Low-Complexity Millimeter Wave CSI Estimation in MIMO-OFDM Hybrid Beamforming Systems

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***Abstract*—Channel state information (CSI) estimation in hybrid analog-digital (HAD) millimeter-wave (mmWave) massive MIMO systems is a challenging problem due to the high channel dimension and reduced number of radio-frequency chains. The problem becomes even harder when we consider wideband channels with higher frequency selectivity than the narrowband channels. Fortunately, by exploiting the sparse scattering nature of the mmWave channels and by adopting a simple setup at the transmitter, it was shown that the received signal can be organized into a third-order tensor that admits a Canonical Polyadic decomposition. Therefore, the channel parameters can be simply recovered once the decomposed factor matrices are estimated, e.g., using an alternating least square (ALS) method. However, ALS has a high computational complexity and a slow convergence rate. To resolve this issue, we propose a low-complexity non-iterative algorithm to recover the channel parameters without requiring the estimation of the decomposed factor matrices by utilizing a compressed sensing technique and tensor algebra. Simulation results are provided to evaluate the effectiveness of the proposed algorithm.**

***Index Terms*—Compressed sensing, Tensor decomposition, hybrid analog-digital, CSI estimation, MIMO.**

I. Introduction

The combination of mmWave and massive MIMO wireless technologies is seen as key enabler in future wireless networks [1]. However, the large number of required radio frequency (RF) chains and power amplifiers render fully-digital (FD) architectures impractical due to the associated high cost and power consumption per antenna. Recently, HAD beamforming architectures have been proposed to facilitate the practical implementation of massive MIMO systems by dividing the beamforming process between the analog and digital domains to reduce the number of the energy-hungry RF chains [2].

To realize the potentials of the HAD-MIMO systems, CSI is required at transmitter, which is harder to estimate with HAD systems than with FD counterparts due to the reduced number of RF chains [3]. Fortunately, it was observed by several measurement campaigns that the massive MIMO channel matrix in mmWave communication has a sparse structure in the angular domain due to the limited number of scatters comparing to the large number of antenna elements [1]. Exploiting this sparse structure, compressed sensing (CS) tools have been used to estimate the MIMO channel, where the problem can be turned into estimating the parameters of dominant channel paths, while yet requiring far less training overhead [3]–[6]. However, most of the existing methods assume narrowband channels, while in practice, the mmWave channels are very likely to operate on wideband channels with frequency selectivity [7]. In this line, the authors of [8] proposed a distributed CS method for uplink multi-user HAD-MIMO-OFDM systems that exploits the angular domain structured sparsity of mmWave wideband frequency-selective fading channels.

More recently, the authors of [9] formulated the downlink mmWave HAD-MIMO-OFDM channel estimation problem as a third-order tensor that admits a Canonical Polyadic (CP) decomposition [10], which is shown to significantly reduce the required training overhead. Similar approach has been taken by [11] for the uplink mmWave HAD-MIMO-OFDM systems. In [9], [11], the decomposed factor matrices are first estimated using an ALS technique, and then the channel parameters are estimated using a simple correlation-based scheme. However, the ALS technique has high computational complexity and slow convergence rate. Therefore, a different approach that requires lower complexity is desired.

In this paper, similar to [9], we consider a downlink mmWave HAD-MIMO-OFDM system model, where we formulate the channel estimation problem as a third-order tensor, which admits a CP decomposition. Unlike [9], [11], we propose a low-complexity non-iterative algorithm to recover the channel parameters without requiring the estimation of the decomposed factor matrices. The proposed algorithm utilizes a CS technique and tensor algebra to recover the channel parameters of each channel-path separately. By doing so, the computational complexity can be significantly reduced, since the estimation of the decomposed factor matrices can be avoided. Simulation results are provided to evaluate the effectiveness of the proposed algorithm. It is shown that the proposed algorithm outperforms the ALS-based algorithm in the more critical scenarios, such as, in the low SNR regime and with the large number of channel paths.

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II. System and Channel Model

We consider a HAD downlink MIMO-OFDM system between a transmitter with M_T antennas and a receiver with M_R antennas, as shown in Fig. 1. The transmitter has $N_T \leq M_T$ RF chains, while the receiver has $N_R \leq M_R$ RF chains. The system has in total K subcarriers and the training period is divided into T OFDM frames [1].

