CS1011：數位電子導論

Inductance

Outline

- Introduction
- Inductance
- Self-inductance
- Inductors
- Inductors in Series and Parallel
- Voltage and Current
- Energy Storage in an Inductor
- Mutual Inductance
- Transformers
- Circuit Symbols
- The Use of Inductance in Sensors

Introduction

- We know that capacitors store energy by producing an electric field within a piece of dielectric material
- Inductors also store energy, in this case it is stored within a magnetic field

Inductance

- A changing magnetic flux induces an e.m.f. in any conductor within it
- Faraday's law:

  The magnitude of the e.m.f. induced in a circuit is proportional to the rate of change of magnetic flux linking the circuit

- Lenz's law:

  The direction of the e.m.f. is such that it tends to produce a current that opposes the change of flux responsible for inducing the e.m.f.

- Various applications in our daily life
  - Generator
  - RFID
  - Wireless charging
  - Metal detector

Inductance

- **When a circuit forms a single loop, the e.m.f. induced is given by the rate of change of the flux**
- **When a circuit contains many loops, the resulting e.m.f. is the sum of those produced by each loop**
- **Therefore, if a coil contains *N* loops, the induced voltage *V* is given by**

$$V = N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

where dΦ/d*t* is the rate of change of flux in Wb/s

- **This property, whereby an e.m.f. is induced as a result of changes in magnetic flux, is known as inductance**

Self-inductance

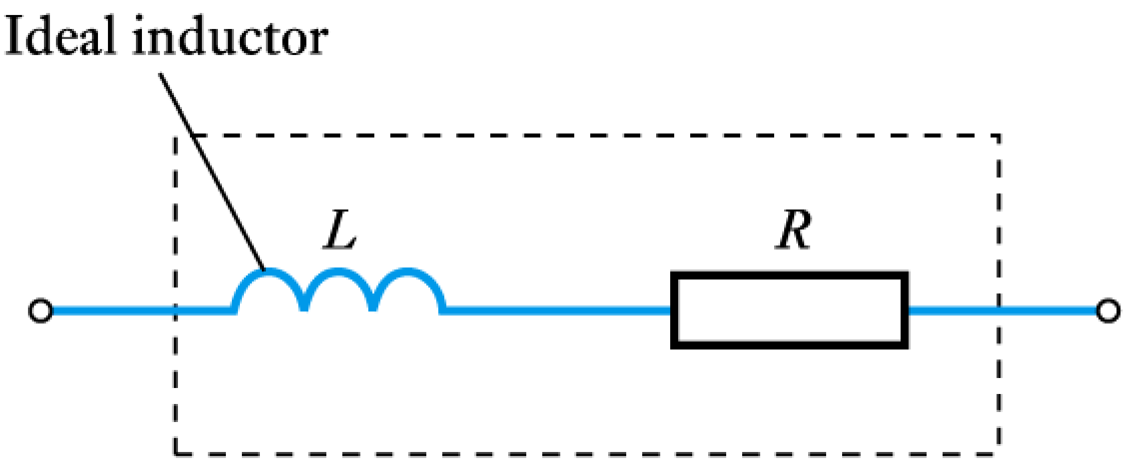
- **A changing current in a wire causes a changing magnetic field about it**
- **A changing magnetic field induces an e.m.f. in conductors within that field**
- **Therefore, when the current in a coil changes, it induces an e.m.f. in the coil itself**
- **This process is known as self-inductance**

$$V = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

where *L* is the inductance of the coil (unit is the Henry)

Equivalent Circuit of an Inductor

- **All circuits possess stray inductance (often unwanted)**
  - **Besides of inductance, all real circuits also possess stray capacitance**

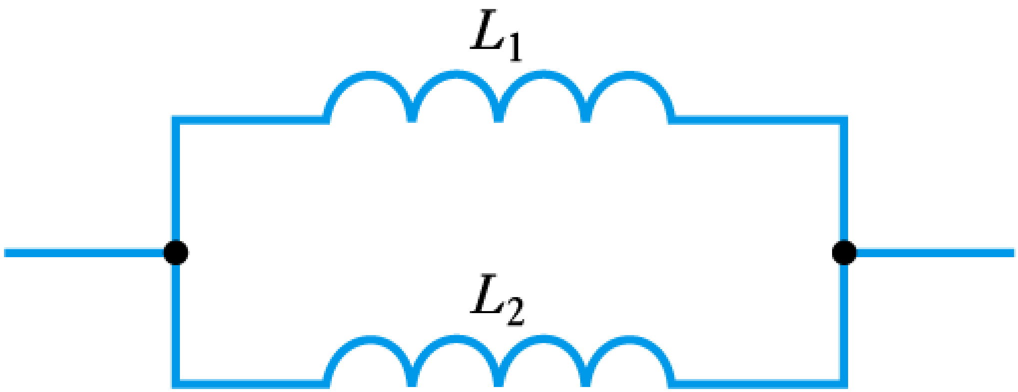


Inductors in Series

- When several inductors are connected together their effective inductance can be calculated in the same way as for resistors – provided that they are not linked magnetically

$$L = L_1 + L_2$$

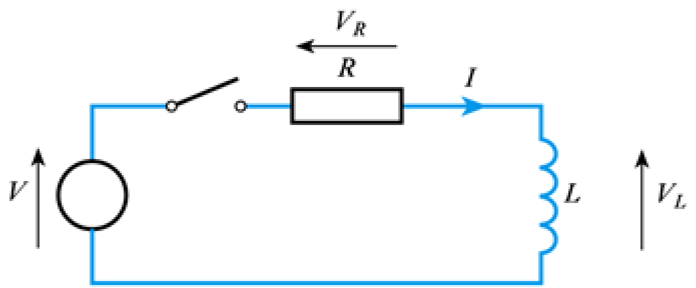
Inductors in Parallel



$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Energizing Inductor

- Inductor is initially un-energized
  - Current through it will be zero
- Switch is closed at $t = 0$
- I is initially zero
  - Hence, V_R is initially 0
  - Hence, V_L is initially V
- As the inductor is energized…
  - I increases
  - V_R increases
  - Hence, V_L decreases
- When the inductor is fully energized
  - $t \rightarrow \infty$
  - $V_R = V$
  - $V_L = 0$
  - $I = V/R$



