HW5

第九組

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第一題 (Problem 10.1)

10.1

①
$dF = y\,dx + dy$

$\Rightarrow y\frac{\partial \gamma(y)}{\partial y} = \frac{\partial y}{\partial y}\gamma(y) = 0$

$\Rightarrow y\frac{\partial \gamma(y)}{\partial y} = -\gamma(y)$

$\Rightarrow \gamma(y) = \frac{1}{y}$

②
$\int_{(0,0)}^{(1,0)} 1\,dx + \frac{1}{y}\,dy + \int_{(1,0)}^{(1,1)} 1\cdot dx + \frac{1}{y}dy = 1$

$\int_{(0,0)}^{(0,1)} 1\,dx + \frac{1}{y}dx + \int_{(0,1)}^{(1,1)} 1\,dx + \frac{1}{y}dy = 1$

第二題 (Problem 11.1)

11-1

$\eta = \frac{dW}{dQ} = 1 - \frac{T_L}{T_H}$

$dS = dQ_H + dQ_L$

$dW' = dQ_H + dQ_L - dE$

$\Rightarrow \eta' = \frac{dW'}{dQ_H} = \eta - \frac{dE}{dQ_H} < \eta$

第三題 (Problem 11.2)

11.2

1. Because it are conjunction with an ideal heat engine, the final temperature will all be the lower initial temperature.

2. if $T_{A,0} > T_{B,0}$

$U_A = \int_{T_{B,0}}^{T_{A,0}} C_A\,dT = C_A(T_{A,0} - T_{B,0})$ ※

3. 設 final temperature 為 T_F. $T_{A,0} > T_{B,0}$

$C_A \times (T_A - T_F) = C_B \times (T_F - T_B)$

$C_A T_{A,0} + C_B T_{B,0} = C_A T_F + C_B T_F$

$T_F = \frac{C_A T_{A,0} + C_B T_{B,0}}{C_A + C_B}$ ※

4. higher then the heat engine is used.
Because when the heat engine is used, the heat will be converted into mechanical energy.

5. $dS = \frac{dQ}{T} = m \times C \times \frac{dT}{T}$ which m is mass. C is heat capacity

$S = \int dS = \int_{T_1}^{T_2} mc\frac{1}{T}dT = mc\ln\left(\frac{T_2}{T_1}\right)$

$S = m_A \times C_A \times \ln\left(\frac{T_F}{T_A}\right) + m_B \times C_B \times \ln\left(\frac{T_F}{T_B}\right)$ ※

第四題 (Problem 11.3)

11.3

1. All will be the same with the lowest initial temperature.
assume T_1 is the lowest temperature, C_1 is the lowest heat capacity

$$U_{max} = \sum_{j=1}^{N} C_j \times (T_j - T_1) \quad \text{※}$$

2. The final temperature is the lowest initial temperature.

$$U_{max} = \sum_{j=1}^{N} \int_{T_1}^{T_j} A_j T^3 dT \quad \text{※}$$

第五題 (Problem 12.1)

12.1 $S = k_B N\left(\frac{3}{2}\ln\left(\frac{U}{N}\right) + \ln\left(\frac{V}{N}\right) + \ln X\right)$

$$\Rightarrow \ln U = \frac{2}{3}\frac{S}{k_B N} - \frac{2}{3}\ln\frac{V}{N} - \frac{2}{3}\ln X + \ln N$$

① $U = e^{\frac{2}{3}\frac{S}{k_B N}} - \left(\frac{V}{N}\right)^{2/3} - (X)^{2/3} + N$

② $U = \frac{3}{2}k_B N T$

$$\Rightarrow T = \frac{2}{3k_B N}\left(e^{\frac{2S}{3k_B N}} - \left(\frac{V}{N}\right)^{2/3} - (X)^{2/3} + N\right)$$

③ $dU = TdS - PdV + \mu dN$

$$dF = dU - TdS - SdT$$
$$= -SdT - PdV + \mu dN$$

④ $F(T,V,N) = U - TS$

$$dF = dU - TdS - SdT = -SdT - PdV + \mu dN$$

第六題 (Problem 12.2)

Problem 12.2

Helmholtz free energy:

$F = U - TS$

$$\Rightarrow dF = dU - TdS - SdT$$
$$= TdS - PdV + \mu dN - TdS - SdT$$
$$= -SdT - PdV + \mu dN$$

Gibbs

$G = F + PV$

$$\Rightarrow dG = dF + PdV + VdP$$
$$= -SdT - PdV + \mu dN + PdV + VdP$$
$$= -SdT + VdP + \mu dN$$

第七題 (Problem 12.3)

Problem 12.3

承上題: $dG = -SdT + VdP + \mu dN$

題目: $G = AT + BT^2 + CP + DP^2 + ETP$

(1)

$$V = \left(\frac{\partial G}{\partial P}\right)_{T,N} = C + 2DP + ET \ \#$$

(2)

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N} = -\frac{2D}{C+2DP+ET} \ \#$$

(3)

$$S = -\left(\frac{\partial G}{\partial T}\right)_{P,N} = -A - 2BT - EP \ \#$$

第八題 (Problem 12.4)

Problem 12.4

(1)

This is only one expression of the fundamental thermondynamic relation. It may be expressed in other ways, using different variables. #.

$\Rightarrow H = U + PV$

$\Rightarrow dH = dU + PdV + VdP$

$\Rightarrow dH = TdS - PdV + \mu dN + PdV + VdP$

$\Rightarrow dH = TdS + VdP + \mu dN$

(2)

定 P.N

$dH = TdS = đQ = C_p dT$

$$\Rightarrow C_p = \left(\frac{\partial H}{\partial T}\right)_{P,N} = \frac{\partial}{\partial T}\left(A + BT + CP + DT^2 + EP^2 + FTP\right)$$

$$\Rightarrow C_p = B + 2DT + FP \ \#$$