The received signal vector on the k-th subcarrier, $k \in \{1,\dots,K\}$, in the t-th OFDM frame, $t \in \{1,\dots,T\}$, is given as

$$\mathbf{y}_{k,[t]} = \mathbf{W}^{\mathrm{T}}\mathbf{H}_k\mathbf{P}_{[t]}\mathbf{s}_{[t]} + \mathbf{W}^{\mathrm{T}}\mathbf{z}_{k,[t]} \in \mathbb{C}^{N_R}, \tag{1}$$

where $\mathbf{P}_{[t]} \in \mathbb{C}^{M_T \times N_T}$ denotes the analog precoding matrix and $\mathbf{s}_{[t]} \in \mathbb{C}^{N_T}$ denotes the unit-norm training vector, which are assumed to be common for the K subcarriers. Moreover, $\mathbf{W} \in \mathbb{C}^{M_R \times N_R}$ denotes the analog decoding matrix, which is assumed to be common for the T OFDM frames and the K subcarriers, and $\mathbf{z}_{k,[t]} \in \mathbb{C}^{M_R}$ denotes the additive white Gaussian noise with variance σ^2. Here, we assume that $\mathbf{P}_{[t]}$ and $\mathbf{W}$ have constant modulus elements, i.e., $|[\mathbf{P}_{[t]}]_{[i,j]}| = 1/\sqrt{M_T}, \forall i,j$, and $|[\mathbf{W}]_{[i,j]}| = 1/\sqrt{M_R}, \forall i,j$. Furthermore, $\mathbf{H}_k \in \mathbb{C}^{M_R \times M_T}$ denotes the MIMO channel matrix on the k-th subcarrier, which is modeled as [1], [2]

$$\mathbf{H}_k = \sum_{\ell=1}^{L} \alpha_\ell \cdot e^{-j2\pi\tau_\ell f_s \frac{k}{K}} \mathbf{a}(\theta_\ell)\mathbf{a}^{\mathrm{T}}(\phi_\ell), \tag{2}$$

where f_s denotes the sampling rate, L denotes the total number of channel paths, for which $\alpha_\ell, \tau_\ell, \theta_\ell$ and ϕ_ℓ denote, respectively, the complex path gain, time delay, angle-of-arrival (AoA), and angle-of-departure (AoD) of the ℓ-th path. In (2), $\mathbf{a}(\theta_\ell)$ and $\mathbf{a}(\phi_\ell)$ denote the array response/steering vectors at receiver and transmitter, respectively. Assuming uniform linear arrays with half-wavelength spacing between elements, $\mathbf{a}(\theta_\ell)$ and $\mathbf{a}(\phi_\ell)$ are given respectively as [1], [2]

$$\mathbf{a}(\theta_\ell) = \left[1, e^{j\pi\cos(\theta_\ell)}, \dots, e^{j\pi(M_{\mathrm{R}}-1)\cos(\theta_\ell)}\right]^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{R}}}, \tag{3}$$
$$\mathbf{a}(\phi_\ell) = \left[1, e^{j\pi\cos(\phi_\ell)}, \dots, e^{j\pi(M_{\mathrm{T}}-1)\cos(\phi_\ell)}\right]^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{T}}}. \tag{4}$$

The channel matrix in (2) can be written in a compact form as

$$\mathbf{H}_k = \mathbf{A}_1\mathbf{D}_k\mathbf{A}_2^{\mathrm{T}}, \tag{5}$$

where we let $d_{\ell,k} = \alpha_\ell \cdot e^{-j2\pi\tau_\ell f_s \frac{k}{K}}$ and

$$\begin{cases} \mathbf{A}_1 = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_L)] \in \mathbb{C}^{M_R \times L} \\ \mathbf{A}_2 = [\mathbf{a}(\phi_1), \dots, \mathbf{a}(\phi_L)] \in \mathbb{C}^{M_T \times L} \\ \mathbf{D}_k = \mathrm{diag}\{d_{1,k}, \dots, d_{L,k}\} \in \mathbb{C}^{L \times L}. \end{cases} \tag{6}$$

[1]During the channel estimation period, we assume that only the analog domain is used, while in the data transmission period, both analog and digital domains are used. See [2], [5] for more details.

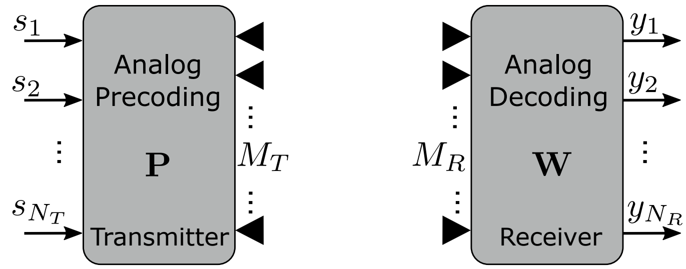

Fig. 1. HAD-MIMO System Model

III. Problem Formulation based CP Decomposition

By stacking $\mathbf{y}_{k,[t]}, \forall t$, next to each other we obtain the signal matrix $\mathbf{Y}_k \in \mathbb{C}^{N_R \times T}$ that is given as

$$\mathbf{Y}_k = [\mathbf{y}_{k,[1]} \dots \mathbf{y}_{k,[T]}] = \mathbf{W}^{\mathrm{T}}\mathbf{H}_k\mathbf{F} + \mathbf{N}_k, \tag{7}$$

where

$$\begin{cases} \mathbf{F} = [\mathbf{P}_{[1]}\mathbf{s}_{[1]} \dots \mathbf{P}_{[T]}\mathbf{s}_{[T]}] \in \mathbb{C}^{M_T \times T} \\ \mathbf{N}_k = \mathbf{W}^{\mathrm{T}}\mathbf{Z}_k \in \mathbb{C}^{N_R \times T} \\ \mathbf{Z}_k = [\mathbf{z}_{k,[1]} \dots \mathbf{z}_{k,[T]}] \in \mathbb{C}^{M_R \times T}. \end{cases} \tag{8}$$

