MIE1622: Assignment #3 – Credit Risk Modeling and Simulation

Name: Alexander Cheng

Student Number: 1001634298

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Part 1: Implement portfolio credit risk simulation model in Python. Refer to ipynb file.

Part 2: Analyze your results.

Python output:

Portfolio 1:

Out-of-sample: VaR 99.0% = \$82696853.28, CVaR 99.0% = \$123389989.06 In-sample MC1: VaR 99.0% = \$84168413.52, CVaR 99.0% = \$124234846.56 In-sample MC2: VaR 99.0% = \$85050766.71, CVaR 99.0% = \$125489858.78 In-sample No: VaR 99.0% = \$46366497.88, CVaR 99.0% = \$52204462.00 In-sample N1: VaR 99.0% = \$46840656.05, CVaR 99.0% = \$52739514.74 In-sample N2: VaR 99.0% = \$47256573.09, CVaR 99.0% = \$53208982.55

Out-of-sample: VaR 99.9% = \$174144664.76, CVaR 99.9% = \$212730444.31 In-sample MC1: VaR 99.9% = \$175382157.59, CVaR 99.9% = \$217105880.96 In-sample MC2: VaR 99.9% = \$176110425.39, CVaR 99.9% = \$215929958.52 In-sample No: VaR 99.9% = \$59526641.86, CVaR 99.9% = \$64296326.87 In-sample N1: VaR 99.9% = \$60138070.70, CVaR 99.9% = \$64957507.29 In-sample N2: VaR 99.9% = \$60674703.77, CVaR 99.9% = \$65537891.96

Portfolio 2:

Out-of-sample: VaR 99.0% = \$73348811.38, CVaR 99.0% = \$116266854.05 In-sample MC1: VaR 99.0% = \$74753668.62, CVaR 99.0% = \$116767847.31 In-sample MC2: VaR 99.0% = \$76270338.94, CVaR 99.0% = \$118382526.99 In-sample No: VaR 99.0% = \$42782078.58, CVaR 99.0% = \$48115955.60 In-sample N1: VaR 99.0% = \$43151429.66, CVaR 99.0% = \$48531521.06 In-sample N2: VaR 99.0% = \$43644854.34, CVaR 99.0% = \$49090084.60

Out-of-sample: VaR 99.9% = \$175928347.73, CVaR 99.9% = \$210293929.72 In-sample MC1: VaR 99.9% = \$172192044.23, CVaR 99.9% = \$213633516.27 In-sample MC2: VaR 99.9% = \$172993636.57, CVaR 99.9% = \$213957542.18 In-sample No: VaR 99.9% = \$54805891.68, CVaR 99.9% = \$59163731.63 In-sample N1: VaR 99.9% = \$55279420.84, CVaR 99.9% = \$59675018.48 In-sample N2: VaR 99.9% = \$55919683.84, CVaR 99.9% = \$60368500.69

These results will be discussed in detail further below.

Plot Loss Distributions:

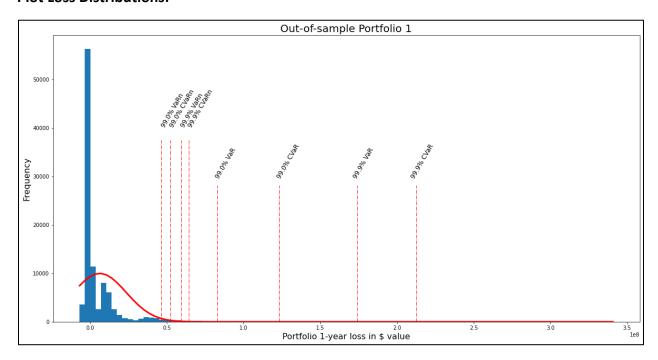


Figure 1 – Loss Distribution of True Distribution (Portfolio 1)

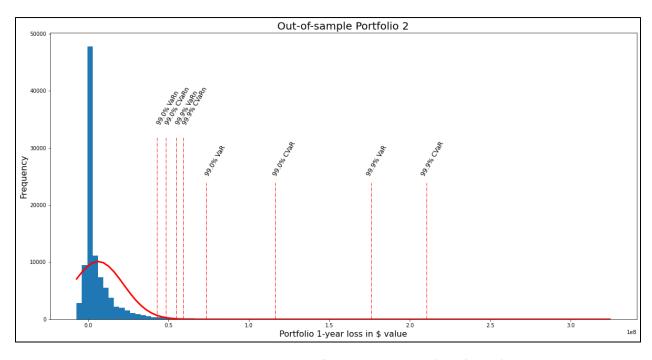


Figure 2 – Loss Distribution of True Distribution (Portfolio 2)

