Pre-Assessment Evaluation

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Background Questions

My name is Andrew Saab, a political science Ph.D. student. I am hovering between American and Comparative politics, but will pursue methods as a subfield with certainty. I hope to refresh some basic mathematical concepts and intuition that tend to be notoriously forgotten even among those with more advanced backgrounds.

Problem Solving Questions

1. Basics

(a)

i. Π is a symbol that denotes the multiplication of a set of factors (e.g., $\Pi a_i = a_1 \times a_2 \times a_3 \times ...$). On the other hand, π denotes the constant that is equivalent to the ratio of the circumference to the diameter of a circle.

ii. Σ denotes a summation of a set of factors (e.g., (e.g., $\Sigma a_i = a_1 + a_2 + a_3 + ...$).

(b)

i. $4 \geq x - 7 \implies 4 + 7 \geq x - 7 + 7 \implies x \leq 11$

ii. $-9x + 2 > 3 \implies -9x + 2 - 2 > 3 - 2 \implies -9x > 1 \implies -x > \frac{1}{9} \implies x < -\frac{1}{9}$

iii. $|x - 2| \leq 2 \implies x - 2 \leq 2$ and $x - 2 \geq -2$

$x \leq 4$ and $x \geq 0$

iv. $2e^{6x} = 18 \implies e^{6x} = 9 \implies lne^{6x} = 9 \implies 6xlne = 9 \implies x = \frac{9}{6} = \frac{3}{2}$ (Note: $lne = 1$)

v. $e^{x^2} = 1 \implies lne^{x^2} = 1 \implies x^2 lne = 1 \implies x^2 = 1 \implies x = 1$ or $x = -1$

vi. $lnx^2 = 5 \implies e^{lnx^2} = e^5 \implies x^2 = e^5 \implies x = \sqrt{e^5}$ or $x = -\sqrt{e^5}$ (Note: $e^{lnx} = x$)

vii. $\sum_{n=1}^{10} 3 + n = \sum_{n=1}^{10} 3 + \sum_{n=1}^{10} n = (10 \times 3) + (1 + 2 + ... + 10) = 30 + 55 = 85$

viii. $4! = 4 \times 3 \times 2 \times 1 = 24$

ix. $\left(\frac{x^4 y^{-3}}{x^2 y^3}\right)^3 = \frac{x^{12} y^{-9}}{x^6 y^9} = x^6 y^0 = x^6$

(c)

i. $m^2 + 3m + 2 = (m + 1)(m + 2)$

ii. $x^2 + 5x + 6 = (x + 2)(x + 3)$

iii. $x^2 + x = x(x + 1)$

2. Set Theory

(a)

i. $\in$ denotes that a partcular element belongs to a set.

ii. $\forall$ is used to express that a particular statement holds for all values satisfying the specified conditions (e.g., $\frac{x}{y} = 1 \; \forall x = y$).

(b)

i. $A \cup B = \{3, 4, 5, hat, triangle, forklift\}$

ii. $C = \{4, 5, 6, 7, 8\}$

iii. $A \cap C = \{4, 5\}$

3. Functions and Pre-Calculus +3

(a) Let $f : E \to R$ and $p \in E$. f is continuous if and only if f has a limit at p and its limit equals to $f(p)$ (i.e., $\lim_{x \to p} f(x) = f(p)$).

Additionally, this is true if and only if $\forall \varepsilon > 0$, $\exists \delta$ s.t. if $x \in E$ and $|x - p| < \delta$ then $|f(x) - f(p)| < \varepsilon$. This means that one can always find a point x within the domain such that the distance between $f(x)$ and $f(p)$ can be chosen to be arbitrarily small.

(b) Unfortunately, I do not have the skills to draw on LaTeX. However, the definition of an increasing function is one such that $f(x + \varepsilon) \geq f(x) \; \forall x \in R$ and $\forall \varepsilon > 0$. A function is strictly increasing if $f(x + \varepsilon) > f(x)$ (i.e., $f(x) = x$).

(c) A tangent line is one that just touches a curve at a particular point. The slope of this tangent line indicates the curve's instantaneous rate of change at the point of contact with the curve. The slope of a tangent line can be calculated by taking the first derivative of a function and plugging the values of the domain to calculate the rate of change at the desired point.

4. Matrix Algebra +1.5

(a) I have taken a course on linear algebra at the undergraduate and graduate level. I have used its applications extensively in econometrics courses.

0.5 (b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$ 3x4 = 3 rows x 4 cols

5. Calculus +4

(a) $\frac{d(4x)}{dx} = 4$

(b) $\frac{d(3m^2 - 8m + 5)}{dm} = 6m - 8$

(c) $\int_0^5 (x^3 + 0.5x^2 + 5x)dx = \frac{1}{4}x^4 + \frac{0.5}{3}x^3 + \frac{5}{2}x^2 + c \; \Big|_0^5$ dont combine dec. w/ frac

Plug in 5 and subtract result from that obtained when plugging in 0.

(d) $\int e^x dx = e^x + c$

Missing part (e & 7 (prob & stats)