HW4

第九組

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第一題 (Problem 5.6)

$$P(V_x)=Ae^{-\beta\frac{1}{2}mV_x^2}$$

1.
$$\int_{-\infty}^{\infty}P(V_x)\,dV_x=1$$

$$\Rightarrow \int_{-\infty}^{\infty}Ae^{-\beta\frac{1}{2}mV_x^2}\,dV_x=1$$

$$\Rightarrow A\cdot\sqrt{\frac{2\pi}{\beta m}}=1 \Rightarrow A=\sqrt{\frac{m\beta}{2\pi}}$$ #

2.
$$P(\vec{V})=P(V_x)P(V_y)P(V_z)$$

$$\Rightarrow P(\vec{V})=A^3e^{-\beta\cdot\frac{1}{2}(V_x^2+V_y^2+V_z^2)}$$

$$\Rightarrow P(\vec{V})=\left(\frac{m\beta}{2\pi}\right)^{3/2}e^{-\beta\cdot\frac{1}{2}\vec{V}\cdot\vec{V}}$$ #

3.
速率無方向性,相於一維之射線

$$P(V)=Be^{-\beta\frac{1}{2}mV^2}$$

$$\int_0^{\infty}Be^{-\beta\frac{1}{2}mV^2}\,dV=1$$

$$\Rightarrow B\cdot\frac{1}{2}\sqrt{\frac{2\pi}{\beta m}}=1 \Rightarrow B=\sqrt{\frac{2m\beta}{\pi}}$$

$$\Rightarrow P(V)=\sqrt{\frac{2m\beta}{\pi}}e^{-\beta\frac{1}{2}mV^2}$$ #

第二題 (Problem 5.7)

Date : :

當 $T_A = T_B$ 時達平衡，此時 $E_A = E_B$

(a)

$$P(E_A) = \delta\left(E_A - \frac{E}{2}\right) \quad \#$$

(b)

$$\langle E_A \rangle = \int_0^E E_A\, \delta\left(E_A - \frac{E}{2}\right) dE_A = \frac{E}{2} \quad \#$$

(c)

When $E_A = \frac{E}{2}$ # have maximum

第三題 (Problem 5.8)

$$P(\vec{p}, z) = X e^{-\beta \frac{|\vec{p}|^2}{2m}}$$

$$\int_{-\infty}^{\infty} P(\vec{p}, z)\, d\vec{p} = 1 \Rightarrow \int_{-\infty}^{\infty} X e^{-\beta \frac{p_z^2}{2m}}\, dp_z = 1$$

$$\Rightarrow X \cdot \sqrt{\frac{2m\pi}{\beta}} = 1 \Rightarrow X = \sqrt{\frac{\beta}{2m\pi}}$$

$$P(\vec{p}) = P(\vec{p}, x)\, P(\vec{p}, y)\, P(\vec{p}, z)$$

$$= X^3 e^{-\beta\left(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}\right)}$$

$$\Rightarrow P(E) = \left(\frac{\beta}{2m\pi}\right)^{3/2} e^{-\beta E} \quad \#$$

第四題 (Problem 5.9)

5-9.

1. $\iiint_0^\infty x e^{\left(-\beta\frac{|\vec{p}|^2}{2m}-\beta mgz\right)} = 1$

$\Rightarrow \int_0^\infty 4\pi p^2 x e^{\left(-\beta\frac{|p|^2}{2m}\right)}dp \int_0^\infty e^{(-\beta mgz)}dz = 1$

$\Rightarrow x\left(\frac{\beta}{m}\right)^{-3/2}\pi^{3/2} - \frac{1}{\beta mg} = 1 \Rightarrow x = \left(\frac{\beta}{2m}\right)^{3/2}\pi^{-3/2} - \beta mg$

2. $\int_0^\infty xze^{\left(-\beta\frac{|\vec{p}|^2}{2m}-\beta mgz\right)}$

$= \int_0^\infty 4\pi^2 p^2 x e^{\left(-\beta\frac{|p|^2}{2m}\right)}dp \cdot \int_0^\infty e^{(-\beta mgz)} z\,dz$

$= x\left(\frac{\beta}{2m}\right)^{-\frac{3}{2}}\pi^{3/2}\frac{1}{(\beta mg)^2} = -\frac{1}{\beta mg}$

