CS1011: 數位電子導論

Filters

Outline

- Introduction
- High-pass/Low-pass Filter Examples
- RC Filters
- RL Filters
- Band-pass Filter
- Band-stop Filter
- LC Oscillator
- RLC Circuit

https://www.allaboutcircuits.com/textbook/alternating-current/chpt-8/what-is-a-filter/

Introduction

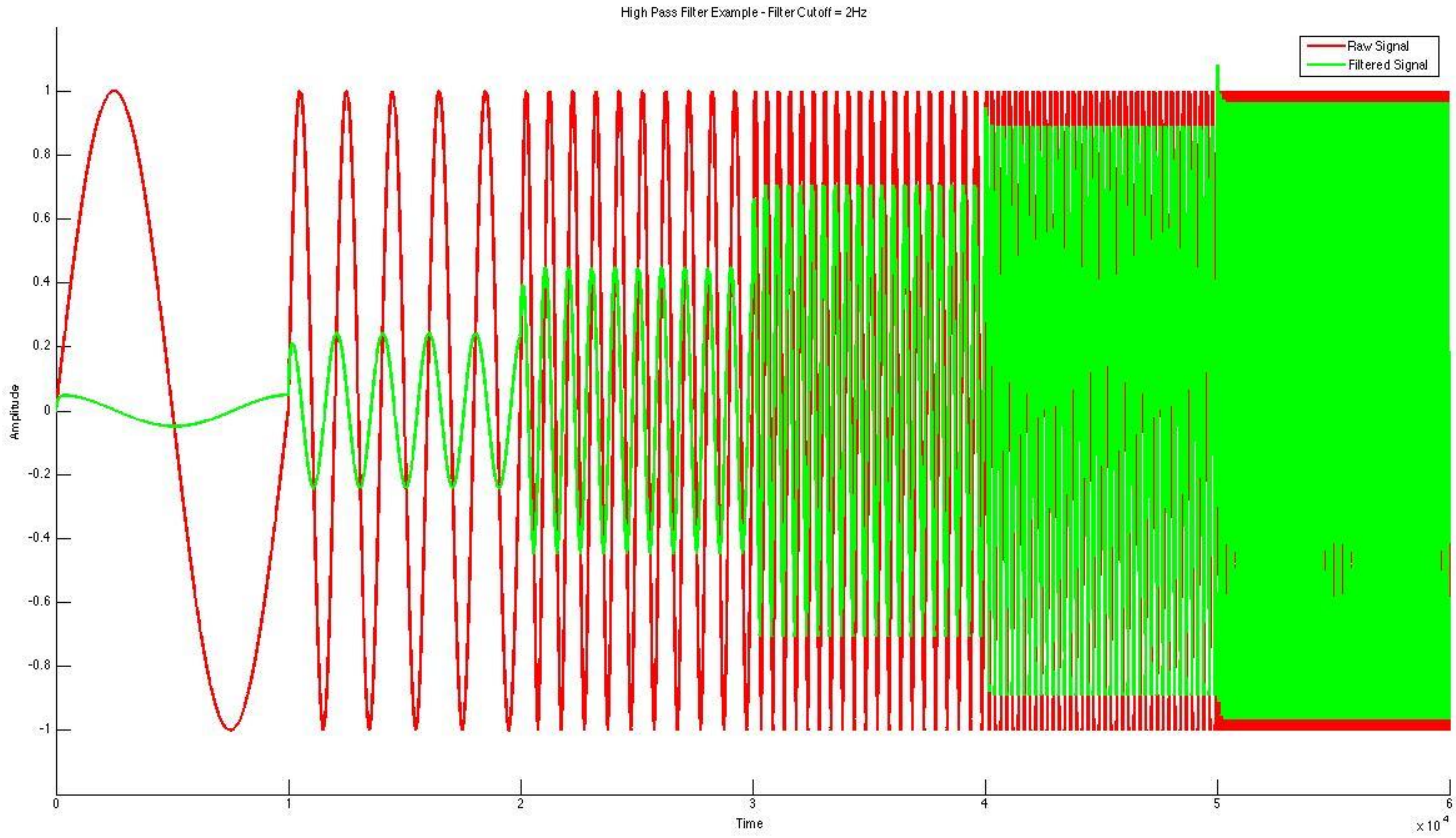
- It is sometimes desirable to have circuits capable of selectively filtering one frequency or range of frequencies out of a mix of different frequencies in a circuit
  - A common need for filter circuits is in high-performance stereo systems, where certain ranges of audio frequencies need to be amplified or suppressed for best sound quality and power efficiency
- A circuit designed to perform this frequency selection is called a *filter*
- Two important behaviors in the capacitor and inductor
  - The voltage across a capacitor cannot change instantaneously, since

