MIE1622 – Assignment #4: Asset Pricing

Name: Alexander Cheng

Student Number: 1001634298

Date: Friday, April 7th, 2023

Part 1: Implement pricing functions in Python (Refer to Appendix)

The number of paths chosen for each Monte Carlo simulation is 100,000. The number of paths need to be large enough for accurate price estimation (according to the law of large numbers the average of the results obtained from a large number of trials should be close to the expected value). The number of time steps chosen (used to calculate the number of calculations of the stock price) for the multi-step was 12, 52, and 252 (Representing months, weeks, and years). The number of steps is selected to represent the time-based periods of common trading strategies. As the number of paths and time steps increase, it will provide a more accurate result but will increase the computation time. The results of the one-step MC simulation and multi-step MC simulation are compared below in Part 2.

Part 2: Analyze your results

Refer to Appendix for Black-Scholes, 1 step Monte Carlo, and Multi-step Monte Carlo simulation for the Call and Put price for the given European and Barrier option. The results are discussed below.

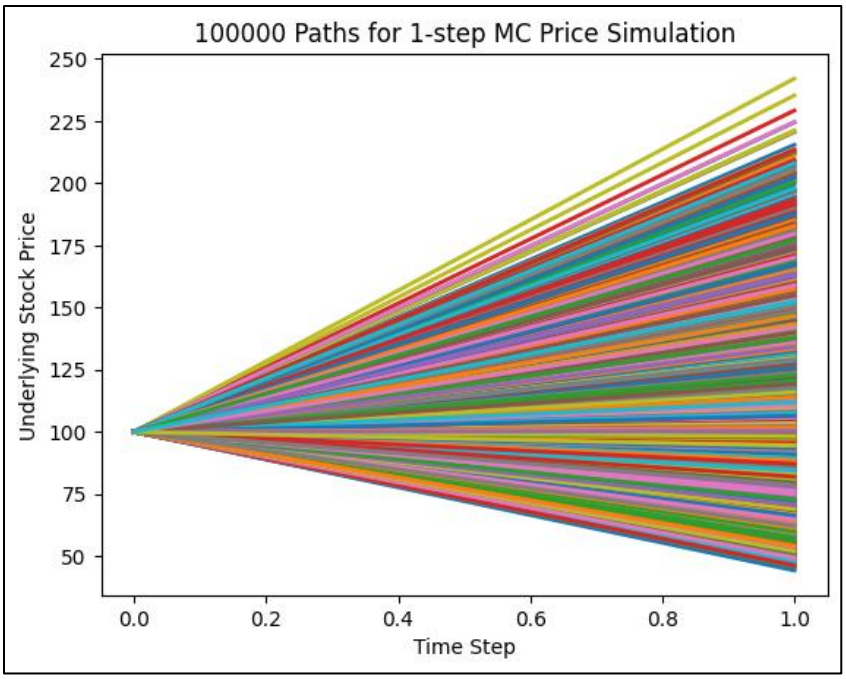

Figure 1 – Plot of 1 Step Monte Carlo Simulation Stock Price

Each line represents a path for the stock price, since this is a 1-time step model, all of the paths start at initial stock price S0, and predicts the value at the end of 1 year.

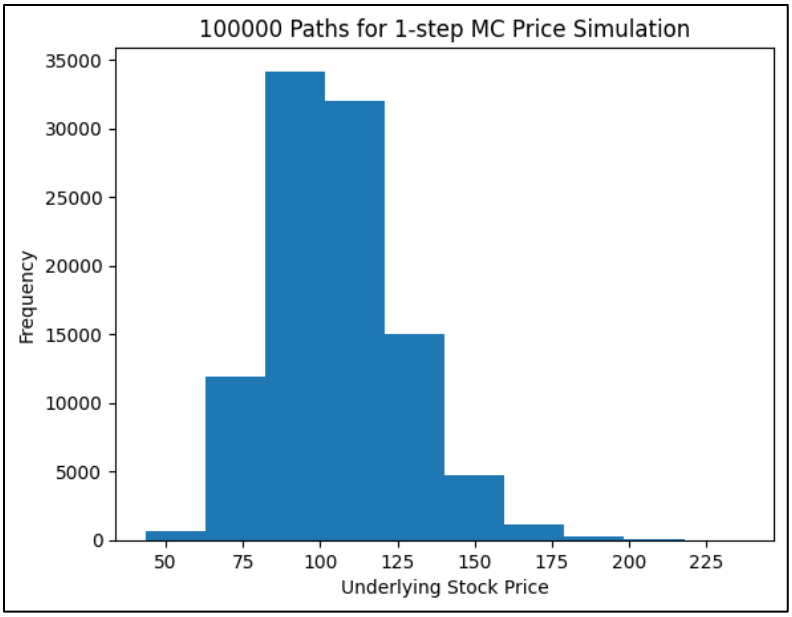

Figure 2 – Histogram of 1 Step Monte Carlo Simulation Stock Price (10 bins, 100,000 paths)

The histogram shows the frequency of the asset price distribution for the 100,000 paths for the 1 step model. The mean stock price at time T is hovering around $100.

