Stochastic optimal power flow based on data-driven distributionally robust optimization

Yi Guo*, Kyri Baker†, Emiliano Dall'Anese†, Zechun Hu‡
and Tyler Summers*

*Control, Optimization & Network Laboratory
The University of Texas at Dallas

†The University of Colorado, Boulder

‡Tsinghua University

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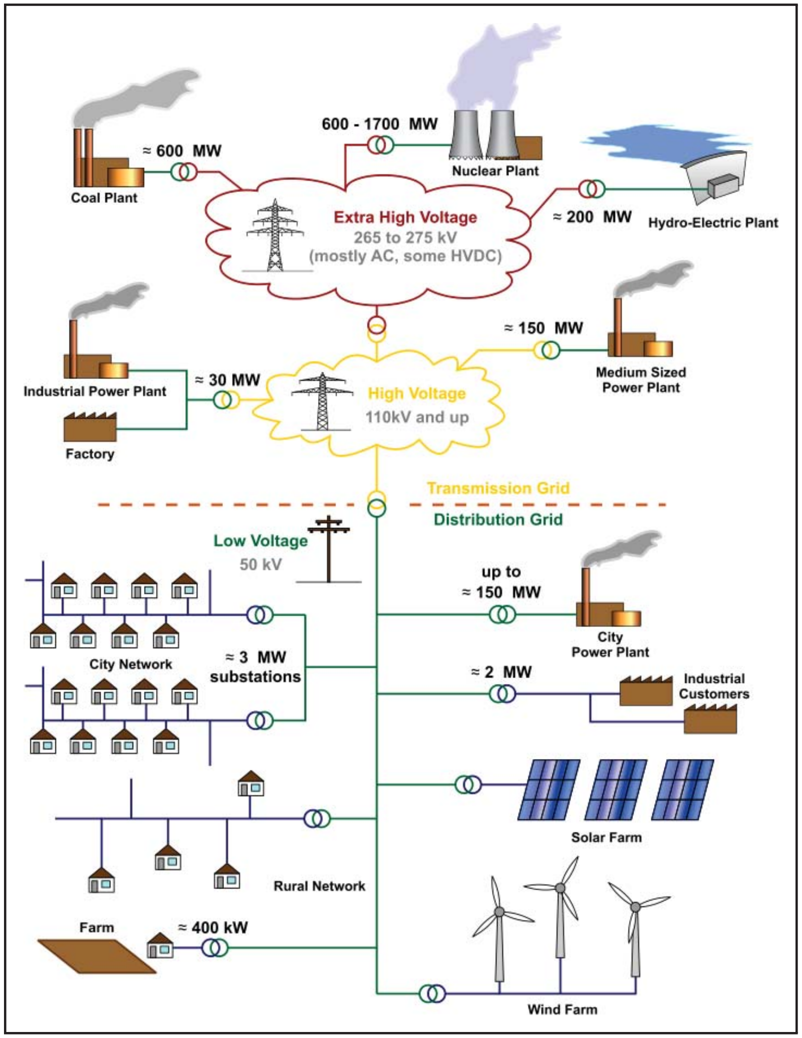
Stochastic optimization in power networks

How to explicitly incorporate information about forecast errors in optimal power flow (OPF) problems?



- rapid changes in modern grids $\Longrightarrow$ increasing uncertainty in net load
- need more sophisticated stochastic optimization & control approaches
- what's the best way to model forecast errors?
- how to balance inherent tradeoffs between efficiency, constraint violation risk, and sampling errors from finite datasets?

