

HW1

第九組

謝愷昀、鄭琮寶、石笨源

Question : write down the annotation of the program

```
1  from visual import *
2  from visual . graph import *
3  import random
4  import sys
5  from types import *
6  from time import clock , time      #import需要用到的library
7
8  trials = 100                      #骰子試驗的次數
9  print "Number of trials = ", trials #印出骰子試驗的次數
10 sides = 6                         #骰子總共有六面
11
12 histogram = zeros(sides , int)    #利用陣列初始化骰子各點分布的次數[0,0,0,0,0,0]
13 print histogram                   #印出初始化骰子的陣列
14 sum = 0.0
15 j=0
16 r=0
17 while j < trials :                #利用迴圈控制試驗次數
18     r=int(random.random()*sides)   #隨機從1~sides中選一個當作此次試驗的結果(程式事實上為:0~sides-1)
19     histogram[r] = histogram[r] + 1 #將被選中的試驗結果+1
20     j=j+1                          #將目前已試驗的次數+1
21 j=0
22 while j < sides :                  #利用迴圈印出各情況分布(histogram[j])和各情況的離差(histogram[j]-trials/sides)
23     print histogram[j], histogram[j]-trials/sides
24     j=j+1
```

Problem 3.1

Question1:

See in other file --- homework1_1.py

Question2:

Program : See in other file --- homework1_1.py

Result:

```
Number of trials = 100
number of dise : 1
    times : 14 times deviation : 2
    probability : 0.14 probability deviation : 0.0266666666667
number of dise : 2
    times : 20 times deviation : 4
    probability : 0.2 probability deviation : 0.0333333333333
number of dise : 3
    times : 20 times deviation : 4
    probability : 0.2 probability deviation : 0.0333333333333
number of dise : 4
    times : 20 times deviation : 4
    probability : 0.2 probability deviation : 0.0333333333333
number of dise : 5
    times : 15 times deviation : 1
    probability : 0.15 probability deviation : 0.0166666666667
number of dise : 6
    times : 11 times deviation : 5
    probability : 0.11 probability deviation : 0.0566666666667
```

Question3:

Program : See in other file --- homework1_1.py

Result:

running...

.....

.....

.....

.....

.....

.....

```
Number of trials = 1070010
  times of distribution : [178098 178018 178051 178390 178908 178545]
  time : 0.949019150855
Number of trials = 1080010
  times of distribution : [178994 179987 179644 180422 180414 180549]
  time : 0.949122631197
Number of trials = 1090010
  times of distribution : [182276 181621 181176 181447 181426 182064]
  time : 0.958758888472
Number of trials = 1100010
  times of distribution : [183829 182918 182956 183625 183118 183564]
  time : 0.973421400387
Number of trials = 1110010
  times of distribution : [185135 185690 184458 185081 184753 184893]
  time : 0.974245047976
Number of trials = 1120010
  times of distribution : [186712 186471 186403 186481 186931 187012]
  time : 0.983720025078
Number of trials = 1130010
  times of distribution : [188101 188325 188561 188255 188375 188393]
  time : 0.993267718095
```

-----change delta

-----change delta

-----change delta

```
Number of trials = 1130020
  times of distribution : [188637 188274 188005 188560 188187 188357]
  time : 0.990513835834
```

-----change delta

Final Answer:

```
Number of trials = 1130020 (minimum unit:10)
  times of distribution : [188457 188283 187558 188751 188572 188409]
  times deviation of distribution : [ 121  53  778  415  236  73]
  probability of distribution : [ 0.16677315  0.16661917  0.16597759  0.16703333  0.16687492  0.16673068]
  probability deviation of distribution : [ 1.06487791e-04  4.74917848e-05  6.89073350e-04
                                           3.66660177e-04  2.08255901e-04  6.40106665e-05]
time: 0.990513835834
```

