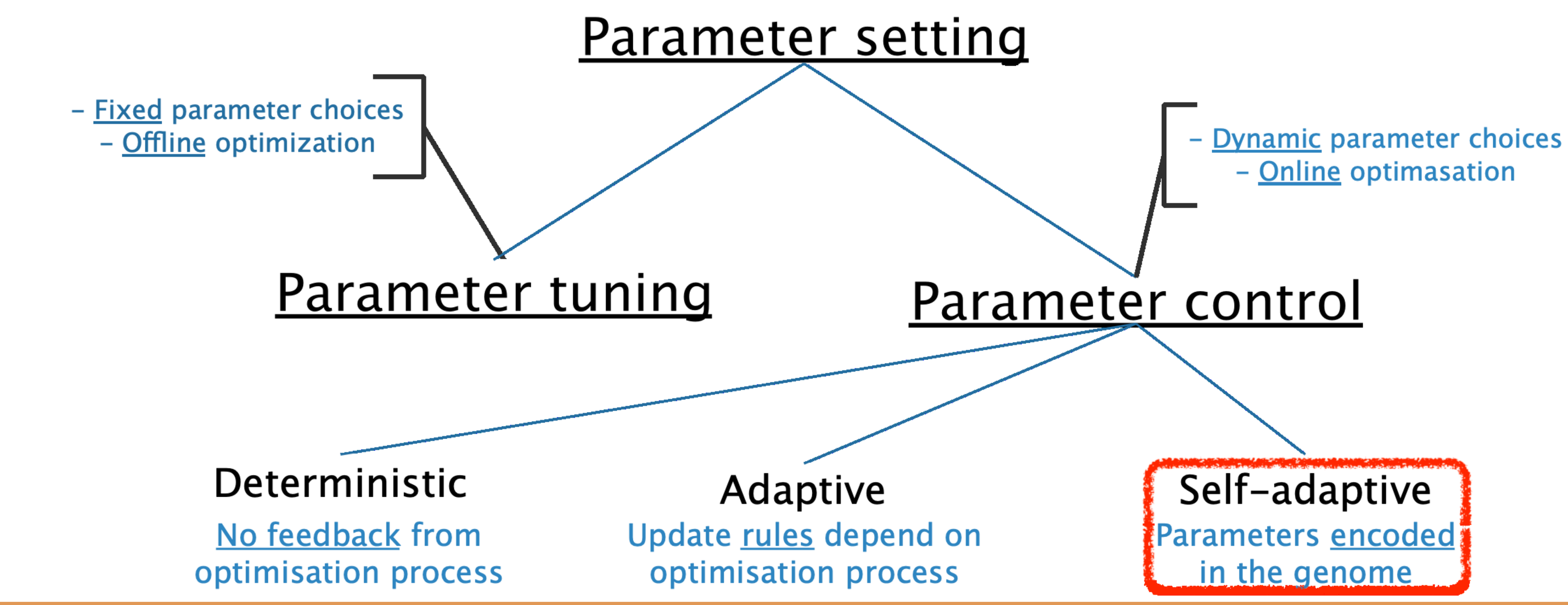
Self-adaptation Can Improve the Noise-tolerance of EAs

