Layout Algebra

Maths behind CuTe's APIs to do tiling, reshape, selection, permutation …

Coalsece

Simplify layout if possible.

Composition

Layout can be considered as a mapping from coordinates to indices.

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Functional composition, R := A o B
R(c) := (A o B)(c) := A(B(c))

Example

A = (6,2):(8,2)
B = (4,3):(3,1)

R( 0) = A(B( 0)) = A(B(0,0)) = A( 0) = A(0,0) =  0
R( 1) = A(B( 1)) = A(B(1,0)) = A( 3) = A(3,0) = 24
R( 2) = A(B( 2)) = A(B(2,0)) = A( 6) = A(0,1) =  2
R( 3) = A(B( 3)) = A(B(3,0)) = A( 9) = A(3,1) = 26
R( 4) = A(B( 4)) = A(B(0,1)) = A( 1) = A(1,0) =  8
R( 5) = A(B( 5)) = A(B(1,1)) = A( 4) = A(4,0) = 32
R( 6) = A(B( 6)) = A(B(2,1)) = A( 7) = A(1,1) = 10
R( 7) = A(B( 7)) = A(B(3,1)) = A(10) = A(4,1) = 34
R( 8) = A(B( 8)) = A(B(0,2)) = A( 2) = A(2,0) = 16
R( 9) = A(B( 9)) = A(B(1,2)) = A( 5) = A(5,0) = 40
R(10) = A(B(10)) = A(B(2,2)) = A( 8) = A(2,1) = 18
R(11) = A(B(11)) = A(B(3,2)) = A(11) = A(5,1) = 42

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1D coord in B -> 2D coord in B -> index in B, 1D coord in A -> 2D coord in A -> index in A

A(B(11))      = A(B(3,2))      = A(11)                      = A(5,1)        = 42
```

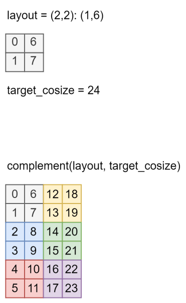
Computing Composition

Composition Tilers

A `Tiler` is one of the following objects.

- A `Layout`.
- A tuple of `Tiler`s.
- A `Shape`, which will be interpreted as a tiler of `Layout`s with stride-1.

Complement



Division (Tiling)

`logical_divide(A, B)` splits a layout `A` into two modes

- in the first mode are all elements pointed to by `B` and
- in the second mode are all elements not pointed to by `B`, which is an iterator over each tile of `B`.

$$
\begin{aligned}
A \oslash B &= (4,2,3):(2,1,8) \;\; \oslash \;\; 4:2 \\
&\to ((2,2),(2,3)):((4,1),(2,8))
\end{aligned}
$$

Coord c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$A(c)$	0	2	4	6	1	3	5	7	8	10	12	14	9	11	13	15	16	18	20	22	17	19	21	23
$(A \oslash B)(c)$	0	4	1	5	2	6	3	7	8	12	9	13	10	14	11	15	16	20	17	21	18	22	19	23

Product (Tiling)

`logical_product(A, B)` results in a two mode layout where

- the first mode is the layout `A` and - the second mode is the layout `B` but with each element replaced by a “unique replication” of layout `A`

$$
\begin{aligned}
A \otimes B &= (2,2):(4,1) \;\; \otimes \;\; 6:1 \\
&\to ((2,2),(2,3)):((4,1),(2,8))
\end{aligned}
$$

Coord c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$A(c)$	0	4	1	5																				
$B(c)$	0	1	2	3	4	5																		
$(A \otimes B)(c)$	0	4	1	5	2	6	3	7	8	12	9	13	10	14	11	15	16	20	17	21	18	22	19	23