

# **MIE1622: Assignment #2 – Risk-Based and Robust Portfolio Selection Strategies**

Name: Alexander Cheng

Student Number: 1001634298

Date: Friday, March 10<sup>th</sup>, 2023

## Part 1: Implement Investment Strategies in Python (Year 2020-2021)

**Rounding procedure:** After determining the optimal weight and determining the number of stocks in the portfolio, we use the numpy round function to round each stock to the nearest integer. We run a while loop, in which in the first run, if there isn't enough cash in the account only those units that were rounded up will go down by 1. If there still isn't enough account, we overestimate and decrease all stocks by 1 (stocks that are not 0). We then add a unit to each stock that should be rounded up until we go through all the stocks.

**Equal Risk Contribution Portfolio Objective:** Gradient derivation can be found in Appendix A

**Leverage Equal Risk Contributions:** The initial shares in the portfolio in period 1 has been doubled (200% long position), in which the portfolio value calculation considers the borrowed risk-free asset by subtracting the borrowed amount (with compounded interest) from the value of the portfolio. This allows the available risk-free asset to be considered as additional capital present in the investment strategy.

### Robust Mean Variance Optimization:

From the mean variance optimization, we can observe that low variations in expected returns and standard deviations can lead to drastic changes in weight allocations. We want to increase the stability by considering the estimation error within the optimization process.

Target Risk Estimation Error: Used the 1/n portfolio as the target return estimation error. This is a rough general band, if we were to choose it tighter, we may not get a solution to the optimization.

$$\varepsilon_{rob} = w_0^T \theta w_0$$

We set theta to the diagonal of our covariance matrix (the variances), and we used the 1/n portfolio to calculate the weights. Our estimation error is set such that our upper band is weighted evenly among all the assets, this is a very high band since the portfolio variance will be quite high. We can minimize this error by choosing other weights from another strategy but will choose to keep the problem simple by using 1/n.

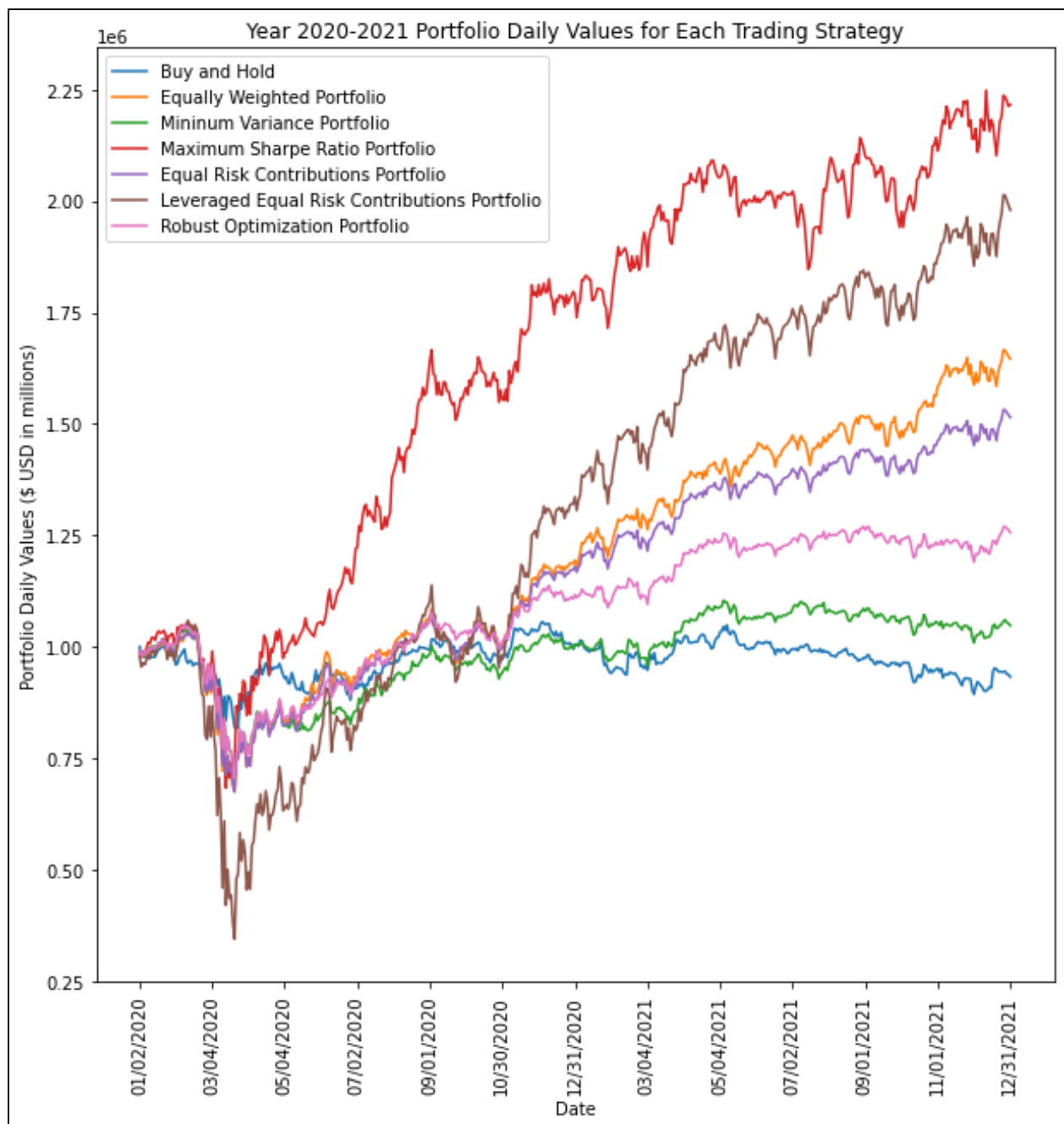
Target Portfolio Return: Used the weights of the minimum variance strategy and calculates the return. This must be greater than the return given by the mean variance portfolio.

$$\varepsilon_{ret} = \mu^T w_{MVP}$$

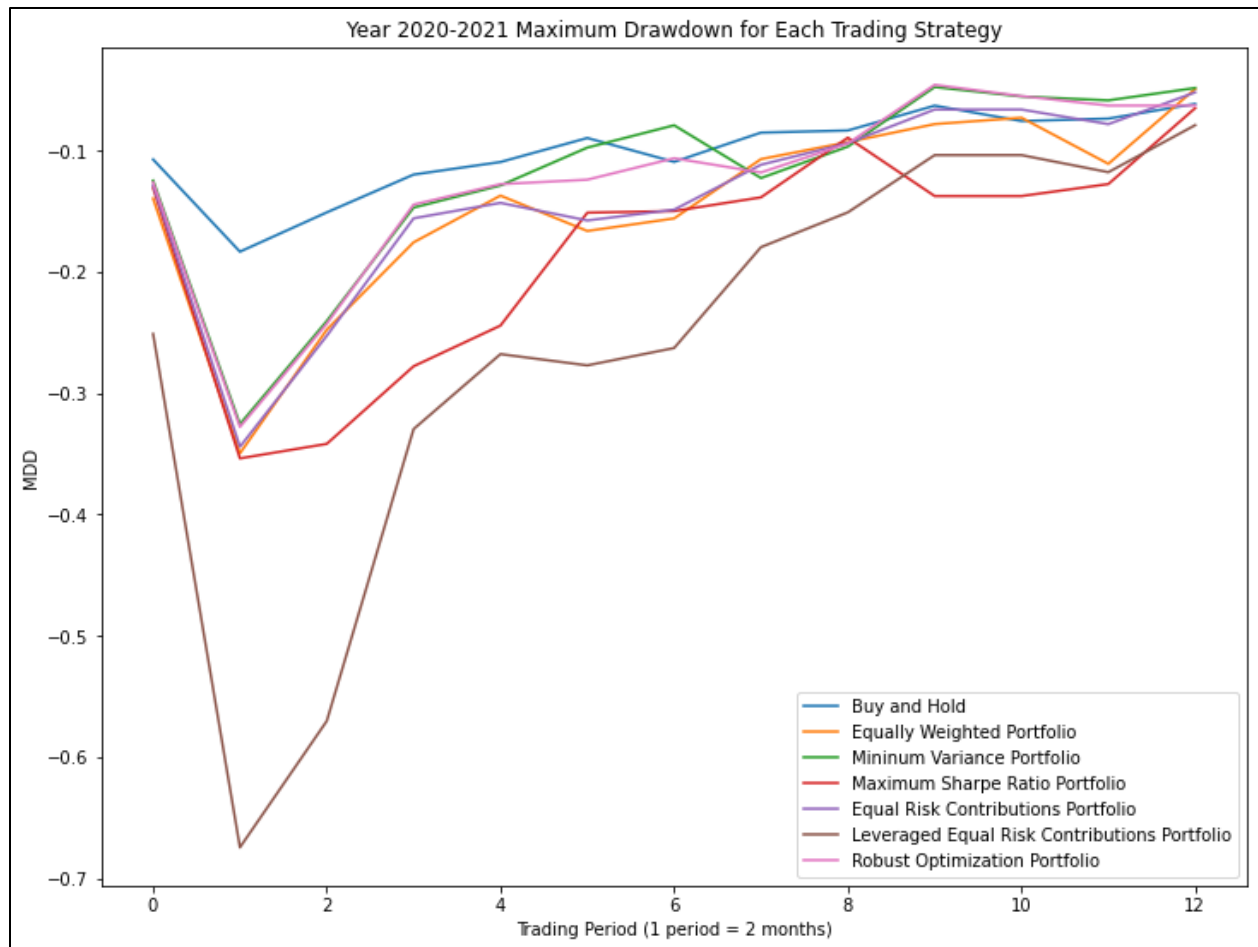
Objectives: Minimize the variance of the portfolio return, maximize the expected portfolio return, and minimize the portfolio return estimation error. We also wanted to compare this robust strategy with a non stable strategy such as the minimum variance strategy.

Constraints: Short selling of assets is not allowed and sum of assets weights must be equal to 1.