$$V = \frac{Q}{C} = \frac{1}{C}\int I \mathrm{d}t$$

  - The current through an inductor cannot change instantaneously, since

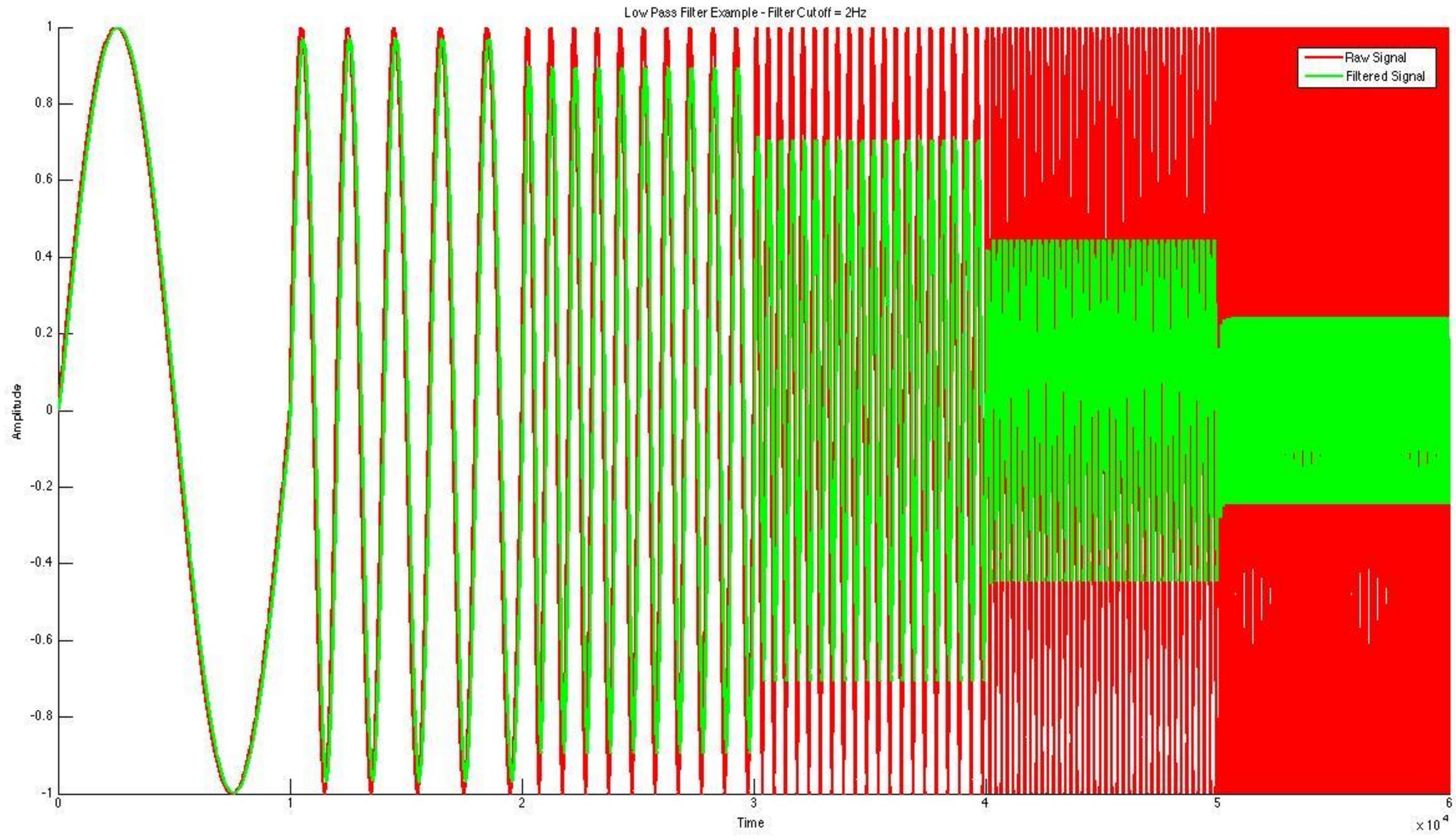
$$V = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

High-pass Filter Example



http://www.nickgillian.com/wiki/pmwiki.php/GRT/HighPassFilter

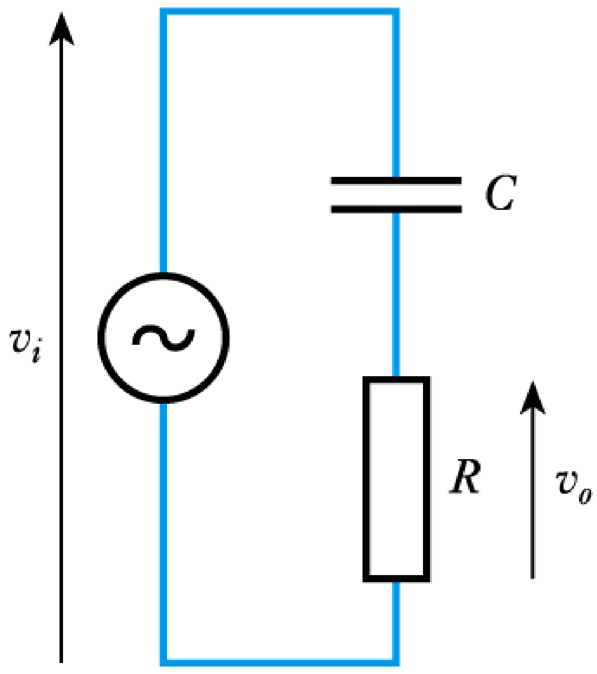
Low-pass Filter Example



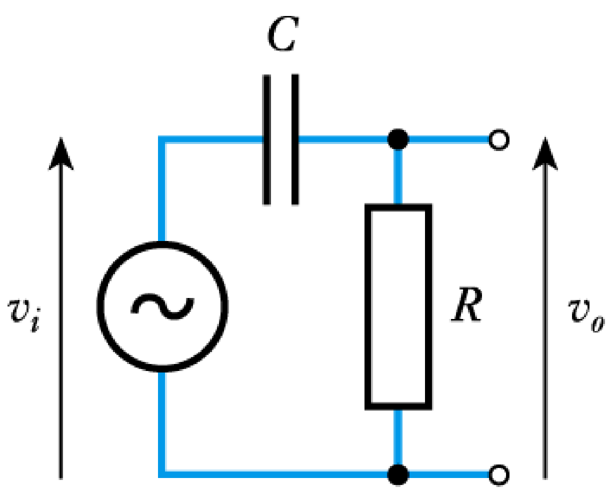
http://www.nickgillian.com/wiki/pmwiki.php/GRT/LowPassFilter

