Optimization of Drone-based Food Delivery Services
Project Report

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Abstract

This paper addresses a drone-enabled vehicle routing problem within the context of a university campus, aiming to optimize food delivery for students. The study employs an Integer Linear Programming (ILP) model in two stages. It focuses on the flow of drones leaving the central hub, picking up one order from each of up to two different restaurants, and returning to the hub. First, the model determines the minimum number of drones to deliver all the food orders. Second, setting the number of drones as a constant, the model then aims to calculate the minimum aggregated distance traveled by all the drones from the hub, to the restaurant(s) and back to the hub. Battery life, weight capacity, and veganism constraints are incorporated to control the performance. The paper presents a sensitivity analysis of constraints, revealing potential improvements in areas of weight capacity and dynamic order systems.

1. Introduction

Students at Columbia University are approaching the end of the semester and spending more time on campus to study for their final examinations. In an effort to achieve outstanding grades, many are forgoing a healthy diet by either skipping meals or purchasing food from the nearest providers, which are usually low on variety. To help students maintain a balanced diet during periods of high pressure by providing easier and more efficient access to food, our team is committed to employing fixed-capacity drones to deliver food orders from the 50 highest-rated restaurants around the Morningside campus, referred to as the central hub. Yet, gathering these food orders presents logistical challenges, including constraints on the weight capacity of drones, which restricts the number of orders each drone can deliver, and the high costs associated with acquiring high-quality drones. We aim to set up a mathematical model to minimize the number of drones while ensuring timely and efficient order fulfillment.

These findings can be valuable for the Columbia community, comprising students, alumni, employees, and affiliates, offering a reference for investments, design, and enhancements in logistics systems and strategies aimed at enhancing academic life. Moreover, this information can be applied as a point of reference for diverse purposes beyond the immediate scope.

This paper is divided into seven sections. In the first section, an overview of the research background, key points, and the overall research objective is presented. The second section describes relevant previous papers related to the vehicle routing problem. Following that, the third section outlines the methodology employed. The fourth section is dedicated to building and describing the mathematical model. Moving forward, the fifth section offers an analysis of the results. Subsequently, the sixth section engages in a discussion of potential improvements. Finally, the last section summarizes all the findings of the paper and puts forth ideas for potential future research.

2. Literature Review

Drone delivery systems are emerging as a cornerstone in smart city logistics, by offering a rapid, flexible alternative to traditional ground transportation, enabling direct aerial routes that can significantly reduce delivery times. Demonstrating this advancement, E-Mart24 Inc. has introduced their Multicopter, an aircraft drone capable of a 10 km flight lasting 30 minutes on a single charge, showcasing the practical application of these systems in commercial delivery services (Song, 2023).

Yet, the deployment of drones in delivery services encounters a range of operational challenges. Janszen et al. (2021) highlight how wind conditions can significantly affect drone energy use, necessitating advanced planning for energy conservation. Dukkanci et al. (2020) underscore the energy and cost demands inherent in drone operations, advocating for strategic approaches to balance expenditures.

Numerous studies have delved into enhancing the performance of drone delivery systems through optimization techniques. Shahzaad et al. (2023) introduce a context-sensitive framework tailored for smart cities, employing a heuristic line-of-sight algorithm to pinpoint the most efficient delivery routes. A series of studies also explored optimization strategies for drone-based food delivery systems. Liu (2019) crafted a dynamic vehicle routing algorithm that employs a mixed integer programming model to optimize on-demand meal delivery, taking into account the unique aspects of geometry and mobility in vehicle routing which allows for the dynamic input of orders across various locations. Pinto et al. (2020) concentrated on optimizing the location of charging stations in a drone meal delivery system, demonstrating how these placements significantly influence overall performance and align with the maximum wait times acceptable to customers.

To conclude, the collective body of research highlights the intricate nature of optimizing drone delivery systems, emphasizing the critical balance between energy efficiency, cost management, and logistical planning in urban settings.

3. Methodology

In our study, we sourced data from the "New York City Restaurant Inspection Data (2017-2021)" available on Kaggle (2023). This dataset includes addresses and inspection scores - ratings - for New York City restaurants. Anchoring the Morningside Campus of Columbia University as the central hub, we employed geospatial analysis to identify restaurants within a 5km radius, extracting the 50 highest-rated to create a distance matrix.

```
[[0.         4.8061581  0.94556118 1.67243466 2.27949263]
 [4.8061581  0.         5.71548088 4.3320953  3.45677807]
 [0.94556118 5.71548088 0.         2.03033786 2.87059944]
 [1.67243466 4.3320953  2.03033786 0.         0.95930093]
 [2.27949263 3.45677807 2.87059944 0.95930093 0.        ]]
```

Table 1: Distance Matrix

Due to the lack of real-world operational data, we simulated additional variables necessary for our model:

(1) the number of orders per restaurant: {0,5},

(2) the weight of food orders: {3,8} lbs, and

(3) the qualitative categorization of vegan restaurants: {0,1}.

