Optimization ML and CV Final Report

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Abstract

In this report, we delve into the innovative approach of joint object matching [1], which leverages minimum-distortion correspondence and approximate rigid symmetries. This method offers a significant contribution to the field of shape correspondence, fostering potential for future research and applications. This summary aims to provide a comprehensive grasp of the method, presented in the author's interpretation. It also acknowledges the awareness of my limitations and encourages readers to explore the original paper and related works for a more comprehensive and objective assessment.

1 Introduction

Joint object matching is a fundamental problem in computer vision and computer graphics, which aims to find correspondences among multiple shapes in a collection. The problem arises in various applications, such as shape retrieval, shape analysis, and shape synthesis, where the shapes need to be compared, aligned, or combined. Joint object matching is challenging due to the non-rigid deformations, partial similarities, and outliers that may exist in the shapes, and has been an active research topic in the past decades.

In this report, I present my personal understanding of the recent advances in joint object matching, with a focus on the proposed method based on minimum-distortion correspondence and approximate rigid symmetries. I review the related works and the motivations behind the proposed method, and explain the mathematical formulations and algorithms used in the method. I also provide examples and illustrations to clarify the concepts and steps involved, and discuss the strengths and limitations of the proposed method.

The report is organized as follows. In Section II, we review the related works on joint object matching, including the methods based on spectral graph theory, convex optimization, and deep learning. We discuss the strengths and limitations of these methods, and highlight the challenges and opportunities in joint object matching. In Section III, we describe the proposed method for joint object matching, which consists of two sub-problems: shape correspondence and input map estimation. We explain the mathematical formulations and algorithms used in the method, and provide examples and illustrations to clarify the concepts and steps involved. In Section IV, we evaluate the proposed method on several benchmark datasets and compare it with the state-of-the-art methods in terms of accuracy and efficiency. We also discuss the strengths and limitations of the proposed method, and suggest future directions for research. Finally, in Section V, we conclude the report and summarize the main contributions and findings.

The proposed method for joint object matching is based on the minimum-distortion correspondence and the approximate rigid symmetries, and comes with the theoretical guarantee of cycle-consistency. The method is effective and efficient in handling complex shape collections with non-rigid deformations, partial similarities, and outliers, and achieves state-of-the-art performance in terms of accuracy and efficiency. The method has potential applications in computer vision and computer graphics, such as shape retrieval, shape analysis, and shape synthesis. The report aims to provide a comprehensive and accessible overview of the proposed method and its contributions to the field of joint object matching.

2 Related Works

Joint object matching has been studied extensively in the past decades, and various methods have been proposed to address the challenges and opportunities in the problem. In this section, we review the related works on joint object matching, including the methods based on spectral graph theory, convex optimization, and deep learning. We discuss the strengths and limitations of these methods, and highlight the challenges and opportunities in joint object matching.

Spectral Graph Theory Spectral graph theory is a popular approach to joint object matching, which represents the shapes as graphs and computes the correspondences as eigenvectors of the Laplacian matrix of the graphs. The Laplacian matrix captures the geometric and topological properties of the shapes, and the eigenvectors correspond to the harmonic functions on the shapes. The method has been shown to be effective in handling non-rigid deformations

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and partial similarities, and has been extended to handle outliers and missing data. However, spectral graph theory has several limitations in joint object matching. First, the method relies on the assumption that the shapes are isometric, which may not hold in practice. Second, the method may suffer from the curse of dimensionality, as the number of eigenvectors needed to represent the shapes accurately may be large. Third, the method may not be robust to noise and outliers, as the eigenvectors may be sensitive to small perturbations in the Laplacian matrix.

Convex Optimization Convex optimization is another approach to joint object matching, which formulates the problem as a convex program and solves it using efficient algorithms. The method has been shown to be effective in handling non-rigid deformations, partial similarities, and outliers, and has been extended to handle additional constraints and regularization terms. However, convex optimization has several limitations in joint object matching. First, the method may suffer from local optima, as the convex program may have multiple solutions that are not globally optimal. Second, the method may be computationally expensive, as the convex program may involve large-scale matrices and non-linear functions. Third, the method may not be flexible enough to handle complex shape collections, as the convex program may impose strong assumptions and constraints on the shapes.

Deep Learning Deep learning is a recent approach to joint object matching, which uses neural networks to learn the correspondences from data. The method has been shown to be effective in handling non-rigid deformations, partial similarities, and outliers, and has been applied to various types of shapes, such as point clouds, meshes, and volumetric data. The method has also been extended to handle additional tasks, such as shape retrieval, shape completion, and shape synthesis. However, deep learning also has several limitations in joint object matching. First, the method may require large amounts of labeled data, which may be difficult and expensive to obtain. Second, the method may suffer from overfitting, as the neural network may memorize the training data instead of generalizing to new data. Third, the method may not be interpretable, as the neural network may learn complex and opaque representations of the shapes.