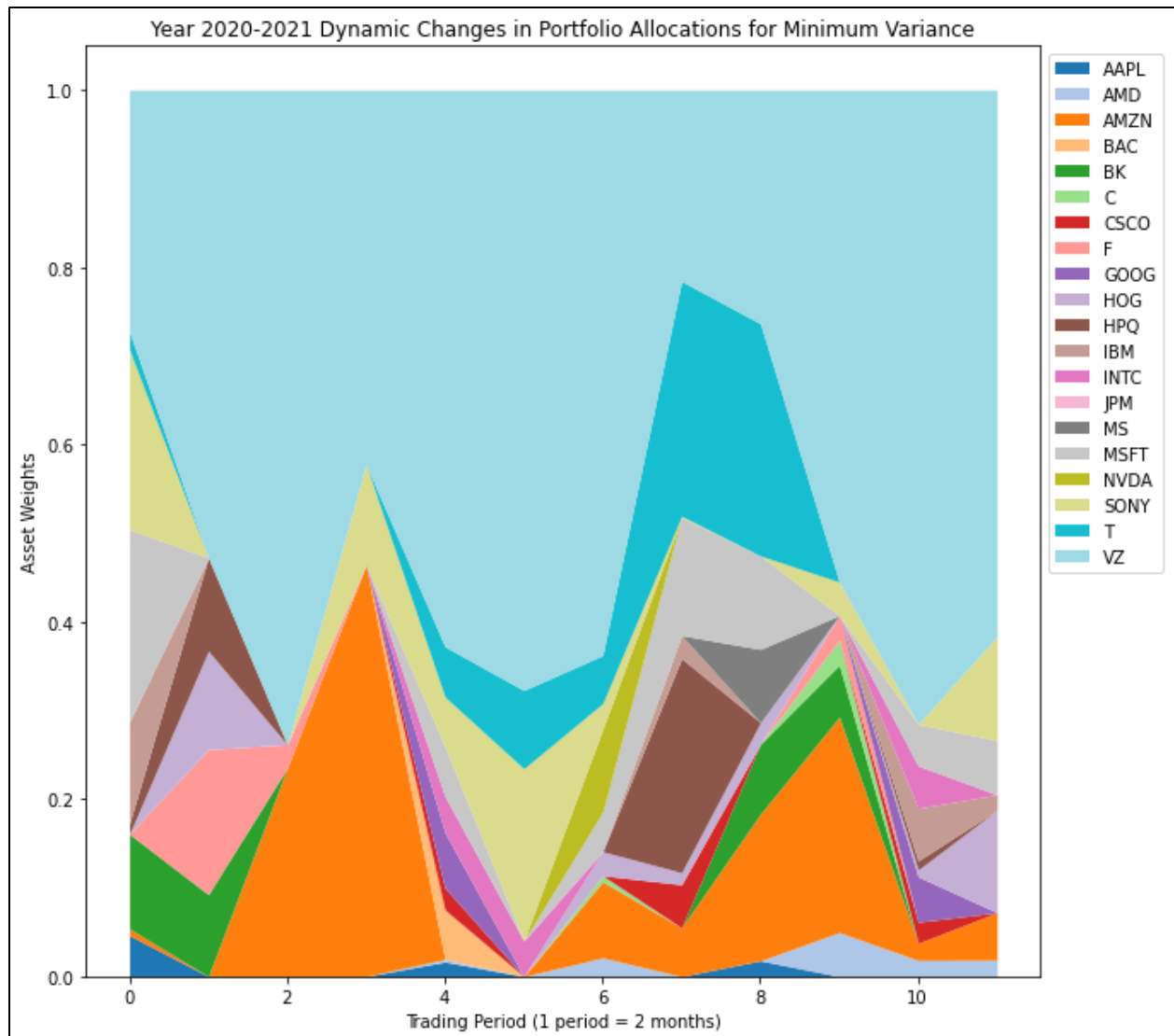
**Part 2: Analyze your results (Year 2021-2022)** – Refer to Appendix A for output.



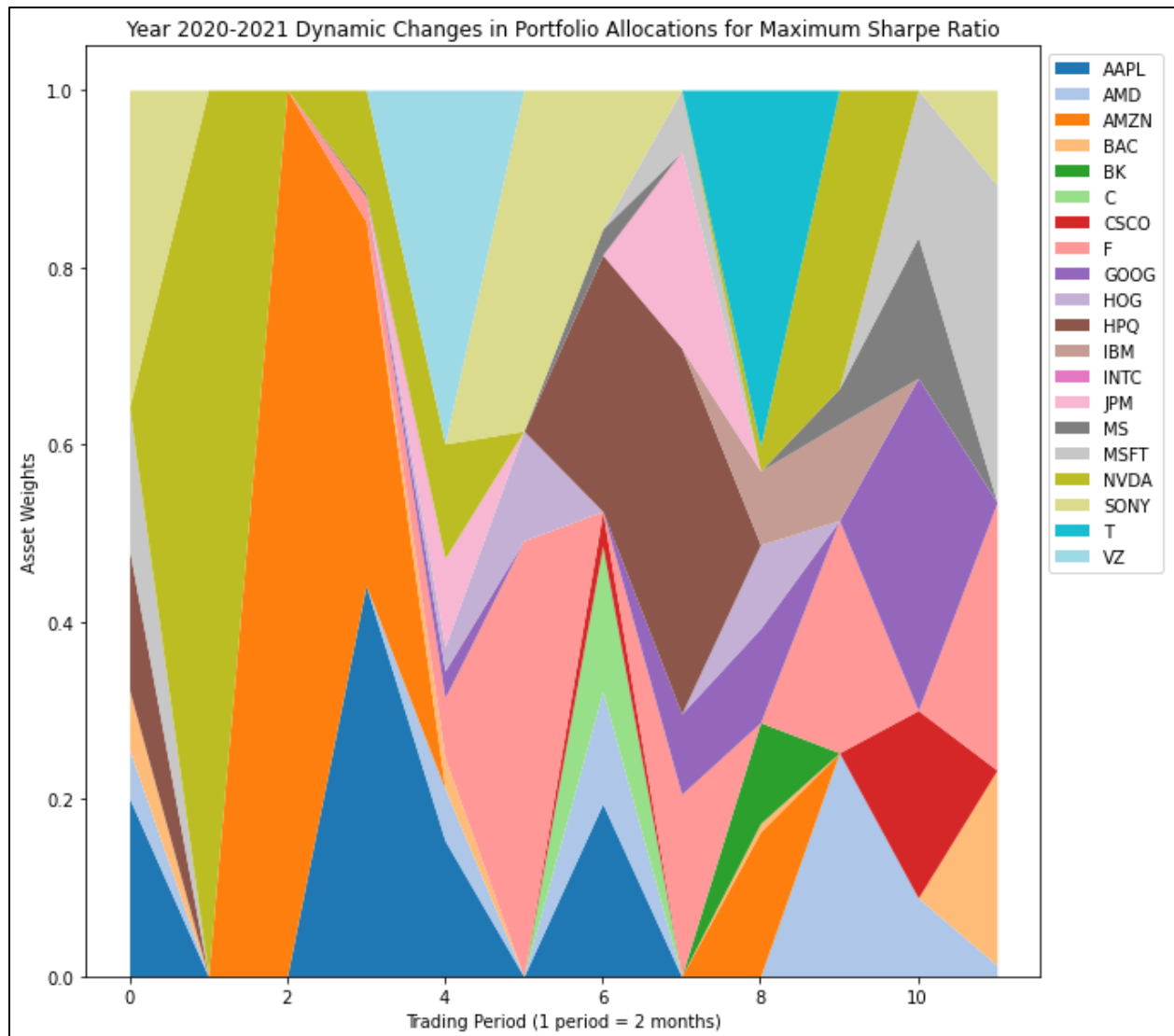
This chart illustrates the daily value of the portfolio for each trading strategy over the years 2020 and 2021 using the daily prices. This will be discussed further below. Main takeaways are that maximum Sharpe's ratio and leverage ERC does very well, many of these strategies start to pick up in value after 10/30/2020.



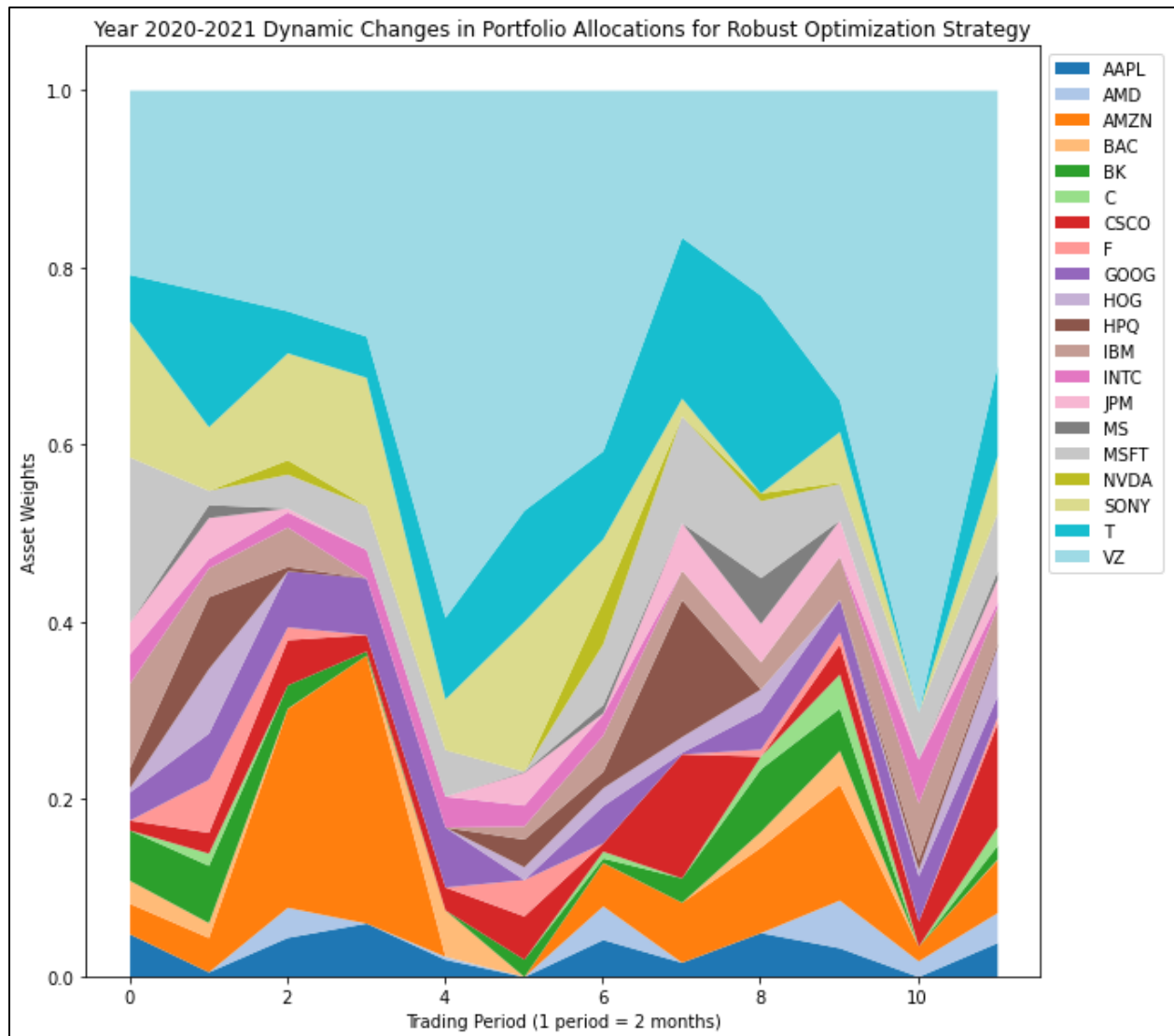
The maximum drawdown is the maximum observed loss from a peak to a trough of a portfolio before a new peak is attained. It indicates the downside risk of a specified time period. We can find this using a rolling window for each period. We can see that the leveraged equal risk contribution has a much larger MDD since it's investing more capital into the stocks. We can observe that the high returns strategies such as the maximum Sharpe ratio and the leveraged ERC have the highest risk (largest MDD).



