CS1011: 數位電子導論

Resistance and DC Circuits

Outline

- Introduction
- Current and Charge
- Voltage Sources
- Current Sources
- Resistance and Ohm's Law
- Resistors in Series and Parallel
- Kirchhoff's Laws
- Superposition
- Nodal Analysis
- Mesh Analysis
- Choice of Techniques
- Solving Simultaneous Circuit Equations (and Supercomputers)

Introduction

- **In earlier lectures, we have seen that many circuits can be analyzed using little more than Ohm's law**
- **However, in some cases we need some additional techniques and these are discussed in this lecture**
- **We begin by reviewing some of the basic elements that we have used in earlier lectures to describe our circuits**

Current and Charge

- An electric current is a flow of electric charge

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

- At an atomic level a current is a flow of electrons
  - Each electron has a charge of 1.6×10^{-19} coulombs
  - Conventional current flows in the opposite direction
- Rearranging above expression gives

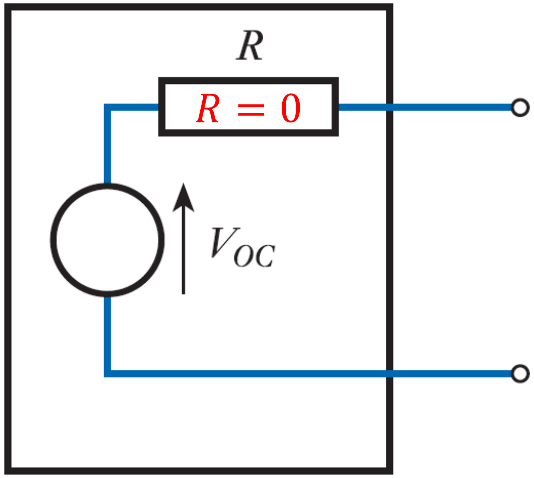
$$Q = \int I \mathrm{d}t$$

- For constant current

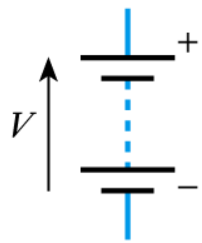
$$Q = I \times t$$

Voltage Sources

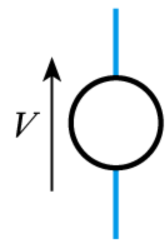
- A voltage source produces an electromotive force (e.m.f.) which causes a current to flow within a circuit
  - Unit of e.m.f. is the volt
- Real voltage sources, such as batteries have resistance associated with them
  - Typically, we use ideal voltage sources (i.e., *zero* output resistance)
  - We also use controlled or dependent voltage sources



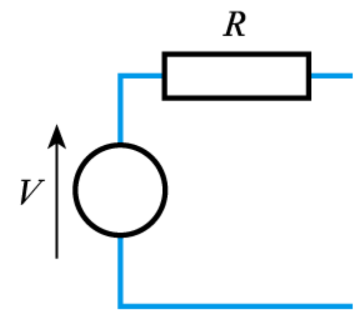
Voltage Source Symbols



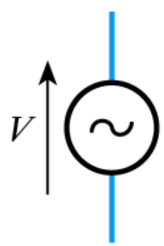
(a) A battery



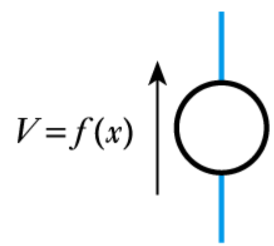
(b) An ideal voltage source



(c) Modelling a battery using an ideal voltage source



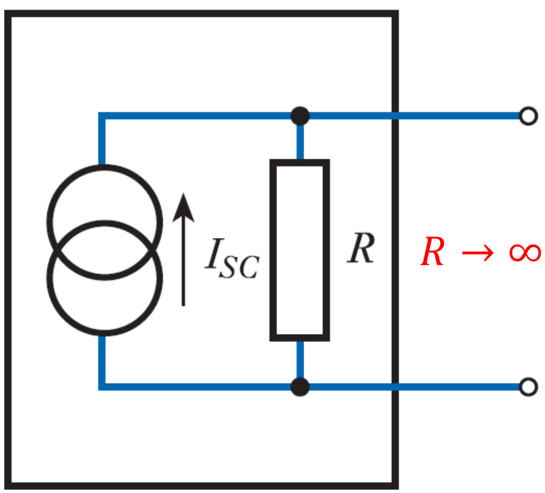
(d) An alternating voltage source

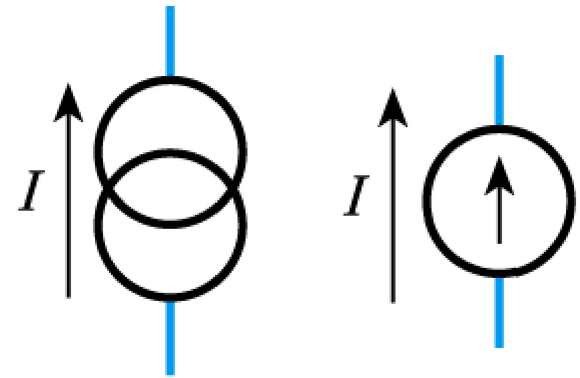


(e) A controlled voltage source

Current Sources

- **We also sometimes use the concept of an ideal current source**
  - **Unrealizable, but useful in circuit analysis**
  - **Can be a fixed current source, or a controlled or dependent current source**
  - **While an ideal voltage source has *zero* output resistance; an ideal current source has *infinite* output resistance**





Resistance and Ohm's Law

Ohm's law

$$V \propto I$$

- **Constant of proportionality is the resistance *R***
- **Hence,**

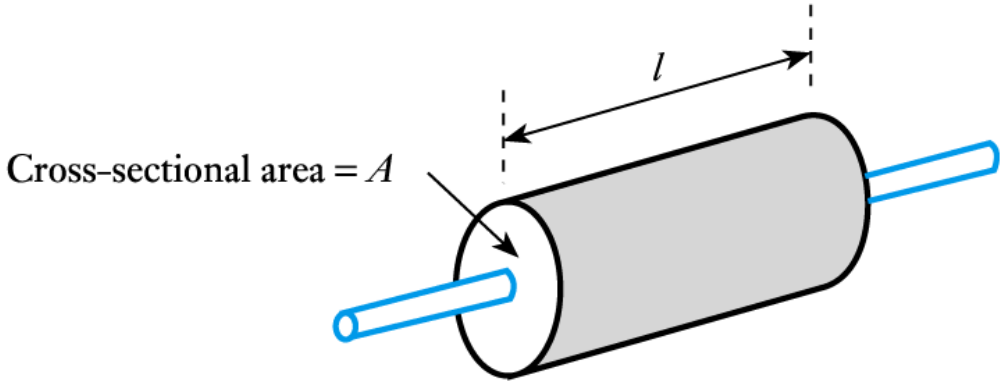
$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