Then, by substituting (5) into (7), we can write

$$\mathbf{Y}_k = \tilde{\mathbf{A}}_1\mathbf{D}_k\tilde{\mathbf{A}}_2^{\mathrm{T}} + \mathbf{N}_k, \tag{9}$$

where $\tilde{\mathbf{A}}_1 = \mathbf{W}^{\mathrm{T}}\mathbf{A}_1 \in \mathbb{C}^{N_R \times L}$ and $\tilde{\mathbf{A}}_2 = \mathbf{F}^{\mathrm{T}}\mathbf{A}_2 \in \mathbb{C}^{T \times L}$. Next, by stacking the matrices $\mathbf{Y}_k, \forall k$, behind each other, we have a tensor $\boldsymbol{\mathcal{Y}} \in \mathbb{C}^{N_R \times T \times K}$, which admits a CP decomposition and can be written as [9], [11]

$$\boldsymbol{\mathcal{Y}} = \sum_{\ell=1} \tilde{\mathbf{a}}_{1,\ell} \circ \tilde{\mathbf{a}}_{2,\ell} \circ \tilde{\mathbf{a}}_{3,\ell} + \mathcal{N}, \tag{10}$$

where $\circ$ denotes the outer-product operation, $\mathcal{N} \in \mathbb{C}^{N_R \times T \times K}$ is the noise tensor, where its k-th slice is given by $\mathbf{N}_k$, $\tilde{\mathbf{a}}_{1,\ell}$, $\tilde{\mathbf{a}}_{2,\ell}$, and $\tilde{\mathbf{a}}_{3,\ell}$ are the ℓ-th columns of $\tilde{\mathbf{A}}_1$, $\tilde{\mathbf{A}}_2$, and $\tilde{\mathbf{A}}_3$, respectively. Here, $\tilde{\mathbf{A}}_3 = \mathbf{A}_3\mathbf{G} \in \mathbb{C}^{K \times L}$, where $\mathbf{G} = \mathrm{diag}\{\alpha_1, \dots, \alpha_L\} \in \mathbb{C}^{L \times L}$ and $\mathbf{A}_3 \in \mathbb{C}^{K \times L}$ is a matrix such that its l-th column is given by

$$\mathbf{a}(\tau_\ell) = [e^{-j2\pi\tau_\ell f_s \frac{1}{K}}, \dots, e^{-j2\pi\tau_\ell f_s \frac{K}{K}}]^{\mathrm{T}} \in \mathbb{C}^{K}. \tag{11}$$

Note that the channel parameters $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell\}_{\ell=1}^{L}$ can be recovered from the decomposed factor matrices $\tilde{\mathbf{A}}_1$, $\tilde{\mathbf{A}}_2$, and $\tilde{\mathbf{A}}_3$ using a simple correlation scheme, where the algorithm starts off by employing a coarse grid and then gradually refine the search in the vicinity of possible grid points [12]. Therefore, estimating $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell\}_{\ell=1}^{L}$ is equivalent to estimating $\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2$, and $\tilde{\mathbf{A}}_3$, for which several methods have been proposed, such as the ALS technique that was used e.g., in [9], [11], which updates one matrix at a time, while assuming the other two are fixed. To be self-contained, we summarize the ALS steps in Algorithm 1, where the superscript (t) indicates the iteration index.

At the convergence of Algorithm 1, the channel parameters $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell\}_{\ell=1}^{L}$ are first recovered from the estimated matrices $\hat{\tilde{\mathbf{A}}}_1$, $\hat{\tilde{\mathbf{A}}}_2$, and $\hat{\tilde{\mathbf{A}}}_3$, which are then used to

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Algorithm 1 Parameter Estimation Based ALS Method.