(a)

(b)

(c)

Energizing Inductor

- Applying Kirchhoff's voltage law

$$iR + v = V$$

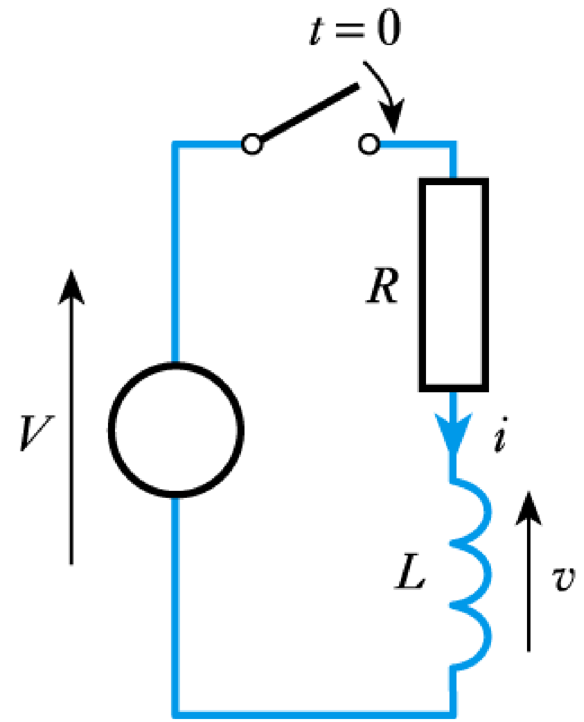
- Now, in an inductor

$$v = L\frac{di}{dt}$$

- Which substituting gives

$$iR + L\frac{di}{dt} = V$$

- Solve the differential equation
  - 1. Guess a general solution
  - 2. Apply the solution
  - 3. Substitute by boundary condition
  - 4. Verify the equation



Energizing Inductor

- **1. Guess a general solution**

$$i(t) = \alpha \cdot e^{\beta t} + \gamma$$

- **2. Apply the solution**

$$iR + L\frac{di}{dt} = V \rightarrow R \cdot \alpha \cdot e^{\beta t} + R \cdot \gamma + L \cdot \alpha\beta \cdot e^{\beta t} = V$$

$$\rightarrow \begin{cases} R \cdot \alpha \cdot e^{\beta t} + L \cdot \alpha\beta \cdot e^{\beta t} = 0 \rightarrow \beta = -\dfrac{R}{L} \\ R \cdot \gamma = V \rightarrow \gamma = \dfrac{V}{R} \end{cases}$$

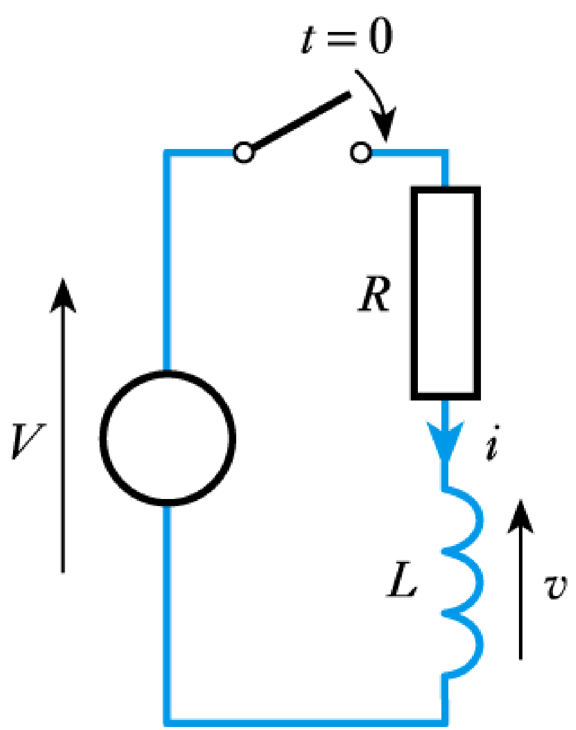
$$\therefore i(t) = \alpha \cdot e^{-\frac{R}{L}t} + \frac{V}{R}$$

- **3. Substitute by boundary condition**

When $t = 0, i = 0$:

$$i(t=0) = \alpha + \frac{V}{R} = 0 \rightarrow \alpha = -\frac{V}{R}$$

$$\therefore i(t) = -\frac{V}{R}e^{-\frac{R}{L}t} + \frac{V}{R} = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$