Multi-stage stochastic optimal power flow (OPF)

grid-connected components:

1. traditional generators
2. fixed, deferrable, and curtailable loads
3. storage devices, e.g., batteries and plug-in electric vehicles

objective: $\inf_{D,e} \quad \mathbb{E} \sum_{t=0}^{T} h_t(x_t, u_t, \xi)$

affine reserve policies $u_t = D_j^t \xi + e_j^t$

devices dynamics: $x_{t+1} = f_t(x_t, u_t, \xi_t)$

power balance constraints: $\sum_{j=1}^{N_d} (r_j + G_j \xi + C_j \mathbf{x}^j) = 0$

line flow constraints: $\sum_{j=1}^{N_d} \Gamma_j (r_j + G_j \xi + C_j \mathbf{x}^j) \leq \bar{p}$

local device constraints: $T_j \mathbf{x}^j + U_j \mathbf{u}^j + Z_j \xi \leq w_j$

voltage constraints: $V_{\min} \leq V \leq V_{\max}$

uncertainty modeling: $\xi \in \Xi, \mathrm{or}, \xi \sim \mathbb{P}, \mathrm{or}, \xi \in [\hat{\xi}_1, \ldots, \hat{\xi}_N]$

Approaches to quantify uncertainties in stochastic OPF

scenario-based approach

- decisions based on limited number of scenarios

  [Vrakopoulou, et.al., 2018] [Li, et.al., 2018]

assumptions on distributions

- forecast errors follow a prescribed distribution (commonly Gaussian)

  [Bienstock, et.al., 2014,], [Roald, et.al., 2013], [Lubin, et.al., 2016]

robust approach

- acknowledged bounds of uncertainties

  [Warrington, et.al., 2013], [Jabr, et.al., 2015]

distributionally robust approach

- decisions robust to data-generating distributions with consistent distribution parameters (e.g., moment-based)

  [Summers, et.al., 2015], [Li, et.al., 2016], [Dall'Anese, et.al., 2013]

limitations:

- in practice, only have access to finite historical data, not distributions, bounds, moments, etc.
- yet finite, sparse datasets have inherent sampling errors

Data-based distributionally robust approach

How to explicitly incorporate, control, and visualize tradeoffs between

- nominal efficiency
- risks of constraint violation
- sample errors inherent in finite forecast error datasets

in multi-period stochastic optimal power flow?

Data-based distributionally robust approach

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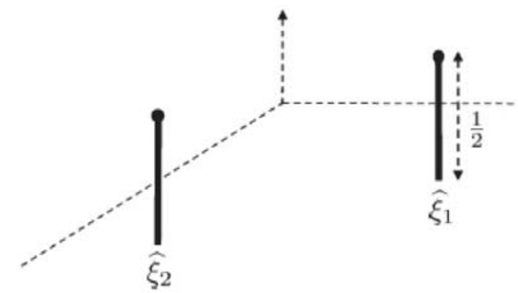
Data-based distributionally robust OPF

- ambiguity sets contain all uncertainty distributions within Wassertein distance ε of empirical distribution
- can guarantee out-of-sample performance by adjusting Wasserstein radius
- decisions robust to *worst-case* data generating distribution in ambiguity sets.

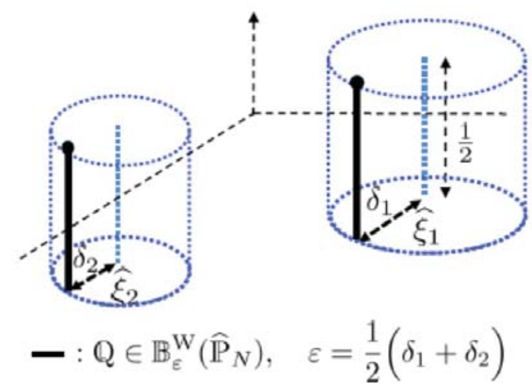
Tradeoffs between sampling errors and performance

$$
\begin{aligned}
\inf_{D,e} \ {\color{red}\sup_{\mathbb{P}\in\mathcal{P}_\varepsilon^N}} \quad & \mathbb{E}^{\mathbb{P}} \sum_{t=0}^{T} h_t(x_t,u_t,\xi_t) \\
\text{subject to} \quad & x_{t+1} = f_t(x_t,u_t,\xi_t) \\
& u_t = \ D_j\xi + e_j \\
& (x_t,u_t) \in \mathcal{X}_t, \xi_t \in \Xi_t.
\end{aligned}
$$