Xiaoyu Qin & Per Kristian Lehre @ University of Birmingham

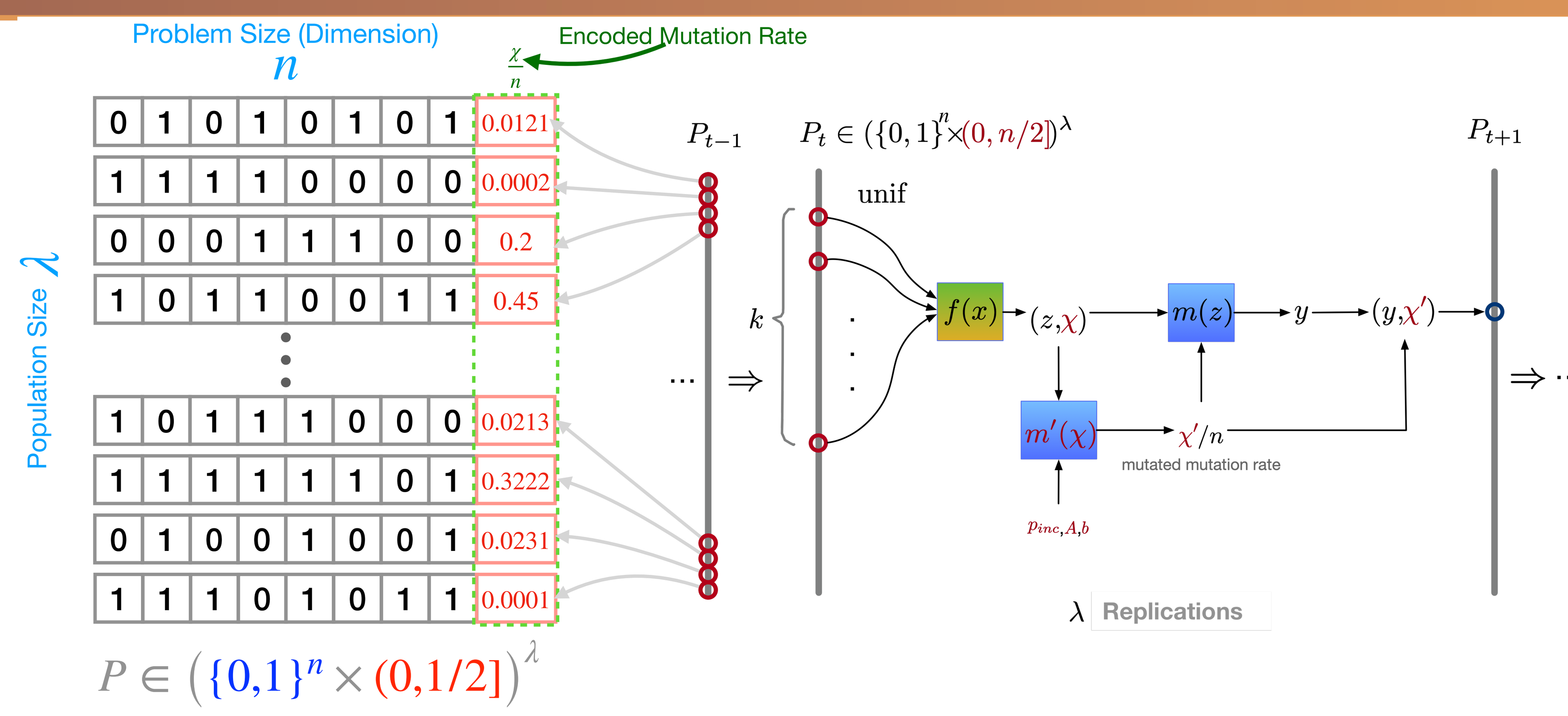
FOGA 2023

Background

- Evolutionary Algorithms (EAs) are parameterised algorithms.
- Parameters setting can dramatically impact performance of EAs [1].
- Parameters setting is instance- and state-dependent [1], e.g., under noise [2].
- Classification scheme of parameters setting in EAs [3]:



Self-adaptive 2-tournament EAs



Algorithm 2 2-tournament EA with self-adaptation

Require: Fitness function $f : \mathcal{X} \to \mathbb{R}$.
Require: Population size $\lambda \in \mathbb{N}$ where $\lambda \geq 2$.
Require: Self-adapting mutation rate strategy $D_{\text{mut}} : \mathbb{R} \to \mathbb{R}$.
Require: Initial population $P_0 \in \mathcal{Y}^\lambda$.

```
for t = 0, 1, 2, ... until termination condition met do
    for i = 1 to λ do
        (x1, χ1/n) ← P_t(i1) where i1 ∽ Uniform([λ]).
        (x2, χ2/n) ← P_t(i2) where i2 ∽ Uniform([λ]).
        if (x1, χ1/n) ⪰ (x2, χ2/n) then
            (z, χ/n) ← (x1, χ1/n),
        else
            (z, χ/n) ← (x2, χ2/n).
        Sample χ'/n ~ D_mut(χ/n).
        P_{t+1}(i) ← (y, χ'/n) where y created by mutating z
    with mutation rate χ'/n.
```

Algorithm 3 Self-adapting mutation rate strategy (SA)

Require: $A > 1$, $\varepsilon > 0$, $p_{\text{inc}} \in (0,1)$.
Require: Mutation rate χ/n.

