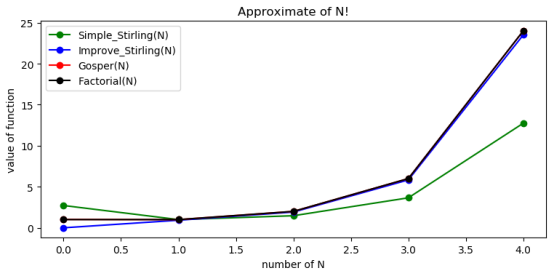
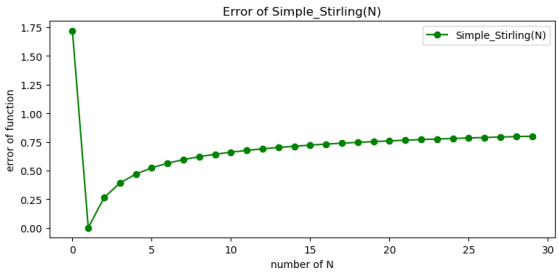
HW2

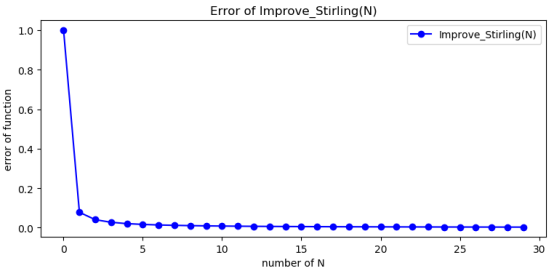
第九組

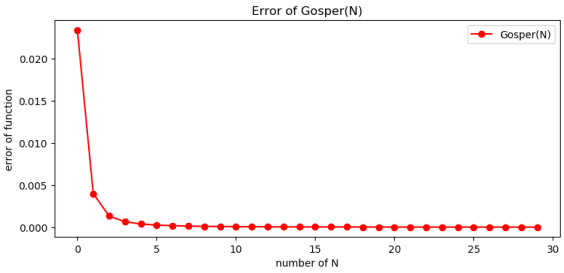
謝愷昀、鄭琮寶、石苯源

Problem 1