empirical distribution supported on $\hat{\Xi}^N$ [Esfahani, Kuhn, 2017]



discrete distribution by the Wasserstein metric



ambiguity set: Wasserstein balls with radius ε

$$
\hat{\mathcal{P}}_\varepsilon^N := \left\{ \mathbb{P} \in \mathcal{M}(\Xi) : d_w(\hat{\mathbb{P}}^{N_s}, \mathbb{P}) \le \varepsilon \right\},
$$

- out-of-sample performance guarantee
- controllable conservativeness

Tradeoffs between efficiency and risks

$$
\begin{aligned}
\inf_{D,e} \sup_{\mathbb{P}\in\hat{\mathcal{P}}_\varepsilon^N} \quad & \sum_{t=0}^{T} \mathbb{E}^{\mathbb{P}} J_{\text{Cost}}^t(x_t, u_t, \xi_t) + \rho J_{\text{Risk}}^t(x_t, u_t, \xi_t) \\
\text{subject to} \quad & x_{t+1} = f_t(x_t, u_t, \xi_t) \\
& u_t = D_j^t \xi + e_j^t \\
& (x_t, u_t) \in \mathcal{X}_t.
\end{aligned}
$$

operational cost functions:

$$
J_{\text{Cost}}^t(x_t, u_t) := f_x^\intercal x_t + \frac{1}{2} x_t{}^\intercal H_x x_t + f_u^\intercal u_t + \frac{1}{2} u_t{}^\intercal H_u u_t,
$$

constraint violation risk functions: conditional value at risk (CVaR):

$$
J_{\text{Risk}}^t := \text{CVaR}[\ell_i(x_t, u_t, \xi_t)] = \mathbb{E}^{\mathbb{P}}[\max_{k=1,2} \langle a_k(y), \xi\rangle + b_k].
$$

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Distributionally robust OPF convex reformulation

the distributionally robust stochastic OPF

$$\inf_{y\in\mathbb{Y},\tau\in\mathbb{R}} \sup_{\mathbb{P}\in\hat{\mathcal{P}}_\varepsilon^N} \mathbb{E}^{\mathbb{P}}[\max_{k=1,2} \langle a_k(y), \xi\rangle + b_k],$$

can be equivalently reformulated as a linear program [Esfahani, Kuhn, 2017]:

$$\inf_{\lambda, s_i, \gamma_{ik}, y, \tau} \quad \lambda\varepsilon + \frac{1}{N_s}\sum_{i=1}^{N_s} s_i,$$

subject to

$$\begin{aligned} b_k + \langle a_k(y), \hat{\xi}_i\rangle + \langle \gamma_{ik}, d - H\hat{\xi}\rangle \le s_i, &\quad \forall i \le N_s, \forall k = 1,2,\\ ||H'\gamma_{ik} - a_k(y)||_\infty \le \lambda, &\quad \forall i \le N_s, \forall k = 1,2,\\ \gamma_{ik} \ge 0, &\quad \forall i \le N_s, \forall k = 1,2. \end{aligned}$$

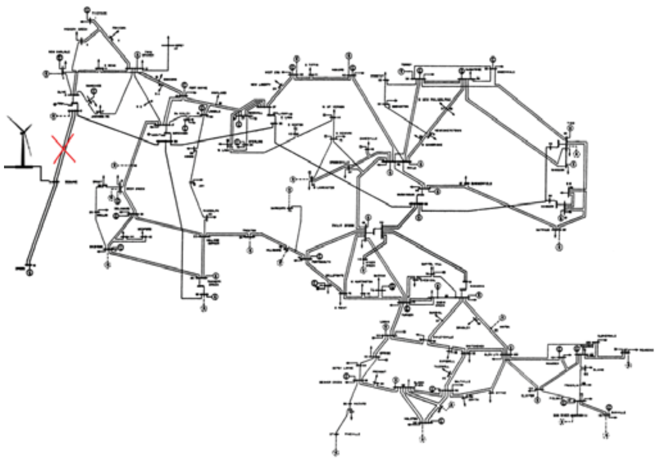
Distributionally robust model predictive control implementation

