CS1011：數位電子導論

Capacitance

Outline

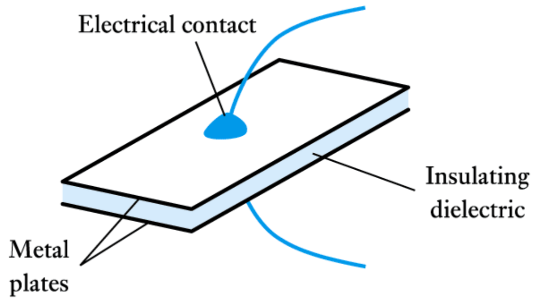
- Introduction
- Capacitors and Capacitance
- Alternating Voltages and Currents
- Capacitors in Series and Parallel
- Voltage and Current
- Energy Stored in a Charged Capacitor
- Circuit Symbols

Introduction

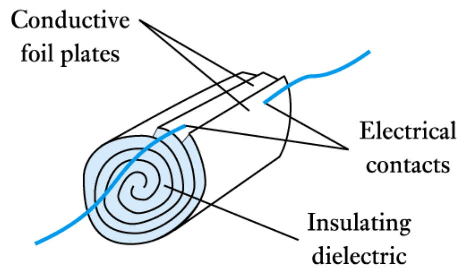
- We noted earlier that an electric current represents a flow of charge
- A capacitor can store electric charge and can therefore store electrical energy
- Capacitors are often used in association with alternating currents and voltages
- Capacitor is a key component in almost all electronic circuits

Capacitors and Capacitance

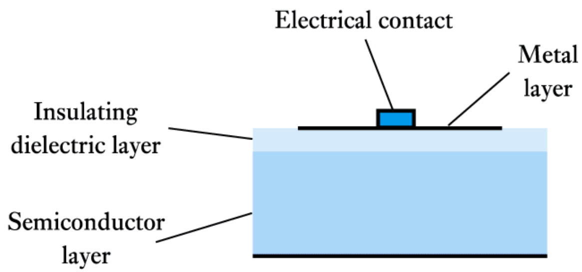
- Capacitors consist of two conducting surfaces separated by an insulating layer called a dielectric



(a) A parallel-plate capacitor



(b) A cylindrical capacitor



(c) An integrated circuit capacitor

- Supercapacitors are widely used in transportation and energy harvesting applications

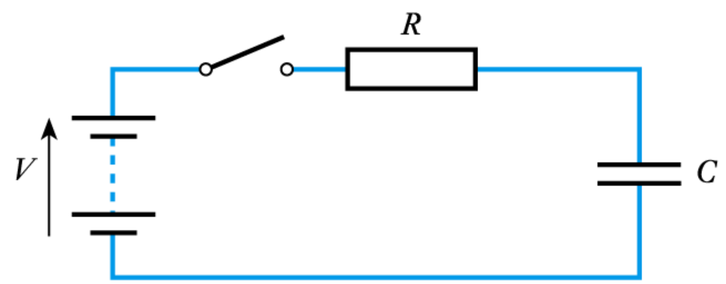
Energy Storage Technology Comparisons

Parameter	Aluminum Electrolytic Capacitor	Double-layer Capacitor	Pseudocapacitor	Hybrid (Li-ion)	Lithium-ion Battery
Temperature Range	−40 ~ +125 °C	−40 ~ +70 °C	−20 ~ +70 °C	−20 ~ +70 °C	−20 ~ +60 °C
Maximum Voltage	4 ~ 630 V	1.2 ~ 3.3 V	2.2 ~ 3.3 V	2.2 ~ 3.8 V	2.5 ~ 4.2 V
Recharge Cycles	< unlimited	100 k ~ 1 000 k	100 k ~ 1 000 k	20 k ~ 100 k	0.5 k ~ 10 k
Capacitance	≤ 2.7 F	0.1 ··· 470 F	100 ··· 12 000 F	300 ··· 3 300 F	–
Energy Density	0.01 ~ 0.3 Wh/kg	1.5 ~ 3.9 Wh/kg	4 ~ 9 Wh/kg	10 ~ 15 Wh/kg	100 ~ 265 Wh/kg
Power Density	> 100 W/g	2 ~ 10 W/g	3 ~ 10 W/g	3 ~ 14 W/g	0.3 ~ 1.5 W/g
Self-discharge	short (days)	Medium (weeks)	Medium (weeks)	Long (months)	Long (months)
Efficiency (%)	99%	95%	95%	90%	90%
Working Life	> 20 years	5 ~ 10 years	5 ~ 10 years	5 ~ 10 years	3 ~ 5 years