1: Inputs: Tensor $\boldsymbol{\mathcal{Y}}$ and number of paths L
2: Initialize: Unfolding $[\mathbf{Y}]_{(1)}, [\mathbf{Y}]_{(2)}, [\mathbf{Y}]_{(3)}$, and initial $\mathbf{A}_2^{(0)}$, $\mathbf{A}_3^{(0)}$
3: **while** not converged **do**
4: $\quad \tilde{\mathbf{A}}_1^{(t+1)} = \arg\min_{\hat{\mathbf{A}}_1} = \|[\mathbf{Y}]_{(1)}^{\mathrm{T}} - (\tilde{\mathbf{A}}_3^{(t)} \diamond \tilde{\mathbf{A}}_2^{(t)})\hat{\mathbf{A}}_1^{\mathrm{T}}\|_F^2$
5: $\quad \tilde{\mathbf{A}}_2^{(t+1)} = \arg\min_{\hat{\mathbf{A}}_2} = \|[\mathbf{Y}]_{(2)}^{\mathrm{T}} - (\tilde{\mathbf{A}}_3^{(t)} \diamond \tilde{\mathbf{A}}_1^{(t+1)})\hat{\mathbf{A}}_2^{\mathrm{T}}\|_F^2$
6: $\quad \tilde{\mathbf{A}}_3^{(t+1)} = \arg\min_{\hat{\mathbf{A}}_3} = \|[\mathbf{Y}]_{(3)}^{\mathrm{T}} - (\tilde{\mathbf{A}}_2^{(t+1)} \diamond \tilde{\mathbf{A}}_1^{(t+1)})\hat{\mathbf{A}}_3^{\mathrm{T}}\|_F^2$
7: **end while**
8: Output: Estimates $\hat{\mathbf{A}}_1$, $\hat{\mathbf{A}}_2$, and $\hat{\mathbf{A}}_3$

calculate $\hat{\mathbf{A}}_1 = [\mathbf{a}(\hat{\theta}_1)\dots\mathbf{a}(\hat{\theta}_L)]$, $\hat{\mathbf{A}}_2 = [\mathbf{a}(\hat{\phi}_1)\dots\mathbf{a}(\hat{\phi}_L)]$, and $\hat{\mathbf{A}}_3 = [\mathbf{a}(\hat{\tau}_1)\dots\mathbf{a}(\hat{\tau}_L)]$. Finally, the path gains $\mathbf{G} = \mathrm{diag}\{\alpha_1,\dots,\alpha_L\}$ are estimated as

$$\hat{\mathbf{G}} = \hat{\mathbf{A}}_3^{+}\Big([\mathbf{Y}]_{(3)}\mathbf{R}^{*}\big((\mathbf{R}^{\mathrm{H}}\mathbf{R})^{-1}\big)^{\mathrm{T}}\Big), \tag{12}$$

where $\mathbf{R} = \big(\mathbf{F}^{\mathrm{T}}\hat{\mathbf{A}}_2 \diamond \mathbf{W}^{\mathrm{T}}\hat{\mathbf{A}}_1\big)$. Here, $\diamond$ and $^{+}$ denote the Khatri-Rao product and the Moore-Penrose pseudo inverse, respectively.

IV. Proposed Parameters Estimation Method

To avoid the ALS computational complexity, in this section we propose a low-complexity non-iterative algorithm to recover the channel parameters without requiring the estimation of the decomposed factor matrices by utilizing a CS technique and tensor algebra.

At first, we note that the tensor $\boldsymbol{\mathcal{Y}} \in \mathbb{C}^{N_R\times T\times K}$ can be written as [10]

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{I}}_{3,L} \times_1 \tilde{\mathbf{A}}_1 \times_2 \tilde{\mathbf{A}}_2 \times_3 \tilde{\mathbf{A}}_3 = \boldsymbol{\mathcal{S}} \times_1 \tilde{\mathbf{A}}_1, \tag{13}$$

where $\times_r$ denotes the r-mode product, $\boldsymbol{\mathcal{I}}_{3,L} \in \mathbb{R}^{L\times L\times L}$ is a diagonal tensor containing ones on its diagonal and zeros elsewhere, and $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{L\times T\times K}$ is a tensor given as

$$\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{I}}_{3,L} \times_2 \tilde{\mathbf{A}}_2 \times_3 \tilde{\mathbf{A}}_3. \tag{14}$$

The unfolding of tensor $\boldsymbol{\mathcal{S}}$ along its first dimension can be written as [12]

$$[\mathbf{S}]_{(1)} = (\tilde{\mathbf{A}}_3 \diamond \tilde{\mathbf{A}}_2)^{\mathrm{T}} = \begin{bmatrix} (\tilde{\mathbf{a}}_{3,1} \diamond \tilde{\mathbf{a}}_{2,1})^{\mathrm{T}} \\ \vdots \\ (\tilde{\mathbf{a}}_{3,L} \diamond \tilde{\mathbf{a}}_{2,L})^{\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{L\times TK}. \tag{15}$$

From (15), we can see that each row of $[\mathbf{S}]_{(1)}$ represents the unfolding of the sub-tensor $\boldsymbol{\mathcal{S}}_\ell = \tilde{\mathbf{a}}_{2,\ell} \circ \tilde{\mathbf{a}}_{3,\ell} \in \mathbb{C}^{1\times T\times K}$ along its first dimension. Therefore, the tensor $\boldsymbol{\mathcal{S}}$ can be written as concatenation of sub-tensors $\boldsymbol{\mathcal{S}}_\ell$ along the first dimension, i.e., we can write $\boldsymbol{\mathcal{S}}$ as [10]

$$\boldsymbol{\mathcal{S}} = [\boldsymbol{\mathcal{S}}_1 \sqcup_1 \boldsymbol{\mathcal{S}}_2 \sqcup_1 \cdots \sqcup_1 \boldsymbol{\mathcal{S}}_L] \in \mathbb{C}^{L\times T\times K}. \tag{16}$$

