Time–Frequency Joint Sparse Channel Estimation for MIMO-OFDM Systems

Wenbo Ding, Fang Yang, *Senior Member, IEEE*, Wei Dai, *Member, IEEE*, and Jian Song, *Senior Member, IEEE*

***Abstract*—This letter proposes a time-frequency joint sparse channel estimation for multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems under the framework of structured compressive sensing (CS). The proposed scheme first relies on a pseudorandom preamble, which is identical for all transmit antennas, to acquire the partial common support by utilizing the sparse common support property of the MIMO channels. Then, a very small amount of frequency-domain orthogonal pilots are used for the accurate channel recovery. Simulation results show that the proposed scheme demonstrates better performance and higher spectral efficiency than the conventional MIMO-OFDM schemes. Moreover, the obtained partial common support can be further utilized to reduce the complexity of the CS algorithm and improve the signal recovery probability under low signal-to-noise-ratio conditions.**

***Index Terms*—MIMO-OFDM, structured compressive sensing, sparse common support, time-frequency joint channel estimation, spectrum-efficient.**

I. Introduction

ORTHOGONAL FREQUENCY division multiplexing (OFDM) and multiple-input multiple-output (MIMO) have attracted considerable interests from both academia and industrial due to the outstanding capability to combat multipath fading and achieve high spectral efficiency [1]. Hence, MIMO-OFDM has been widely adopted in the latest communication standards, such as LTE, IEEE 802.11, etc., and it is also considered as a key technique for future wireless communications [2].

As one of the major challenges for practical MIMO-OFDM systems, accurate channel estimation is essential to guarantee the system performance [2]. The conventional channel estimation methods for MIMO-OFDM systems can be categorized into two types: the time-domain preamble based [3] and the frequency-domain orthogonal pilot aided [4] methods. However, the overhead of the required preamble or pilots will significantly increase as the number of transmit antennas becomes large, which decreases the spectral efficiency. Recently, some greedy compressive sensing (CS) methods [5], [6] have been introduced for channel estimation in MIMO systems, which could improve the spectral efficiency but requires the knowledge of the accurate channel sparsity *in prior* [7].

In this letter, we propose a spectrum-efficient OFDM scheme, namely time-frequency training OFDM (TFT-OFDM), for sparse MIMO systems based on a time-frequency joint channel estimation method under the framework of structured CS (SCS) [8]. First, by exploiting the sparse common support property of the MIMO wireless channels [9], the time-domain preamble is used to acquire the partial common support and sparsity of the channels, while the exact channel recovery will depend on a small number of frequency-domain pilots by using SCS. Second, the overhead of both preamble and pilots in the proposed scheme is far less than the conventional MIMO-OFDM systems [3], [4], and hence higher spectral efficiency can be achieved. Third, the partial common support obtained can be further utilized to reduce the complexity of the standard CS algorithms as well as improve the channel recovery probability under low signal-to-noise ratio (SNR) conditions.

Notation: The lowercase and uppercase boldface letters denote vectors and matrices, respectively. $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^\dagger$, $diag(\cdot)$, and $\|\cdot\|_p$ denote the transpose, Hermitian transpose, matrix inversion, matrix pseudo inversion, diagonal matrix, and l_p norm operations, respectively. Finally, Φ_Π represents the submatrix comprised of the Π columns of the matrix Φ.

II. System Model

A. MIMO Channel Model With Sparse Common Support

In an $N_t \times N_r$ MIMO system consisting of N_t transmit and N_r receive antennas, the L-length channel impulse response (CIR) associated with the p-th transmit antenna and a certain receive antenna[1] during the i-th symbol can be modeled as [4]

$$\mathbf{h}_i^{(p)} = \left[h_{i,1}^{(p)}, h_{i,2}^{(p)}, \cdots, h_{i,L}^{(p)}\right]^T. \tag{1}$$

Due to the physical properties of outdoor electromagnetic propagation, the CIRs in wireless communications usually contain a few significant paths, i.e., the CIRs are sparse [7], [10]. Moreover, when the scale of the antenna array is negligible compared to the signal wavelength in the commonly used collocated MIMO systems, channels associated with different transmit-receive antenna pairs share the similar path arrival times.[2] Therefore, the corresponding CIRs in MIMO systems

[1]As the processing for channel estimation is identical for every receive antenna, the receive antenna index will be omitted for simplification from now on.

[2]This assumption could be invalid in the distributed MIMO system where the distance of the transmit antennas is comparable to the signal wavelength.