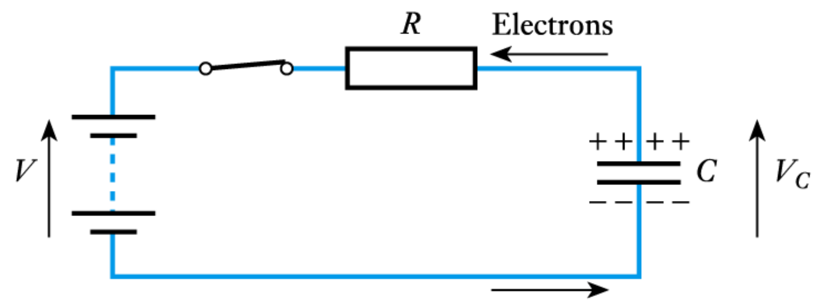
Wikipedia: https://en.wikipedia.org/wiki/Supercapacitor

A Simple Capacitor Circuit

- When switch is closed, electrons flow from top plate into battery and from battery onto bottom plate
- Charge produces an electric field across the capacitor and a voltage across it



(a)



(b)

Capacitance Property

- For a given capacitor, the stored charge q is directly proportional to the voltage V across it
- The constant of proportionality is the capacitance C and thus

$$C = \frac{Q}{V}$$

- If the charge is measured in coulombs and the voltage in volts, then the capacitance is in farads

Example

- **A 10 μF capacitor has 10 V across it. What quantity of charge is stored in it?**

