Formal Grammar

Concepts of Programming Languages
Lecture 12

CAS CS 320

Outline

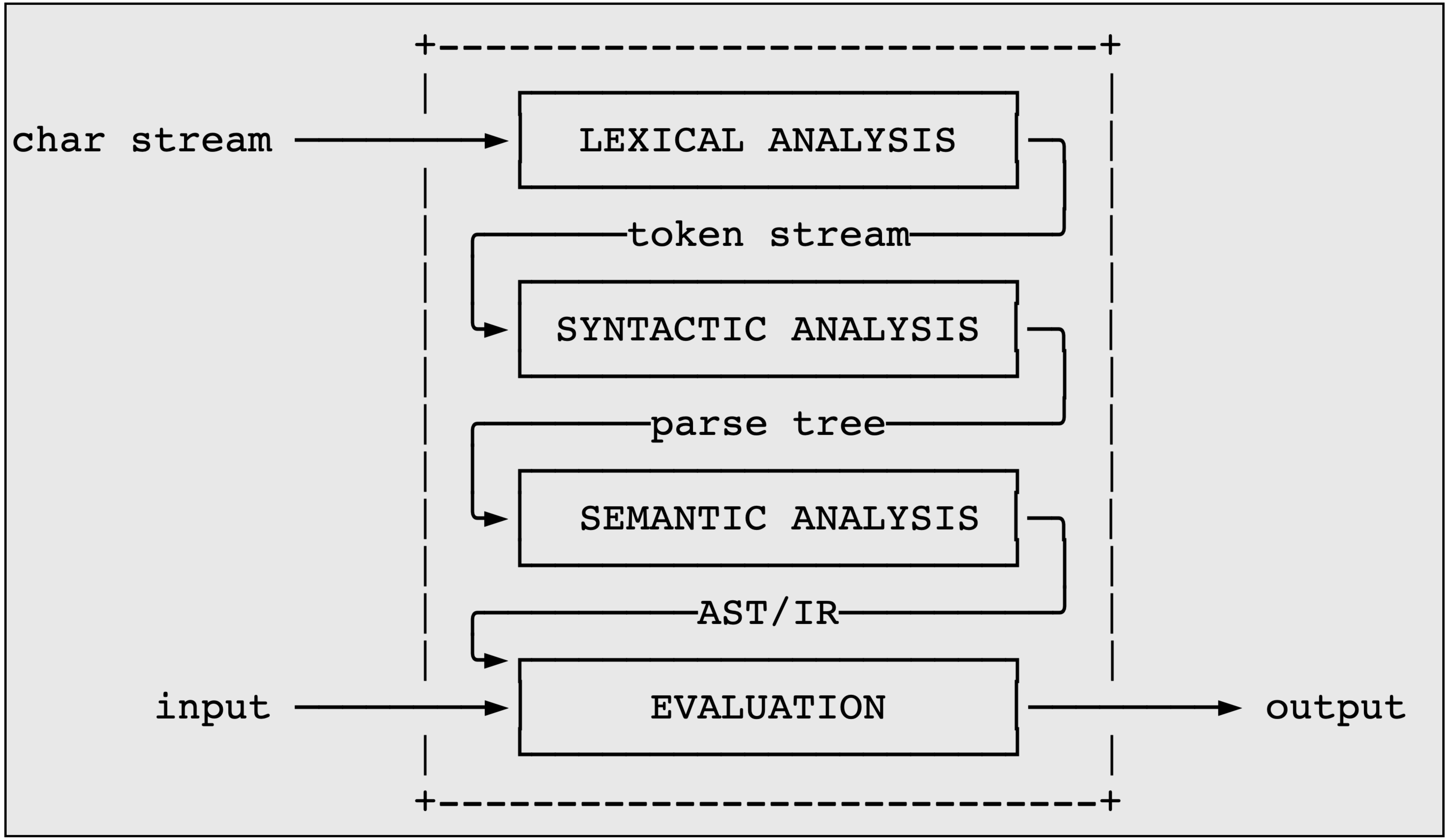
- » Discuss briefly the **interpretation pipeline**, and how it will look in the context of this course

- » Introduce **formal grammars**, a mathematical framework for thinking about syntax and parsing

- » Discuss **ambiguity** in grammar, and look at ways of avoiding it

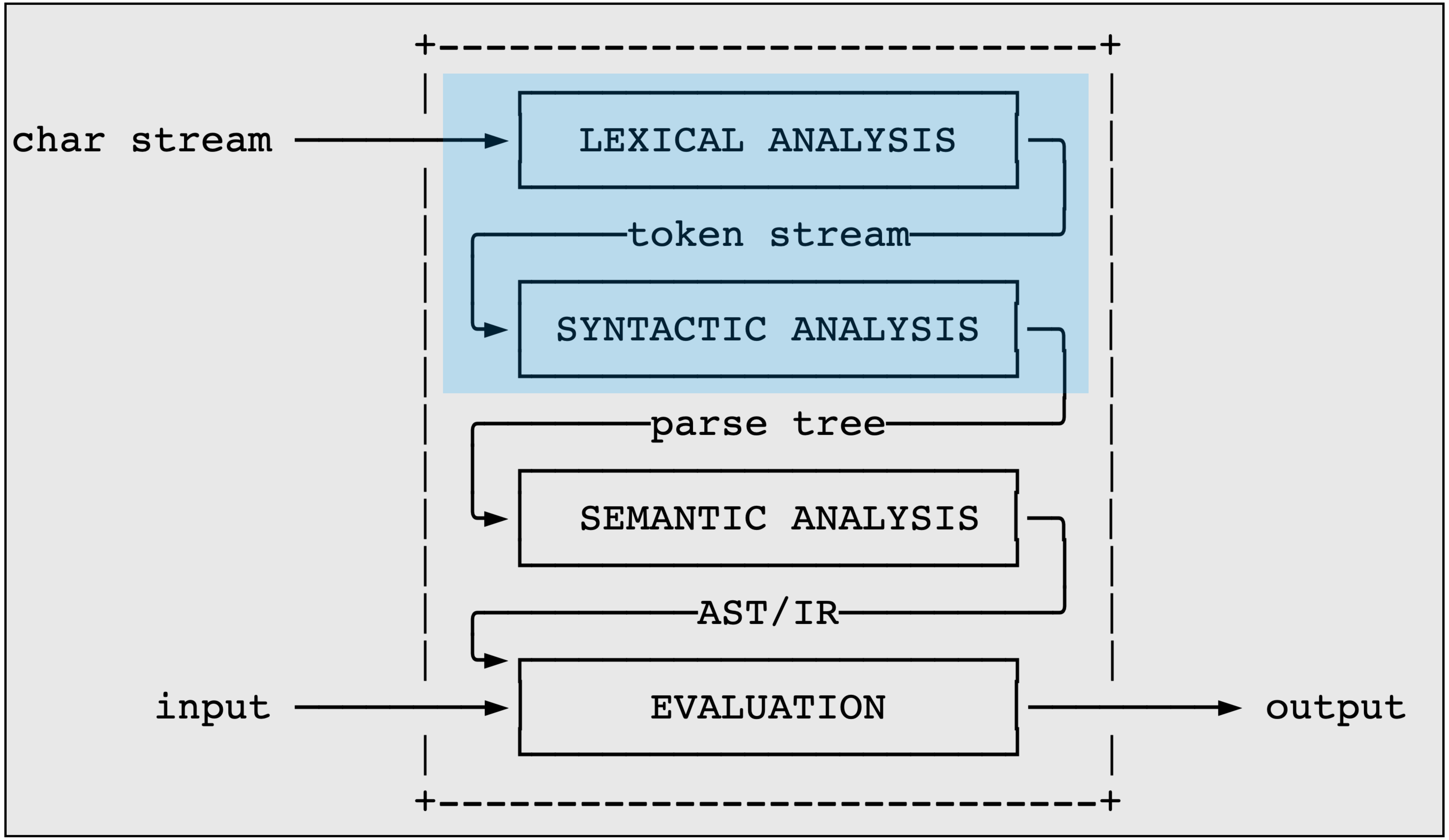
The Interpretation Pipeline

The Picture


char stream
LEXICAL ANALYSIS
token stream
SYNTACTIC ANALYSIS
parse tree
SEMANTIC ANALYSIS
AST/IR
input
EVALUATION
output

The Picture

parsing (this week)



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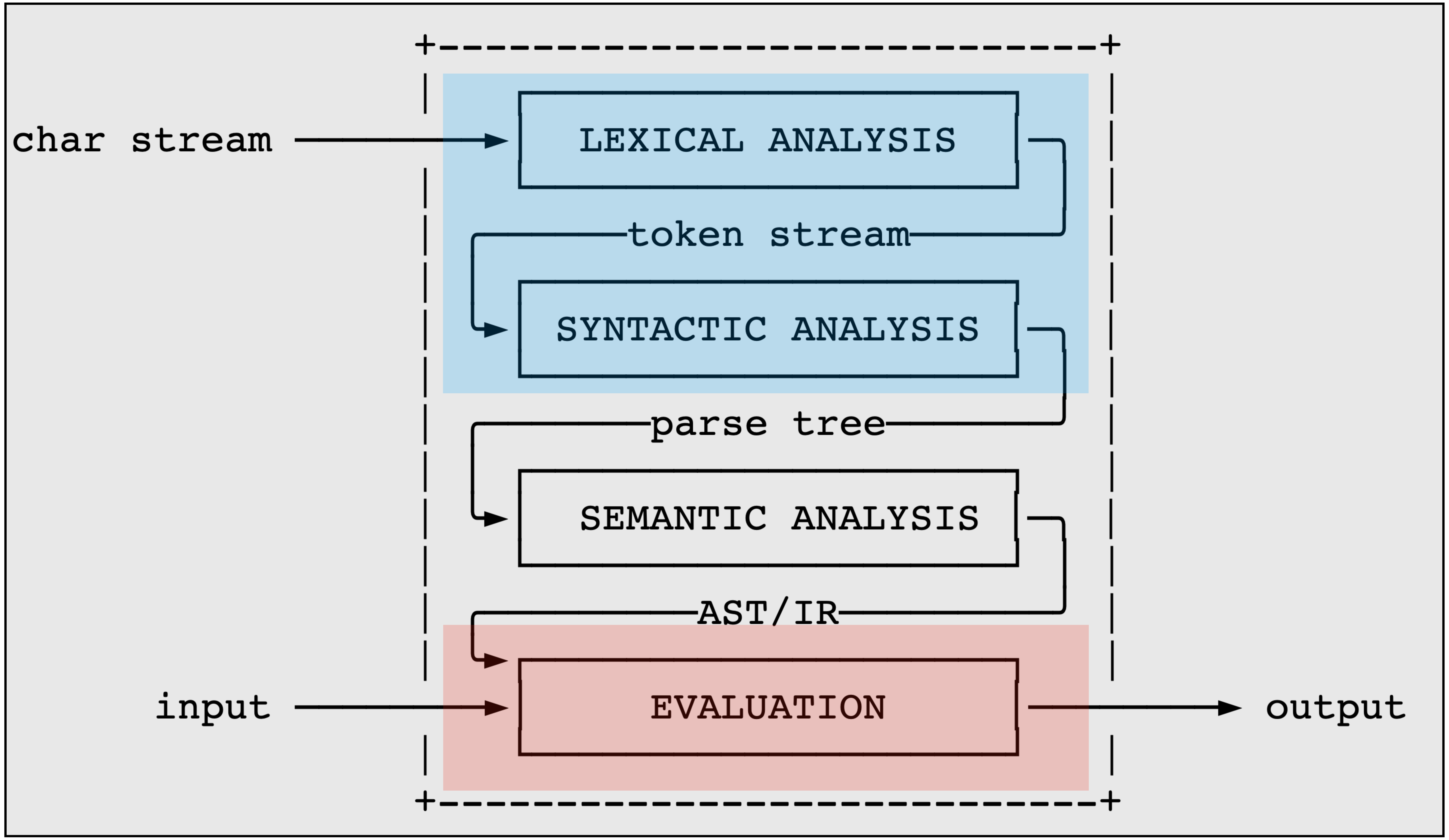
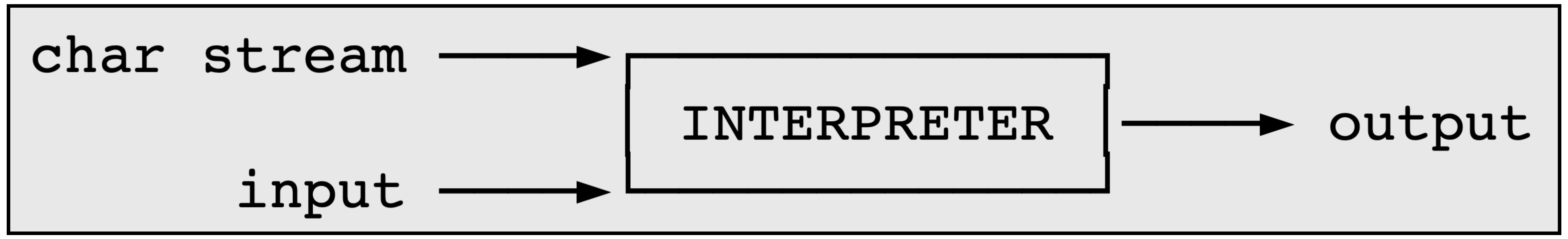
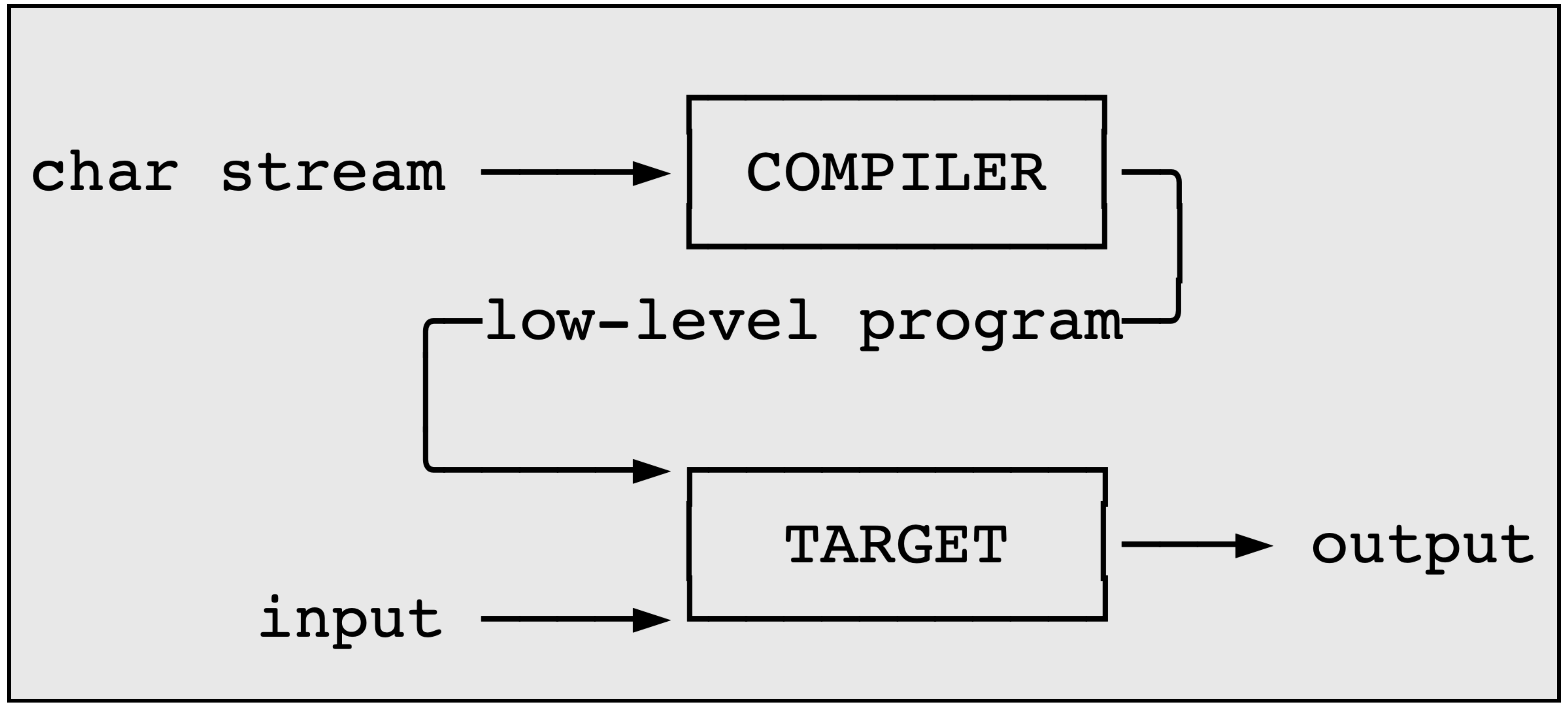


semantics (next week)

A Note on Compilation





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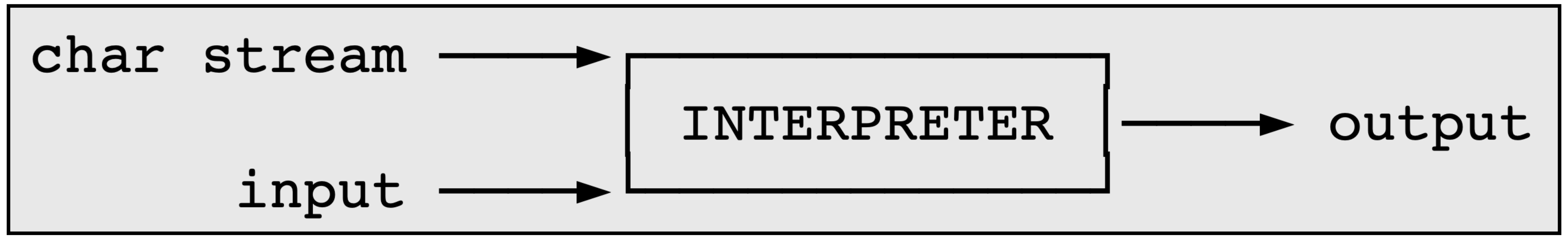