High-pass (Low-cut) RC Filter

- **Consider the following circuit**
  - **Capacitors are widely used in electronic circuits for blocking direct current (low-cut) while allowing alternating current to pass (high-pass)**



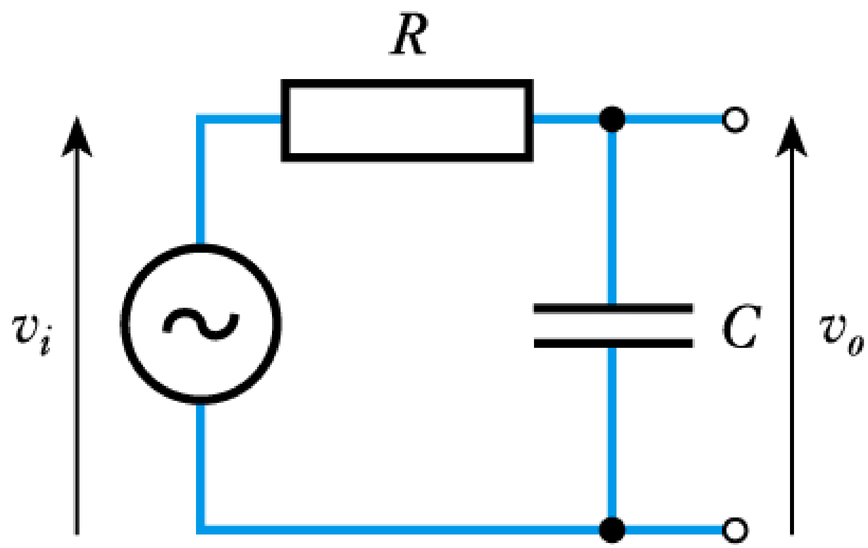
(a)



(b)

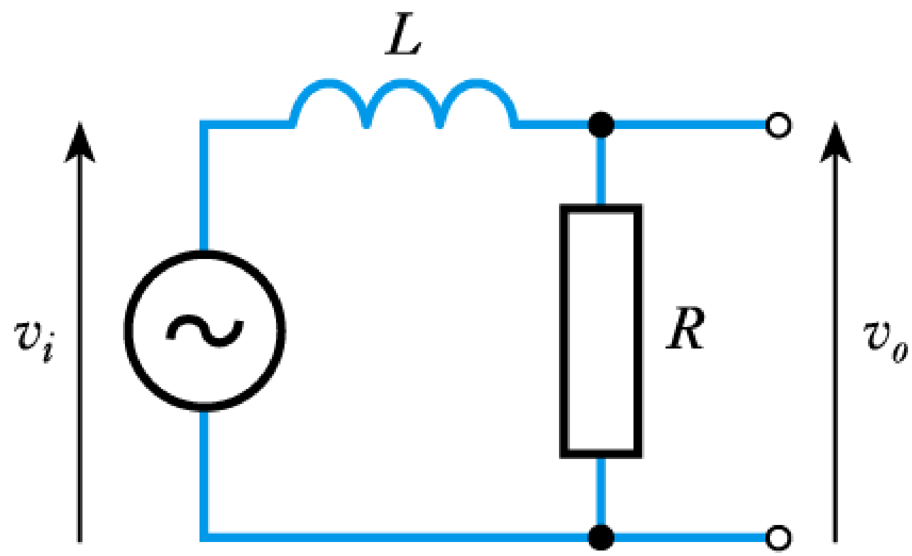
Low-pass (High-cut) RC Filter

- **Transposing the *C* and *R* gives a very different behavior**



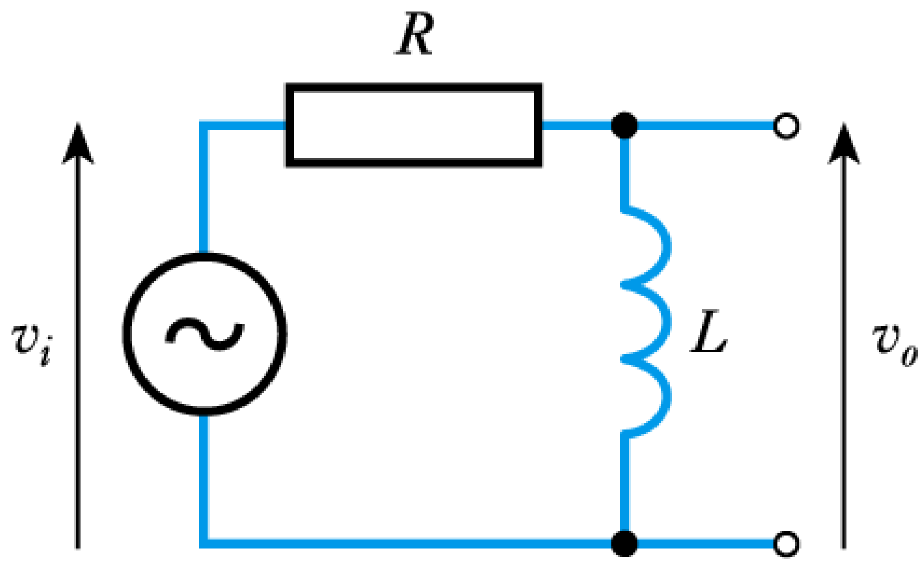
Low-pass (High-cut) RL Filter

- Low-pass networks can also be produced using *RL* circuits
  - Inductors are widely used in alternating current electronic equipment for blocking AC (high-cut) while allowing DC to pass (low-pass)