Given the ℓ-th sub-tensor $\boldsymbol{\mathcal{S}}_\ell \in \mathbb{C}^{1\times T\times K}$ and the ℓ-th column of $\tilde{\mathbf{A}}_1$, the signal tensor $\boldsymbol{\mathcal{Y}}$ can be written as $\boldsymbol{\mathcal{Y}} = \sum_{\ell=1}\boldsymbol{\mathcal{Y}}_\ell$, where $\boldsymbol{\mathcal{Y}}_\ell$ denotes the ℓ-th sub-tensor of $\boldsymbol{\mathcal{Y}}$, which is calculated as $\boldsymbol{\mathcal{Y}}_\ell = \boldsymbol{\mathcal{S}}_\ell \times_1 \tilde{\mathbf{a}}_{1,\ell} \in \mathbb{C}^{N_R\times T\times K}$. Note that $\boldsymbol{\mathcal{Y}}_\ell$ contains information only from the ℓ-th path. Therefore, the parameters of the ℓ-th path, i.e., θ_ℓ, ϕ_ℓ, and τ_ℓ can be simply recovered using, e.g., a simple correlation scheme from the 3-modal unfolding of the ℓ-th sub-tensor $\boldsymbol{\mathcal{Y}}_\ell$. Due to the structure of each unfolding of sub-tensor $\boldsymbol{\mathcal{Y}}_\ell$ (i.e., $[\mathbf{Y}_\ell]_{(1)} = \tilde{\mathbf{a}}_{1,\ell}(\tilde{\mathbf{a}}_{3,\ell}\diamond\tilde{\mathbf{a}}_{2,\ell})^{\mathrm{T}}$, $[\mathbf{Y}_\ell]_{(2)} = \tilde{\mathbf{a}}_{2,\ell}(\tilde{\mathbf{a}}_{3,\ell}\diamond\tilde{\mathbf{a}}_{1,\ell})^{\mathrm{T}}$, $[\mathbf{Y}_\ell]_{(3)} = \tilde{\mathbf{a}}_{3,\ell}(\tilde{\mathbf{a}}_{2,\ell}\diamond\tilde{\mathbf{a}}_{1,\ell})^{\mathrm{T}}$), any column of $[\mathbf{Y}_\ell]_{(r)}$ can be used to recover the r-th mode's associated parameter. To achieve this end, we first need to estimate $\tilde{\mathbf{A}}_1$, i.e., $\hat{\tilde{\mathbf{A}}}_1$.

A. SVD Based Approach

One choice is to use the singular value decomposition (SVD) of $[\mathbf{Y}]_{(1)} \in \mathbb{C}^{N_R\times TK}$ [12]. Let the SVD of $[\mathbf{Y}]_{(1)}$ be given as $[\mathbf{Y}]_{(1)} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$. Here, $\mathbf{U}$ and $\mathbf{V}$ contain the left and the singular column vectors, respectively, and $\boldsymbol{\Sigma}$ is a diagonal matrix containing the singular values, which are assumed to be arranged in a decreasing order. Let, $\mathbf{U}_s \in \mathbb{C}^{N_R\times L}$ denote the matrix containing the first L left singular vectors of $\mathbf{U}$. Then, using the observation that $\mathrm{span}(\tilde{\mathbf{A}}_1) = \mathrm{span}(\mathbf{U}_s)$, there exists a non-singular matrix $\mathbf{T} \in \mathbb{C}^{L\times L}$ that satisfy $\tilde{\mathbf{A}}_1 = \mathbf{U}_s\mathbf{T}$. Here, $\mathbf{T}$ can be calculated from the eigenvectors of $\mathbf{U}_e = \underline{\mathbf{U}}_s^{+}\overline{\mathbf{U}}_s \in \mathbb{C}^{L\times L}$, where $\underline{\mathbf{U}}_s$ (resp. $\overline{\mathbf{U}}_s$) is a sub-matrix of $\mathbf{U}_s$ after removing its first (resp. last) row [12]. However, our preliminary results showed that using such approach provides a bad estimates (as it will also be shown in the numerical results section). To overcome this issue, we propose to estimate $\tilde{\mathbf{A}}_1$ using a CS approach as follows.