上述四張圖可以看出

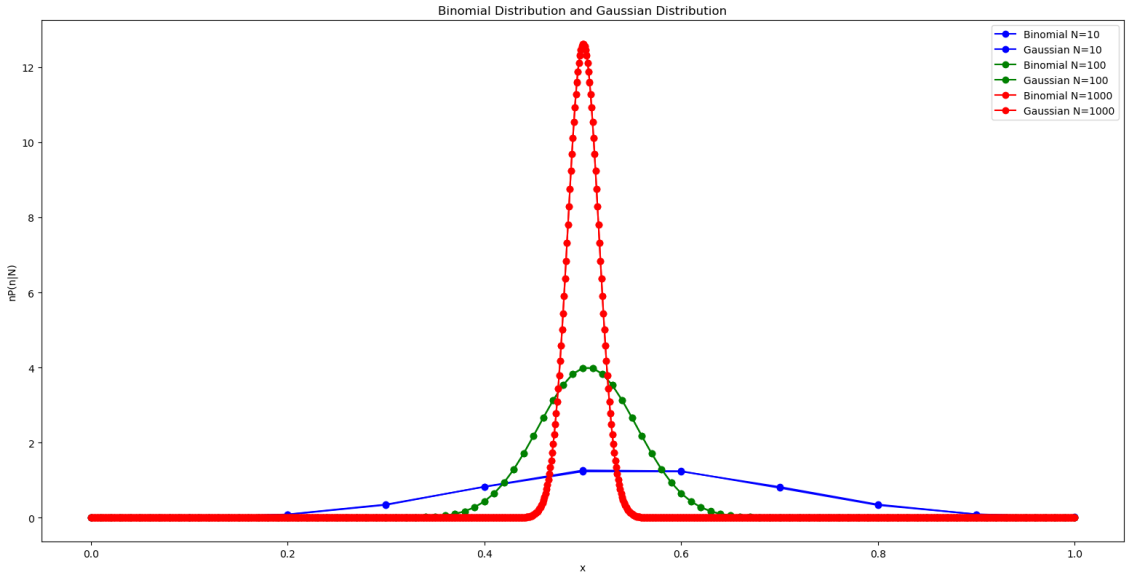
Simple_Stirling : 隨 N 變大誤差會上升到一個定值

Improve_Stirling : 隨 N 變大誤差會持續下降

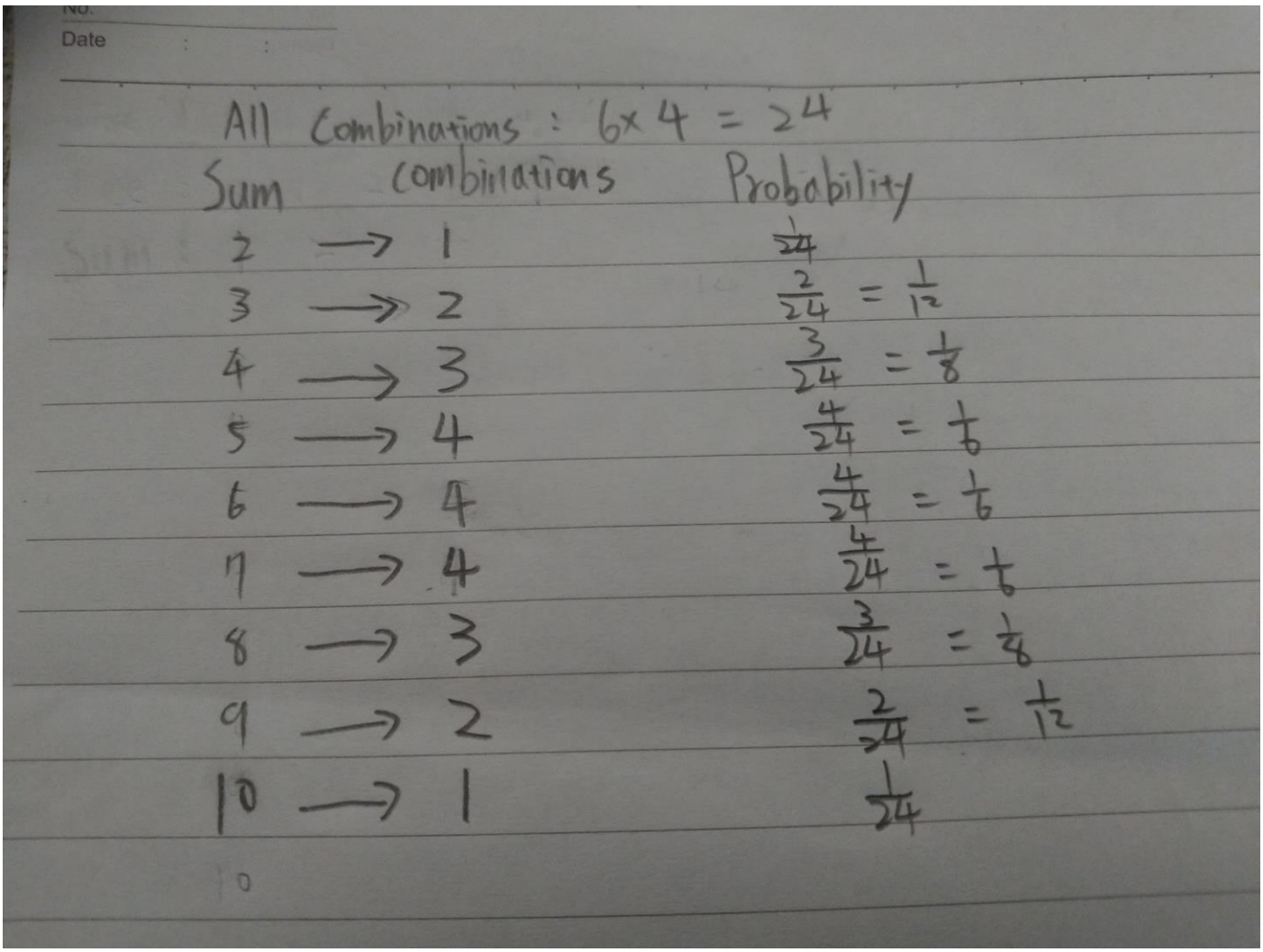
Gosper: 隨 N 變大誤差會持續下降