$$
\begin{aligned}
\inf_{D,e} \sup_{\mathbb{P}\in\hat{\mathcal{P}}^{N_s}} \quad & \sum_{\tau=t}^{t+\mathcal{H}_t} \mathbb{E}^{\mathbb{P}} J^{\tau}_{\text{Cost}}(x_\tau, u_\tau, \xi_\tau) + \rho J^{\tau}_{\text{Risk}}(x_\tau, u_\tau, \xi_\tau) \\
\text{subject to} \quad & x_{\tau+1} = f_\tau(x_\tau, u_\tau, \xi_\tau) \\
& u_\tau = D^{\tau}_{j}\xi + e^{\tau}_{j} \\
& (x_\tau, u_\tau) \in \mathcal{X}_\tau .
\end{aligned}
$$

- repeatedly solve finite-horizon distributionally robust optimization problem to obtain feedback policy

- at time step t, forecast uncertainties across power network over horizon $\mathcal{H}_t$
- solve above problem over horizon $\mathcal{H}_t$
- implement reserve control policies for each device at current time step t
- move to time $t+1$, and repeat

Case studies



- modified 118-bus transmission system
- wind energy injection
- optimal power output adjustment and reserve policies for generators
- generation costs
- CVaR of line flow constraint violations

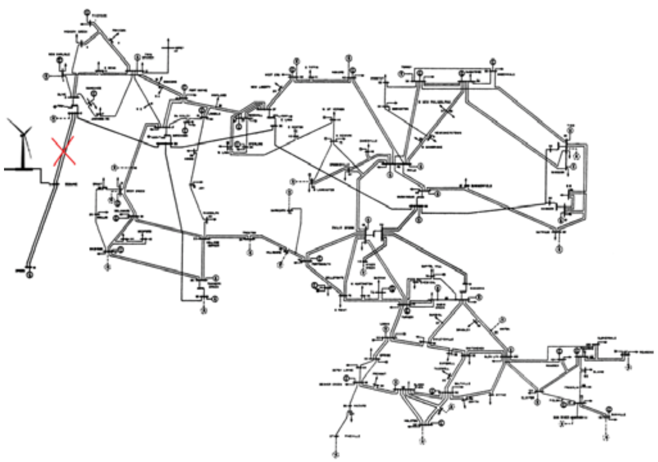
Given a dataset $\hat{\Xi}$ of forecast errors

CVaR OPF
decisions based on sample-avaerage approximation

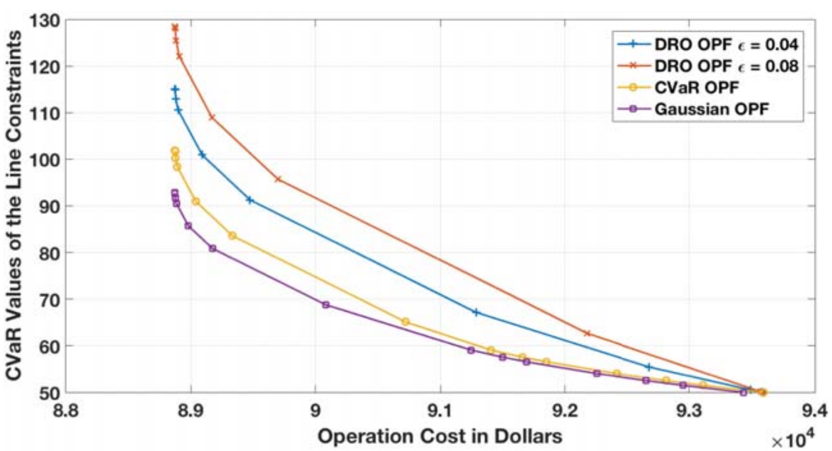
Gaussian OPF
decisions based on assuming Gaussian distribution