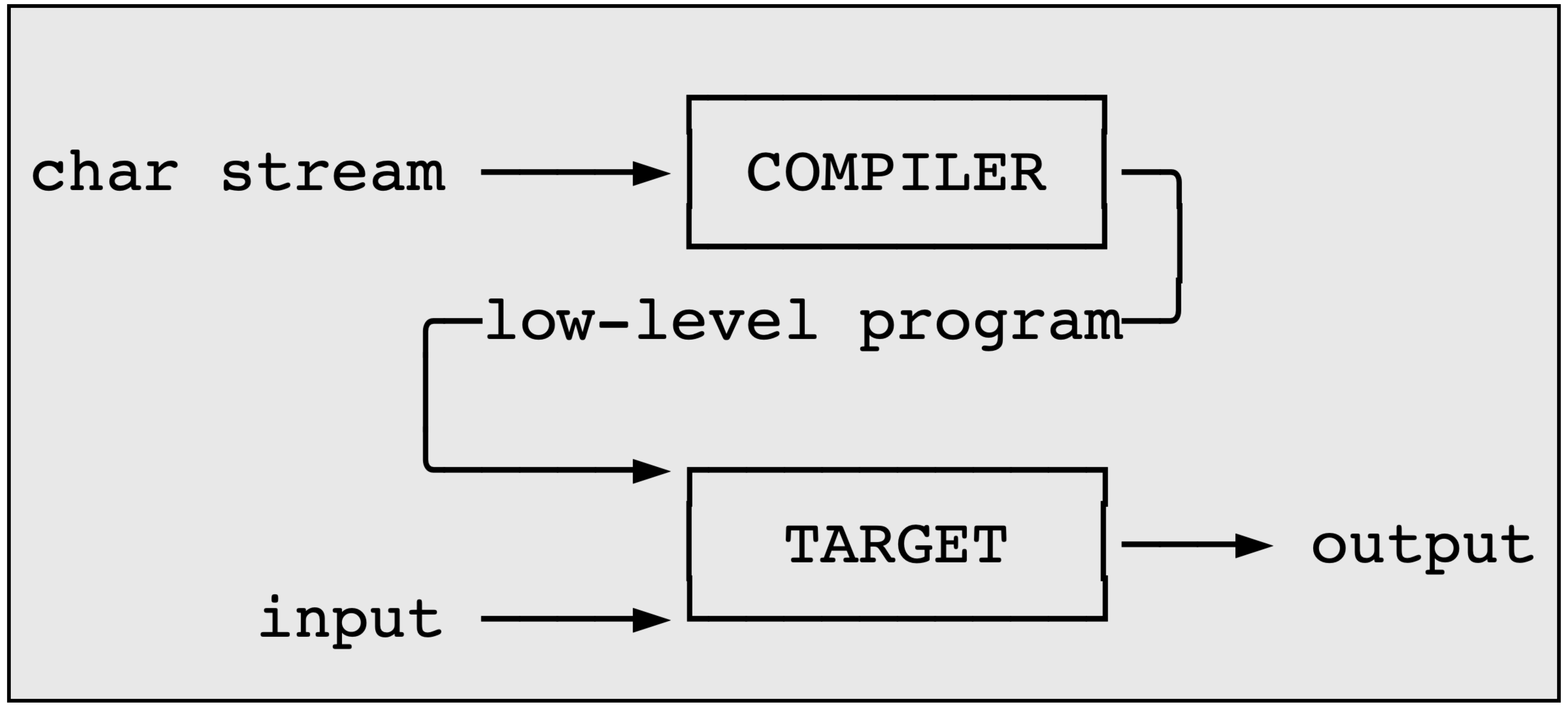

We will be building programs that directly read an evaluate programs (**interpreters**)

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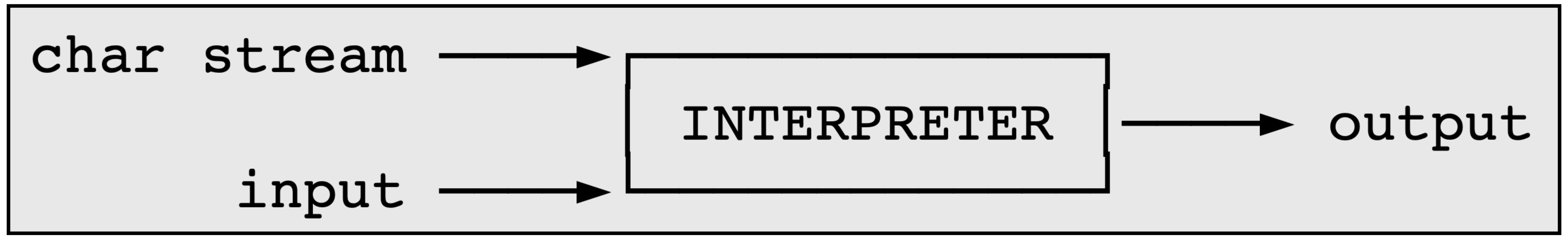
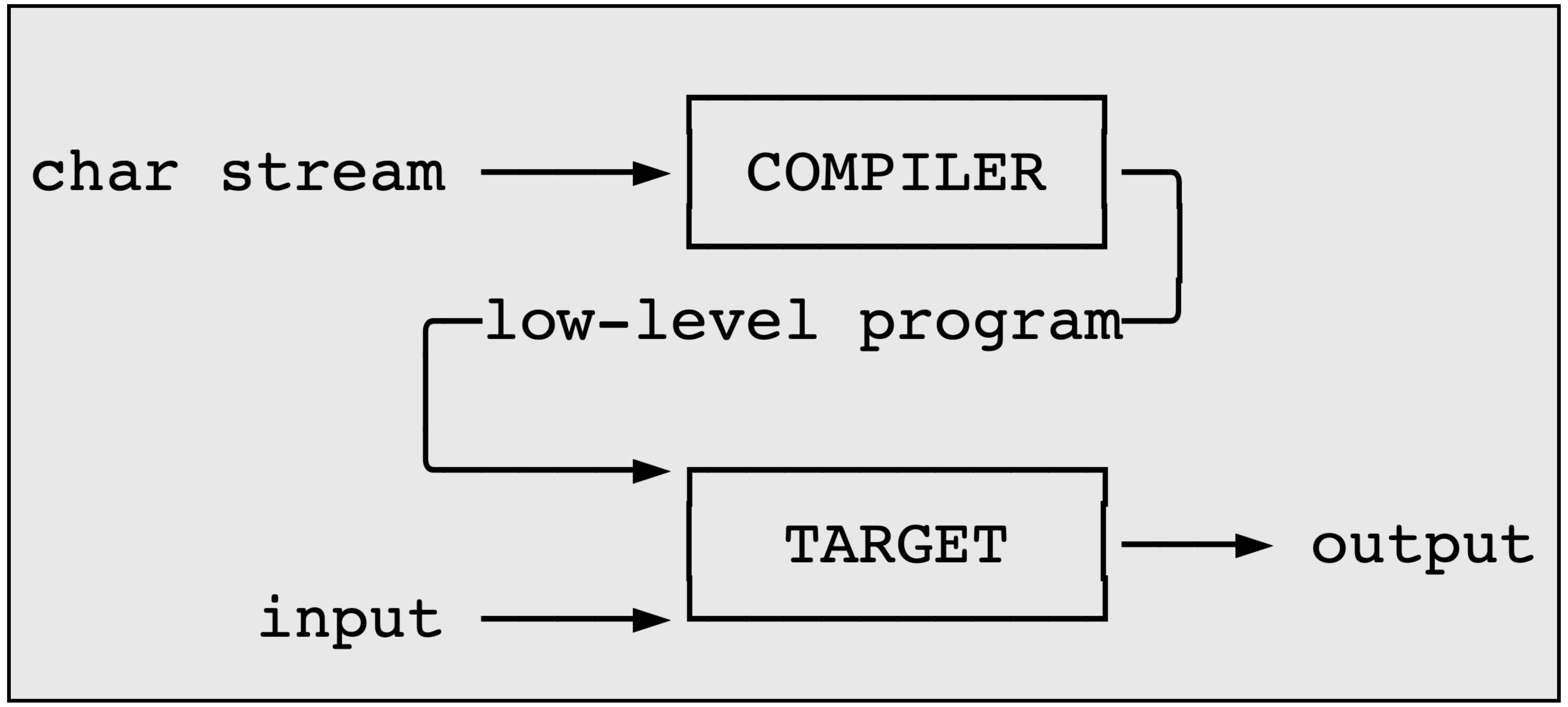




We will be building programs that directly read an evaluate programs (**interpreters**)

In a different course you may write a program which *translates* programs into another language which can then be evaluated elsewhere (**compilers**, we'll cover this briefly)

The Mini-Projects

There will be **three** mini–projects, each 2 weeks long.

For each project, you will build an interpreter.

<u>You'll be given:</u>

- » the syntax
- » the type rules (not in project 1)
- » the semantics

Today

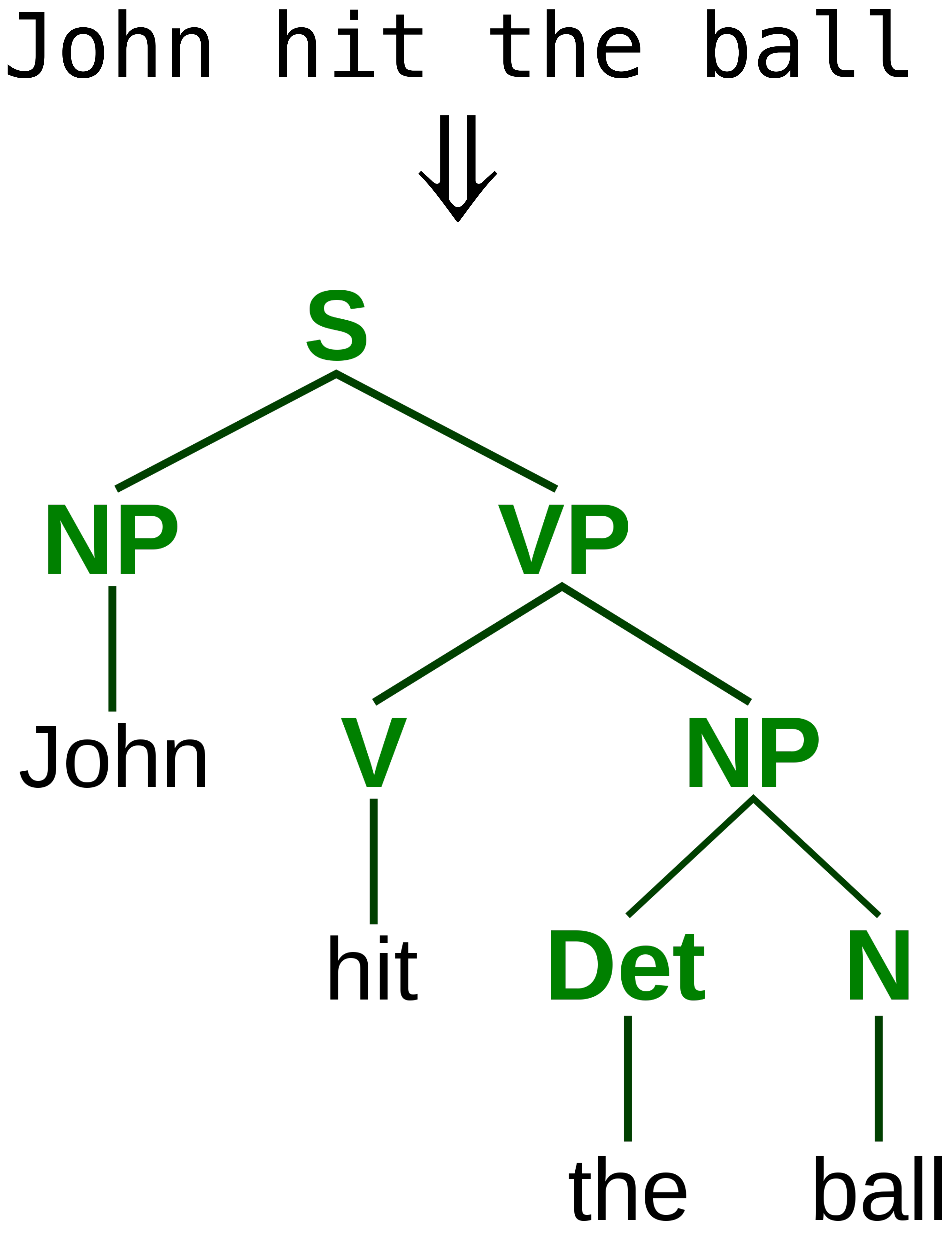
We need a formal language for describing the syntax of programming languages

This is part of the study of **formal language theory**

Nearly every PL out there (including OCaml) is described using **Backus–Naur Form (BNF) Grammars**

Formal Grammar

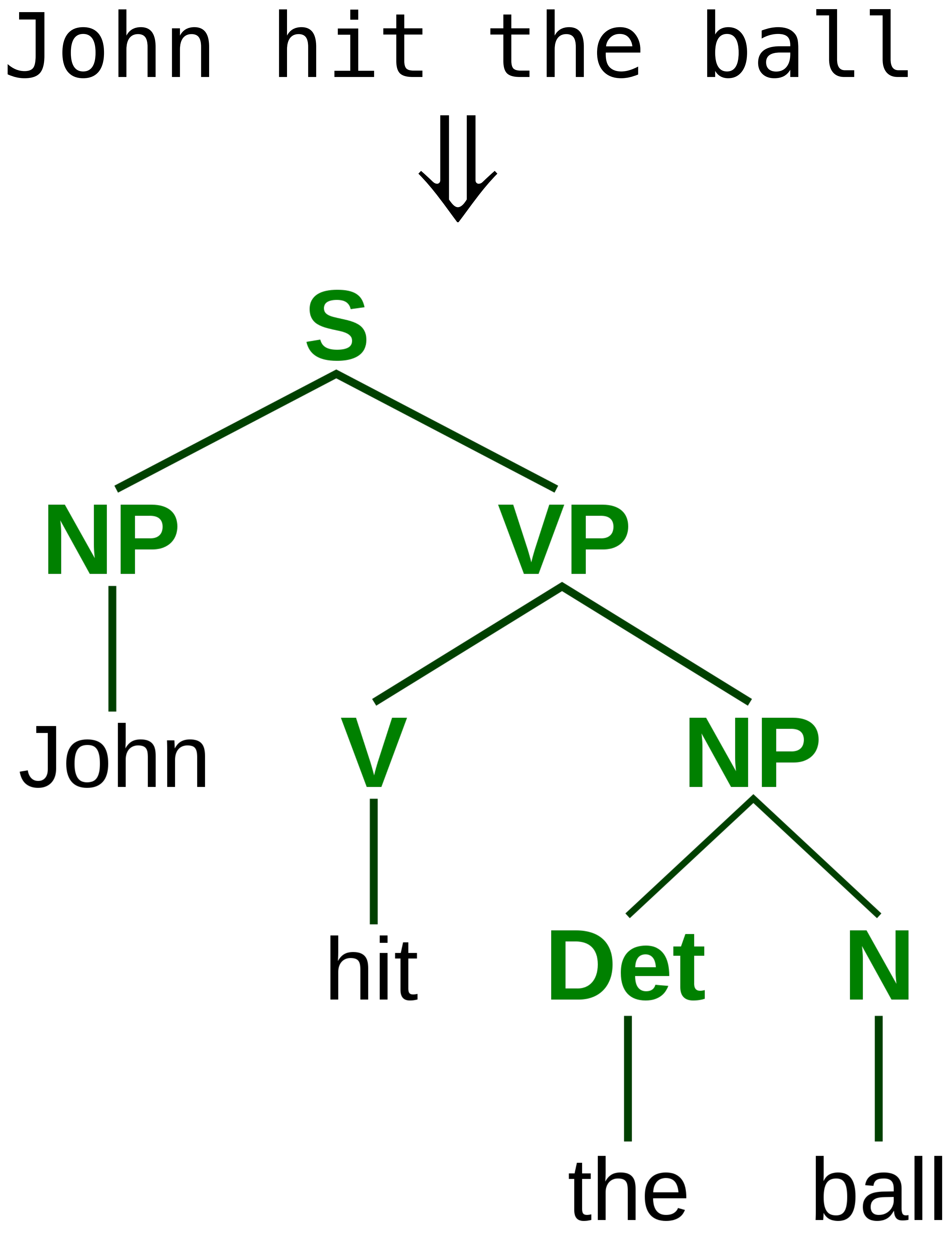
What is Grammar?



https://commons.wikimedia.org/wiki/File:ParseTree.svg

What is Grammar?

Grammar refers to the rules which govern what statements are well-formed



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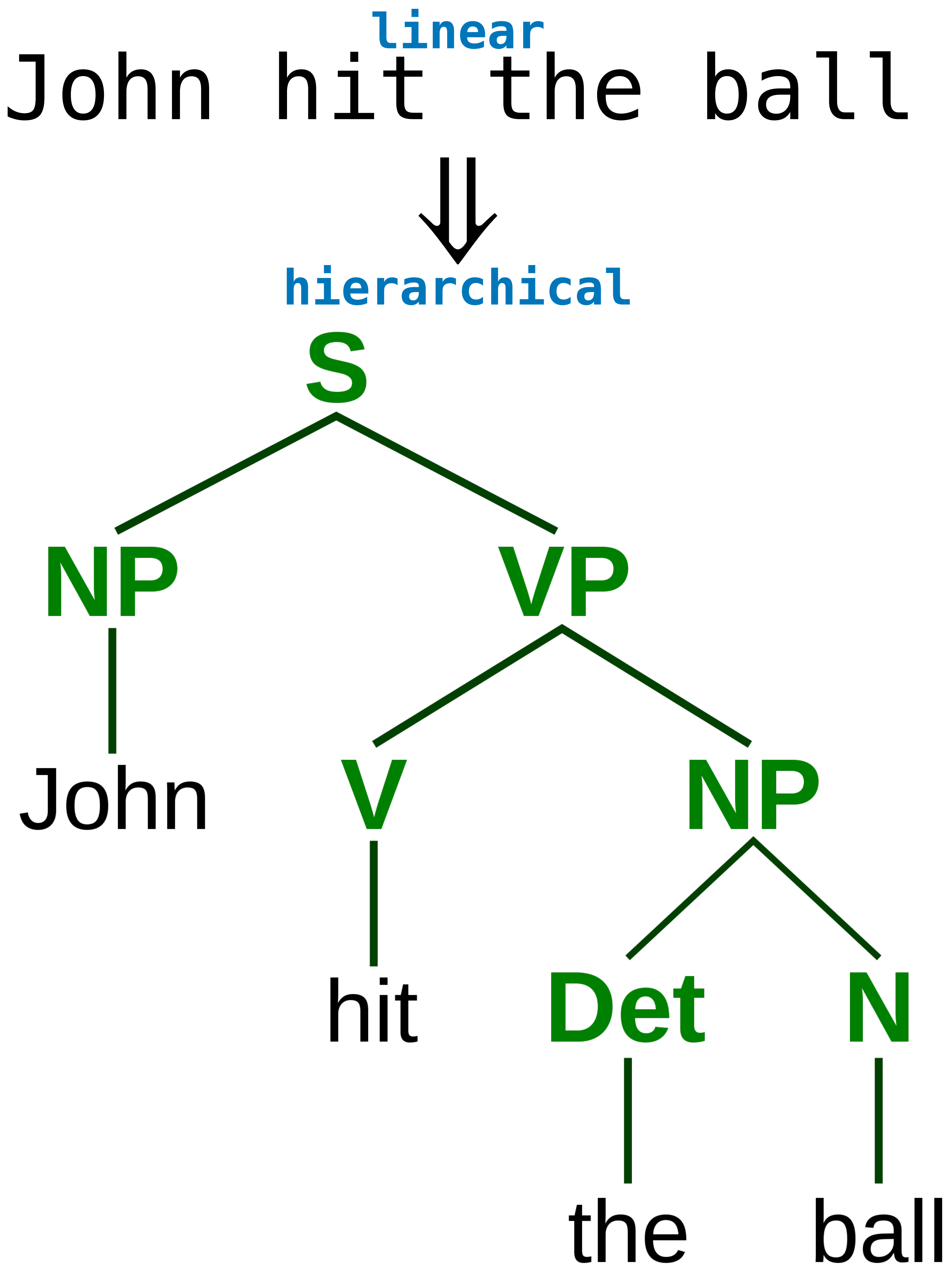
What is Grammar?