De-Energizing Inductor

- Applying Kirchhoff's voltage law

$$iR + v = 0$$

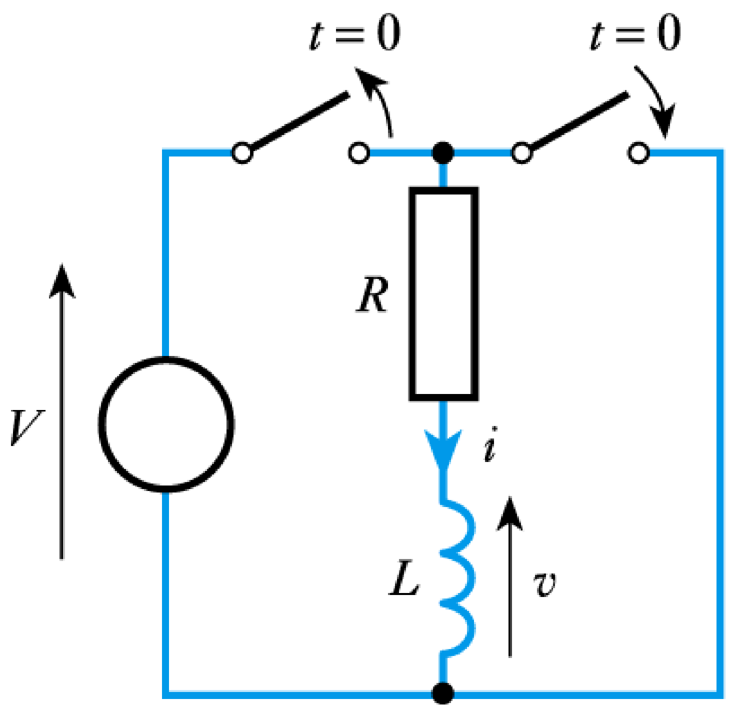
- Now, in an inductor

$$v = L\frac{di}{dt}$$

- Which substituting gives

$$iR + L\frac{di}{dt} = 0$$

- Solve the differential equation
  - 1. Guess a general solution
  - 2. Apply the solution
  - 3. Substitute by boundary condition
  - 4. Verify the equation



De-Energizing Inductor

- **1. Guess a general solution**

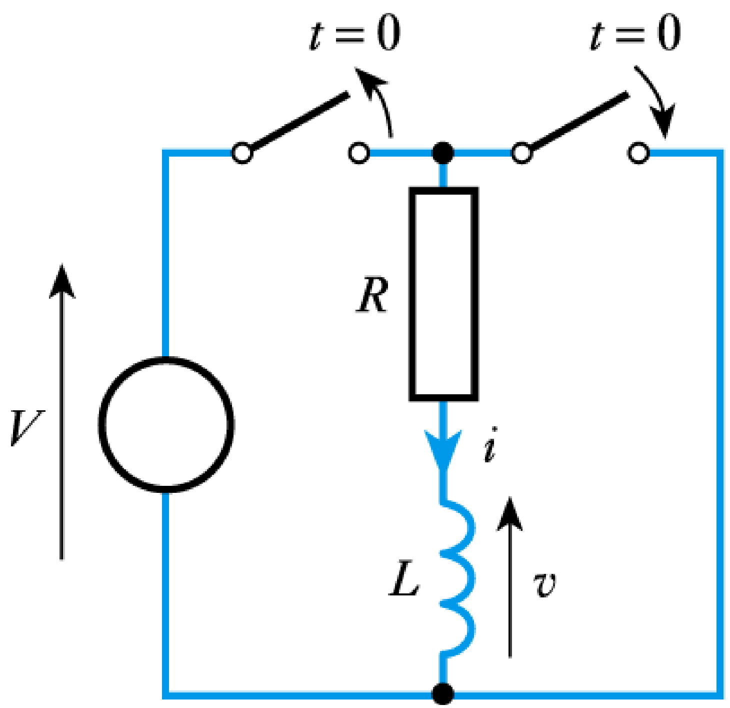
$$i(t) = \alpha \cdot e^{\beta t} + \gamma$$

- **2. Apply the solution**

$$iR + L\frac{di}{dt} = 0 \rightarrow R \cdot \alpha \cdot e^{\beta t} + R \cdot \gamma + L \cdot \alpha\beta \cdot e^{\beta t} = 0$$

$$\rightarrow \begin{cases} R \cdot \alpha \cdot e^{\beta t} + L \cdot \alpha\beta \cdot e^{\beta t} = 0 \rightarrow \beta = -\dfrac{R}{L} \\ R \cdot \gamma = 0 \rightarrow \gamma = 0 \end{cases}$$

$$\therefore i(t) = \alpha \cdot e^{-\frac{R}{L}t}$$



- **3. Substitute by boundary condition**

When $t = 0, i = \dfrac{V}{R}$:

$$i(t=0) = \alpha = \frac{V}{R}$$

$$\therefore i(t) = \frac{V}{R} e^{-\frac{R}{L}t}$$

Time Constant

- Inductor energizing

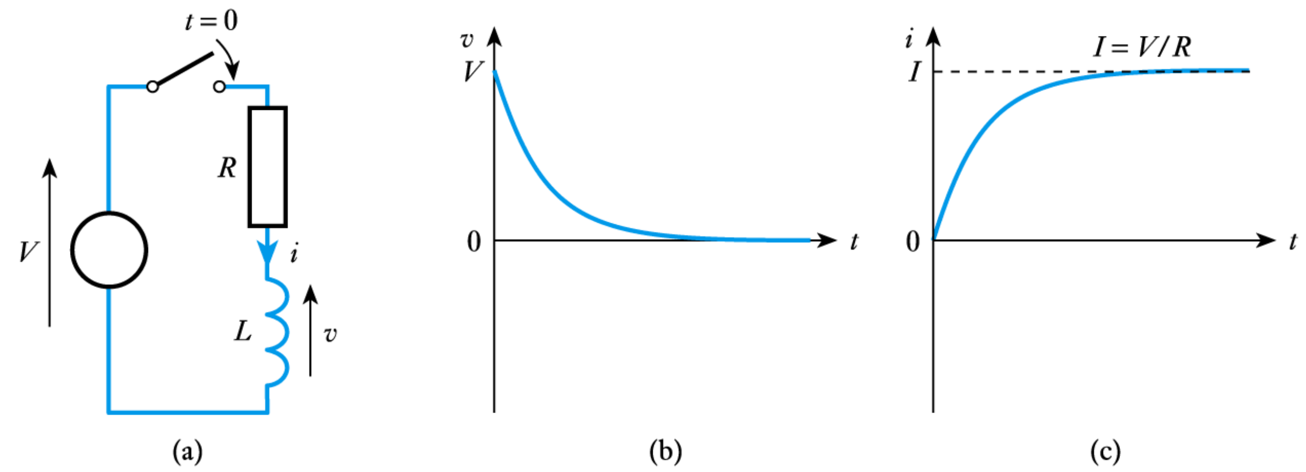
$$i(t)=\frac{V}{R}(1-e^{-\frac{R}{L}t})=\frac{V}{R}(1-e^{-\frac{t}{\mathrm{T}}})$$