B. CS Based Approach

First, we note that unfolding of tensor $\boldsymbol{\mathcal{Y}}$ along its first dimension can be expressed as

$$[\mathbf{Y}]_{(1)} = \tilde{\mathbf{A}}_1\mathbf{Q}^{\mathrm{T}} \in \mathbb{C}^{N_R\times TK}, \tag{17}$$

where $\mathbf{Q} = (\tilde{\mathbf{A}}_3 \diamond \tilde{\mathbf{A}}_2) \in \mathbb{C}^{TK\times L}$. Recall that $\tilde{\mathbf{A}}_1 = \mathbf{W}^{\mathrm{T}}\mathbf{A}_1$, where $\mathbf{A}_1 = [\mathbf{a}(\theta_1)\dots\mathbf{a}(\theta_L)] \in \mathbb{C}^{M_R\times L}$ and assume that the AoAs $\{\theta_\ell\}_{\ell=1}^L$ lie perfectly on a finite grid $\bar{\boldsymbol{\theta}} = [\bar{\theta}_1,\dots,\bar{\theta}_{G_R}]$ such that $\theta_\ell \in \bar{\boldsymbol{\theta}}, \forall\ell$, where $G_R \gg L$ denotes the number of grid points. Then, we can write $\tilde{\mathbf{A}}_1 = \mathbf{W}^{\mathrm{T}}\mathbf{C}\boldsymbol{\Pi}$, where $\mathbf{C} = [\mathbf{a}(\bar{\theta}_1)\dots\mathbf{a}(\bar{\theta}_{G_R})] \in \mathbb{C}^{M_R\times G_R}$ denotes the AoAs codebook and $\boldsymbol{\Pi} \in \mathbb{R}^{G_R\times L}$ is a sparse matrix containing only one nonzero entry in each column. Note that $\mathbf{X} = \boldsymbol{\Pi}\mathbf{Q}^{\mathrm{T}} \in \mathbb{R}^{G_R\times TK}$ is a row-sparse matrix, where its ℓ-th nonzero row is given by $(\tilde{\mathbf{a}}_{3,\ell} \diamond \tilde{\mathbf{a}}_{2,\ell})^{\mathrm{T}} \in \mathbb{C}^{1\times TK}$. More importantly, the position of the ℓ-th nonzero row indicates the ℓ-th AoA.

Therefore, to get an estimate of $\{\hat{\theta}_\ell\}$, a CS approach can be used as

$$\{\hat{\theta}_\ell\}_{\ell=1}^L = \arg\min_{\mathbf{X}} \|[\mathbf{Y}]_{(1)} - \mathbf{W}^{\mathrm{T}}\mathbf{C}\mathbf{X}\|_F^2. \tag{18}$$

The formulation in (18) fulfills a sparse recovery problem [9] and can be solved using, e.g., the orthogonal matching pursuit (OMP) technique [3]. Using $\{\hat{\theta}_\ell\}_{\ell=1}^L$, an estimate of $\tilde{\mathbf{A}}_1$ can be obtained as $\hat{\tilde{\mathbf{A}}}_1 = \mathbf{W}^{\mathrm{T}}[\mathbf{a}(\hat{\theta}_1)\dots\mathbf{a}(\hat{\theta}_L)] \in \mathbb{C}^{M_R\times L}$, which can be then used to obtain an estimate of tensor $\boldsymbol{\mathcal{S}}$ as

$$\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{Y}} \times_1 \hat{\tilde{\mathbf{A}}}_1^{+}. \tag{19}$$

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TABLE I
COMPUTATIONAL ANALYSIS OF ALGORITHMS 1 AND 2.

Algorithm	Computational Complexity
Alg. 1	$I_{\max}(6N_rTKL + 4L^2(N_r + T + K) + 2L^3)$
Alg. 2-CS	$2LG_RN_rTK$
Alg. 2-SVD	$7N_r(TK)^2$

Using $\hat{\boldsymbol{\mathcal{S}}}$ and $\hat{\tilde{\mathbf{A}}}_1$, we can obtain an estimate of sub-tensor $\boldsymbol{\mathcal{Y}}_\ell$ as

$$\hat{\boldsymbol{\mathcal{Y}}}_\ell = \hat{\boldsymbol{\mathcal{S}}}_\ell \times_1 \hat{\tilde{\mathbf{a}}}_\ell. \tag{20}$$

To sum up, the proposed algorithm to estimate the channel parameters $\{\theta_\ell, \phi_\ell, \tau_\ell, \alpha_\ell\}_{\ell=1}^L$ is summarized in Algorithm 2.

Algorithm 2 Proposed Parameter Estimation Method.