Grammar refers to the rules which govern what statements are well-formed

Grammar gives **linear** statements (in natural language or code) their **hierarchical** structure



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Grammar vs. Semantics

I taught my car in the refrigerator. ✓

vs.

My the car taught I refrigerator. ✗

Grammar vs. Semantics

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Grammar is not (typically) interested in meaning, just structure

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(As we will see, it is useful to separate these two concerns)

Grammars for Programming Languages

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<u>Formal grammars</u> for PL
tell us which programs
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Grammars for Programming Languages

```
# let f x = x + 1;;
val f : int -> int = <fun>
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Grammars for Programming Languages

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Line 1, characters 14-15:
1 | let rec x = x x x x ;;
                  ^
Error: This expression has type ...
       but an expression was ex ...
       The type variable 'a occ ...
```

Grammars for Programming Languages

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# let rec f x = f x + 1 - 1;;
val f : 'a -> int = <fun>
```

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# let rec f x = f x + 1 - 1;;
val f : 'a -> int = <fun>
# let x = List.hd [];;
Exception: Failure "hd".
```

Grammars for Programming Languages

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(In OCaml, well-formed programs are the ones we can type-check)

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val f : 'a -> int = <fun>
# let x = List.hd [];;
Exception: Failure "hd".
# let x = ;;
Line 1, characters 8-10:
1 | let x = ;;
            ^^
Error: Syntax error
```

How do we formally represent well-formed sentences?

An Example

`the cow jumped over the moon`

An Example

`the cow jumped over the moon`

How do we know this a well-formed sentence?

An Example

```
<article> cow jumped over the moon
```

An Example

```
<article> <noun> jumped over the moon
```

An Example

```
<noun-phrase> jumped over the moon
```

An Example

`<noun-phrase> jumped over <article> moon`

An Example

```
<noun-phrase> jumped over <article> <noun>
```

An Example

<noun-phrase> jumped over <noun-phrase>

An Example

```
<noun-phrase> jumped over <noun-phrase>
```

a thing jumped over a thing

An Example

`<noun-phrase> jumped <prep> <noun-phrase>`

An Example

```
<noun-phrase> jumped <prep-phrase>
```

An Example

```
<noun-phrase> <verb> <prep-phrase>
```

An Example

```
<noun-phrase> <verb-phrase>
```

An Example

`<noun-phrase> <verb-phrase>`

a thing did a thing

An Example

<sentence>

An Example

`<sentence>`

We know it's a sentence because it has the right kind of hierarchical structure

A Derivation