1: $\chi' = \begin{cases} \min(A\chi, \varepsilon n A^{\lfloor \log_A(\frac{1}{2\varepsilon}) \rfloor}) & \text{with prob. } p_{\text{inc}}, \\ \max(\chi/A, \varepsilon n) & \text{otherwise.} \end{cases}$

Algorithm 4 Self-adapting two mutation rates (SA-2mr)

Require: $\chi_{\text{high}} > \chi_{\text{low}} > 0$, $p_c \in (0,1)$.
Require: Mutation rate χ/n.

1: Set $\chi' := \begin{cases} \chi_{\text{high}} \text{ if } \chi = \chi_{\text{low}}, \chi_{\text{low}} \text{ if } \chi = \chi_{\text{high}} & \text{with prob. } p_c \\ \chi & \text{otherwise.} \end{cases}$

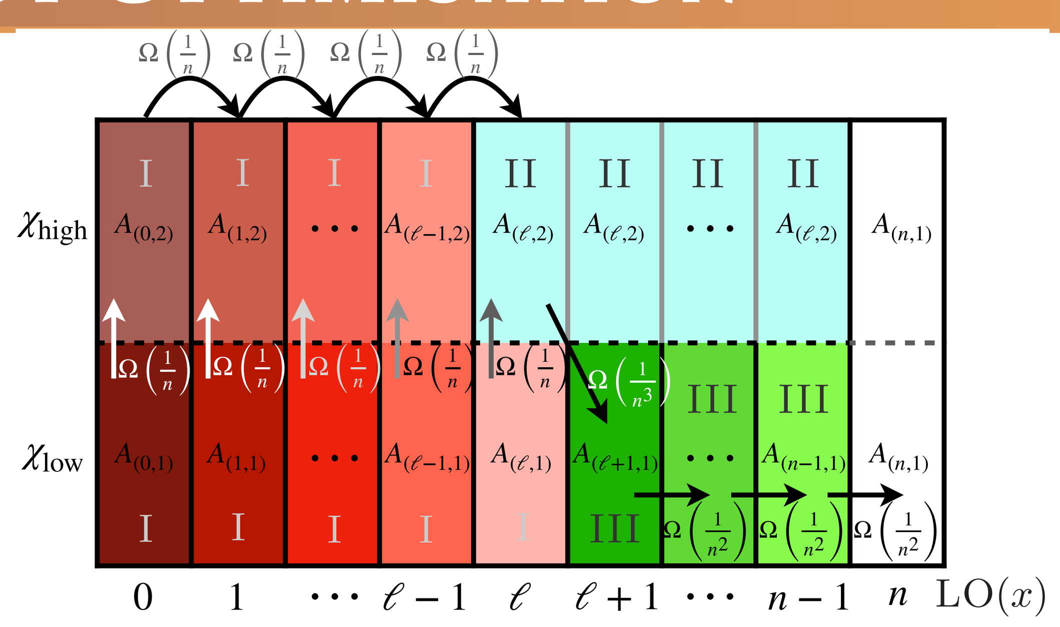
2: Return new mutation rate χ'/n.

Runtime Analysis in Noisy Optimisation

Table 1: Theoretical results of EAs on LeadingOnes under symmetric noise (C, q) ($C \in \mathbb{R}$, constant $0 < \chi_{\text{high}} < \ln(2)$, $\chi_{\text{low}} = a/n$, $\lambda = c\log(n)$ where $a, c > 0$ are constants, $p_c \in o(1) \cap \Omega(1/n)$ in 2-tour' EA with SA-2mr)

