107 學年(下)學期 熱統計物理(一) 作業-2

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1 Approximation to $N!$

Table 3.4 in the text book shows the comparison for different approximations to the factorial function $N!$. Please write a Python program and reproduce the comparison among the simple Stirling approximation Eq. (1), the improved Stirling approximation Eq. (2), and the Gosper's approximation Eq. (3).

The simple Stirling approximation:

$$N! \approx N^N \exp(1-N) \tag{1}$$

The improved Stirling approximation:

$$N! \approx e^{-N} N^N \sqrt{2\pi N} \tag{2}$$

The Gosper's approximation:

$$N! \approx e^{-N} N^N \sqrt{\left(2N + \frac{1}{3}\right)\pi} \tag{3}$$

2 Binomial Distribution

The binomial distribution Eq. (4) is given by the text book, page 26,

$$P(n|N) = \left(\begin{array}{c} N \\ n \end{array} \right) p^n (1-p)^{N-n} \tag{4}$$

and the Gaussian function Eq. (5) (also see the text book, page 28)

$$P(n|N) \approx \frac{1}{\sqrt{2\pi p(1-p)N}} \exp\left[-\frac{(n-pN)^2}{2p(1-p)N} \right] \tag{5}$$

is a good approximation to the binomial distribution as long as n and N are large enough. Write a Python program to plot the binomial distribution Eq. (4) and the Gaussian function Eq. (5) for $p = 0.5$ and $N = 10, 100, 1000$. Set $x = \frac{n}{N}$ is the renormalized variable and plot $nP(n|N)$ as a function of x, as shown in Fig. (1)

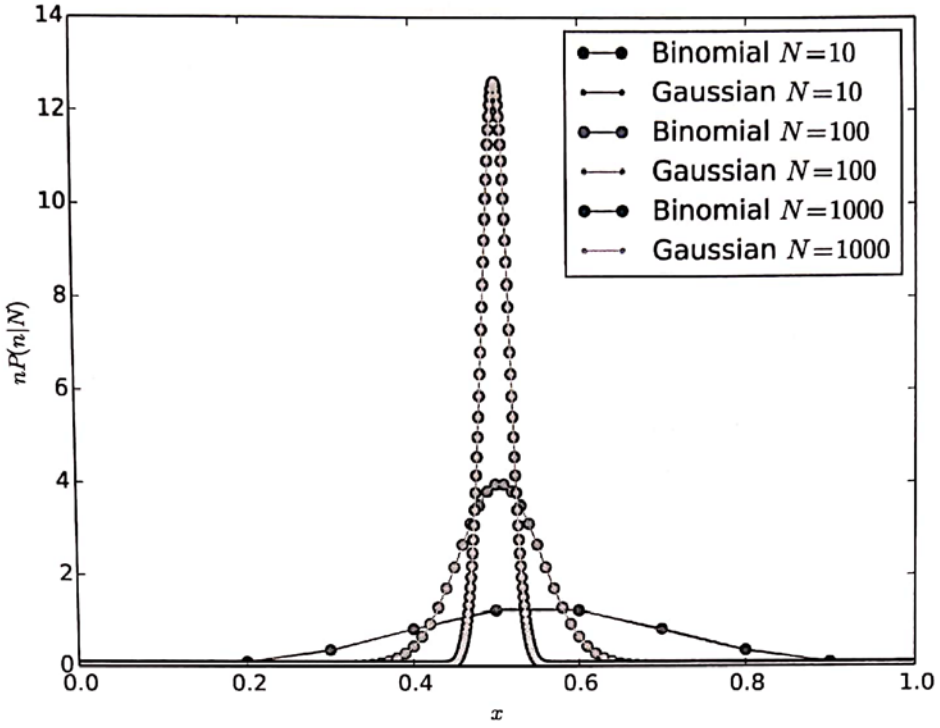

Figure 1: Note that $\langle n \rangle = \sum_{n=0}^{N} nP(n|N)$. You may check your program by examining $\sum_{n=0}^{N} P(n|N) = 1$ and $\langle n \rangle = pN$

3 **Problem 3.7 in the text book, page 37.**

4 **Problem 3.8 in the text book, page 37.**

5 **Problem 3.9 in the text book, page 37-38.**

6 **Problem 3.10 in the text book, page 38.**

7 **Problem 3.11 in the text book, page 38-39.**

8 **Problem 3.12 in the text book, page 39.**