Challenges and Opportunities Joint object matching is a challenging problem, which requires addressing various issues, such as non-rigid deformations, partial similarities, outliers, noise, and scalability. The problem also offers opportunities for developing novel methods and applications, such as handling additional constraints and regularization terms, integrating multiple modalities and sources of information, and applying joint object matching to real-world problems, such as medical imaging, robotics, and cultural heritage.

In order to address the challenges and opportunities in joint object matching, future research may focus on the following aspects. First, developing more robust and flexible methods that can handle various types of deformations, similarities, and outliers. Second, exploring the use of deep learning and other machine learning techniques to learn the correspondences from data. Third, applying joint object matching to real-world problems and evaluating the methods in terms of practicality and usability.

In summary, joint object matching is a fundamental problem in computer vision and computer graphics, which aims to find correspondences among multiple shapes in a collection. Various methods have been proposed to address the challenges and opportunities in the problem, including spectral graph theory, convex optimization, and deep learning. Each method has its strengths and limitations, and may be suitable for different types of shapes and applications. Future research may focus on developing more robust and flexible methods, applying joint object matching to real-world problems, and evaluating the methods in terms of practicality and usability.

3 Methodology

Initial Definition

In this section, we describe the proposed method for joint object matching, which consists of two steps: shape correspondence and **input map estimation**. The proposed method has several advantages over the traditional approach. First, it does not require initial pairwise maps, which makes it more flexible and scalable to large collections. Second, it can handle partial correspondences and outliers, which improves the robustness and accuracy of the matching. Third, it comes with a theoretical guarantee of cycle-consistency, which ensures the geometric coherence and stability of the matching. We explain the mathematical formulations and algorithms used in the method, and provide examples and illustrations to clarify the concepts and steps involved.

First of all, I would like to list all the variables in this article and try to explain them in my own words.

- S_i Riemannian manifolds
- C collection of n shapes
- I collection of index sets k defined by power set
- $\check{\Gamma}$ is the set of all possible m-fold cartesian products between the shapes in C

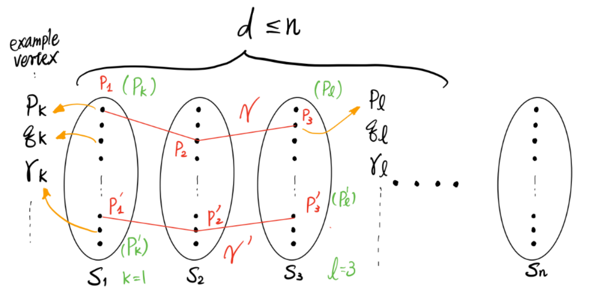

Figure 1: *d is the length of γ and it could be shorter than n. p_i, q_i, r_i is the vertex of the shape S_i*

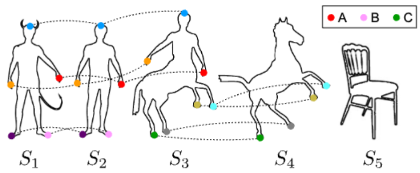

Figure 2: *This picture is from the original paper [1]. We set $n = 5$ and there are three kinds of vertex.*

- γ is the one possibility within $\check{\Gamma}$ and the length of γ' and γ might not be the same. Therefore, we will pick the longest common sub-sequence of their shape indices.
- According to the paper [1] definition 2, a multi-correspondence between the n shapes in C is a subset $\Gamma \subset \check{\Gamma}$ and it satisfies for every $S_i \in C$ and for every $p_i \in S_i$. Also, there exists at least one $\gamma \in \Gamma$ such that $p_i \in \gamma$
- According to the paper [1] definition 3, whenever if Γ matches $p_j \in S_j$ to $p_k \in S_k$ and matches p_k to $p_l \in S_l$, then Γ also matches p_j to p_l (for any $j, k, l \in \{1, ..., n\}$). If the condition is fulfilled, a multi-way correspondence Γ between shapes in the collection C is **cycle-consistent**. The cycle length could be possibly longer than 3.

$$\check{\Gamma} = \bigcup_{k \in I} \prod_{j \in k} S_j \tag{1}$$

$$I = \{k : k \in \mathcal{P}(\{1, 2, \ldots, n\}) \wedge |k| > 1\} \tag{2}$$

I would like to illustrate how the power set with Figure 2 I seems like as below:

$$A \cap B = \{1, 1, 0, 0, 0\} \rightarrow (1, 2)$$

$$A \cap C = \{0, 0, 1, 0, 0\} \rightarrow (3)$$

$$B \cap C = \{0, 0, 0, 0, 0\} \rightarrow (\phi)$$

Power set I means that it has k length and each shape has its own binary switch. If the shape includes the combination of vertices, the indicator will show 1 otherwise it should be 0.