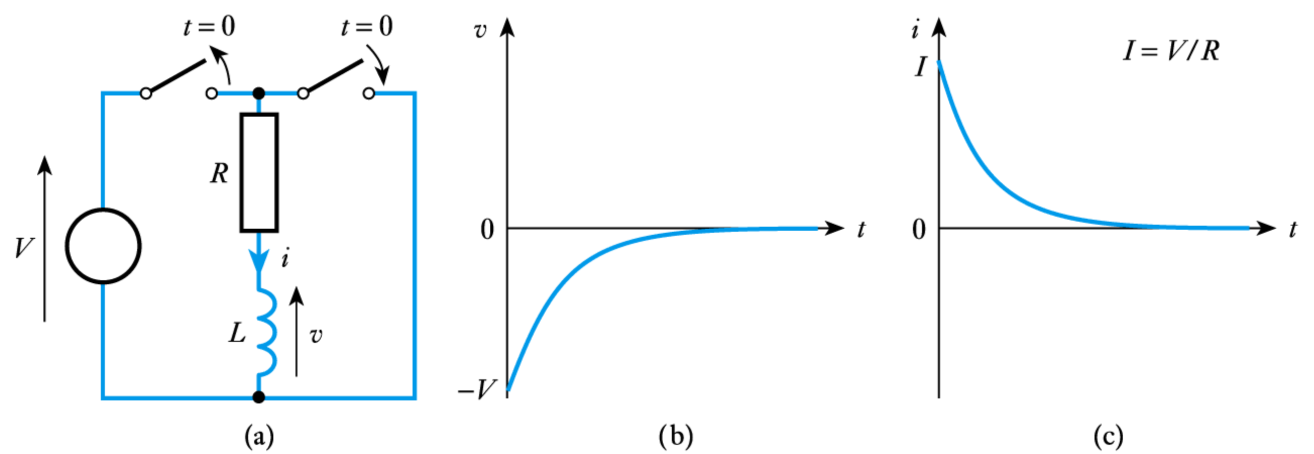
- Inductor de-energizing

$$i(t)=\frac{V}{R}e^{-\frac{R}{L}t}=\frac{V}{R}e^{-\frac{t}{\mathrm{T}}}$$

- In a capacitor-resistor circuit, the time required to charge to a particular voltage is determined by the time constant *RC*
- In an inductor-resistor circuit, the time taken for the current to rise to a certain value is determined by *L/R*
- This value is the time constant Τ (greek tau, τ)

Energizing and De-energizing Summary





Energy Storage in an Inductor

- **In a small amount of time d*t*, the energy added to the magnetic field is the product of the instantaneous voltage, the instantaneous current, and the time**

$$\text{Energy added} = vi\,\mathrm{d}t = L\frac{\mathrm{d}i}{\mathrm{d}t}i\mathrm{d}t = Li\,\mathrm{d}i$$

- **Thus, when the current is increased from zero to *I***

$$E = L\int_0^I i\,di = \frac{1}{2}LI^2$$

STUDENT

Mutual Inductance

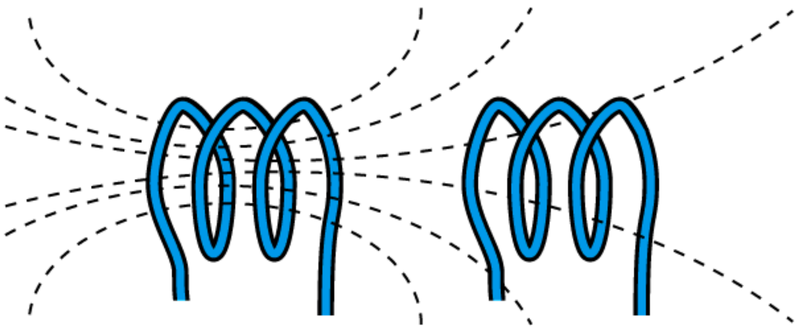
- **When two coils are linked magnetically, a changing current in one will produce a changing magnetic field which will induce a voltage in the other – this is mutual inductance**
- **When a current I_1 in one circuit, induces a voltage V_2 in another circuit, then**

$$V_2 = M \frac{\mathrm{d}I_1}{\mathrm{d}t}$$

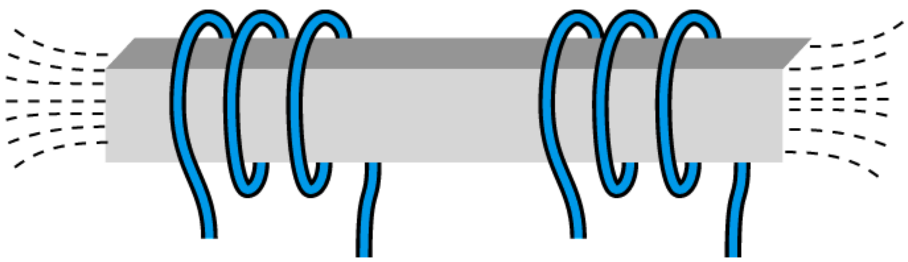
where M is the mutual inductance between the circuits. The unit of mutual inductance is the Henry (as for self-inductance)

Mutual Inductance

- **The coupling between the coils can be increased by wrapping the two coils around a core**
  - **The fraction of the coupled magnetic field is the coupling coefficient**



