Robotics Project: Part 2

- ## Interface

Using MATLAB R2019a to finish the project. Open the 'joint_motion.m' and 'cartesian_motion.m' and run the script. The trajectory, joint variables, and the information in Cartesian space will show up in the graph and workspace. The trajectory is $A \rightarrow A' \rightarrow (B) \rightarrow C' \rightarrow C$.

- ## Program Architecture

  - Joint motion

    Define the A matrix of point A, B, C, and the [n o a p] of point A, B, C.
    Use function 'inverse_kinematics.m' to find joint valuables $[\theta_1\ \ \theta_2\ \ d_3\ \ \theta_4\ \ \theta_5\ \ \theta_6]$.
    Line portion motion planning from A to A' (-0.5 ~ -0.2).
    Transition portion motion planning from A' to C' (-0.2 ~ 0.2).
    Line portion motion planning from C' to C (0.2~0.5).
    Use function 'forward_kinematics.m' to transform to Cartesian space in every motion planning.
    Plot the result.

  - Cartesian motion

    Define the A matrix of point A, B, C, and the [n o a p] of point A, B, C.
    Line portion motion planning from A to A' (-0.5 ~ -0.2).
    Transition portion motion planning from A' to C' (-0.2 ~ 0.2).
    Line portion motion planning from C' to C (0.2~0.5).
    Plot the result.

- ## Mathematics

  - Joint motion

Let $A = \begin{bmatrix} \theta_{A1} \\ \theta_{A2} \\ \theta_{A3} \\ \theta_{A4} \\ \theta_{A5} \\ \theta_{A6} \end{bmatrix}, B = \begin{bmatrix} \theta_{B1} \\ \theta_{B2} \\ \theta_{B3} \\ \theta_{B4} \\ \theta_{B5} \\ \theta_{B6} \end{bmatrix}, C = \begin{bmatrix} \theta_{C1} \\ \theta_{C2} \\ \theta_{C3} \\ \theta_{C4} \\ \theta_{C5} \\ \theta_{C6} \end{bmatrix}$. Define $\begin{cases} \Delta C = C - B \\ \Delta B = A - B \end{cases}$

For linear portion, update the trajectory, velocity, and acceleration with

$$\begin{cases} q(h) = \Delta C \cdot h + B \\ \dot{q}(h) = \frac{\Delta C}{T} \\ \ddot{q}(h) = 0 \end{cases},$$

where $h = \frac{t}{T}, t_{acc} \leq t \leq T - t_{acc}$.

For transition portion, update the trajectory, velocity, and acceleration with

$$\begin{cases} q(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(2-h)h^2 - 2\Delta B\right] h + B + \Delta B \\ \dot{q}(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(1.5-h)2h^2 - \Delta B\right] \frac{1}{t_{acc}} \\ \ddot{q}(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(1-h)\right] \frac{3h}{t_{acc}^2} \end{cases},$$

where $h = \frac{t + t_{acc}}{2t_{acc}}$, $-t_{acc} \leq t \leq t_{acc}$.

- Cartesian motion

Let the $D(1)$ matrix with

$$D(1) = POS1^{-1} * POS2 = \begin{bmatrix} {}^1n \cdot {}^2n & {}^1n \cdot {}^2o & {}^1n \cdot {}^2a & {}^1n \cdot ({}^2p - {}^1p) \\ {}^1o \cdot {}^2n & {}^1o \cdot {}^2o & {}^1o \cdot {}^2a & {}^1o \cdot ({}^2p - {}^1p) \\ {}^1a \cdot {}^2n & {}^1a \cdot {}^2o & {}^1a \cdot {}^2a & {}^1a \cdot ({}^2p - {}^1p) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First, we need to find the difference of $x, y, z, \phi, \theta, \psi$ between two points with

