Research on an Iterative Algorithm of LS Channel Estimation in MIMO OFDM Systems

Yantao Qiao, Songyu Yu, Pengcheng Su, and Lijun Zhang

***Abstract*—An iterative Least Square (LS) channel estimation algorithm for MIMO OFDM systems was proposed in this paper. Compared to common LS channel estimation, this algorithm can greatly improve estimation accuracy, and the low-pass filtering in time domain reduces AWGN and ICI significantly. MIMO OFDM system with this algorithm also works well in mobile situations. Simulation results have shown good MSE performance for this algorithm.**

***Index Terms*—Channel estimation, iterative algorithm, MIMO, OFDM.**

I. Introduction

MULTI-INPUT Multi-output (MIMO) channel adopts multi-antenna array on both ends of wireless link. Diverse techniques have been applied to MIMO channel in both space and time domains, which can effectively increase system capacity and improve the reliability of wireless link [1]. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) technology receives increasingly wider applications in wireless multimedia communications [2], due to its high data transmission rate, high bandwidth efficiency and excellent performance in resisting multi-path fading. By introducing OFDM to MIMO channel and utilizing space and time diversities, both the capacity and the performance can be enhanced simultaneously [3]. For wideband wireless communication, it is necessary to dynamically estimate the channel before demodulating the MIMO OFDM signals, since radio channel is frequency selective and time-dependent.

For OFDM channel estimation based on pilot, it can be classified as preamble method and PSAM method (Pilot Symbol Assisted Modulation or comb-type pilot method) in accordance with the difference of insertion position of pilots [4], [5]. Based on the criterion of realization, it can be classified as Minimum Mean Square Error (MMSE), Least Square (LS) and Maximum Likelihood Estimator (MLE) [6] and so on. From the filters adopted and the structure, it can be categorized as two-dimensional filtering [7], [8], two one-dimensional concatenation filtering and so on.

With the recent development in diversity technique, especially the space-time coding, the combination of estimation technique with diversity technique and its application in OFDM has become the new research front [9]–[13]. With the introduction of transmission diversity to MIMO channel, the old channel estimation methods can no longer be used, since the received signal is the combination of all transmission signals. In the old channel estimation method, the other transmitted signals are treated as interference when the channel from a transmitter antenna to receiver antennas is to be estimated, and poor estimation will be obtained under the condition of multiple transmitter antennas. For this reason, it is necessary to introduce the new channel estimation schemes. Channel estimation methods based on the diversity were investigated in literatures [9]–[13] and [16].

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The authors are with the Institute of Image Communication & Information Processing, Shanghai Jiaotong University, Shanghai, P. R. China 200030 (e-mail: qiaoytmail@sjtu.edu.cn; syyu@cdtv.org.cn; supc@sjtu.edu.cn, johnzlj@sjtu.edu.cn).

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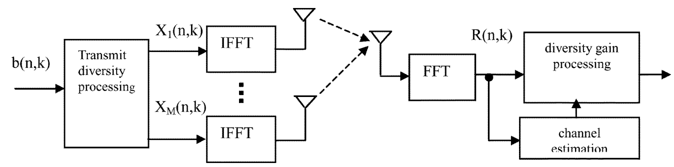

Fig. 1. MIMO OFDM system structure.

However, in multi-antenna OFDM system, the received signals on each sub-carrier are the overlap of independent fading signals from multiple antennas, and the channel estimation algorithm for single antenna cannot identify multiple channel fading coefficients. In this paper, we proposed a training sequence design scheme with high bandwidth efficiency, and an iterative Least Square (LS) channel estimation algorithm for MIMO OFDM systems. Results have shown good performance for this algorithm.