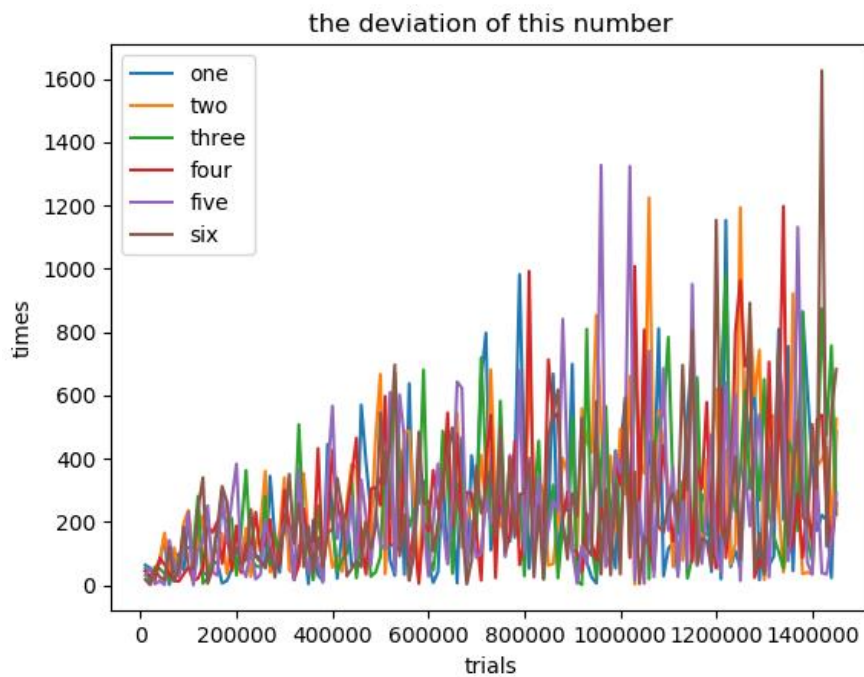
Number of trials = 1130020 (more approach closer to one second)

(一開始 delta 為 10000 後來以十倍遞減到 10 為止)

Question4:

Program : See in other file --- homework1_1.py

Result:



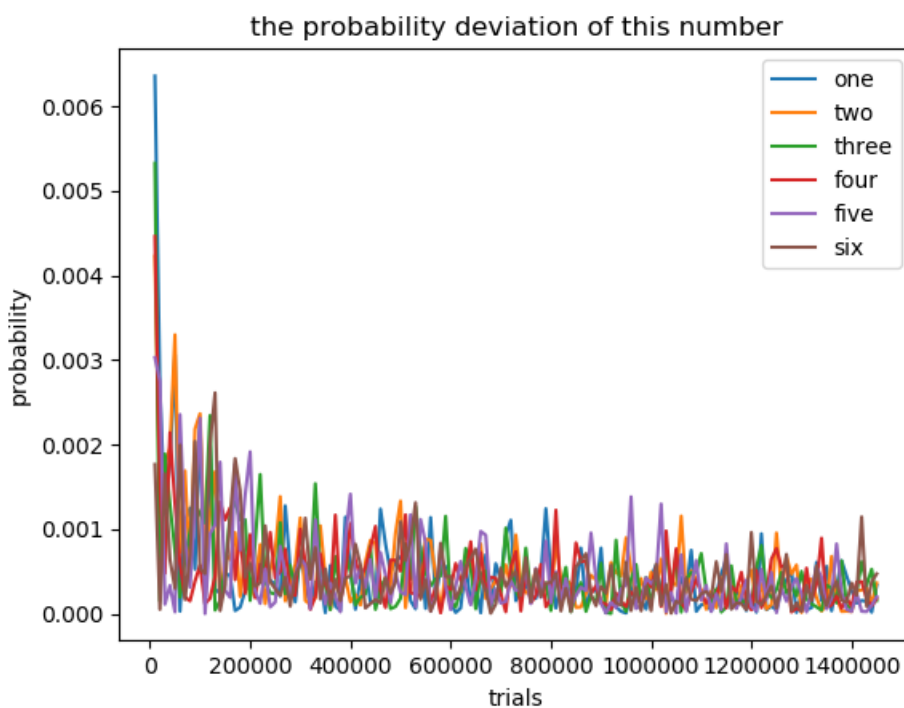
Meaningless because of the trials

(看起來是遞增但是其實是因為每次的 trials 都不一樣所以數量級不同無法比較)

Question5:

Program : See in other file --- homework1_1.py

Result:



the ratio of the number of times approach closer to $1/6$

Problem 3.2

假設有四個數字:3、4、5、60

事件 A:隨機抽一個數為 3 的倍數

事件 B:隨機抽一個數為 4 的倍數

事件 C:隨機抽一個數為 5 的倍數

已知事件 A、B、C 兩兩獨立，因為:

$$P(AB)=P(A)*P(B)=1/4$$

$$P(AC)=P(A)*P(C)=1/4$$

$$P(BC)=P(B)*P(C)=1/4$$

但是事件 A、B、C 不相互獨立，因為:

$$P(ABC)=1/4 \text{ 不等於 } P(A)*P(B)*P(C)=1/8$$

因為事件 AB 不與事件 C 獨立，當事件 AB 發生時事件 C 必定會發生

因為當事件 AB 發生，代表一定抽到 60，則事件 C 發生

(上述為題目之反例)

