數值方法_作業十二_E14101082_陳政謙

HW 12

1. Given the problem

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=xy\,,\quad 0<x<\pi\,,\quad 0<y<\pi/2$$

$$u(0,y)=\cos y\,,\quad u(\pi,y)=-\cos y\,,\quad 0\le y\le \pi/2\,,$$

$$u(x,0)=\cos x\,,\quad u(x,\pi/2)=0\,,\quad 1\le y\le 2$$

To calculate $u(x,y)$ by using $h=k=0.1\pi$.

```
=== u(x,y) at interior nodes after 67 iterations ===
u(0.31, 0.31) = 0.753239
u(0.31, 0.63) = 0.564643
u(0.31, 0.94) = 0.368106
u(0.31, 1.26) = 0.172819
u(0.63, 0.31) = 0.555933
u(0.63, 0.63) = 0.347681
u(0.63, 0.94) = 0.176388
u(0.63, 1.26) = 0.053111
u(0.94, 0.31) = 0.333266
u(0.94, 0.63) = 0.132703
u(0.94, 0.94) = -0.004920
u(0.94, 1.26) = -0.058847
u(1.26, 0.31) = 0.085847
u(1.26, 0.63) = -0.086797
u(1.26, 0.94) = -0.182281
u(1.26, 1.26) = -0.166708
u(1.57, 0.31) = -0.173154
u(1.57, 0.63) = -0.305563
u(1.57, 0.94) = -0.353842
u(1.57, 1.26) = -0.269867
u(1.88, 0.31) = -0.424216
u(1.88, 0.63) = -0.511084
u(1.88, 0.94) = -0.511579
u(1.88, 1.26) = -0.364121
u(2.20, 0.31) = -0.645185
u(2.20, 0.63) = -0.686121
u(2.20, 0.94) = -0.641964
u(2.20, 1.26) = -0.441277
u(2.51, 0.31) = -0.814449
u(2.51, 0.63) = -0.809906
u(2.51, 0.94) = -0.724348
u(2.51, 1.26) = -0.486296
u(2.83, 0.31) = -0.915774
u(2.83, 0.63) = -0.858872
u(2.83, 0.94) = -0.725465
u(2.83, 1.26) = -0.467861
```

2. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{4K}\frac{\partial T}{\partial t}, \quad \frac{1}{2}\le r\le 1, \quad 0\le t,$$

$$T(1,t)=100+40t, \quad 0\le t\le 10; \qquad \frac{\partial T}{\partial r}+3T=0 \quad \text{at} \quad r=\frac{1}{2}$$

$$T(r,0)=200(r-0.5), \quad 0.5\le r\le 1,$$

and use $\Delta t=0.5$, $\Delta r=0.1$, and $K=0.1$ to calculate $T(r,t)$

By (a) the forward-difference method

(b) the backward-difference method

© the Crank-Nicolson algorithm.

```
=== T(r, t=10) FORWARD ===
[-4.05108557e+22 -5.26641124e+22  1.19111062e+23 -1.46390274e+23
  1.29283732e+23 -7.41604679e+22  5.00000000e+02]

=== T(r, t=10) BACKWARD ===
[363.657457 472.754695 484.800004 489.130399 492.588585 496.147296
 500.      ]

=== T(r, t=10) CRANK-NICOLSON ===
[363.66407  472.763291 484.802012 489.116963 492.603307 496.139804
 500.      ]
```

3. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0, \quad \frac{1}{2}\le r\le 1, \quad 0\le t\le \pi/3,$$

$$T(r,0)=0, \quad T(r,\pi/3)=0, \quad T(1/2,\theta)=50, \quad T(1,\theta)=100.$$

```
=== T(r, θ) after 78 iterations ===
T(0.50, 0.00) = 0.000000
T(0.50, 0.17) = 50.000000
T(0.50, 0.35) = 50.000000
T(0.50, 0.52) = 50.000000
T(0.50, 0.70) = 50.000000
T(0.50, 0.87) = 50.000000
T(0.50, 1.05) = 0.000000
T(0.60, 0.00) = 0.000000
T(0.60, 0.17) = 32.256692
T(0.60, 0.35) = 45.860166
T(0.60, 0.52) = 49.678613
T(0.60, 0.70) = 45.860166
T(0.60, 0.87) = 32.256692
T(0.60, 1.05) = 0.000000
T(0.70, 0.00) = 0.000000
T(0.70, 0.17) = 32.944392
T(0.70, 0.35) = 50.593746
T(0.70, 0.52) = 55.835007
T(0.70, 0.70) = 50.593746
T(0.70, 0.87) = 32.944392
T(0.70, 1.05) = 0.000000
T(0.80, 0.00) = 0.000000
T(0.80, 0.17) = 43.104344
T(0.80, 0.35) = 62.454940
T(0.80, 0.52) = 67.725257
T(0.80, 0.70) = 62.454940
T(0.80, 0.87) = 43.104344
T(0.80, 1.05) = 0.000000
T(0.90, 0.00) = 0.000000
T(0.90, 0.17) = 63.536510
T(0.90, 0.35) = 79.718194
T(0.90, 0.52) = 83.305331
T(0.90, 0.70) = 79.718194
T(0.90, 0.87) = 63.536510
T(0.90, 1.05) = 0.000000
T(1.00, 0.00) = 0.000000
T(1.00, 0.17) = 100.000000
```

```
T(0.90, 0.35) = 79.718194
T(0.90, 0.52) = 83.305331
T(0.90, 0.70) = 79.718194
T(0.90, 0.87) = 63.536510
T(0.90, 1.05) = 0.000000
T(1.00, 0.00) = 0.000000
T(1.00, 0.17) = 100.000000
T(1.00, 0.35) = 100.000000
T(1.00, 0.52) = 100.000000
T(1.00, 0.70) = 100.000000
T(1.00, 0.87) = 100.000000
T(1.00, 1.05) = 0.000000
```

4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \le x \le 1, \quad 0 \le t$$

$$p(0,t) = 1, \quad p(1,t) = 2, \quad p(x,0) = \cos(2\pi x), \quad \frac{\partial p}{\partial t}(x,0) = 2\pi \sin(2\pi x), \quad 0 \le x \le 1$$

To calculate p by using $\Delta x = \Delta t = 0.1$.

```
=== p(x, t=1) ===
p(0.0, 1.0) = 1.000000
p(0.1, 1.0) = 2.190983
p(0.2, 1.0) = 2.690983
p(0.3, 1.0) = 3.309017
p(0.4, 1.0) = 3.809017
p(0.5, 1.0) = 4.000000
p(0.6, 1.0) = 3.809017
p(0.7, 1.0) = 3.309017
p(0.8, 1.0) = 2.690983
p(0.9, 1.0) = 2.190983
p(1.0, 1.0) = 2.000000
```