$$(\text{誤差} = \frac{|\text{實驗} - \text{理論}|}{\text{理論}})$$

Problem 2

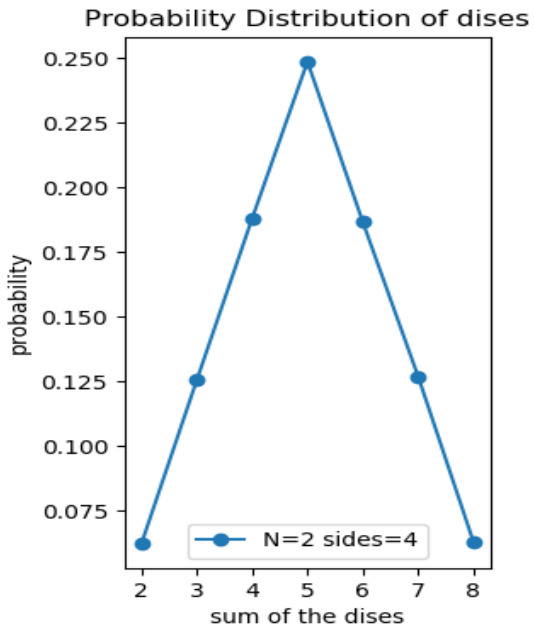


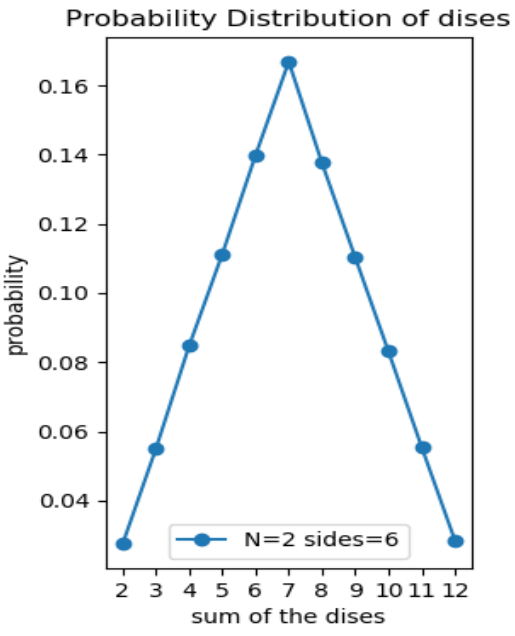
Problem 3



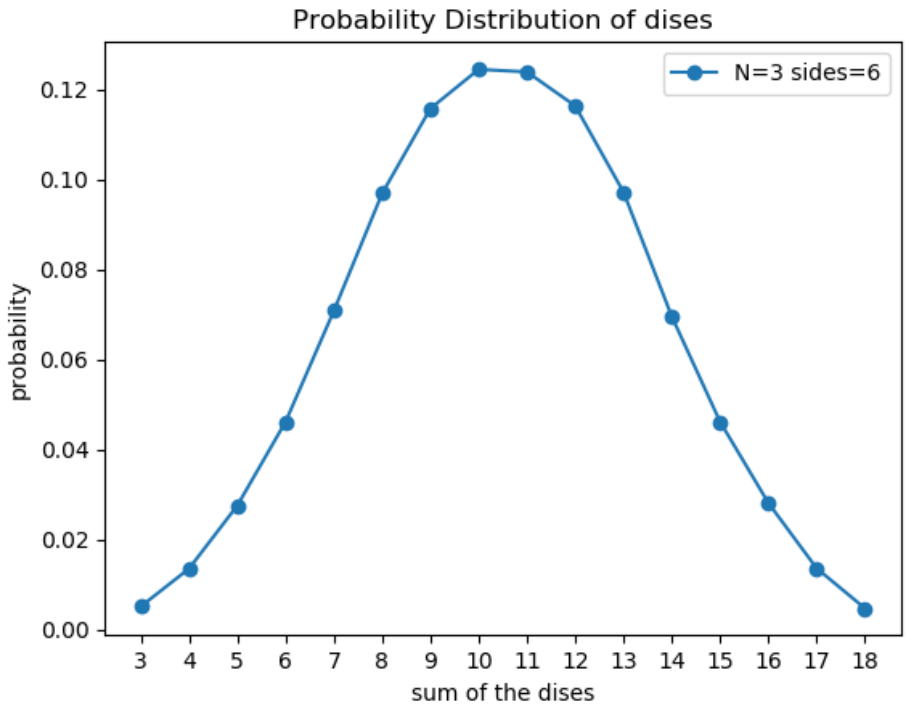
Problem 4

Question1:





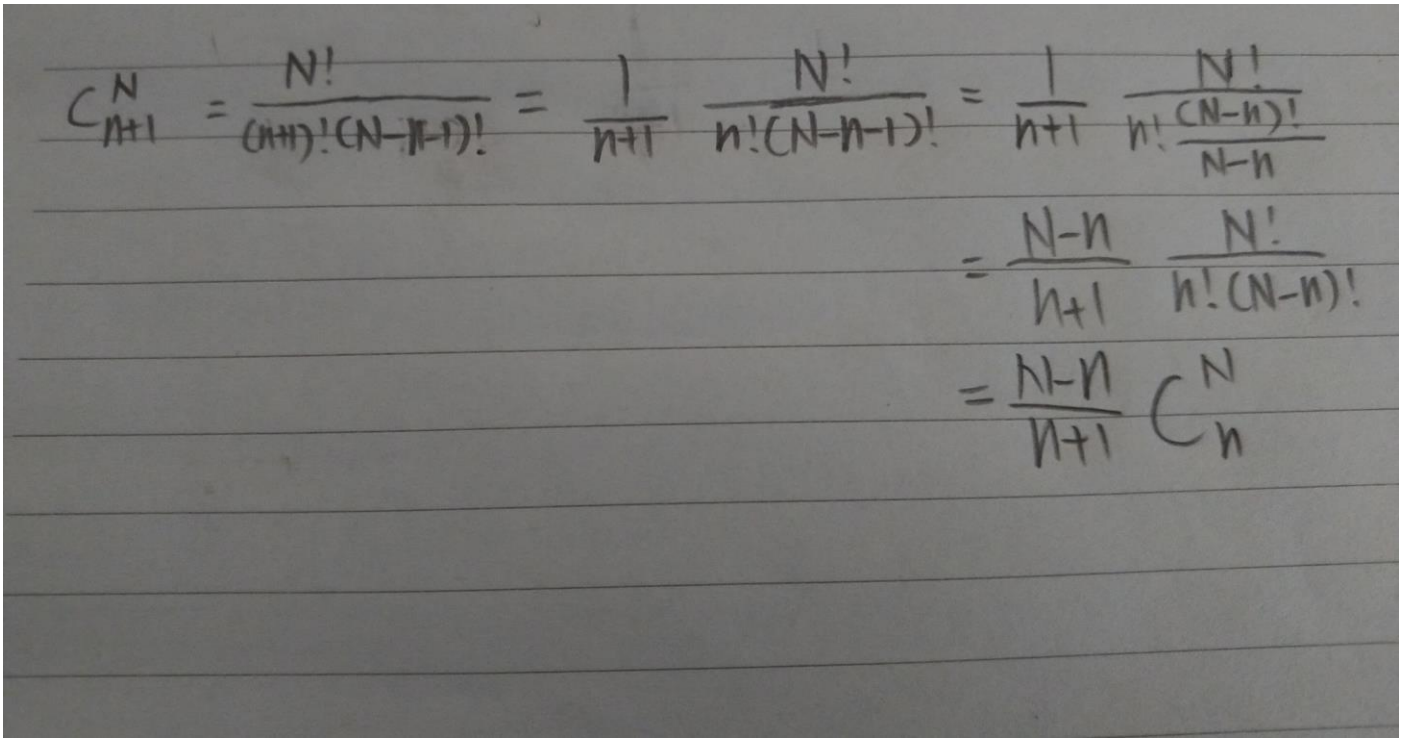
Question2:



比第一題較接近常態分布

Problem 5

Question1:



Question2:

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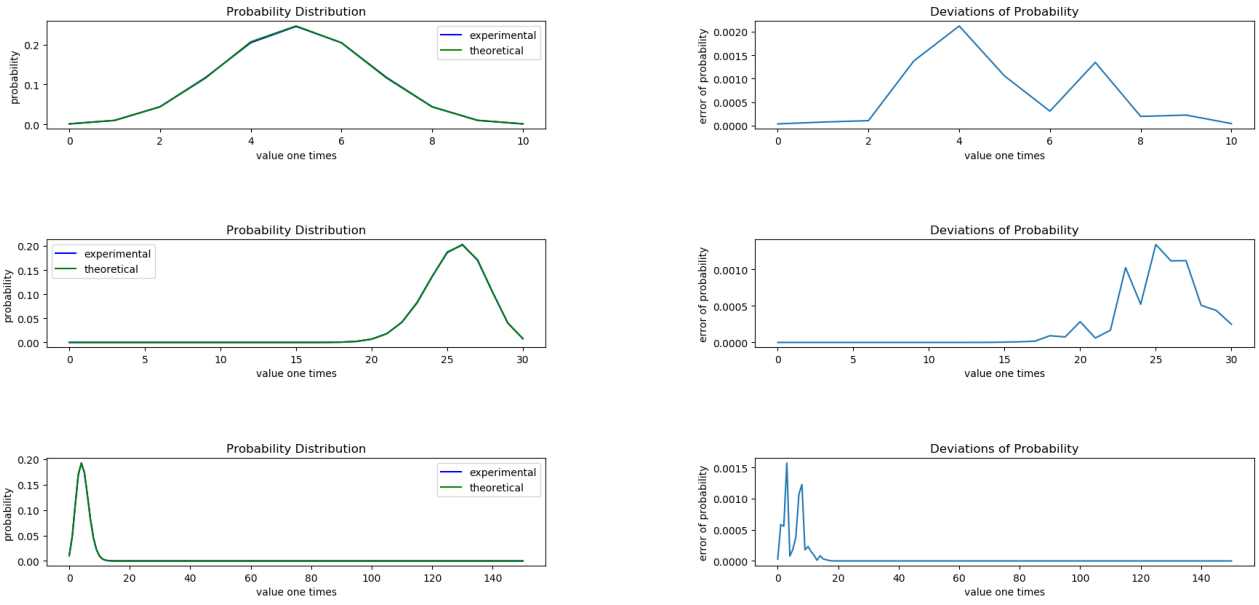
question two and three:

  trials = 100000  number of dise = 10  probability = 0.5  times = 0.16520092319368018
  prediction:
    mean: 5.0
    variance: 2.5
    standard deviation: 1.5811388300841898
  experiment:
    mean: 4.9979499999999994
    variance: 2.4799057974999994
    standard deviation: 1.5747716651946717
  result : experiment results are simlar to prediction answers

  trials = 100000  number of dise = 30  probability = 0.85  times = 0.38449696810408795
  prediction:
    mean: 25.5
    variance: 3.8250000000000006
    standard deviation: 1.9557607215607948
  experiment:
    mean: 25.511049999999994
    variance: 3.8213578975
    standard deviation: 1.954829378104391
  result : experiment results are simlar to prediction answers

  trials = 100000  number of dise = 150  probability = 0.03  times = 1.3547975294272305
  prediction:
    mean: 4.499999999999982
    variance: 4.364999999999983
    standard deviation: 2.089258241577614
  experiment:
    mean: 4.504200000000002
    variance: 4.36932236
    standard deviation: 2.0902924101665774
  result : experiment results are simlar to prediction answers
```

Question3:



上圖為題目數據圖，左為機率分布，右為誤差分布

Problem 6

Question 1 and 2:

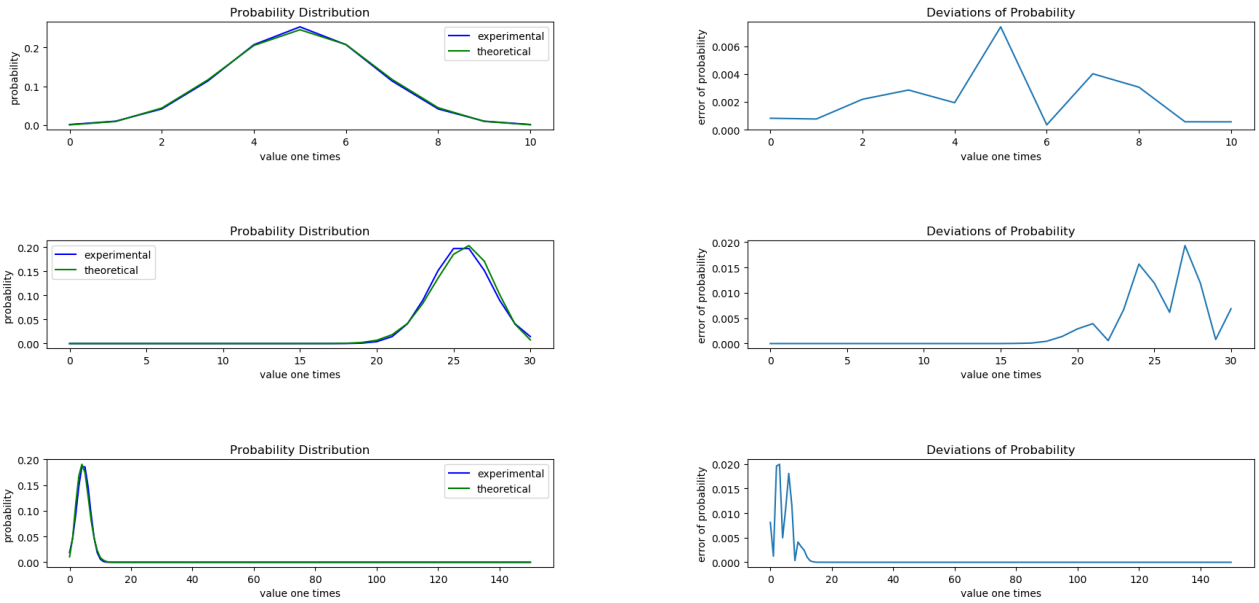
```
question one,two,three:

  trials = 100000  number of dise = 10  probability = 0.5  times = 0.16780871208567305
  prediction:
    mean: 4.99796914978672
    variance: 2.4849770620289613
    standard deviation: 1.5763810015440307
  experiment:
    mean: 5.003960000000001
    variance: 2.4837643184
    standard deviation: 1.5759962939042718
  result : experiment results are simlar to prediction answers

  trials = 100000  number of dise = 30  probability = 0.85  times = 0.3866513521646766
  prediction:
    mean: 25.34776688282773
    variance: 3.6781541218791185
    standard deviation: 1.917851433734928
  experiment:
    mean: 25.499149999999997
    variance: 3.8083992775
    standard deviation: 1.9515120490276252
  result : experiment results are simlar to prediction answers