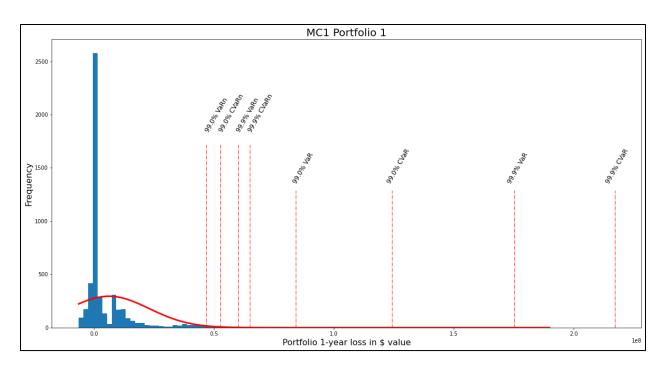


Figure 3 – Loss Distribution of Monte Carlo Scenario 1 (Portfolio 1)

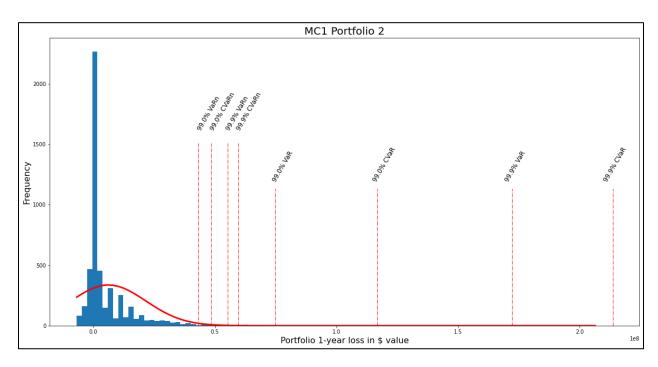


Figure 4 – Loss Distribution of Monte Carlo Scenario 1 (Portfolio 2)

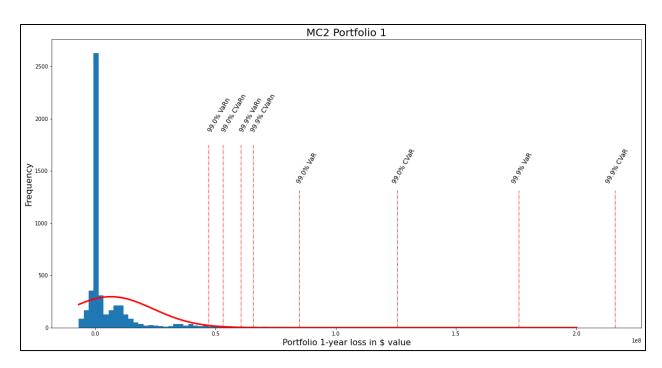


Figure 5 – Loss Distribution of Monte Carlo Scenario 2 (Portfolio 1)

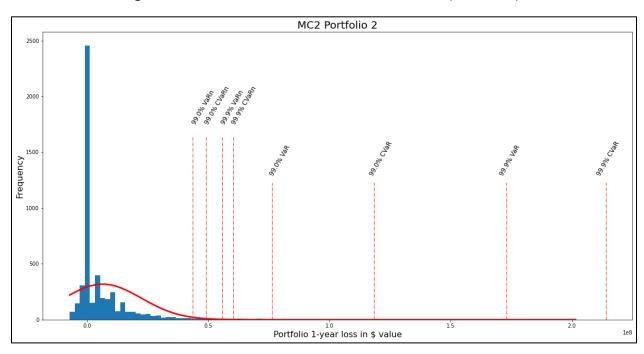


Figure 6 – Loss Distribution of Monte Carlo Scenario 2 (Portfolio 2)

Sampling Error:

Analyze sampling error when comparing non-Normal approximations to the true (out-of-sample) loss distribution.

	Case	Loss (\$)	True Value (\$)	Error (%)
0	99.0% MC1 VaR (Portfolio 1)	84168413.52	82696853.28	1.78
1	99.0% MC1 CVaR (Portfolio 1)	124234846.56	123389989.06	0.68
2	99.0% MC2 VaR (Portfolio 1)	85050766.71	82696853.28	2.85
3	99.0% MC2 CVaR (Portfolio 1)	125489858.78	123389989.06	1.70
4	99.9% MC1 VaR (Portfolio 1)	175382157.59	174144664.76	0.71
5	99.9% MC1 CVaR (Portfolio 1)	217105880.96	212730444.31	2.06
6	99.9% MC2 VaR (Portfolio 1)	176110425.39	174144664.76	1.13
7	99.9% MC2 CVaR (Portfolio 1)	215929958.52	212730444.31	1.50
8	99.0% MC1 VaR (Portfolio 2)	74753668.62	73348811.38	1.92
9	99.0% MC1 CVaR (Portfolio 2)	116767847.31	116266854.05	0.43
10	99.0% MC2 VaR (Portfolio 2)	76270338.94	73348811.38	3.98
11	99.0% MC2 CVaR (Portfolio 2)	118382526.99	116266854.05	1.82
12	99.9% MC1 VaR (Portfolio 2)	172192044.23	175928347.73	-2.12
13	99.9% MC1 CVaR (Portfolio 2)	213633516.27	210293929.72	1.59
14	99.9% MC2 VaR (Portfolio 2)	172993636.57	175928347.73	-1.67
15	99.9% MC2 CVaR (Portfolio 2)	213957542.18	210293929.72	1.74