DRO OPF
decisions robust to the worst-case distributions within ambiguity set

Results: tradeoffs and controllable conservativeness

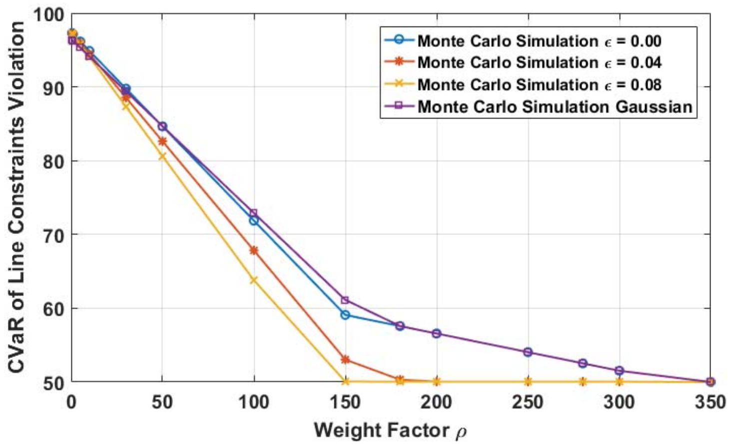


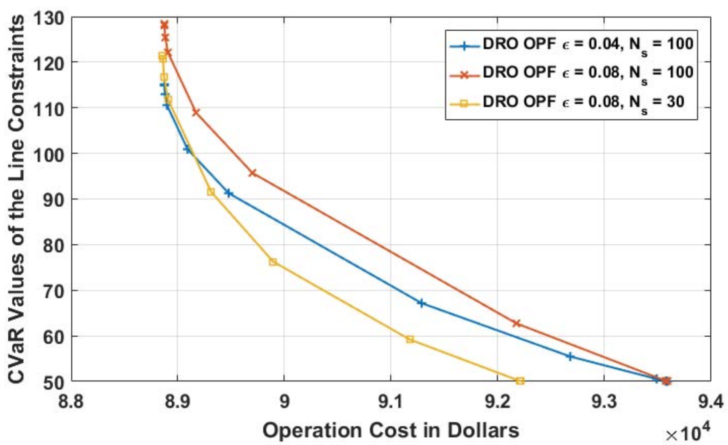
- only one line flow modelled by DRO OPF
- no local device or other constraints



- tradeoffs between risks and efficiency by ρ
- tradeoffs between sampling errors and performance by ε
- three levels $\varepsilon = 0.00, 0.04, 0.08$
- CVaR OPF = DRO OPF for $\varepsilon = 0$
- Gaussian OPF underestimates the risk of the line constraint violation
- offers operators more coherent method to achieve appropriate tradeoffs