II. System Model

A MIMO OFDM system with M transmitter antennas and one receiver antenna is shown in Fig. 1. At time step n, a data block $\{b(n,k), k=0,1,\ldots,N_s-1\}$ (N_s is the number of the sub-carrier) can be converted into M different symbol blocks $\{X_i(n,k), k=0,1,\ldots,N_s-1, i=1,\ldots,M\}$. Each symbol block is transmitted over different antenna on N_s sub-carriers. In other words, communication link established by OFDM exist between every pair of transmitter and receiver antennas. The received information on each sub-carrier is the overlap of M distorted transmitted signal after demodulation (by FFT transform). The channel is actually a Rayleigh multi-path propagation channel, and assumed to be invariable during an OFDM symbol period T, but variable from one OFDM symbol to another.

During the nth OFDM symbol period, the channel impulse response coefficient from the ith transmitter antenna can be expressed as:

$$h_i(n,\tau)=\sum_{l=0}^{L-1} h_i(l)\exp\left(j\frac{2\pi}{N_s}f_{Dl,i}Tn\right)\delta(\tau-\tau_{i,l}) \qquad i=1,\ldots,M. \quad (1)$$

The multi-path delay spread is normalized to L time units by the sampling time. $h_i(l)$, the complex impulse response of the Lth path from ith transmitter antenna to receiver antenna, is a complex Gauss process with zero mean and covariance σ_l^2. $\sum_{l=0}^{L-1}\sigma_l^2 = 1$ will be obtained if the channel response is normalized. τ is the delay spread index, and $\tau_{i,l}$ is the lth-path delay time normalized by sampling time. $f_{Dl,i}$ is the lth-path Doppler frequency shift which arouses ICI of the received signals.

After signals from M transmitter antennas pass through a channel with addictive Gaussian noise, the received time-domain OFDM signal can be expressed as

$$y(n) = \sum_{i=1}^{M}(x_i(n) \otimes h_i(n,\tau)) + w(n) \qquad n = 0,1,\ldots,N_s-1 \quad (2)$$

where $w(n)$ represents additive Gaussian noise. At the receiver side, the signal on the kth sub-carrier after FFT transform can be expressed as:

$$Y(n,k) = \sum_{i=1}^{M}(X_i(k)H_i(k) + I_i(k)) + W(k) \qquad k = 0,1,\ldots,N_s-1 \quad (3)$$

where $H_i(k)$ is the channel frequency response coefficient from ith transmitter antenna to receiver antenna at the kth sub-carrier, during the nth OFDM symbol interval.

$$\begin{aligned} &H_i(k) \\ &= \sum_{l=0}^{L-1} h_i(l)\exp(j\pi f_{Dl,i}T)\frac{\sin(\pi f_{Dl,i}T)}{\pi f_{Dl,i}T}\exp\left(-j2\pi\frac{k\tau_{i,l}}{N_s}\right) \\ &\quad k = 0,1,\ldots,N_s-1. \end{aligned} \quad (4)$$

The inter-carrier interference is

$$\begin{aligned} I_i(k) = &\frac{1}{N_s}\sum_{l=0}^{L-1}\sum_{\substack{K=0 \\ K\neq k}}^{N_s-1} h_i(l)X_i(K) \\ &\bullet \frac{1-\exp(j2\pi(f_{Dl,i}T-k+K))}{1-\exp(j\frac{2\pi}{N_s}(f_{Dl,i}T-k+K))}\exp\left(-j\frac{2\pi\tau_{i,l}}{N_s}k\right) \end{aligned} \quad (5)$$

and

$$\begin{aligned} Y(k) &= DFT\{y(n)\} = \sum_{n=0}^{Ns-1} y(n)\exp\left(-j2\pi\frac{kn}{N_s}\right) \\ X_i(k) &= DFT\{x_i\{n\}\} = \sum_{n=0}^{Ns-1} x_i(n)\exp\left(-j2\pi\frac{kn}{N_s}\right) \\ W(k) &= DFT\{w(n)\} = \sum_{n=0}^{Ns-1} w(n)\exp\left(-j2\pi\frac{kn}{N_s}\right) \\ &\quad n = 0,1,\ldots,N_s-1. \end{aligned} \quad (6)$$

