

1. b

$$0.01 \left(1 - \frac{12}{N}\right) \geq 0.006$$

$$0.4 \geq \frac{12}{N}$$

$$N \geq 30$$

2. C

由講義第9章第10頁得知.

invertible  $X^T X$ .  $\rightarrow$  unique solution  $\rightarrow \nabla E_{in}(w) = 0$

3. C

1a) rank 不变

1b) rank 不变

1c) rank 会改变.

1d) rank 不变

1e) rank 不变.

4. e

根据 Hoeffding's inequality  $P[|v - u| > \epsilon] \leq 2 \exp(-2\epsilon^2 N)$

$$\frac{1}{N} \sum z(\hat{y} - y_n)^2$$

$$\frac{\partial}{\partial \hat{y}} E_{in}(\hat{y}) = \frac{1}{N} \sum z(\hat{y} - y_n)$$

$$\hat{y} = 0 \quad v = \frac{1}{N} \sum y_n$$

$$\nabla E_{in}(\hat{y}) = \frac{1}{N} \sum (-z) \cdot y_n$$

$-2v$

$$-\nabla E_{in}(\hat{y}) = 2v$$

5. c

$$\text{likelihood}(h) = P(X_1)h(X_1) \times P(X_2)h(X_2) \dots P(X_N)h(X_N)$$

$\therefore$  uniform distribution

$$\therefore P(x) = \frac{1}{\theta}$$

$$\begin{aligned} \text{likelihood} &= \frac{1}{\theta} \cdot y_1 \times \frac{1}{\theta} \cdot y_2 \dots \\ &= \prod_{n=1}^N \frac{y_n}{\theta} \end{aligned}$$

6. b

$$-\nabla E_{in} = \frac{1}{N} \sum yx$$

$$E_{in} = \frac{1}{N} \sum (-y w^T x)$$

$$\hat{\sum} y_n = -1, \quad w^T x = 1.$$

$$w^T x \neq y_n.$$

$$\begin{aligned} \text{Err}(w, x, y) &= \max(0, -y w^T x) \\ &= \max(0, 1) \\ &= 1 \end{aligned}$$

7. a

$$\frac{\partial}{\partial w} \text{Err}_{\text{exp}}(w, x, y) = \frac{\partial}{\partial w} \exp(-y w^T x)$$

$$\nabla \text{Err}_{\text{exp}}(w, x, y) = -y x \exp(-y w^T x)$$

$$-\nabla \text{Err}_{\text{exp}}(w, x, y) = y x \exp(-y w^T x)$$

8. b.

$$E(w) \approx E(w) + bE(u)^T(v) + \frac{1}{2}(v)^T A_{E(w)}(v) \quad w \leftarrow u+v.$$

$$E'(v) \approx bE(u)^T + A_{E(w)} \cdot v = 0.$$

$$v = -bE(u)^T \cdot A_{E(w)}^{-1}$$

$$= -A_{E(w)}^{-1} bE(u)$$

9. b.

$$\begin{aligned} E_{in}(w) &= \frac{1}{N} |xw - y|^2 \\ &= \frac{1}{N} (w^T x^T x w - 2w^T x^T y + y^T y) \end{aligned}$$

$$\nabla E_{in} = \frac{1}{N} (2x^T x w - 2x^T y)$$

$$\nabla^2 E_{in} = \frac{2}{N} (x^T x)$$

10. b.

$$e_{err}(w, x, y) = -\sum [y=k] \ln h_k(x)$$

$$= - \left( [y=k] \frac{1}{h_k(x)} - (1 - [y=k]) \frac{1}{1 - h_k(x)} \right) \cdot \frac{\partial h_k(x)}{\partial w_i}$$

$$= - \underbrace{h_k(x)(1 - h_k(x))}_{\substack{\uparrow \\ \text{from } [y=k] \frac{1}{h_k(x)} - (1 - [y=k]) \frac{1}{1 - h_k(x)}}} \cdot x_i \left( [y=k] \frac{1}{h_k(x)} - (1 - [y=k]) \frac{1}{1 - h_k(x)} \right)$$

$$= -x_i \left( [y=k] (1 - h_k(x)) - (1 - [y=k]) h_k(x) \right)$$

$$= -x_i \left( [y=k] - \cancel{[y=k]} h_k(x) - h_k(x) + \cancel{[y=k]} h_k(x) \right)$$

$$= (h_k(x) - [y=k]) \cdot x_i$$

Code

14~20

d、c、c、b、a、b、d

讀檔案

```
def load_data(file):
    f = open(file, "r")
    data = f.read()
    f.close()

    all_data = []
    all_label = []

    for line in data.split("\n"):
        sub_data = []
        i = 0
        for value in line.split("\t"):
            if value != "" and i == 10:
                value = float(value)
                all_label.append(value)
                i += 1
            elif value != "":
                value = float(value)
                sub_data.append(value)
                i += 1
        if sub_data != []:
            all_data.append(sub_data)

    for i in range(len(all_data)):
        all_data[i].insert(0, 1)

    all_data = np.array(all_data)
    all_label = np.array(all_label)
    return all_data, all_label

train_data, train_label = load_data("./hw3_train.dat.txt")
test_data, test_label = load_data("./hw3_test.dat.txt")
```

