

Demo

CSR to CVR

Matrix

- $15 * 15$
- $\text{nnz} = 51$

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6
	10		9								8			
				7										
			1				5			9				7
2			5		8	4	4				1			
		8												1
				6			7					6		
			5			10		9						
								4		6				11

Matrix

Thread_0
nnz = 26

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

Thread_1
nnz = 25

	10		9								8			
				7										
			1				5			9				7
2			5		8	4	4				1			
		8												1
				6			7					6		
			5			10		9						
								4		6				11

Thread_0

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

CVR

lane	val							tail	col_idx						
0	1	1	2	3	5	7	7	0	1	3	4	6	12	11	13
1	8	1	3	2	1	3	4	1	0	3	4	7	8	10	14
2	5	5	8	3	3	6	0	6	1	5	10	0	7	14	14
3	9	1	4	4	2	3	0	5	11	12	1	2	6	13	14

rec	pos	7	10	16	22	23	24	25
	wb	4	2	0	2	3	2	1

lr_rec = 16

Thread_0, initialize

val	1	1	2	3	5	8	1	3	2	1	3	4	5	5	8	9
col_idx	1	3	4	6	12	0	3	4	7	8	10	14	1	5	10	11

row_ptr	0	5	12	15	17
---------	---	---	----	----	----

lane	val		col_idx		tail	tracker		
						valid	rowID	count
0	-	-	-	-	-	0	0	5
1	-	-	-	-	-	5	1	7
2	-	-	-	-	-	12	2	3
3	-	-	-	-	-	15	4	2

rowID = row

valid =
row_ptr[row]

count =
row_ptr[row+1]
- row_ptr[row]

Convert

val	1	1	2	3	5	8	1	3	2	1	3	4	5	5	8	9
col_idx	1	3	4	6	12	0	3	4	7	8	10	14	1	5	10	11
row_ptr	0	5	12	15	17											

Diagram illustrating the conversion process. The `row_ptr` array (0, 5, 12, 15, 17) is used to map rows to the `val` and `col_idx` arrays. Red arrows show the mapping: `row_ptr[0] = 0` maps to `val[0] = 1` and `col_idx[0] = 1`; `row_ptr[1] = 5` maps to `val[5] = 8` and `col_idx[5] = 0`; `row_ptr[2] = 12` maps to `val[12] = 5` and `col_idx[12] = 1`; `row_ptr[3] = 15` maps to `val[15] = 9` and `col_idx[15] = 11`.

lane	val		col_idx		tail	tracker		
						valid	rowID	count
0	1	-	1	-	-	0	0	5
1	8	-	0	-	-	5	1	7
2	5	-	1	-	-	12	2	3
3	9	-	11	-	-	15	4	2

Convert

val	1	1	2	3	5	8	1	3	2	1	3	4	5	5	8	9
col_idx	1	3	4	6	12	0	3	4	7	8	10	14	1	5	10	11

row_ptr	0	5	12	15	17
---------	---	---	----	----	----

lane	val		col_idx		tail	tracker		
						valid	rowID	count
0	1	-	1	-	-	1	0	4
1	8	-	0	-	-	6	1	6
2	5	-	1	-	-	13	2	2
3	9	-	11	-	-	16	4	1

valid++

count--

Convert

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

[illegible]

Convert

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val			col_idx			tail	tracker		
								valID	rowID	count
0	1	-	-	1	-	-	-	1	0	4
1	8	-	-	0	-	-	-	6	1	6
2	5	-	-	1	-	-	-	13	2	2
3	9	-	-	11	-	-	-	16	4	1

Trigger recording

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val			col_idx			tail	tracker		
								valID	rowID	count
0	1	1	-	1	3	-	-	2	0	3
1	8	1	-	0	3	-	-	7	1	5
2	5	5	-	1	5	-	-	14	2	1
3	9	1	-	11	12	-	-	17	4	0

Record

lane	val			col_idx			tail	tracker		
	$i = 0$	$i = 1$						valid	rowID	count
0	1	1	-	1	3	-	-	2	0	3
1	8	1	-	0	3	-	-	7	1	5
2	5	5	-	1	5	-	-	14	2	1
lane = 3	3	9	1	-	11	12	-	17	4	0

$pos = i * n_lanes + lane$

$wb = rowID$

rec	pos	7	-
	wb	4	-

Feed

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val			col_idx			tail	tracker		
								valID	rowID	count
0	1	1	-	1	3	-	-	2	0	3
1	8	1	-	0	3	-	-	7	1	5
2	5	5	-	1	5	-	-	14	2	1
3	9	1	-	11	12	-	-	17	5	4

Convert

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val			col_idx			tail	tracker		
								valID	rowID	count
0	1	1	-	1	3	-	-	2	0	3
1	8	1	-	0	3	-	-	7	1	5
2	5	5	-	1	5	-	-	14	2	1
3	9	1	-	11	12	-	-	17	5	4