This chart illustrates the weight allocation of each stock in the Minimum variance portfolio for the 12 trading periods. We can observe that the Verizon (VZ) stock dominates the portfolio through the 12 trading periods. This stock offers the lowest variance among all the stocks. (Assume trading period 0 is period 1).



This chart illustrates the weight allocation of each stock in the Maximum Sharpe's Ratio portfolio for the 12 trading periods. We can observe that there are much more fluctuations among the weight allocation of the stocks. This could be a larger issue if there are larger trading costs.



This chart illustrates the weight allocation of each stock in the Robust Mean-Variance Optimization portfolio for the 12 trading periods. We can observe that the Verizon (VZ) stock still dominates the portfolio through the 12 trading periods similarly to minimum variance. We can observe that the stocks invested are more diversified than the Minimum Variance and the Maximum Sharpe Ratio strategy but also reduced the amount of trading (typically the same assets between periods), this is beneficial if trading costs are high.

Compare trading strategies and discuss their performance relative to each other. Which strategy would you select for managing your own portfolio and why? **(Year 2020 – 2021)**

Initial portfolio value = \$ 1000013.0

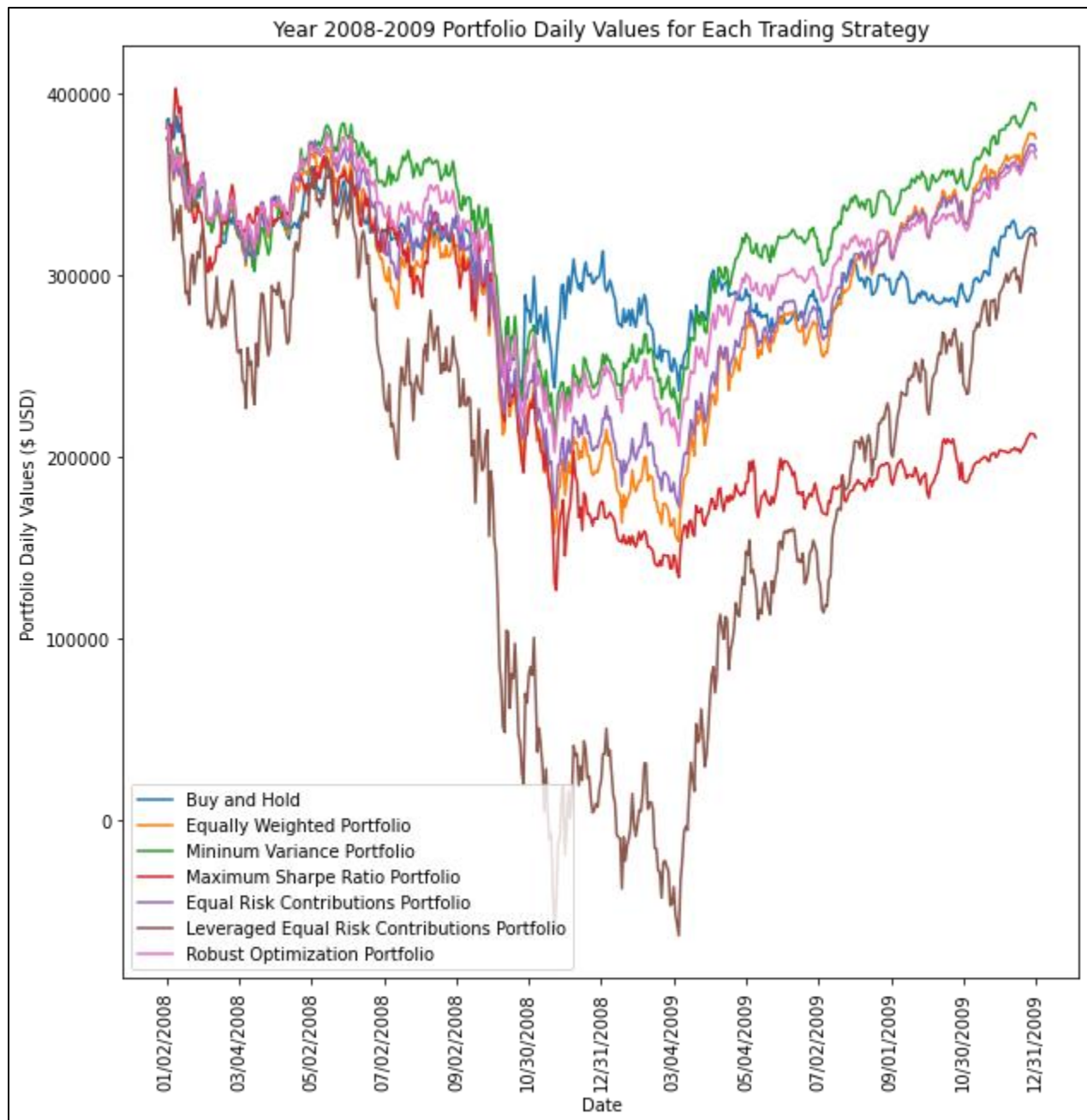
Strategy	Final Portfolio Value (in USD)
Buy and hold	\$ 932471.35
Equally Weighted	\$ 1656226.85
Minimum Variance	\$ 1048387.97
Maximum Sharpe Ratio	\$ 2218942.31
Equal Risk Contributions	\$ 1515001.16
Leveraged Equal Risk Contributions	\$ 1980093.88
Robust Optimization Portfolio	\$ 1256306.78

We could see that the maximum Sharpe ratio strategy has outperformed the other six methods over the 12 trading periods (2 years) in terms of portfolio value. The leveraged equal risk contribution strategy had the second highest portfolio value in which it observed the largest loss in the first few months and then started to rapidly increase its value after the second year. From the MDD graph and what we will see in the next two sections (recession periods), leveraging is risky and need to be careful how we use it. We can observe that the simple equally weighted strategy did well, seeing the third highest portfolio value which performed better than the equal risk contribution strategy came in fourth place. Our robust mean variance strategy had the fifth highest and did better than the minimum variance strategy. Lastly is the buy and hold strategy.