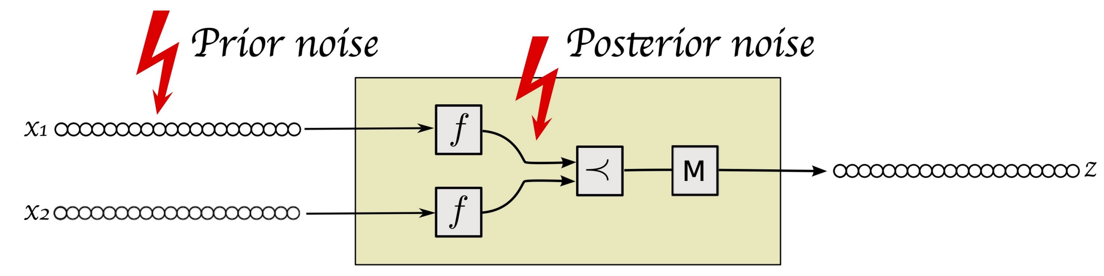
Algorithm	Noise-free	Under Noise
(1+1) EA	$O(n^2)$	$e^{\Omega(n)}$ † (Thm. 2.3)
2-tour' EA with $\frac{\chi_{\text{high}}}{n}$	$O(n^2)$	$e^{\Omega(n)}$ ‡ (Thm. 2.1)
2-tour' EA with $\frac{\chi_{\text{low}}}{n}$	$\Omega(n^2\log(n))$ (Cor. 3.1)	$O(n^3)$ § (Thm. 2.2)
2-tour' EA with UM-2mr	$O(n^2)$ (Thm. 4.1)	$e^{\Omega(n)}$ ‡ (Thm. 4.2)
2-tour' EA with SA-2mr	$O(n^2)$ (Thm. 5.1)	$O(n^3)$ § (Thm. 5.2)

- High/Low Mut. Rates Lead to Failed Optimisation under Noise or or Slow Optimisation without Noise.



- Uniformly Mixing Mut. Rates No Help under Noise.
- Self-adapting Mut. Rates Guarantee Efficiency.

Noisy Optimisation



- Real-world optimisation often involves uncertainty, such as noise.
- **Symmetric Noise Model** (C, q) [2, 6]: Given $q \in [0,1]$ (noise level), an arbitrary number $C \in \mathbb{R}$, then

$$f^n(x) = \begin{cases} f(x) & \text{with probability } 1-q, \\ C - f(x) & \text{with probability } q. \end{cases}$$

Tab. Previous analysis of EAs on LO under symmetric noise [a]

Algorithm	Exp. Runt. T
2-tour. EA with mut. rate $\chi/n < \ln(2(1-q))$	$O(n^2)$
2-tour. EA with mut. rate $\chi/n > \ln(2(1-q))$	$e^{\Omega(n)}$
(μ,λ) EA with mut. rate $\chi/n < \ln(\frac{\lambda}{\mu}(1-q))$	$O(n^2)$
(μ,λ) EA with mut. rate $\chi/n > \ln(\frac{\lambda}{\mu}(1-q))$	$e^{\Omega(n)}$

- It is challenging to find the appropriate mutation rate if the occurrence of noise is unpredictable (or the noise level is unknown).