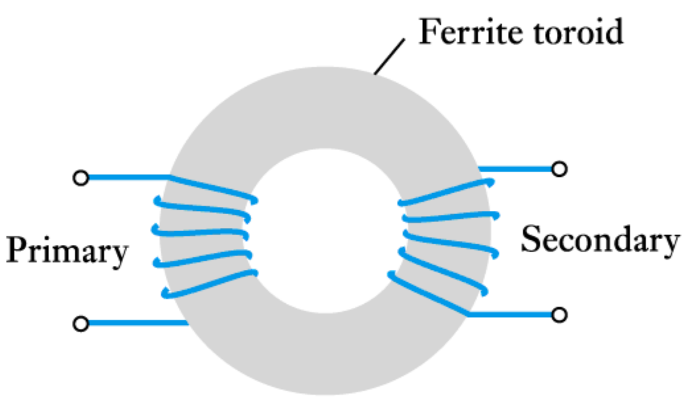
(a) Two loosely coupled coils



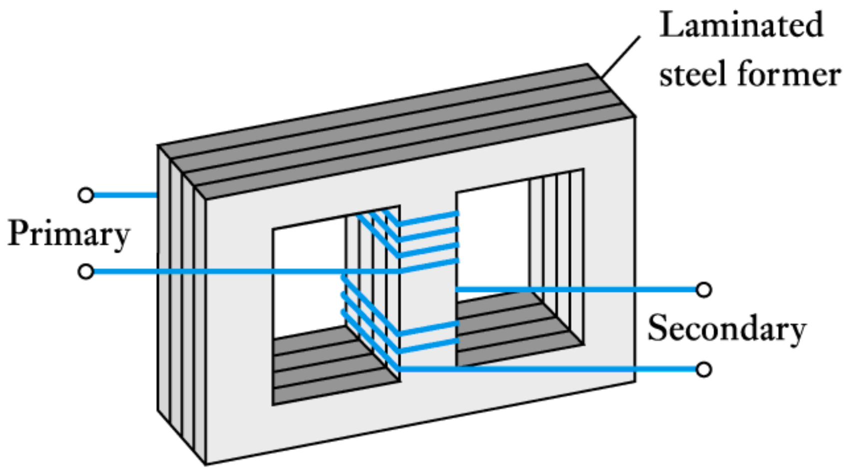
(b) Two coils on a ferromagnetic core

Coupling Coefficient

- **Coupling is particularly important in transformers**
  - **The following arrangements give a coupling coefficient very close to 1**



(c) Coils on a ferrite toroid

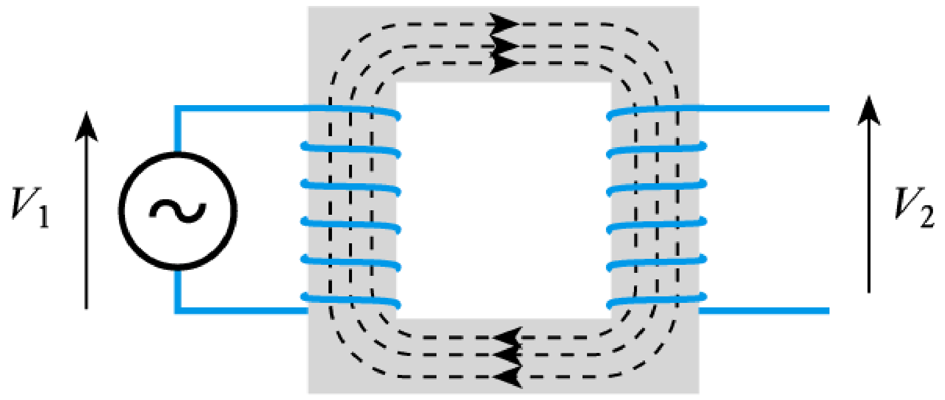


(d) Coils wound on a laminated steel core

Transformers (1/2)

- **Most transformers approximate to ideal components**
  - **That is, they have a coupling coefficient ≈ 1**
  - **For such a device, when *unloaded*, their behavior is determined by the turns ratio**
  - **For alternating voltages**

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

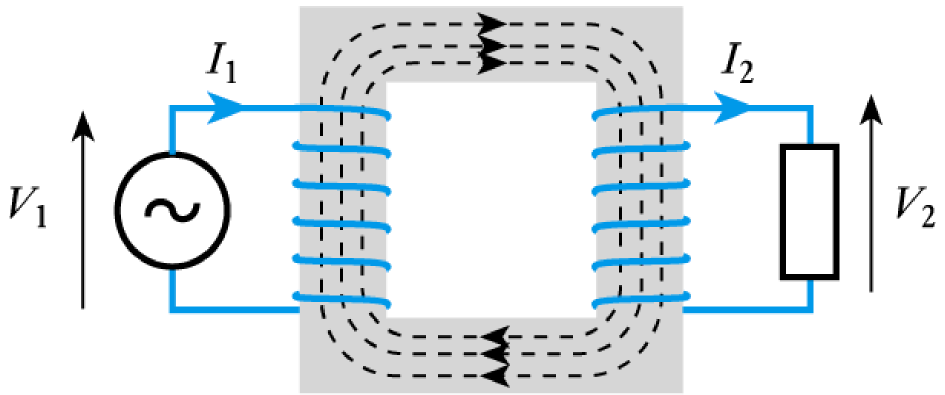


(a) An unloaded transformer

Transformers (2/2)