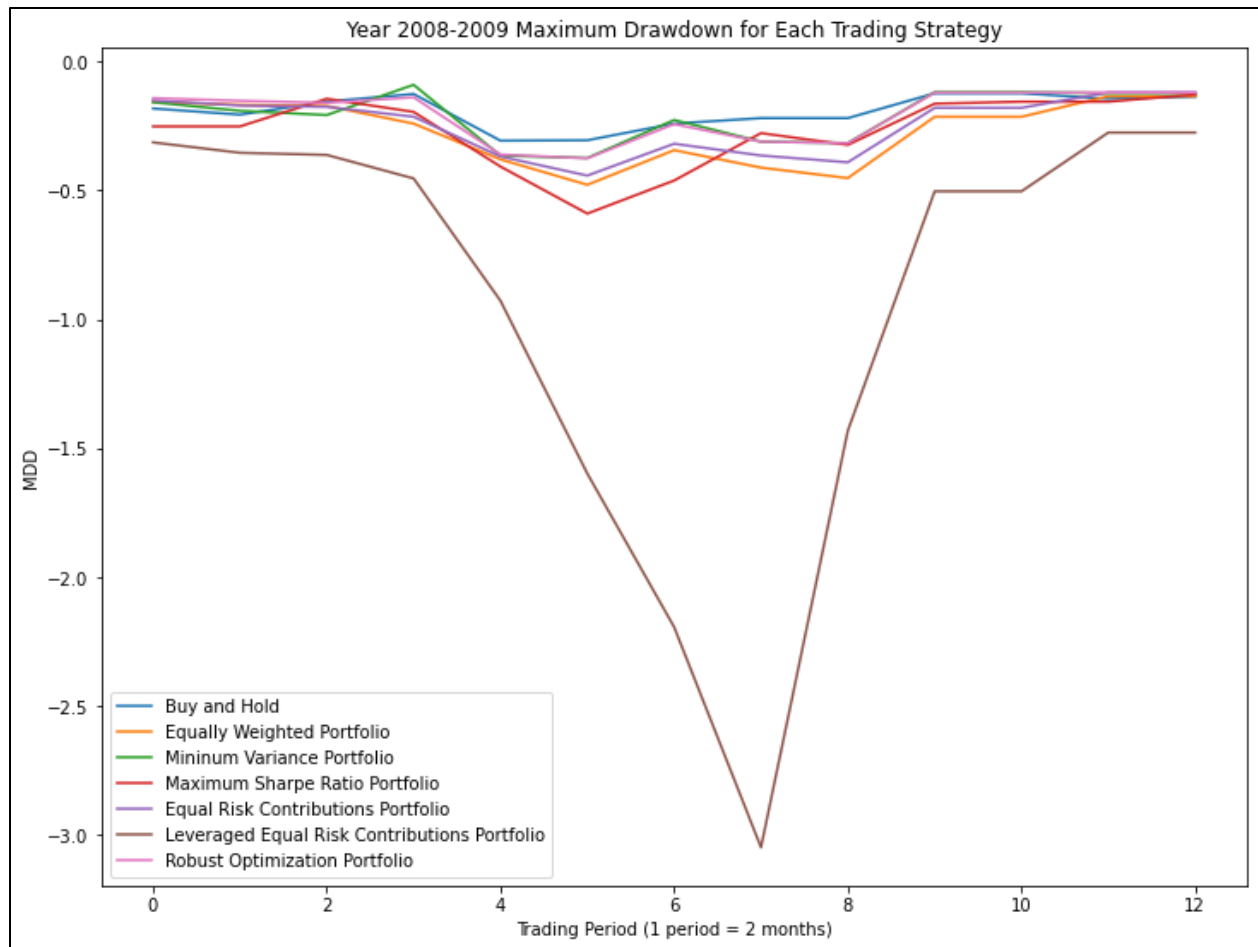
Thus, I would still select the maximum Sharpe ratio portfolio to manage my own portfolio from the year 2020-2021. Sharpe ratio allows us to quantify and optimize the return per unit risk. We can observe that there are much more fluctuations among the weight allocation of the stocks in the Maximum Sharpe ratio strategy so this strategy may not be feasible if there are larger trading costs (Currently it is 0.5%). Taking large risks as with using the maximum Sharpe ratio is the best strategy in times of non-crisis non-recession periods.



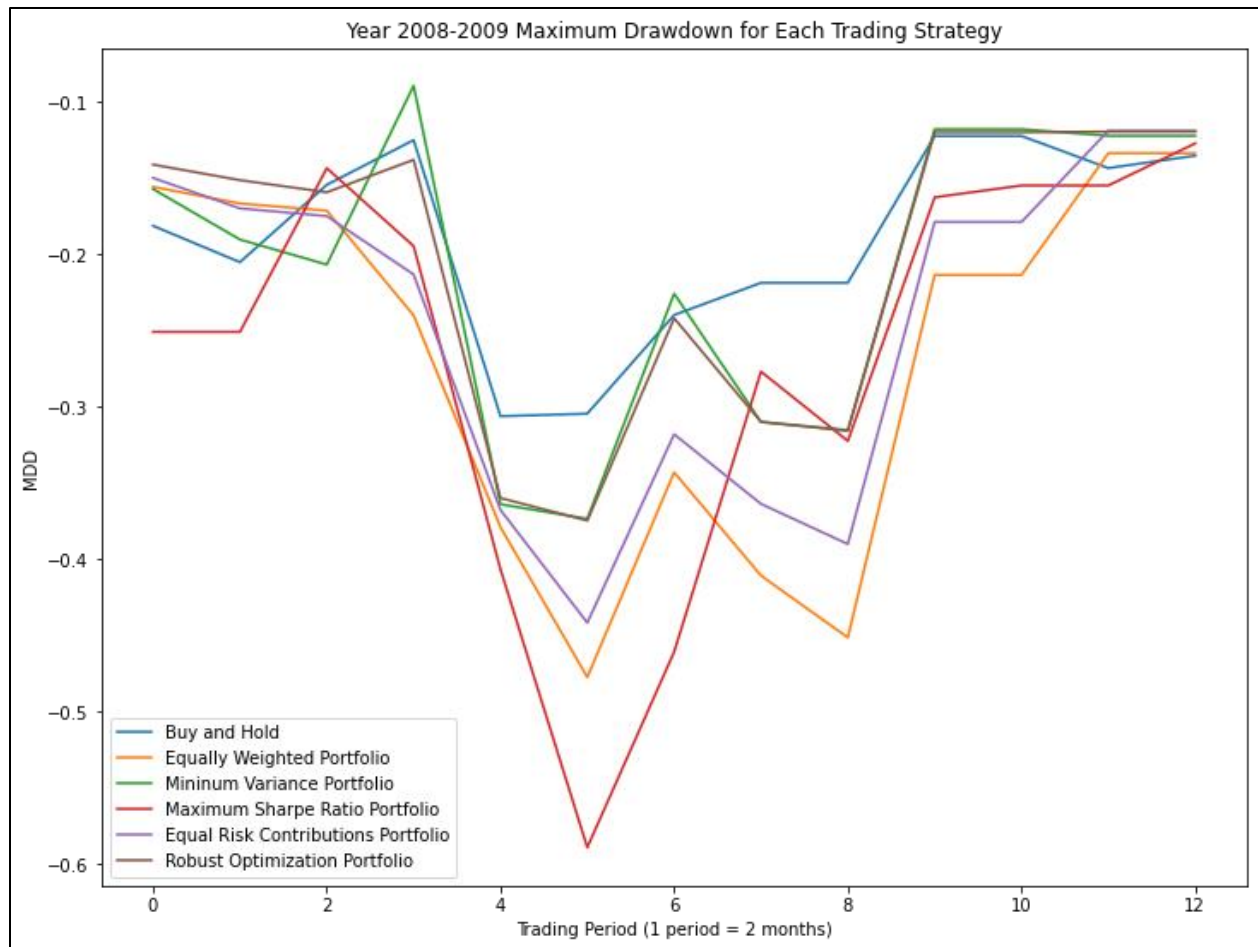
### Part 3: Analyze your results (Year 2008 – 2009)



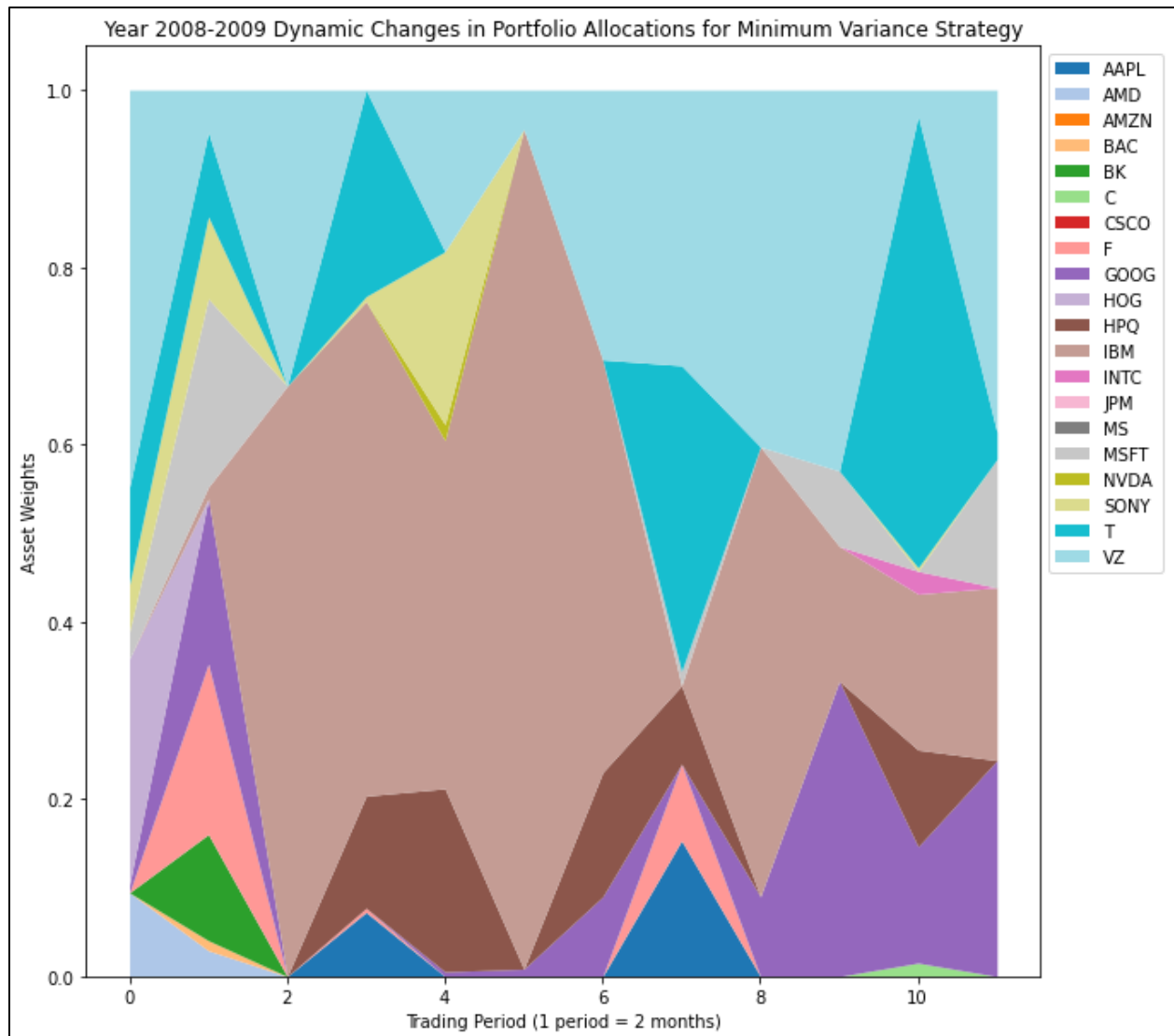
This chart illustrates the daily value of the portfolio for each trading strategy over the market crash years 2008 and 2009 using the daily prices. This will be discussed further below. Main takeaways are that maximum Sharpe's ratio and leveraged ERC performed terribly during the time of crisis. Minimum variance which performed poorly in 2020-2021 was the best strategy if used for the 2008-2009 recession period.