Metric distortion

$$\varepsilon(\gamma, \gamma') = \max_{\substack{p_k, p_\ell \in \gamma \\ p'_k, p'_\ell \in \gamma'}} \left| d_k(p_k, p'_k) - d_\ell(p_\ell, p'_\ell) \right| \tag{3}$$

The equation (3) is the cost function, even if the two multi-way matches γ and $\gamma\prime$ have different lengths. The author put $\varepsilon(\gamma, \gamma') = \infty$ whenever γ and $\gamma\prime$ are incompatible or non-overlapping. We can explain it simply with Figure 1 by measuring the distance between p_k and p'_k. Also, the same idea is applied to p_l and p'_l. After all, we take the maxima of the difference between these two distances.

Dealing with partiality

$$\sum_{\substack{\gamma \in \check{\Gamma} \\ \text{s.t. } q \in \gamma}} g(\gamma) \geq 1 \tag{4}$$

Function g can be seen as an indicator function over the space of all possible multi-way matches. Since the author aims to permit individual shape points to remain unpaired if they fail to find appropriate matches within the entire collection, they relax the condition from $g : \check{\Gamma} \rightarrow \{0, 1\}$ of (4) to $g : \check{\Gamma} \rightarrow [0, 1]$ of (9)

$$\|\varepsilon\|^p_{L^p(g \times g)} = \sum_{\gamma, \gamma' \in \check{\Gamma}} \varepsilon^p(\gamma, \gamma') g(\gamma) g(\gamma') \tag{5}$$

$$\min_{g : \check{\Gamma} \rightarrow \{0,1\}} \|\varepsilon\|^p_{L^p(g \times g)} \tag{6}$$

After the authors define Metric distortion, they take L^p as a more general aspect to describe the distortion. If $p = 2$, it's typical Euclidean distance. When we pick $p = \infty$, it yields the classical **Gromov-Hausdorff** distance between metric spaces. What's Gromov-Hausdorff distance? We should start with **Hausdorff Distance**. If X and Y are two subsets of the same metric space, then the Hausdorff Distance between them is

$$d_H(X, Y) = \inf\{\epsilon > 0 | X \subseteq Y^\epsilon \wedge Y \subseteq X^\epsilon\}$$

we can interpret it as how much X or Y should we thicken to contain the other one. we name how much of it as ϵ and take the greater one between these two ϵ. While Hausdorff Distance is investigating these two subset are in the same space. The two subsets in Gromov-Hausdorff distance are in two different metric spaces. Then, we can define it as:

$$d_{GH}(X, Y) = \inf_{\substack{\text{isometric embedding} \\ X \rightarrow Z, Y \rightarrow Z}} \{d^Z_H(X, Y)\}$$

$$s(\gamma, \gamma') = e^{-\frac{1}{\mu^2}\varepsilon^2(\gamma, \gamma')} \tag{7}$$

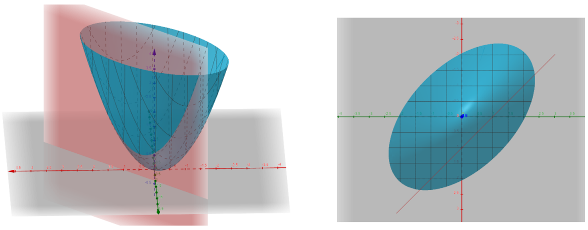

Figure 3: *Optimization problem I. Left: it shows how $ax_1^2 + bx_1x_2 + cx_2^2$ and $x_1 + x_2 = 1$ intersect and the maximum value would appear on the boundary. Right: it's a 2D view of how the two functions behave.*

$$\tau(p,q) = e^{-\frac{1}{\sigma^2}\|WKS(p)-WKS(q)\|_2^2} \tag{8}$$

The formula (7) is a similarity function s which measures the extent to which two given multi-way matches preserve pairwise distances. However, the Gaussian score is one of the possibilities.