```
<sentence>
<noun-phrase> <verb-phrase>
<noun-phrase> <verb> <prep-phrase>
<noun-phrase> jumped <prep-phrase>
<noun-phrase> jumped <prep> <noun-phrase>
<noun-phrase> jumped over <noun-phrase>
<noun-phrase> jumped over <article> <noun>
<noun-phrase> jumped over <article> moon
<noun-phrase> jumped over the moon
<article> <noun> jumped over the moon
<article> cow jumped over the moon
the cow jumped over the moon
```

A Derivation

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<noun-phrase> <verb-phrase>
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<noun-phrase> jumped <prep-phrase>
<noun-phrase> jumped <prep> <noun-phrase>
<noun-phrase> jumped over <noun-phrase>
<noun-phrase> jumped over <article> <noun>
<noun-phrase> jumped over <article> moon
<noun-phrase> jumped over the moon
<article> <noun> jumped over the moon
<article> cow jumped over the moon
```

```
the cow jumped over the moon
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<noun-phrase> jumped over <article> <noun>
<noun-phrase> jumped over <article> moon
<noun-phrase> jumped over the moon
<article> <noun> jumped over the moon
<article> cow jumped over the moon
```

```
the cow jumped over the moon
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A Derivation

<sentence>
<noun-phrase> <verb-phrase>
<noun-phrase> <verb> <prep-phrase>
<noun-phrase> jumped <prep-phrase>
<noun-phrase> jumped <prep> <noun-phrase>
<noun-phrase> jumped over <noun-phrase>
<noun-phrase> jumped over <article> <noun>
<noun-phrase> jumped over <article> moon
<noun-phrase> jumped over the moon
<article> <noun> jumped over the moon

<article>

the cow jumped over the moon

A Derivation

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<noun-phrase> jumped over <article> <noun>

<noun-phrase> jumped over <article> moon

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<article> <noun> jumped over the moon

<article>

the cow jumped over the moon

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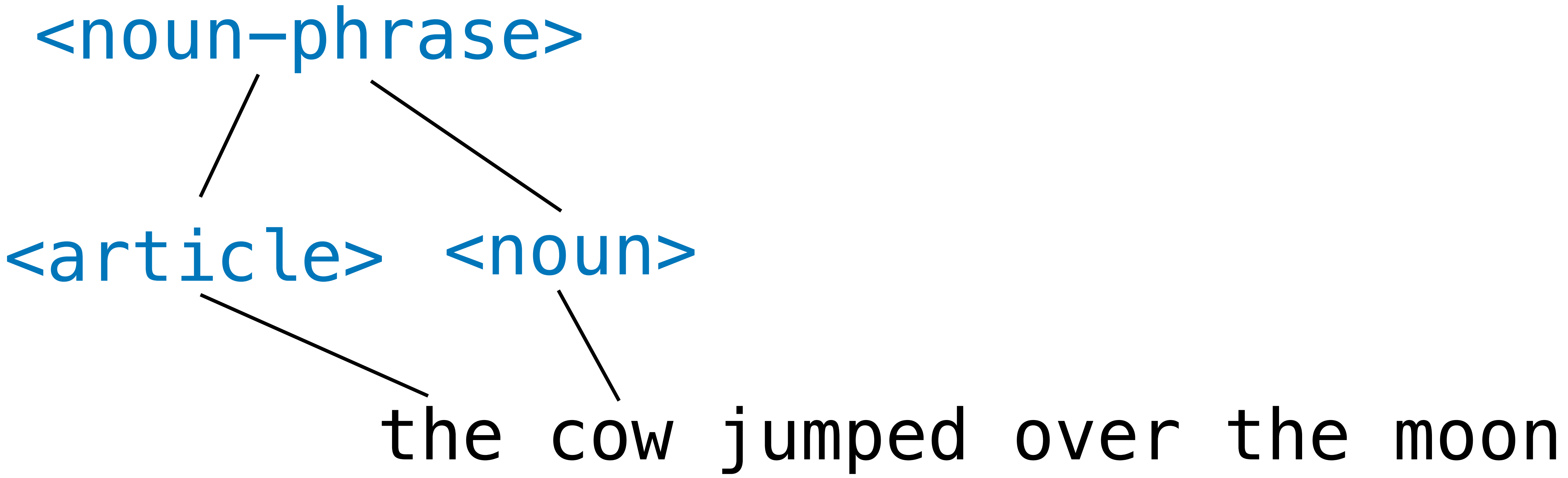

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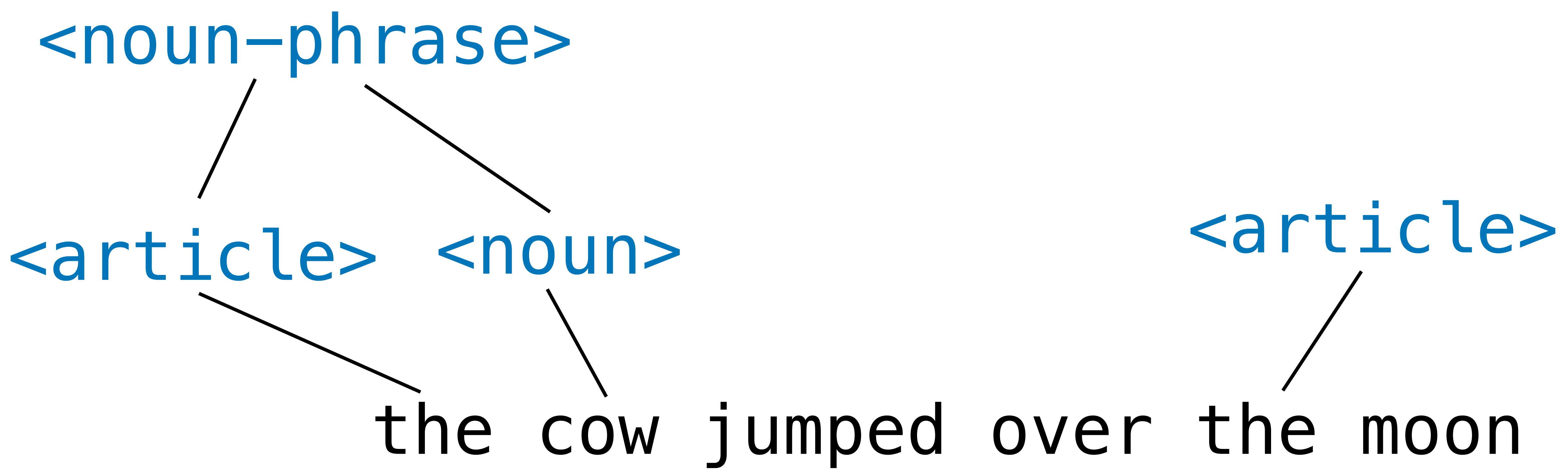

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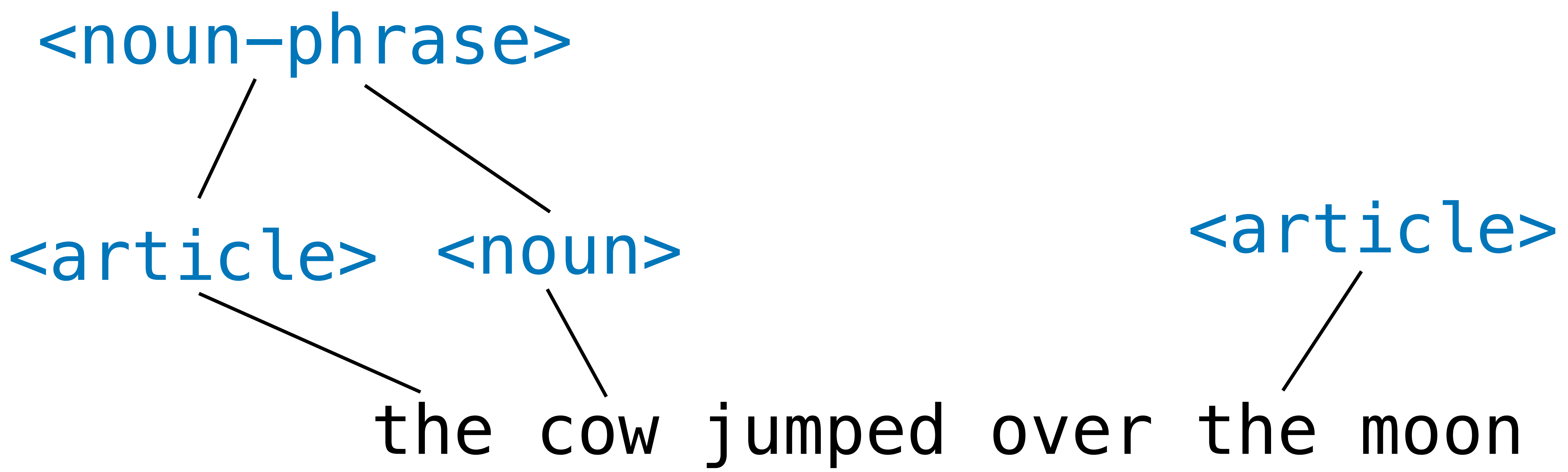

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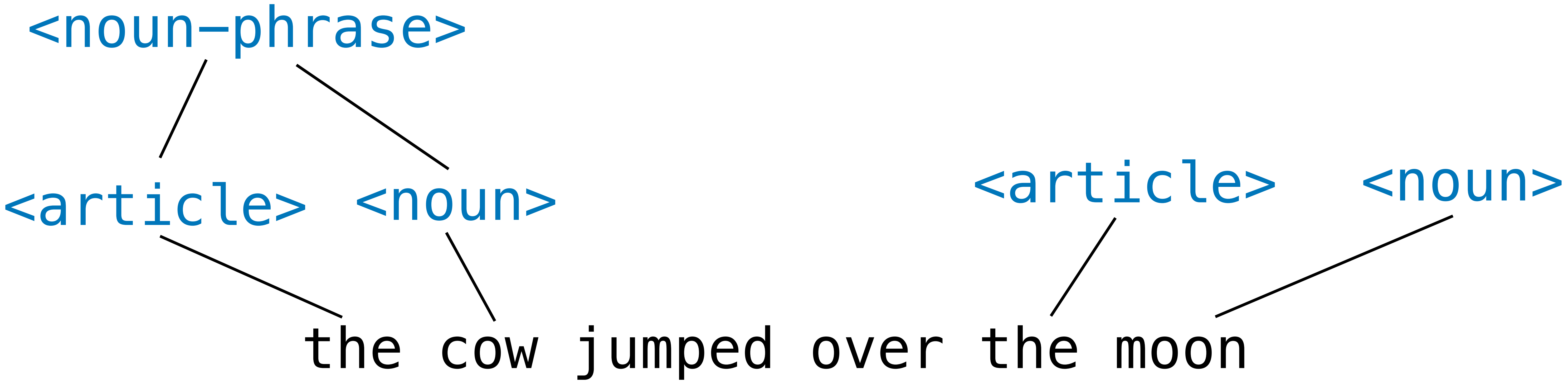

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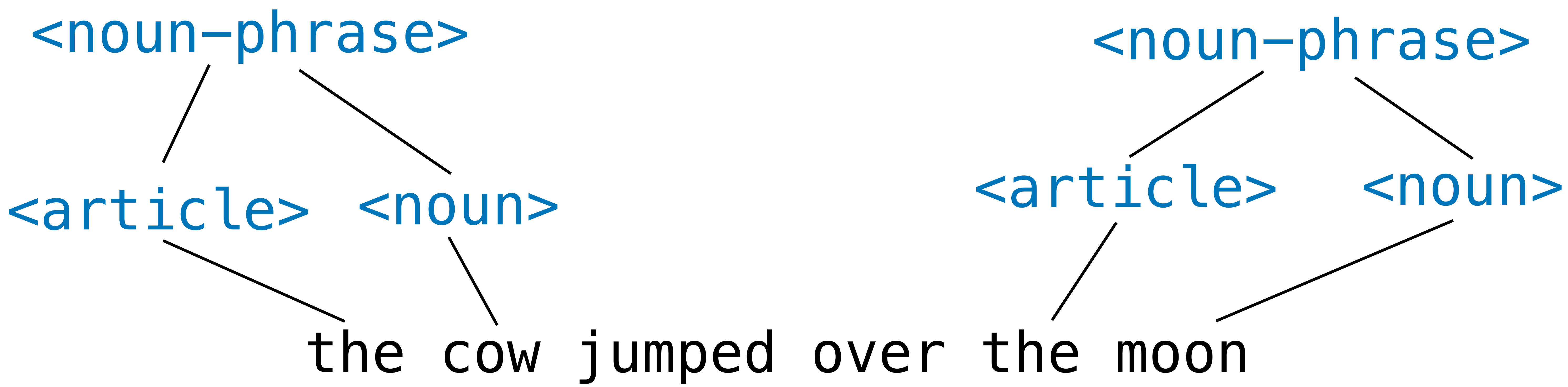

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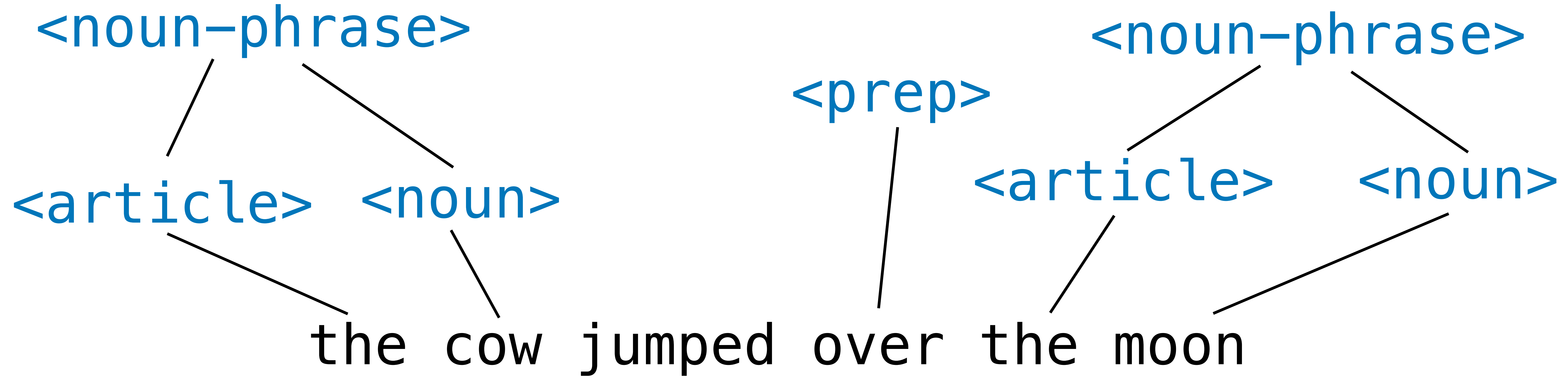

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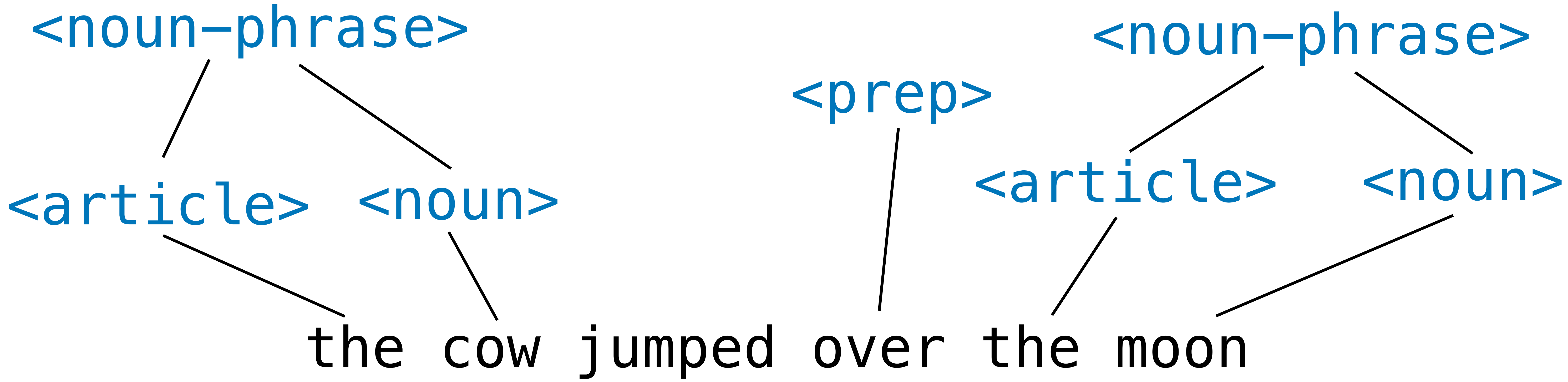

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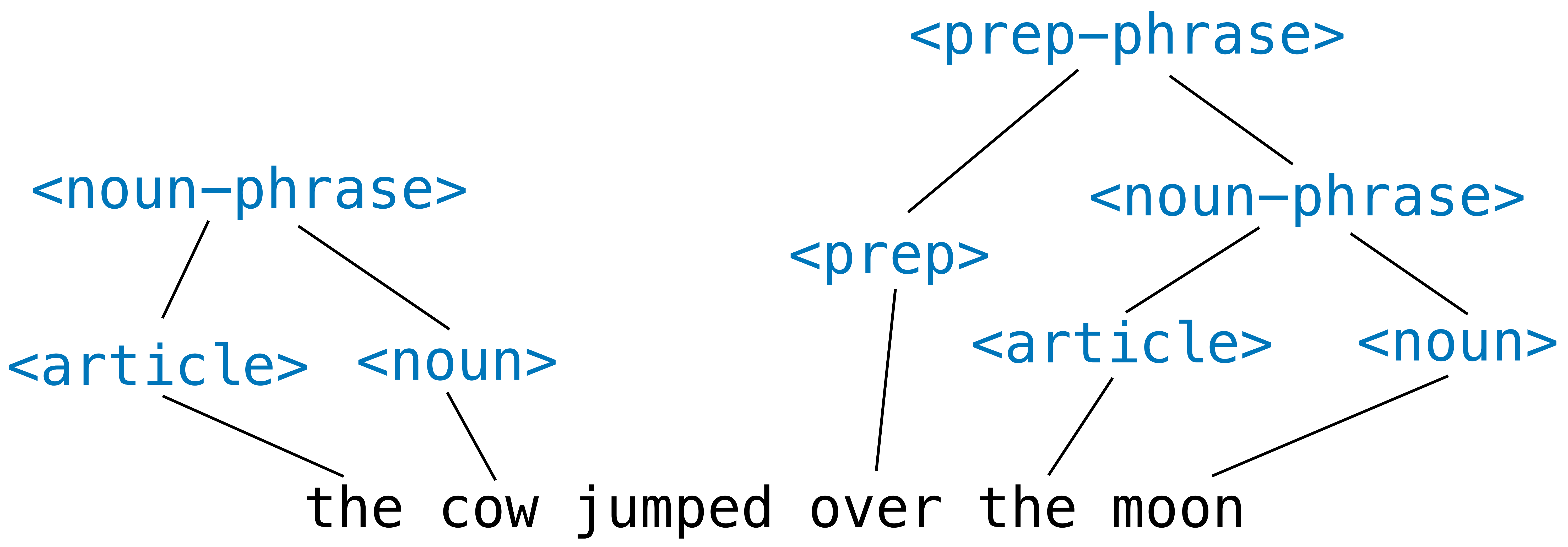

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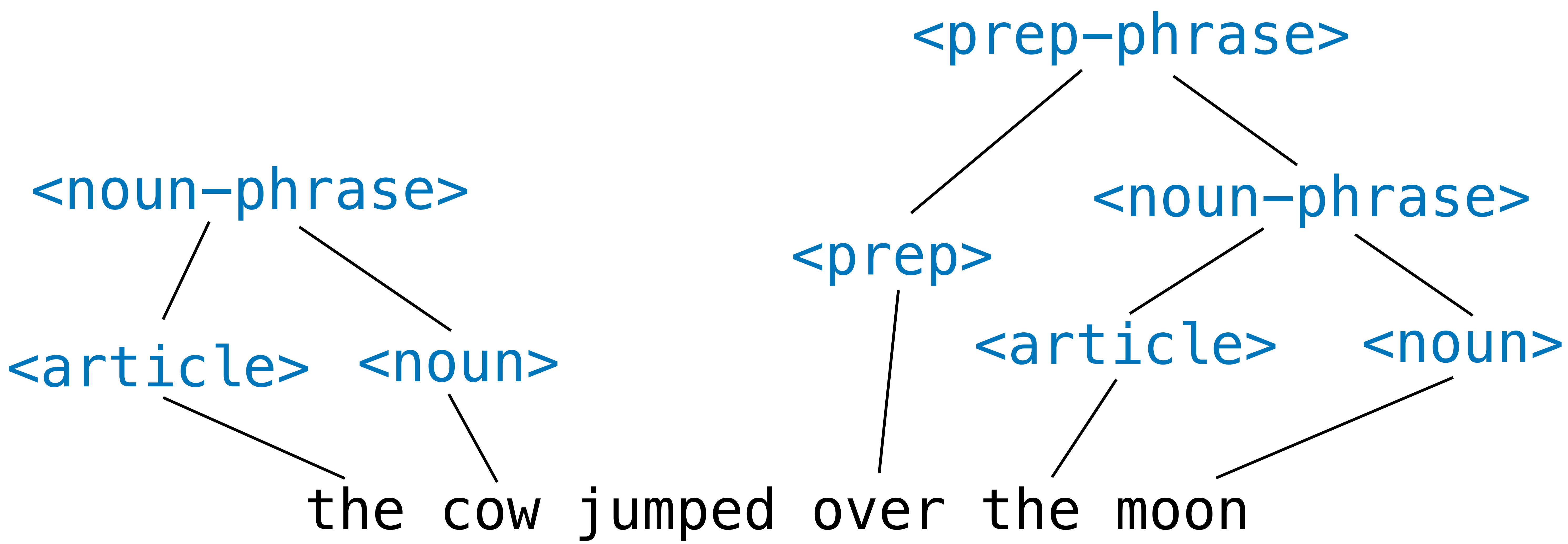

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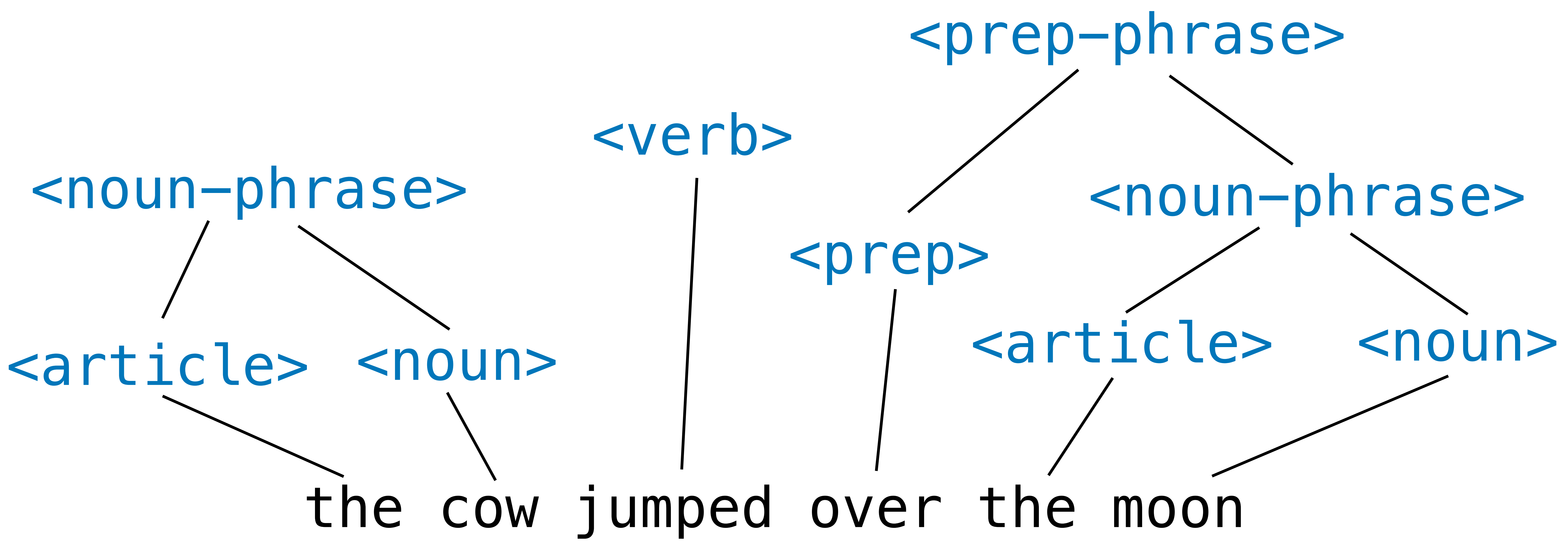

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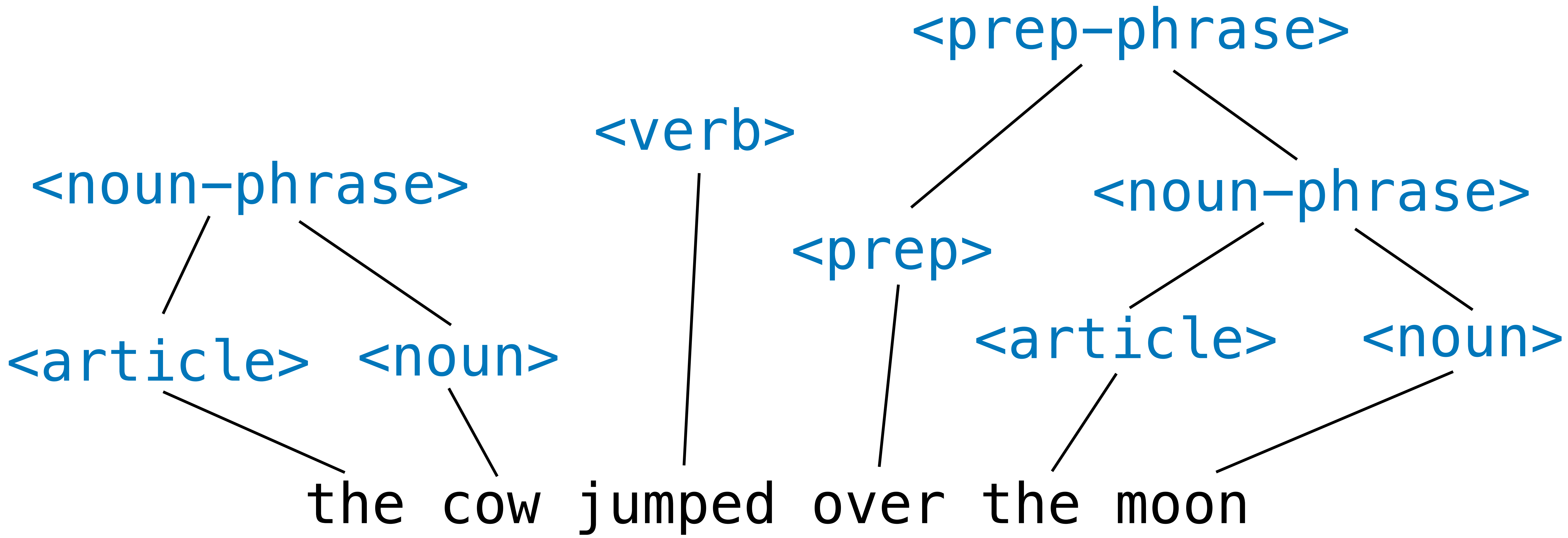

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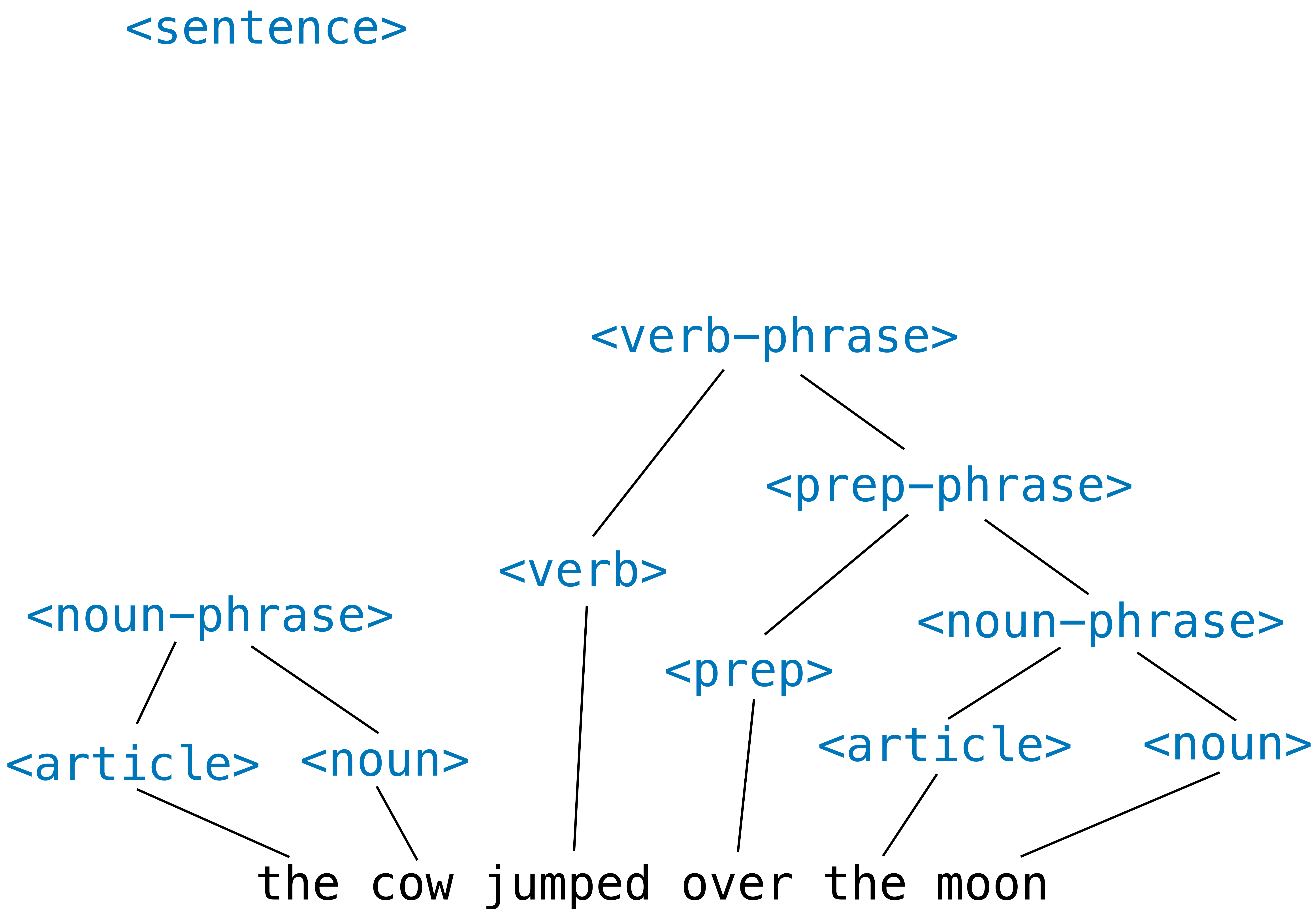

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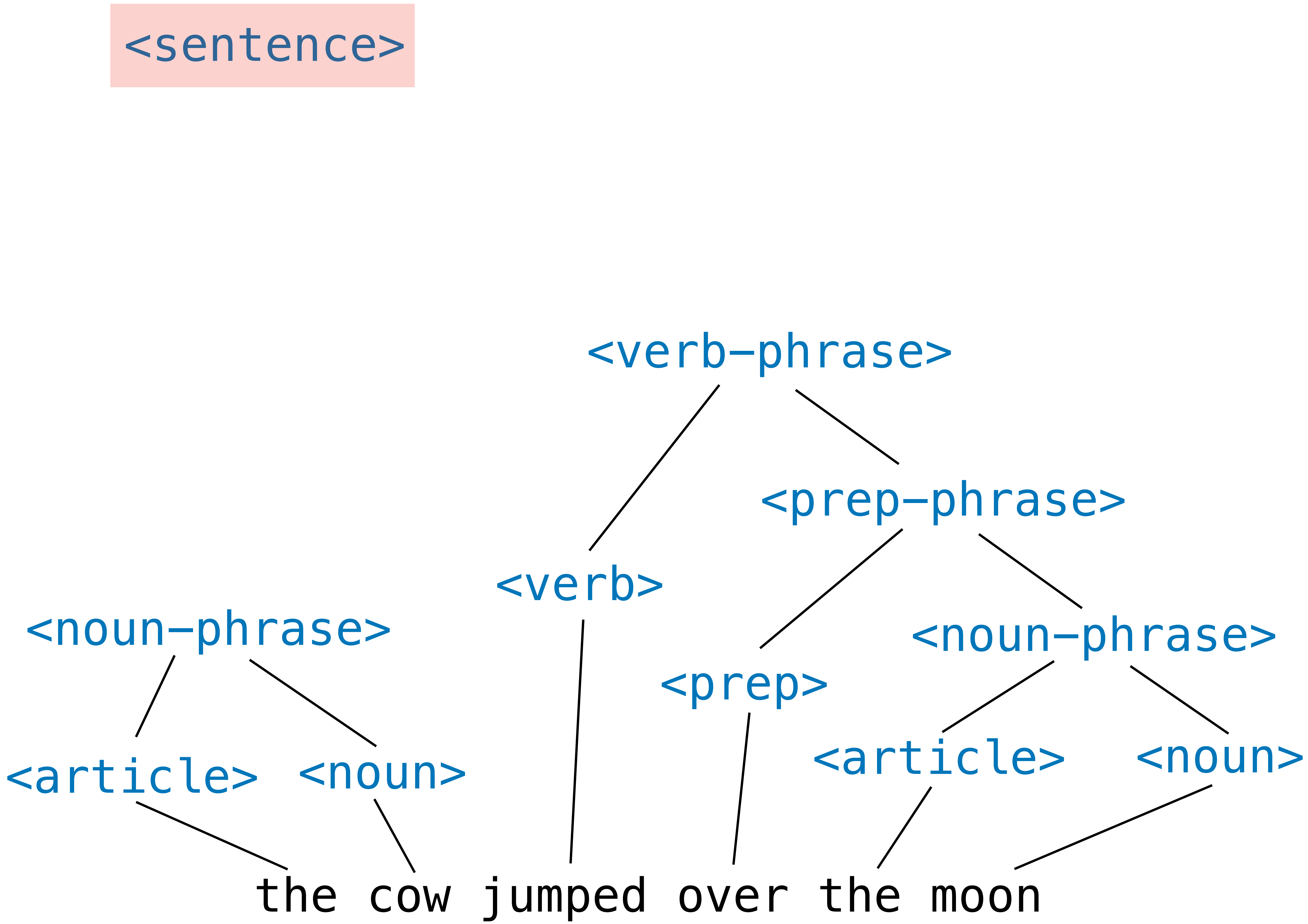

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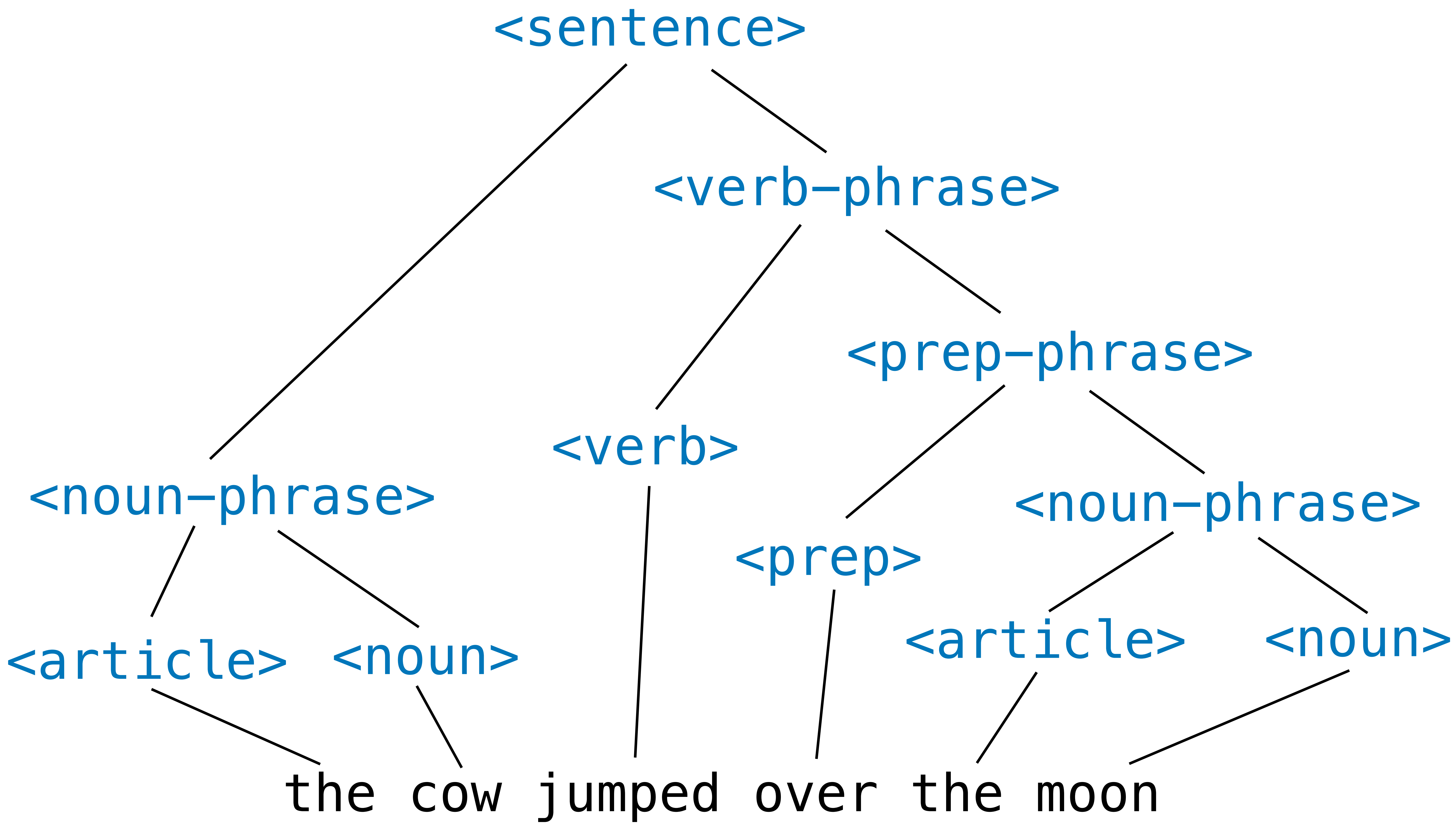
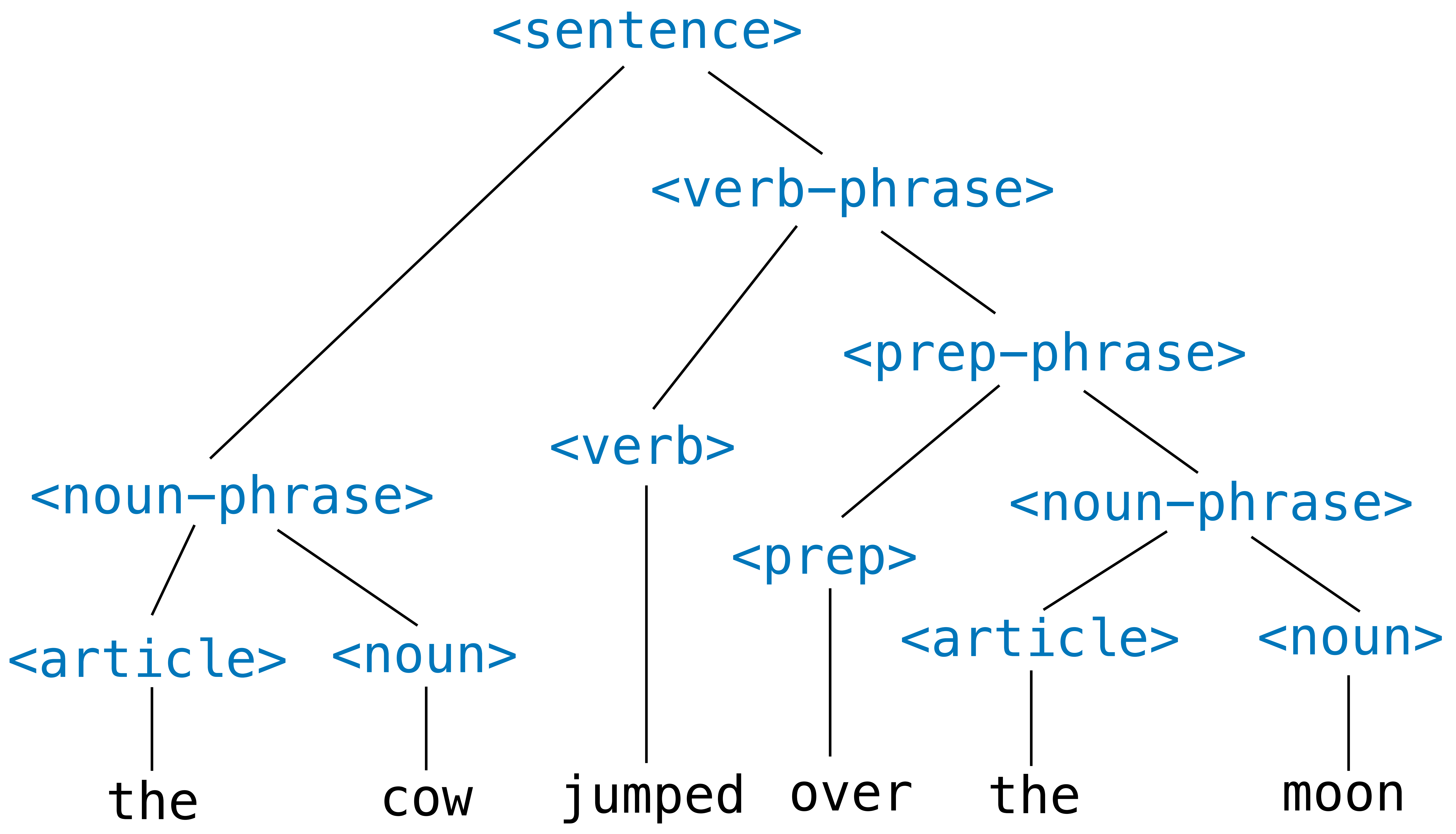


A Parse Tree



A derivation encodes hierarchical structure

Definitions (Symbols and Sentences)