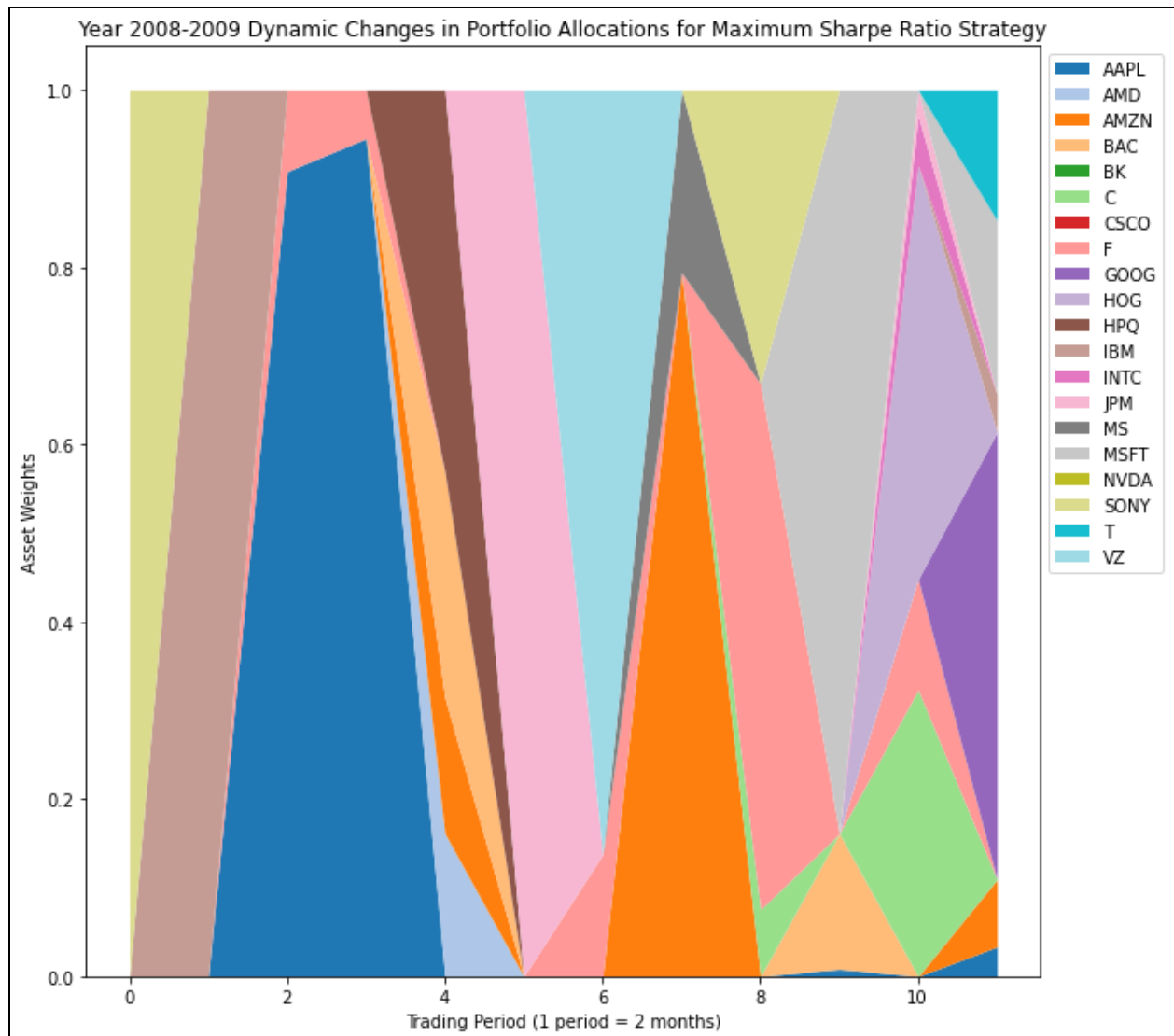
Maximum drawdown was plotted, we can observe that the leveraged ERC strategy has a significant loss, this plot is plotted with and without the leverage ERC for clarity of the other six strategies.



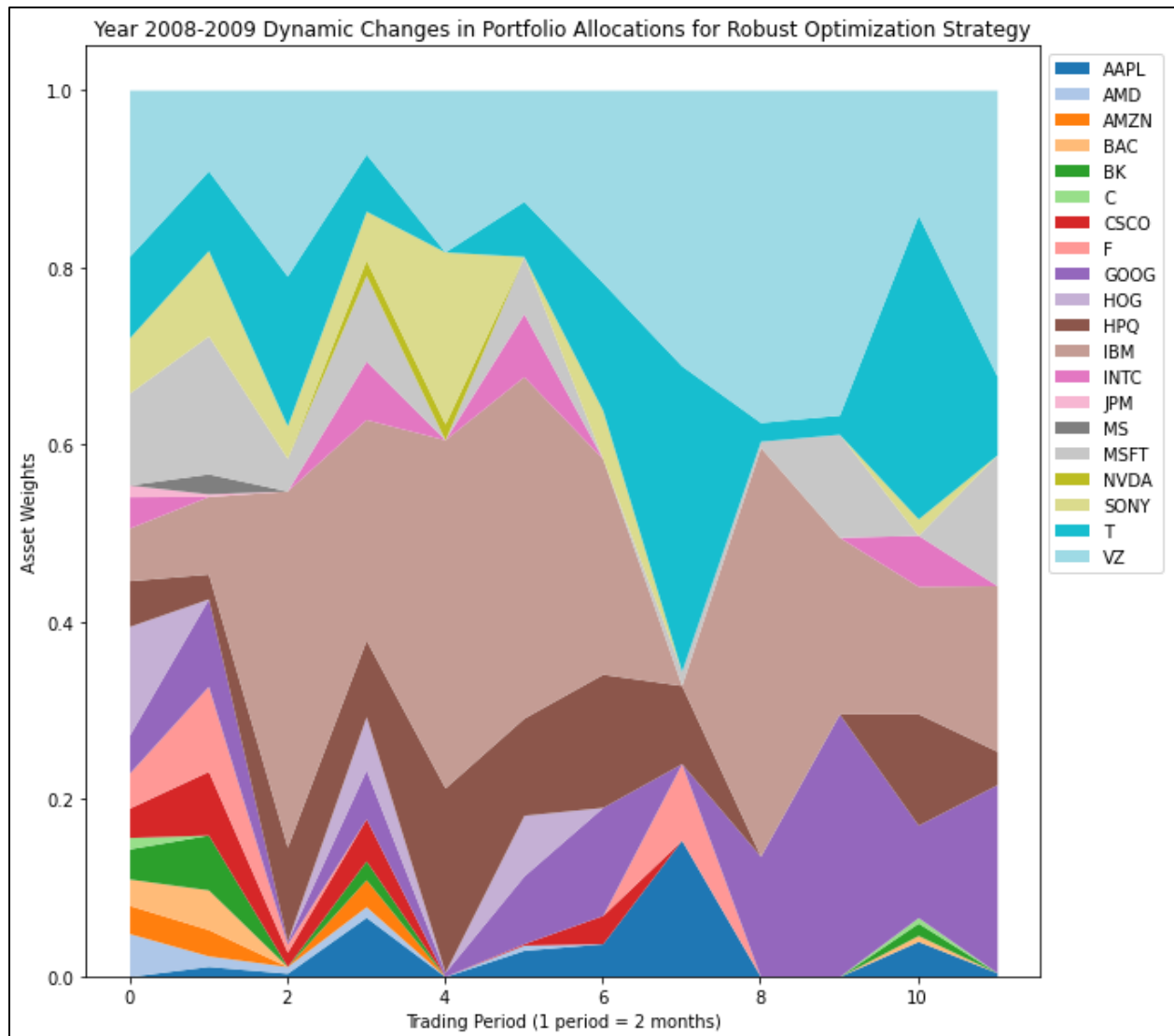
The maximum drawdown is the maximum observed loss from a peak to a trough of a portfolio before a new peak is attained. It indicates the downside risk of a specified time. We can find this using a rolling window for each period. We can see that the leveraged equal risk contribution has a much larger MDD since it's investing more capital into the stocks at such a volatile period. Two MDD graphs were displayed, the second one is without the leverage portfolio to visualize the scale easier. We can observe that the maximum drawdown for all of the strategies is significantly more than experienced in 2020-2021 (Essentially doubled, or twice the loss through out the time period from average of -0.2 to -0.4 for most strategies).



This chart illustrates the weight allocation of each stock in the Minimum variance portfolio for the 12 trading periods. We can observe that the IBM stock dominates the portfolio through most of the trading periods. There are not many diversities of stocks within the portfolio. This stock offers the lowest variance among all the stocks and is also optimal if trading costs are high.



This chart illustrates the weight allocation of each stock in the Maximum Sharpe's Ratio portfolio for the 12 trading periods. We can observe that there are much more fluctuations among the weight allocation of the stocks. We also see that it tends to gravitate towards one stock which dominates the portfolio which makes the portfolio much more volatile. This could be a larger issue if there are larger trading costs.



This chart illustrates the weight allocation of each stock in the Robust Mean-Variance Optimization portfolio for the 12 trading periods. We can observe that the Verizon, IBM, and AT&T stock dominates the portfolio through the 12 trading periods. We can observe that the stocks invested are more diversified than the Minimum Variance and the Maximum Sharpe Ratio strategy. The amount of trading is significantly less compared to the Maximum Sharpe's ratio strategy, but more trading compared to the minimum variance strategy.