[a] Arbitrary value $C \in \mathbb{R}$, Constant $q \in [0, 1/2)$

Experimental Analysis

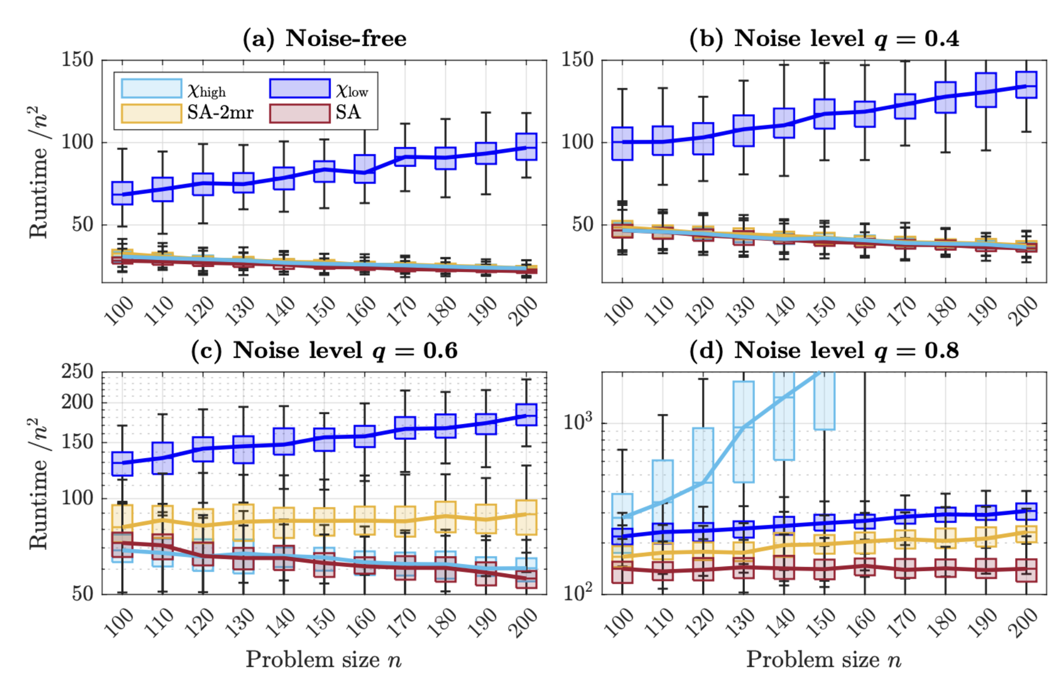
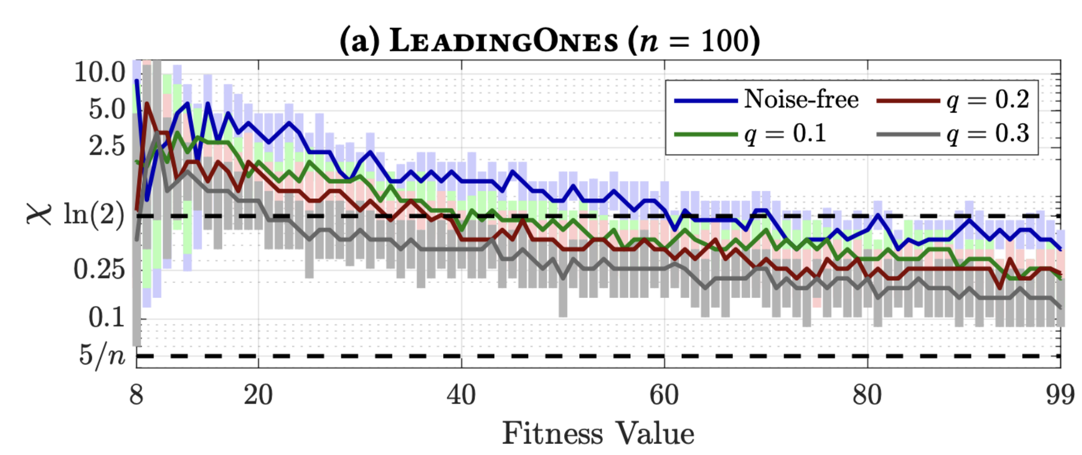


Figure 6: Runtimes of 2-tournament EAs on LeadingOnes under one-bit noise with different noise levels.



Conclusion

- Although the noise model examined in the theoretical study is relatively simplistic and artificial,
- Our findings still provide a compelling indication that the self-adaptive EA adapts to the presence of noise.

References

[1] Benjamin Doerr and Carola Doerr. Theory of parameter control for discrete black-box optimization: Provable performance gains through dynamic parameter choices. *Theory of Evolutionary Computation*, pages 271–321, 2020. Publisher: Springer.

[2] Per Kristian Lehre and Xiaoyu Qin. More Precise Runtime Analyses of Non-elitist EAs in Uncertain Environments. In *Proceedings of the Genetic and Evolutionary Computation Conference*, page 9, Lille, France, 2021. ACM.

[3] A.E. Eiben, R. Hinterding, and Z. Michalewicz. Parameter control in evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, 3(2):124–141, July 1999.

[4] Jens Jagerskupper and Tobias Storch. When the Plus Strategy Outperforms the Comma Strategyand When Not. In *2007 IEEE Symposium on Foundations of Computational Intelligence*, pages 25–32, Honolulu, HI, April 2007. IEEE.

[5] Duc-Cuong Dang, Anton Eremeev, and Per Kristian Lehre. Non-elitist Evolutionary Algorithms Excel in Fitness Landscapes with Sparse Deceptive Regions and Dense Valleys. In *Proceedings of the Genetic and Evolutionary Computation Conference*, Lille, France, 2021. ACM.

[6] Chao Qian, Chao Bian, Wu Jiang, and Ke Tang. Running Time Analysis of the (1+1)-EA for OneMax and LeadingOnes Under Bit-Wise Noise. *Algorithmica*, 81(2):749–795, February 2019.