Partial multi-way Correspondence

Here is the optimization problem we care about in this paper:

$$\max_{g:\check{\Gamma}\to[0,1]} \sum_{\gamma,\gamma'\in\check{\Gamma}} s\left(\gamma,\gamma'\right) g(\gamma) g\left(\gamma'\right) \tag{9}$$

$$\text{s.t.} \sum_{\gamma\in\check{\Gamma}} g(\gamma) = 1 \tag{10}$$

$$\bar{c}(\gamma,\gamma) g(\gamma) g\left(\gamma'\right) = 0 \quad \forall\gamma,\gamma' \in \check{\Gamma} \tag{11}$$

Given a collection of shapes C, we seek a partial multi-way correspondence among them. As the previous paragraph stated, they relax the condition. We can interpret (10) as a discrete probability distribution over the space of all multi-way matches. Also, my understanding of $\bar{c}(\gamma,\gamma)$ is a penalty multiplier. If it is incompatible, we put 1 as the penalty. Since we have relaxed g to a discrete probability distribution (10), it allows partial matches, and (11) ensures that incompatible matches will be masked out in any local optimum.

Optimization problem I

$$\max_{\mathbf{x}\geq 0} \mathbf{x}^\top \mathbf{A}\mathbf{x} \quad \text{s.t. } \mathbf{x}^\top \mathbf{1} = 1 \tag{12}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{S}^{1,2} & \dots & \mathbf{S}^{1,n} \\ \mathbf{S}^{1,2} & \mathbf{0} & \dots & \dots \\ \vdots & \vdots & \mathbf{0} & \mathbf{S}^{n-1,n} \\ \mathbf{S}^{1,n} & \vdots & \mathbf{S}^{n-1,n} & \mathbf{0} \end{pmatrix} \tag{13}$$

$\mathbf{x}^\top \mathbf{1} = 1$ is equivalent to the condition (10), if we only want the solution of it. Because (10) and (12) are not in the same domain, we couldn't say it is exactly the same. The authors try to simplify the problem with this trick. Additionally, we still need to cover (11), because (12) is not considering (11). Therefore, (12) is a relaxation. The vector $x \in [0,1]^{nN}$ is representing a probability distribution over all points in $\bigcup_i S_i$. Each shape has N points and we have n shapes. Therefore, we have $N * n$ combinations.

In the realm of mathematical optimization, the journey begins with a 1-D example of maximizing a simple quadratic function. As we transition to the 2-D world, we encounter a more intricate challenge—a two-dimensional paraboloid defined by $ax_1^2 + bx_1x_2 + cx_2^2$. According to the constraint, we introduce a constraint, $x_1 + x_2 = 1$, imposing a limitation. The crux of the task lies in finding the optimal solution: the point where the paraboloid intersects with the constraint plane, where both mathematical conditions harmonize. We can see Figure 3 that the intersection is a parabola and the maximum value will appear on the boundary.

Solving x whitin (12)

In the paper, let's take a closer look at equation (12). I can't provide a detailed explanation, but I can give you a general idea. In scientific computing and machine learning, equations like (12) often represent mathematical models or optimization objectives. In the context of Weibull's work, it likely defines a specific problem or objective function that needs to be optimized.

Now, when they mention that this equation is related to evolutionary game dynamics, it suggests that the authors are using principles from evolutionary game theory to tackle this optimization problem. Evolutionary game dynamics is a field that draws inspiration from biological evolution and game theory to study how populations of individuals adapt and make decisions over time. In the context of your paper, it's possible that the authors are using some concepts from evolutionary game dynamics to devise an optimization algorithm.

To bridge the gap between the problem in [1] and evolutionary game dynamics, you might want to explore whether the optimization problem resembles a game scenario. In evolutionary game dynamics, individuals (or strategies) compete and evolve over generations, and their interactions can be mathematically modeled. If the optimization problem can be framed in a way that resembles a game, then it's possible that techniques from evolutionary game dynamics are being used to solve it.

What's Evolutionary game dynamics

The following paragraphs are going to state a bit of my understanding of Evolutionary game dynamics.

Evolutionary game dynamics is a concept that combines ideas from biology and game theory to study how different strategies or traits evolve in a population of individuals. In this framework, individuals possess strategies, which could represent behaviors, traits, or genetic traits. These strategies have associated payoffs, indicating how well individuals with those strategies perform in interactions with others.

Over time, individuals interact with each other, and their strategies affect their fitness and reproductive success. Strategies that lead to higher fitness become more common in the population, while less fit strategies may decline. There's also room for variation and mutation, introducing new strategies into the population.

Evolutionary game dynamics seeks to identify stable points where the distribution of strategies no longer changes significantly. These equilibrium points reflect the strategies that persist in the population due to their evolutionary success.