From the above equations, we can see that $H_i(k)$ is a sinusoidal function of k, the rate of change is determined by $\tau_{i,l}/N_s$. A smaller $\tau_{i,l}/N_s$ will result in a slower variation of $H_i(k)$. In general, $\tau_{i,l} \ll N_s$, and $H_i(k)$ changes rather slowly with respect to k. $I_i(k)$ is the ICI component in the kth sub-carrier, depending on the signal values $X_i(K)$, which is modulated on all other sub-carriers. When data $X_i(K)$ are zero-mean random variables, $I_i(k)$ appears to be a fast-changing function of k.

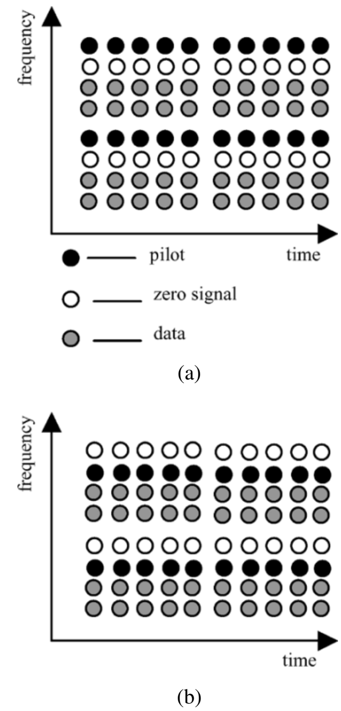

Fig. 2. Pilot arrangment. (a) Antenna 1. (b) Antenna 2.

III. The Design of Training Sequence

Channel estimation based on comb-typed pilot arrangement was used in this design. N_p pilot tone sub-carriers are divided into M groups: $K_i = \{mM(N_s/N_p) + i - 1, m = 0,1,\ldots,N_p/M-1\}$, $i = 1,2,\ldots,M$. For the ith antenna, send out pilot tone on $k_i \in K_i$ subcarrier, send zero signals on $k_i \in K_j$ $\{j = 1,2,\ldots,M (j \neq i)\}$ subcarrier and transmit data signal on other subcarriers, which is shown in Fig. 2.

So, on sub-carrier k of the ith antenna, only the sub-carrier itself can send out signals, and (3) can be simplified as following accordingly:

$$Y(n,k) = X_i(k)H_i(k) + I_i(k) + W(k). \quad (7)$$

IV. An Iterative Algorithm of LS Channel Estimation

For the ith ($i = 1,\ldots,M$) transmitter antenna, since the value of pilot signals is known as c, a rough estimation of channel frequency response at the pilot sub-carriers $k_j \in K_i$ $\{j = 0,\ldots,N_p/M-1\}$ can be obtained as

$$\hat{H}_i(k_j) = \frac{Y(k_j)}{c} = H_i(k_j) + \frac{(I_i(k_j) + W(k_j))}{c} \qquad j = 0,1,\ldots,\frac{N_p}{M}-1. \quad (8)$$

From the above analysis, we can see frequency response $H_i(k_j)$ changes quite slow compared to pilot tone sub-carrier k_j, while the noise component $(I_i(k_j) + W(k_j))/c$ changes very fast. Therefore, they are separable. Since the channel parameters are unknown and time dependent, it is difficult to remove the noise component in frequency domain. The ICI and AWGN in time domain are random processes with zero mean [14]. As expected, the signal component is located at the lower "frequency" region, while the noise component is spread over the whole "transform" region, and time-domain low-pass filtering will be applied to effectively suppress the influence of noise [15]. Channel impulse response estimation of size N_p/M in time domain can be obtained after an N_p/M point IFFT transform to the $\hat{H}_i(k_j)$, using the initial Least Square (LS) method. Since the maximum channel delay spread is L, we can assume that channel impulse response estimation after the Lth sample point is the noise component that is removable, so the filtering in time-domain can be implemented. Accurate channel estimation on all sub-carriers can be obtained by the following iterative algorithm:

1) The channel frequency response $\hat{H}_i(k_j)$ on the pilot sub-carriers can be obtained using the Least Square (LS) method described above;
2) An IFFT transform of size N_p/M in time domain is performed on $\{\hat{H}_i(k_j), j = 0, 1, \ldots, N_p/M - 1\}$ and we have

$$\hat{h}_i(n) = \frac{1}{\frac{N_p}{M}} \sum_{j=0}^{N_p/M-1} \hat{H}_i(k_j) \exp\left(j2\pi \frac{jn}{\frac{N_p}{M}}\right) \quad n = 0, 1, \ldots, \frac{N_p}{M} - 1. \quad (9)$$

The adopted method is from **(8)**. Dividing the received signals at the pilot sub-carriers by known transmitted pilot signals, a rough estimation of channel frequency response at the pilot sub-carriers can be achieved. we can also conclude that the nonzero values of $\hat{h}_i(n)$, $n = L, L+1, \ldots, N_p/M-1$ are removable estimation noise, i.e., $\hat{h}_i^1(n) = \hat{h}_i(n)$, $n = 0, 1, \ldots, L-1$ is the estimated value of channel impulse response, in which the superscript represents the iterative time step. The noise component is reduced to LM/N_p of its original value, and AWGN and ICI are reduced significantly in time-domain.

3) Iterative procedure: for the mth ($m \geq 1$) iteration, convert $\hat{h}_i^m(n)$ to frequency domain using FFT transform of size N_s, after padding with zeroes, i.e.,

$$\hat{H}_i^m(k) = \sum_{n=0}^{L} \hat{h}_i^m(n) \exp\left(-j2\pi \frac{kn}{N_s}\right) \quad k = 0, 1, \ldots, N_s - 1. \quad (10)$$

4) Replace the frequency response at the pilot tone $\hat{H}_i^m(k_j), j = 0, 1, \ldots N_p/M - 1$ with the result from step **(1)** (iteration is not necessary for the last iterative process).
5) Compute

$$delta = \max\left\{\left|\hat{H}_i^{m+1}(k) - \hat{H}_i^m(k)\right|\right\} \quad k = 0, 1, \ldots, N_s - 1. \quad (11)$$

If delta is below a predetermined threshold, the iteration can be terminated and the final decision is available; otherwise, $H_i^{m+1}(k)$ can be converted back to the time domain, i.e.,

$$\hat{h}_i^{m+1}(n) = \sum_{k=0}^{N_s-1} \hat{H}_i^{m+1}(k) \exp\left(-j2\pi \frac{kn}{N_s}\right) \quad n = 0, 1, \ldots, N_s - 1. \quad (12)$$

Now we are able to rerun the time-domain noise suppression again, and this time we have $\hat{h}_i^{m+1'}(n) = \hat{h}_i^{m+1}(n)$, $n = 0, 1, \ldots, L-1$, and the noise component is reduced. Repeat steps **(3)** $\sim$ **(5)** until final result can be obtained.

Following the similar analysis as before, the remaining ICI and AWGN are spread over the whole time domain, and the part of the channel impulse response estimation after the Lth sample point will be further removed. After each iteration, part of noise will be removed, and the performance of channel estimation can be further improved.

By iterative procedure described above, accurate channel estimation values can be obtained, and performance of the channel estimation can be evaluated by the mean squared error

$$MSE = \sum_{i=1}^{M} \underset{k}{E} \frac{\left\{\left|\hat{H}_i(k) - H_i(k)\right|^2\right\}}{M} \quad (13)$$

with frequency response at the pilot tone $\hat{H}_i^m(k_j), j = 0, 1, \ldots N_p/M - 1$ from step (1) (iteration is not necessary for the last iterative process).