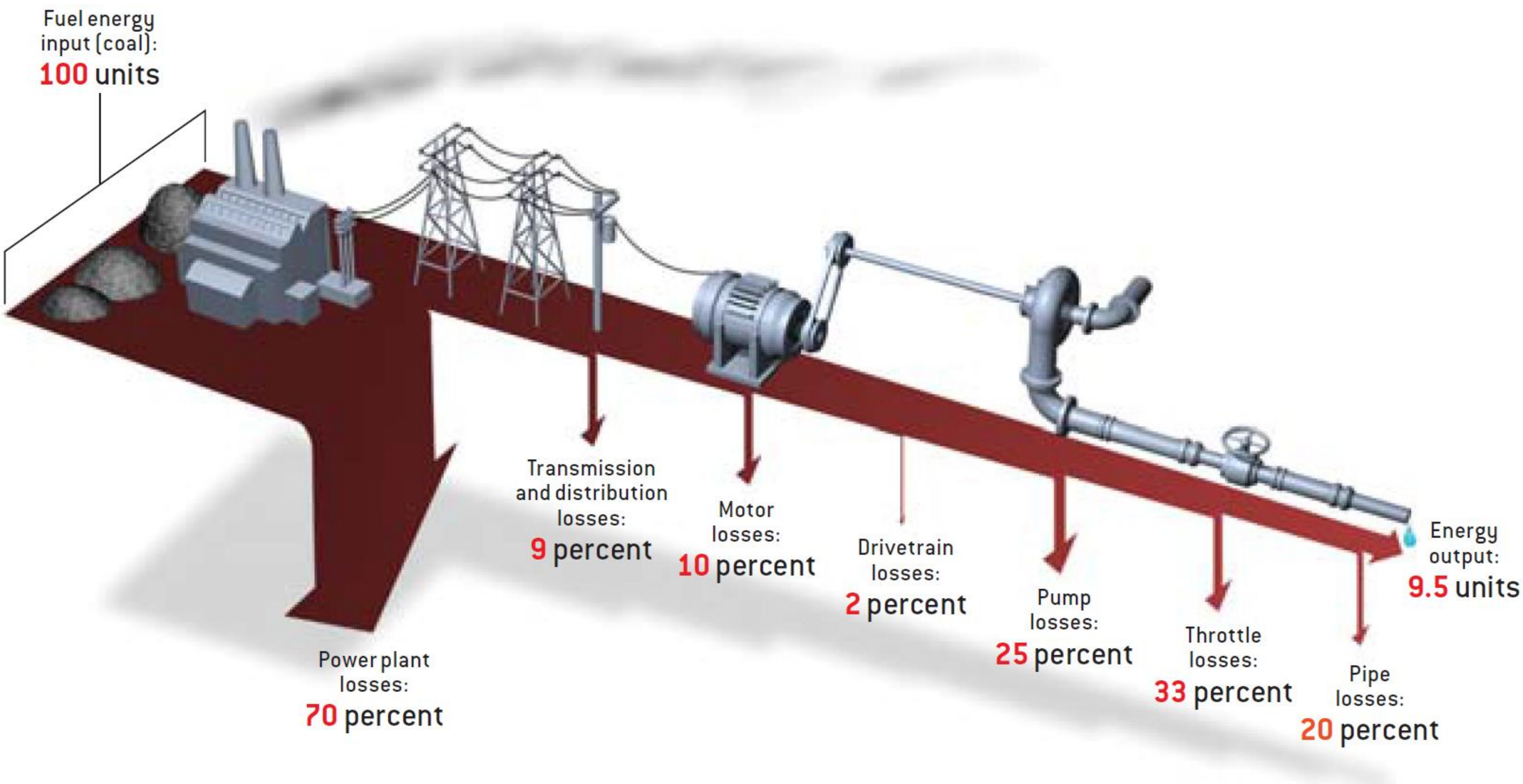
- **When used with a resistive load, current flows in the secondary**
  - **This current itself produces a magnetic flux which opposes that produced by the primary**
  - **Thus, current in the secondary reduces the output voltage**
  - **For an ideal transformer**

$$V_1 I_1 = V_2 I_2$$



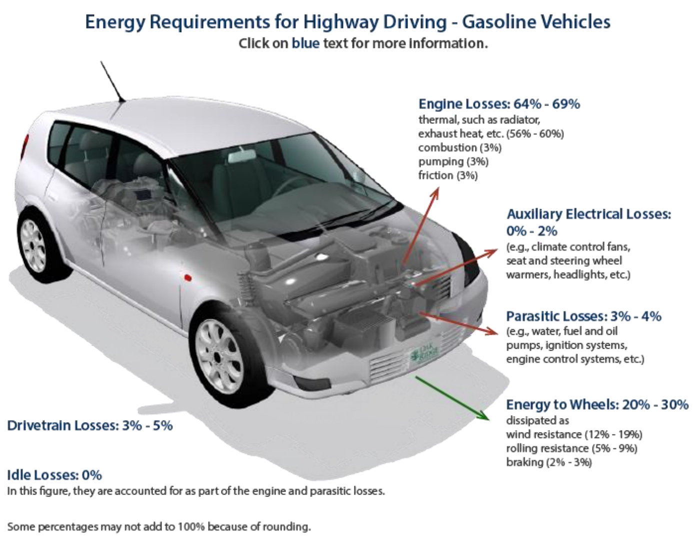
(b) A transformer with a resistive load

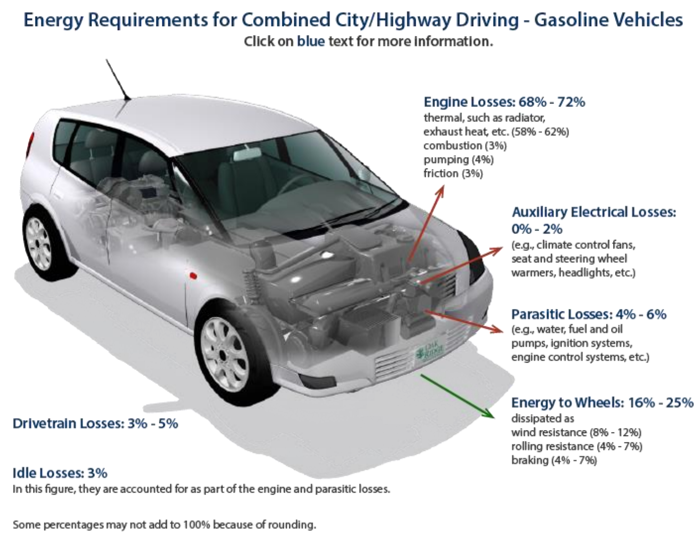
Save Our Planet!