$$C = \frac{Q}{V}$$

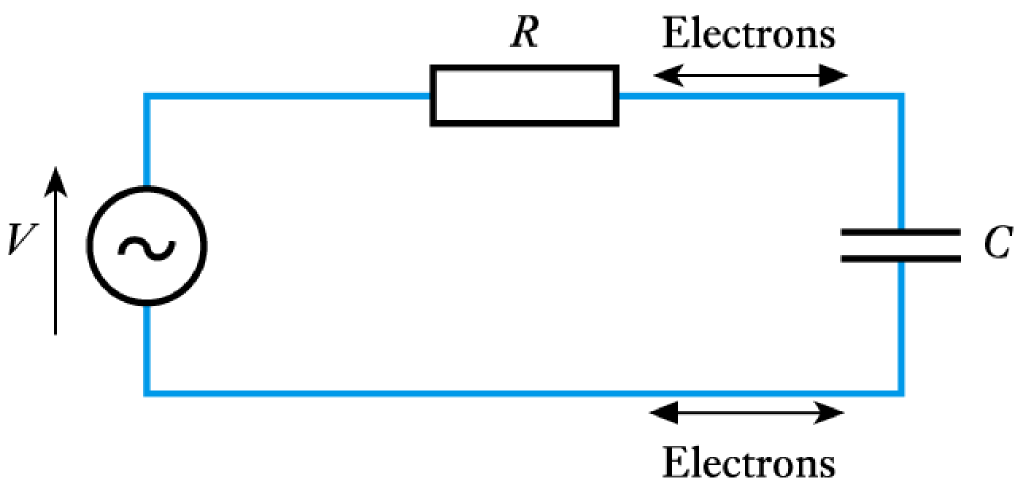
$$Q = CV$$

$$= 10^{-5} \times 10$$

$$= 100 \ \mu\text{C}$$

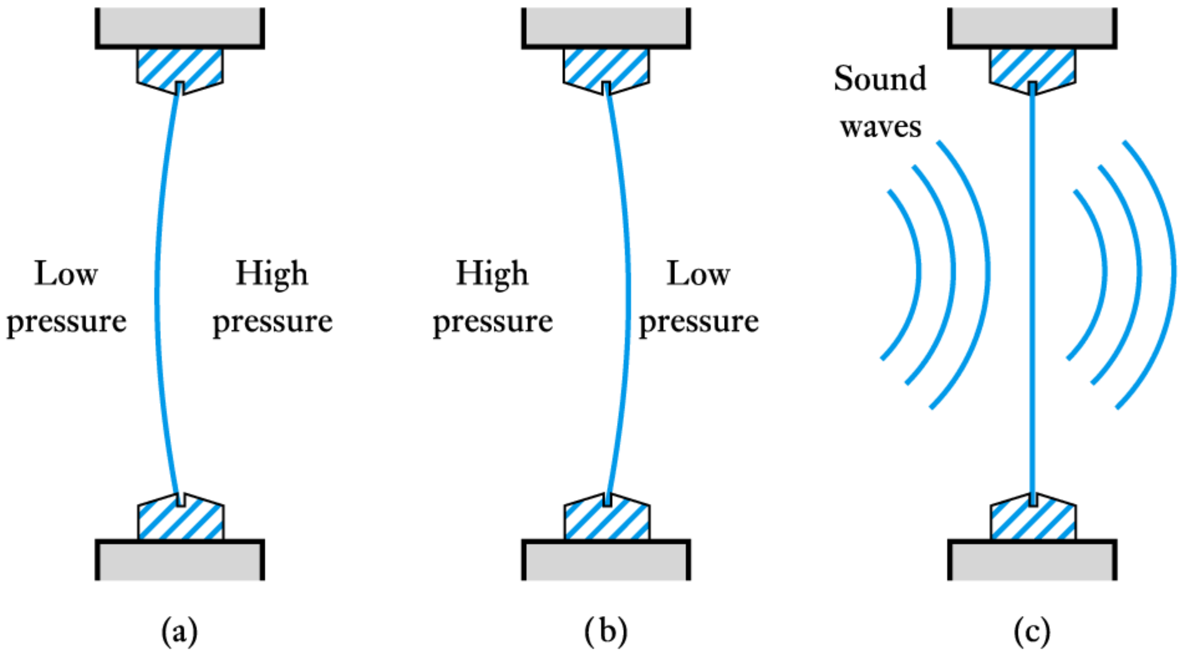
Alternating Voltages and Currents

- A constant current cannot flow through a capacitor
  - However, since the voltage across a capacitor is proportional to the charge on it, an alternating voltage must correspond to an alternating charge, and hence to current flowing into and out of the capacitor
  - This can give the impression that an alternating current flows through the capacitor



A Mechanical Analogy of a Capacitor

- A mechanical analogy may help to explain this
  - Consider a window - air cannot pass through it, but sound (which is a fluctuation in air pressure) can



Capacitors in Parallel

- Consider a voltage *V* applied across two capacitors
- The charge on each capacitor is

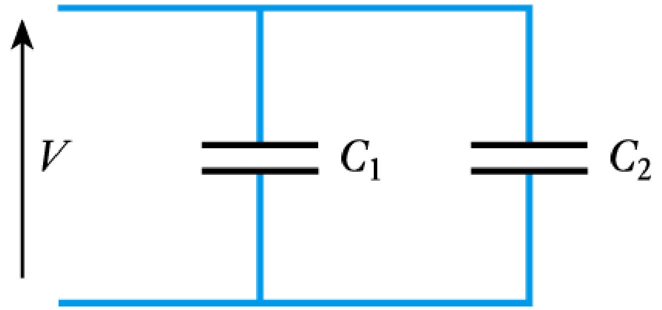
$$Q_1 = VC_1 \qquad\qquad Q_2 = VC_2$$

- If the two capacitors are replaced with a single capacitor C which has a similar effect as the pair, then

$$\text{Charge stored on } C = Q_1 + Q_2$$

$$VC = VC_1 + VC_2$$

$$C = C_1 + C_2$$



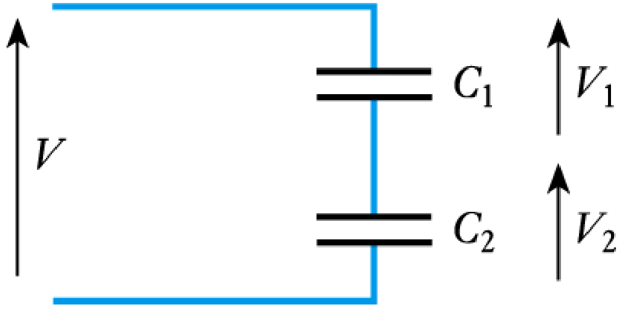
Capacitors in Series

- Consider a voltage *V* applied across two capacitors in series
- The only charge that can be applied to the lower plate of C_1 is that supplied by the upper plate of C_2.
- Therefore the charge on each capacitor must be identical. Let this be *Q*, and therefore if a single capacitor *C* has the same effect as the pair, then