Manuscript received September 23, 2014; revised October 31, 2014; accepted November 14, 2014. Date of publication November 20, 2014; date of current version January 7, 2015. This work was supported by the National Natural Science Foundation of China under Grants 61401248 and 61471219. The associate editor coordinating the review of this paper and approving it for publication was P. Salvo Rossi.

W. Ding is with the Department of Electronic Engineering and the Research Institute of Information Technology, Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: dwb11@mails.tsinghua.edu.cn).

F. Yang and J. Song are with the Department of Electronic Engineering and the Research Institute of Information Technology, Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing 100084, China, and also with Shenzhen City Key Laboratory of Digital TV System, Shenzhen 518057, China (e-mail: fangyang@tsinghua.edu.cn; jsong@tsinghua.edu.cn).

W. Dai is with the Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, U.K. (e-mail: wei.dai1@imperial.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LCOMM.2014.2372006

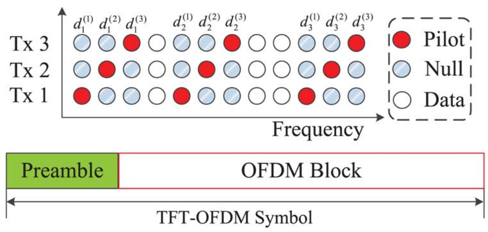

Fig. 1. The frame structure of the proposed MIMO-TFT-OFDM systems.

usually share a sparse common support in spite of the different path gains [9].

B. TFT-OFDM Signal Structure for MIMO Systems

As shown in Fig. 1, unlike conventional MIMO-OFDM schemes where the training information for channel estimation only exists in either time [3] or frequency domain [4], the proposed MIMO-TFT-OFDM has both time-domain preamble and frequency-domain pilots for every symbol. The i-th TFT-OFDM symbol for the p-th transmit antenna consists of an M-length preamble $\mathbf{c} = [c_1, c_2, \cdots, c_M]^T$ and an N-length OFDM block $\mathbf{x}_i^{(p)} = [x_{i,1}^{(p)}, x_{i,2}^{(p)}, \cdots, x_{i,N}^{(p)}]^T = \mathbf{F}^H \tilde{\mathbf{x}}_i^{(p)}$, where $\mathbf{F}$ is the $N \times N$ discrete Fourier transform (DFT) matrix. The preamble is an identical pseudo random sequence for all transmit antennas, which has good autocorrelation property and can be used for partial common support acquisition. $\tilde{\mathbf{x}}_i^{(p)}$ denotes the frequency-domain OFDM block which contains a small amount N_P of pilots on the location of $D^{(p)} = \{d_n^{(p)}\}_{n=1}^{N_P}$ and $N_P(N_t - 1)$ null pilots (zeros) on the pilot location reserved for the other transmit antennas. For the $(p+1)$-th transmit antenna, the pilot location $D^{(p+1)}$ follows the following pattern,

$$d_n^{(p+1)} = d_n^{(p)} + 1 = d_n^{(1)} + p, \quad 1 \le n \le N_P. \tag{2}$$

Instead of the equally spaced comb-type pilots in the conventional MIMO-OFDM systems [4], the pilot location $D^{(1)} = \{d_n^{(1)}\}_{n=1}^{N_P}$ is randomized to guarantee the restricted isometry property of the observation matrix in the CS [8].

III. Time-Frequency Joint Channel Estimation for Sparse MIMO Systems

In this section, by fully exploiting the time-frequency training feature of MIMO-TFT-OFDM scheme and the channel sparse common support property, we proposed the time-frequency joint sparse channel estimation based on the sparsity adaptive simultaneous orthogonal matching pursuit (SA-SOMP) algorithm. The proposed channel estimation is composed of three steps as follows.

A. Step 1: Partial Common Support Acquisition

Different from the conventional time-domain estimation for MIMO systems where the preamble length M has to be no less than N_t times of the channel length L for accurate channel estimation, i.e., $M \ge N_t \times L$ [3], the proposed scheme utilizes the preamble merely to acquire the partial channel support information. Hence, the preamble length here only needs to be no less than the channel length, i.e., $M \ge L$.