1: Inputs: Tensor $\boldsymbol{\mathcal{Y}}$ and number of paths L
2: Estimate $\hat{\tilde{\mathbf{A}}}_1$ using SVD or CS approach
3: Estimate $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{Y}} \times_1 \hat{\tilde{\mathbf{A}}}_1^+$
4: **for** $\ell = 1$ to L **do**
5: Obtain the ℓ-th sub-tensor $\hat{\boldsymbol{\mathcal{S}}}_\ell$ from $\hat{\boldsymbol{\mathcal{S}}}$ according to (16)
6: Estimate the ℓ-th sub-tensor $\hat{\boldsymbol{\mathcal{Y}}}_\ell = \hat{\boldsymbol{\mathcal{S}}}_\ell \times_1 \hat{\tilde{\mathbf{a}}}_\ell$
7: **for** $r = 1$ to 3 **do**
8: Unfold $\hat{\boldsymbol{\mathcal{Y}}}_\ell$ along its r-th dimension to get $[\hat{\mathbf{Y}}_\ell]_{(r)}$
9: Estimate the r-th mode parameter
10: **end for**
11: **end for**
12: Compute $\hat{\mathbf{A}}_1$, $\hat{\mathbf{A}}_2$, $\hat{\mathbf{A}}_3$ and $\mathbf{R} = \left(\mathbf{F}^{\mathrm{T}}\hat{\mathbf{A}}_2 \diamond \mathbf{W}^{\mathrm{T}}\hat{\mathbf{A}}_1\right)$
13: Compute $\hat{\mathbf{G}} = \hat{\mathbf{A}}_3^+\left([\mathbf{Y}]_{(3)}\mathbf{R}^*\left((\mathbf{R}^{\mathrm{H}}\mathbf{R})^{-1}\right)^{\mathrm{T}}\right)$
14: Outputs: $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell, \hat{\alpha}_\ell\}$

Note that in step 2 of Algorithm 2, the estimated parameters $\{\hat{\theta}_\ell\}_{\ell=1}^L$ when using the CS approach (18) requires a finite grid. However, in step 9, the estimated parameters $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell\}_{\ell=1}^L$ can be gradually refined [12] to overcome the grid quantization errors.

C. Complexity Analysis

Table I shows the computational complexity analysis of the major steps of the Algorithms 1 and 2. Here, we note that the major computational complexity of Algorithm 1 comes from the iterative steps 4-6, while for Algorithm 2 comes from estimating $\tilde{\mathbf{A}}_1$ using (18), where we assume an OMP technique is used with a codebook of G_R atoms, or using SVD of $[\mathbf{Y}]_{(1)}$. The remaining steps to estimate $\{\hat{\theta}_\ell, \hat{\phi}_\ell, \hat{\tau}_\ell, \hat{\alpha}_\ell\}_{\ell=1}^L$ are common for both algorithms, thus are not considered. We assume that the computational complexity of the matrix product between $[n \times m]$ and $[m \times r]$ matrices equals to $2nmr$, the inversion of $[n \times n]$ matrix equals to $\frac{2}{3}n^3$, and the SVD of $[n \times m]$ matrix equals to $7nm^2$. In Table I, $I_{\max}$ denotes the maximum number of iterations required by Algorithm 1.

V. NUMERICAL RESULTS

In this section, we show some simulation results to evaluate the performance of the proposed algorithm. Table II summarizes the system parameters. The analog precoder matrix $\mathbf{F}$ and the analog decoder matrix $\mathbf{W}$ are randomly generated with their entries uniformly chosen from a unit circle. We show the simulation results in terms of the normalized mean-square-error (NMSE) that is defined as

TABLE II
SYSTEM PARAMETERS.

Parameter	Value
Channel parameters $\{\alpha_\ell, \phi_\ell, \tau_\ell\}_{\ell=1}^L$	on gird $[100 \times 100 \times 100]$
# transmit antennas M_T	64
# receive antennas M_R	32
# subcarriers K	6
# OFDM frames T	10
Path gains α_ℓ	$\mathcal{CN}(0,1), \forall l$
Additive noise $\mathbf{z}_{k,[t]}$	$\mathcal{CN}(0,\sigma^2), \forall k, t$
Signal-to-noise ratio (SNR)	$\frac{1}{\sigma^2}$
Sampling rate f_s	1

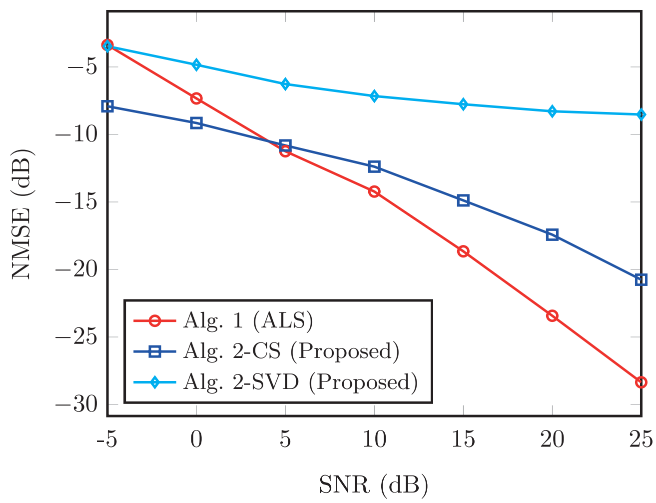

Fig. 2. NMSE vs. SNR.

$$\mathrm{NMSE} = \sum_{k=1}^{K} \frac{\|\mathbf{H}_k - \hat{\mathbf{H}}_k\|_F^2}{\|\mathbf{H}_k\|_F^2}, \tag{21}$$

where $\mathbf{H}_k$ denotes the k-th subcarrier true matrix and $\hat{\mathbf{H}}_k$ being its estimate.