```
<noun-phrase> jumped over <noun-phrase>
```

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`<noun-phrase> jumped over <noun-phrase>`

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Terminal symbols are symbols cannot be further expanded (e.g. *moon*)

A **sentential form** is a sequence of terminal or nonterminal symbols

A **sentence** is a sequence of *only terminal* symbols

Production Rules

```
<non-term> ::= sent-form1 | sent-form2 | ...
```

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```
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```

A **(BNF) production rule** describes what we can replace a non-terminal symbol with in a derivation

The "|" means: we can replace it with one or the other sentential forms on either side of the "|"

Recall: Let-Expressions (Syntax Rule)

```
<expr> ::= let <var> = <expr> in <expr>
```

If x is a valid variable name, and e_1 is a well-formed expression and e_2 is a well-formed expression then

$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2$$

is a well-formed expression

Example

```
<sentence>      ::= <noun-phrase> <verb-phrase>

<verb-phrase>   ::= <verb> <prep-phrase>

<noun>          ::= cow | moon
```

BNF Grammar

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

Note. We don't specify the symbols of a grammar, they are implicit in the rules

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

Note. We don't specify the symbols of a grammar, they are implicit in the rules

Note. We don't specify the start symbol, it's the left nonterminal symbol in the **first rule**

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

Example

```
<expr>   ::= <op1> <expr>
           | <op2> <expr> <expr>
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

Example

```
<expr>   ::= <op1> <expr>
           | <op2> <expr> <expr>
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

tokens (terminal symbols)

Example

```
<expr>   ::= <op1> <expr>
           | <op2> <expr> <expr>
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

abstractions (non-terminal symbols)

tokens (terminal symbols)

Example

production rules

```
<expr>   ::= <op1> <expr>
           | <op2> <expr> <expr>
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

abstractions (non-terminal symbols)

tokens (terminal symbols)

Example

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase> | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow | moon
<verb>        ::= jumped
<prep>        ::= over
```

What are the nonterminal and terminal symbols of this grammar?

Derivations and Parse Trees

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
and not x y
```

Derivations and Parse Trees

Definition. A **derivation** is a sequence of sentential forms (beginning at the start symbol) in which each form is the result of **replacing a non-terminal symbol in the previous form** according to a production rule

<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
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Derivations and Parse Trees

Definition. A **derivation** is a sequence of sentential forms (beginning at the start symbol) in which each form is the result of **replacing a non-terminal symbol in the previous form** according to a production rule

Definition. A **leftmost derivation** is a derivation in which the leftmost nonterminal symbol is replaced in each line

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
and not x y
```

Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

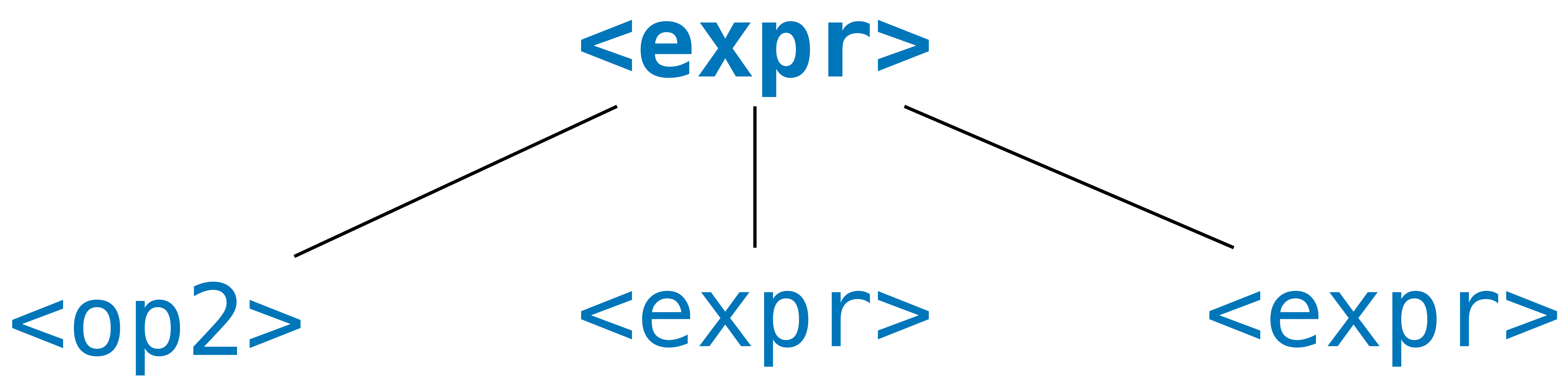
<expr>

<expr>

Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

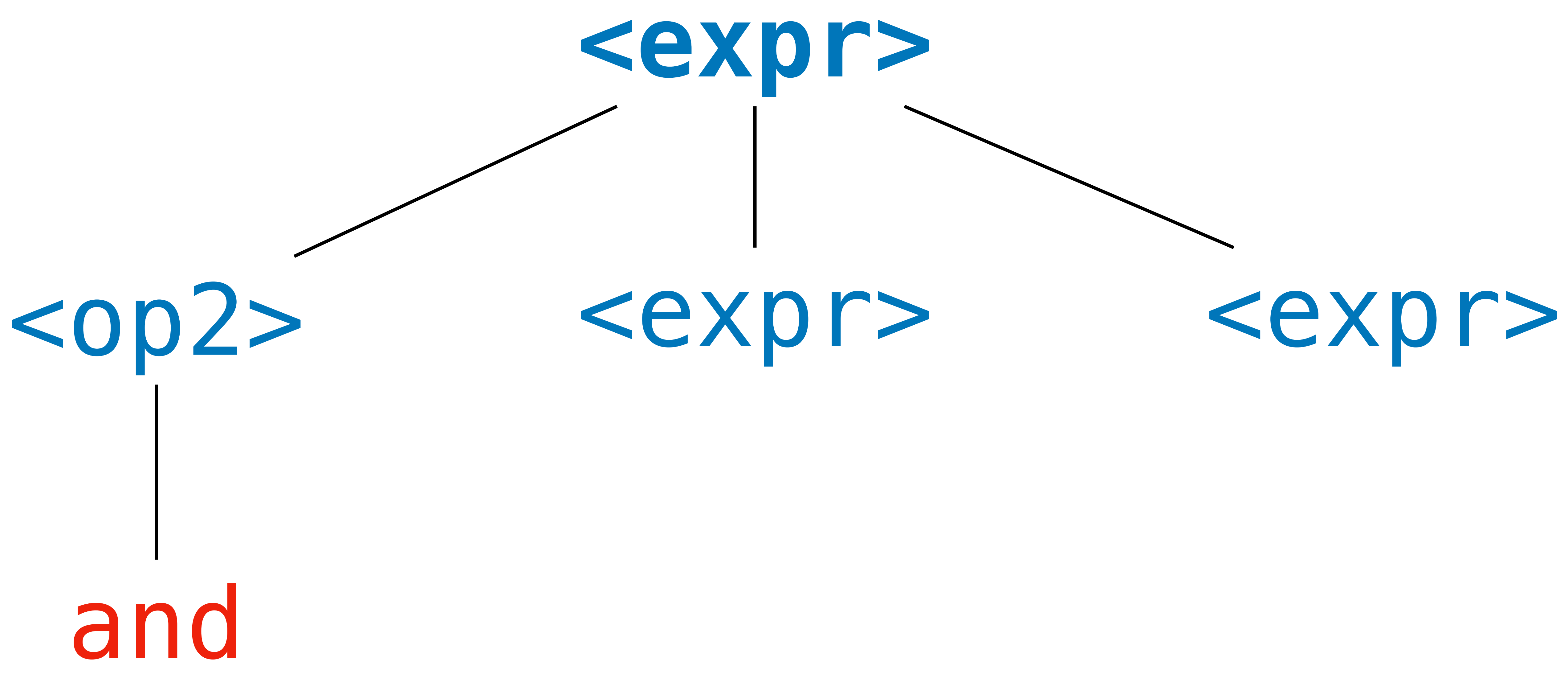
<expr>
<op2> <expr> <expr>



Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

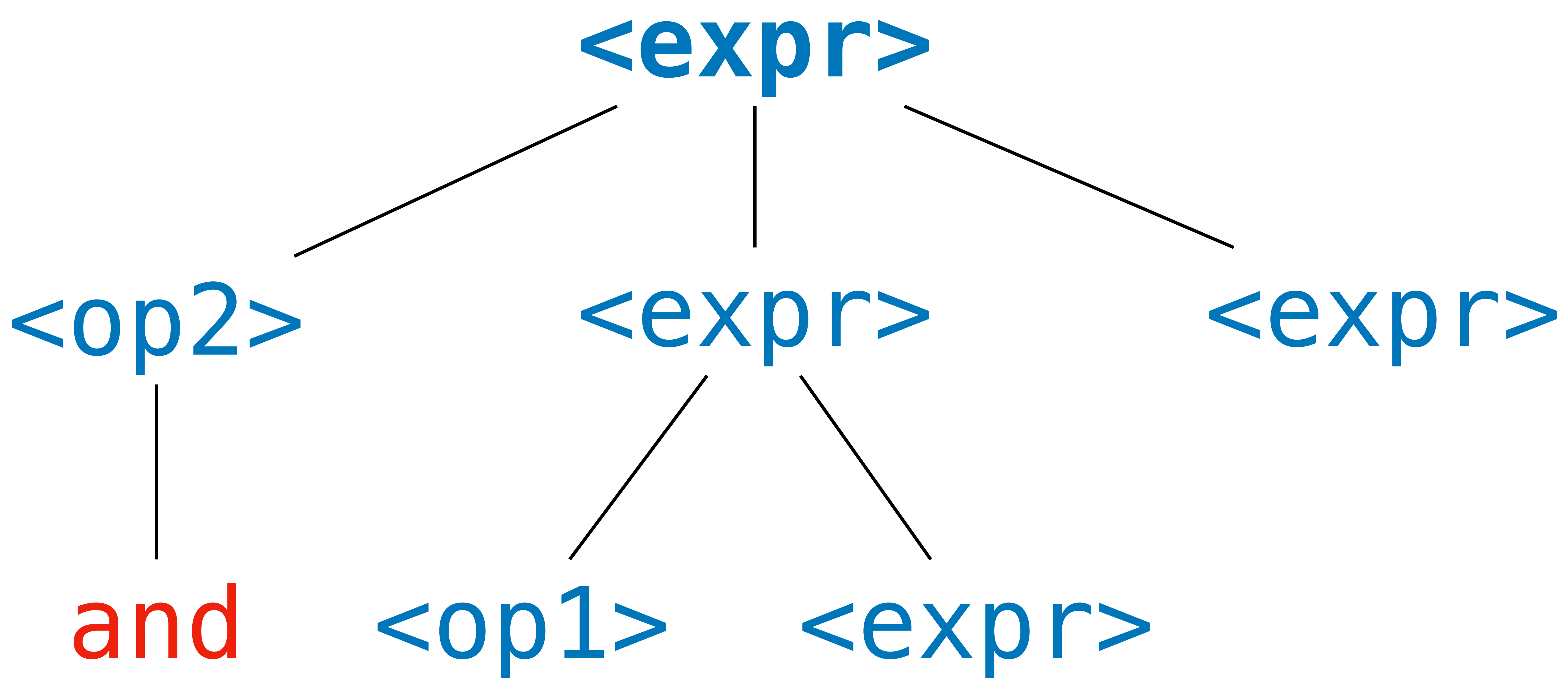
```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
```



Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

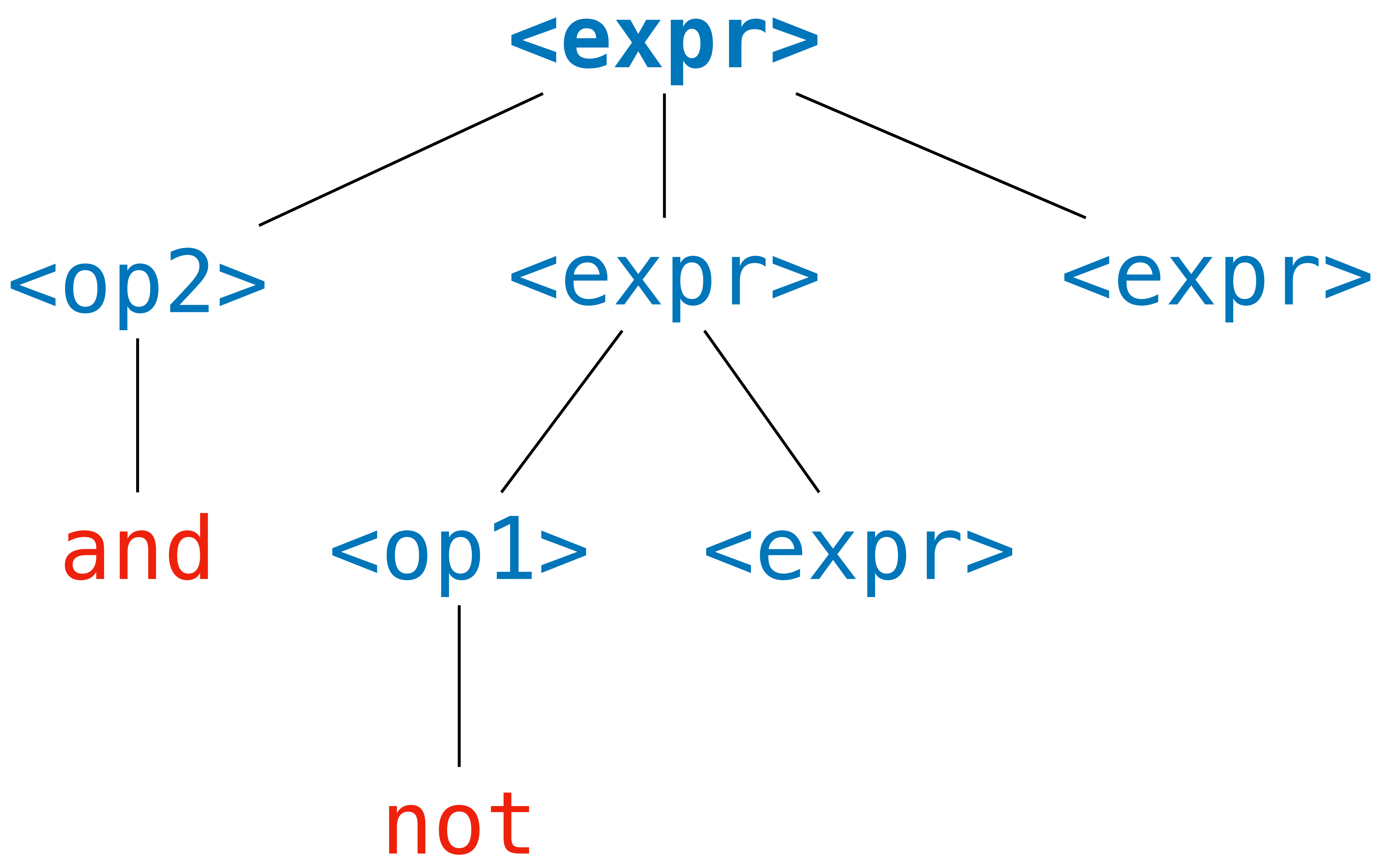
```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
```



Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
```



Derivations and Parse Trees

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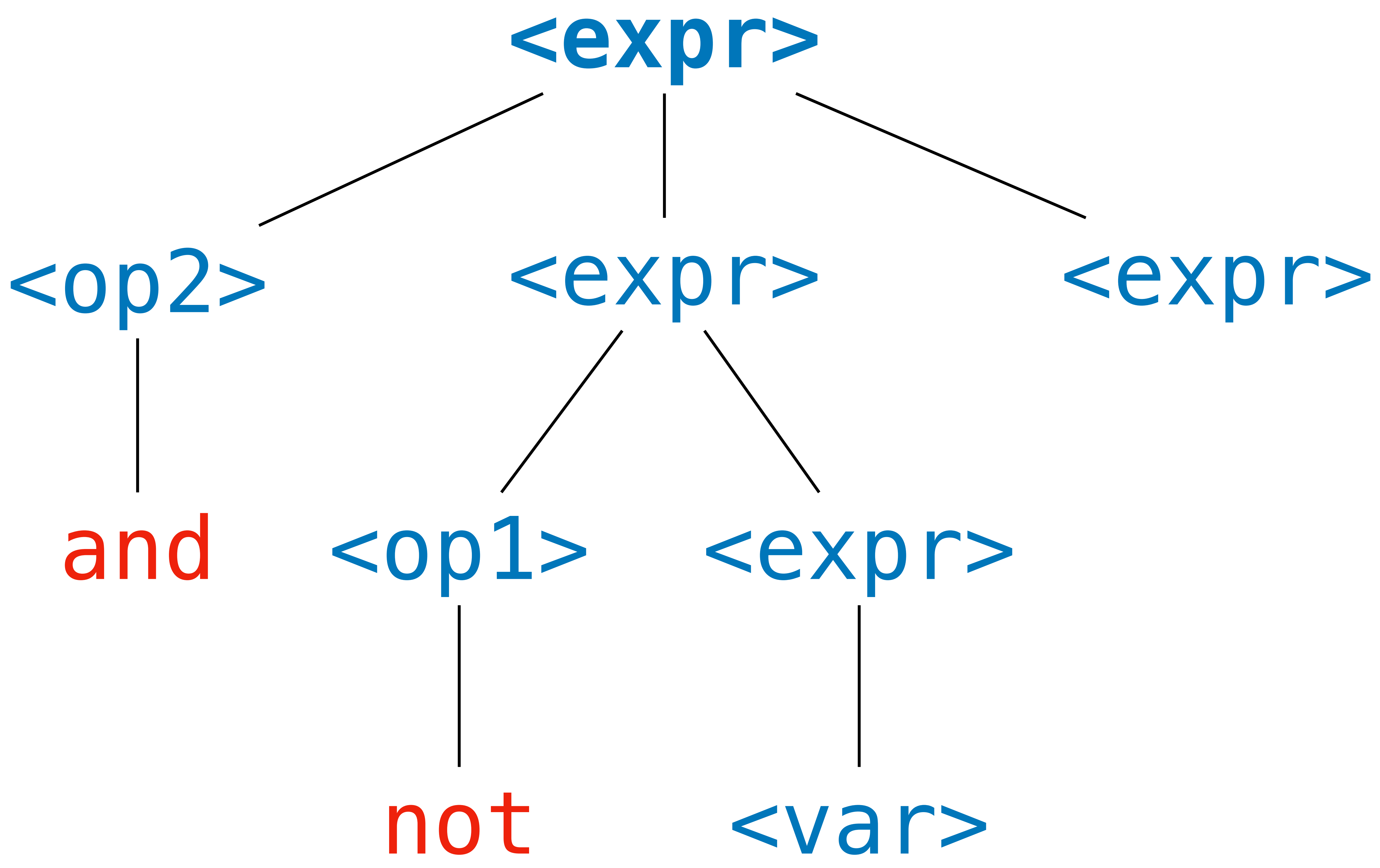
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
```



Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
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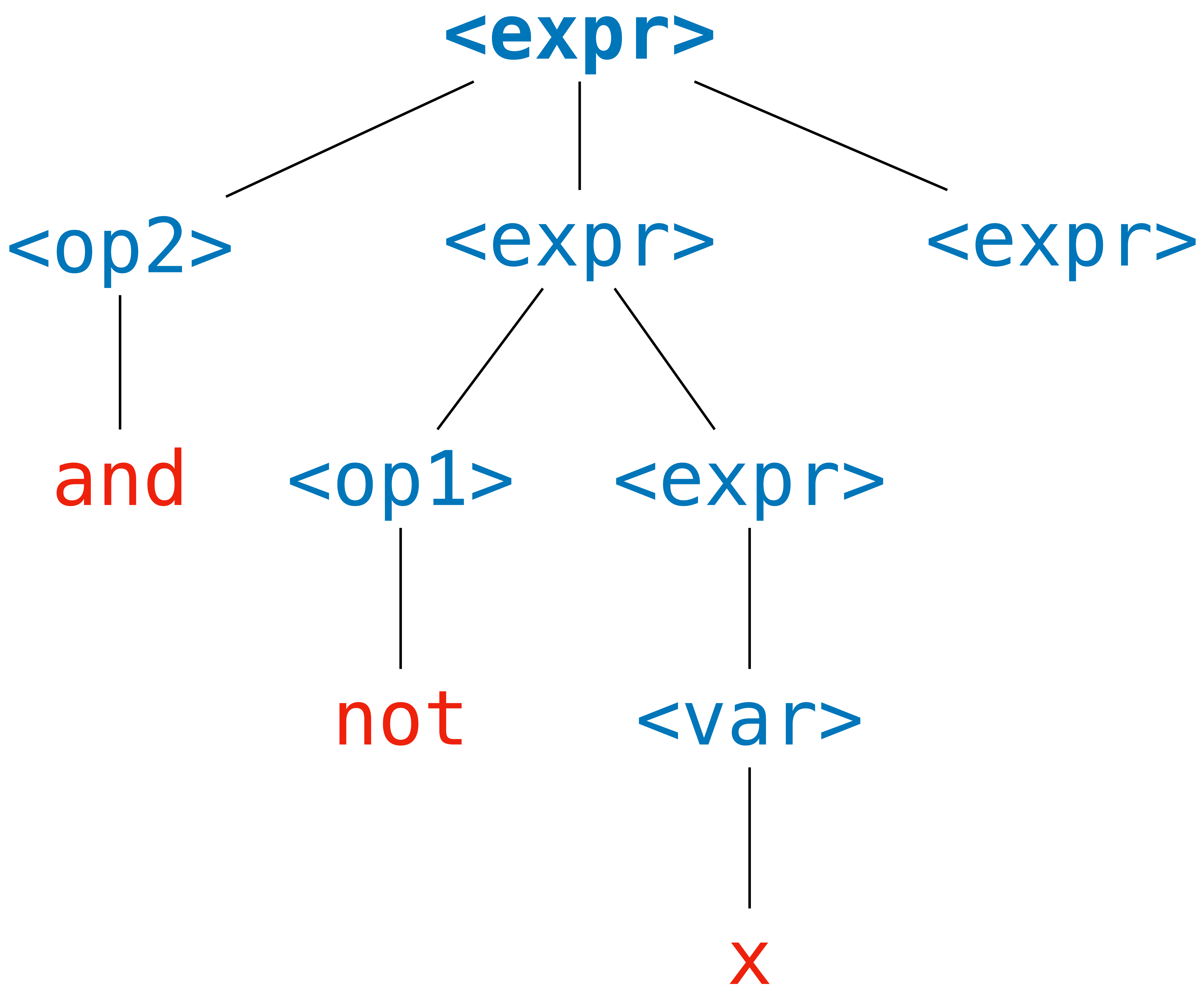
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
```



Derivations and Parse Trees

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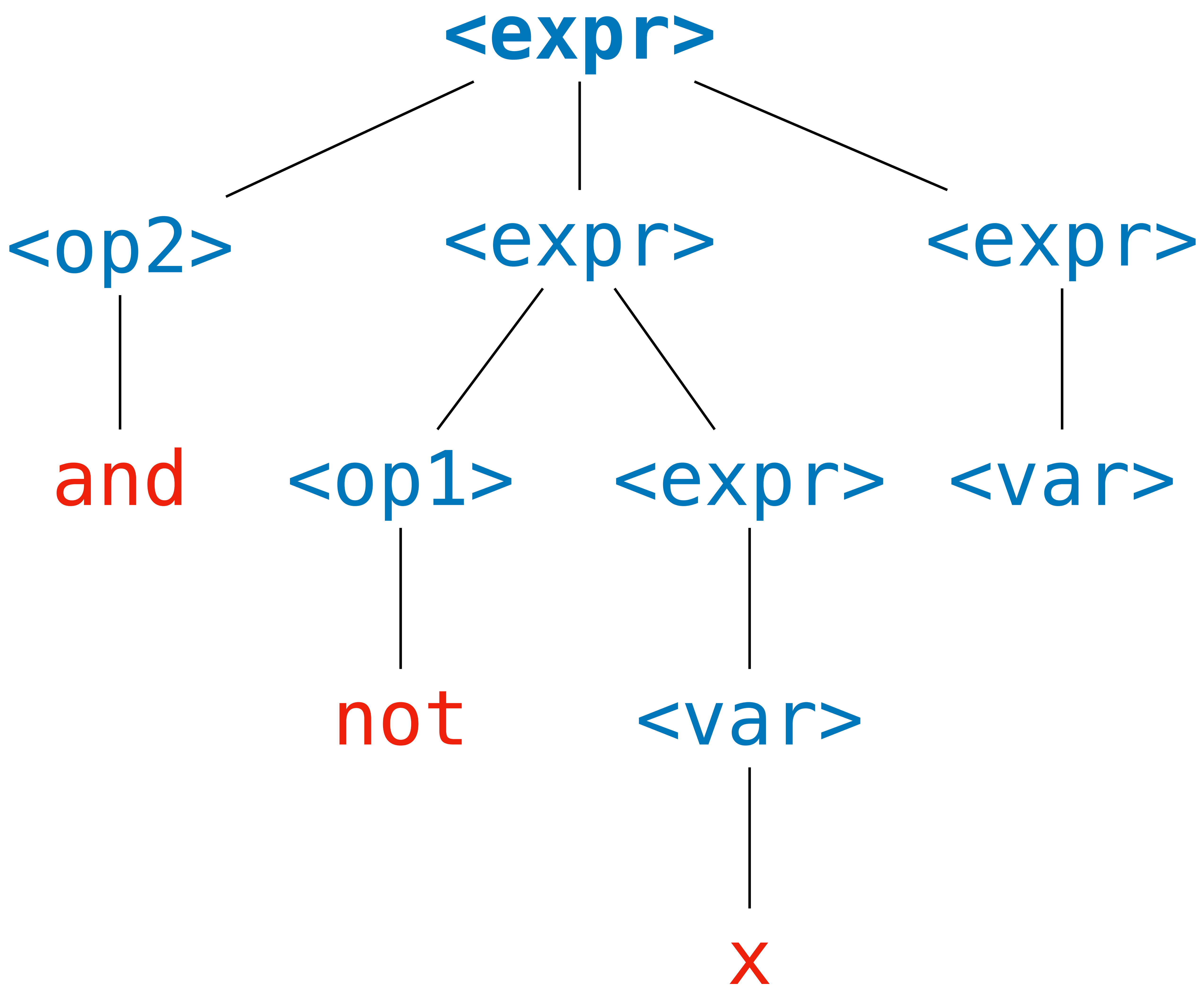
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
```



