數值方法_作業五_E14101082_陳政謙

1. The initial-value problem

   $y' = 1 + (y/t) + (y/t)^2$, $1 \le t \le 2$, $y(1) = 0$ has the exact solution $y(t) = t\tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

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--- Problem 1: y' = 1+(y/t)+(y/t)^2, y(1)=0, 1 <= t <= 2 ---

 --- Part (a): Euler's Method (h = 0.10000000) ---
 t_i       w_i (Euler)    y_i (Exact)    |Error|
 ------------------------------------------------
 1.00000000  0.00000000  0.00000000  0.00000000
 1.10000000  0.10000000  0.10515982  0.00515982
 1.20000000  0.20991736  0.22124277  0.01132542
 1.30000000  0.33047056  0.34912113  0.01865057
 1.40000000  0.46235355  0.48968166  0.02732812
 1.50000000  0.60628547  0.64387533  0.03758986
 1.60000000  0.76304149  0.81275274  0.04971125
 1.70000000  0.93347503  0.99749413  0.06401910
 1.80000000  1.11853673  1.19943864  0.08090191
 1.90000000  1.31929261  1.42011584  0.10082322
 2.00000000  1.53694328  1.66128176  0.12433848
```

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--- Part (b): Taylor's Method Order 2 (h = 0.10000000) ---
t_i       w_i (Taylor2)  y_i (Exact)    |Error|
-----------------------------------------------------
1.00000000  0.00000000  0.00000000  0.00000000
1.10000000  0.10500000  0.10515982  0.00015982
1.20000000  0.22091916  0.22124277  0.00032362
1.30000000  0.34861239  0.34912113  0.00050875
1.40000000  0.48895375  0.48968166  0.00072791
1.50000000  0.64288278  0.64387533  0.00099255
1.60000000  0.81143817  0.81275274  0.00131457
1.70000000  0.99578673  0.99749413  0.00170740
1.80000000  1.19725172  1.19943864  0.00218692
1.90000000  1.41734353  1.42011584  0.00277230
2.00000000  1.65779466  1.66128176  0.00348709
```

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

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--- Problem 2: System u1', u2', u1(0)=4/3, u2(0)=2/3 ---

--- Runge-Kutta Order 4 (h = 0.10000000, t_end = 1.00000000) ---
t_j       w1_j (RK4)   u1_j (Exact) |Error u1|   w2_j (RK4)   u2_j (Exact) |Error u2|
--------------------------------------------------------------------------------
0.00000000  1.33333333  1.33333333  0.00000000   0.66666667  0.66666667  0.00000000
0.10000000  -2.64518125  1.79306259  4.43824384   7.84454215  -1.03200245  8.87654460
0.20000000  -18.45168697  1.42390240  19.87558936   38.87658182  -0.87468103  39.75126284
0.30000000  -87.47324626  1.13157652  88.60482278   176.48474018  -0.72499857  177.20973875
0.40000000  -394.07739612  0.90940859  394.98680471   789.36548692  -0.60821421  789.97370113
0.50000000  -1760.04904441  0.73878784  1760.78783225   3521.06009139  -0.51565767  3521.57574906
0.60000000  -7848.70111385  0.60570965  7849.30682350   15698.17331104  -0.44041076  15698.61372180
0.70000000  -34990.43518256  0.49986025  34990.93504281   69981.49274609  -0.37740382  69981.87014991
0.80000000  -155983.48939112  0.41367148  155983.90306260   311967.48322576  -0.32295352  311967.80617928
0.90000000  -695350.55145103  0.34161435  695350.89306538   1390701.51176668  -0.27440884  1390701.78617552
1.00000000  -3099761.00761206  0.27967491  3099761.28728697   6199522.34472266  -0.22988784  6199522.57461050
```

```
--- Runge-Kutta Order 4 (h = 0.05000000, t_end = 1.00000000) ---
t_j       w1_j (RK4)   u1_j (Exact) |Error u1|   w2_j (RK4)   u2_j (Exact) |Error u2|
--------------------------------------------------------------------------------
0.00000000  1.33333333  1.33333333  0.00000000   0.66666667  0.66666667  0.00000000
0.05000000  1.73641638  1.91205863  0.17564226   -0.55779025  -0.90907659  0.35128634
0.10000000  1.71222044  1.79306259  0.08084215   -0.87031502  -1.03200245  0.16168743
0.15000000  1.57269316  1.60196676  0.02927360   -0.90290746  -0.96145871  0.05855125
0.20000000  1.41407174  1.42390240  0.00983066   -0.85501507  -0.87468103  0.01966595
0.25000000  1.26442992  1.26764562  0.00321570   -0.78878439  -0.79522077  0.00643638
0.30000000  1.13052570  1.13157652  0.00105082   -0.72289179  -0.72499857  0.00210678
0.35000000  1.01264500  1.01299856  0.00035355   -0.66234738  -0.66305963  0.00071225
0.40000000  0.90927824  0.90940859  0.00013035   -0.60794846  -0.60821421  0.00026575
0.45000000  0.81857063  0.81862953  0.00005890   -0.55926657  -0.55938925  0.00012268
0.50000000  0.73875202  0.73878784  0.00003582   -0.51558139  -0.51565767  0.00007629
0.55000000  0.66824657  0.66827466  0.00002809   -0.47616417  -0.47622475  0.00006058
0.60000000  0.60568443  0.60570965  0.00002521   -0.44035621  -0.44041076  0.00005455
0.65000000  0.54988557  0.54990941  0.00002383   -0.40758384  -0.40763534  0.00005150
0.70000000  0.49983737  0.49986025  0.00002289   -0.37735451  -0.37740382  0.00004931
0.75000000  0.45467271  0.45469474  0.00002203   -0.34924820  -0.34929551  0.00004731
0.80000000  0.41365032  0.41367148  0.00002116   -0.32290824  -0.32295352  0.00004529
0.85000000  0.37613748  0.37615771  0.00002023   -0.29803287  -0.29807605  0.00004318
0.90000000  0.34159510  0.34161435  0.00001925   -0.27436787  -0.27440884  0.00004096
0.95000000  0.30956480  0.30958300  0.00001821   -0.25170005  -0.25173868  0.00003864
1.00000000  0.27965780  0.27967491  0.00001710   -0.22985162  -0.22988784  0.00003621
```