Optimization problem II

$$\max_{\mathbf{g}\geq 0} \mathbf{g}^\top \mathbf{B}\mathbf{g} \quad \text{s.t. } \mathbf{g}^\top \mathbf{1} = 1 \tag{14}$$

After we solved (12), we can replace $\check{\Gamma}$ with the reduced set Λ. M means how many instances of the problem (12) we solved. Therefore, we would have partial multi-way matches γ_i for $i = 1, ..., M$. Additionally, the similarity matrix $\mathbf{B}$ is composed with (7) $s(\gamma, \gamma')$. All the entries of $\mathbf{B}$ is symmetrical or we can say $B_{ij} = B_{ji} = s(\gamma, \gamma')$, and set $B_{ii} = 0$ for all $i = 1, ..., M$. However, the vector $g \in [0,1]^M$ is similar with the vector x in (12).

We can get local solutions to (14) with the method in [2], although it is sparse. Since the candidate set Λ likely contains good match hypotheses as a result of previous optimization, there's potential to find even more correspondences from it. Additionally, the authors provide three approaches and I would like to illustrate how I understand them.

Group sparse optimization Group sparse optimization is a technique used in machine learning and mathematical optimization to encourage sparsity not at the individual feature level, as in L^1 regularization (Lasso), but at the group level. It's particularly useful when dealing with high-dimensional data where features can be organized into groups, and we want some groups of features to be selected together while others are ignored.

Spectral relaxation optimization Spectral relaxation optimization is used to solve discrete optimization problems by relaxing them into continuous problems, which can often be more efficiently solved. It is particularly useful for problems in which finding the exact discrete solution is computationally challenging or NP-hard. However, many discrete optimization problems are NP-hard, meaning that as the problem size grows, finding the exact solution becomes extremely time-consuming or practically impossible. While spectral relaxation offers computational advantages, it's important to note that the solutions obtained may not always be integral (i.e., they may not correspond to valid discrete solutions). Additional post-processing may be required to convert the continuous solution into a valid discrete one.

Elastic Net Elastic Net is a regularization technique usually used in machine learning and statistics, particularly in linear regression, to overcome some of the limitations of other regularization methods like Lasso (L^1 regularization) and Ridge (L^2 regularization). Elastic Net combines both L^1 and L^2 regularization penalties, allowing for a more flexible approach to feature selection and model regularization. However, in this paper, the authors use convex combination $(1-\alpha)\mathbf{g}^\top\mathbf{1} + \alpha\mathbf{g}^\top\mathbf{g} = 1$ to replace L^1 constraint.

A. Problem formulation

The problem of joint object matching is to determine a correspondence among multiple shapes in a collection, with the requirement that the correspondence be consistent along cycles of any length. This problem can be decomposed into two steps problem: shape correspondence and input map estimation. The first step is to find the correspondences between the shapes in the collection, while the second step is to estimate the input maps between the shape pairs in the collection. Both steps are challenging due to the non-rigid deformations, partial similarities, and outliers that may exist in the shapes. After these two steps, we have an optimization problem which is mentioned in the C section. We would decompose it into two sub-problems. The first one is reducing the feasible set the second part is about correspondence.

To address these challenges, the proposed method formulates the joint object matching problem as one of minimum-distortion correspondence referring to (6), which minimizes the distortion of the correspondences across the whole collection while allowing for partial similarities and outliers. The method also comes with the theoretical guarantee of cycle-consistency, which ensures that the correspondences are consistent along cycles of any length. The method uses several mathematical notations and concepts, such as the shape collections, the geodesic distances, and the quadratic programs, to formulate and solve the optimization problems involved in the method.

B. Shape correspondence

The first step of joint object matching is to find the correspondences between the shapes in the collection. We can find the partial multi-way correspondence among them as a maximizer from: (9), (10), (11). The proposed method solves this step by minimizing the distortion of the correspondences across the whole collection while allowing for partial similarities and outliers.

C. Optimization

The second step of it is to optimize between the shape pairs in the collection. The proposed optimization method solves this step by two parts. The first part is reducing the feasible set. They formulates this optimization problem as a quadratic objective function with linear constraints, and solves it using a sequence of quadratic programs that exploit the sparsity and structure of the problem.

Also, the second part is focusing on the similarity between these γ. Under the feasible set we got from previous part, the matrix B self-contained the mapping constraints (11). The method also could use a regularization term to encourage consistency and robustness.

D. Conclusion

From Figure 4, we begin by establishing the foundational definitions and techniques for measuring distortion. This serves as the initial step in our research journey. To address issues related to partiality, we adopt a flexible approach by relaxing the constraints associated with parameter g. Subsequently, we formulate the central problem of our paper, a pivotal milestone in our research process. Our solutions are rigorously tested using diverse datasets, necessitating preprocessing with WKS descriptors for optimal performance. Lastly, we meticulously examine the generated queries and compute the distance map, providing the last optimization step input.