3. $P(E) = 4\pi x\int_0^\infty dp\int_0^\infty dz\, p^2 e^{-\beta\left(\frac{|p|^2}{2m}+mgz\right)} = \frac{16\pi x\sqrt{2m}}{3g} e^{-\beta E} E^{3/2}$

第五題 (Problem 7.1)

7-1

∵ $S(\lambda U,\lambda V,\lambda N)=\lambda S(U,V,N)$

1. $A[(\lambda U)(\lambda V)(\lambda N)]^{1/3} = \lambda A(U,V,N)^{1/3}$

2. $A\left[\frac{(\lambda N)(\lambda U)}{(\lambda V)}\right]^{2/3} = \lambda^{2/3}A\left(\frac{NU}{V}\right)^{2/3}$

3. $A\left[\frac{(\lambda U)(\lambda V)}{(\lambda N)}\right] = \lambda A\left(\frac{UV}{N}\right)$

∴ 2不合

4. $A\left[\frac{(\lambda V)^3}{(\lambda N)(\lambda V)}\right] = \lambda A\left(\frac{V^3}{NV}\right)$

∵ U上升 S 必須上升 ∴ 4不合

∵ $S=S_A+S_B$，令 $U_A=U_B$，$V_A=V_B$，$N=N_A+N_B$，$N_A, N_B\approx 10^{23}$

1. $\frac{S}{S_A+S_B} = \frac{(N_A+N_B)^{1/3}}{N_A^{1/3}+N_B^{1/3}} = \frac{1}{1+3N_A^{1/3}N_B^{1/3}} \approx 0$

3. $\frac{S}{S_A+S_B} = \frac{4}{(N_A+N_B)\left(\frac{1}{N_A}+\frac{1}{N_B}\right)} \neq 0$ ∴ 1不合 ∴ 3 #

第六題 (Problem 7.2)

7.2

$$S = k_B N\left(\frac{3}{2}\ln\left(\frac{E}{N}\right) + \ln(V) + \chi\right)$$

$$\frac{\partial S}{\partial N} = \frac{3}{2}k_B \ln\left(\frac{E_A}{N}\right) + k_B \ln(V) + k_B\chi + \frac{-3}{2}k_B$$

$$\frac{\partial S}{\partial N} = \frac{P}{T}$$

for $P_A = P_B \quad T_A = T_B$

$$A:\ \frac{3}{2}k_B \ln\left(\frac{E_A}{N_A}\right) + k_B \ln(V_A) + k_B\chi - \frac{3}{2}k_B$$

$$B:\ \frac{3}{2}k_B \ln\left(\frac{E_B}{N_B}\right) + k_B \ln(V_B) + k_B\chi - \frac{3}{2}k_B$$

平衡時 S 最大

$$\frac{3}{2}k_B \ln\left(\frac{E_A}{N_A}\right) + k_B \ln(V_A) = \frac{3}{2}k_B \ln\left(\frac{E_B}{N_B}\right) + k_B \ln(V_B)$$

$$\frac{3}{2}\ln\left(\frac{E_A}{E_B} \times \frac{N_B}{N_A}\right) = \ln\left(\frac{V_B}{V_A}\right)$$

平衡時 $\Rightarrow N_A : N_B = E_A : E_B$

$\Rightarrow E_A : E_B = N_A : N_B = V_A : V_B$.

第七題 (Problem 8.1)

8.1

(1) $$\frac{\partial S}{\partial E} = \frac{3}{2}\frac{k_B N}{E} \qquad \frac{\partial S_x}{\partial U_x} = \frac{1}{2}\frac{k_B N_x}{\sqrt{V_x}\sqrt{U_x}}$$

$$\frac{3}{2}\frac{k_B N}{E} = \frac{1}{2}\frac{k_B N_x}{\sqrt{V_x}\sqrt{U_x}}$$

$$\frac{3k_B N}{E} = \frac{k_B N_x}{\sqrt{V_x}\sqrt{U_x}}$$

$$U_x = \left(\frac{N_x E}{3N\sqrt{V_x}}\right)^2$$

(2) $$\frac{1}{2}\frac{k_B N_x}{\sqrt{V_x}\sqrt{U_x}} = \frac{1}{T}$$

$$T = \frac{2\sqrt{V_x}\sqrt{U_x}}{k_B N_x}$$