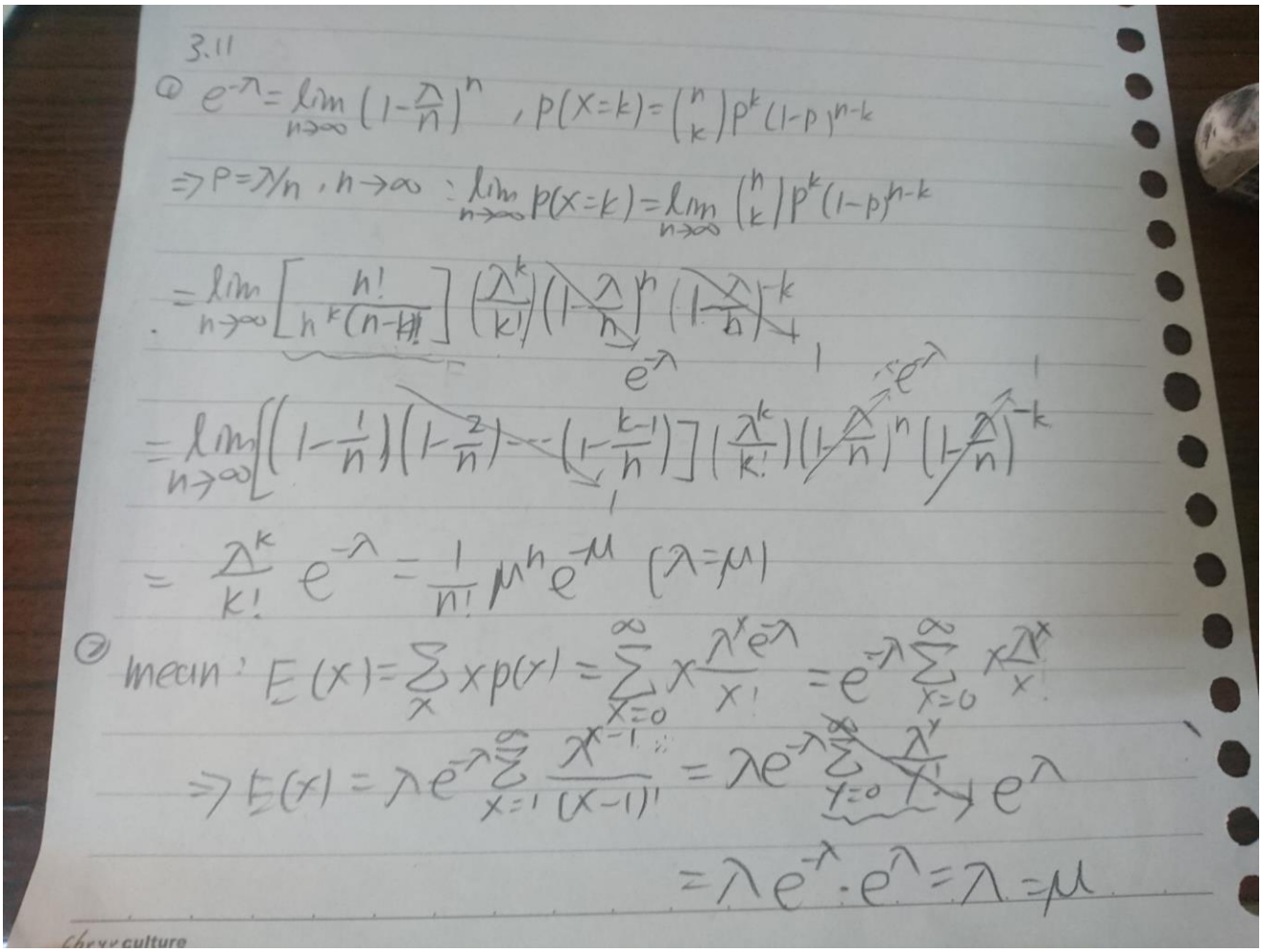
  trials = 100000  number of dise = 150  probability = 0.03  times = 1.3207677048328605
  prediction:
    mean: 4.510132930468253
    variance: 4.098502676058834
    standard deviation: 2.0244759015752285
  experiment:
    mean: 4.50042
    variance: 4.371679823600001
    standard deviation: 2.090856241734472
  result : experiment results are simlar to prediction answers
```

Question3:



上圖為題目數據圖，左為機率分布，右為誤差分布

Problem 7



3.11

① $e^{-\lambda} = \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n$, $P(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$

$\Rightarrow p = \lambda/n$, $n\to\infty$: $\lim_{n\to\infty} P(X=k) = \lim_{n\to\infty}\binom{n}{k}p^k(1-p)^{n-k}$