x	${}^1n \cdot ({}^2p - {}^1p)$
y	${}^1o \cdot ({}^2p - {}^1p)$
z	${}^1a \cdot ({}^2p - {}^1p)$
ϕ	$S\phi = -S\psi C\psi V\theta({}^1n \cdot {}^2n) + [(C\psi)^2 V\theta + C\theta]({}^1o \cdot {}^2n) - S\psi S\theta({}^1a \cdot {}^2n)$ $C\phi = -S\psi C\psi V\theta({}^1n \cdot {}^2o) + [(C\psi)^2 V\theta + C\theta]({}^1o \cdot {}^2o) - S\psi S\theta({}^1a \cdot {}^2o)$ $\tan\phi = \frac{S\phi}{C\phi},\ -\pi \le \phi \le \pi$
θ	$\tan\theta = \frac{[({}^1n \cdot {}^2a)^2 + ({}^1o \cdot {}^2a)^2]^{\frac{1}{2}}}{{}^1a \cdot {}^2a}$
ψ	$\tan\psi = (\frac{{}^1o \cdot {}^2a}{{}^1n \cdot {}^2a})$

Then, update $D(r)$ matrix every time step,

$$D(r) = T_r(r) * R_a(r) * R_o(r)$$

Where $T_r(r) = \begin{bmatrix} 1 & 0 & 0 & r_x \\ 0 & 1 & 0 & r_y \\ 0 & 0 & 1 & r_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $R_a(r) = \begin{bmatrix} S\psi^2 V(r\theta) & -S\psi C\psi V(r\theta) & C\psi S(r\theta) & 0 \\ -S\psi C\psi V(r\theta) & C\psi^2 V(r\theta) + C\theta & S\psi S(r\theta) & 0 \\ -C\psi S(r\theta) & -S\psi S(r\theta) & C(r\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$$R_o(r) = \begin{bmatrix} C(r\phi) & -S(r\phi) & 0 & 0 \\ S(r\phi) & C(r\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which represents the difference between position1 and position2.

For linear portion, update the trajectory, velocity, and acceleration with

$$\begin{cases} q(h) = \Delta C \cdot h + B \\ \dot{q}(h) = \frac{\Delta C}{T} \\ \ddot{q}(h) = 0 \end{cases},$$

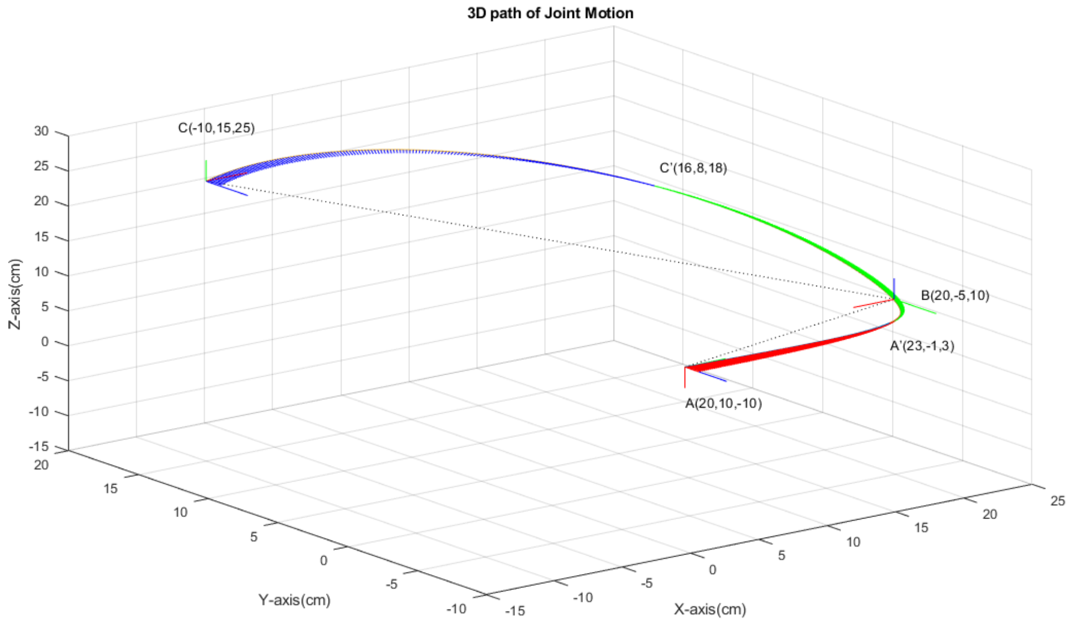
where $h = \frac{t}{T}, t_{acc} \le t \le T - t_{acc}$.