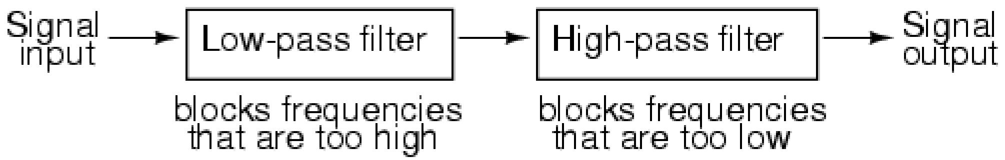
High-pass (Low-cut) RL Filter

- **Transposing the *L* and *R* gives a very different behavior**

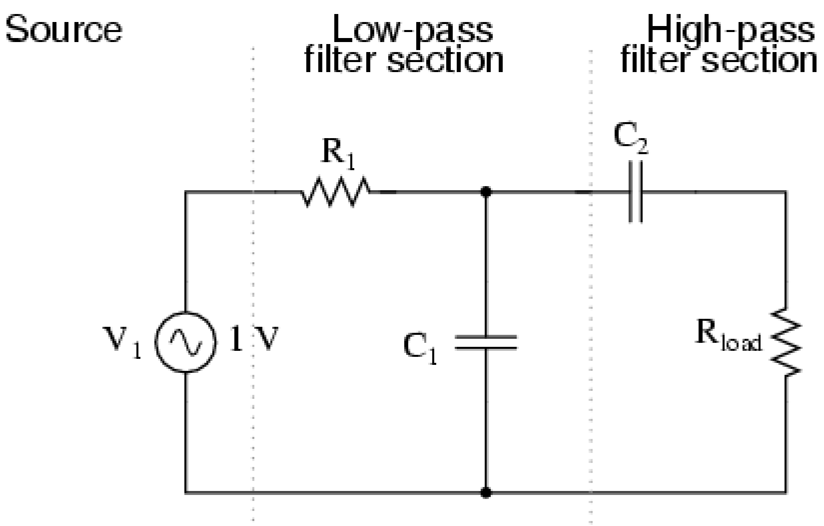


Band-pass Filter

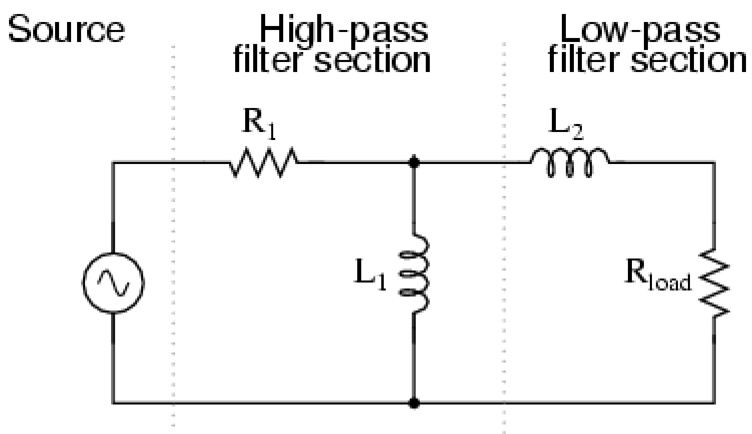
- System level block diagram of a band-pass filter