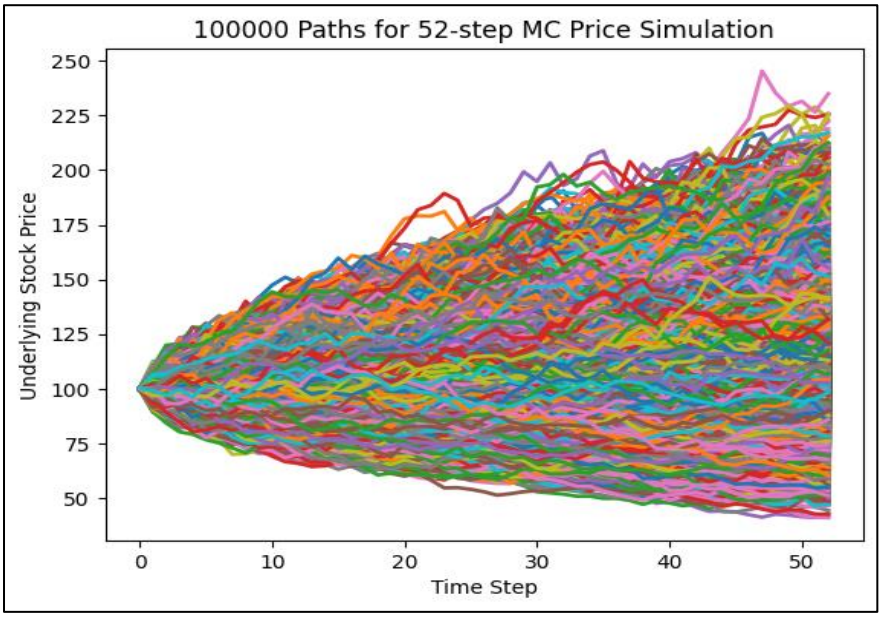

Figure 3 – Plot of 1 Step Monte Carlo Simulation Stock Price

Each line represents a path for the stock price, since this is a 52-time step model, all of the paths start at initial stock price S0, and predicts the future value at 52 different equally spaced time steps to ultimately predict the stock price at the end of 1 year.

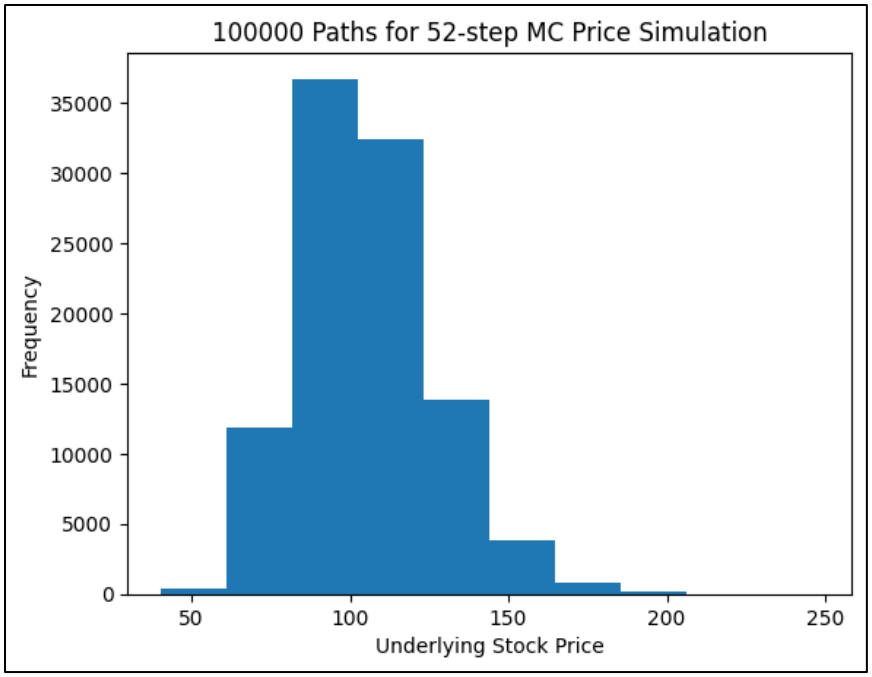

Figure 4 – Histogram of 52 Step Monte Carlo Simulation Stock Price (10 bins, 100,000 paths)

This shows the asset price distribution for the 100,000 paths for the 52-step model. The mean stock price at time T is similarly also hovering around $100. The number of bins can be chosen to make the histogram look smooth since there are 100,000 paths/samples.