Convert

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val							tail	tracker									
									valID	rowID	count							
0	1	1	-	-	-	-	-	2	0	3								
1	8	1	-	-	-	-	-	7	1	5								
2	5	5	-	-	-	-	-	14	2	1								
3	9	1	-	-	-	-	-	17	5	4								
	col_idx							<div>rec</div>										
	1	3	-	-	-	-	-							pos	7	-	-	-
	0	3	-	-	-	-	-							wb	4	-	-	-
	1	5	-	-	-	-	-							pos	-	-	-	-
	11	12	-	-	-	-	-							wb	-	-	-	-

Record

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	-	-	-	-	-	3	0	2
1	8	1	3	-	-	-	-	-	8	1	4
2	5	5	8	-	-	-	-	-	15	2	0
3	9	1	4	-	-	-	-	-	18	5	3

col_idx						
1	3	4	-	-	-	-
0	3	4	-	-	-	-
1	5	10	-	-	-	-
11	12	1	-	-	-	-

rec	pos	7	10	-	-
	wb	4	2	-	-
	pos	-	-	-	-
	wb	-	-	-	-

Feed

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	-	-	-	-	-	3	0	2
1	8	1	3	-	-	-	-	-	8	1	4
2	5	5	8	-	-	-	-	-	21	6	5
3	9	1	4	-	-	-	-	-	18	5	3

col_idx						
1	3	4	-	-	-	-
0	3	4	-	-	-	-
1	5	10	-	-	-	-
11	12	1	-	-	-	-

rec	pos	7	10	-	-
	wb	4	2	-	-
	pos	-	-	-	-
	wb	-	-	-	-

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	-	-	-	-	-	3	0	2
1	8	1	3	-	-	-	-	-	8	1	4
2	5	5	8	-	-	-	-	-	21	6	5
3	9	1	4	-	-	-	-	-	18	5	3

Last row

lane	val							tail	tracker		
									valid	rowID	count
0	1	1	2	-	-	-	-	0	3	0	2
1	8	1	3	-	-	-	-	1	8	1	4
2	5	5	8	-	-	-	-	6	21	2	5
3	9	1	4	-	-	-	-	5	18	3	3

tail[] = rowID[]

rowID[] = lane

Feeding will be replaced by
stealing since then.

Convert

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val							tail	tracker									
									valID	rowID	count							
0	1	1	2	-	-	-	-	0	3	0	2							
1	8	1	3	-	-	-	-	1	8	1	4							
2	5	5	8	-	-	-	-	6	21	2	5							
3	9	1	4	-	-	-	-	5	18	3	3							
	col_idx							<div>rec</div>										
	1	3	4	-	-	-	-							pos	7	10	-	-
	0	3	4	-	-	-	-							wb	4	2	-	-
	1	5	10	-	-	-	-							pos	-	-	-	-
	11	12	1	-	-	-	-							wb	-	-	-	-

Convert

	1		1	2		3					5		
8			1	3			2	1		3			4
	5				5					8			
											9	1	
	4	4				2							3
3							3				7		7 6

lane	val							tail	tracker									
									valID	rowID	count							
0	1	1	2	3	-	-	-	0	4	0	1							
1	8	1	3	2	-	-	-	1	9	1	3							
2	5	5	8	3	-	-	-	6	22	2	4							
3	9	1	4	4	-	-	-	5	19	3	2							
	col_idx							<div>rec</div>										
	1	3	4	6	-	-	-							pos	7	10	-	-
	0	3	4	7	-	-	-							wb	4	2	-	-
	1	5	10	0	-	-	-							pos	-	-	-	-
	11	12	1	2	-	-	-							wb	-	-	-	-

Record

	1		1	2		3					5			
8			1	3			2	1		3			4	
	5				5					8				
										9	1			
	4	4				2						3		
3							3				7		7	6

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	3	5	-	-	0	5	0	0
1	8	1	3	2	1	-	-	1	10	1	2
2	5	5	8	3	3	-	-	6	23	2	3
3	9	1	4	4	2	-	-	5	20	3	1

col_idx						
1	3	4	6	12	-	-
0	3	4	7	8	-	-
1	5	10	0	7	-	-
11	12	1	2	6	-	-

lr_rec = 16

rec	pos	7	10	16	-
	wb	4	2	0	-
	pos	-	-	-	-
	wb	-	-	-	-

Steal

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	3	5	-	-	0	5	0	0
1	8	1	3	2	1	-	-	1	10	1	2
2	5	5	8	3	3	-	-	6	23	2	3
3	9	1	4	4	2	-	-	5	20	3	1

$$\text{average} = (2 + 3 + 1 + n_lanes - 1) / n_lanes = 2$$

Steal

lane	val							tail	tracker		
									valid	rowID	count
0	1	1	2	3	5	-	-	0	5	0	0
1	8	1	3	2	1	-	-	1	10	1	2
2	5	5	8	3	3	-	-	6	23	2	3
3	9	1	4	4	2	-	-	5	20	3	1

average = 2

count[2] > average
candidate = 2

Steal

lane	val							tail	tracker		
									validID	rowID	count
0	1	1	2	3	5	-	-	0	23	2	2
1	8	1	3	2	1	-	-	1	10	1	2
2	5	5	8	3	3	-	-	6	25	2	1
3	9	1	4	4	2	-	-	5	20	3	1