- Filter examples



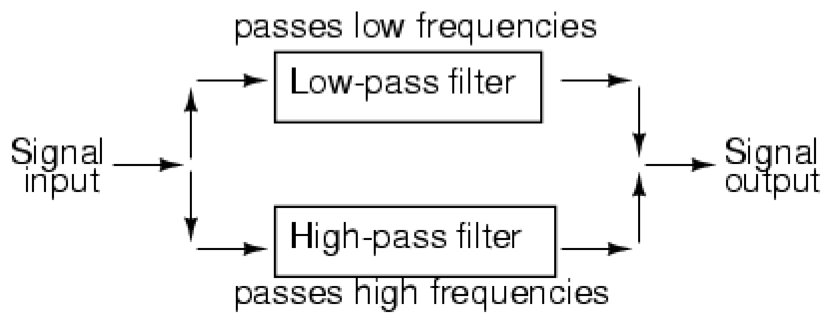
Capacitive band-pass filter



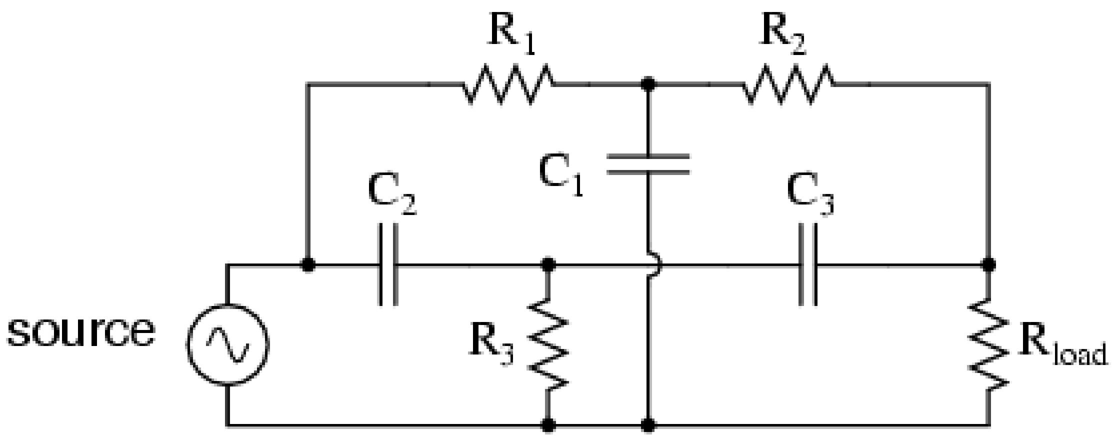
Inductive band-pass filter

Band-stop Filter

- **System level block diagram of a band-stop filter**



- **Filter example**



"Twin-T" band-stop filter

LC Oscillator

- Applying Kirchhoff's voltage law

$$v_C + v_L = 0$$

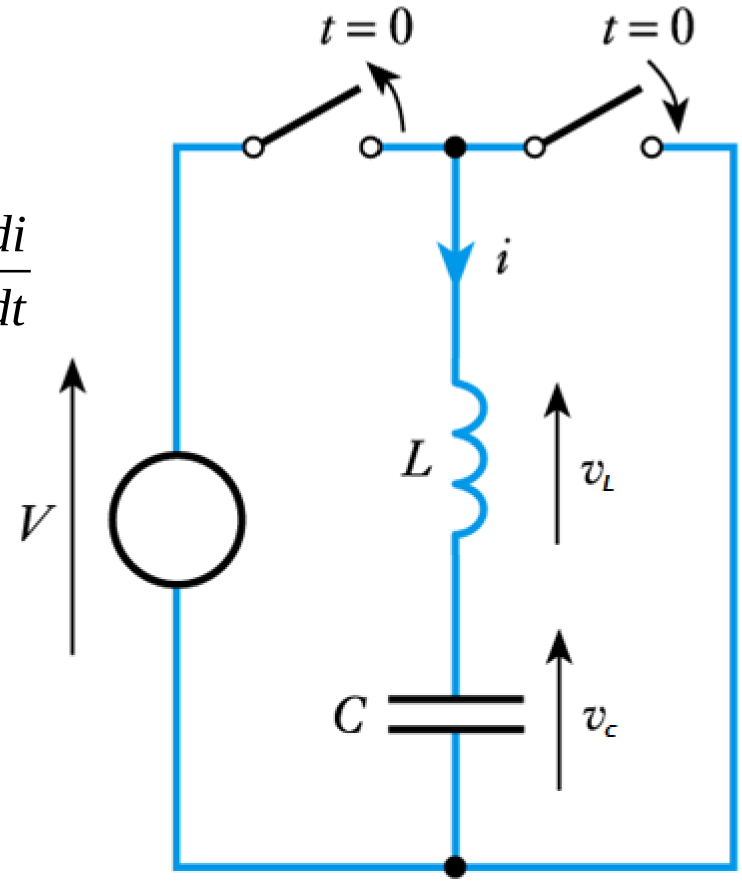
- (a) in a capacitor (b) in an inductor

$$i = C\frac{dv}{dt} \qquad v_L = L\frac{di}{dt}$$

- Which substituting gives

$$v + LC\frac{d^2v}{dt^2} = 0$$

- Solve the differential equation
  - 1. Guess a general solution
  - 2. Apply the solution
  - 3. Substitute by boundary condition
  - 4. Verify



LC Oscillator

1. Guess a general solution

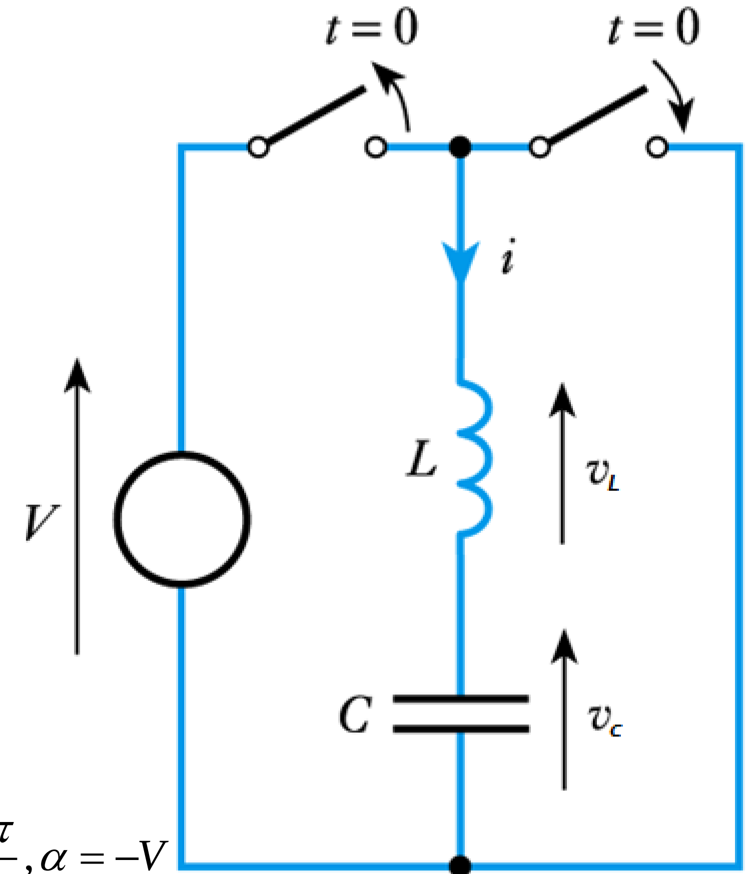
$$v(t) = \alpha \cdot \sin\left(\omega t + \phi\right) + \gamma$$