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```
def linear_regression(x, y):
    x_pinv = np.linalg.pinv(x)
    w = np.dot(x_pinv, y)
    return w

w = linear_regression(train_data, train_label)
se_arr = []
for i in range(len(train_data)):
    pred = np.dot(train_data[i], w)
    se = (pred - train_label[i])**2
    se_arr.append(se)
Ein = np.mean(se_arr)
print(E_in)
```

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```
def SGD_LR(x, y, lr, Ein):
    epoch = 0
    Ein_SGD = 100
    w = np.zeros(11)
    while Ein_SGD > Ein*1.01:
        num = random.randint(0, 999)
        w = w + lr * 2 * (y[num] - np.dot(w, x[num].T))
    * x[num]
    Ein_SGD = np.square(np.dot(w, x.T) - y)
    Ein_SGD = np.mean(Ein_SGD)
    epoch += 1
    return epoch

epoch_arr = []
for i in range(1000):
    print(i)
    epoch = SGD_LR(train_data, train_label, 0.001, Ein)
    epoch_arr.append(epoch)
ans = np.mean(epoch_arr)
print(ans)
```

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```
def theta(s):
    return 1 / (1 + np.exp(-s))

def SGD_Logistic(x, y, lr):
    w = np.zeros(11)
    for i in range(500):
        num = random.randint(0, 999)
        grad = theta(-
y[num]*np.dot(w, x[num].T))*(y[num]*x[num])
        w = w + lr*grad
    ce_arr = []
    for j in range(len(x)):
        ce = np.log(1 + np.exp(-
y[j]*np.dot(w, x[j].T)))
        ce_arr.append(ce)
    Ein = np.mean(ce_arr)
    return Ein

Ein_arr = []
for i in range(1000):
    Ein = SGD_Logistic(train_data, train_label, 0.001)
    Ein_arr.append(Ein)
ans = np.mean(Ein_arr)
print(ans)
```

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```
def SGD_Logistic_17(x, y, lr):
    w = linear_regression(x, y)
    for i in range(500):
        num = int(np.random.uniform(0, len(x)))
        grad = theta(-
y[num]*np.dot(w, x[num].T))*(y[num]*x[num])
        w = w + lr*grad
    ce_arr = []
    for j in range(len(x)):
        ce = np.log(1 + np.exp(-
y[j]*np.dot(w, x[j].T)))
```

```

        ce_arr.append(ce)
    Ein = np.mean(ce_arr)
    return Ein

Ein_arr = []
for i in range(1000):
    Ein = SGD_Logistic_17(train_data, train_label, 0.001)
    Ein_arr.append(Ein)
ans = np.mean(Ein_arr)
print(ans)

```

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```

def sign(number):
    if number > 0:
        return 1
    else:
        return -1

def Ein_Eout_diff(train_x, train_y, test_x, test_y):
    w = linear_regression(train_x, train_y)
    error = 0
    for i in range(len(train_x)):
        pred = sign(np.dot(w, train_x[i].T))
        if pred != train_y[i]:
            error += 1
    Ein = error / len(train_x)
    error = 0
    for i in range(len(test_x)):
        pred = sign(np.dot(w, test_x[i].T))
        if pred != test_y[i]:
            error += 1
    Eout = error / len(test_x)
    return np.abs(Ein - Eout)

ans = Ein_Eout_diff(train_data, train_label, test_data,
                    test_label)
print(ans)

```

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```
def trasform(x):
    new_x = []
    for i in range(len(x)):
        tmp = []
        tmp.append(1)
        for j in range(len(x[i])):
            if x[i][j] != 1:
                tmp.append(x[i][j])
                tmp.append(x[i][j]**2)
                tmp.append(x[i][j]**3)
        new_x.append(tmp)
    new_x = np.array(new_x)
    return new_x

new_train_data = trasform(train_data)
new_test_data = trasform(test_data)

ans = Ein_Eout_diff(new_train_data, train_label, new_test_data, test_label)
print(ans)
```

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```
def trasform(x):
    new_x = []
    for i in range(len(x)):
        tmp = []
        tmp.append(1)
        for j in range(len(x[i])):
            if x[i][j] != 1:
                tmp.append(x[i][j])
                tmp.append(x[i][j]**2)
                tmp.append(x[i][j]**3)
                tmp.append(x[i][j]**4)
                tmp.append(x[i][j]**5)
                tmp.append(x[i][j]**6)
                tmp.append(x[i][j]**7)
                tmp.append(x[i][j]**8)
```



```
        tmp.append(x[i][j]**9)
        tmp.append(x[i][j]**10)
    new_x.append(tmp)
new_x = np.array(new_x)
return new_x

new_train_data = trasform(train_data)
new_test_data = trasform(test_data)

ans = Ein_Eout_diff(new_train_data, train_label, new_test_data, test_label)
print(ans)
```