Below is a snapshot of the suggested final data matrix, encapsulating these simulated variables, which underpins our analysis:

	Index	OBJECTID	CAMIS	DBA	SCORE	Latitude	Longitude	NEAR_DIST	Order_Count	Weights	IsVegetarian
1202	1	1203	50094092	THE STRANGER	133.0	40.766982	-73.983502	4806.158099	4	6.29	0
212	2	213	50073625	KRISPY KRUNCHY CHICKEN AND PIZZA	104.0	40.813932	-73.955868	945.561176	5	7.29	1
1605	3	1606	50081725	TERANGA TAC	101.0	40.796298	-73.949613	1672.434656	0	6.62	1
951	4	952	50090198	THAI PARAGON	100.0	40.788180	-73.953473	2279.492633	3	6.27	0
591	5	592	50093284	CATRINA DELI RESTAURANT	97.0	40.836547	-73.939326	3805.284506	3	5.54	1
1958	6	1959	50073673	TANOSHI SUSHI SAKE BAR	96.0	40.767669	-73.953133	4487.529612	3	6.88	1

Table 2: Restaurants Order Table

Our model is divided into two main sections:

(1) the first part focuses on minimizing the number of drones required to fulfill all orders;

(2) the second part optimizes the shortest delivery distance while keeping the minimized number of drones from part one constant.

Additionally, we examined constraints related to weight and veganism requirements through sensitivity analysis to further understand their impacts.

4. Model and Implementation

Assumptions:

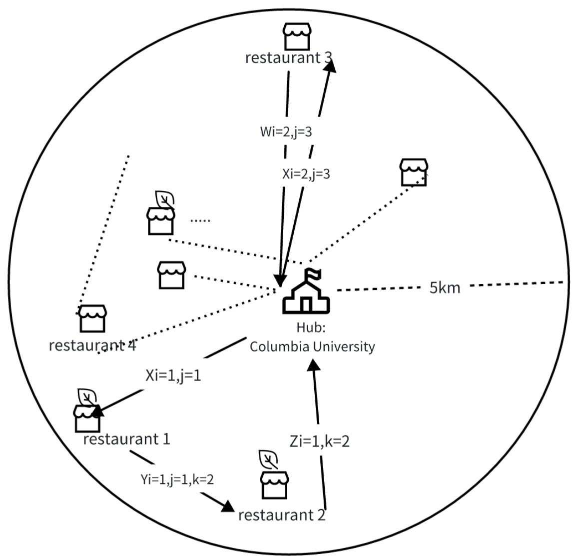
Below are the assumptions that directly influence the constraints of our drone delivery model:

1. Battery Capacity: Battery capacity allows each drone to fly for at most 10 km.
2. Carrying Capacity: Based on our research, a drone can carry a maximum of 12 pounds of food.
3. Single Deployment: Drones depart from the hub only once per operational cycle.
4. Restricted Restaurant Visits: Each drone visits at most two restaurants, collecting only one order unit per restaurant.
5. Return to Hub: drones return to the hub for Columbia students to collect their orders.
6. Single Loop Operation: Drones perform only one delivery loop before returning to the hub.

7. Vegetarian Delivery Requirements: Vegetarian food must be delivered separately, in drones not used for non-vegetarian food.

The rest of the assumptions relate to practical aspects of drone operations:

8. Static Order Generation: Orders are generated at a fixed point in time, with immediate drone deployment.
9. Linear Flight Path: Drones are assumed to fly in straight lines at a constant altitude.
10. Unrestricted Airspace: The operational area is free from any aviation control restrictions.
11. Uniform Order Weight: Each restaurant's orders are assumed to be of uniform weight. Below is the drone route map according to our model:



Graph 1: Drone Route Map

Part 1: Determine the minimum number of drones

Decision Variables:

Let X_{ij} be a binary variable such that

X_{ij} = 1 if drone i goes from the hub 0 to the first restaurant j, 0 otherwise

Let Y_{ijk} be a binary variable such that

Y_{ijk}= 1 if drone i goes from the first restaurant j to the second restaurant k, 0 otherwise

Let W_{ij} be a binary variable such that

W_{ij}= 1 if drone i goes from the first restaurant j to the hub 0, 0 otherwise

Let Z_{ik} be a binary variable such that

Z_{ik}= 1 if drone i goes from the second restaurant k to the hub 0, 0 otherwise

<u>Parameters:</u>
D_j : Number of orders for restaurant $j \in Restaurants$
d_{jk} : Distance from location $j \in \{hub,\ Restaurants\}$ to location $k \in \{Restaurants,\ hub\},\ j \neq k$
w_j : Weight of food order from restaurant $j \in Restaurants$
V_j = 1 if the restaurant $j \in Restaurants$ is vegan, 0 otherwise
n = Number of restaurants, 50 restaurants
M = Maximum number of drones required
Max_weight = Maximum weight carried by each drone, 12 lbs

<u>Objective Function:</u> Our initial goal is to minimize the fleet of drones needed to deliver a predetermined number of food orders from different restaurants.