- **Current through a resistor causes power dissipation**

$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2 R$$

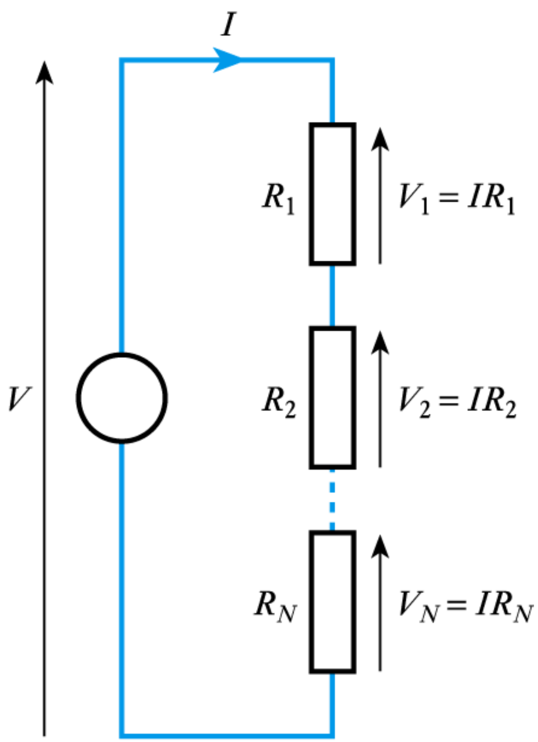
Resistors

- Resistance of a given sample of material is determined by its electrical characteristics and its construction
- Electrical characteristics described by its **resistivity ρ** or its **conductivity σ** (where $\sigma = 1/\rho$)



$$R = \frac{\rho l}{A}$$

Resistors in Series

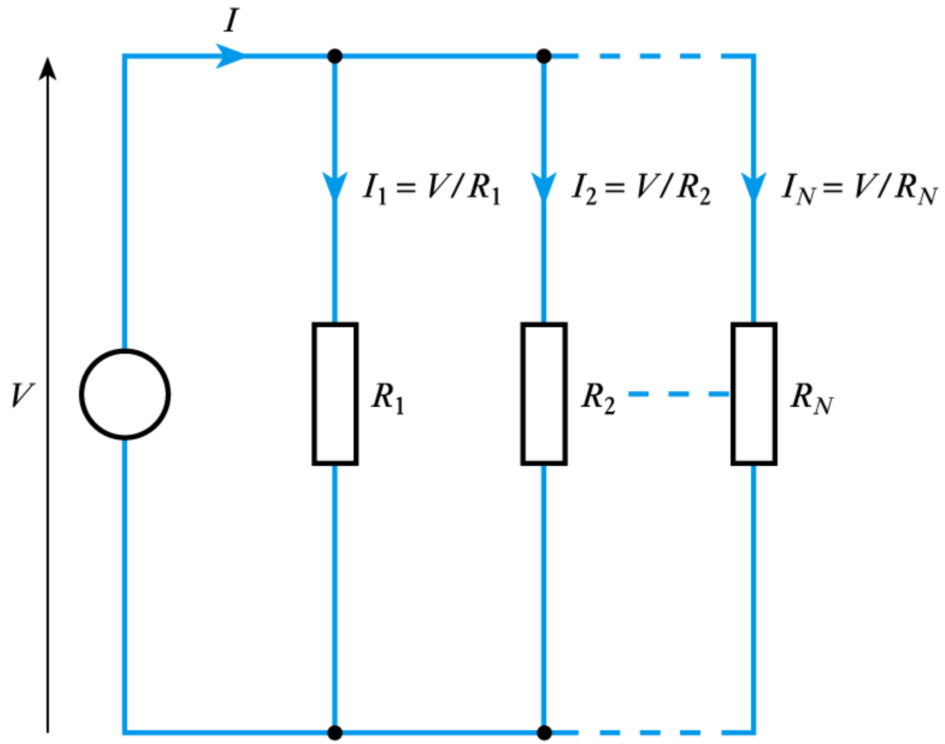


$$
\begin{aligned}
V &= IR_1 + IR_2 + \cdots + IR_N \\
&= I(R_1 + R_2 + \cdots + R_N) \\
&= IR
\end{aligned}
$$

where

$R = (R_1 + R_2 + \ldots + R_N)$.