$$V = V_1 + V_2$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$



Voltage and Current

- The voltage across a capacitor is directly related to the charge on the capacitor

$$V = \frac{Q}{C} = \frac{1}{C}\int I \mathrm{d}t$$

- Alternatively, since Q = CV we can see that

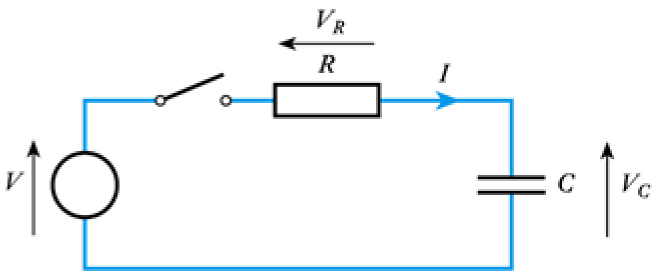
$$\frac{\mathrm{d}Q}{\mathrm{d}t} = C\frac{\mathrm{d}V}{\mathrm{d}t}$$

and since dQ/dt is equal to current, it follows that

$$I = C\frac{\mathrm{d}V}{\mathrm{d}t}$$

Charging Capacitors

- Capacitor is initially discharged
  - Voltage across it will be zero
- Switch is closed at $t = 0$
- V_C is initially zero
  - Hence, V_R is initially V
  - Hence, I is initially V/R
- As the capacitor charges…
  - V_C increases
  - V_R decreases
  - Hence, I decreases
- When the capacitor is fully charged
  - $t \rightarrow \infty$
  - $V_C = V$
  - $V_R = 0$
  - $I = 0$



(a)

(b)

(c)

Charging Capacitors

- Applying Kirchhoff's voltage law

$$iR + v = V$$

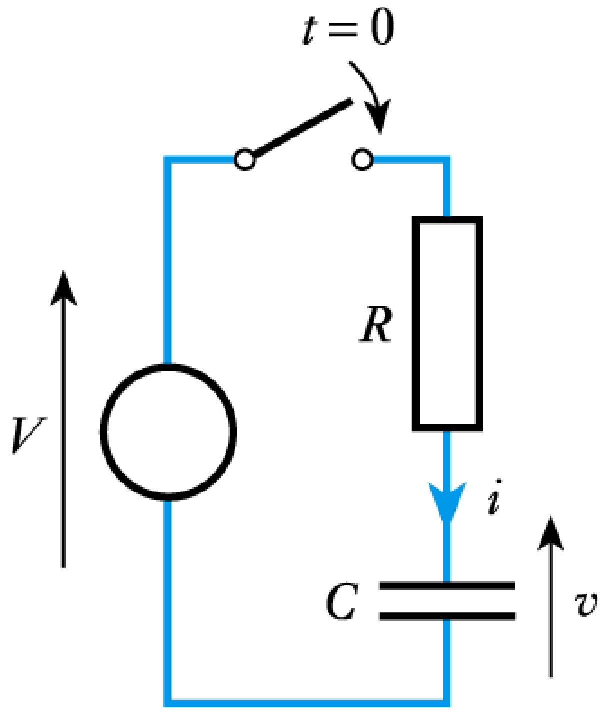
- Now, in a capacitor

$$i = C\frac{dv}{dt}$$

- Which substituting gives

$$RC\frac{dv}{dt} + v = V$$

- Solve the differential equation
  - 1. Guess a general solution
  - 2. Apply the solution
  - 3. Substitute by boundary condition
  - 4. Verify the equation



Charging Capacitors

- **1. Guess a general solution**

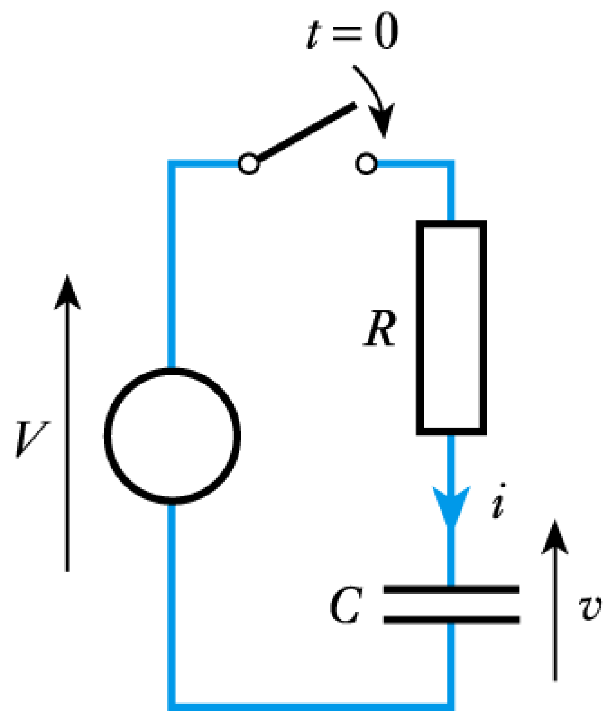
$$v(t) = \alpha \cdot e^{\beta t} + \gamma$$

