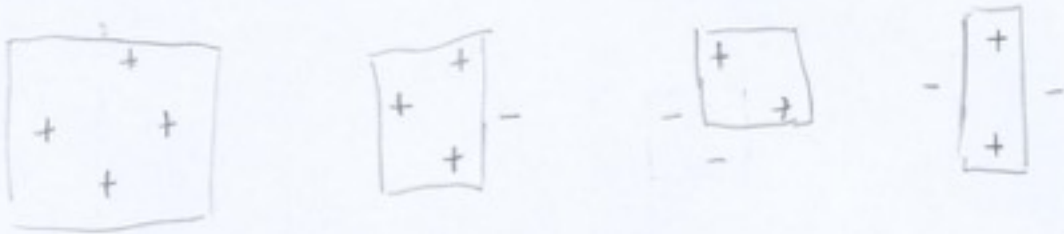
Machine learning assignment 2 103062512

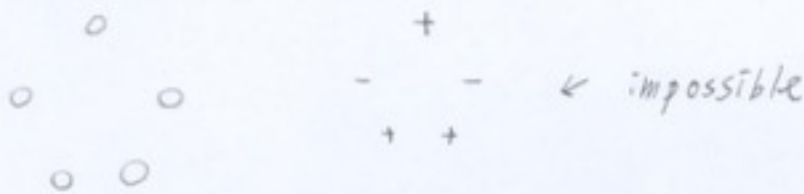
1.

1° show that VC-dim is at least 4.



2° show that an axis-aligned rect can't shatter 5 points or more

For any 3 points which share the same line, it is impossible to shatter. So the only case we need to consider is



Thus, the VC-dim is 4. #

2. $\frac{b-f(z)}{b-a} e^{\eta a} + \frac{f(z)-a}{b-a} e^{\eta b}$

$= (1-\theta)e^{\eta a} + \theta e^{\eta b}$ where $\theta = \frac{f(z)-a}{b-a}$

$\geq e^{\eta[(1-\theta)a+\theta b]}$

$= e^{\eta f(z)}$ for some $f(z) \in [a,b]$

$\therefore E[e^{\eta f(z)}] \leq \frac{b-E[f(z)]}{b-a}e^{\eta a} + \frac{E[f(z)]-a}{b-a} e^{\eta b}$ #

3.

$$S_H(N) \le \sum_{k=0}^{v} C_k^N$$

since $N > v$, we have $1 > \frac{v}{N} \geq 0$

$$\left(\frac{v}{N}\right)^v S_H(N) \le \left(\frac{v}{N}\right)^v \sum_{k=0}^{v} C_k^N \le \sum_{k=0}^{v} \left(\frac{v}{N}\right)^k C_k^N$$

$$= e^v$$

$$\therefore \left(\frac{v}{N}\right)^v S_H(N) \le e^v$$

$$\Rightarrow S_H(N) \le \left(\frac{eN}{v}\right)^v \#$$

4.

$$Var[x] = E[x^2] - (E[x])^2$$

let $x = 1(g^*(x) \neq r)$ and $P(1(g^*(x) \neq r)) = p$

$$\Rightarrow E[x] = 1 \cdot p + 0 \cdot (1-p) = p$$

$$E[x^2] = 1^2 \cdot p + 0^2 \cdot (1-p) = p$$

$$\therefore Var[x] = p - p^2 = -(p - \tfrac{1}{2})^2 + \tfrac{1}{4} \le \tfrac{1}{4}$$

$$\Rightarrow Var[x] \le \frac{1}{4} \#$$