Problem 3.3

Question1:

$$\text{mean} = \frac{1}{s} \sum_{n=1}^s n = \frac{s(1+s)}{2s} = \frac{s+1}{2}$$

$$\text{variances} = \frac{1}{s} \sum_{n=1}^s \left(n - \frac{s+1}{2}\right)^2 = \frac{1}{s} \sum_{n=1}^s n^2 - \frac{s+1}{s} \sum_{n=1}^s n + \frac{1}{s} \sum_{n=1}^s \left(\frac{s+1}{2}\right)^2 = \frac{s^2-1}{12}$$

$$\text{standard deviation} = \sqrt{\frac{1}{s} \sum_{n=1}^s \left(n - \frac{s+1}{2}\right)^2} = \sqrt{\frac{s^2-1}{12}}$$

Question2 and 3:

Program : See in other file --- homework1_3.py

Result:

```
sides = 10 trials = 500000 times = 0.36056606544
prediction:
  mean: 5.5
  variance: 8.25
  standard deviation: 2.87228132327
experiment:
  mean: 5.498586
  variance: 8.2424660006
  standard deviation: 2.87096952276
result : experiment results are similar to prediction answers

sides = 20 trials = 500000 times = 0.360802392167
prediction:
  mean: 10.5
  variance: 33.25
  standard deviation: 5.76628129734
experiment:
  mean: 10.487732
  variance: 33.3302534962
  standard deviation: 5.77323596401
result : experiment results are similar to prediction answers
```

以上為兩種不同的例子，實驗皆與預測相似

Question4:

Program : See in other file --- homework1_3.py

Result:

running...

.....

.....

.....

.....

.....

```
sides = 20 trials = 19323 error = 0.960482520791%
sides = 20 trials = 19323 error = 0.339343935059%
sides = 20 trials = 19323 error = 0.619789394824%
sides = 20 trials = 19323 error = 0.155400872355%
sides = 20 trials = 19323 error = 0.35836467563%
sides = 20 trials = 19323 error = 0.386386341308%
sides = 20 trials = 19323 error = 1.27046704928%
sides = 20 trials = 19324 error = 0.376536456023%
sides = 20 trials = 19324 error = 0.185311135425%
sides = 20 trials = 19324 error = 0.333165764753%
sides = 20 trials = 19324 error = 0.450143136293%
sides = 20 trials = 19324 error = 1.15885661779%
sides = 20 trials = 19325 error = 0.966435141657%
sides = 20 trials = 19325 error = 0.788408784815%
sides = 20 trials = 19325 error = 0.97442209575%
sides = 20 trials = 19325 error = 0.388591141502%
sides = 20 trials = 19325 error = 0.357543276043%
sides = 20 trials = 19325 error = 1.3348905488%
sides = 20 trials = 19326 error = 0.407288149394%
sides = 20 trials = 19326 error = 0.647126987199%
sides = 20 trials = 19326 error = 0.738147520323%
sides = 20 trials = 19326 error = 0.458176260046%
sides = 20 trials = 19326 error = 0.610291500036%
sides = 20 trials = 19326 error = 0.273272877786%
sides = 20 trials = 19326 error = 0.817026147607%
sides = 20 trials = 19326 error = 1.48515496445%
sides = 20 trials = 19327 error = 0.38756538472%
sides = 20 trials = 19327 error = 0.440076632531%
sides = 20 trials = 19327 error = 0.232342121927%
sides = 20 trials = 19327 error = 0.499917460646%
sides = 20 trials = 19327 error = 0.30941895667%
sides = 20 trials = 19327 error = 0.702939632934%
sides = 20 trials = 19327 error = 0.221501132144%
sides = 20 trials = 19327 error = 0.710192826847%
sides = 20 trials = 19327 error = 0.358685078525%
sides = 20 trials = 19327 error = 0.967875657686%
-----change delta
```

Final Answer:

```
sides = 20 trials = 19325 error = 0.967875657686%
```

Number of trials = 19325 (error<=1%)

(一開始 delta 為 1000 後來以十倍遞減到 1 為止)

Problem 3.4

當 A 與 B 獨立，則下式成立：

$$\langle F(A)G(B) \rangle = \langle F(A) \rangle \langle G(B) \rangle \text{ - 式 1}$$

從式 1 可知 $F(A)$ 與 $G(B)$ 也獨立

接著設 $F(A)=X$ 、 $G(B)=Y$

將上式代入式 1 得：

$$\langle XY \rangle = \langle X \rangle \langle Y \rangle \text{ - 式 2}$$

由此可知式 1 與式 2 等價

同理可證，可推得 X 與 Y 獨立

Problem 3.5

Question1:

沒骰到 3 的機率為 $\frac{5}{6}$

\therefore 骰 10 次都沒骰到的機率為 $(\frac{5}{6})^{10}$

Question2:

(1) 至少骰到一次 6 的機率 = $1 - \text{都沒骰到 6 的機率}$

\therefore 骰 4 次至少出現一次 6 的機率為 $1 - (\frac{5}{6})^4$

(2) 沒出現兩面 6 的機率為 $\frac{35}{36}$

\therefore 同時骰 2 個 24 次，至少出現過一次兩面 6 的機率為 $1 - (\frac{35}{36})^{24}$

而 $(\frac{5}{6})^4 < (\frac{35}{36})^{24} \quad \therefore (1) \text{ 的機率} > (2) \text{ 的機率}$