- **2. Apply the solution**

$$RC\frac{dv}{dt} + v = V \rightarrow RC \cdot \alpha\beta \cdot e^{\beta t} + \alpha \cdot e^{\beta t} + \gamma = V$$

$$\rightarrow \begin{cases} RC \cdot \alpha\beta \cdot e^{\beta t} + \alpha \cdot e^{\beta t} = 0 \rightarrow \beta = -\dfrac{1}{RC} \\ \gamma = V \end{cases}$$

$$\therefore v(t) = \alpha \cdot e^{-\frac{t}{RC}} + V$$



- **3. Substitute by boundary condition**

When $t = 0, v = 0$:

$$v(t) = \alpha \cdot e^{-\frac{t}{RC}} + V = \alpha + V = 0 \rightarrow \alpha = -V$$

$$\therefore v(t) = -Ve^{-\frac{t}{RC}} + V = V(1 - e^{-\frac{t}{RC}})$$

Discharging Capacitors

- Applying Kirchhoff's voltage law

$$iR + v = 0$$

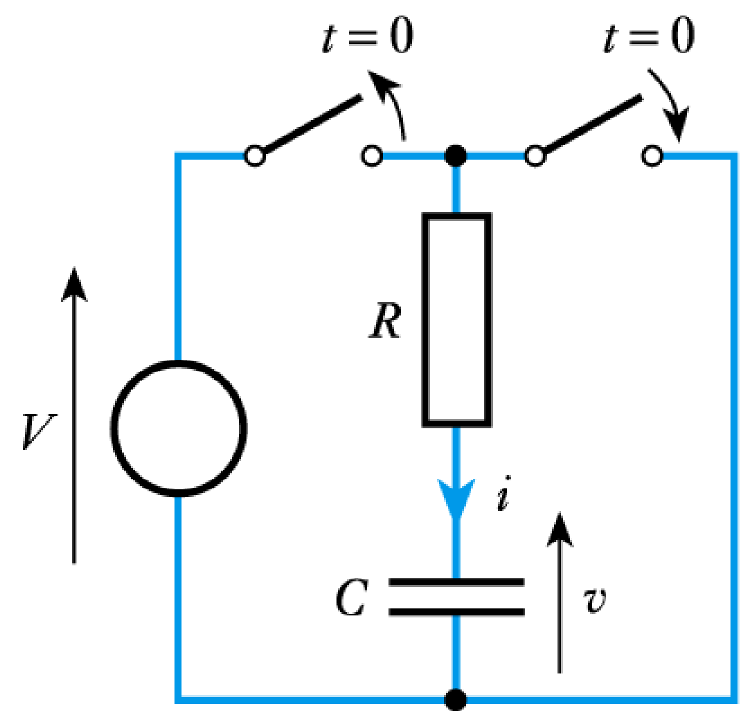
- Now, in a capacitor

$$i = C\frac{dv}{dt}$$

- Which substituting gives

$$RC\frac{dv}{dt} + v = 0$$

- Solve the differential equation
  - 1. Guess a general solution
  - 2. Apply the solution
  - 3. Substitute by boundary condition
  - 4. Verify the equation