Minimize Z = $\sum_{i=0}^{M} \sum_{j=0}^{n} X_{ij}$

<u>Constraints:</u>
1. Order Constraint: The total number of food orders from restaurant j must be picked up and delivered.
   $\sum_{j=1}^{M} X_{ij} + \sum_{j=1}^{M} \sum_{k=1}^{n} Y_{ijk} = D_j, \forall i$
2. Battery Constraint: The total distance traveled (in km) by a drone i is less than or equal to 10 km.
   $\sum_{j=1}^{n} X_{ij} d_{jk} + \sum_{j=1}^{n} \sum_{k=1, k\neq j}^{n} Y_{ijk} d_{jk} + \sum_{j=1}^{n} W_{ij} d_{jk} + \sum_{k=1, k\neq j}^{n} Z_{ik} d_{jk} \leq 10, \forall i$
3. Weight Constraint: The sum of the weights of the food orders a single drone i can carry is limited by the maximum weight capacity constant of a drone, assuming all drones are identical.
   $\sum_{j=1}^{n} X_{ij} w_j + \sum_{j=1}^{n} \sum_{k=1,\ k\neq j}^{n} Y_{ijk} w_k \leq Max\_weight, \forall i$
4. Vegan Constraint:
   a. If restaurant $j \in Restaurants$ is vegan, then restaurant $k \in Restaurants$ must also be vegan:
      $Y_{ijk} V_j (1 - V_k) = 0, \forall i, j \neq k$
   b. If restaurant $j \in Restaurants$ is non-vegan, then restaurant $k \in Restaurants$ must also be non-vegan:
      $Y_{ijk} V_k (1 - V_j) = 0, \forall i, j \neq k$

5. Logical Constraint:

X_{ij}	Y_{ijk}	W_{ij}	Z_{ik}
1	1	0	1
1	0	1	0
0	0	0	0

a. Capacity Constraint: The number of orders a single drone i can carry in one circuit from the hub to the restaurant(s) back to the hub is at most 2 orders.
$$\sum_{j=1}^{n} X_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{n} Y_{ijk} \le 2, \forall i$$
b. If a drone i is not deployed, no distance can be traveled for drone i.
c. Each drone can only take off once from the hub
$$\sum_{j=1}^{n} X_{ij} \le 1, \forall i$$
d. If a drone i is employed
   i. If drone i goes to restaurant j and then immediately returns to the hub
   $X_{ij} = W_{ij}, \forall j$
   ii. If drone i goes to restaurant k after restaurant j before returning to the hub
   $X_{ij} = Y_{ijk}$, $\forall j$ and $Y_{ijk} = Z_{ik}, \forall k$
e. For any drone i that is deployed, the drone i must travel to a different location from its current location.
$\forall i : \forall j \ \forall k, \ k \neq j$
f. A drone i deployed will necessarily reach a restaurant. Then, it can either return to the hub or travel to a second restaurant before returning to the hub.
$$\forall i, j, \ W_{ij} + \sum_{k=1}^{n} Y_{ijk} = X_{ij}$$
g. For any drone i that is deployed, the drone i must return to the hub immediately after picking up a second order.
$\forall \ Y_{ijk} = Z_{ik}$
h. The number of drones deployed is at most the total number of food orders available.
$Y_{ijk} \le MX_{ij}, \forall i, j, k$

Part 2: Determine the minimum distance traveled by all drones

After establishing the minimum number of 56 drones (M = 56), our second model focuses on identifying optimal routes for each drone, yielding the aggregated shortest distance traveled. The parameters and constraints remain the same.

$$Minimize\ Z = \sum_{j=1}^{n} X_{ij} d_j + \sum_{j=1}^{n} \sum_{k=1, k\neq j}^{n} Y_{ijk} d_{ijk} + \sum_{j=1}^{n} W_{ij} d_j + \sum_{k=1, k\neq j}^{n} Z_{ik} d_k\ , \forall i$$

5. Results and Analysis

In our model, we considered three key constraints: order demand, drone capacity, and battery life. They were supported by logical constraints to determine the minimum number of drones required. Our recommendation is to deploy 56 drones out of 111 (the upper bound) to collect and deliver all food orders from designated restaurants. Table 3 presents a segment of our findings, while a comprehensive list of results is available in Appendix 1. Analysis reveals that each utilized drone carried two orders before returning to the hub while adhering to the battery capacity by traveling a distance under 10 kilometers.