Resistors in Parallel



$$I = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_N}$$

$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)$$

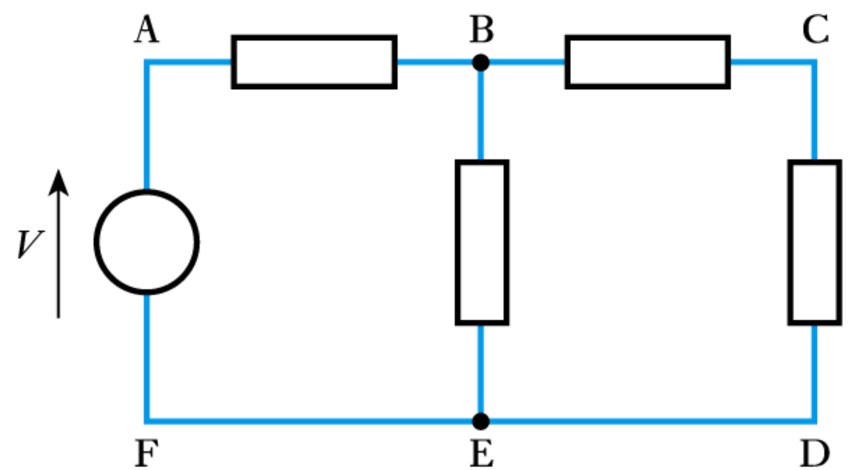
$$= V\left(\frac{1}{R}\right)$$

where

$1/R = 1/R_1 + 1/R_2 + \ldots + 1/R_N$

Kirchhoff's Laws

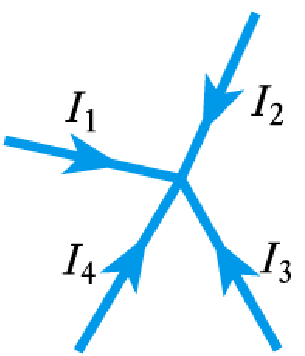
- **Node**
  - A point in a circuit where two or more circuit components are joined
- **Loop**
  - Any closed path that passes through no node more than once
- **Mesh**
  - A loop that contains no other loop
- **Examples:**
  - A, B, C, D, E and F are ***nodes***
  - The paths ABEFA, BCDEB and ABCDEFA are ***loops***
  - ABEFA and BCDEB are ***meshes***



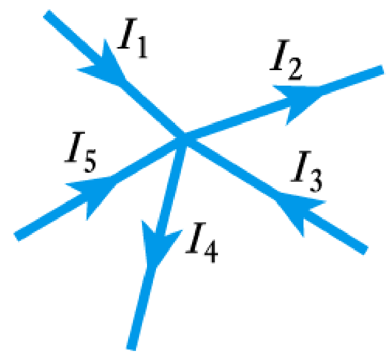
Current Law

▣ Current Law

- ***At any instant, the algebraic sum of all the currents flowing into any node in a circuit is zero***
  - » **If currents flowing *into* the node are positive, currents flowing *out of* the node are negative, then** $\sum I = 0$



$$I_1 + I_2 + I_3 + I_4 = 0$$

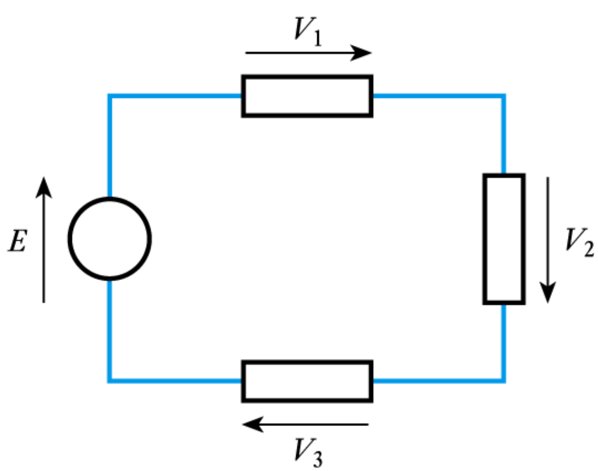


$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

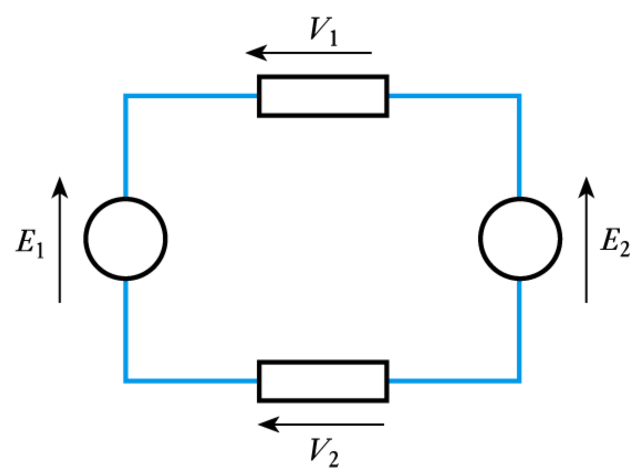
Voltage Law

▣ Voltage Law

- ***At any instant the algebraic sum of all the voltages around any loop in a circuit is zero***
  - » **If clockwise voltage arrows are positive and anticlockwise arrows are negative then** $\sum V = 0$



$E + V_1 + V_2 + V_3 = 0$



$E_1 - V_1 - E_2 + V_2 = 0$

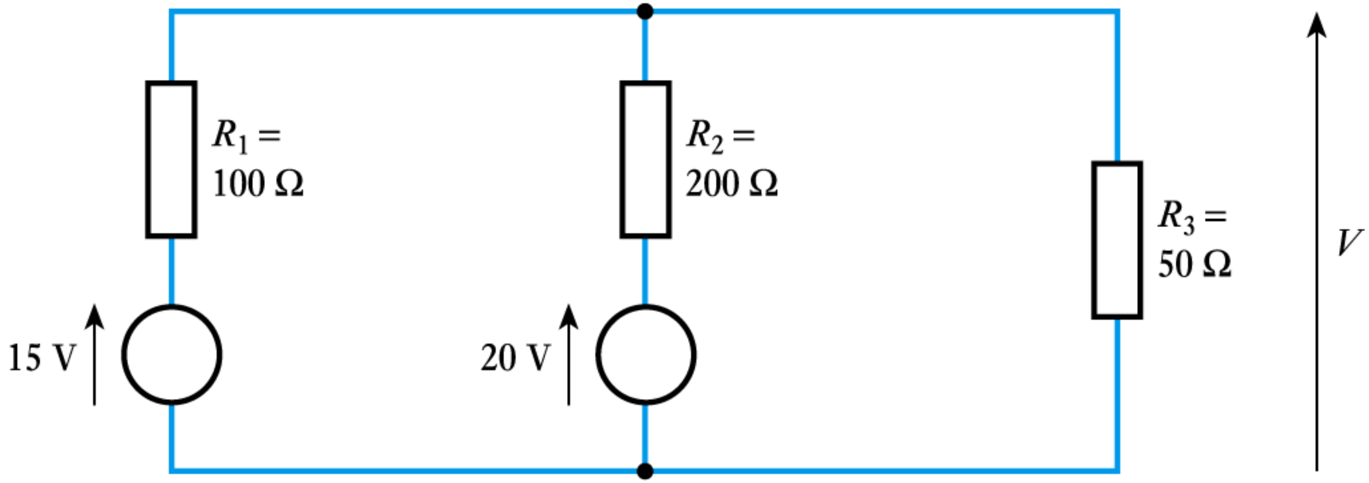
Superposition

Principle of superposition

- ***In any linear network of resistors, voltage sources and current sources, each voltage and current in the circuit is equal to the algebraic sum of the voltages or currents that would be present if each source were to be considered separately. When determining the effects of a single source the remaining sources are replaced by their internal resistance.***