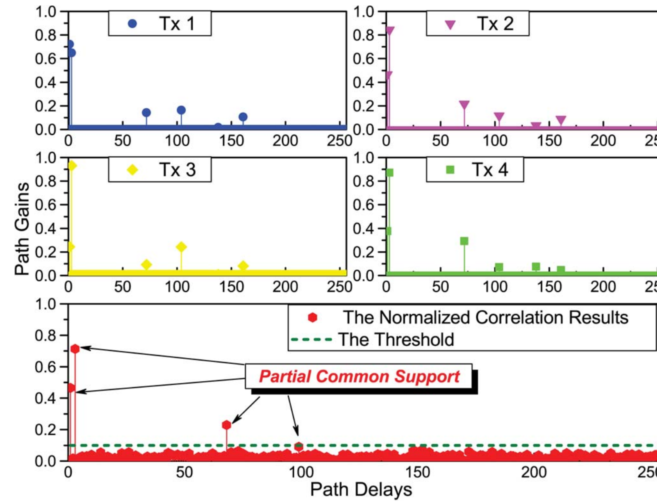

Fig. 2. Illustration of the partial common support information acquisition by the time-domain preamble in a 4×4 MIMO system.

At the receiver, we adopt the received preamble $\hat{\mathbf{c}}_i$ without interference removal to correlate with the local preamble $\mathbf{c}$ to obtain the partial sparse common support,

$$\bar{\mathbf{z}}_i = \frac{1}{MN_t} \mathbf{c} \otimes \hat{\mathbf{c}}_i = \frac{1}{N_t} \sum_{p=1}^{N_t} \mathbf{h}_i^{(p)} + \mathbf{v}_i, \tag{3}$$

where $\otimes$ is the circular correlation operation, $\mathbf{v}_i$ denotes the interference including the channel noise and the inter-block interference (IBI), and the correlation result $\bar{\mathbf{z}}_i$ is the rough estimate of the superposition of all the CIRs associated with the N_t transmit antennas.

Due to the channel sparse common support property and the good correlation property of the preamble, the partial common support information $T_0 = \{l : \|\bar{z}_{i,l}\|_2 \ge a\}_{l=0}^{L-1}$ can be obtained by appropriately selecting the threshold $a = 3\|\bar{\mathbf{z}}_i\|_2/L = 3(\sum_{l=1}^{L} |\bar{z}_{i,l}|^2)^{1/2}/L$ [11], as shown in Fig. 2, where the ITU Vehicular B (ITU-VB) channel model [10] $(L = 161)$ with a SNR of 10 dB is considered. Meanwhile, the channel sparsity level which is essential to the CS algorithm can also be estimated by adding a compensation factor, i.e., $K = \|T_0\|_0 + b$. Such information is used to reduce the complexity of the standard CS algorithm for practical applications, which will be detailed in ***Step 3***.

B. Step 2: Cyclicity Reconstruction of OFDM Block

In the proposed MIMO-TFT-OFDM scheme, the standard cyclic prefix (CP) [4] is replaced by the known preamble to improve the spectral efficiency while sacrificing the cyclicity property of the received OFDM block under multipath fading channels. Hence, cyclicity reconstruction is required to obtain the pilot for accurate channel recovery.

For single input and single output (SISO) systems, the cyclicity reconstruction of the OFDM block is achieved by the mature overlapping and adding operation (OLA) [11], [12]. Then the cyclicity reconstruction of the OFDM block in MIMO systems can be implemented by extending the OLA operations above to every transmit antenna. After that, the orthogonal pilots can be extracted for the accurate channel estimation in the following ***Step 3***.

C. Step 3: Accurate Channel Estimation Based on SA-SOMP

After cyclicity reconstruction, the received OFDM block in the frequency domain can be written as,

$$\tilde{\mathbf{y}}_i = \sum_{p=1}^{N_t} diag\left(\tilde{\mathbf{x}}_i^{(p)}\right)\mathbf{F}_L\mathbf{h}_i^{(p)} + \mathbf{w}_i, \tag{4}$$

where $\mathbf{F}_L$ is the $N \times L$ partial DFT matrix containing the first L columns of $\mathbf{F}$. We only focus on the received frequency-domain pilots located at $D^{(p)}$, and (4) can be simplified as

$$\mathbf{u}_i^{(p)} = \mathbf{F}^{(p)}\mathbf{h}_i^{(p)} + \hat{\mathbf{w}}_i^{(p)}, \quad 1 \le p \le N_t, \tag{5}$$

where $\mathbf{u}_i^{(p)} = [\tilde{y}_{i,d_1^{(p)}}/\tilde{x}^{(p)}_{i,d_1^{(p)}}, \tilde{y}_{i,d_2^{(p)}}/\tilde{x}^{(p)}_{i,d_2^{(p)}}, \cdots, \tilde{y}_{i,d_{N_P}^{(p)}}/\tilde{x}^{(p)}_{i,d_{N_P}^{(p)}}]^T$, and $\mathbf{F}^{(p)}$ is the $N_P \times L$ partial DFT matrix with the $(n+1,k+1)$-th entry being $\exp(-j2\pi d_n^{(p)}k/N)/\sqrt{N}$.

