

Assessment Evaluation - POST Math Prefresher - 9/2017 ANSWER KEY

We've just spent a magical week together. This is to see how much stuck from the course and to help you identify areas that could use more attention and development. Do your best. **Note: THERE ARE TWO SIDES!**

Background questions:

Name, discipline and subfield?

Questions

1. Basics

(a) Explain the significance or use of the following symbols:

- i. Π ANS: This is for multiplication of a series or the symbol to represent 3.14...
- ii. Σ ANS: This is for the summation of a series

(b) Solve: (no need to simplify but show steps/work if possible)

- i. $4 \geq x - 7$ ANS: $x \leq 11$
 - ii. $-9x + 2 > 3$ ANS: $x < \frac{-1}{9}$
 - iii. $|x - 2| \leq 2$ ANS: $0 \leq x \leq 4$
 - iv. $2e^{6x} = 18$ ANS: $e^{6x} = 9$, $x = \frac{\ln(9)}{6}$
 - v. $e^{x^2} = 1$ ANS: $x^2 = 0$ so $x = 0$
 - vi. $\ln(x^2) = 5$ ANS: $\ln(x) = 2.5$, so $x = e^{2.5}$
 - vii. $\sum_{n=1}^{10} 3 + n$ ANS: $3 * 10 + (10 * (11))/2 = 30 + 55 = 85$
 - viii. $4!$ ANS: $4 * 3 * 2 * 1$
 - ix. $(\frac{x^4 y^{-3}}{x^2 y^3})^3$ ANS: $(\frac{x^2}{y^6})^3 = \frac{x^6}{y^{18}}$
- (c) Factor
- i. $m^2 + 3m + 2$ ANS: $(m + 2)(m + 1)$
 - ii. $x^2 + 7x + 6$ ANS: $(x + 1)(x + 6)$
 - iii. $x^4 + x^2$ ANS: $x^2(x^2 + 1)$

2. Set Theory

(a) Explain the meaning of the following symbols:

i. \in ANS: 'an element of'

ii. \forall ANS: 'for all'

(b) Suppose $A = \{3, 6, 12\}$, $B = \{\text{hat, bulldozer, forklift}\}$ and $C = \{x | x \text{ is a natural number} | x > 3 \text{ and } x < 9\}$

i. What is $A \cup B$? ANS: $\{3, 6, 12, \text{hat, bulldozer, forklift}\}$

ii. Write the elements of C ANS: $\{4, 5, 6, 7, 8\}$

iii. What is $A \cap C$? ANS: $\{6\}$

iv. What is $A \setminus C$? ANS: $\{3, 12\}$

3. Functions & Pre-Calculus

(a) What is a continuous function? ANS: One you can draw without picking up a pencil – where the limit from the left equals that from the right and equals the value at the point (and the value exists!)

(b) Draw an increasing function. ANS: one where y gets larger as x increases (x and y move together–positive slope)

(c) What is a tangent line? What does it do? ANS: The tangent line is one that touches the graph of a function at only one point.

(d) What is a derivative? ANS: an instantaneous rate of change – the slope of the tangent line at a particular point

4. Matrix Algebra

(a) Give an example of a 3×4 matrix $\begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \end{bmatrix}$

(b) Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 2 & 1 \\ 6 & 2 & 9 \end{bmatrix}$$

i. Which matrices can be added together? A and B can be added together because they have the same dimensions 2×3

- ii. Add the matrices from the above response. $\begin{bmatrix} 3 & 5 & 2 \\ 1 & 4 & 5 \end{bmatrix}$
- iii. Which matrices can be multiplied together? ANS: A and B can each be multiplied by C (SO, $A * C$) but cannot multiply A and B by each other or C by A or B (so, yes AC, no CA) We know that because A is 2×3 and C is 3×3 , we can multiply them (middle two numbers same). We also know that the dimensions of the final matrix will be the number of rows in A (2) and the number of columns in C (3) – so 2×3 .
- iv. Multiply the matrices from the above response. Can do $A * C$ or $B * C$. $A * C$ as example: $\begin{bmatrix} (3 * 1 + 0 + 1 * 6) & (3 * 4 + 4 * 2 + 1 * 2) & (3 * 1 + 4 * 1 + 1 * 9) \\ (0 + 0 + 6 * 1) & (0 + 2 * 2 + 1 * 2) & (0 + 2 * 1 + 1 * 9) \end{bmatrix}$

5. Calculus

- (a) what is the derivative of 4? ANS: 0
- (b) what is the derivative of $2x$? ANS: 2
- (c) calculate the derivative of $7m^2 - m + 2$ ANS: $14m - 1$
- (d) calculate the integral $\int_0^5 (x^3 + 0.5x^2 + 5x) dx$ ANS: $x^4/4 + x^3/6 + 5/2x^2|_0^5$
- (e) calculate the integral $\int e^x dx$ ANS: $e^x + c$