Drone ID	ROUTE	TOTAL DISTANCE
0	Hub -> 47 -> 5 -> Hub	8.60430428
2	Hub -> 30 -> 9 -> Hub	7.38845327
5	Hub -> 46 -> 9 -> Hub	6.92352536
7	Hub -> 1 -> 34 -> Hub	9.63330996
9	Hub -> 35 -> 37 -> Hub	9.84807256
12	Hub -> 49 -> 6 -> Hub	8.98851948
14	Hub -> 18 -> 5 -> Hub	8.02220704
16	Hub -> 1 -> 25 -> Hub	9.93540548
18	Hub -> 10 -> 11 -> Hub	7.66747889
19	Hub -> 36 -> 45 -> Hub	9.87130112

Table 3: Results of 10 Drones

However, repeated trials with Gurobi did not yield results in line with our expectations, even though the model's logical constraints on various decision variables were correctly formulated. Consequently, we downsized the model to involve only 20 restaurants, with each restaurant having a maximum of two orders. Nevertheless, inconsistencies persisted. For instance, the number of times a restaurant was visited occasionally exceeded its total number of orders, contradicting our order constraint, as depicted in Graph 2. In the case of Drone 13, which was assigned only one order, it was recorded to take off twice, defying our stipulation that it should only take off once, as shown in Graph 3. Another discrepancy, illustrated in Graph 4, was observed where, out of 18 orders, only 15 restaurants were visited.

	OBJECTID	CAMIS	DBA	SCORE	Latitude	Longitude	NEAR_DIST	Order_Count	Index
1202	1203	50094092	THE STRANGER	133.0	40.766983	-73.983502	4806.158099	2	1
212	213	50073625	KRISPY KRUNCHY CHICKEN AND PIZZA	104.0	40.813932	-73.955868	945.561176	0	2
1605	1606	50081725	TERANGA TAC	101.0	40.796298	-73.949613	1672.434656	1	3
951	952	50090198	THAI PARAGON	100.0	40.788180	-73.953473	2279.492633	0	4
591	592	50093284	CATRINA DELI RESTAURANT	97.0	40.836547	-73.939326	3805.284506	0	5
1958	1959	50073673	TANOSHI SUSHI SAKE BAR	96.0	40.767669	-73.953133	4487.529612	0	6
1733	1734	50017185	OASIS JIMMA JUICE BAR	90.0	40.814647	-73.959057	873.386614	1	7
1835	1836	50072849	DUN HUANG UPPER WEST SIDE, MELLOW TEA	83.0	40.810944	-73.958052	577.042111	1	8
862	863	50077742	ASHOKA	80.0	40.779958	-73.950214	3231.163146	2	9
1334	1335	50092520	SUSHI JIN	77.0	40.776284	-73.952084	3574.137358	1	10

Graph 2

	Drone	Route	Total_Distance
0	0	Hub -> Restaurant 1 -> Restaurant 14 -> Hub	9.616075
1	1	Hub -> Restaurant 18 -> Restaurant 11 -> Hub	9.307706
2	2	Hub -> Restaurant 10 -> Restaurant 9 -> Hub	7.243152
3	3	Hub -> Restaurant 14 -> Restaurant 1 -> Hub	9.616075
4	4	Hub -> Restaurant 19 -> Restaurant 3 -> Hub	7.861477
5	6	Hub -> Restaurant 12 -> Restaurant 13 -> Hub	10.021481
6	9	Hub -> Restaurant 12 -> Restaurant 7 -> Hub	9.956087
7	11	Hub -> Restaurant 19 -> Restaurant 9 -> Hub	7.660110
8	13	Hub -> Restaurant 8 -> Restaurant 13 -> Hub	9.378913

Graph 3

```
print(max_drones)
```

18

	Drone	Route	Total_Distance
0	0	Hub -> Restaurant 11 -> Restaurant 3 -> Hub	3.542753
1	2	Hub -> Restaurant 11 -> Restaurant 4 -> Hub	5.065689
2	3	Hub -> Restaurant 19 -> Restaurant 9 -> Hub	7.660110
3	4	Hub -> Restaurant 18 -> Restaurant 2 -> Hub	8.036599
4	6	Hub -> Restaurant 3 -> Restaurant 4 -> Hub	4.911228
5	9	Hub -> Restaurant 14 -> Restaurant 15 -> Hub	9.899824
6	10	Hub -> Restaurant 1 -> Hub	9.612316
7	17	Hub -> Restaurant 6 -> Restaurant 9 -> Hub	9.107184

Graph 4

We have experimented with numerous solutions and identified that the issue might originate from Gurobi's internal loop algorithm. A deeper examination of Gurobi's iterative algorithm could provide valuable insights for enhancing our results. Additionally, the complexity of the logical constraints is a significant challenge, and we may still be missing some necessary conditions. Appendix 4 lists all the logical constraints we have considered so far. We attempted to apply the model using all identified logical constraints, even though some conditions were repetitive, but the outcomes still did not meet our expectations. This indicates that further discussion and analysis are required.

Ideally, if the logical constraints are correct and complete, the objective of minimizing distance should yield the minimum number of drones required under a given number of available units (this only requires changing the objective and setting the number of drones to the minimum identified in the first model). Incorporating constraints for vegan and weight considerations is also feasible by building upon the foundation of Model 1. After adding constraints for vegan and weight, given the same order table, it is expected that more drones will be utilized.