From the designed pilot pattern according to (2), the recursive representation of $\mathbf{F}^{(p)}$ can be written as

$$\mathbf{F}^{(p+1)} = \mathbf{F}^{(p)}\Theta = \mathbf{F}^{(1)}\Theta^p, \tag{6}$$

where Θ is the diagonal matrix given by

$$\Theta = diag\left(\left[1, e^{-j\frac{2\pi}{N}}, \cdots, e^{-j\frac{2\pi}{N}(L-1)}\right]^T\right). \tag{7}$$

Stacking the noisy measurements $\mathbf{u}_i^{(p)}$ into one matrix $\mathbf{U}_i = [\mathbf{u}_i^{(1)}, \mathbf{u}_i^{(2)}, \cdots, \mathbf{u}_i^{(N_t)}]$, a matrix formulation of (5) for all antennas can be derived as

$$\mathbf{U}_i = \mathbf{F}^{(1)}\mathbf{H}_i + \hat{\mathbf{W}}_i, \tag{8}$$

where $\mathbf{H}_i = [\mathbf{h}_i^{(1)}, \Theta\mathbf{h}_i^{(2)}, \cdots, \Theta^{N_t-1}\mathbf{h}_i^{(N_t)}]$ contains all the CIR information which needs to be estimated and $\hat{\mathbf{W}}_i$ denotes the noise. Fortunately, the $\mathbf{h}_i^{(p)}$ usually share the same sparse support and moreover, the matrix $\Theta^{p-1}, p \in \mathbb{N}$ will not change the sparse support or the absolute value of $\mathbf{h}_i^{(p)}$. For multiple sparse signals $\mathbf{H}_i$ formulated in (8), the SOMP algorithm [8] developed from the standard OMP algorithm [5] could improve the signal recovery performance by taking the correlation of the sparse signals into account. However, it requires the known signal sparsity level K *a priori* for the recovery performance guarantee, which is impractical in real systems. Moreover, its computational complexity is high due to the high-dimensional matrix inversion in every iteration.

Based on the basic principle of SOMP [8], we propose the SA-SOMP algorithm to estimate all of the sparse signals in the sparse matrix $\mathbf{H}_i$, which is described in **Algorithm 1**. The proposed algorithm exploits the obtained partial common support information in ***Step 1*** for more accurate initial configuration and hence reduces the iteration number in the conventional SOMP. The SA-SOMP algorithm has the complexity of $O((K-\|T_0\|_0)N_tN_P(L+K^2))$, which is less than that of the standard SOMP algorithm, $O(KN_tN_P(L+K^2))$. Moreover, the channel sparsity level can be well estimated by the partial common support information, making the algorithm adaptive to variable channel sparsity under different conditions.

Algorithm 1 Sparsity adaptive SOMP (SA-SOMP).

Inputs:
1) Initial channel common support T_0
2) Channel sparsity level K;
3) Noisy measurements $\mathbf{M} \triangleq \mathbf{U}_i$;
4) Observation matrix $\Phi \triangleq \mathbf{F}^{(1)}$.

Output: The K-sparse estimate $\bar{\mathbf{H}} \triangleq \bar{\mathbf{H}}_i$.

Initial Configuration:
1: $\Pi \leftarrow T_0$;
2: $\bar{\mathbf{H}}^{(0)} \leftarrow \mathbf{0}$; $\bar{\mathbf{H}}_{\Pi}^{(0)} \leftarrow \Phi_{\Pi}^{\dagger}\mathbf{M}$;
3: $\mathbf{R} \leftarrow \mathbf{M} - \Phi\bar{\mathbf{H}}^{(0)}$;
Iteration:
4: **for** $k = 1 : K - \|T_0\|_0$ **do**
5: $\mathbf{P} \leftarrow \Phi^H\mathbf{R}$;
6: $\Pi \leftarrow \Pi \cup \{\arg\max_i \sum_j \|p_{i,j}\|_1\}$;
7: $\bar{\mathbf{H}}^{(k)} \leftarrow \mathbf{0}$; $\bar{\mathbf{H}}^{(k)} \leftarrow \Phi_{\Pi}^{\dagger}\mathbf{M}$;
8: $\mathbf{R} \leftarrow \mathbf{M} - \Phi\bar{\mathbf{H}}^{(k)}$;
9: **end for**
10: $\bar{\mathbf{H}} \leftarrow \bar{\mathbf{H}}^{(k)}$.

