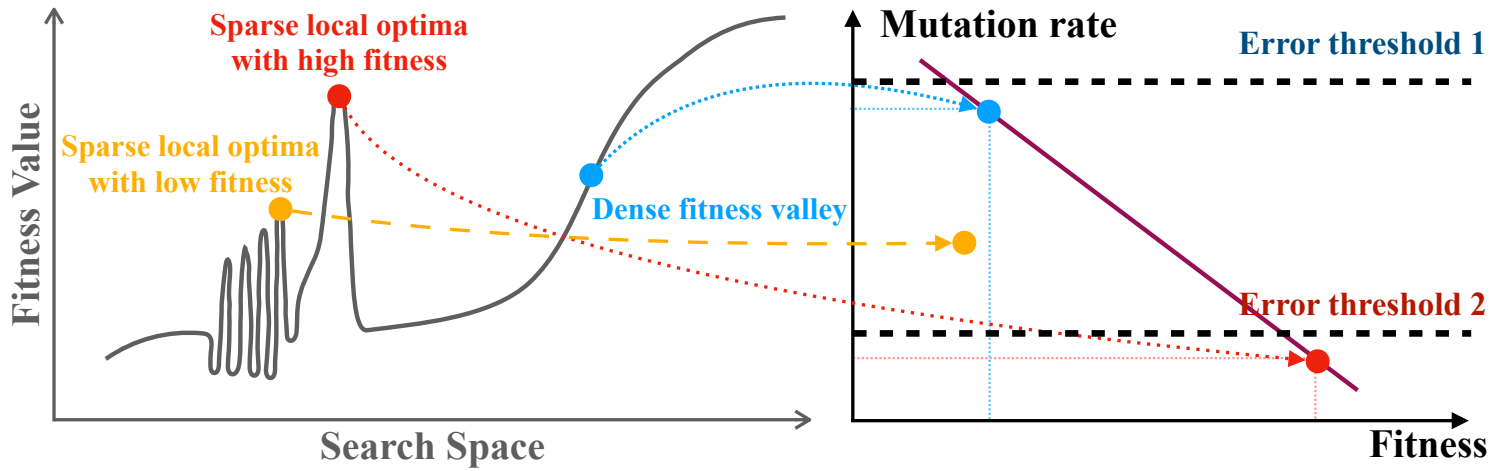

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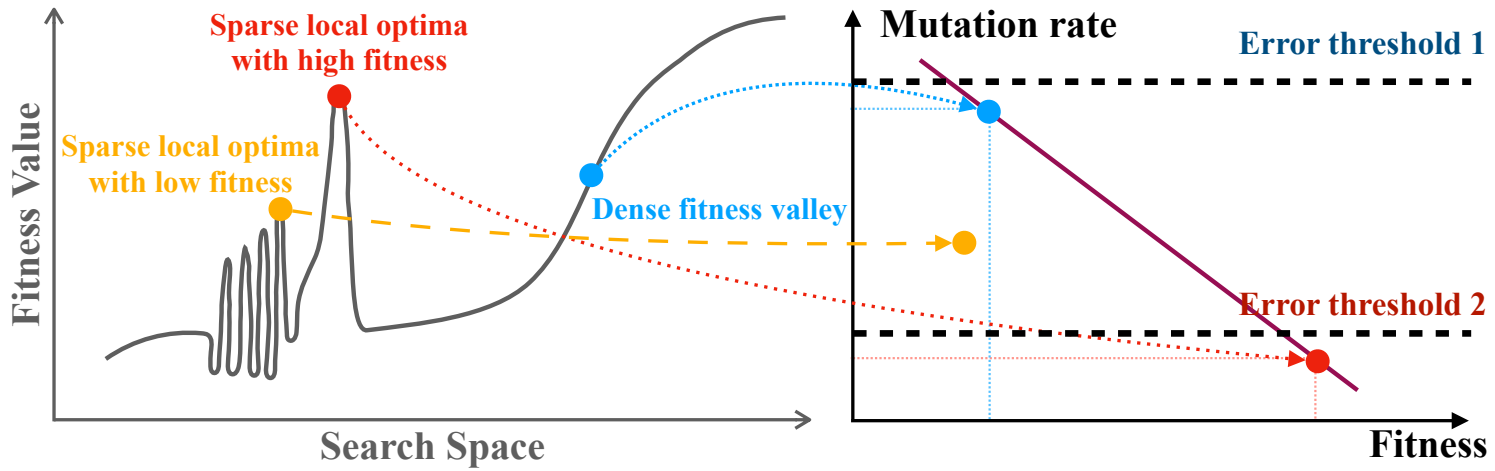