Compare trading strategies and discuss their performance relative to each other. Which strategy would you select for managing your own portfolio and why? **(Year 2008 – 2009)**

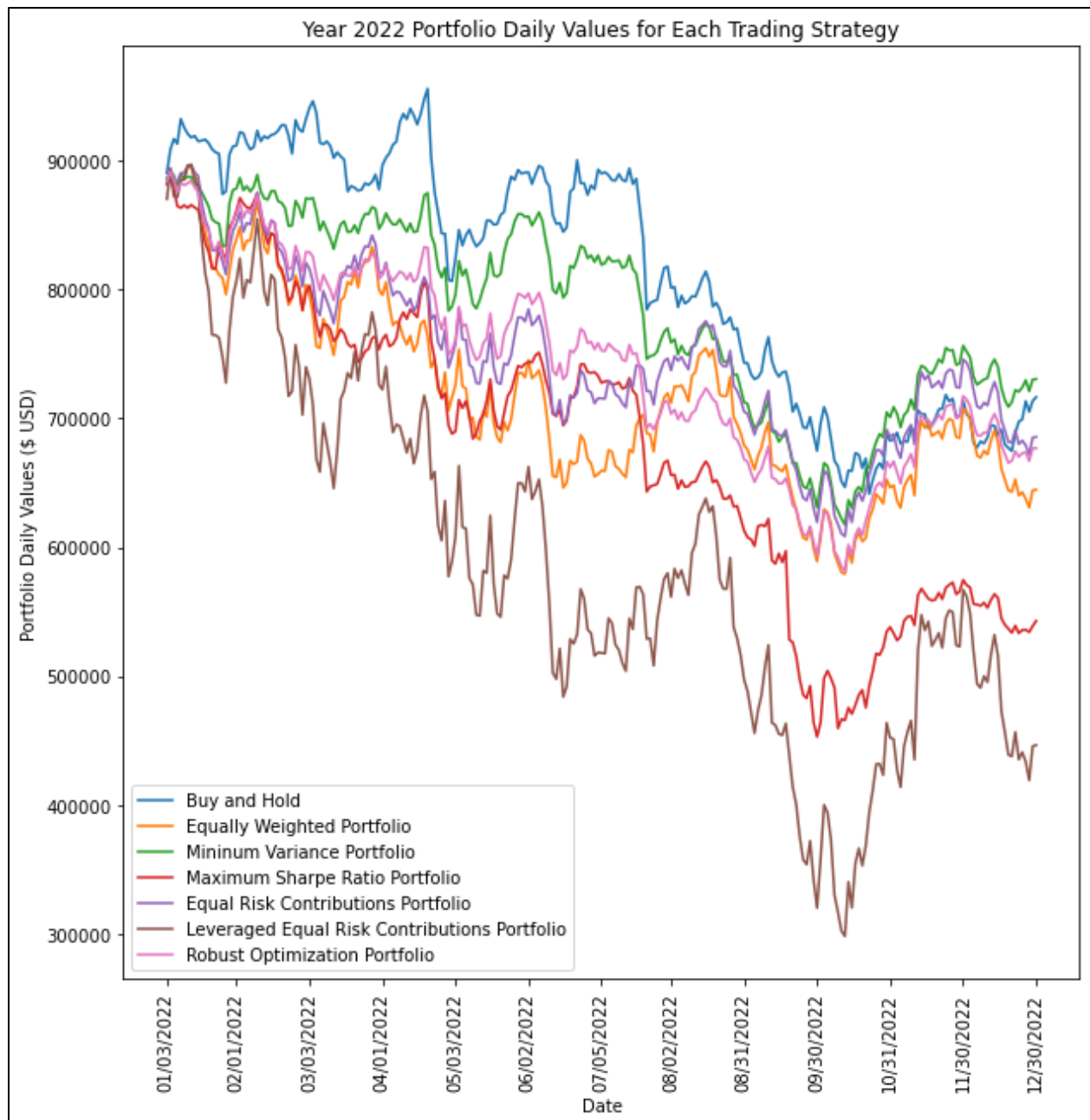
Initial portfolio value = \$ 385097.15

Strategy	Final Portfolio Value (in USD)
Buy and hold	\$ 323101.02
Equally Weighted	\$ 375644.43
Minimum Variance	\$ 391102.48
Maximum Sharpe Ratio	\$ 210603.75
Equal Risk Contributions	\$ 368937.71
Leveraged Equal Risk Contributions	\$ 316610.61
Robust Optimization Portfolio	\$ 364555.49

It is the opposite of the 2021/2021 data, we could see that the maximum Sharpe ratio strategy has performed the worst compared to the other six methods over the 12 trading periods (2 years) in terms of portfolio value. The leveraged equal risk contribution strategy had the second lowest portfolio value in which is observed the largest loss after a year of investing and then started to rapidly increase its value after the 1.5 year mark. From the MDD graph, we can observe that leveraging is risky especially during a recession period. Luckily the portfolio started to pick up value, but as an investor and seeing that amount of loss is very unmanageable and stressful to continue investing in that strategy. Our buy and hold strategy had the third lowest portfolio value. We can observe that the simple equally weighted strategy performed roughly the same as the equal risk contribution strategy and the robust minimum variance strategy.

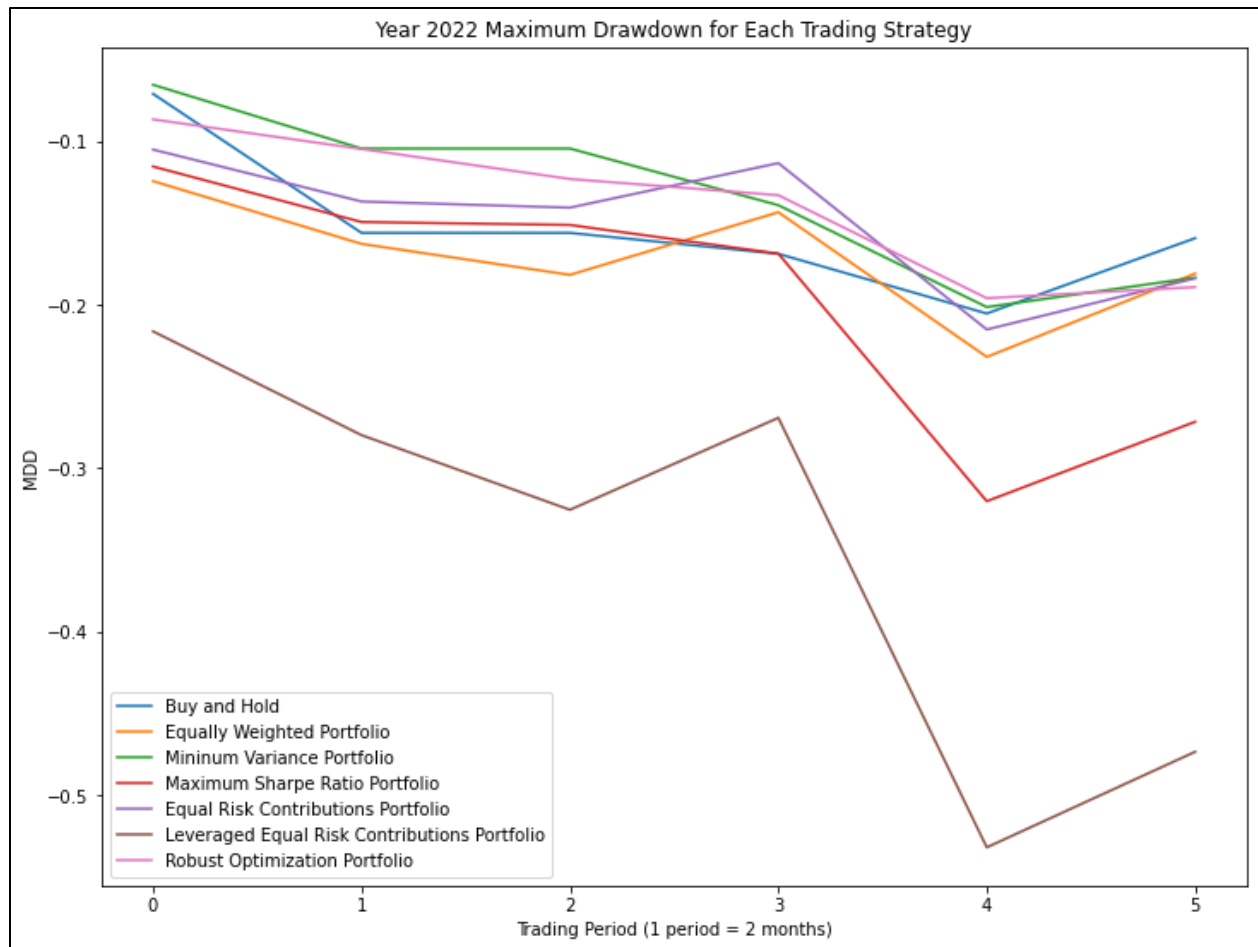
The minimum variance strategy seemed to perform the best in time of crisis and uncertainty. Thus, I would still select the Minimum variance optimization strategy to manage my own portfolio for the year of 2008-2009 and in recession periods. We can observe that it was the only strategy that provided a net positive return that was higher than the initial portfolio value.

#### Part 4: Analyze your results (Year 2022)



This chart illustrates the daily value of the portfolio for each trading strategy over the 'Recession' year 2022 using the daily prices. This will be discussed further below. Main takeaways are that it is following somewhat similarly to 2008 in which the maximum Sharpe's ratio and leveraged ERC strategy performed terribly during the recession period. Minimum variance which performed poorly in 2020-2021 was the best strategy if used for the 2022 recession period.





We can see that the leveraged equal risk contribution has a much larger MDD since it's investing more capital into the stocks. Relative to the 2008-2009 recession period, the MDD seems to be much less severe because it seems to be going in a downhill trend (i.e., No peaks). From both recession periods, we see that the high risk / high reward strategies such as the maximum Sharpe ratio and the leveraged ERC which worked the best for 2020-2021 performed terribly in times of recession and uncertainty.