After the estimate $\bar{\mathbf{H}}$ of $\mathbf{H}_i$ has been obtained, the CIR associated with the p-th transmit antenna can be restored as

$$\hat{\mathbf{h}}^{(p)} = \left(\Theta^{p-1}\right)^{-1}\bar{\mathbf{h}}_p, \tag{9}$$

where $\bar{\mathbf{h}}_p$ is the p-th column of $\bar{\mathbf{H}}$. Note that the inversion of a diagonal matrix $\left(\Theta^{p-1}\right)^{-1}$ only involves a low complexity.

IV. Simulation Results

This section investigates the performance of the proposed MIMO-TFT-OFDM scheme. The simulation parameters are summarized as follows: the system bandwidth is $f_S = 8$ MHz located at the central frequency of 835 MHz; the OFDM block length and the preamble length are $N = 4096$ and $M = 256$, respectively; the 6-tap ITU-VB channel model specified by 3GPP [9] is considered. The threshold is set according to [11] and the compensation factor is $b = 2$.

To evaluate the performance gain of the proposed SA-SOMP algorithm compared with conventional SOMP and OMP algorithms, Fig. 3 presents the correct CIR recovery probability when different numbers of pilots N_P are used under the static ITU-VB channel with the fixed SNR of 20 dB in a 4 × 4 MIMO system. Here, the correct recovery is defined as the estimation mean square error (MSE) is lower than 10^{-2}. It can be seen from Fig. 3 that by utilizing the obtained partial common support information and the sparse common support property, the required number of pilots in the SA-SOMP is less

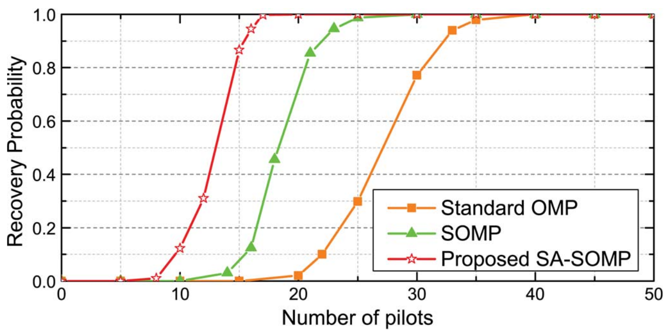

Fig. 3. Comparison of the CIR recovery probabilities at the SNR = 20 dB.

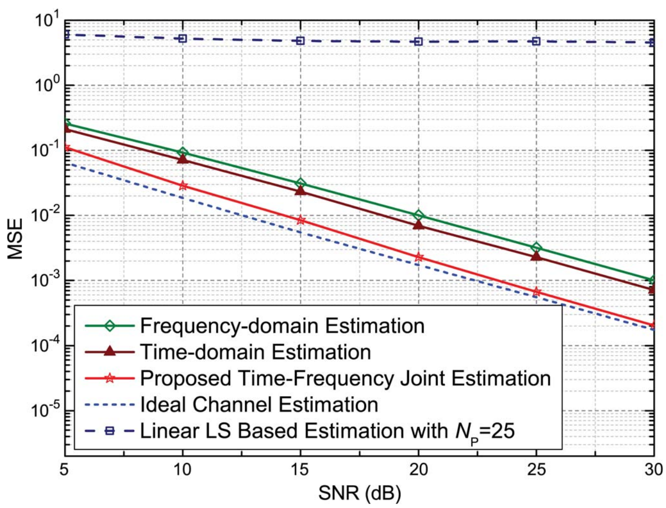

Fig. 4. Comparison of the MSE performance among different schemes under the time-varying ITU-VB channel in a 4 × 4 MIMO system.

than that in the SOMP algorithm and far less than that in the standard OMP algorithm, e.g., $N_P = 15$ could ensure the correct CIR recovery probability. In practice, considering the tradeoff between the robustness of the CIR recovery performance and the pilot overhead, we choose the number of pilots for each transmit antenna to be $N_P = 25$ for the proposed scheme.