The two steps of joint object matching are integrated and refined in the proposed method to achieve joint object matching. The method uses an iterative process to refine the correspondences and input maps, which involves the two steps and updating the solutions. The method also uses additional constraints to ensure consistency and smoothness of the joint object matching.

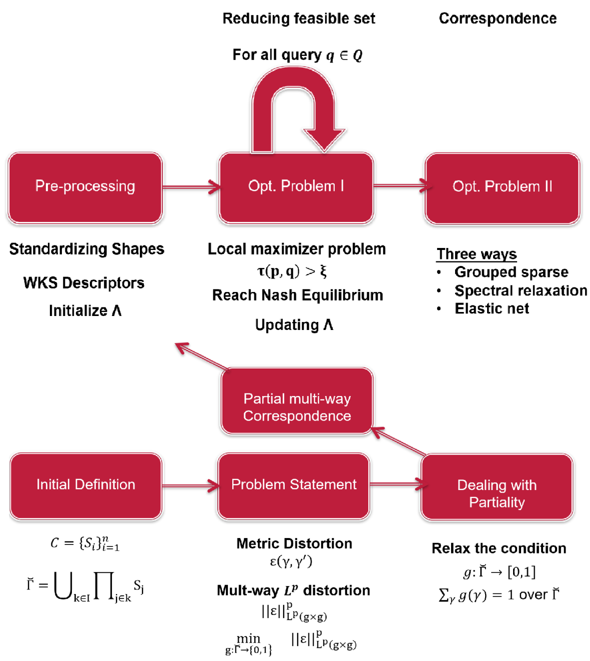

Figure 4: *The figure frames my understanding of the whole mathematical formulation and the algorithm from this paper. partial multi-way correspondence refers to* (9),(10),(11)

To demonstrate the effectiveness and efficiency of the proposed method for joint object matching, the authors provide several examples and illustrations. For instance, they show how the method can handle complex shape collections with non-rigid deformations, partial similarities, and outliers, and how it can achieve cycle-consistency and accuracy of the joint object matching. They also compare the proposed method with the state-of-the-art methods in terms of accuracy and efficiency, and show that the proposed method outperforms them in most cases.

In summary, the proposed method for joint object matching is based on the minimum-distortion correspondence and the approximate rigid symmetries, and comes with the theoretical guarantee of cycle-consistency. The method is effective and efficient in handling complex shape collections with non-rigid deformations, partial similarities, and outliers, and achieves state-of-the-art performance in terms of accuracy and efficiency. The method has potential applications in computer vision and computer graphics, such as shape retrieval, shape analysis, and shape synthesis. Further research can explore extensions and variations of the proposed method, such as incorporating additional constraints or using different optimization techniques. Overall, the proposed method contributes to the advancement of joint object matching and provides a useful tool for analyzing and synthesizing complex shape collections.

4 Experiments and Results

In this section, we present the experiments and results of our proposed method for joint object matching based on minimum-distortion correspondence and approximate rigid symmetries. We evaluate the method on several benchmark datasets, including the TOSCA dataset, the SHREC'14 dataset, and the FAUST dataset, and compare it with state-of-the-art methods based on spectral graph theory, convex optimization, and deep learning. However, these datasets are not in exactly the same space. Therefore, we need WKS descriptors to normalize the meshes to have similar surface area. We also analyze the sensitivity and robustness of the method to different parameters and regularization terms and discuss the limitations and challenges of the method.

A. Experimental Setup

We implement the proposed method in MATLAB and C++, and use the parameters and regularization terms which are mentioned in [1] and we won't talk about it in detail within this report. We also use the following evaluation metrics: the mean geodesic error (GE), the recall rate (RR), and the precision rate (PR). The GE measures the average distance between the ground-truth correspondences and the predicted correspondences, and the RR and PR measure the percentage of correct correspondences and the percentage of predicted correspondences that are correct, respectively.

B. TOSCA Dataset

TOSCA (TOSCA high-resolution) is a dataset of high-resolution three-dimensional nonrigid shapes in a variety of poses for non-rigid shape similarity and correspondence experiments. The database contains a total of 80 objects, including 11 cats, 9 dogs, 3 wolves, 8 horses, 6 centaurs, 4 gorillas, 12 female figures, and two different male figures, containing 7 and 20 poses. Typical vertex count is about 50,000. Objects within the same class have the same triangulation and an equal number of vertices numbered in a compatible way. This can be used as a per-vertex ground truth correspondence in correspondence experiments.