Comparing the error between the true distribution and the two methods of Monte Carlo simulations for both portfolios, the non-normal approximations of VaR and CVaR for both confidence levels are very similar to the true distribution. The first Monte Carlo simulation strategy (1000 systematic scenarios and 5 idiosyncratic scenarios for each systematic) does better than 1 idiosyncratic scenario with 5000 systematic scenarios. It's using very little data points to calculate the values at 99.9% confidence levels (1 point vs 5 points). The sampling error between the two Monte Carlo approximations are very small.

Modelling Error:

If we wrongly assume that the data follows a normal distribution. We can compare the difference between losses against the true distribution.

	Case	Loss (\$)	True Value (\$)	Error (%)
0	99.0% N1 VaR (Portfolio 1)	46840656.05	82696853.28	-43.36
1	99.0% N1 CVaR (Portfolio 1)	52739514.74	123389989.06	-57.26
2	99.0% N2 VaR (Portfolio 1)	47256573.09	82696853.28	-42.86
3	99.0% N2 CVaR (Portfolio 1)	53208982.55	123389989.06	-56.88
4	99.9% N1 VaR (Portfolio 1)	60138070.70	174144664.76	-65.47
5	99.9% N1 CVaR (Portfolio 1)	64957507.29	212730444.31	-69.46
6	99.9% N2 VaR (Portfolio 1)	60674703.77	174144664.76	-65.16
7	99.9% N2 CVaR (Portfolio 1)	65537891.96	212730444.31	-69.19
8	99.0% N1 VaR (Portfolio 2)	43151429.66	73348811.38	-41.17
9	99.0% N1 CVaR (Portfolio 2)	48531521.06	116266854.05	-58.26
10	99.0% N2 VaR (Portfolio 2)	43644854.34	73348811.38	-40.50
11	99.0% N2 CVaR (Portfolio 2)	49090084.60	116266854.05	-57.78
12	99.9% N1 VaR (Portfolio 2)	55279420.84	175928347.73	-68.58
13	99.9% N1 CVaR (Portfolio 2)	59675018.48	210293929.72	-71.62
14	99.9% N2 VaR (Portfolio 2)	55919683.84	175928347.73	-68.21
15	99.9% N2 CVaR (Portfolio 2)	60368500.69	210293929.72	-71.29

For both portfolios, we can observe that the values of VaR and CVaR are significantly underestimated for both values of alphas compared to the true estimated loss. This is very important since we would be underestimating our losses and assume our portfolio will perform much better than it actually is. The normal distribution model does not have long flat tails like the true loss distribution. The normal distribution underestimates the 99% VaR and CVaR loss (true loss is doubled) and the 99.9% by significantly more (true loss is tripled the estimated loss).

Part 3: Discuss possible strategies for minimizing impacts of sampling and model errors:

If you report the in-sample VaR and CVaR to decision-makers in your bank, what consequences for the bank capital requirements it may have?

If the in-sample non-normal approximations of VaR and CVar are reported to the decision makers in the bank, they will have an accurate prediction / representation of the true estimated credit risks and losses. This will allow them to make correct risk strategies. The bank will not be using underestimated loss as if they were to use the normal approximation. This may potentially lead to significant losses it terms of financial crisis where many uncertainties happen (Being more risk adverse without understanding the actual risk tolerance).

However, the difference between VaR and CVaR vary significantly, where CVaR is almost 1.5x of VaR for the 99% confidence interval and 1.3x of VaR for the 99.9% confidence interval. With such a large difference between VaR and CVaR, it will drastically affect the amount of capital the bank will have to keep within the bank for risk management.

Can you suggest techniques for minimizing impacts of sampling and model errors?

From part 2, we can observe that the impact of sampling error is not a huge concern (Within 2% error). To minimize the impacts of sampling errors, we can increase the sampling size and the number of trials. By increasing the number of idiosyncratic scenarios as well ad the number of systematic scenarios, we can reduce sampling error.

To minimize the modelling errors, we should not assume a type of distribution without taking a further look of the data. By running a quick Monte Carlo simulation of randomly generating the scenarios and plotting the losses, we can see which distribution pattern it fits and then choose a model it likely fits to perform more calculations and to save computation.