2. Apply the solution

$$v + LC\frac{d^2v}{dt^2} = 0 \rightarrow \alpha \cdot \sin(\omega t + \phi) + \gamma - \alpha\omega^2 LC \cdot \sin(\omega t + \phi) = 0$$

$$\rightarrow \begin{cases} \alpha \cdot \sin(\omega t + \phi) - \alpha\omega^2 LC \cdot \sin(\omega t + \phi) = 0 \rightarrow \omega = \dfrac{1}{\sqrt{LC}} \\ \gamma = 0 \end{cases}$$

$$\therefore v(t) = \alpha \cdot \sin\left(\frac{t}{\sqrt{LC}} + \phi\right)$$

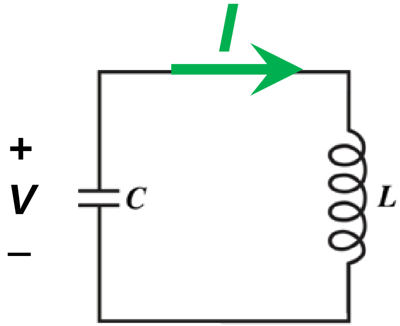


3. Substitute by boundary condition

When $t = 0, v = V, i = 0$:

$$\begin{cases} v(t=0) = \alpha \sin\phi = V \\ i(t=0) = C\dfrac{dv}{dt} = C \cdot \alpha\omega \cos(\phi) = 0 \end{cases} \rightarrow \phi = \frac{\pi}{2}, \alpha = V \text{ or } \phi = -\frac{\pi}{2}, \alpha = -V$$

$$\therefore v(t) = V \sin\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right) = V \cos\frac{t}{\sqrt{LC}}$$


I
+
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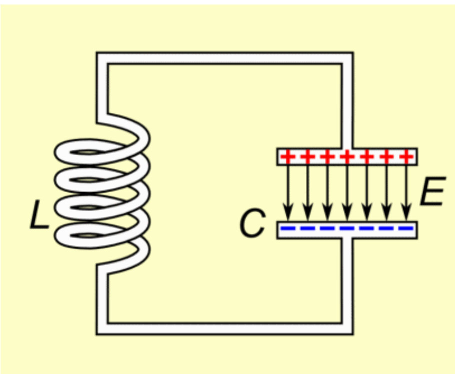
L
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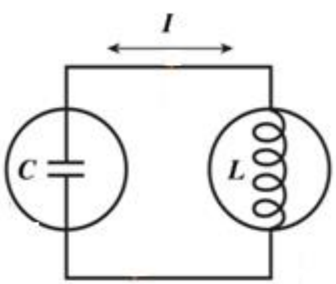
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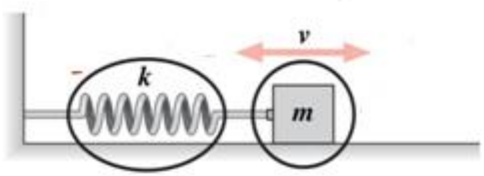
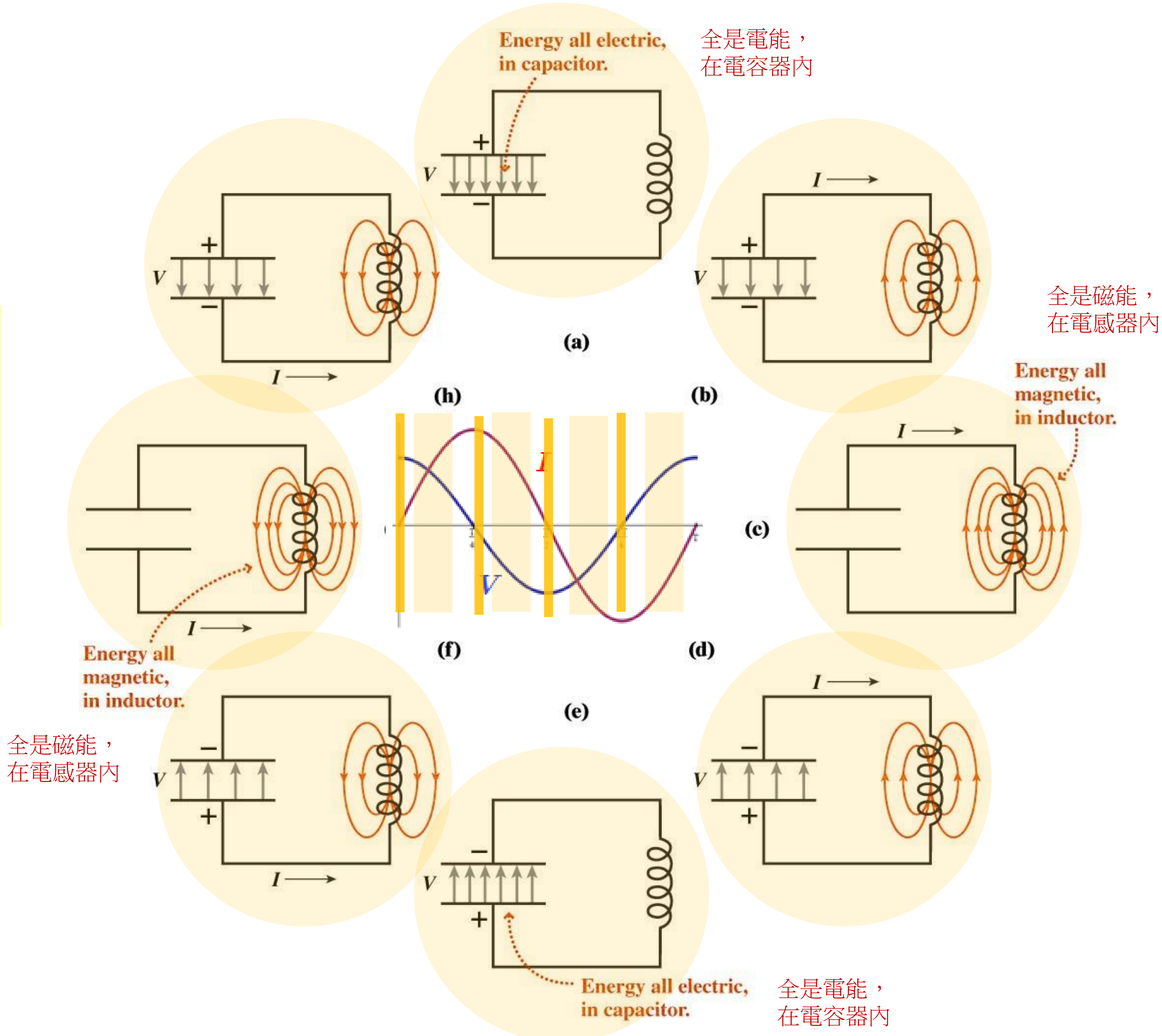