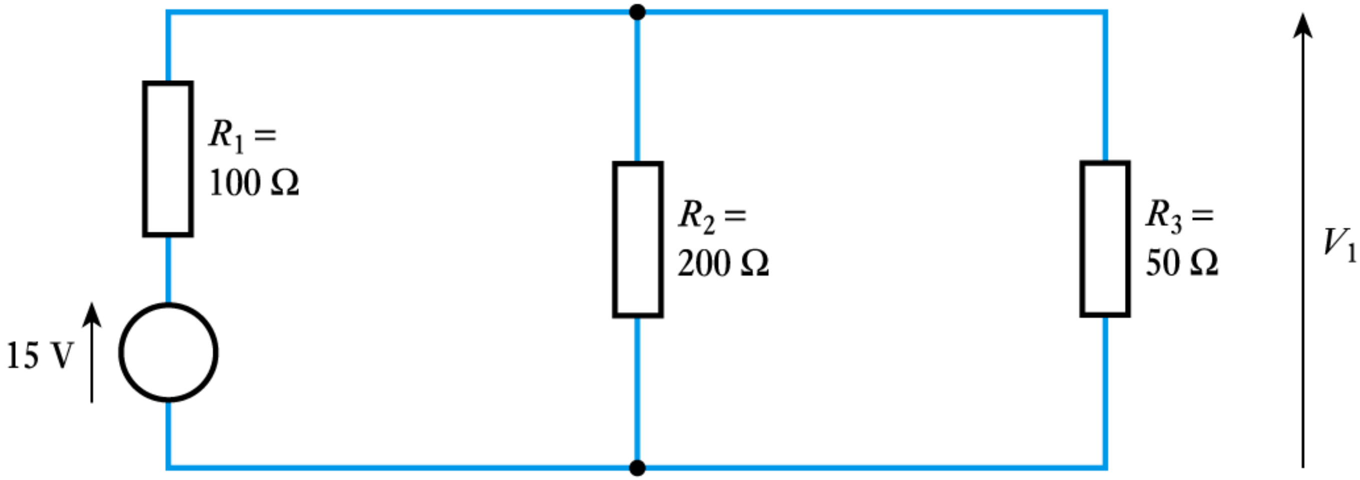
Superposition Example (1/4)

- Use the principle of superposition to calculate the output voltage *V* of the following circuit.



Superposition Example (2/4)

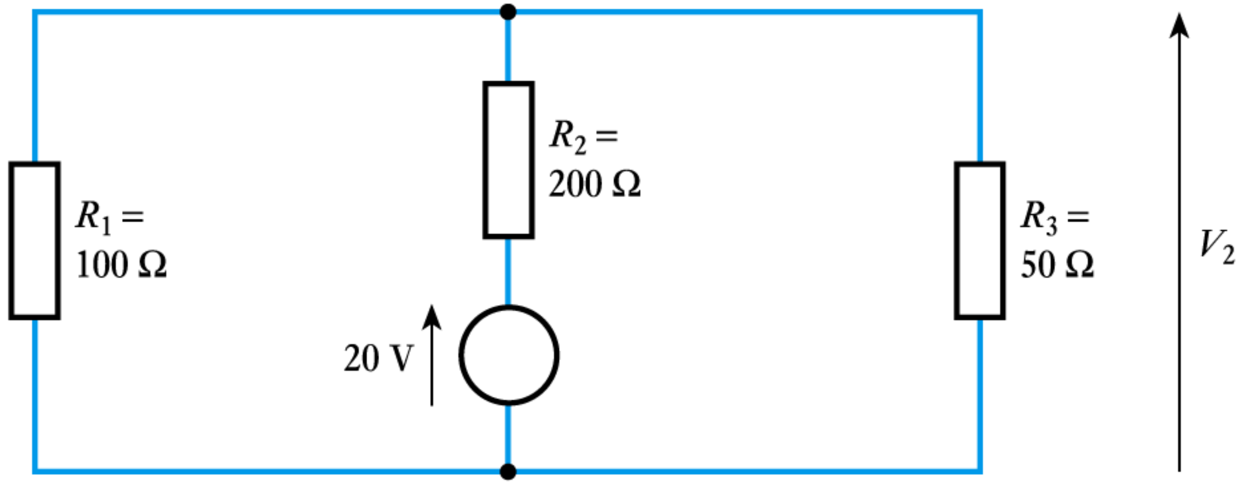
- First consider the effect of the 15V source alone



$$V_1 = 15 \frac{200 // 50}{100 + 200 // 50} = 15 \frac{40}{100 + 40} = 4.29 \text{ V}$$

Superposition Example (3/4)

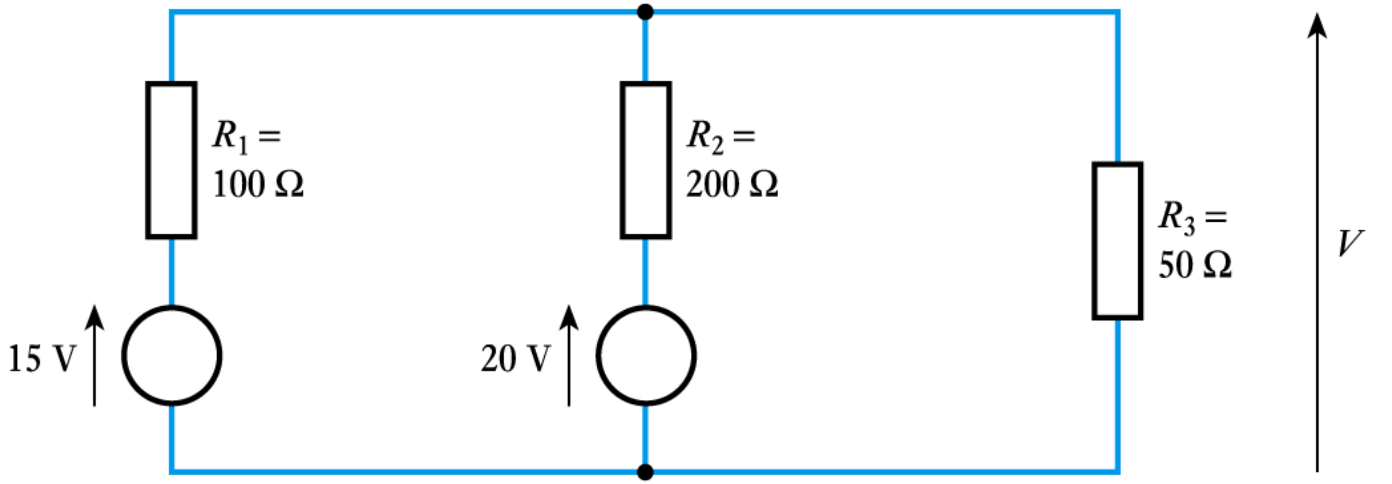
- Next consider the effect of the 20V source alone



