Principles and Applications of Digital Image Processing

【Fall, 2020】
工科所 R08525116 吳承哲

Homework 2

Part 1: (30%)

Solve the problems 2.12, 2.16, 2.18, 2.37, 3.12, 3.18 in the textbook.

2.12

$i(x,y) = 255 e^{-[(x-x_0)^2+(y-y_0)^2]}$, $0 < i(x,y) < \infty$

$r(x,y) = 1$

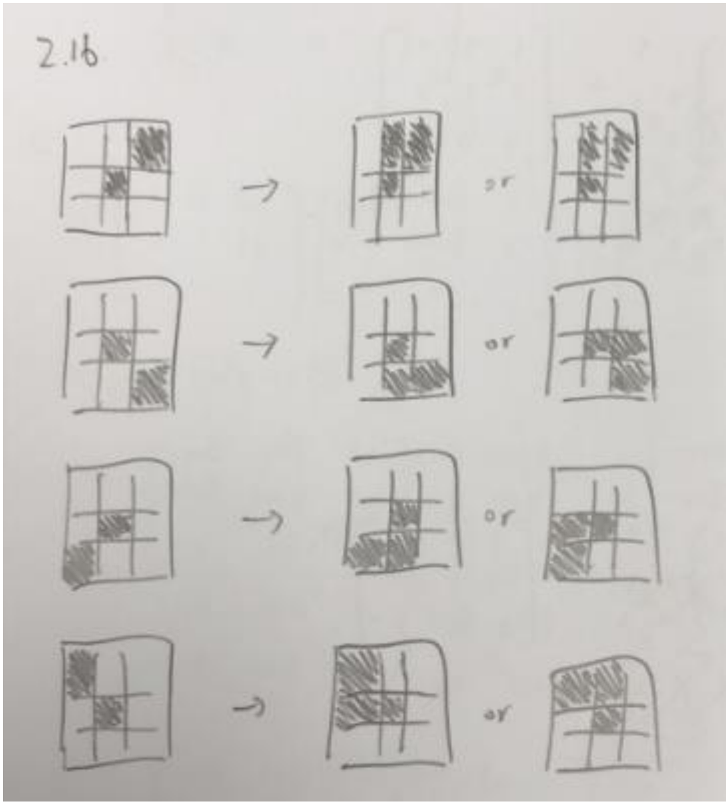
$f(x,y) = i(x,y) \cdot r(x,y)$

$= 255 e^{-[(x-x_0)^2+(y-y_0)^2]}$

$\Delta G = \frac{255+1}{2^k} \geq 8$

$k \leq 5$ #

2.16



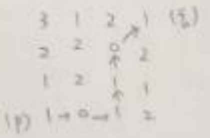
2.18

(a)

$D_4(p,q) = |x_p - x_q| + |y_p - y_q| = 3+3 = 6$ #

$D_8(p,q) = \max(|x_p - x_q|, |y_p - y_q|) = \max(3,3) = 3$ #

$D_m(p,q) = 5$ #



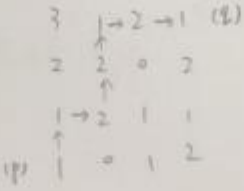
(b)

D_4, D_8 are independent of V

$D_4(p,q) = 6$ #

$D_8(p,q) = 3$ #

$D_m(p,q) = 6$ #



2.37

(a)

$$A = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \det(A) = C_x C_y \quad \operatorname{adj} A = \begin{bmatrix} C_y & 0 & 0 \\ 0 & C_x & 0 \\ 0 & 0 & C_x C_y \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{adj} A}{\det(A)} = \frac{1}{C_x C_y}\begin{bmatrix} C_y & 0 & 0 \\ 0 & C_x & 0 \\ 0 & 0 & C_x C_y \end{bmatrix} = \begin{bmatrix} C_x^{-1} & 0 & 0 \\ 0 & C_y^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ #

(b)

$$A = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \quad \det(A) = 1 \quad \operatorname{adj} A = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$ #

(c)

Vertical

$$A = \begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \det(A) = 1 \quad \operatorname{adj} A = \begin{bmatrix} 1 & -s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ #

Horizontal

$$A = \begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \det(A) = 1 \quad \operatorname{adj} A = \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ #

(d)

$$A=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \det(A)=\cos^2\theta+\sin^2\theta = 1$$

$$\operatorname{adj}A=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{matrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\\ 0 & 0\end{matrix}$$

$$\begin{matrix}\cos\theta & -\sin\theta & 0 & \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta & 0 & \sin\theta & \cos\theta\end{matrix} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$ #

(e)

$$A=\begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta & t_x\\ \sin\theta & \cos\theta & t_y\\ 0 & 0 & 1\end{bmatrix}$$

$$\det(A)=\cos^2\theta+\sin^2\theta=1$$

$$\operatorname{adj}A=\begin{bmatrix}\cos\theta & -\sin\theta & t_x\\ \sin\theta & \cos\theta & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{matrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\\ 0 & 0\end{matrix}$$

$$\begin{matrix}\cos\theta & -\sin\theta & t_x & \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta & t_y & \sin\theta & \cos\theta\end{matrix}$$

$$=\begin{bmatrix}\cos\theta & \sin\theta & -t_y\sin\theta - t_x\cos\theta\\ -\sin\theta & \cos\theta & t_x\sin\theta - t_y\cos\theta\\ 0 & 0 & 1\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\cos\theta & \sin\theta & -t_y\sin\theta - t_x\cos\theta\\ -\sin\theta & \cos\theta & t_x\sin\theta - t_y\cos\theta\\ 0 & 0 & 1\end{bmatrix}$$ #

3.12

3.18

(a)

0	0	0
0	1	0
0	1	0

1	2	1
2	4	2
1	2	1

(b)

$$f_p = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

$$w \bigstar f = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

$$= \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) f(3-s, 4-t)$$

$5\times5 \to 7\times7$

center point $(3,7)$ $(3,9)$

$$w = \begin{bmatrix} 1&2&1 \\ 2&4&2 \\ 1&2&1 \end{bmatrix}$$

$w(0,0)f_p(3,4) = 4$ $(w \bigstar f)(3,4) = 4+2 = 6$

$w(-1,0)f_p(4,4) = 2$

$$(w \bigstar f)(x,y) = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 0&0&1&2&1&0&0 \\ 0&0&3&6&3&0&0 \\ 0&0&4&8&4&0&0 \\ 0&0&3&6&3&0&0 \\ 0&0&1&2&1&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

(c) 因為 $w = \begin{bmatrix} 1&2&1 \\ 2&4&2 \\ 1&2&1 \end{bmatrix}$ 為對稱矩陣

$$(w \star f)(x,y) = (w \bigstar f)(x,y)$$