average = 2

count[2] > average

candidate = 2

valid = valid[candidate]

rowID = candidate

count = average

valid[candidate] += average

count[candidate] -= average

Convert

lr_rec = 16

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	3	5	-	-	0	23	2	2
1	8	1	3	2	1	-	-	1	10	1	2
2	5	5	8	3	3	-	-	6	25	2	1
3	9	1	4	4	2	-	-	5	20	3	1

col_idx						
1	3	4	6	12	-	-
0	3	4	7	8	-	-
1	5	10	0	7	-	-
11	12	1	2	6	-	-

rec	pos	7	10	16	-
	wb	4	2	0	-
	pos	-	-	-	-
	wb	-	-	-	-

Record

lr_rec = 16

	1		1	2		3					5		
8			1	3			2	1		3			4
	5				5					8			
											9	1	
	4	4				2							3
3							3				7	7	6

lane	val							tail	tracker								
									valID	rowID	count						
0	1	1	2	3	5	7	-	0	24	2	1						
1	8	1	3	2	1	3	-	1	11	1	1						
2	5	5	8	3	3	6	-	6	26	2	0						
3	9	1	4	4	2	3	-	5	21	3	0						
	col_idx							<div>rec</div>									
	1	3	4	6	12	11	-						pos	7	10	16	22
	0	3	4	7	8	10	-						wb	4	2	0	2
	1	5	10	0	7	14	-						pos	23	-	-	-
	11	12	1	2	6	13	-						wb	3	-	-	-

Steal(the last round)

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	3	5	7	-	0	24	2	1
1	8	1	3	2	1	3	-	1	11	1	1
2	5	5	8	3	3	6	-	6	26	2	0
3	9	1	4	4	2	3	-	5	21	3	0

average = $(1 + 1 + n_lanes - 1) / n_lanes = 1$
count[] <= average

Steal(the last round)

lane	val							tail	tracker		
									valID	rowID	count
0	1	1	2	3	5	7	-	0	24	2	1
1	8	1	3	2	1	3	-	1	11	1	1
2	5	5	8	3	3	6	-	6	-1	2	1
3	9	1	4	4	2	3	-	5	-1	3	1

average = 1
count[] <= average

valID = -1
count = 1
(padding)

Convert

lr_rec = 16

	1		1	2		3					5		
8			1	3			2	1		3			4
	5				5					8			
											9	1	
	4	4				2							3
3							3				7	7	6

lane	val							tail	tracker								
									valID	rowID	count						
0	1	1	2	3	5	7	-	0	24	2	1						
1	8	1	3	2	1	3	-	1	11	1	1						
2	5	5	8	3	3	6	-	6	-1	2	1						
3	9	1	4	4	2	3	-	5	-1	3	1						
	col_idx							<div>rec</div>									
	1	3	4	6	12	11	-						pos	7	10	16	22
	0	3	4	7	8	10	-						wb	4	2	0	2
	1	5	10	0	7	14	-						pos	23	-	-	-
	11	12	1	2	6	13	-						wb	3	-	-	-

Record

lr_rec = 16

	1		1	2		3					5		
8			1	3			2	1		3			4
	5				5					8			
											9	1	
	4	4				2						3	
3							3				7		7 6

lane	val							tail	tracker																							
									valID	rowID	count																					
0	1	1	2	3	5	7	7	0	25	2	0																					
1	8	1	3	2	1	3	4	1	12	1	0																					
2	5	5	8	3	3	6	0	6	0	2	0																					
3	9	1	4	4	2	3	0	5	0	3	0																					
	col_idx							<div>rec</div> <table><tr><td>pos</td><td>7</td><td>10</td><td>16</td><td>22</td></tr><tr><td>wb</td><td>4</td><td>2</td><td>0</td><td>2</td></tr><tr><td>pos</td><td>23</td><td>24</td><td>25</td><td>-</td></tr><tr><td>wb</td><td>3</td><td>2</td><td>1</td><td>-</td></tr></table>					pos	7	10	16	22	wb	4	2	0	2	pos	23	24	25	-	wb	3	2	1	-
	pos	7	10	16	22																											
	wb	4	2	0	2																											
	pos	23	24	25	-																											
	wb	3	2	1	-																											
1	3	4	6	12	11	13																										
0	3	4	7	8	10	14																										
1	5	10	0	7	14	14																										
11	12	1	2	6	13	14																										

Thread_0

	1		1	2		3						5		
8			1	3			2	1		3				4
	5				5					8				
											9	1		
	4	4				2							3	
3							3				7		7	6

CVR

lane	val							tail	col_idx						
0	1	1	2	3	5	7	7	0	1	3	4	6	12	11	13
1	8	1	3	2	1	3	4	1	0	3	4	7	8	10	14
2	5	5	8	3	3	6	0	6	1	5	10	0	7	14	14
3	9	1	4	4	2	3	0	5	11	12	1	2	6	13	14

rec	pos	7	10	16	22	23	24	25
	wb	4	2	0	2	3	2	1

lr_rec = 16