$$V_2 = 20\frac{100\,//\,50}{200 + 100\,//\,50} = 20\frac{33.3}{200 + 33.3} = 2.86\text{ V}$$

Superposition Example (4/4)

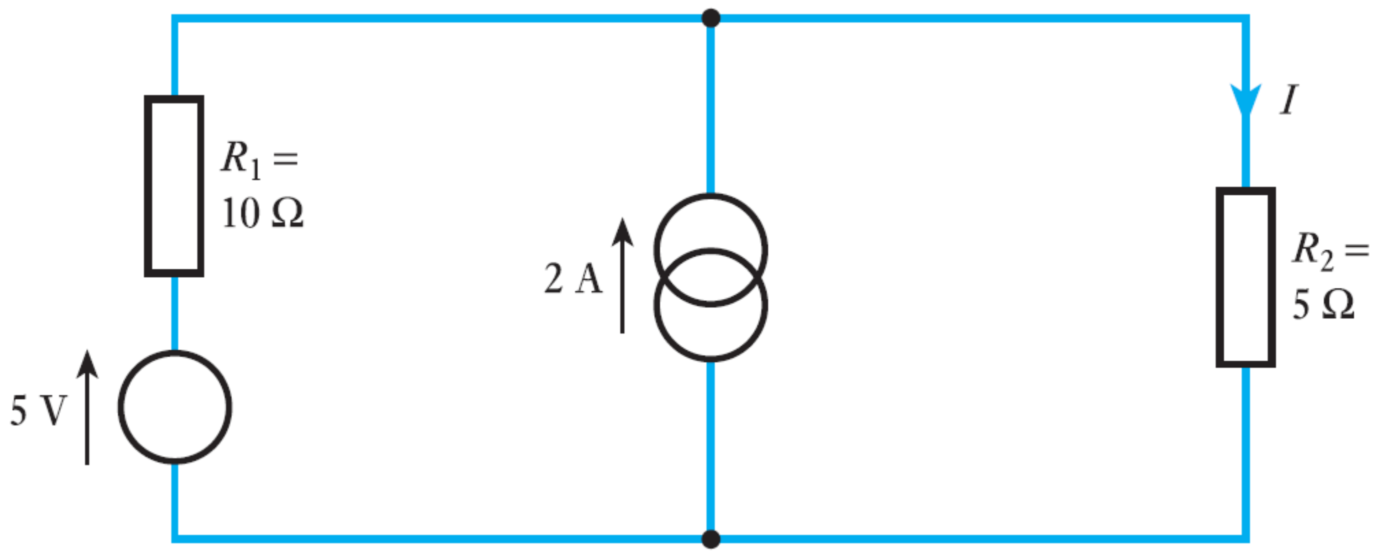
- The output of the circuit is the sum of these two voltages



$$V = V_1 + V_2 = 4.29 + 2.86 = 7.15 \text{ V}$$

Superposition Example

- **Use the principle of superposition to calculate the output current *I* in the following circuit**



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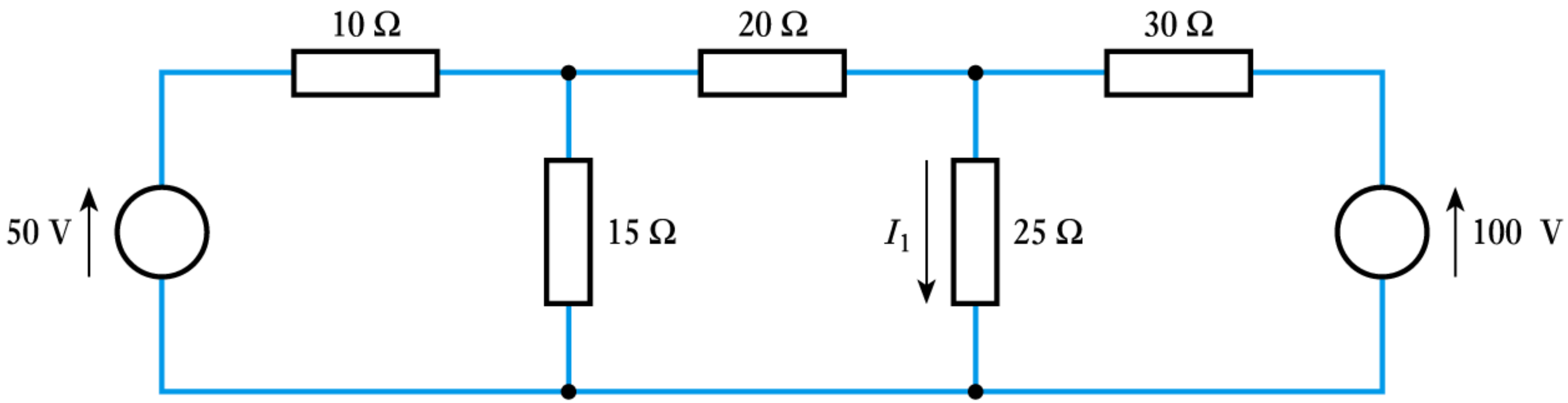
Nodal Analysis

- **Six steps:**
  1. **Chose one node as the reference node**
  2. **Label remaining nodes V_1, V_2, etc.**
  3. **Label any known voltages**
  4. **Apply Kirchhoff's current law to each unknown node**
  5. **Solve simultaneous equations to determine voltages**
  6. **If necessary, calculate required currents**

