

Homework3

工科所 R08525116 吳承哲

Part 1

3.22

3.22
a)
 $V = [1 \ 2 \ 1]^T$ $W^T = [2 \ 1 \ 3]$
藉由定義, separable kernel 由 VW^T 形成
因此 VW^T 為 separable
b)
 $W = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \ 3 \ 1]$
 $W_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
 $W_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \#$

3.28

3.28.
a) 由 table 3.6 得知, Gaussian 的 convolution 還是 Gaussian.
b)
 $\sigma = \sqrt{1.5^2 + 2^2 + 4^2}$
 $= 4.717 \#$
c)
 $\lceil 6\sigma \rceil \times \lceil 6\sigma \rceil$
 $\rightarrow \lceil 28.302 \rceil \times \lceil 28.302 \rceil = 29 \times 29 \#$

3.44

3.44

a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rank} = 2 \rightarrow \text{Nonseparable} \#$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rank} = 2 \rightarrow \text{Nonseparable} \#$$

b)

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{rank} = 2 \rightarrow \text{Nonseparable} \#$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{rank} = 2 \rightarrow \text{Nonseparable} \#$$

c)

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{rank} = 1 \quad V = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad W = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \#$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{rank} = 1 \quad V = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \quad W = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \#$$

4.3

4.3

$$a) \quad \delta(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau) \delta(t-t_0-\tau) d\tau = \delta(t-t_0) \#$$

b)

$$\delta(t-t_0) * \delta(t+t_0) = \int_{-\infty}^{\infty} \delta(\tau-t_0) \delta(t+t_0-\tau) d\tau$$

$$\hat{=} \alpha = \tau - t_0$$

$$\tau = \alpha + t_0$$

$$\Rightarrow \int_{-\infty}^{\infty} \delta(\alpha) \delta(t-\alpha) d\alpha = \delta(t) \#$$

4.51

4.51

Laplacian kernel: $f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$

從 table 4.4 得知。

$$G(u, v) = F(u, v) \left[e^{j\frac{2\pi u}{M}} + e^{-j\frac{2\pi u}{M}} + e^{j\frac{2\pi v}{N}} + e^{-j\frac{2\pi v}{N}} - 4 \right]$$

$$= F(u, v) H(u, v)$$

$H(u, v)$ is the filter transfer function in frequency-domain

$$H(u, v) = 2 \left(\cos \frac{2\pi u}{M} + \cos \frac{2\pi v}{N} - 2 \right)$$

$H(u, v)$ 在 $[-\frac{M}{2}, \frac{M}{2}]$ 的週期中。 $\begin{cases} \text{在原點附近} \rightarrow \text{靠近} 0. \\ \text{在極值附近} \rightarrow \text{有最大的負值} \end{cases}$

\therefore Laplacian kernel \rightarrow high pass filter *