In the sensitivity analysis section, we explore the impact of battery constraints on drone allocation. The minimum battery distance required should be equal to the distance of the longest loop after optimizing paths in Model 2. For instance, if we adjust the battery constraints to 20km (assuming two restaurants are located at the ends of a diameter of a 5km radius circle), then drones will no longer be limited by battery constraints.

Similarly, with weight constraints, if we continuously increase the weight limit, fewer drones will be used. The maximum right-hand side (RHS) for weight constraints is 18 lbs (the maximum combined weight of two orders). If we adjust both weight and battery to their maximum required values, theoretically, all vegan and non-vegan orders would be paired:

6. Discussion

a. Conclusion

In this project, we sought to model a vehicle routing problem based on a real-world problem. By formulating a series of assumptions, we built two models based on different objectives and we did the sensitivity analysis. Our first model found a minimum number of drones to fulfill an order placed at a fixed point in time. This is reasonable because optimizing the number of drones means optimizing the fixed costs that we need to invest in. The other one is to optimize the route of these drones to pick up the orders based on a known minimum number. In this way, we could minimize our operating costs for drone delivery, which could also make our project more beneficial. We believe both of the models will make a lot of practical sense, whereas both of them need some future improvements when considering more and more realistic scenario factors.

b. Limitations & Future Improvements

Our initial drone delivery model serves as a promising starting point, yet there's significant scope for refinement. The following advancements are envisaged to enhance the model's practicality and adaptability, aligning it more closely with real-world logistical complexities:

- **Varied Order Weights**: Introducing distinct weights for individual orders, by introducing an associated order index parameter a. This would mirror the real-world scenario of diverse order sizes more accurately. It involves updating our decision variables to X_{ija} and Z_{ika}. This change enables the model to track and manage individual order weights. Accordingly, the weight constraint can be formulated as:
  $\forall i$: $\sum_j \sum_a x_{ija} w_a + \sum_k \sum_a z_{ika} w_a \leq Capacity_i$, $i \in Drones$, $j, k \in Restaurants$, $a \in Orders$
  and w_a stands for the weight of order a.
- **Broadening Drone Reach Beyond Two Restaurants**: The current model facilitates a straightforward approach to modeling by allowing drones to cater to up to two eateries per sortie. Prospective model evolutions could remove this restraint, broadening the scope of each drone's operational capacity, and permitting drones to extend their service

to a greater number of eateries within a single flight path, significantly amplifying both the scale and efficacy of delivery operations.

- **Evolving Towards a Dynamic and Scalable Drone Delivery System:** Our vision for the drone delivery system encompasses a multi-faceted development strategy, poised to enhance responsiveness, scalability, and efficiency. This comprehensive enhancement includes

    (1) ***Dynamic Adaptation to Order Timing*** through transitioning to a model that dynamically responds to the ebb and flow of order placements;
    (2) ***Optimized Drone Utilization and Energy Management*** by recycling drones within daily operational cycles and factoring their recharging times;
    (3) ***Expansion with Multiple Launch Hubs*** to broaden delivery horizons and accelerate service, and allowing drones to end up charging in different Hubs.

- **Integrating Practical Operational Constraints:** Future models could delve into more intricate operational aspects such as airspace restrictions, the interplay between flight altitude and power usage, and weather-related impacts. Addressing these factors will not only ensure adherence to aviation norms but also optimize the energy efficiency of drone operations.

These advancements strive to propel the drone delivery paradigm to greater levels of efficacy and applicability. As drones become more widespread, an abundance of data will become available, offering valuable insights that can further refine these models and better align them with the complexities of contemporary logistics and delivery ecosystems.

To summarize, we believe that our research is of high practical implications. This is not only because we built our model from a realistic demand and problem, but also because we considered as many realistic assumptions as possible when modeling, such as capacity limitations of drones, load-bearing constraints, battery limitations, and order satisfaction for restaurants as well as segregation of vegetarian and non-vegetarian orders. We also present a set of model limitations as well as possible future improvements for further optimization by future researchers.