k
v
m


Energy all electric,
in capacitor.
全是電能，
在電容器內
全是磁能，
在電感器內
Energy all
magnetic,
in inductor.
(a)
(b)
(c)
(d)
(e)
(f)
(h)
I
V
Energy all
magnetic,
in inductor.
全是磁能，
在電感器內
Energy all electric,
in capacitor.
全是電能，
在電容器內

RLC Circuit (Series Style)

- Applying Kirchhoff's voltage law

$$v_R + v_C + v_L = 0$$

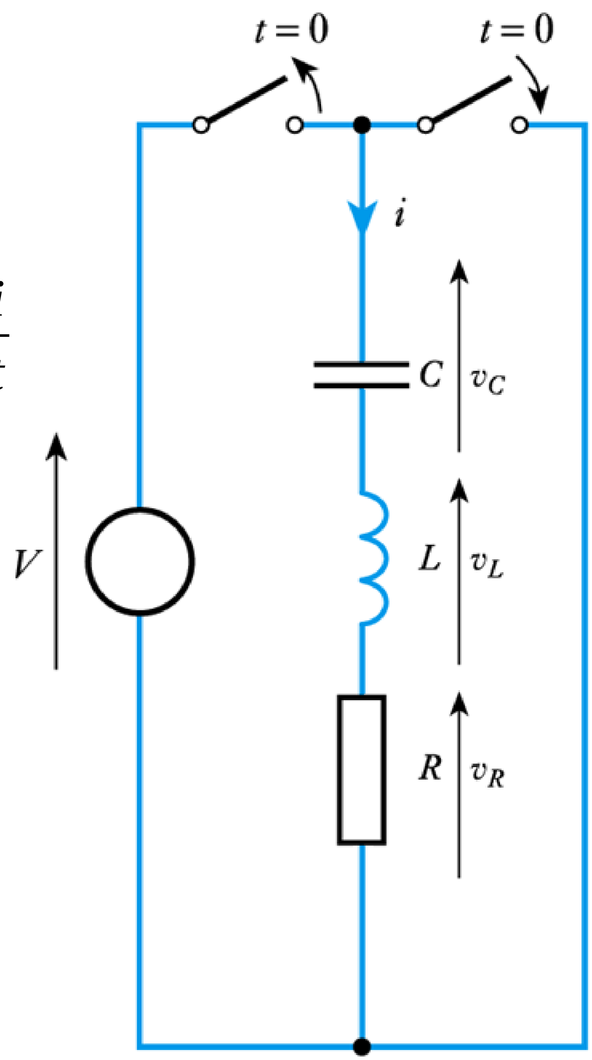
- (a) in a capacitor (b) in an inductor

$$i = C\frac{dv}{dt} \qquad v_L = L\frac{di}{dt}$$

- Which substituting gives

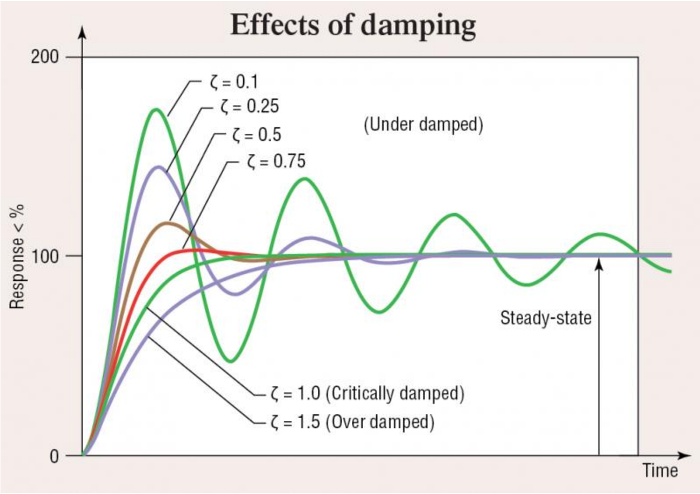
$$RC\frac{dv}{dt} + v + LC\frac{d^2v}{dt^2} = 0$$

- General 2-order differential equation
  - Let's skip the mathematics

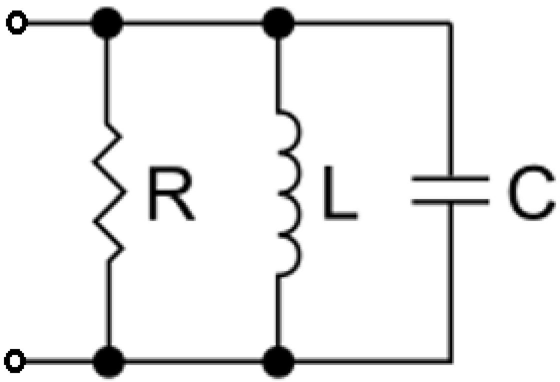


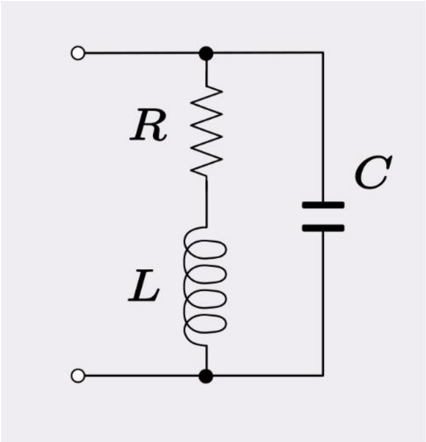
RLC Circuit (Series Style)