In Fig. 2, we show NMSE versus signal-to-noise ratio (SNR) comparing between Algorithms 1 and 2, while assuming $L = 4$ and $N_R = 8$. In Fig. 3, we show NMSE versus number of RF chains N_R, while assuming $L = 4$ and SNR $= 10$dB. In Fig. 4, we show NMSE versus the number of channel paths L, while assuming $N_R = 8$ and SNR $= 10$dB.

From Fig. 2, we can observe that Alg. 2-SVD, i.e., when the SVD approach is used to estimate the factor matrix $\tilde{\mathbf{A}}_1$, Alg. 2 has the worst performance compared to Alg. 1, especially in the high SNR regime. Nonetheless, when the CS approach (18) is used to estimate $\tilde{\mathbf{A}}_1$, i.e., Alg. 2-CS, the performance of Alg. 2 significantly improves and

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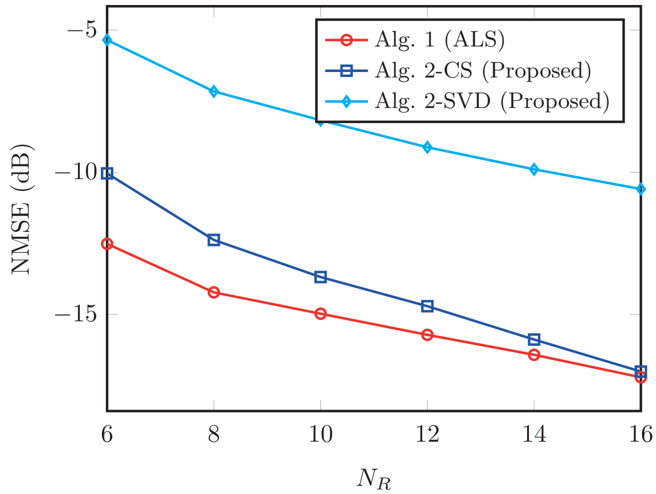

Fig. 3. NMSE vs. number of RF chains N_R [SNR = 10 dB].

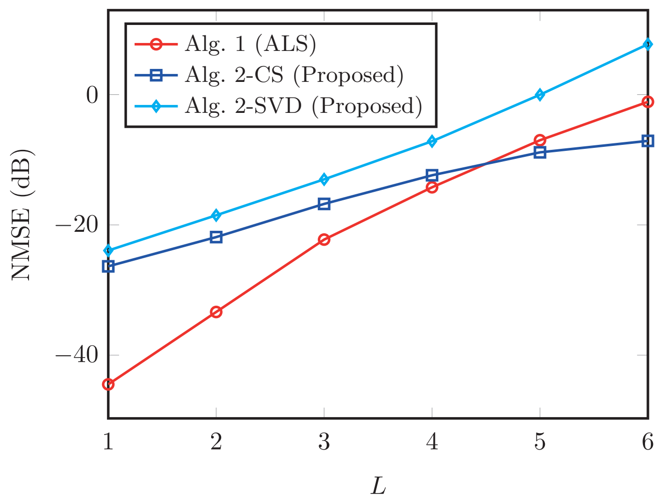

Fig. 4. NMSE vs. number of channel paths L [SNR = 10 dB].

outperforms Alg. 1 in the low SNR regime, which is a more critical region especially in the mmWave communication.

Moreover, we can see from Fig. 3 that Alg. 2-CS, unlike Alg. 2-SVD, approaches the Alg. 1 performance when we increase the number of receiver RF chains N_R to 16. Meanwhile, Alg. 2-CS is shown to outperform the Alg. 1 performance when we increase the number of channel paths larger than 4, as we see from Fig. 4.

VI. Conclusions

We have proposed a low-complexity CSI estimation method for wideband mmWave HAD-MIMO channels with frequency selectivity. By exploiting the channel sparsity and by adopting a simple setup at the transmitter, the received signal is organized into a third-order tensor that admits a Canonical Polyadic decomposition. Unlike the classical ALS solution to estimate the decomposed factor matrices, the proposed method utilizes the compressed sensing technique and tensor algebra to recover the channel parameters of each path separately in a non-iterative way and without need to estimate the factor matrices. Simulation results are provided evaluating the effectiveness of the proposed algorithm. It is shown that the proposed algorithm outperforms the ALS-based algorithm in the more critical scenarios, i.e., in the low SNR regime and with the large number of channel paths.

VII. Acknowledgment

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