For transition portion, update the trajectory, velocity, and acceleration with

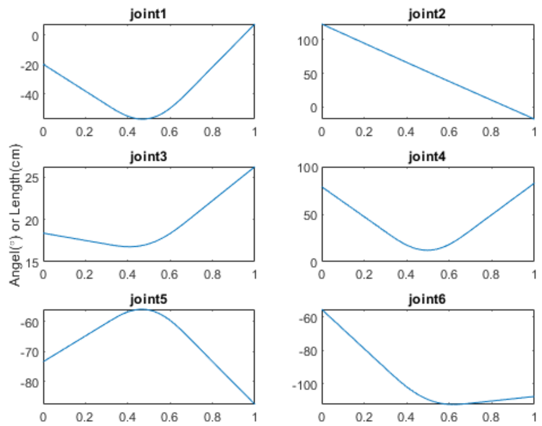
$$\begin{cases} q(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(2-h)h^2 - 2\Delta B\right] h + B + \Delta B \\ \dot{q}(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(1.5-h)2h^2 - \Delta B\right] \frac{1}{t_{acc}} \\ \ddot{q}(h) = \left[\left(\Delta C \frac{t_{acc}}{T} + \Delta B\right)(1-h)\right] \frac{3h}{t_{acc}^2} \end{cases},$$

where $h = \frac{t + t_{acc}}{2t_{acc}},\ -t_{acc} \le t \le t_{acc}$.

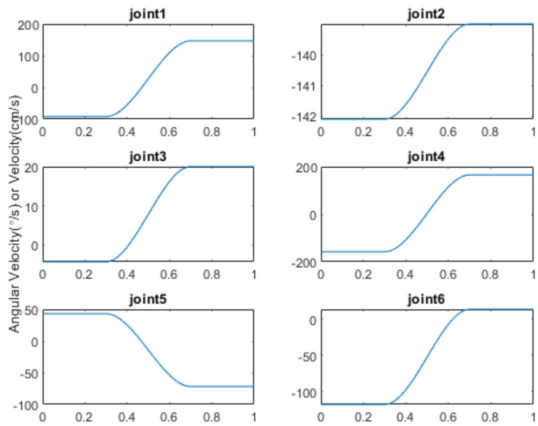
- **Trajectory profile**
  - **Joint motion**



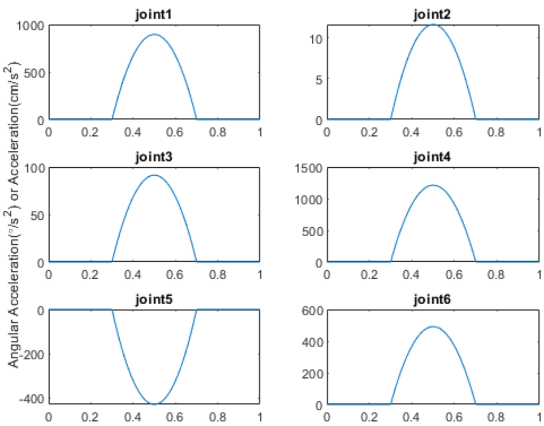
▲ **Trajectory profile after motion planning in joint space**