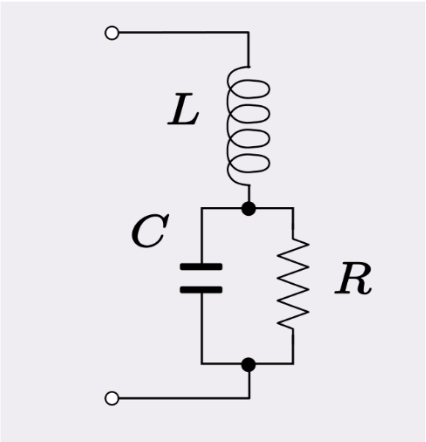
- **Damping factor:** $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$
  - ζ < 1: under damp
  - ζ = 1: critical damp
  - ζ > 1: over damp



- **Miscellaneous RLC circuits for various applications**



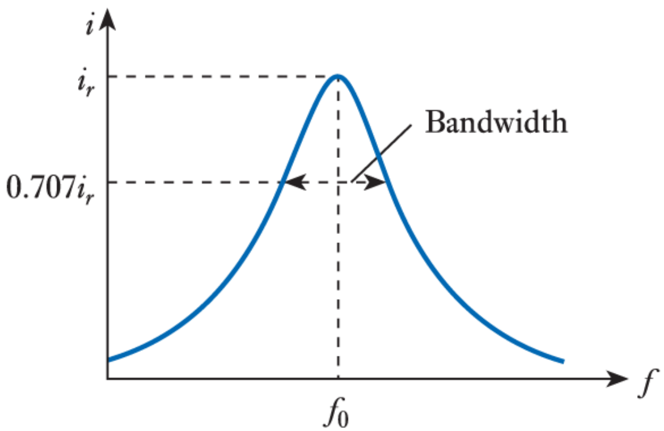


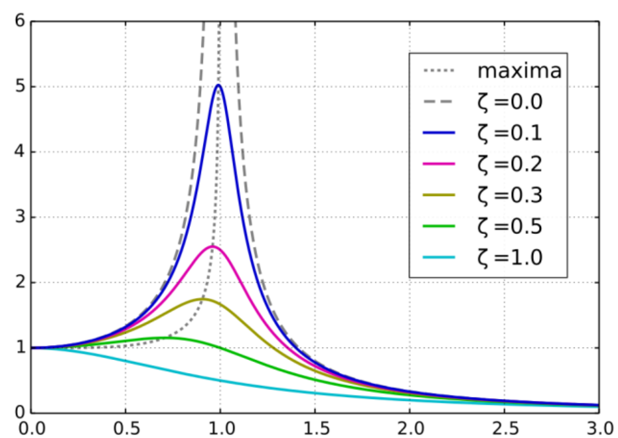


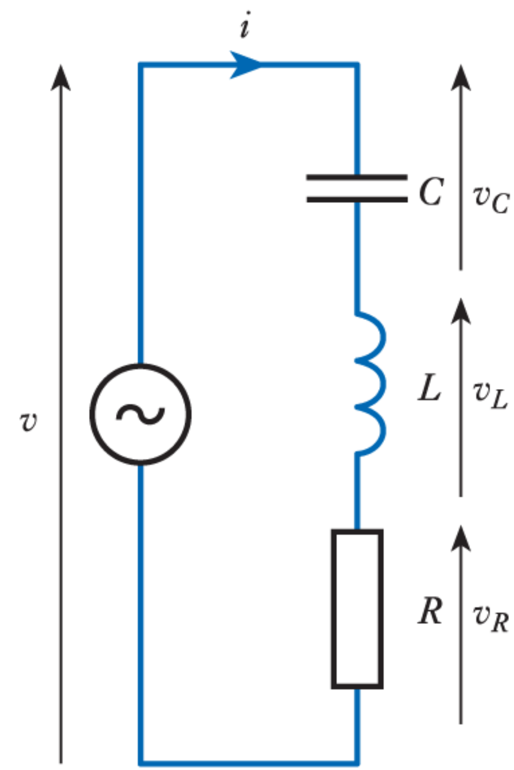
http://www.wikiwand.com/en/RLC_circuit

RLC Circuit Resonance (Series Style)

- Given an alternating voltage source, the voltage response of the RLC circuit could oscillate
  - Resonance frequency: $\omega_o = \frac{1}{\sqrt{LC}}, f_o = \frac{1}{2\pi\sqrt{LC}}$







Analogies Between Electrical and Mechanical Systems

Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

Key Points

- **Passive circuit elements, *R*, *L* and *C*, can be combined in a number of different topologies, with practical importance in real circuits**
- **Filter is a circuit capable of selectively filtering one frequency or range of frequencies out of a mix of different frequencies**
- **The behavior of *LC* and *RLC* circuits are described by second-order differential equations in circuit analysis**
- **The resonance effect of the *LC* circuit has many important applications in signal processing and communications systems**
- ***RLC* circuits can be used as a band-pass filter, band-stop filter, low-pass filter or high-pass filter, based on the configurations**