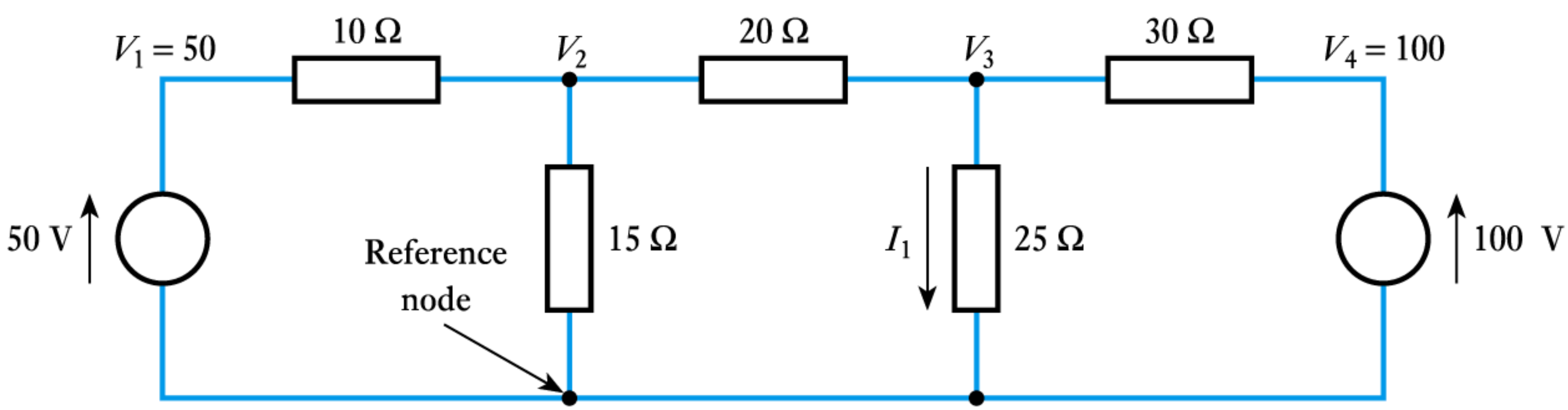
Nodal Analysis Example

- Use the nodal analysis to determine the current I_1 in the following circuit



Nodal Analysis Example (1/2)

- First, we pick a reference node and label the various node voltages, assigning values where these are known

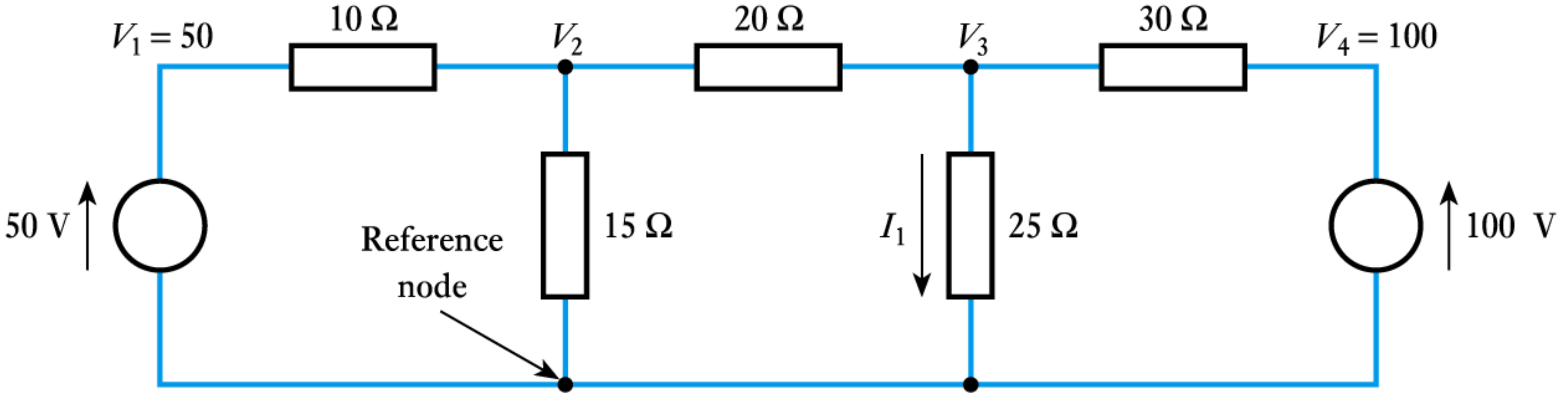


Nodal Analysis Example (2/2)

- Next, we sum the currents flowing into the nodes for which the node voltages are unknown. This gives

$$\frac{50 - V_2}{10} + \frac{V_3 - V_2}{20} + \frac{0 - V_2}{15} = 0 \qquad \frac{V_2 - V_3}{20} + \frac{100 - V_3}{30} + \frac{0 - V_3}{25} = 0$$