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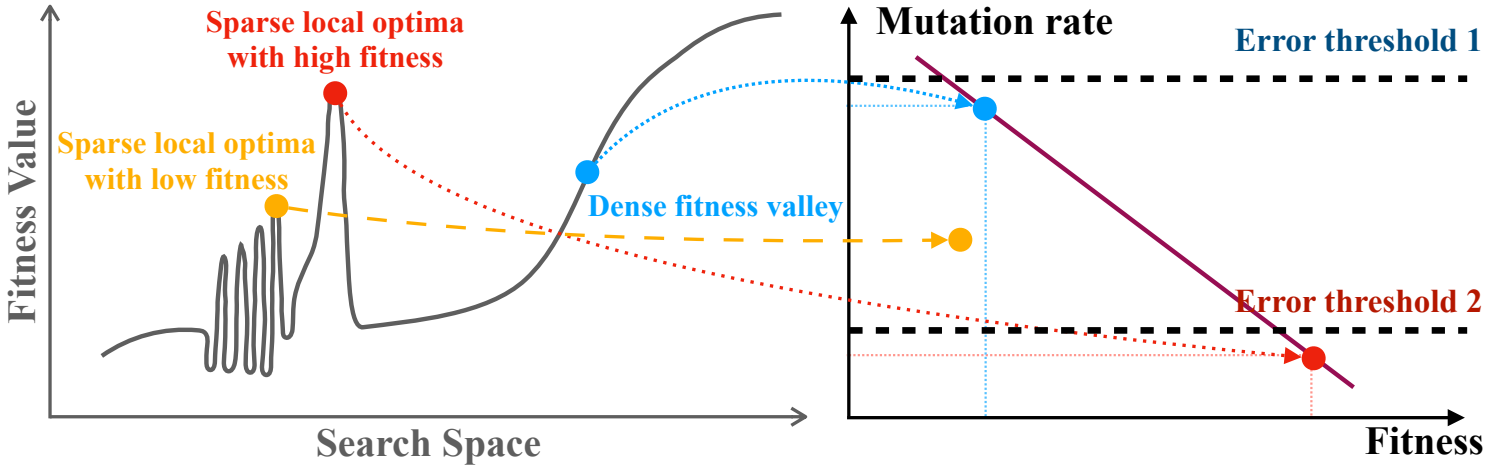

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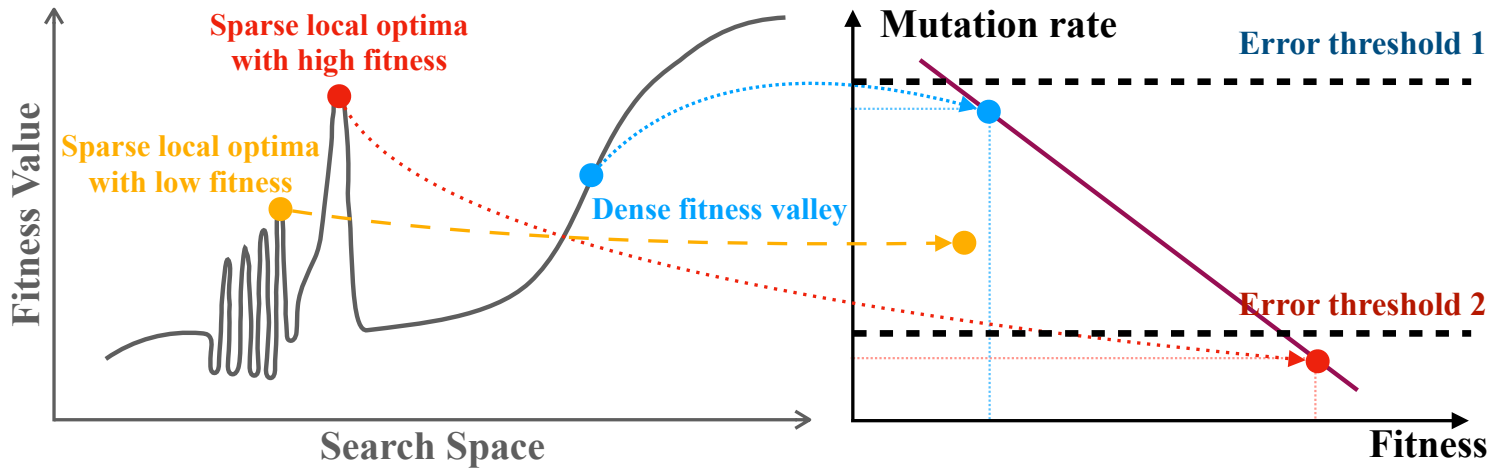