Discharging Capacitors

- **1. Guess a general solution**

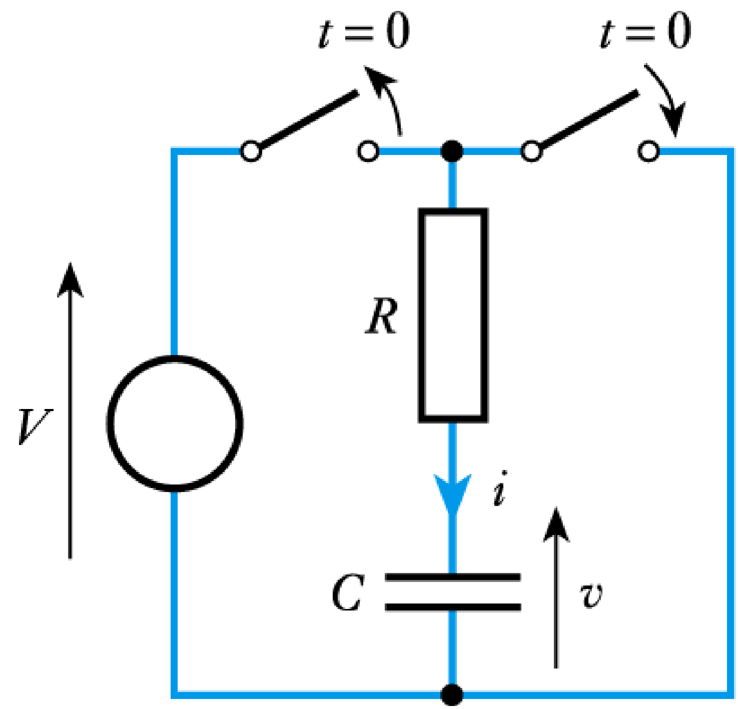
$$v(t) = \alpha \cdot e^{\beta t} + \gamma$$

- **2. Apply the solution**

$$RC\frac{dv}{dt} + v = 0 \rightarrow RC \cdot \alpha\beta \cdot e^{\beta t} + \alpha \cdot e^{\beta t} + \gamma = 0$$

$$\rightarrow \begin{cases} RC \cdot \alpha\beta \cdot e^{\beta t} + \alpha \cdot e^{\beta t} = 0 \rightarrow \beta = -\dfrac{1}{RC} \\ \gamma = 0 \end{cases}$$

$$\therefore v(t) = \alpha \cdot e^{-\frac{t}{RC}}$$



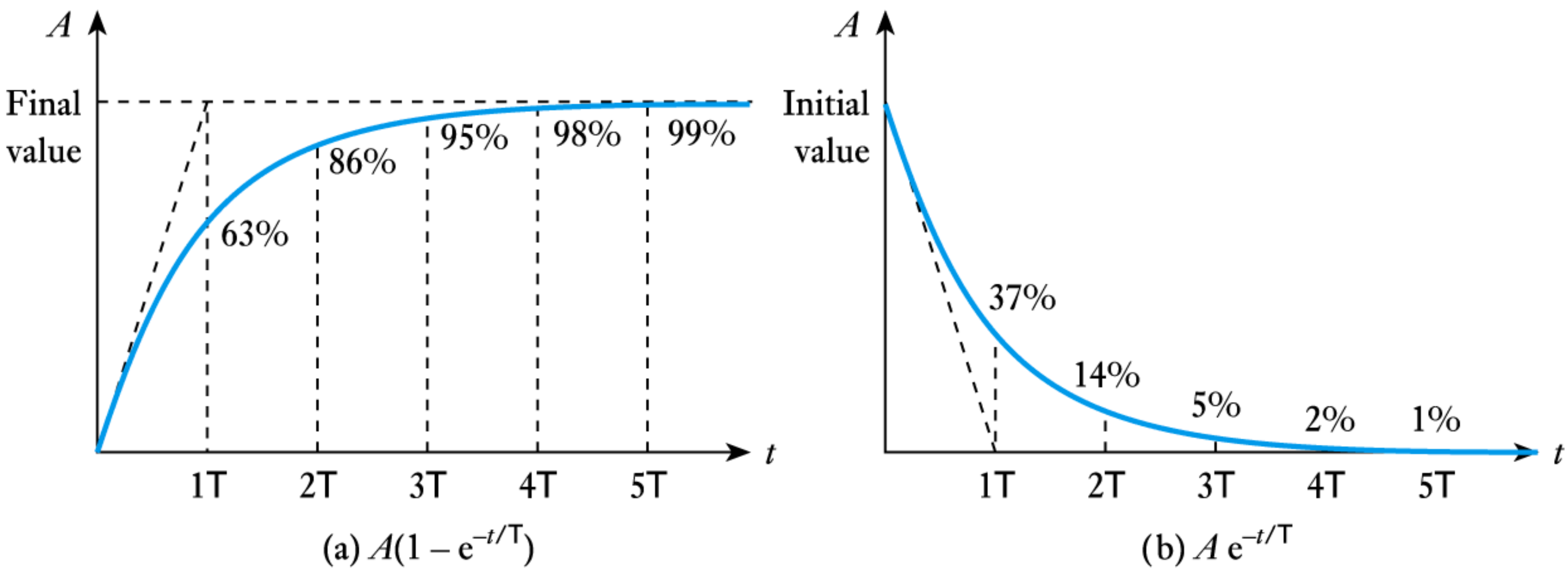
- **3. Substitute by boundary condition**