Fig. 4 illustrates the MSE performance of different schemes under the time-varying ITU-VB channel with the mobile speed of 90 km/h in a 4 × 4 MIMO system. The conventional time- and frequency-domain channel estimation methods based on preamble and comb-type pilots are also evaluated for comparison. The preamble based scheme adopts the preamble of length $M = 1024$ for channel estimation, and each preamble is followed by only one OFDM block to combat the time-varying channel [3]. The comb-type pilot based scheme uses $N_P = 256$ embedded orthogonal pilots for each transmit antenna, and hence the overall pilot overhead is 1024 [4]. A 256-length CP is added in both of the two schemes to avoid the IBI and the average power of the preamble or the pilot are set equal to that of the transmitted data for all the tested schemes. The MSE of the ideal channel estimation is illustrated as the benchmark, which shows the optimal estimation performance when the exact location of nonzero channel taps is known *a priori*. In addition, the performance of the linear LS based estimation method with the same pilot number is provided for comparison, which turns out to fail due to the underdetermined

TABLE I
SPECTRAL EFFICIENCY COMPARISON

Solution	Spectral Efficiency
Time-domain preamble based scheme [3]	76.19%
Frequency-domain comb-type pilot based scheme [4]	70.59%
Proposed MIMO TFT-OFDM scheme	91.82%

matrix of $\mathbf{F}^{(1)}$. It could be seen that for the mobile ITU-VB channel, the MSE performance of the proposed TFT-OFDM scheme performs 5 dB and 7 dB better than the preamble and comb-type pilot based schemes, respectively. Besides, the spectral efficiency of the three different schemes has been specified in Table I, and the proposed MIMO-TFT-OFDM scheme demonstrates much higher spectral efficiency than the two conventional counterparts.

V. CONCLUSION

In this letter, a spectrum-efficient MIMO-TFT-OFDM alternative to the standard MIMO-OFDM scheme is proposed by designing a time-frequency joint sparse channel estimation with high accuracy under the framework of SCS. The proposed SA-SOMP algorithm has better performance and lower complexity than the standard SOMP and OMP algorithms. Simulation results show that the proposed MIMO-TFT-OFDM scheme demonstrates better performance, more robustness and higher spectral efficiency than the conventional time- and frequency-domain based MIMO-OFDM schemes.

REFERENCES

[1] G. Stuber *et al.*, "Broadband MIMO-OFDM wireless communications," *Proc. IEEE*, vol. 92, no. 2, pp. 271–294, Feb. 2004.
[2] J. Ketonen, M. Juntti, and J. Cavallaro, "Performance-complexity comparison of receivers for a LTE MIMO-OFDM system," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3360–3372, Jun. 2010.
[3] S. K. Mohammed, A. Zaki, A. Chockalingam, and B. S. Rajan, "Highrate space-time coded large-MIMO systems: Low-complexity detection and channel estimation," *IEEE J. Sel. Topics Signal Process.*, vol. 3, no. 6, pp. 958–974, Dec. 2009.
[4] H. Minn and N. Al-Dhahir, "Optimal training signals for MIMO OFDM channel estimation," *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1158–1168, May 2006.
[5] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.
[6] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2230–2249, May 2009.
[7] N. Wang, G. Gui, Z. Zhang, T. Tang, and J. Jiang, "A novel sparse channel estimation method for multipath MIMO-OFDM systems," in *Proc. IEEE VTC—Fall*, Sep. 2011, pp. 1–5.
[8] J. A. Tropp, A. C. Gilbert, and M. J. Strauss, "Algorithms for simultaneous sparse approximation Part I: Greedy pursuit," *Signal Process.*, vol. 86, no. 3, pp. 572–588, Mar. 2006.
[9] Y. Barbotin, A. Hormati, S. Rangan, and M. Vetterli, "Estimation of sparse MIMO channels with common support," *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3705–3716, Dec. 2012.
[10] L. Dai, J. Wang, Z. Wang, P. Tsiaflakis, and M. Moonen, "Spectrum- and energy-efficient OFDM based on simultaneous multi-channel reconstruction," *IEEE Trans. Signal Process.*, vol. 61, no. 23, pp. 6047–6059, Dec. 2013.
[11] X. Zhou, F. Yang, and J. Song, "Novel transmit diversity scheme for TDS-OFDM system with frequency-shift m-sequence padding," *IEEE Trans. Broadcast.*, vol. 58, no. 2, pp. 317–324, Jun. 2012.
[12] Z. Gao, C. Zhang, Z. Wang, and S. Chen, "Priori-information aided iterative hard threshold: A low-complexity high-accuracy compressive sensing based channel estimation for TDS-OFDM," *IEEE Trans. Wireless Commun.*, to be published.