$= \lim_{n\to\infty}\left[\frac{n!}{n^k(n-k)!}\right]\left(\frac{\lambda^k}{k!}\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-k}$

$\to e^{-\lambda}$, $\to 1$, $\to e^{-\lambda}$, $\to 1$

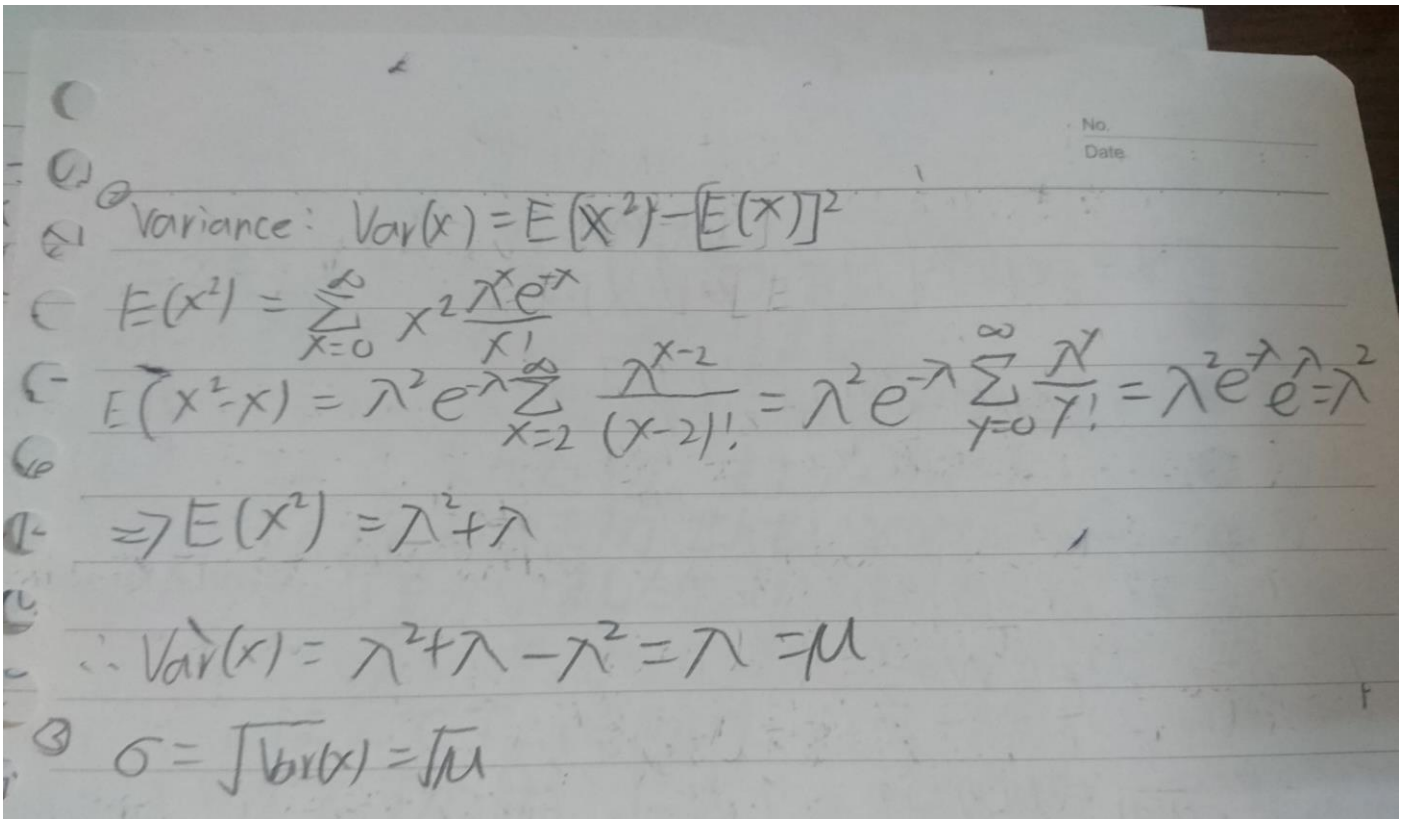
$= \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right)\right]\left(\frac{\lambda^k}{k!}\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-k}$

$= \frac{\lambda^k}{k!}e^{-\lambda} = \frac{1}{n!}\mu^n e^{-\mu}$ $(\lambda=\mu)$

② mean: $E(X) = \sum_x x p(x) = \sum_{x=0}^{\infty} x\frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda}\sum_{x=0}^{\infty} x\frac{\lambda^x}{x!}$

$\Rightarrow E(X) = \lambda e^{-\lambda}\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!} = \lambda e^{-\lambda}\sum_{y=0}^{\infty}\frac{\lambda^y}{y!} \to e^{\lambda}$