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Empirical Study on Combinatorial Optimisation Problems

Contain many local optimums

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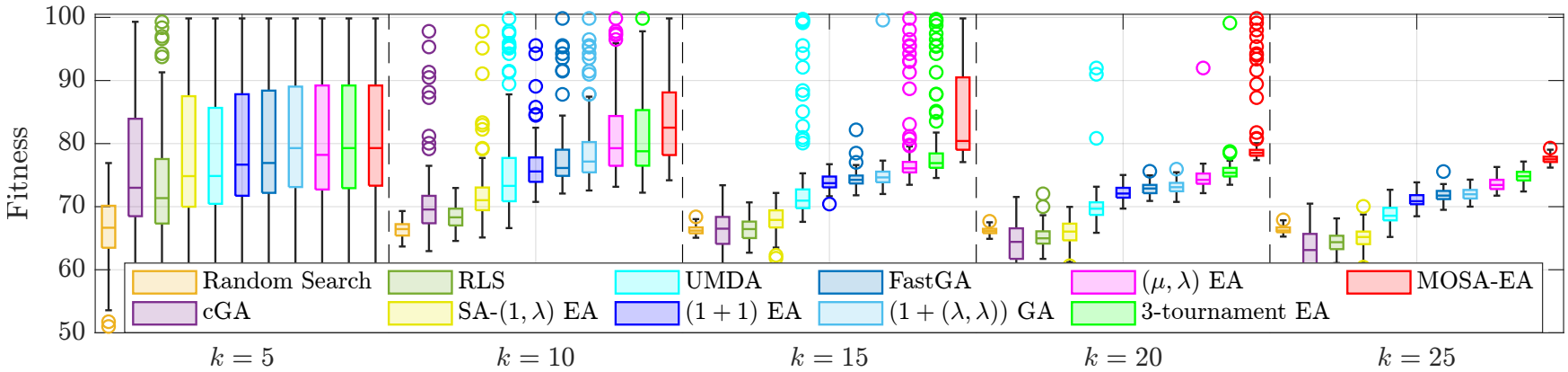
Maximising random NK-LANDSCAPE problems (Qin and Lehre, 2022):



In the complexity class NP

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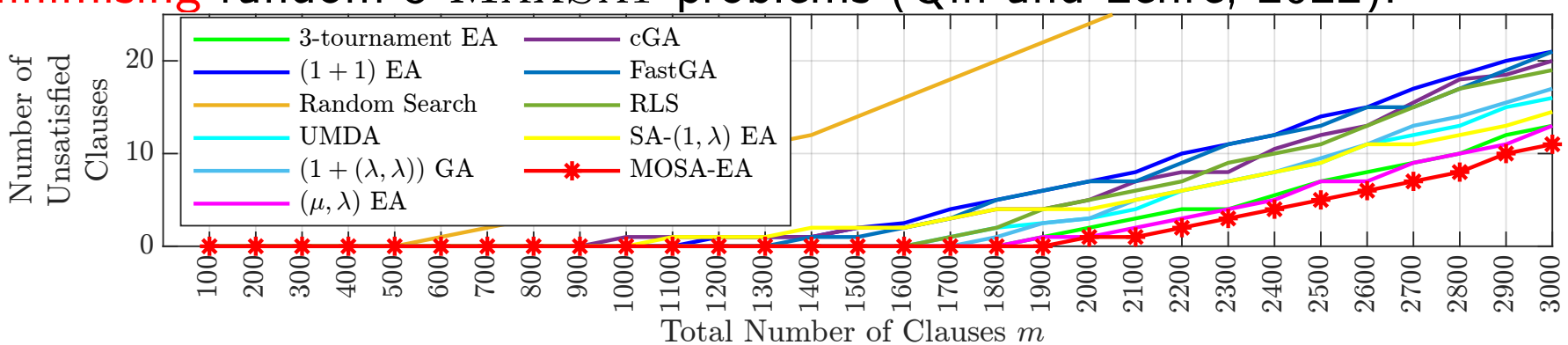
Minimising random 5-MAXSAT problems (Qin and Lehre, 2022):