▲ **Position of every joint**

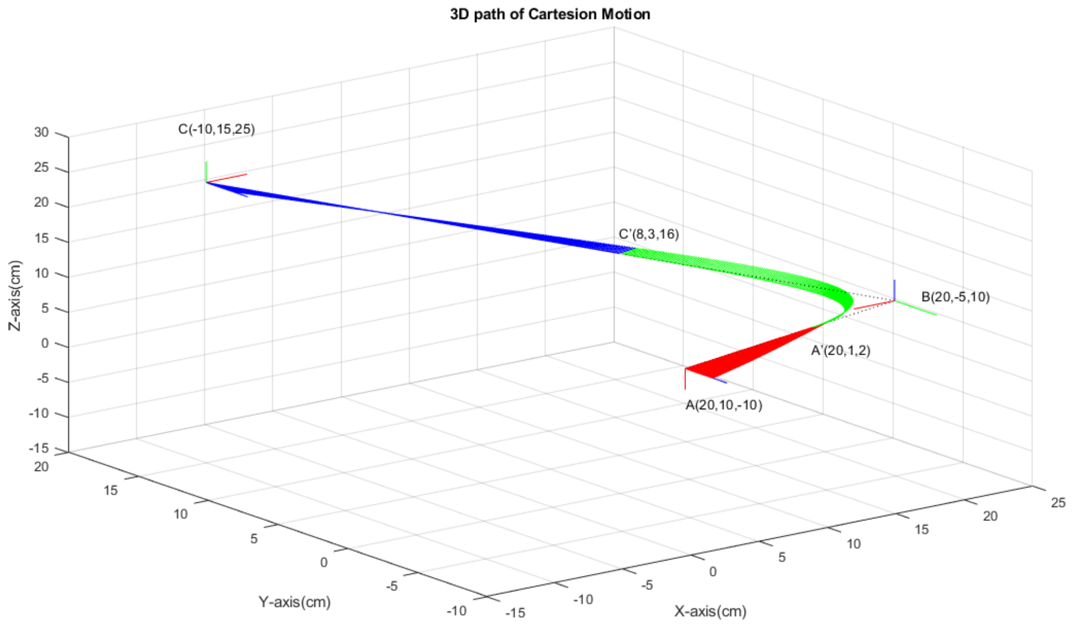


▲ **Velocity of every joint**

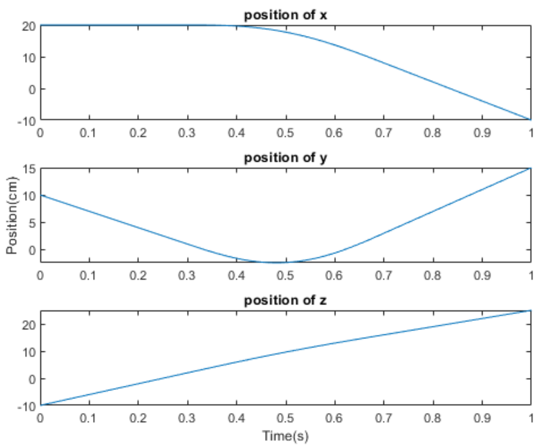


▲ **Acceleration of every joint**

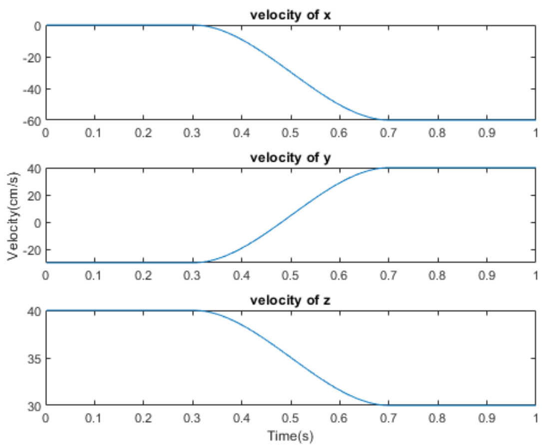
➢ Cartesian motion



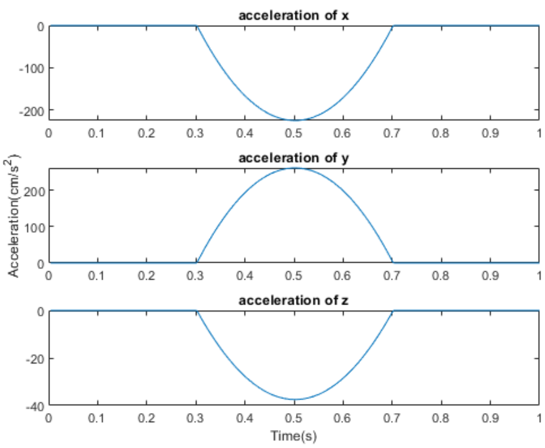
▲ **Trajectory profile after motion planning in Cartesian space**



▲ **Position of x, y, z axis**



▲ **Velocity of x, y, z axis**



▲ **Acceleration of x, y, z axis**

- **Bonus**
  - **Joint motion**

    No singular problem and configuration problem.

    The trajectory may be not easy to visualize.
  - **Cartesian motion**

    Exists singular problem and configuration problem.

    The trajectory is easy to visualize.