Compare trading strategies and discuss their performance relative to each other. Which strategy would you select for managing your own portfolio and why? **(Year 2022)**

Initial portfolio value = \$ 890077.15

Strategy	Final Portfolio Value (in USD)
Buy and hold	\$ 716351.42
Equally Weighted	\$ 644566.45
Minimum Variance	\$ 730346.54
Maximum Sharpe Ratio	\$ 542874.92
Equal Risk Contributions	\$ 685426.42
Leveraged Equal Risk Contributions	\$ 446630.30
Robust Optimization Portfolio	\$ 676758.91

## Appendix

### Gradient: Equal Risk Contribution Portfolio Objective Derivation

Portfolio Variance is the risk measure:  $Var(w) = w^T Q w$

Risk Contribution:  $RC_i = C_i var(w) = w_i \frac{df(w)}{d(w_i)} = 2w_i (Qw)_i$

Portfolio Standard Deviation can also be the risk measure:  $\sigma(w) = \sqrt{w^T Q w}$

Risk Contribution:  $RC_i = C_i std(w) = w_i \frac{df(w)}{d(w_i)} = w_i \frac{(Qw)_i}{\sqrt{w^T Q w}}$

Equal Risk Contribution:  $d(w) = \sum_{i=1}^n \sum_{j=1}^n (RC_i - RC_j)^2 = \sum_{i=1}^n \sum_{j=1}^n ((C_i f(w) - C_j f(w))^2$

Minimize deviation between the deviation of the risk contribution between each asset from the portfolio: Solve non-linear optimization problem: Plugged in variance.

$$Min(w) = \sum_{i=1}^n \sum_{j=1}^n (w_i (Qw)_i - w_j (Qw)_j)^2$$

If we derive this function with respect to w: Using Chain rule

$$Min(w) = 2 * 2 * \sum_{i=1}^n \sum_{j=1}^n (w_i (Qw)_i - w_j (Qw)_j) * (Qw)_i$$

### Gradient Code:

```
def gradient(self, x):
```

```
    # The callback for calculating the gradient
```

```
    grad = np.zeros(n)
```

```
    y = x * np.dot(Q,x)
```

```
    # Insert your gradient computations here
```

```
    for i in range(n):
```

```
        for j in range(n):
```

```
            grad[i] = grad[i] + 2*2*(y[i] - y[j])*np.dot(Q,x)[i]
```

```
    # You can use finite dif*ferences to check the gradient
```

```
    return grad
```

Constraints: Sum of all asset weights must be equal to 1 and short selling of assets are not allowed.

## **Output: Year 2021 – 2022**

Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = \$ 1000013.00, value end = \$ 893956.82

Strategy "Equally Weighted Portfolio", value begin = \$ 990881.79, value end = \$ 892364.23

Strategy "Minimum Variance Portfolio", value begin = \$ 992756.22, value end = \$ 915849.88

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 990063.46, value end = \$ 922016.34

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 991597.39, value end = \$ 898259.85

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 979011.20, value end = \$ 792347.88

Strategy "Robust Optimization Portfolio", value begin = \$ 992167.76, value end = \$ 917120.45

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = \$ 945076.08, value end = \$ 949228.39

Strategy "Equally Weighted Portfolio", value begin = \$ 930690.93, value end = \$ 862192.32

Strategy "Minimum Variance Portfolio", value begin = \$ 955714.76, value end = \$ 850652.49

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 961925.93, value end = \$ 1017080.69

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 937510.50, value end = \$ 852698.61

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 866680.69, value end = \$ 697055.16

Strategy "Robust Optimization Portfolio", value begin = \$ 958934.78, value end = \$ 864852.26

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = \$ 937916.75, value end = \$ 913415.30

Strategy "Equally Weighted Portfolio", value begin = \$ 830750.85, value end = \$ 933850.30

Strategy "Minimum Variance Portfolio", value begin = \$ 826383.68, value end = \$ 853443.62

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 974233.42, value end = \$ 1175637.72

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 822445.39, value end = \$ 917591.56

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 632356.56, value end = \$ 823407.35

Strategy "Robust Optimization Portfolio", value begin = \$ 836233.43, value end = \$ 916760.26

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = \$ 905419.70, value end = \$ 994693.42

Strategy "Equally Weighted Portfolio", value begin = \$ 927447.26, value end = \$ 1060445.57

Strategy "Minimum Variance Portfolio", value begin = \$ 855748.90, value end = \$ 980875.15

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1219489.44, value end = \$ 1607444.05

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 913866.92, value end = \$ 1053737.89

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 812039.88, value end = \$ 1091706.41

Strategy "Robust Optimization Portfolio", value begin = \$ 923472.75, value end = \$ 1051063.93

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = \$ 993194.54, value end = \$ 971914.18

Strategy "Equally Weighted Portfolio", value begin = \$ 1068060.61, value end = \$ 998834.67

Strategy "Minimum Variance Portfolio", value begin = \$ 982627.60, value end = \$ 942075.73

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1641529.42, value end = \$ 1554592.34

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1061628.91, value end = \$ 996450.02

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1103258.59, value end = \$ 972890.89

Strategy "Robust Optimization Portfolio", value begin = \$ 1055917.97, value end = \$ 1011280.01

Period 6: start date 11/02/2020, end date 12/31/2020

Strategy "Buy and Hold", value begin = \$ 983801.02, value end = \$ 1004435.74

Strategy "Equally Weighted Portfolio", value begin = \$ 1007654.20, value end = \$ 1194038.48

Strategy "Minimum Variance Portfolio", value begin = \$ 950551.92, value end = \$ 1005253.01

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1553235.25, value end = \$ 1790601.57

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1004646.38, value end = \$ 1180372.67

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 985020.65, value end = \$ 1336413.02

Strategy "Robust Optimization Portfolio", value begin = \$ 1019121.50, value end = \$ 1116589.15

Period 7: start date 01/04/2021, end date 02/26/2021

Strategy "Buy and Hold", value begin = \$ 1005601.39, value end = \$ 956244.08

Strategy "Equally Weighted Portfolio", value begin = \$ 1180451.55, value end = \$ 1266829.54

Strategy "Minimum Variance Portfolio", value begin = \$ 1003278.97, value end = \$ 974346.31

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1738720.52, value end = \$ 1854192.65

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1167346.23, value end = \$ 1220406.48

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1306087.11, value end = \$ 1412116.37

Strategy "Robust Optimization Portfolio", value begin = \$ 1110415.44, value end = \$ 1107893.25

Period 8: start date 03/01/2021, end date 04/30/2021

Strategy "Buy and Hold", value begin = \$ 957791.35, value end = \$ 1019731.32

Strategy "Equally Weighted Portfolio", value begin = \$ 1297169.82, value end = \$ 1398708.51

Strategy "Minimum Variance Portfolio", value begin = \$ 974664.73, value end = \$ 1087408.46

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1902630.85, value end = \$ 2061846.72

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1244376.30, value end = \$ 1355988.27

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1455770.61, value end = \$ 1679283.36

Strategy "Robust Optimization Portfolio", value begin = \$ 1116908.34, value end = \$ 1235677.06

Period 9: start date 05/03/2021, end date 06/30/2021

Strategy "Buy and Hold", value begin = \$ 1022204.61, value end = \$ 987842.85

Strategy "Equally Weighted Portfolio", value begin = \$ 1397485.37, value end = \$ 1459062.86

Strategy "Minimum Variance Portfolio", value begin = \$ 1087214.65, value end = \$ 1076067.43

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2053343.91, value end = \$ 2015834.59

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1354832.83, value end = \$ 1385002.90

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1672742.89, value end = \$ 1733211.39

Strategy "Robust Optimization Portfolio", value begin = \$ 1234354.11, value end = \$ 1231777.45

Period 10: start date 07/01/2021, end date 08/31/2021

Strategy "Buy and Hold", value begin = \$ 993283.49, value end = \$ 975250.19

Strategy "Equally Weighted Portfolio", value begin = \$ 1466470.24, value end = \$ 1517619.10

Strategy "Minimum Variance Portfolio", value begin = \$ 1076109.76, value end = \$ 1085950.69

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2014835.81, value end = \$ 2120423.56

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1391221.46, value end = \$ 1443019.64

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1741350.23, value end = \$ 1845030.40

Strategy "Robust Optimization Portfolio", value begin = \$ 1234383.55, value end = \$ 1270239.21

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = \$ 974520.08, value end = \$ 949068.41

Strategy "Equally Weighted Portfolio", value begin = \$ 1513369.24, value end = \$ 1563152.36

Strategy "Minimum Variance Portfolio", value begin = \$ 1080423.44, value end = \$ 1056572.30

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2101160.93, value end = \$ 2143092.19

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1437078.64, value end = \$ 1452054.31

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1828785.22, value end = \$ 1858717.46

Strategy "Robust Optimization Portfolio", value begin = \$ 1262258.25, value end = \$ 1234728.98

Period 12: start date 11/01/2021, end date 12/31/2021

Strategy "Buy and Hold", value begin = \$ 951350.41, value end = \$ 932471.35

Strategy "Equally Weighted Portfolio", value begin = \$ 1584445.60, value end = \$ 1646232.49

Strategy "Minimum Variance Portfolio", value begin = \$ 1053826.83, value end = \$ 1047867.41

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2112459.87, value end = \$ 2216770.75

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1462333.67, value end = \$ 1515001.16

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1874911.73, value end = \$ 1980093.88

Strategy "Robust Optimization Portfolio", value begin = \$ 1228963.76, value end = \$ 1256306.78

## Output: Year 2008 – 2009

Period 1: start date 01/02/2008, end date 02/29/2008

Strategy "Buy and Hold", value begin = \$ 385097.15, value end = \$ 325918.34

Strategy "Equally Weighted Portfolio", value begin = \$ 381647.53, value end = \$ 326938.14

Strategy "Minimum Variance Portfolio", value begin = \$ 383268.36, value end = \$ 327358.28

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 381265.54, value end = \$ 332652.59

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 381846.82, value end = \$ 329299.76

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 375707.93, value end = \$ 270599.90

Strategy "Robust Optimization Portfolio", value begin = \$ 382274.61, value end = \$ 332192.51

Period 2: start date 03/03/2008, end date 04/30/2008

Strategy "Buy and Hold", value begin = \$ 325807.08, value end = \$ 349997.20

Strategy "Equally Weighted Portfolio", value begin = \$ 322065.15, value end = \$ 354843.42

Strategy "Minimum Variance Portfolio", value begin = \$ 322920.70, value end = \$ 366009.61

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 325785.77, value end = \$ 344234.61

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 324469.25, value end = \$ 361346.72