- Solving these two equations gives

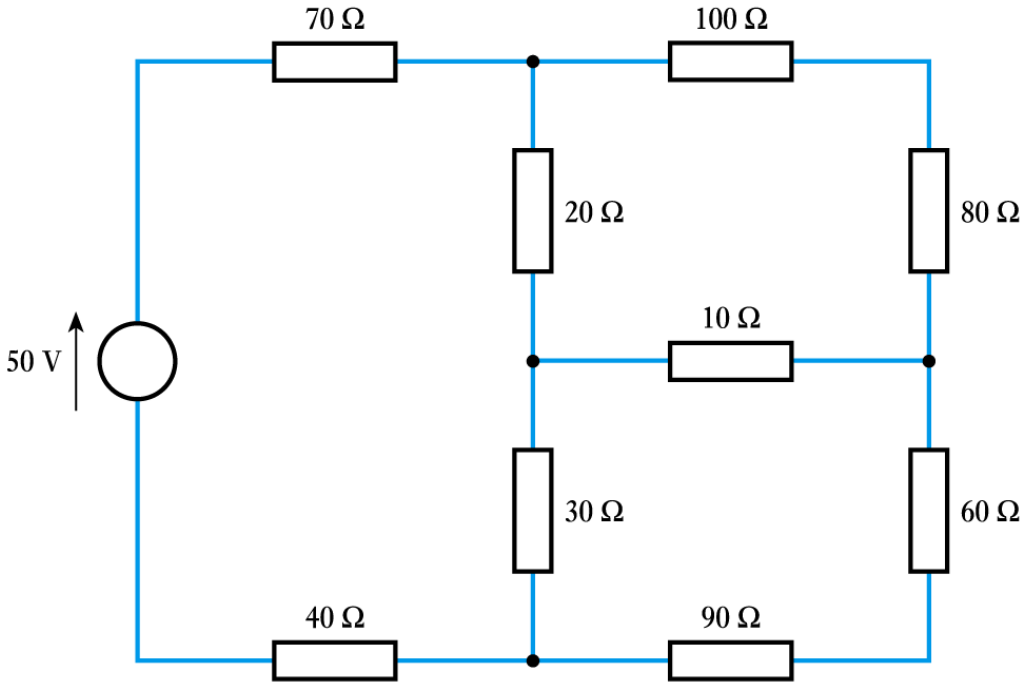


Mesh Analysis

- **Four steps:**
  1. **Identify the meshes and assign a clockwise-flowing current to each. Label these I_1, I_2, etc.**
  2. **Apply Kirchhoff's voltage law to each mesh**
  3. **Solve the simultaneous equations to determine the currents I_1, I_2, etc.**
  4. **Use these values to obtain voltages if required**

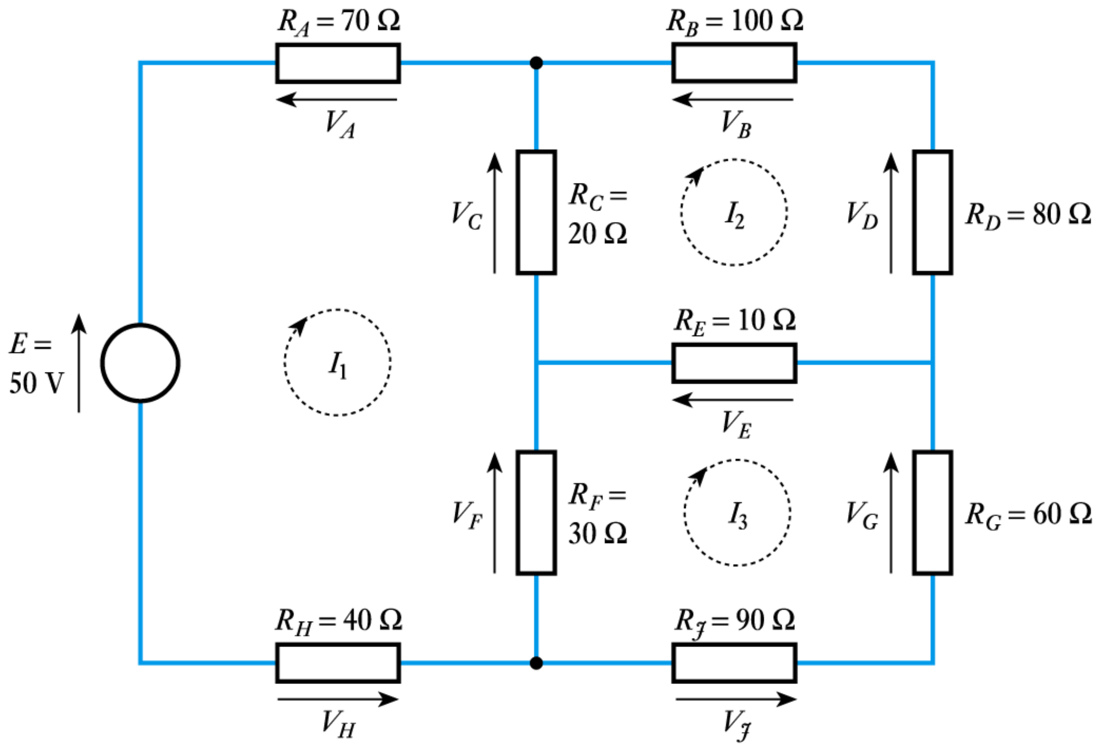
Mesh Analysis Example

- Use of the mesh analysis to determine the voltage across the 10 Ω resistor



Mesh Analysis Example (1/4)

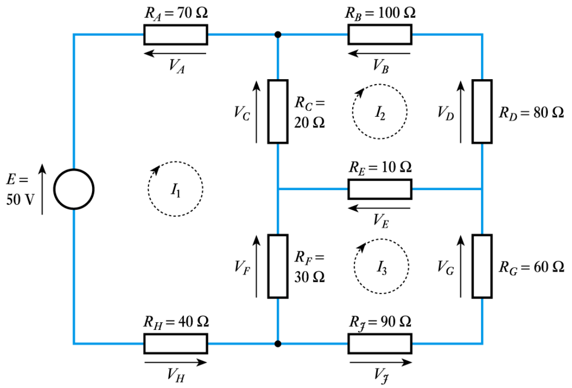
- First, assign loops currents and label voltages



Mesh Analysis Example (2/4)

- Next apply Kirchhoff's law to each loop. This gives

$$E - V_A - V_C - V_F - V_H = 0$$
$$V_C - V_B - V_D + V_E = 0$$
$$V_F - V_E - V_G - V_J = 0$$



- which gives the following set of simultaneous equations

$$50 - 70I_1 - 20(I_1 - I_2) - 30(I_1 - I_3) - 40I_1 = 0$$
$$20(I_1 - I_2) - 100I_2 - 80I_2 + 10(I_3 - I_2) = 0$$
$$30(I_1 - I_3) - 10(I_3 - I_2) - 60I_3 - 90I_3 = 0$$

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Mesh Analysis Example (3/4)

- **These can be rearranged to give**

$$
\begin{aligned}
50 - 160I_1 + 20I_2 + 30I_3 &= 0 \\
20I_1 - 210I_2 + 10I_3 &= 0 \\
30I_1 + 10I_2 - 190I_3 &= 0
\end{aligned}
$$

- **which can be solved to give**

$$
\begin{aligned}
I_1 &= 326\,\text{mA} \\
I_2 &= 34\,\text{mA} \\
I_3 &= 53\,\text{mA}
\end{aligned}
$$

Mesh Analysis Example (4/4)

- **The voltage across the 10 Ω resistor is therefore given by**

$$\begin{aligned} V_E &= R_E(I_3 - I_2) \\ &= 10(0.053 - 0.034) \\ &= 0.19 \text{ V} \end{aligned}$$