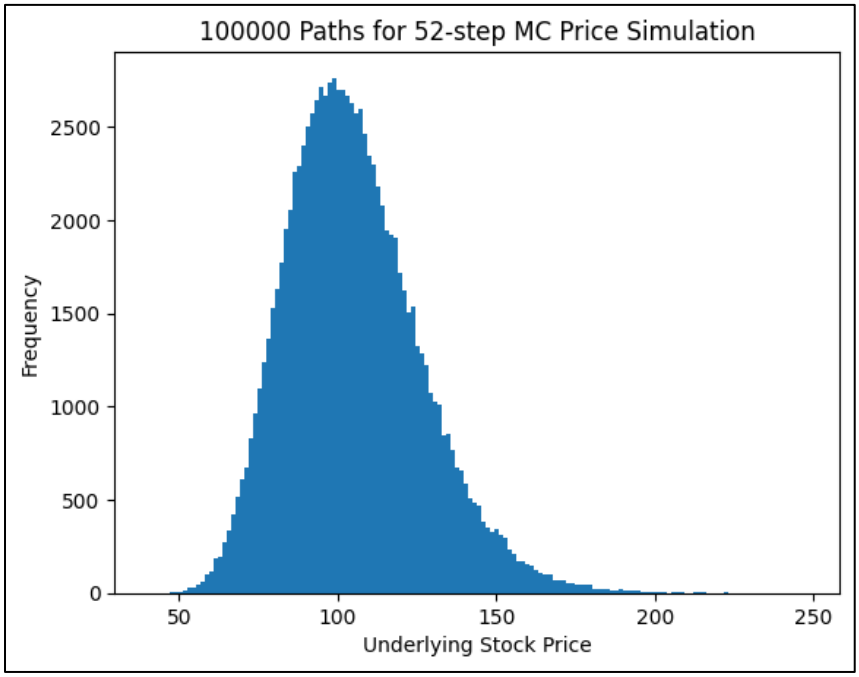

Figure 5 – Histogram of 52 Step Monte Carlo Simulation Stock Price (150 bins, 100,000 paths)

The number of bins can be chosen to make the histogram look smooth since there are 100,000 paths/samples. 150 bins were used instead of 10 bins, from here it looks more like a normal distribution curve.

Compare three pricing strategies for European option:

The three strategies have very similar results when estimating the price of a European option. The one-step and multi-step MC simulations give similar results to the Black-Scholes equation in which is used as the true value. I can observe that as I increase the number of time steps, the value approaches the value given by the Black Scholes equation. However, one may use fewer time steps or even the one-step MC simulation to save computing power. This may be required when there are many more stock prices to be predicted.

Difference between call and put prices obtained for European and Barrier options:

A knock-in option becomes a standard European option if the barrier was crossed by the stock price and worthless if the stock price does not reach the barrier before expiration. This means the knock-in option has a higher chance to become worthless ($0) compared to a standard European option, thus will have a lower expected value and thus lower price. The difference between call option prices for European and Barrier options are very similar. However, the put option prices for European and Barrier options are drastically different. The barrier option is much more stringent and tighter since the barrier ($110) is higher than the strike at expiry ($105). The only way in which the put value makes a profit is for the underlying stock price to go beyond $110 before expiration (pass the barrier) and fall below $105 at expiration (asset price less than strike price). Since the probability of making a profit is quite low, the put option price is also very low. The barrier put option may be worth more if the time to expiry increase, allowing the asset more time to reach the barrier.

Price of Barrier options with volatility increased by 10%:

```
One-step MC price of an Barrier call option with volatility increased by 10% is 8.658251329622082
One-step MC price of an Barrier put option with volatility increased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility increased by 10% is 8.657093178136849
Multi-step MC price of an Barrier put option with volatility increased by 10% is 1.5856543858665149
```

Price of Barrier options with volatility decreased by 10%:

```
One-step MC price of an Barrier call option with volatility decreased by 10% is 6.9518301941217135
One-step MC price of an Barrier put option with volatility decreased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility decreased by 10% is 7.103190938498456
Multi-step MC price of an Barrier put option with volatility decreased by 10% is 0.9781872790602056
```

For the price of Barrier options with increased volatility, the call option is higher. However, the put option is lower. For the price of Barrier options with decreased volatility, both the put and call options is lower. This is because the higher the volatility, the higher the chance for the price of the underlying

stock to cross the barrier before expiration, thus the higher the chance for the Barrier option to become a standard European option (i.e., not worthless anymore). The put option may have decreased in value due the option conditions (such as positive drift, in which the asset price is trending upwards but the put option requires the asset price to go upwards to meet the barrier and then drop below the strike price). Vice versa with the lower volatility, since the volatility is lower, there is less chance that price of the underlying stock will cross the barrier.

