1 The Dirac δ function

The delta function can also be described by a Gaussian function

$$\delta(x) = \lim_{\alpha \to 0} \delta_\alpha(x) \equiv \lim_{\alpha \to 0} \frac{1}{\sqrt{2\pi\alpha}} \exp\left[-\frac{1}{2\alpha^2}x^2\right] \tag{1}$$

Use the definition of the δ function Eq.(1) to prove the following identities.

(a)

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \tag{2}$$

(b)

$$\int_{-\infty}^{\infty} \delta(cx)dx = \frac{1}{|c|}, \quad c \in \mathbb{R} \tag{3}$$

(c)

$$\int_{-\infty}^{\infty} \delta(x - x_0)f(x)dx = f(x_0) \tag{4}$$

(d)

$$\int_{-\infty}^{\infty} \delta'(x)f(x) = -f'(x) \tag{5}$$

(e) Use Python to draw $\delta_\alpha(x)$ for α=1, 0.1, 0.01, and 0.001.

2 Problem 5.1 in the text book, page 57.

3 Problem 5.2 in the text book, page 58.

4 Problem 5.3 in the text book, page 58.

5 Problem 5.4 in the text book, page 59.

6 Problem 5.5 in the text book, page 59.