Results: out-of-sample performance

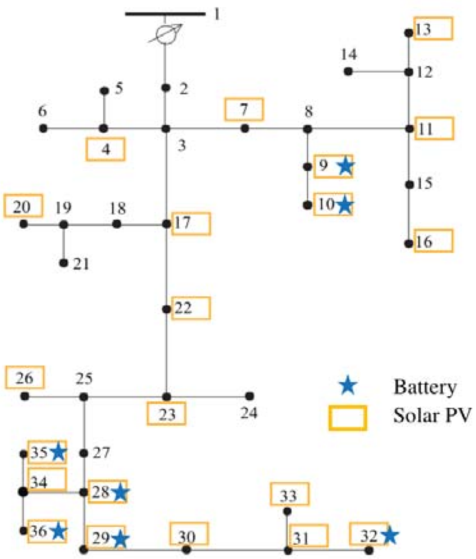




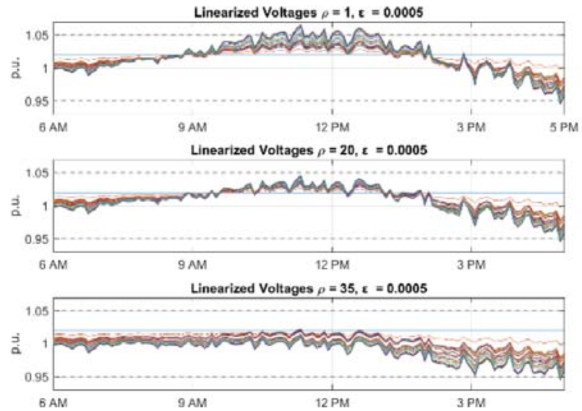
- 1000 scenarios for Monte Carlo simulations
- superior out-of-sample performance
- controllable conservativeness
- DRO OPF ensure smaller line constraint violation
- benefits saturate with smaller weight factor ρ

- adjustable conservativeness (Wasserstein radius ε) and weight factor ρ help to systematically trade off efficiency, violation risks, and sampling error effects

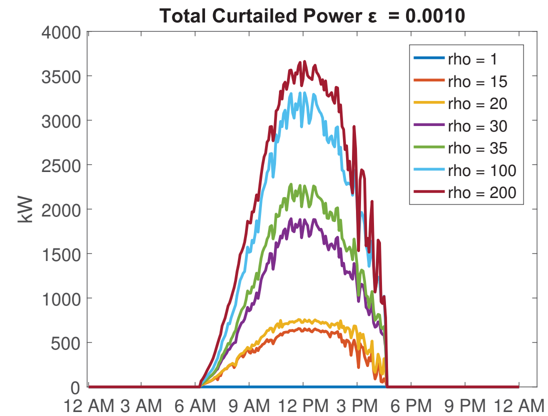
Over-voltage problem in distribution networks



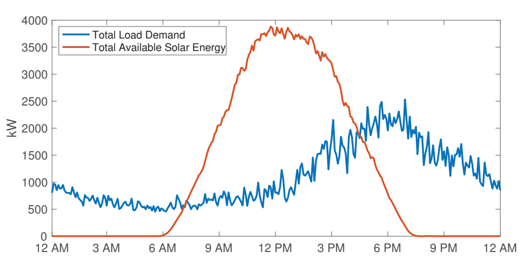
37-node distribution network



voltage profiles



controllable conservativeness



loads and solar

- 21 local onsite PVs, 7 storage devices
- optimal control for inverter-based RESs and batteries
- operational costs included active power certaiment, reactive compensation, electricity purchased by customers, excessive solar fed back to utility
- CVaR of upper voltage magnitude constraints violation
- adjustable weight factor ρ and Wasserstein radius ε tradeoff the voltage profiles, efficiency and sample errors

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Most recent and ongoing research

Overvoltage mitigation in distribution networks

- leveraging linearized AC power flow
- loads model, renewable energy model, and energy storage model
- power factor constraints
- results: controllable conservativeness decisions on RESs and storage devices, superior out-of-sample performance

N-1 security problem in transmission systems

- modified 118 bus transmission system with multiple wind energy injections
- N-1 security problem formulation
- multiple line constraints modelled by DRO OPF
- results: controllable conservativeness decisions on reserve policies and power output adjustment for generators, superior out-of-sample performance

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Conclusions

- propose a multi-period MPC-based distributionally robust OPF with the latest knowledge of the uncertain forecast errors.
- successfully visualize and balance sampling error effect, out-of-sample performance, efficiency and operational risks.
- decisions are robust to the *worst-case* data-generating distribution contained in ambiguity set with adjustable conservativeness.
- convexity, tractability, controllable conservativeness, and superior out-of-sample performance.

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Thanks !

Questions?