Derivations and Parse Trees

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<expr>  ::= <op1> <expr>
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          | <var>
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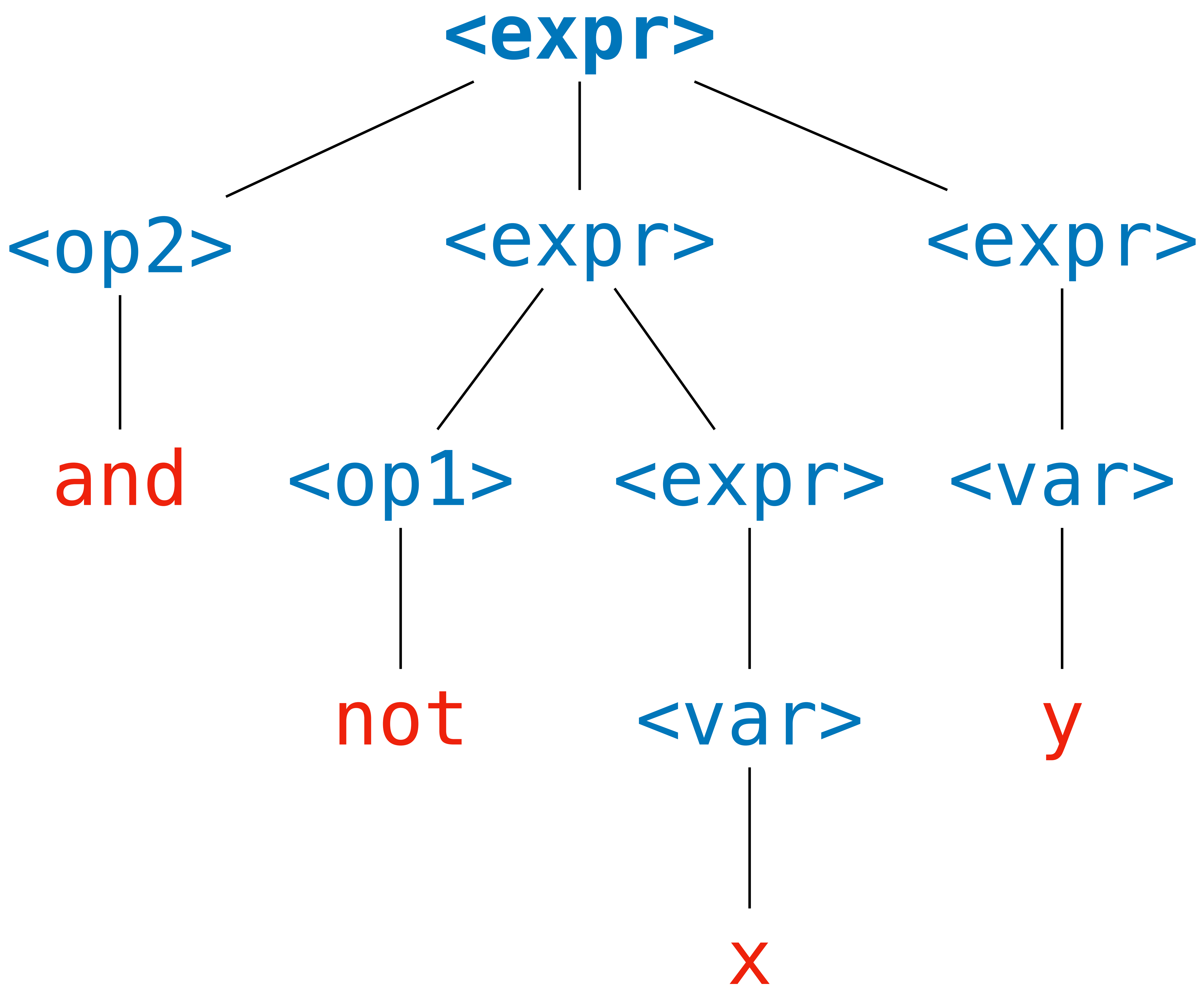
and not x <var>
and not x y
```



Derivations and Parse Trees

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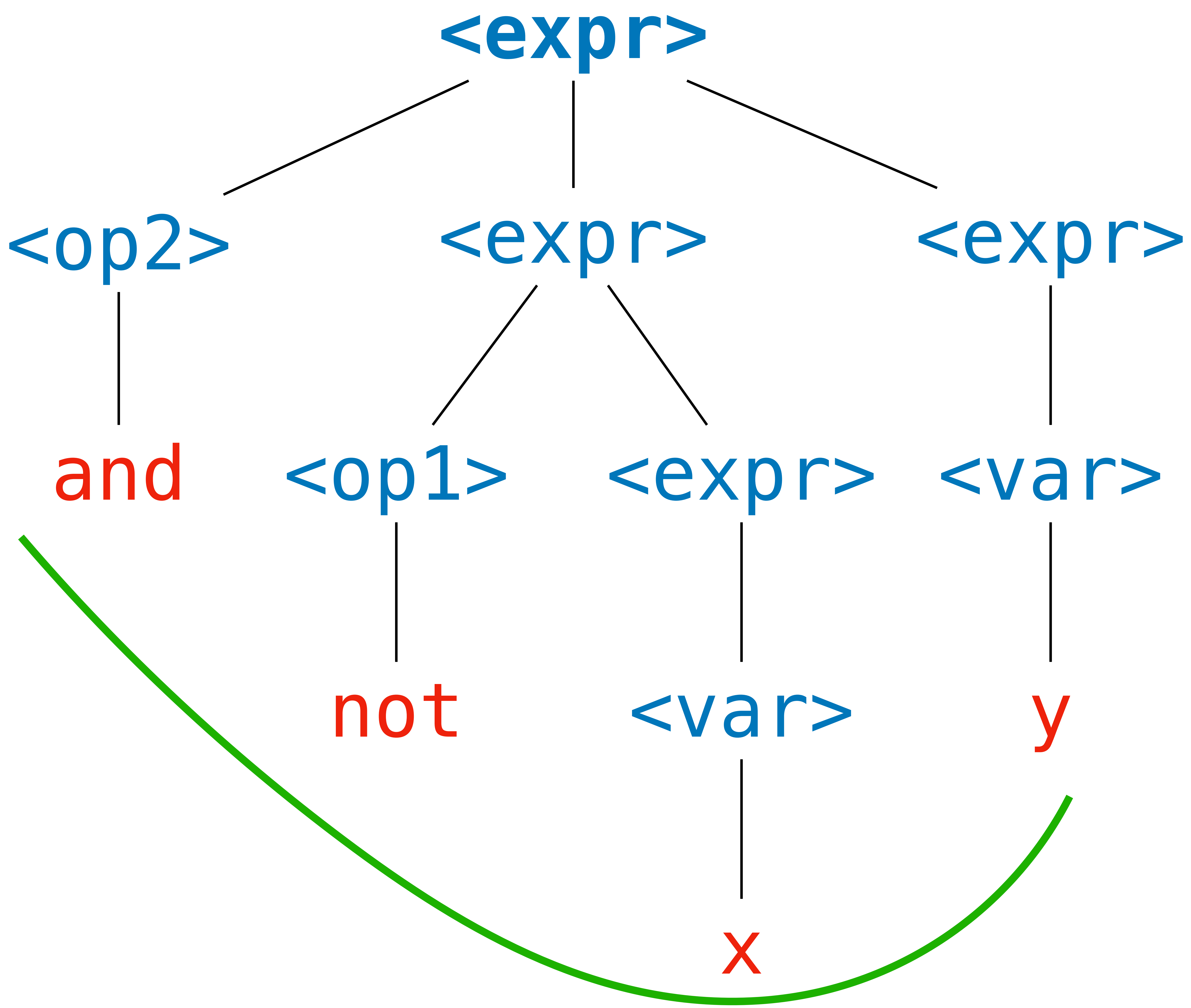

The point: parse trees and derivations represent the same hierarchical structure

Why do we care?

$\boxed{\texttt{char stream}} \xrightarrow{\texttt{lexer}} \boxed{\texttt{token stream}} \xrightarrow{\texttt{parser}} \boxed{\texttt{parse tree}} \longrightarrow \cdots$

Why do we care?

`char stream` $\xrightarrow{\text{lexer}}$ `token stream` $\xrightarrow{\text{parser}}$ `parse tree` $\longrightarrow$ . . .

`a.k.a. list of terminal symbols`

Why do we care?

`char stream` $\xrightarrow{\text{lexer}}$ `token stream` $\xrightarrow{\text{parser}}$ `parse tree` $\longrightarrow$ . . .

`token stream`: a.k.a. list of terminal symbols

`parse tree`: a.k.a. derivation in a grammar

Why do we care?

We will parse **token streams** into **parse trees**

Why do we care?

`char stream` —lexer→ `token stream` —parser→ `parse tree` → . . .

token stream: a.k.a. list of terminal symbols

parse tree: a.k.a. derivation in a grammar

We will parse **token streams** into **parse trees**

It is much easier to evaluate something hierarchical than something which is linear

Practice Problem

```
<expr>  ::= <op1> <expr>
          | <expr> <op2> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

Give a derivation of ***not x and y or z*** *in the above grammar, both as a sequence of sentential forms and as a parse tree*

(In Python, if ***x*** *and* ***y*** *and* ***z*** *are* ***True****, what does this expression evaluate to?)*

Answer

```
<expr>  ::= <op1> <expr>
          | <expr> <op2> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

not x and y or z

Example

```
let x = 2 in if x = z then x else y
```

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How can we demonstrate that this is a well-formed expression?

Example

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```

How can we demonstrate that this is a well-formed expression?

Answer: Well build a derivation/parse tree for it with the root **<expr>**!

Recall: Let-Expressions (Syntax Rule)

```
<expr> ::= let <var> = <expr> in <expr>
```

If x is a valid variable name, and e_1 is a well-formed expression and e_2 is a well-formed expression then

$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2$$

is a well-formed expression

Recall: If-Expressions (Syntax Rule)

```
<expr> ::= if <expr> then <expr> else <expr>
```

If e_1 is a well-formed expression and e_2 is a well-formed expression and e_3 is a well-formed expression, then

if e_1 then e_2 else e_3

is a well-formed expression

```
let x = 2 in if x = z then x else y
```

Ambiguity

Ambiguity in Natural Language

The duck is ready for dinner.

John saw the man on the mountain with a telescope.

He said on Tuesday there would be an exam.

(examples taken from around the internet)

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Natural language has ambiguities that can confuse the meaning of a sentence

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We have informal tactics for avoiding these pitfalls

(examples taken from around the internet)

Ambiguity in Natural Language

*The duck is ready **to eat** dinner.*

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The duck is ready ***to eat*** *dinner.*

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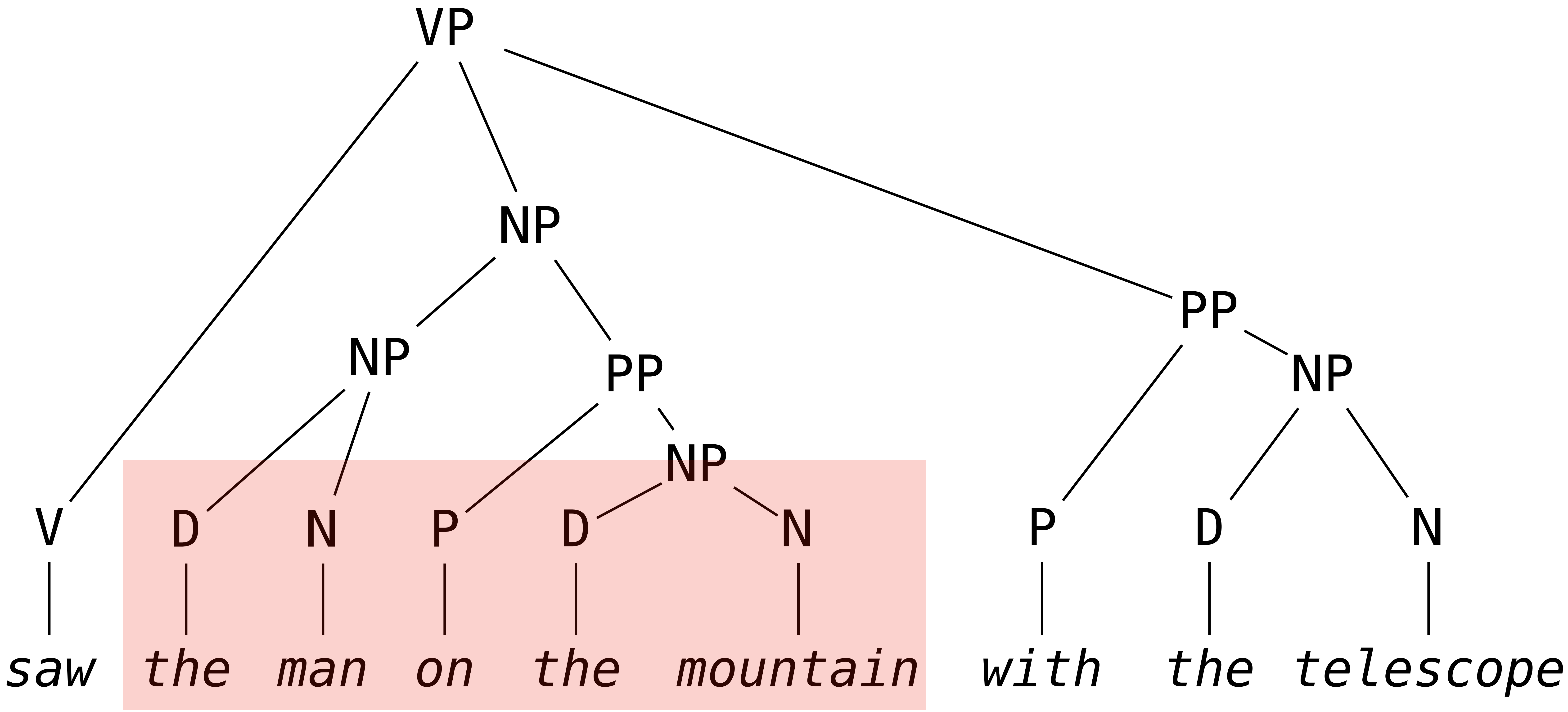
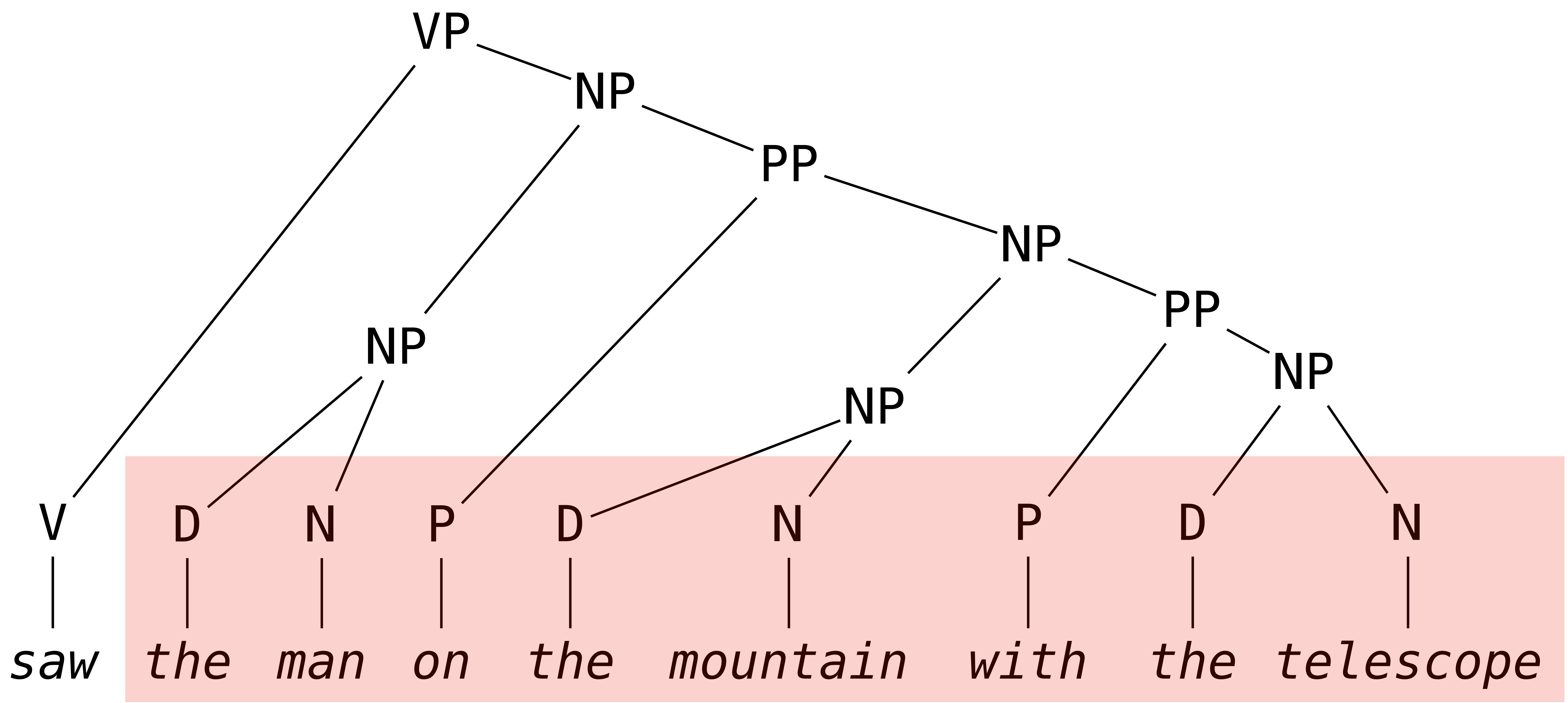
Aside: Ambiguity and Linearity



Ambiguity is caused by writing down hierarchical structures in a linear fashion

Aside: Ambiguity and Linearity



There is no ambiguity in the grammatical parse tree of this statement

The hierarchical structure changes the meaning of the sentence

Ambiguity in Formal Grammar

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```
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         | <var>
<op>   ::= +
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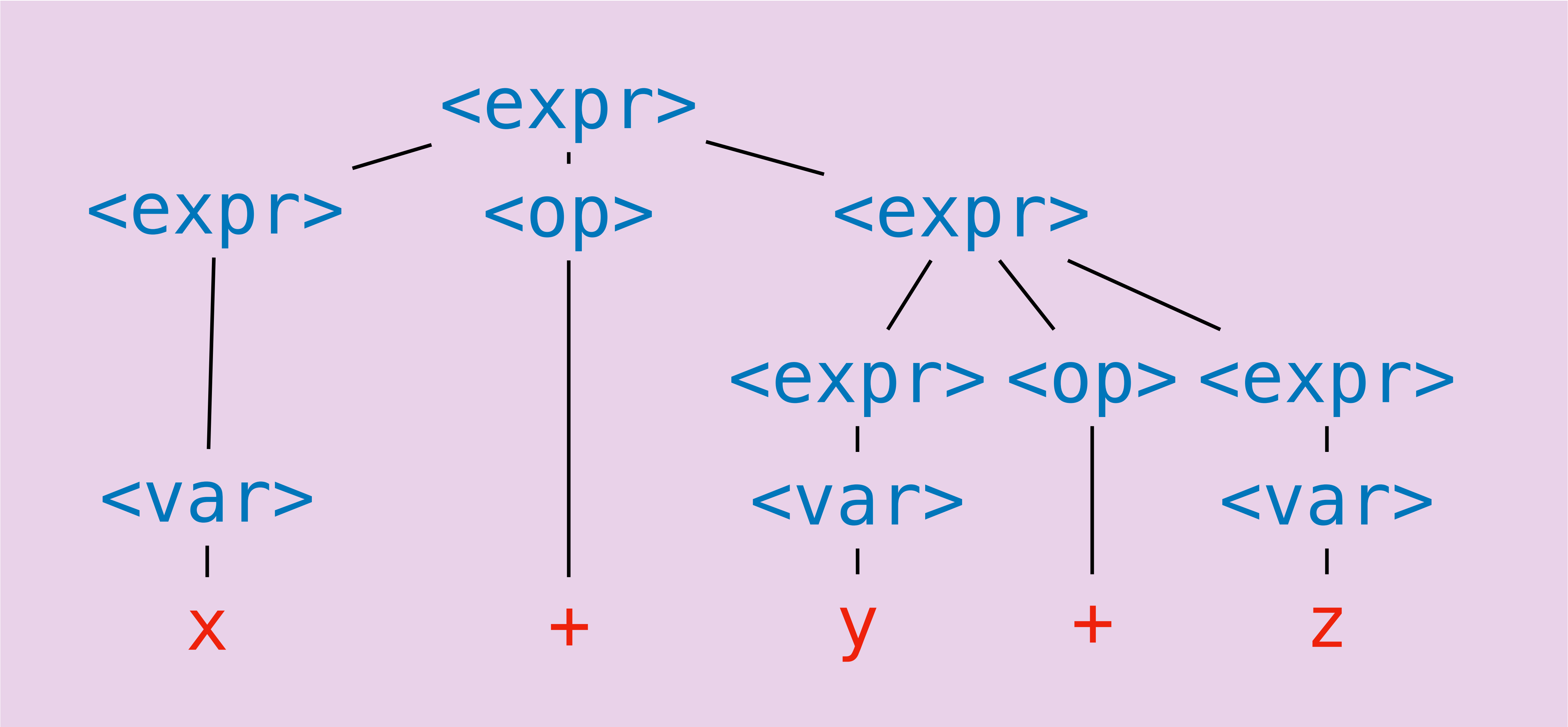

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```
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         | <var>
<op>   ::= +
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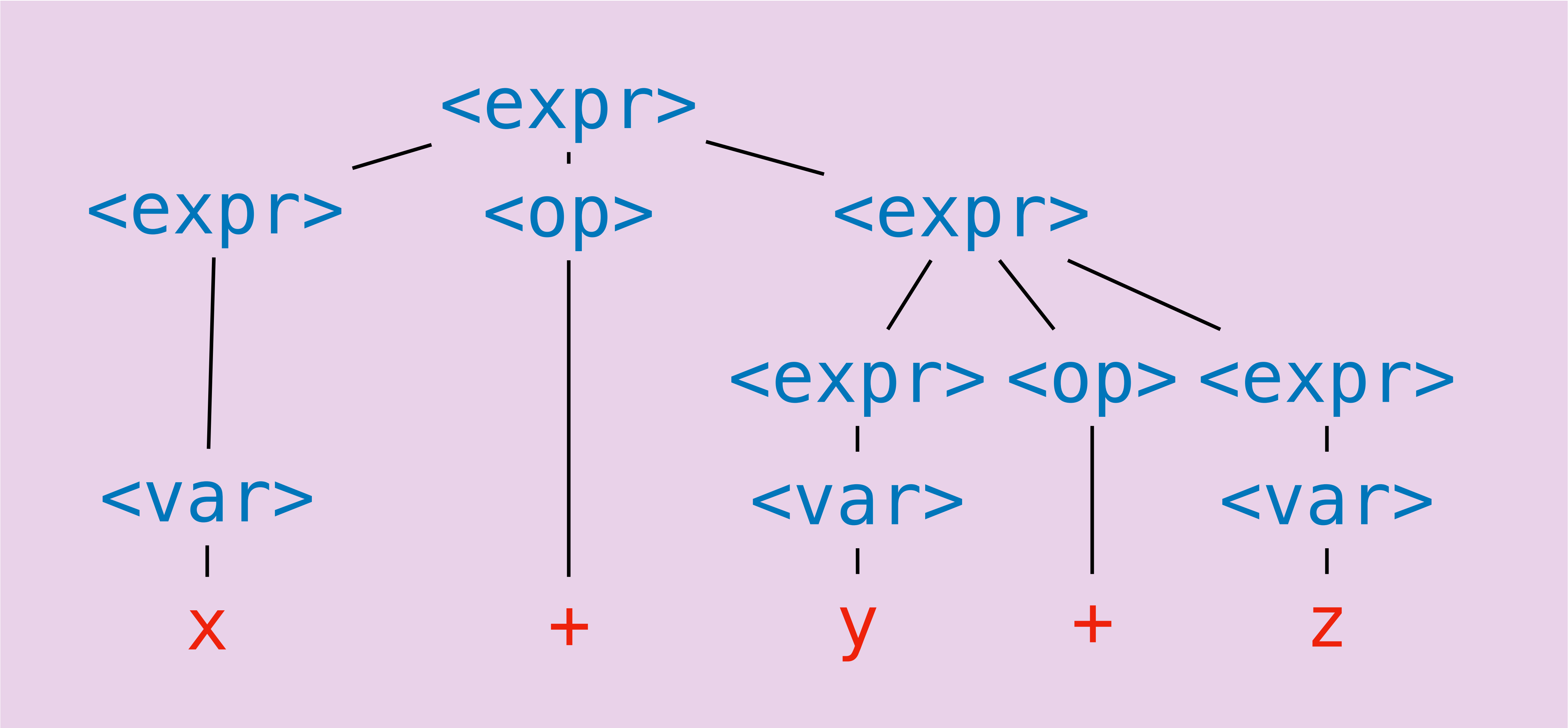
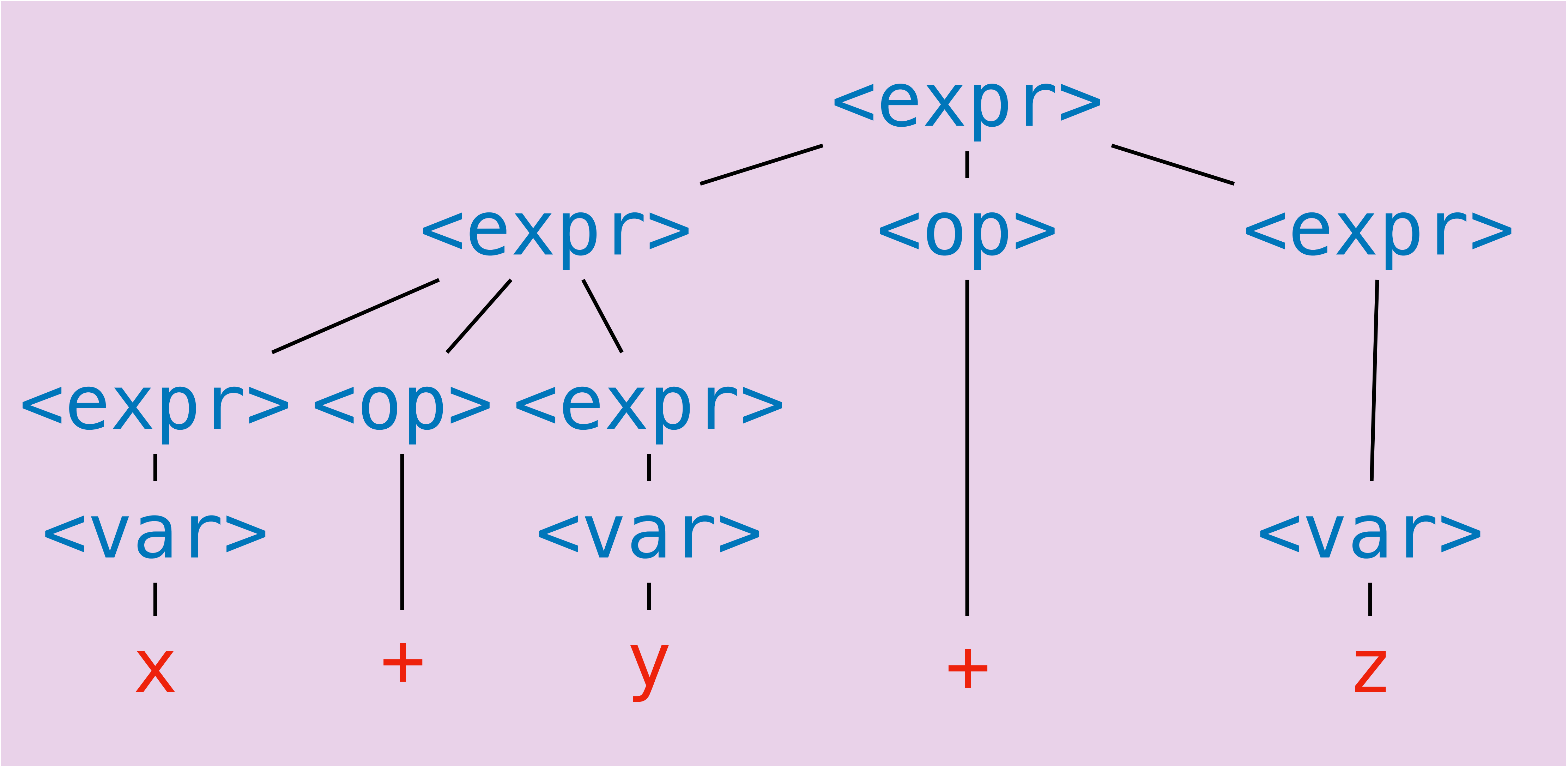
x + y + z can be derived as





Example