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 258023.51, value end = \$ 331891.79

Strategy "Robust Optimization Portfolio", value begin = \$ 328178.03, value end = \$ 362298.38

Period 3: start date 05/01/2008, end date 06/30/2008

Strategy "Buy and Hold", value begin = \$ 357929.49, value end = \$ 322881.56

Strategy "Equally Weighted Portfolio", value begin = \$ 366452.44, value end = \$ 308979.57

Strategy "Minimum Variance Portfolio", value begin = \$ 373420.44, value end = \$ 351741.54

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 349024.91, value end = \$ 313033.39

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 372138.70, value end = \$ 322669.61

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 350563.60, value end = \$ 251678.11

Strategy "Robust Optimization Portfolio", value begin = \$ 370781.52, value end = \$ 340331.75

Period 4: start date 07/01/2008, end date 08/29/2008

Strategy "Buy and Hold", value begin = \$ 324349.75, value end = \$ 326489.53

Strategy "Equally Weighted Portfolio", value begin = \$ 309434.67, value end = \$ 315913.44

Strategy "Minimum Variance Portfolio", value begin = \$ 352084.80, value end = \$ 356555.45

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 325404.17, value end = \$ 315390.45

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 322317.70, value end = \$ 326455.12

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 248011.28, value end = \$ 256279.71

Strategy "Robust Optimization Portfolio", value begin = \$ 338726.80, value end = \$ 340123.74

Period 5: start date 09/02/2008, end date 10/31/2008

Strategy "Buy and Hold", value begin = \$ 333252.73, value end = \$ 274022.75

Strategy "Equally Weighted Portfolio", value begin = \$ 316692.21, value end = \$ 231375.31

Strategy "Minimum Variance Portfolio", value begin = \$ 348646.41, value end = \$ 269323.05

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 306616.50, value end = \$ 229459.58

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 326213.86, value end = \$ 241964.43

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 252821.60, value end = \$ 84276.55

Strategy "Robust Optimization Portfolio", value begin = \$ 335384.90, value end = \$ 259093.31

Period 6: start date 11/03/2008, end date 12/31/2008

Strategy "Buy and Hold", value begin = \$ 282342.11, value end = \$ 305967.56

Strategy "Equally Weighted Portfolio", value begin = \$ 229962.96, value end = \$ 198775.23

Strategy "Minimum Variance Portfolio", value begin = \$ 269733.03, value end = \$ 248523.41

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 226771.37, value end = \$ 175554.09

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 241112.92, value end = \$ 212148.44

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 79583.36, value end = \$ 21514.44

Strategy "Robust Optimization Portfolio", value begin = \$ 259965.10, value end = \$ 241328.31



Period 7: start date 01/02/2009, end date 02/27/2009

Strategy "Buy and Hold", value begin = \$ 313366.90, value end = \$ 258275.19

Strategy "Equally Weighted Portfolio", value begin = \$ 207261.94, value end = \$ 169845.60

Strategy "Minimum Variance Portfolio", value begin = \$ 256572.57, value end = \$ 244388.15

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 174923.22, value end = \$ 145630.88

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 221021.22, value end = \$ 188701.87

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 36265.90, value end = \$ -28379.23

Strategy "Robust Optimization Portfolio", value begin = \$ 249412.90, value end = \$ 226684.65

Period 8: start date 03/02/2009, end date 04/30/2009

Strategy "Buy and Hold", value begin = \$ 248688.22, value end = \$ 286368.72

Strategy "Equally Weighted Portfolio", value begin = \$ 161619.15, value end = \$ 259939.51

Strategy "Minimum Variance Portfolio", value begin = \$ 234752.02, value end = \$ 320581.66

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 138601.74, value end = \$ 180660.31

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 180804.55, value end = \$ 271282.28

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ -47209.48, value end = \$ 133623.58

Strategy "Robust Optimization Portfolio", value begin = \$ 218590.98, value end = \$ 298521.05

Period 9: start date 05/01/2009, end date 06/30/2009

Strategy "Buy and Hold", value begin = \$ 287805.37, value end = \$ 285824.08

Strategy "Equally Weighted Portfolio", value begin = \$ 259507.31, value end = \$ 273156.85

Strategy "Minimum Variance Portfolio", value begin = \$ 318011.44, value end = \$ 321342.79

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 179564.68, value end = \$ 185441.09

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 270794.80, value end = \$ 281310.14

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 129584.01, value end = \$ 150649.69

Strategy "Robust Optimization Portfolio", value begin = \$ 296211.89, value end = \$ 300275.44

Period 10: start date 07/01/2009, end date 08/31/2009

Strategy "Buy and Hold", value begin = \$ 286766.63, value end = \$ 298338.27

Strategy "Equally Weighted Portfolio", value begin = \$ 272845.29, value end = \$ 321639.02

Strategy "Minimum Variance Portfolio", value begin = \$ 321050.98, value end = \$ 342363.71

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 180897.54, value end = \$ 195509.76

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 281314.02, value end = \$ 320028.09

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 147559.12, value end = \$ 224994.78

Strategy "Robust Optimization Portfolio", value begin = \$ 300177.22, value end = \$ 321067.62

Period 11: start date 09/01/2009, end date 10/30/2009

Strategy "Buy and Hold", value begin = \$ 291703.36, value end = \$ 290193.57

Strategy "Equally Weighted Portfolio", value begin = \$ 310065.61, value end = \$ 328220.99

Strategy "Minimum Variance Portfolio", value begin = \$ 334164.65, value end = \$ 350926.72

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 187077.93, value end = \$ 186948.32

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 310296.19, value end = \$ 329115.53

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 202419.13, value end = \$ 240053.39

Strategy "Robust Optimization Portfolio", value begin = \$ 313617.84, value end = \$ 327933.00

Period 12: start date 11/02/2009, end date 12/31/2009

Strategy "Buy and Hold", value begin = \$ 288596.05, value end = \$ 323101.02

Strategy "Equally Weighted Portfolio", value begin = \$ 329572.02, value end = \$ 375644.43

Strategy "Minimum Variance Portfolio", value begin = \$ 348323.75, value end = \$ 391102.48

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 186009.68, value end = \$ 210603.75

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 329765.68, value end = \$ 368937.71

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 238216.81, value end = \$ 316610.61

Strategy "Robust Optimization Portfolio", value begin = \$ 326096.71, value end = \$ 364555.49

## Output: Year 2022

Period 1: start date 01/03/2022, end date 02/28/2022

Strategy "Buy and Hold", value begin = \$ 890077.15, value end = \$ 924072.93

Strategy "Equally Weighted Portfolio", value begin = \$ 881998.87, value end = \$ 802771.38

Strategy "Minimum Variance Portfolio", value begin = \$ 885903.36, value end = \$ 863773.70

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 881221.32, value end = \$ 800281.21

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 882741.73, value end = \$ 817665.70

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 869847.27, value end = \$ 739757.29

Strategy "Robust Optimization Portfolio", value begin = \$ 883508.50, value end = \$ 825486.75

Period 2: start date 03/01/2022, end date 04/29/2022

Strategy "Buy and Hold", value begin = \$ 921940.14, value end = \$ 807230.89

Strategy "Equally Weighted Portfolio", value begin = \$ 783266.45, value end = \$ 705860.50

Strategy "Minimum Variance Portfolio", value begin = \$ 855301.71, value end = \$ 783180.34

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 783828.96, value end = \$ 694300.44

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 801873.00, value end = \$ 739191.61

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 702586.89, value end = \$ 577325.07

Strategy "Robust Optimization Portfolio", value begin = \$ 813765.80, value end = \$ 749219.23

Period 3: start date 05/02/2022, end date 06/30/2022

Strategy "Buy and Hold", value begin = \$ 806237.92, value end = \$ 877550.83

Strategy "Equally Weighted Portfolio", value begin = \$ 716150.91, value end = \$ 654771.07

Strategy "Minimum Variance Portfolio", value begin = \$ 786494.37, value end = \$ 821074.11

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 687921.90, value end = \$ 735767.23

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 747897.43, value end = \$ 711427.92

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 589094.16, value end = \$ 515854.34

Strategy "Robust Optimization Portfolio", value begin = \$ 752790.31, value end = \$ 753378.22

Period 4: start date 07/01/2022, end date 08/31/2022

Strategy "Buy and Hold", value begin = \$ 892738.72, value end = \$ 742946.10

Strategy "Equally Weighted Portfolio", value begin = \$ 656654.59, value end = \$ 679066.98

Strategy "Minimum Variance Portfolio", value begin = \$ 826322.07, value end = \$ 712545.00

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 732876.36, value end = \$ 612899.86

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 715546.33, value end = \$ 705070.50

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 518425.60, value end = \$ 497373.49

Strategy "Robust Optimization Portfolio", value begin = \$ 758140.31, value end = \$ 668196.19

Period 5: start date 09/01/2022, end date 10/31/2022

Strategy "Buy and Hold", value begin = \$ 742641.68, value end = \$ 682506.51

Strategy "Equally Weighted Portfolio", value begin = \$ 675580.30, value end = \$ 646270.98

Strategy "Minimum Variance Portfolio", value begin = \$ 711068.32, value end = \$ 700722.40

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 607671.61, value end = \$ 538152.79

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 703416.73, value end = \$ 685372.60

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 488354.84, value end = \$ 452278.62

Strategy "Robust Optimization Portfolio", value begin = \$ 666283.83, value end = \$ 661895.44

Period 6: start date 11/01/2022, end date 12/30/2022

Strategy "Buy and Hold", value begin = \$ 683477.34, value end = \$ 716351.42

Strategy "Equally Weighted Portfolio", value begin = \$ 648058.49, value end = \$ 644566.45

Strategy "Minimum Variance Portfolio", value begin = \$ 708683.86, value end = \$ 730346.54

Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 533307.53, value end = \$ 542874.92

Strategy "Equal Risk Contributions Portfolio", value begin = \$ 687738.59, value end = \$ 685426.42

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 451286.92, value end = \$ 446630.30

Strategy "Robust Optimization Portfolio", value begin = \$ 666620.13, value end = \$ 676758.91