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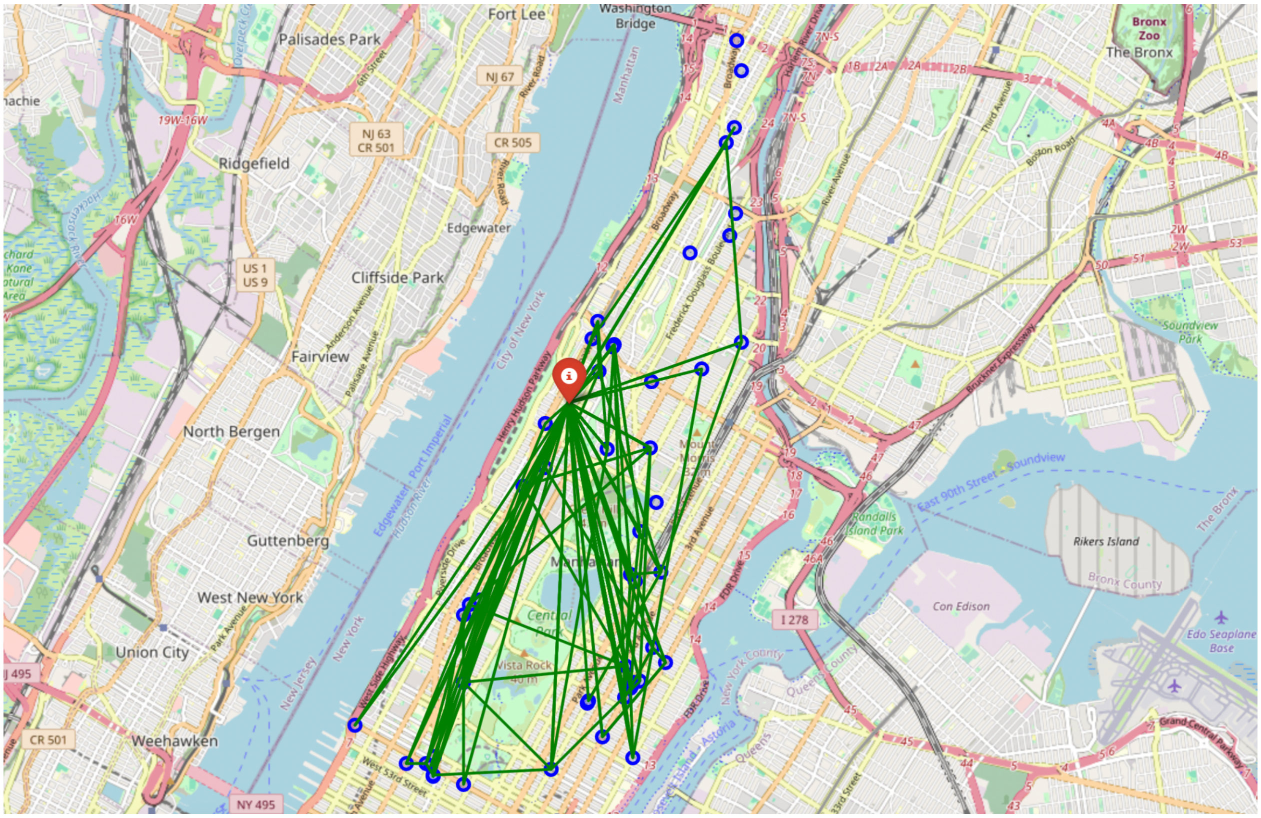
Appendix