When $t = 0, v = V$:

$$v(t) = \alpha \cdot e^{-\frac{t}{RC}} = \alpha = V \rightarrow \alpha = V$$

$$\therefore v(t) = Ve^{-\frac{t}{RC}}$$

The Nature of Exponential Curves



Time Constant

- **Capacitor charging**

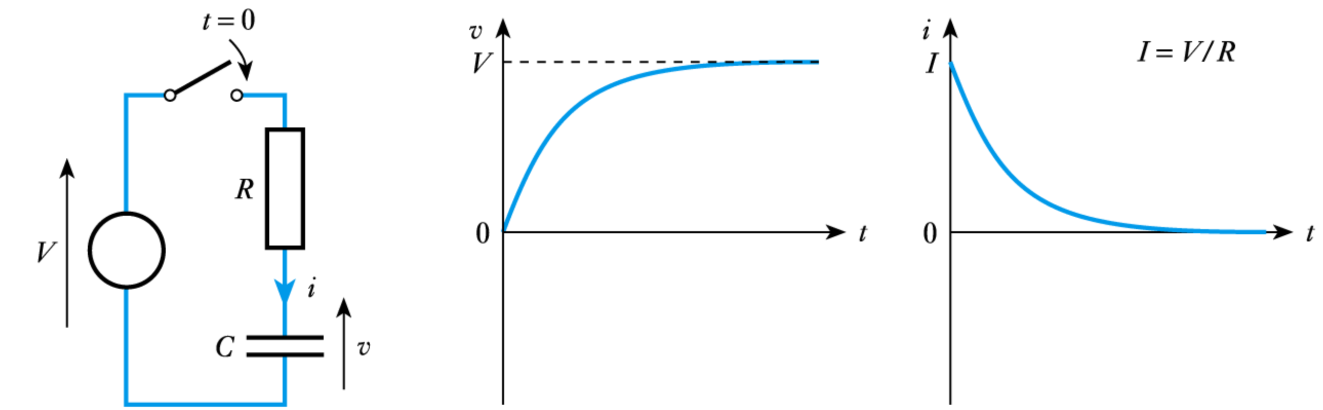
$$v(t) = -Ve^{-\frac{t}{RC}} + V = V(1 - e^{-\frac{t}{RC}}) = V(1 - e^{-\frac{t}{\mathrm{T}}})$$

- **Capacitor discharging**

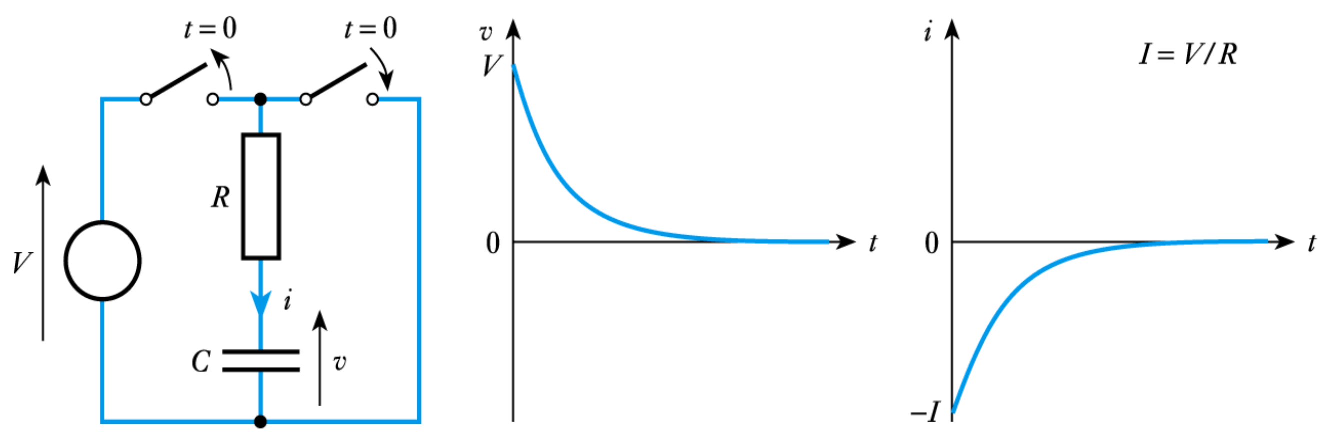
$$v(t) = Ve^{-\frac{t}{RC}} = Ve^{-\frac{t}{\mathrm{T}}}$$