- **Since the calculated voltage is positive, the polarity is as shown by the arrow with the left hand end of the resistor more positive than the right hand end**

Choice of Techniques

- **How do we choose the right technique?**
  - **Nodal and mesh analysis will work in a wide range of situations but are not necessarily the simplest methods**
  - **No simple rules**
  - **Often involves looking at the circuit and seeing which technique seems appropriate**

Solving Simultaneous Circuit Equations

- Both nodal analysis and mesh analysis produce a series of simultaneous equations
  - Can be solved 'by hand' or by using matrix methods
  - e.g.,

$$50 - 160I_1 + 20I_2 + 30I_3 = 0$$
$$20I_1 - 210I_2 + 10I_3 = 0$$
$$30I_1 + 10I_2 - 190I_3 = 0$$

  - can be rearranged as

$$160I_1 - 20I_2 - 30I_3 = 50$$
$$20I_1 - 210I_2 + 10I_3 = 0$$
$$30I_1 + 10I_2 - 190I_3 = 0$$

Solving Simultaneous Circuit Equations

- These equations can be expressed as

$$\begin{bmatrix} 160 & -20 & -30 \\ 20 & -210 & 10 \\ 30 & 10 & -190 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

- which can be solved by hand (e.g. Cramer's rule)
- Or can use automated tools
  - E.g., scientific calculators
  - Computer-based packages such as MATLAB or Mathcad

Cramer's Rule

2-variable linear equation

$$\begin{cases} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \end{cases} \longrightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

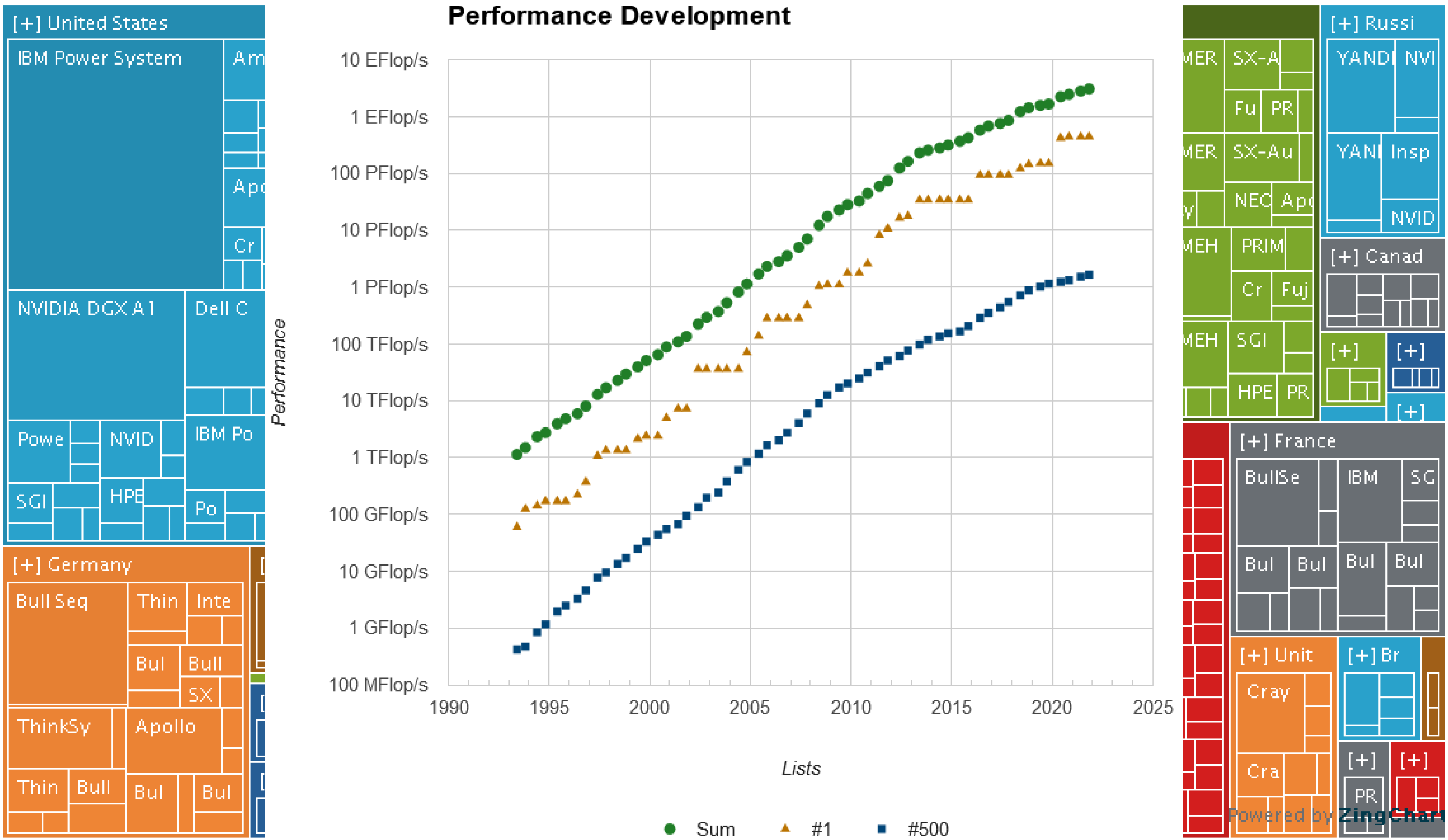
$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - b_1 c_2}{a_1 b_2 - b_1 a_2}, \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1 c_2 - c_1 a_2}{a_1 b_2 - b_1 a_2}.$$

3-variable linear equation

$$\begin{cases} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 \\ a_3 x + b_3 y + c_3 z &= d_3 \end{cases} \longrightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}.$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \text{ and } z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}.$$

TOP500 Supercomputers for LINPACK



TOP500 Supercomputers: https://www.top500.org

Key Points

- An electric current is a flow of charge
- A voltage source produces an e.m.f. which can cause a current to flow
- Current in a conductor is directly proportional to voltage
- At any instant the sum of the currents into a node is zero
- At any instant the sum of the voltages around a loop is zero
- Nodal and mesh analysis provide systematic methods of applying Kirchhoff's laws