Appendix 1: Results of All Drones' Routes

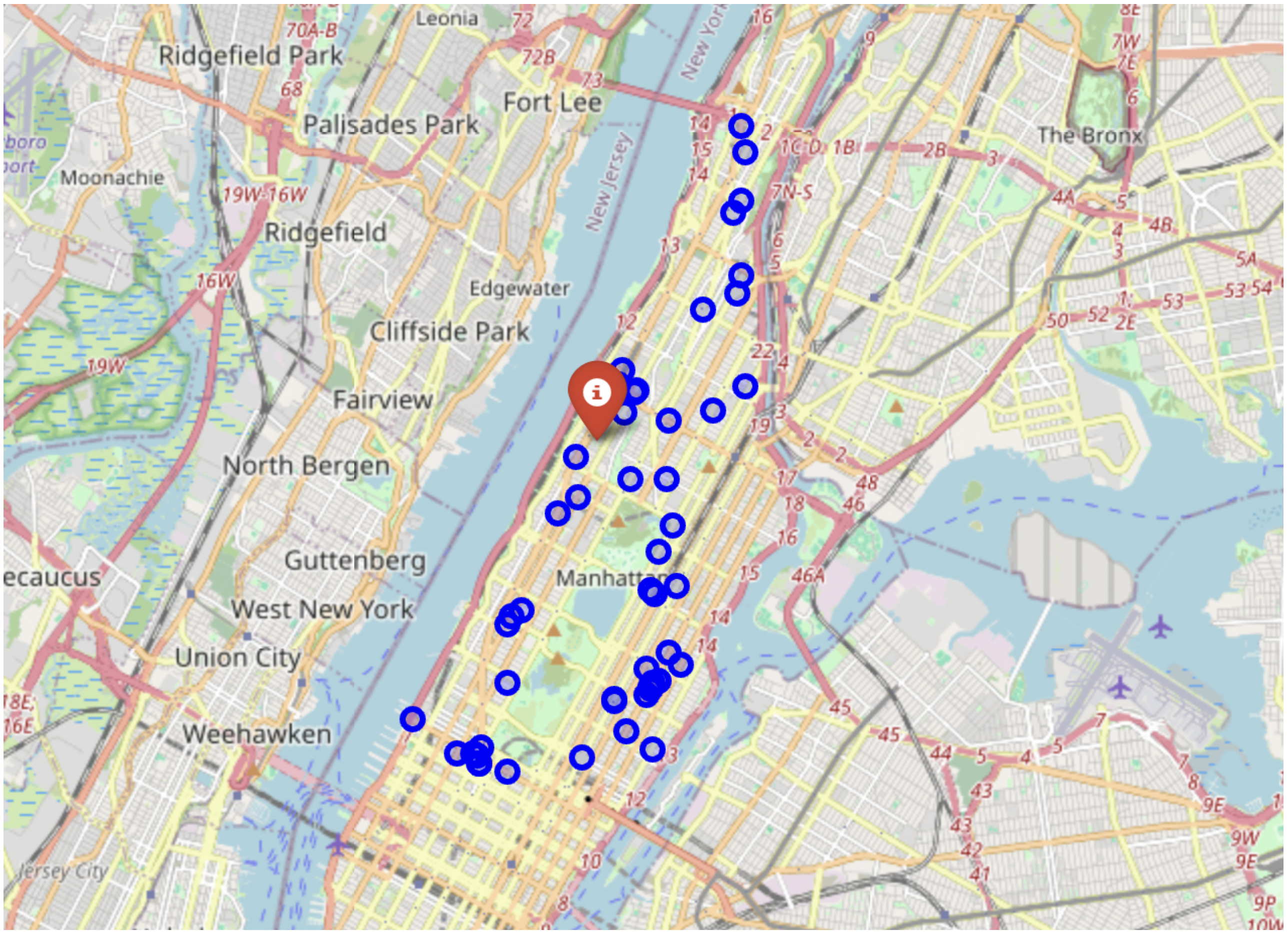
	Drone	Route	Total_Distance
0	0	Hub -> 47 -> 5 -> Hub	8.60430428
1	2	Hub -> 30 -> 9 -> Hub	7.38845327
2	5	Hub -> 46 -> 9 -> Hub	6.92352536
3	7	Hub -> 1 -> 34 -> Hub	9.63330996
4	9	Hub -> 35 -> 37 -> Hub	9.84807256
5	12	Hub -> 49 -> 6 -> Hub	8.98851948
6	14	Hub -> 18 -> 5 -> Hub	8.02220704
7	16	Hub -> 1 -> 25 -> Hub	9.93540548
8	18	Hub -> 10 -> 11 -> Hub	7.66747889
9	19	Hub -> 36 -> 45 -> Hub	9.87130112
10	21	Hub -> 6 -> 9 -> Hub	9.10718442
11	22	Hub -> 2 -> 27 -> Hub	5.13123112
12	25	Hub -> 1 -> 35 -> Hub	9.62507196
13	28	Hub -> 15 -> 21 -> Hub	9.58445616
14	29	Hub -> 48 -> 4 -> Hub	5.06638587
15	31	Hub -> 10 -> 24 -> Hub	9.68727896
16	34	Hub -> 15 -> 21 -> Hub	9.58445616
17	35	Hub -> 46 -> 43 -> Hub	8.84683025
18	37	Hub -> 46 -> 43 -> Hub	8.84683025
19	39	Hub -> 8 -> 31 -> Hub	2.28555929
20	40	Hub -> 40 -> 4 -> Hub	6.02912721
21	44	Hub -> 7 -> 5 -> Hub	7.62590148
22	45	Hub -> 19 -> 28 -> Hub	7.47170966
23	46	Hub -> 47 -> 48 -> Hub	7.74850271
24	47	Hub -> 6 -> 19 -> Hub	8.97808524
25	52	Hub -> 44 -> 31 -> Hub	3.44087181
26	53	Hub -> 25 -> 39 -> Hub	9.81050223
27	55	Hub -> 48 -> 16 -> Hub	7.97393474

28	57	Hub -> 28 -> 27 -> Hub	6.88484604
29	61	Hub -> 33 -> 35 -> Hub	9.62141878
30	63	Hub -> 49 -> 11 -> Hub	7.83275437
31	71	Hub -> 24 -> 36 -> Hub	10.9323777
32	73	Hub -> 28 -> 14 -> Hub	9.06113955
33	77	Hub -> 21 -> 9 -> Hub	6.67713846
34	78	Hub -> 19 -> 4 -> Hub	7.53620978
35	80	Hub -> 27 -> 2 -> Hub	5.13123112
36	83	Hub -> 44 -> 2 -> Hub	3.00916963
37	84	Hub -> 41 -> 45 -> Hub	9.94540548
38	85	Hub -> 48 -> 8 -> Hub	5.5925977
39	86	Hub -> 11 -> 35 -> Hub	3.99084424
40	88	Hub -> 19 -> 2 -> Hub	9.06653512
41	89	Hub -> 44 -> 2 -> Hub	3.00916963
42	90	Hub -> 45 -> 37 -> Hub	9.87253998
43	91	Hub -> 35 -> 28 -> Hub	7.09078093
44	92	Hub -> 11 -> 34 -> Hub	6.93498646
45	93	Hub -> 8 -> 14 -> Hub	8.47775847
46	95	Hub -> 28 -> 34 -> Hub	8.21279573
47	96	Hub -> 30 -> 19 -> Hub	7.49805269
48	97	Hub -> 14 -> 31 -> Hub	9.55753552
49	100	Hub -> 9 -> 23 -> Hub	8.50577542
50	102	Hub -> 24 -> 44 -> Hub	9.37538468
51	103	Hub -> 37 -> 20 -> Hub	9.83943175
52	105	Hub -> 21 -> 36 -> Hub	9.83503424
53	106	Hub -> 1 -> 36 -> Hub	9.89040915
54	109	Hub -> 25 -> 21 -> Hub	9.79073728
55	110	Hub -> 14 -> 24 -> Hub	9.77609901