$= \lambda e^{-\lambda}\cdot e^{\lambda} = \lambda = \mu$



② variance: $Var(x) = E(X^2) - [E(X)]^2$

$E(x^2) = \sum_{x=0}^{\infty} x^2\frac{\lambda^x e^{-\lambda}}{x!}$

$E(x^2-x) = \lambda^2 e^{-\lambda}\sum_{x=2}^{\infty}\frac{\lambda^{x-2}}{(x-2)!} = \lambda^2 e^{-\lambda}\sum_{y=0}^{\infty}\frac{\lambda^y}{y!} = \lambda^2 e^{-\lambda}e^{\lambda} = \lambda^2$

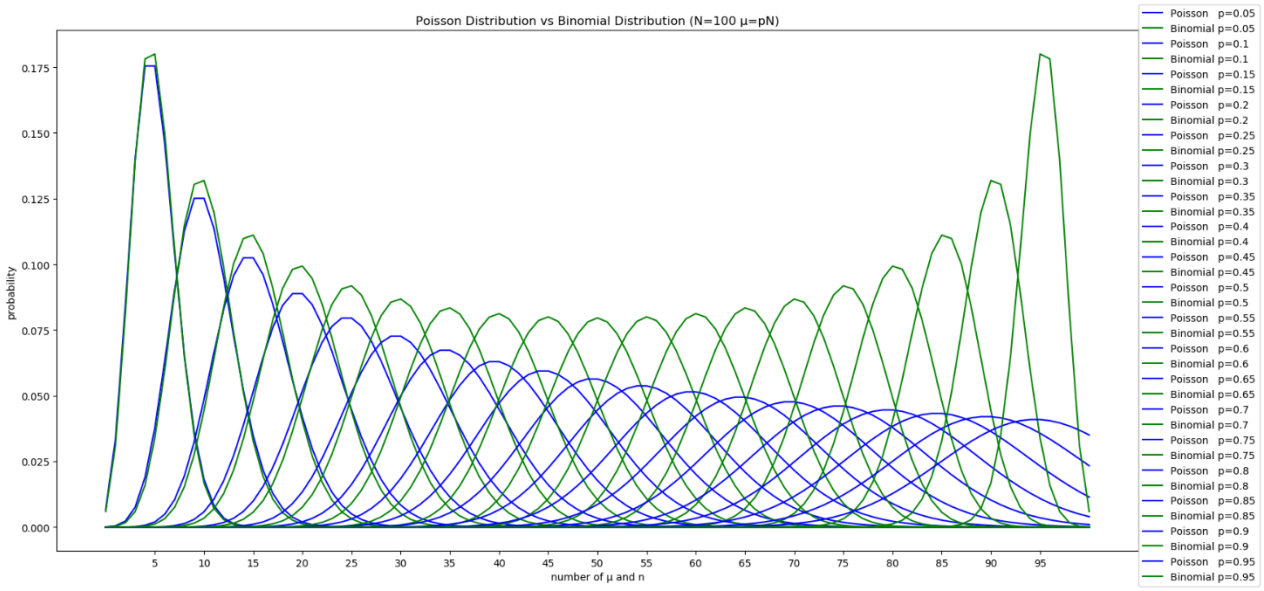
$\Rightarrow E(x^2) = \lambda^2 + \lambda$

$\therefore Var(x) = \lambda^2 + \lambda - \lambda^2 = \lambda = \mu$

③ $\sigma = \sqrt{Var(x)} = \sqrt{\mu}$

Problem 8

Question1:



從上圖可以知道 p 越小 Binomial 和 Poisson 的分布會越相似

Question2:

從第一題的圖可以知道當 μ=5，當 N=100 Binomial 和 Poisson 會相似

Ps

Problem n 對應程式檔為 homework2_n.py