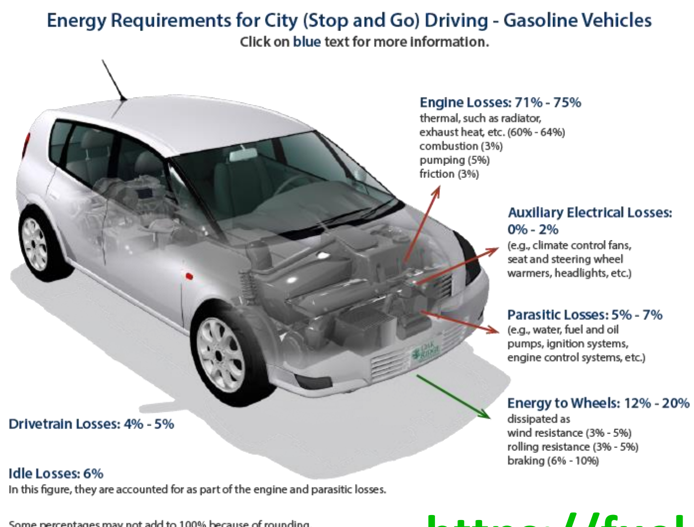


More Profit with Less Carbon, Scientific American, September, 2005

Where the Gasoline Energy Goes?





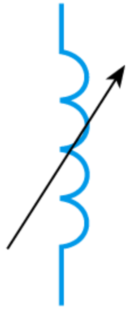


https://fueleconomy.gov/feg/atv.shtml, 2022

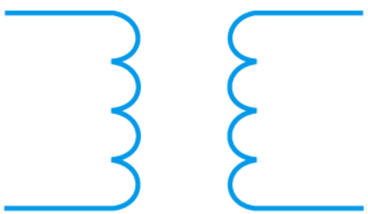
Circuit Symbols

(a) An unspecified inductor

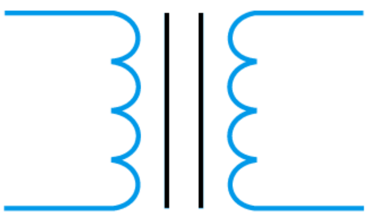
(b) An inductor with a magnetic core



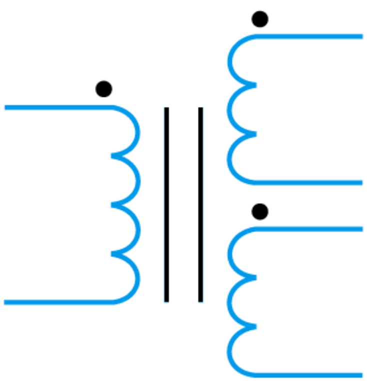
(c) A variable inductor



(d) An air-cored transformer



(e) A transformer with a magnetic core

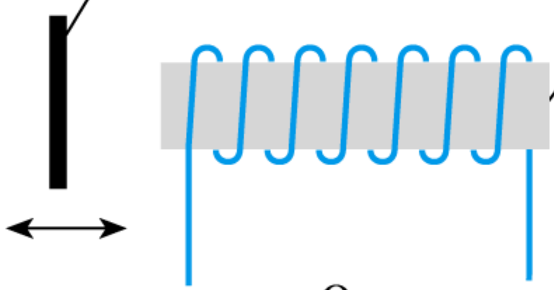


(f) A transformer with two secondary coils

The Use of Inductance in Sensors

▣ Inductive proximity sensors

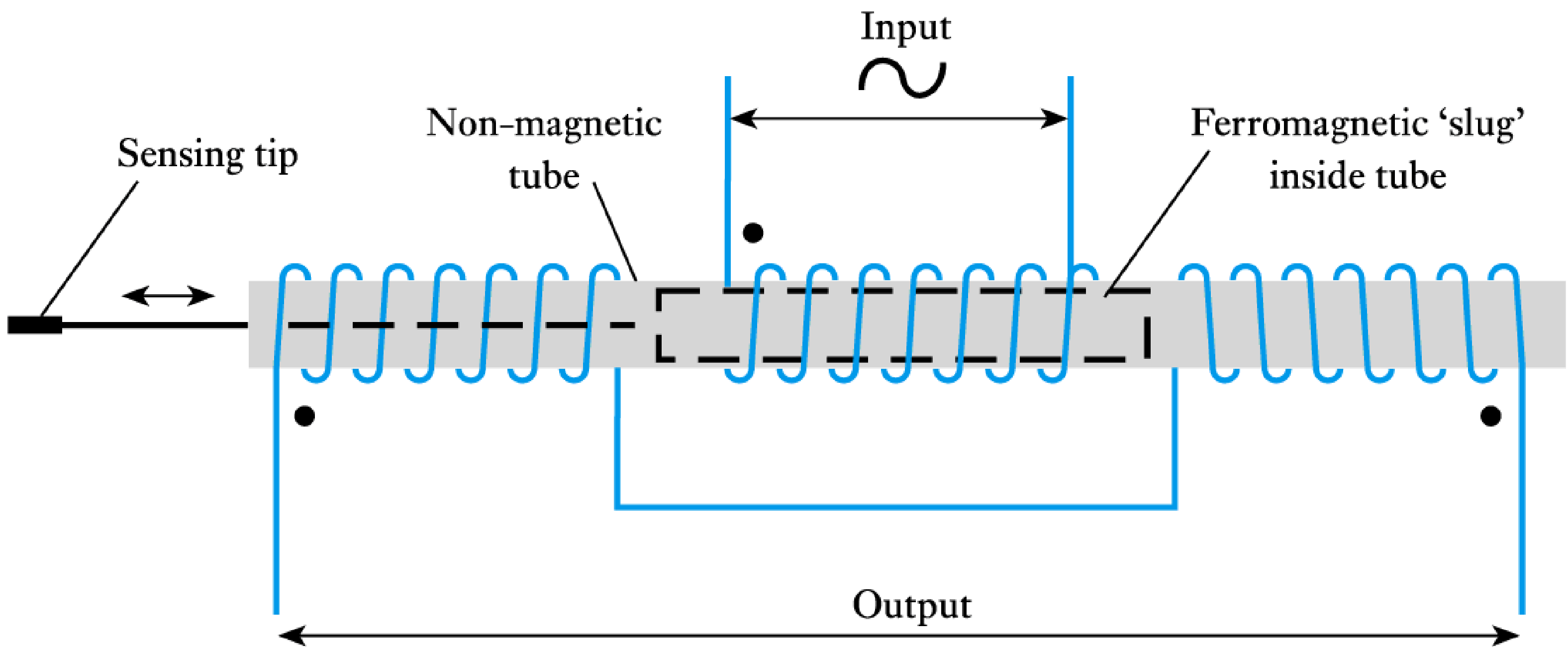
- Basically a coil wrapped around a ferromagnetic rod
- A ferromagnetic plate coming close to the coil changes its inductance allowing it to be sensed
- Can be used as a linear sensor or as a binary switch