Problem 3.6

Question1 and 2:

Program : See in other file --- homework1_6.py

Result:

question (a):

```
sides = 10 trials = 500000 number of dice = 2 times = 0.81654426931
```

prediction:

```
mean: 11.0
```

```
variance: 16.5
```

```
standard deviation: 4.06201920232
```

experiment:

```
mean: 10.996266
```

```
variance: 16.4580160572
```

```
standard deviation: 4.05684804463
```

result : experiment results are similar to prediction answers

question (b):

```
sides = 20 trials = 500000 number of dice = 2 times = 0.825848643685
```

prediction:

```
mean: 21.0
```

```
variance: 66.5
```

```
standard deviation: 8.15475321515
```

experiment:

```
mean: 20.99952
```

```
variance: 66.6212717696
```

```
standard deviation: 8.16218547753
```

result : experiment results are similar to prediction answers

(以上為題目要求的兩種例子，實驗皆與預測相似)

Question3:

(1)

Program : See in other file --- homework1_6.py

Result:

```
sides = 10 trials = 500000 number of dice = 5 times = 1.33960290584
```

prediction:

```
width of the distribution: 15.1252613614
```

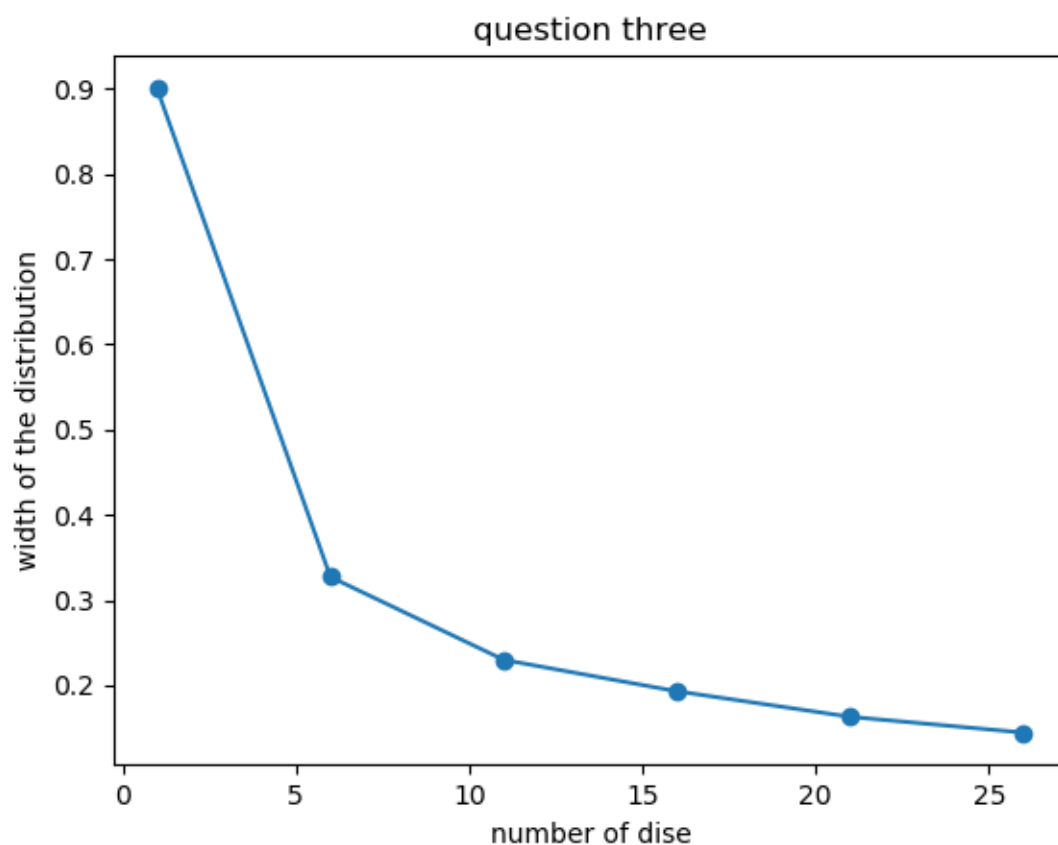
experiment:

```
width of the distribution: 15
```

result : experiment results are similar to prediction answers

比較實驗與理論的半高寬相似

(2)



width of the distribution decrease with increasing numbers of dice

因此骰子數目與圖形的半高寬負相關

(因為當骰子個數不同時，其分布之半高寬無法比較，因此在比較前必須先歸一)

Ps:全部程式的執行結果詳見 result.py