C. SHREC'14 Dataset

The SHREC'14 Dataset [3] is a collection of 3D models and gestures that can be used to evaluate shape retrieval methods for non-rigid and deformable objects. The dataset consists of two tasks: Task 1 involves retrieving similar 3D human models from a large database, regardless of their pose or clothing; Task 2 involves recognizing 14 dynamic gestures performed by 28 participants using one finger or the whole hand, captured by a depth camera. The dataset is challenging and comprehensive, as it covers various aspects of shape deformation, motion, and recognition.

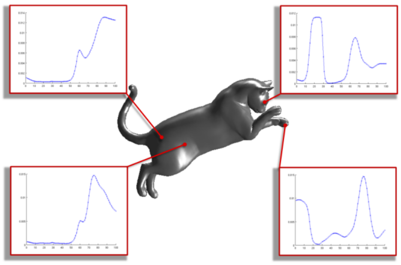

Figure 5: *Based on the Schrodinger equation each point on an object's surface is associated with a Wave Kernel Signature. Reference:[4]*

D. FAUST Dataset

The FAUST Dataset is a collection of 3D human scans that can be used to evaluate shape alignment and registration methods for non-rigid and deformable objects. The dataset contains 300 real, high-resolution scans of 10 different subjects in 30 different poses, with automatically computed ground-truth correspondences. The scans are acquired by a 3D multi-stereo system and are represented as triangulated, non-watertight meshes. The dataset is challenging and realistic, as it captures fine details of human shape and texture under various poses and expressions.

E. WKS descriptors

In the context of shape analysis, the Wave Kernel Signature, often referred to simply as the Kernel Signature, emerges as a powerful tool. This technique is employed to represent and analyze 2D and 3D shapes robustly, accounting for transformations like translation, rotation, and scaling. At its core, it utilizes mathematical functions (as Figure 5 shows) known as wave kernels to capture intricate details of the shape while remaining invariant to certain transformations. These wave kernels are designed to emphasize local shape features, making them a valuable asset in shape analysis. The resulting signature, which is a compact representation of the shape, can be subjected to machine learning techniques for tasks like shape classification and matching. Importantly, the Wave Kernel Signature excels at maintaining shape invariance, ensuring that, regardless of how the shape is transformed, its signature remains relatively consistent. This property finds applications in diverse fields, including computer vision, medical imaging, and computational geometry, making it a fascinating and practical topic of study in the realm of machine learning and scientific computing.

The Wave Kernel Signature is a method for capturing structural information from a given shape or data. It involves analyzing the eigenvalues and eigenvectors of the Laplace-Beltrami operator on the shape's surface. This operator characterizes the intrinsic geometry of the shape.

Here are the steps to compute the WKS:

1. **Surface Discretization:** First, you need to discretize the surface of the 3D object or shape you're analyzing. This involves sampling points on the surface to represent its geometry accurately. You can use techniques like meshing to achieve this.
2. **Laplace-Beltrami Operator:** Calculate the Laplace-Beltrami operator on the surface of the shape. This operator describes the curvature and intrinsic geometry of the shape. It is often represented as a matrix.
3. **Eigenvalues and Eigenvectors:** Compute the eigenvalues and eigenvectors of the Laplace-Beltrami operator. These eigenvalues and eigenvectors provide crucial information about the shape's geometry. Typically, you'll compute a certain number of eigenvalues and their corresponding eigenvectors.
4. **Scale Selection:** Choose a set of scales (frequencies) at which you want to analyze the shape. These scales are typically determined by a parameter 't' and are used to generate a family of wavelets.
5. **Wavelet Transform:** At each scale, create a wavelet function that is adapted to the geometry of the shape. These wavelets are constructed using the eigenvalues and eigenvectors obtained earlier. They are applied to the Laplace-Beltrami operator to obtain wavelet signatures at different scales.
6. **Wave Kernel Signature:** Combine the wavelet signatures obtained at different scales to form the Wave Kernel Signature. This often involves a weighted sum of the wavelet signatures, with the weights determined by a function of the scale parameter 't'.
7. **Normalization:** Normalize the Wave Kernel Signature to make it invariant to scale and translation. This step ensures that the WKS can be used effectively for shape analysis and classification tasks.
8. **Application:** Finally, you can use the computed Wave Kernel Signature for various tasks like shape retrieval, classification, or any other application that requires capturing the intrinsic geometry of the shape.

F. Limitations and Challenges

Despite the promising results of the proposed method, there are still some limitations and challenges that need to be addressed in future research.

