

Assignment 3 Written Solutions

CAS CS 320: *Concepts of Programming Languages*

1 Typing Derivation (I)

$$\frac{\frac{}{\{x : \text{int}, y : \text{bool}\} \vdash y : \text{bool}} \text{(var)} \quad \frac{}{\{x : \text{int}, y : \text{bool}\} \vdash [] : \text{int list}} \text{(nil)} \quad \frac{\frac{}{\{x : \text{int}, y : \text{bool}\} \vdash x : \text{int}} \text{(var)} \quad \frac{}{\{x : \text{int}, y : \text{bool}\} \vdash [] : \text{int list}} \text{(nil)}}{\{x : \text{int}, y : \text{bool}\} \vdash x :: [] : \text{int list}} \text{(cons)} \quad \frac{}{\{x : \text{int}, y : \text{bool}\} \vdash \text{if } y \text{ then } [] \text{ else } x :: [] : \text{int list}} \text{(if)}$$

2 Typing Derivation (II)

$\Gamma = \{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\}$ where τ can be any type.

$$\frac{\frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash g : \text{int} \rightarrow \tau} \text{(var)} \quad \frac{\frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash f : \tau \rightarrow \text{int}} \text{(var)} \quad \frac{\frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash g : \text{int} \rightarrow \tau} \text{(var)} \quad \frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash 0 : \text{int}} \text{(int)}}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash g \ 0 : \tau} \text{(app)} \quad \frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash f \ (g \ 0) : \text{int}} \text{(app)} \quad \frac{}{\{f : \tau \rightarrow \text{int}, g : \text{int} \rightarrow \tau\} \vdash g \ (f \ (g \ 0)) : \tau} \text{(app)}$$

3 URM Programs

$P = J \ 0 \ 1 \ 2 \ I \ 0 \ I \ 1$

$$\frac{\frac{}{\langle P, [(0,5);(1,5)], 0 \rangle \rightarrow \langle P, [(0,5);(1,5)], 2 \rangle} \text{(jump-eq)} \quad \frac{\frac{}{\langle P, [(0,5);(1,5)], 2 \rangle \rightarrow \langle P, [(0,5);(1,6)], 3 \rangle} \text{(incr)} \quad \frac{}{\langle P, [(0,5);(1,6)], 3 \rangle \rightarrow \langle P, [(0,5);(1,6)], 3 \rangle} \text{(refl)}}{\langle P, [(0,5);(1,5)], 0 \rangle \rightarrow^* \langle P, [(0,5);(1,6)], 3 \rangle} \text{(trans)}$$