Appendix 2: Routes for all 56 drones



Appendix 3: Mapping of Top 50 Restaurants near Columbia University



Appendix 4： Complete Logical Constraints

1. All drones can only ply to the same place at most once

$$\forall i, \sum_{j=1}^{n} X_{ij} \leq 1, \sum_{j=1}^{n}\sum_{k=1}^{n} Y_{ijk} \leq 1 \ j \neq k, \sum_{j=1}^{n} W_{ij} \leq 1, \ \sum_{k=1}^{n} Z_{ik} \leq 1$$

2. If a drone doesn't take off: $X_{ij} = 0$:

$$Wij = Y_{ijk} = Z_{ij} = 0$$

This leads to:

$$\forall i \ \sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} W_{ij} = \sum_{j=1}^{n}\sum_{k=1}^{n} Y_{ijk} = \ \sum_{k=1}^{n} Z_{ik} = 0$$

3. If a drone take off: $\forall i, \sum_{j=1}^{n} X_{ij} \leq 1$

3.1. If only visits 1 restaurant:

- For each individual visits:

$$If \ X_{ij} = 1 \ and \ W_{ij} = 1, \forall i,j \Rightarrow Y_{ijk} = 0 \ and \ Z_{ik} = 0, \forall k$$

- Also for each drone:

$$If \ \sum_{j}^{n} X_{ij} = 1 \ and \sum_{j}^{n} W_{ij} = 1 \ \Rightarrow \sum_{j}^{n}\sum_{k}^{n} Y_{ijk} = 0 \ and \ \sum_{k}^{n} Z_{ik} = 0 \ , \forall i$$

3.2. If visits 2 restaurants:

- For each individual visits

$$If \ X_{ij} = 1 \ and \ Y_{ijk} = 1 \ \Rightarrow W_{ij} = 0 \ and \ Z_{ik} = 1 \ \forall i,j,k$$

- For each drone:

$$If \ \sum_{j}^{n} X_{ij} = 1 \ and \sum_{j}^{n}\sum_{k}^{n} Y_{ijk} = 1 \ \Rightarrow \sum_{j}^{n} W_{ij} = 0 \ and \ \sum_{k}^{n} Z_{ik} = 1 \ \ \forall i$$

4. Relationship between W, Y, Z

$$if \ W_{ij} = 1 \ \ \Rightarrow Y_{ijk} = 0, Z_{ik} = 0, \forall i,j,k$$

$$if \sum_{j}^{n} W_{ij} = 1 \ \ \Rightarrow \sum_{j}^{n}\sum_{k}^{n} Y_{ijk} = 0 \, , \sum_{k}^{n} Z_{ik} = 0 \, , \forall i$$

$$if \ W_{ij} = 0 \ \Rightarrow Y_{ijk}, Z_{ik} = \{1,0\}, depending \ on \ X$$

5. Relationship between Y and Z:

- For each individual visits:

$$if \ Y_{ijk} = 1 \ \Rightarrow Z_{ik} = 1 \ \forall i, \forall k$$

$$if \ Y_{ijk} = 0 \ \Rightarrow Z_{ik} = 0 \ \forall i, \forall k$$

- For each drone:

$$if \sum_{j}^{n}\sum_{k}^{n} Y_{ijk} = 1 \ \Rightarrow \sum_{k}^{n} Z_{ik} = 1 \ \forall i$$

$$if \sum_{j}^{n}\sum_{k}^{n} Y_{ijk} = 0 \ \Rightarrow \sum_{k}^{n} Z_{ik} = 0 \ \forall i$$

6. Each drone can only carry up to 2 orders= relationship between X to Y and Z

$$\sum_{j=1}^{n} X_{ij} + \sum_{k=1}^{n} Z_{ik} \leq 2, \forall i$$

$$\sum_{j=1}^{n} X_{ij} + \sum_{j}^{n} \sum_{k}^{n} Y_{ijk} \leq 2, \ \forall i$$

7. For each individual Y_{ijk}:

$$if\ Y_{ijk} = 1, j \neq k$$