Discuss possible strategies to obtain the same prices from two procedures:

To be able to obtain the same prices from the Monte Carlo Simulations as the Black-Scholes equation, I can update the number of paths and number of steps. The possible numbers of steps are 1, 12, 52, and 252 which correspond to the number of years, months, weeks, and trading days in 1 year. The possible numbers of paths are 10000, 100000, 500000, and 1000000. I want to check which number of steps could achieve the same up-to-cent price with a smaller number of paths. Black Scholes Equation tells us that the value of the European call option is $8.0213, and the value of the put option is $7.9004. We can see that the 252 time step period with 500,000 paths provides us the call and put option values that is similar to the Black Scholes equation within a cent.

Appendix: Python Output:

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.060450530055613
One-step MC price of an European put option is 7.840920573848786
Multi-step MC price of an European call option is 7.990448517773648
Multi-step MC price of an European put option is 7.873521968183865
One-step MC price of an Barrier call option is 7.849650504998772
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.93214621191454
Multi-step MC price of an Barrier put option is 1.274492390265891
```

Figure 6 – Put and Call Option Values with numSteps = 12, numPaths = 100,000

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.015161880747636
One-step MC price of an European put option is 7.9274454869754445
Multi-step MC price of an European call option is 7.94785269469662
Multi-step MC price of an European put option is 7.886176659380042
One-step MC price of an Barrier call option is 7.829684272035303
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 8.010029597902415
Multi-step MC price of an Barrier put option is 1.717386354505425
```

Figure 7 – Put and Call Option Values with numSteps = 52, numPaths = 100,000

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.149968778804695
One-step MC price of an European put option is 7.8257668716518305
Multi-step MC price of an European call option is 7.972309141645514
Multi-step MC price of an European put option is 7.918148549202533
One-step MC price of an Barrier call option is 7.787853705917305
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.991006857456702
Multi-step MC price of an Barrier put option is 2.018322737248009
```

Figure 8 – Put and Call Option Values with numSteps = 252, numPaths = 100,000

```
10000-path 1-step MC price of an European call option is 8.25255060930108
10000-path 1-step MC price of an European put option is 7.703936002022148
100000-path 1-step MC price of an European call option is 8.09364322133528
100000-path 1-step MC price of an European put option is 7.870056750479562
500000-path 1-step MC price of an European call option is 8.024695977723002
500000-path 1-step MC price of an European put option is 7.911981150735373
1000000-path 1-step MC price of an European call option is 8.008145834723738
1000000-path 1-step MC price of an European put option is 7.928188911472129
10000-path 12-step MC price of an European call option is 8.044704989934967
10000-path 12-step MC price of an European put option is 7.9792124797642705
100000-path 12-step MC price of an European call option is 8.018831149678638
100000-path 12-step MC price of an European put option is 7.85494537050269
500000-path 12-step MC price of an European call option is 8.013049020225928
500000-path 12-step MC price of an European put option is 7.888007827469914
1000000-path 12-step MC price of an European call option is 7.99203169652598
1000000-path 12-step MC price of an European put option is 7.902230732653788
10000-path 52-step MC price of an European call option is 8.165530510899481
10000-path 52-step MC price of an European put option is 7.844056647752369
100000-path 52-step MC price of an European call option is 8.039302278740346
100000-path 52-step MC price of an European put option is 7.8762498811561175
500000-path 52-step MC price of an European call option is 8.001477938011673
500000-path 52-step MC price of an European put option is 7.902957726168387
1000000-path 52-step MC price of an European call option is 8.032194770271351
1000000-path 52-step MC price of an European put option is 7.91214343103016
10000-path 252-step MC price of an European call option is 8.170634053877206
10000-path 252-step MC price of an European put option is 7.810443730313972
100000-path 252-step MC price of an European call option is 7.9927916233818115
100000-path 252-step MC price of an European put option is 7.883454256529297
500000-path 252-step MC price of an European call option is 8.013618649587345
500000-path 252-step MC price of an European put option is 7.916657631321474
1000000-path 252-step MC price of an European call option is 8.017238190639196
1000000-path 252-step MC price of an European put option is 7.915747598054443
```

Figure 9 – Put and Call Option Values of different time steps and number of paths