For every transmitter antenna, the channel estimation value of all subcarriers of receiver antennas is subtracted by the actual value, the differences are averaged. The final MSE can be obtained by summarizing all these averages and dividing the summation by transmitter antennas numbers.

V. Simulation Results

A MIMO OFDM system with two transmitter antennas and one receiver antenna was used in our simulation. The system has QPSK modulation of 10 MHz bandwidth with carrier frequency of 1 GHz. The total number of sub-carriers is $\mathrm{N} = 1024$, and guard interval $N_g = 226$. A five-path Rayleigh fading channel was selected, the maximum delay spread of the channel (normalized by sampling time) is $L = 136$. As shown in Fig. 2, the number of pilot sub-carrier at two antennas is $N_p = 512$, which is divided into $M = 2$ groups. SNR is defined as ratio of signal energy E_s to AWGN covariance , i.e., $SNR = E_s/N_0$. It should be pointed out that N_0 only reflects AWGN level. The ICI influence can be distinguished by selecting different Doppler spreads, or vehicle speeds.

For comparison purpose, we also performed simulations under the same condition, using the algorithm proposed in literature [16]. For two vehicle speeds, with 10 iterations and without iteration at all, our algorithm has shown better performance in channel estimation than those in [16]. With iteration

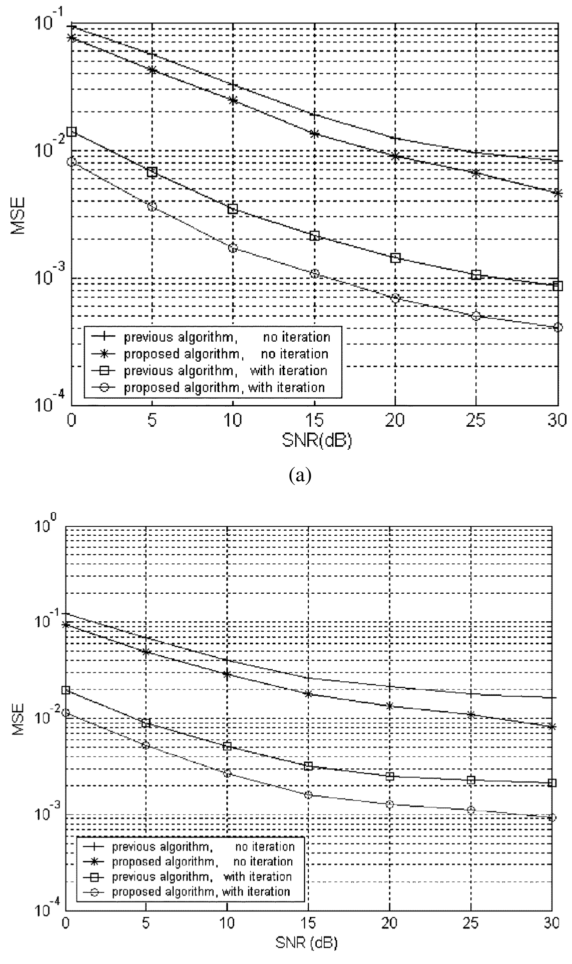

Fig. 3. MSE performance of channel estimation at two vehicle speeds. (a) MSE performance at vehicle speed = 5 km/h. (b) MSE performance at vehicle speed = 150 km/h.