The Use of Inductance in Sensors

▣ Linear variable differential transformers (LVDTs)

- The LVDT consists of three coils around a hollow tube. The central coil forms the primary of the transformer. The two secondary coils are connected in series such that their output voltages are out of phase
- A movable 'slug' of ferromagnetic material increases mutual inductance

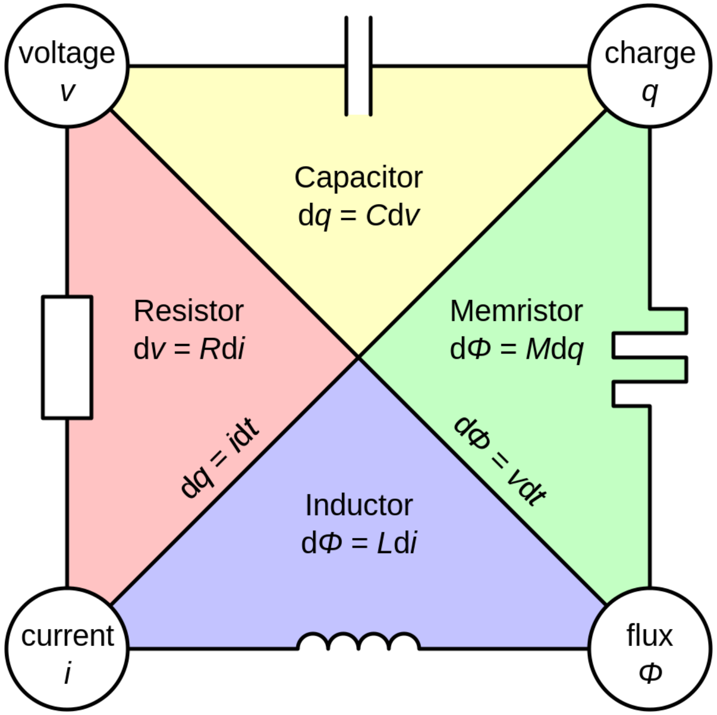


Key Points

- Inductors store energy within a magnetic field
- The induced voltage is proportional to the rate of change of the current
- Inductors can be made by coiling wire in air, but greater inductance is produced if ferromagnetic materials are used
- When an inductor is energized through a resistor, the energizing rate is determined by the time constant L/R
  - Hence, the current through an inductor cannot change instantaneously
- The energy stored in an inductor is equal to $\frac{1}{2}LI^2$
- When a transformer is used with alternating signals, the voltage gain is equal to the turns ratio

The Missing Circuit Element

Current and Charge	$dq = i \cdot dt$
Faraday's Law	$d\varphi = v \cdot dt$
Resistor	$dv = R \cdot di$
Capacitor	$dq = C \cdot dv$
Inductor	$d\varphi = L \cdot di$



Memristor（憶阻器）

- 2008年惠普實驗室(HP Labs)的資深院士R. Stanley Williams成功地證實了有關「憶阻器(memristor)」的學說—所謂的憶阻器是指電子電路中除了電阻、電容與電感之外的第四種被動元素，早在1971年就由美國加州柏克萊大學教授Leon Chua所提出，不過當時僅是初步發現，直到日前才由HP正式發表。而此一成果也意味著相關教科書必須重新改寫。

- 憶阻器概念的創始人Chua表示：「我的處境跟1869年發明化學元素週期表的俄羅斯化學家Dmitri Mendeleev很類似；Mendeleev當時假設該週期表上有許多失落的元素，而現在所有的化學元素都已經被發現了。同樣的，來自HP Labs的Stanley Williams發現失落的電路元素—憶阻器。」

- Chua當時是以數學推論電子電路在電阻、電容與電感之外還有第四種元件；他將其命名為憶阻器的緣故，是因為該元素會透過電阻的改變「記憶」電流的變化。而現在HP則宣稱發現了首個憶阻器的實例—它是由一片雙層的二氧化鈦(bi-level titanium dioxide)薄膜所形成，當電流通過時，其電阻值就會改變。

HP TiO_2 Memristor