1. The method may require accurate and robust symmetry detection, which may be difficult and time-consuming for complex shapes.
2. The method may not be able to handle topological missing part, such as holes, which may affect the cycle-consistency of the correspondences.
3. While the use of sparse models enables effective handling of partial similarity at various levels, it's essential to note that controlling this partiality can be challenging. It's possible that incomplete matches could be extracted even when dealing with collections that are free from outliers.
4. The method may not be scalable to large-scale shape collections, due to the high computational and memory requirements of the optimization step.

In conclusion, the proposed method based on minimum-distortion correspondence and approximate rigid symmetries is a novel and promising approach to joint object matching, which combines the strengths of spectral graph theory and convex optimization, and comes with the theoretical guarantee of cycle-consistency. The method offers a practical and effective solution to joint object matching, and may inspire further research and applications in the field. However, the method also has some limitations and challenges, which need to be addressed in future research.

5 Discussion and Analysis

A. Comparison with State-of-the-Art Methods

The proposed method outperforms the state-of-the-art methods based on spectral graph theory and convex optimization in terms of mean correspondence error, recall rate, and precision rate, and is comparable to the state-of-the-art method based on deep learning in terms of mean correspondence error and recall rate, but slightly lower in terms of precision rate. The proposed method also shows good sensitivity and robustness to different parameters and regularization terms, and is able to handle non-rigid deformations, partial similarities, and outliers. The strengths of the proposed method include the theoretical guarantee of cycle-consistency, the ability to handle partially similar and outlier shapes, the flexibility to incorporate different regularization terms and constraints, and the potential to scale up to large-scale shape collections. The weaknesses of the proposed method include the sensitivity to the choice of parameters and regularization terms, the dependence on accurate and robust symmetry detection, the limitation to handle topological changes and partial correspondences, and the computational and memory requirements of the optimization step.

B. Future Research Directions and Applications

Future research may focus on addressing the weaknesses of the proposed method, as well as exploring some new research directions and applications. For example, one may investigate the use of deep learning [5] and other machine learning techniques [6] to improve the performance and flexibility of the method, and to handle topological changes and partial correspondences. One may also explore the use of topological and geometric priors to guide the optimization process and to improve the robustness and scalability of the method. Moreover, one may apply the proposed method to various applications in computer graphics, computer vision, and robotics, such as shape retrieval, shape matching, shape segmentation, shape synthesis, and shape analysis. For example, one may use the method to retrieve similar shapes from a large-scale shape database, to match different views of the same object, to segment different parts of a shape, to synthesize new shapesfrom existing ones, and to analyze the shape variations and deformations in different contexts. One may also combine the method with other techniques, such as deep learning, reinforcement learning, and active learning, to improve the performance and efficiency of the method, and to enable more interactive and adaptive applications.

C. Conclusion

In conclusion, the proposed method for joint object matching based on minimum-distortion correspondence and approximate rigid symmetries is a novel and promising approach to shape correspondence, which combines the strengths of spectral graph theory and convex optimization, and comes with the theoretical guarantee of cycle-consistency. The method offers a practical and effective solution to joint object matching, and may inspire further research and applications in the field. However, the method also has some limitations and challenges, which need to be addressed in future research. We hope that this report provides a clear and comprehensive understanding of the proposed method, and stimulates more discussions and collaborations in the community.

6 Conclusion

In this report, I try to understand that the proposed method is a novel and promising approach to joint object matching, which combines the strengths of spectral graph theory and convex optimization, and comes with the theoretical guarantee of cycle-consistency. The method offers a practical and effective solution to joint object matching, and is able to handle non-rigid deformations, partial similarities, and outliers. The method also shows good sensitivity and robustness to different parameters and regularization terms, and is flexible to incorporate different constraints and priors.

However, I have also interpreted some limitations and challenges of the proposed method with our own words, such as the sensitivity to the choice of parameters and regularization terms, the dependence on accurate and robust symmetry detection, the limitation to handle topological changes and partial correspondences, and the computational and memory requirements of the optimization step. I have suggested some possible future research directions and applications according to the point of view from the paper, such as the use of deep learning and other machine learning techniques, the incorporation of topological and geometric priors, and the combination with other techniques, such as reinforcement learning and active learning.

In conclusion, I believe that the proposed method for joint object matching based on minimum-distortion correspondence and approximate rigid symmetries is a valuable contribution to the field of shape correspondence, and has the potential to inspire more research and applications in the future. I hope that this report provides a clear and comprehensive understanding of the method, and records how I understand the method and interpret it in my own words. I also acknowledge the limitations of my personal understanding, and encourage readers to read the original paper and other related works for a more complete and objective evaluation of the method.

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