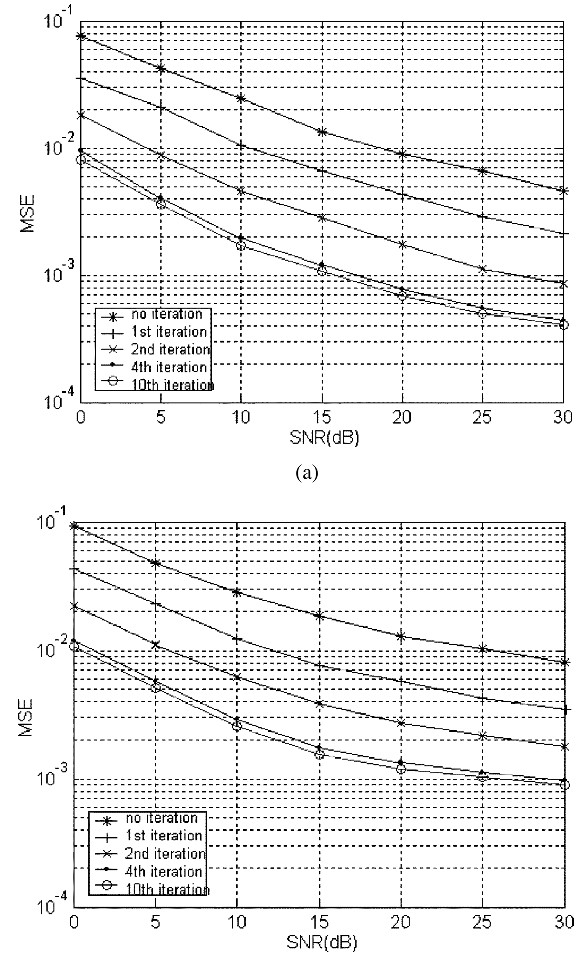

Fig. 4. MSE performance of different iteration numbers at two vehicle speeds. (a) MSE performance at vehicle speed = 5 km/h. (b) MSE performance at vehicle speed = 150 km/h.

technique adopted, the performance has been improved significantly, which indicates that our algorithm can resist the noise effect more efficiently, compared to the algorithm in [16].

In our simulations, we first chose a low vehicle speed of 5 km/h, and ICI influence is considered to be small. Simulations were carried out for different *SNR*. The performance of two schemes was compared: with iteration (iterative time is 10 in this study) and without iteration (corresponding to steps (1) $\sim$ (3) of the proposed algorithm). The MSE performance is shown in Fig. 3(a). As mentioned above, the ICI caused by vehicle speed is very small and can be neglected. From Fig. 3(a), it can be easily observed that the iterative algorithm significantly improves the MSE performance of channel estimation, compared to the algorithm without iteration.

Next, we studied the case with vehicle speed $\mathrm{V} = 150\ \mathrm{km/h}$, where system performance is strongly affected by ICI. In iterative algorithm, the low-pass filtering in time domain can effectively reduce ICI, thus noticeable improvement can be obtained after many iterations. Fig. 3(b) presents the MSE performance of channel estimation for $\mathrm{V} = 150\ \mathrm{km/h}$. Again, iterative algorithm has smaller mean squared estimation error of channel parameters. Furthermore, for higher SNR, MSE doesn't decrease as rapidly as those shown in Fig. 3(a). A larger gain in terms of MSE performance results from iteration. Compared to Fig. 3(a), the MSE of Fig. 3(b) is higher than those of Fig. 3(a) due to higher vehicle speed, with or without iterations. The rate of decreasing is slower than in Fig. 3(a), which indicates it has been affected by vehicle speed.

In addition, in order to investigate the effects of iterations on channel estimation performance, the MSE performance for two vehicle speeds is shown in Fig. 4(a) and (b). Simulation results show that the improvement for the first several iterations is obvious, while it makes little difference in MSE for 4 iterations and 10 iterations. It can be observed from Fig. 4 that after four iterations, little improvement has been achieved with increasing number of iterations, and it MSE for 4 iterations and 10 iterations. It can be observed converges to its final performance. For this reason, four iterations were carried out in this study.

VI. Conclusion

In this paper, an iterative LS channel estimation algorithm for MIMO-OFDM systems was proposed. The algorithm can estimate channel state information accurately. By means of low-pass filtering in the time domain to reduce AWGN and ICI significantly, this algorithm has improved the performance of

MIMO OFDM systems in mobile communications, which was verified by simulation results in terms of MSE performance.

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