- **Charging/discharging current is determined by *R* and the voltage *v* across it**
- **Increasing *R* will increase the time taken to charge/discharge *C***
- **Increasing *C* will also increase time taken to charge/discharge *C***
- **Time required to charge/discharge to a particular voltage is determined by the product *RC***
- **This product is the time constant T (greek tau, τ)**

Charging and Discharging Summary



(a) (b) (c)



(a) (b) (c)

Energy Stored in a Charged Capacitor

- **To move a charge *Q* through a potential difference Δ*V* requires an amount of energy *Q* Δ*V***
- **As we charge up a capacitor, we repeatedly add small amounts of charge Δ*Q* by moving them through a voltage equal to the voltage on the capacitor**
- **Since *Q* = *CV*, it follows that Δ*Q* = *C* Δ*V*, so the energy needed *E* is given by**

$$E = \int_0^V CV \mathrm{d}V = \frac{1}{2}CV^2$$

- **Alternatively, since *V* = *Q*/*C***

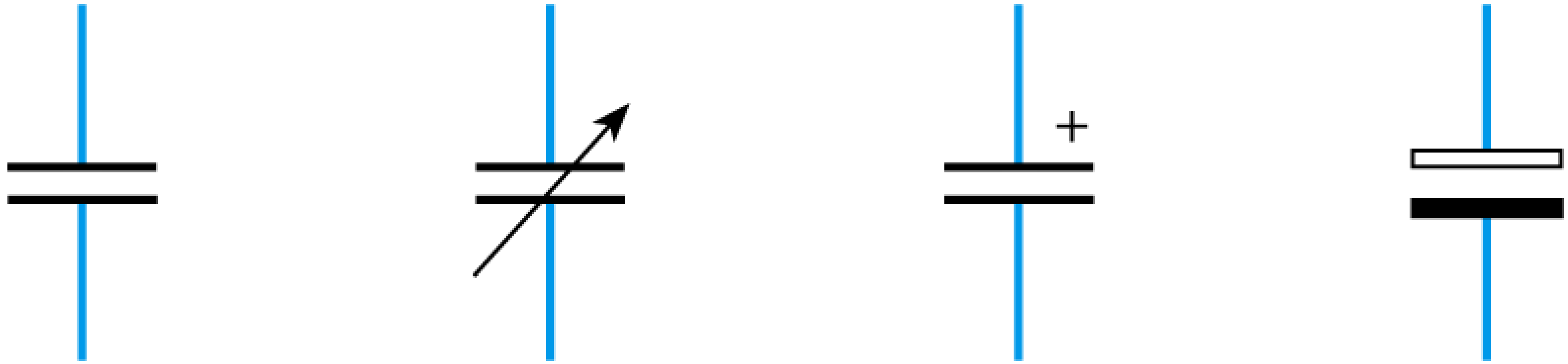
$$E = \frac{1}{2}CV^2 = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}\frac{Q^2}{C}$$

Example

- Calculate the energy stored in a 10 μF capacitor when it is charged to 100 V.

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\times 10^{-5}\times 100^2 = 50\,\mathrm{mJ}$$

Circuit Symbols



(a) A fixed capacitor

(b) A variable capacitor

(c) A polarised capacitor

(d) A polarised capacitor

Key Points

- **A capacitor consists of two plates separated by a dielectric**
- **The charge stored on a capacitor is proportional to *V***
- **A capacitor blocks DC but appears to pass AC**
- **The capacitance of several capacitors in parallel is equal to the sum of their individual capacitances**
- **The capacitance of several capacitors in series is equal to the reciprocal of the sum of the reciprocals of the individual capacitances**
- **When a capacitor is charged through a resistor, the charging rate is determined by the time constant *RC***
  - **Hence, the voltage across a capacitor cannot change instantaneously**
- **The energy stored in a capacitor is $\frac{1}{2}CV^2$ or $\frac{1}{2}Q^2/C$**