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Theory of Evolutionary Computation Group

School of Computer Science

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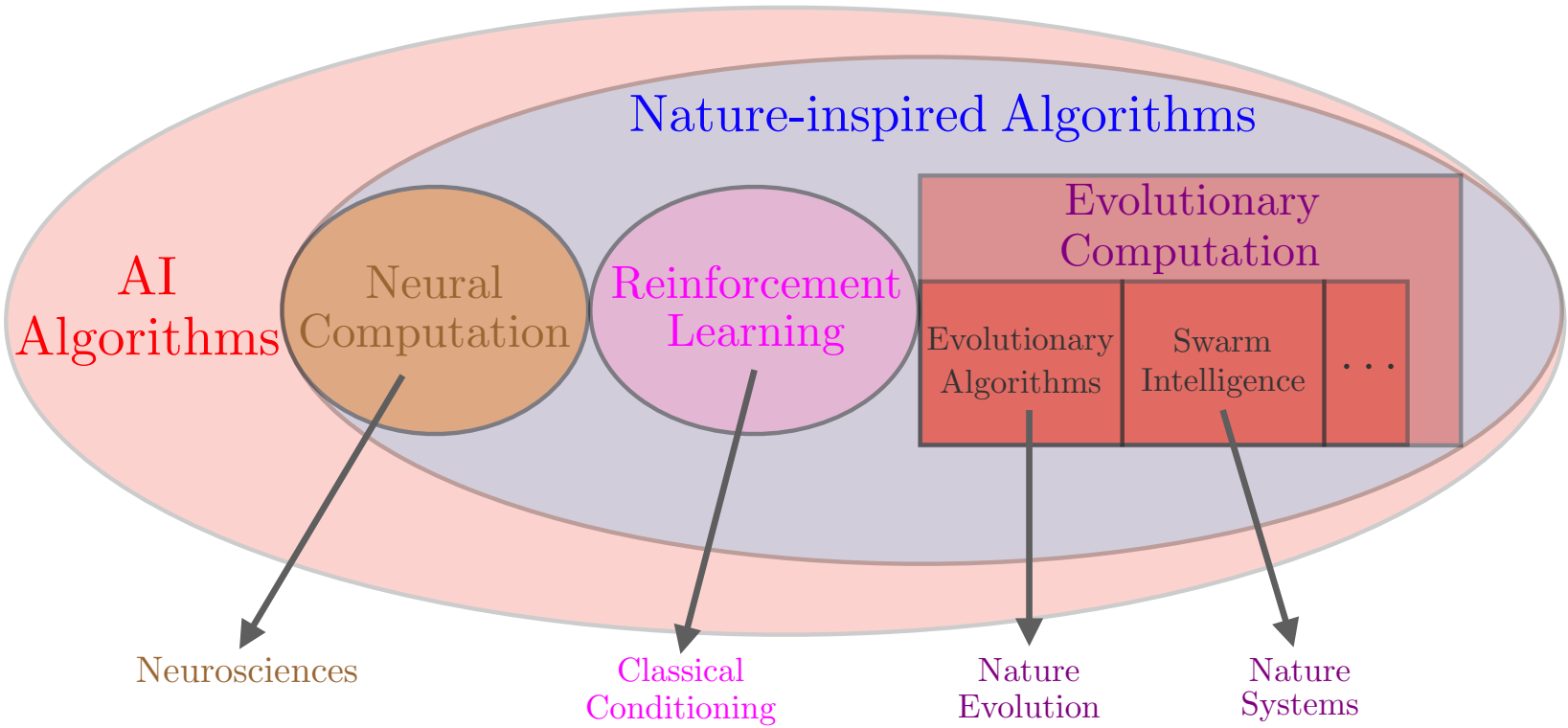
Appendix: Broader Evolutionary Computation



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