```
<expr> ::= x
         | if <expr> then <expr>
         | if <expr> then <expr> else <expr>
```

What can we do about ambiguity?

Fixity

Fixity

The fixity of an operator refers to where an operator is placed relative to its arguments:

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```
prefix        f x , (- x)
```

Fixity

The fixity of an operator refers to where an operator is placed relative to its arguments:

```
prefix         f x , (- x)

postfix        a! (get from ref)
```

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The fixity of an operator refers to where an operator is placed relative to its arguments:

prefix	f x , (- x)
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infix	a * b, a + b, a mod b

Fixity

The fixity of an operator refers to where an operator is placed relative to its arguments:

prefix	f x , (- x)
postfix	a! (get from ref)
infix	a * b, a + b, a mod b
mixfix	if b then x else y

Polish Notation

```
- / + 2 * 1 - 2 3
```

is equivalent to

```
- (2 + (1 * (- 2) / 3)
```

To avoid ambiguity, we can make all operators prefix (or postfix) operators. *We don't even need parentheses*

(This how early calculators worked)

Example

```
<expr> ::= <bool>
         | <var>
         | ifthen <expr> <expr>
         | ifthenelse <expr> <expr> <expr>
<bool>::= tru | fls
<var> ::= x | y | z
```

No more ambiguity. But programs written like this are notoriously difficult to read...

Lots of Parentheses

```
<expr>   ::= (<op1> <expr>)
           | (<expr> <op2> <expr>)
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

Lots of Parentheses

```
<expr>   ::= (<op1> <expr>)
           | (<expr> <op2> <expr>)
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

If we want infix operators, we *could* add parentheses around all operators

Lots of Parentheses

```
<expr>   ::= (<op1> <expr>)
           | (<expr> <op2> <expr>)
           | <var>
<op1>    ::= not
<op2>    ::= and | or
<var>    ::= x | y | z
```

```
(((not x) and (not (not y)) or z)

((((x or y) or z) or x) or y)

(not ((not x) and (not y)) or
     (x and z))

(x and y)
```

If we want infix operators, we *could* add parentheses around all operators

But we run into a similar issue: *Too many parentheses are difficult to read*

Can we get away without (or with fewer) parentheses?

Two Ingredients (or Flavors of Ambiguity)

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Associativity:

How should arguments be grouped in an expression like **1 + 2 + 3 + 4**?

Two Ingredients (or Flavors of Ambiguity)

<u>Associativity:</u>

How should arguments be grouped in an expression like **1 + 2 + 3 + 4**?

<u>Precedence:</u>

How should arguments be grouped in an expression like **1 + 2 * 3 + 4**?

Associativity

The associativity of an infix operator refers to how its arguments are grouped in the absence of parentheses:

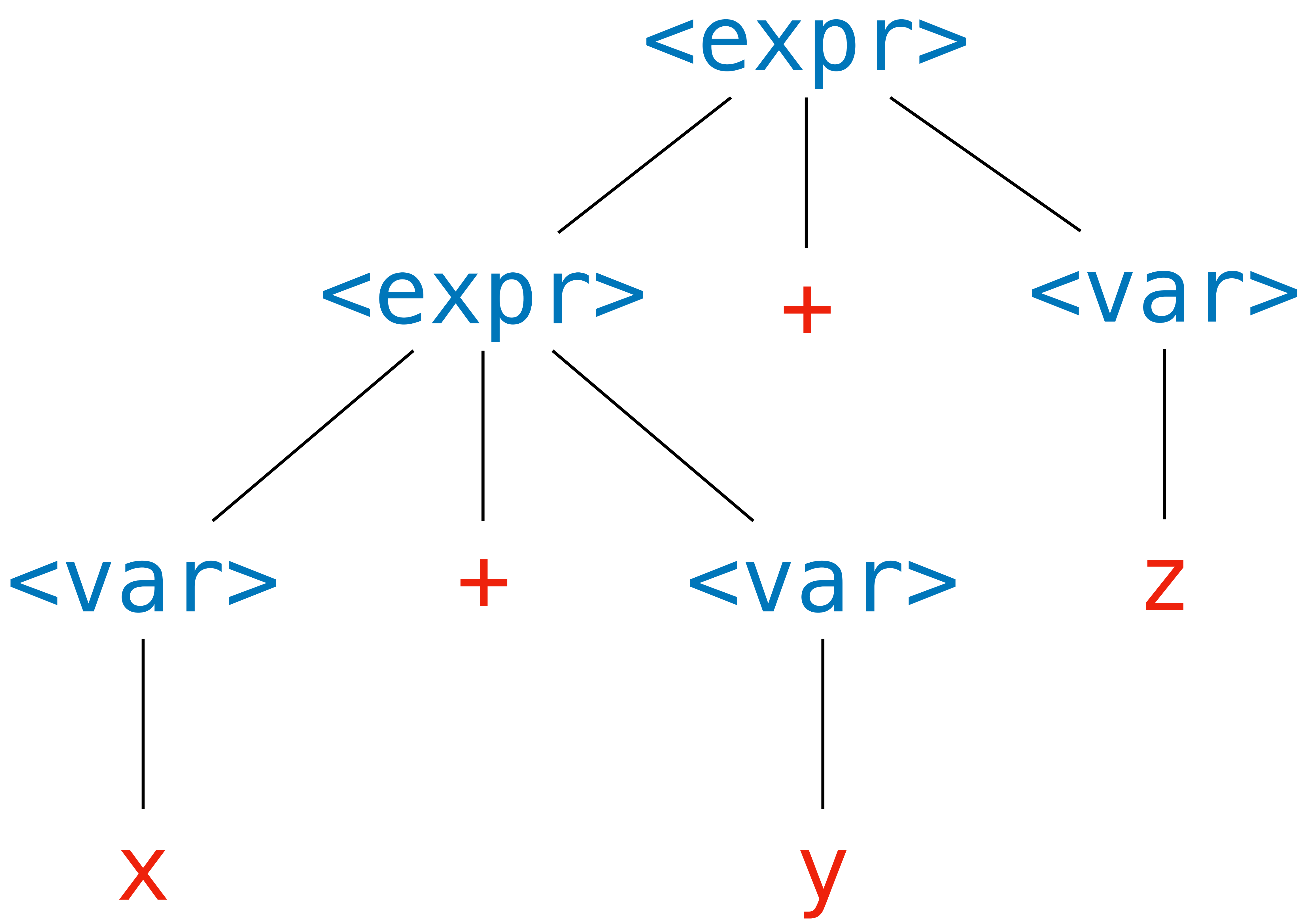
```
left associative      1 +  2 +  3 ⇒ (1 +  2) +  3

right associative     a -> b -> c ⇒  a -> (b -> c)
```

Associativity

```
<expr> ::= <expr> + <expr>
         | <var>
<var>  ::= x | y | z
```

x + y + z ⇒



"*add the sum of x and y to z*"

How do we enforce that we get a tree of this shape?

The Culprit

```
<expr> ::= <expr> + <expr>
         | <var>
<var>  ::= x | y | z
```

The Culprit

```
<expr> ::= <expr> + <expr>
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Any time we have a rule like this, we should be suspicious...

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<expr> + <expr> ⇒ <expr> + <expr> + <expr>
```

The Culprit

```
<expr> ::= <expr> + <expr>
         | <var>
<var>  ::= x | y | z
```

Any time we have a rule like this, we should be suspicious...

<expr> + <expr> ⇒ <expr> + <expr> + <expr>

Which <expr> did we replace?

The Solution: Breaking Symmetry

```
<expr> ::= <expr> + <var>
         | <var>
<var>  ::= x | y | z
```

```
<expr>
<expr> + <var>
<expr> + z
<expr> + <var> + z
<expr> + y + z
<var> + y + z
x + y + z
```

We make sure that one of the arguments must be "simpler"

By enforcing that the second argument is a <var>, we will get the left-associative parse tree

Example Parse Tree

```
<expr> ::= <expr> + <var>
         | <var>
<var>  ::= x | y | z
```

```
<expr>
<expr> + <var>
<expr> + z
<expr> + <var> + z
<expr> + y + z
<var> + y + z
x + y + z
```

And Right Associativity

```
<type> ::= <base> -> <type>
         | <base>
<base> ::= ()
```

```
<type>
<base> -> <type>
() -> <type>
() -> <base> -> <type>
() -> () -> <type>
() -> () -> <base>
() -> () -> ()
```

For right associativity, we break symmetry by "factoring out" the *left* argument.

Example Parse Tree

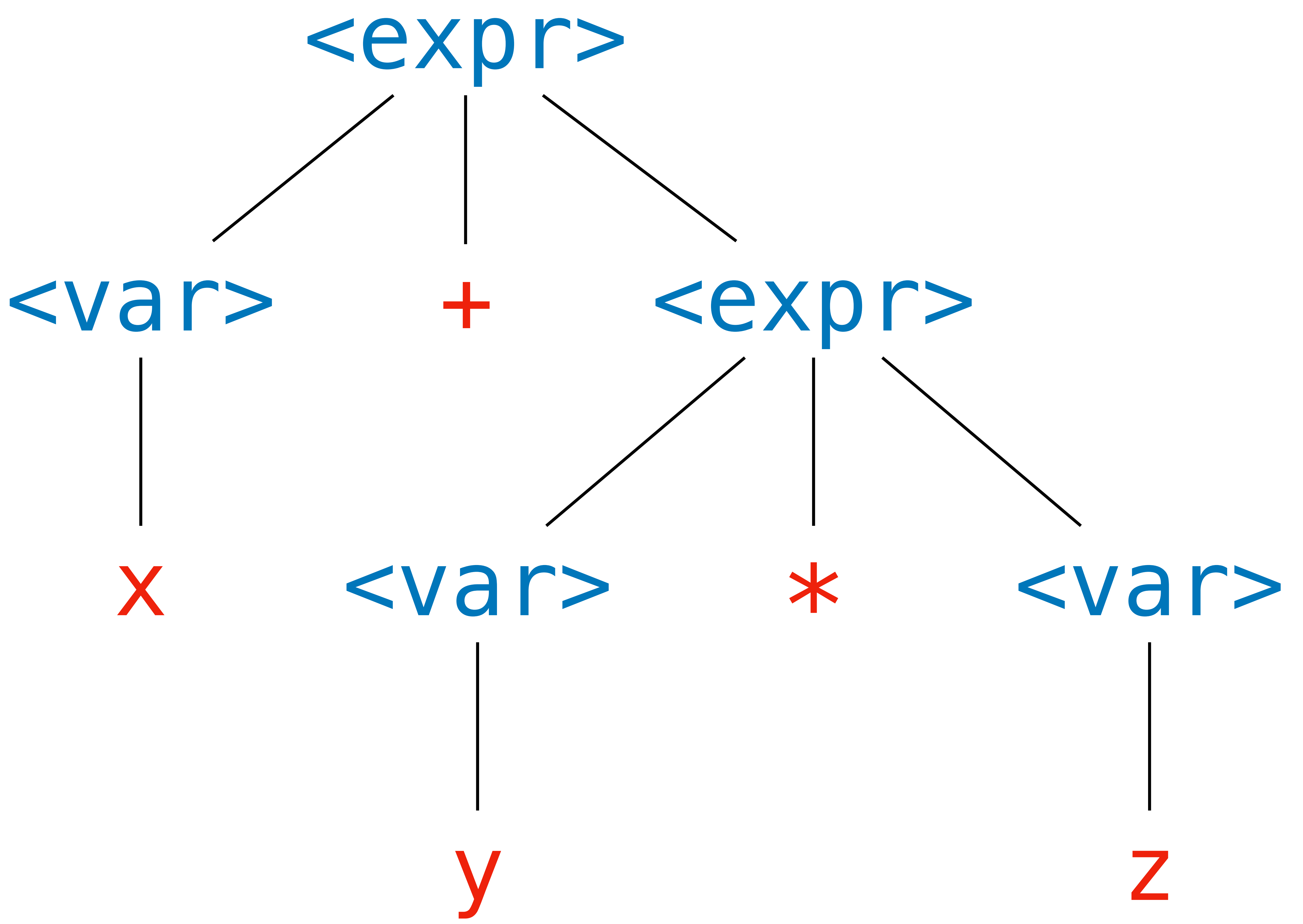
```
<type> ::= <base> -> <type>
         | <base>
<base> ::= ()
```

```
<type>
<base> -> <type>
() -> <type>
() -> <base> -> <type>
() -> () -> <type>
() -> () -> <base>
() -> () -> ()
```

Multiple Operators

```
<expr> ::= <expr> <op> <var>
         | <var>
<op>   ::= + | *
<var>  ::= x | y | z
```

x + y * z ⇒



"*add x to the product of y and z*"

Question. *What if we have multiple operators? Which one should "bind tighter"?*

Precedence

$$2 + 3 \times 6 = 2 + (3 \times 6) = 20$$

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The precedence of an operator refers to order in which an operator should be considered, relative to other operators

Example. **PEMDAS (paren, exp, mul, div, add, sub)**

Higher precedence means it "binds tighter"

Dealing with Precedence within the Grammar

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```

We factor out the * part of the <expr> rule

Note that we handle *lower* precedence terms first, since terms *deeper* in the parse tree are evaluated first

A Note on Associativity and Precedence

```
%token PLUS
%token MINUS
%token TIMES
%token DIVIDE

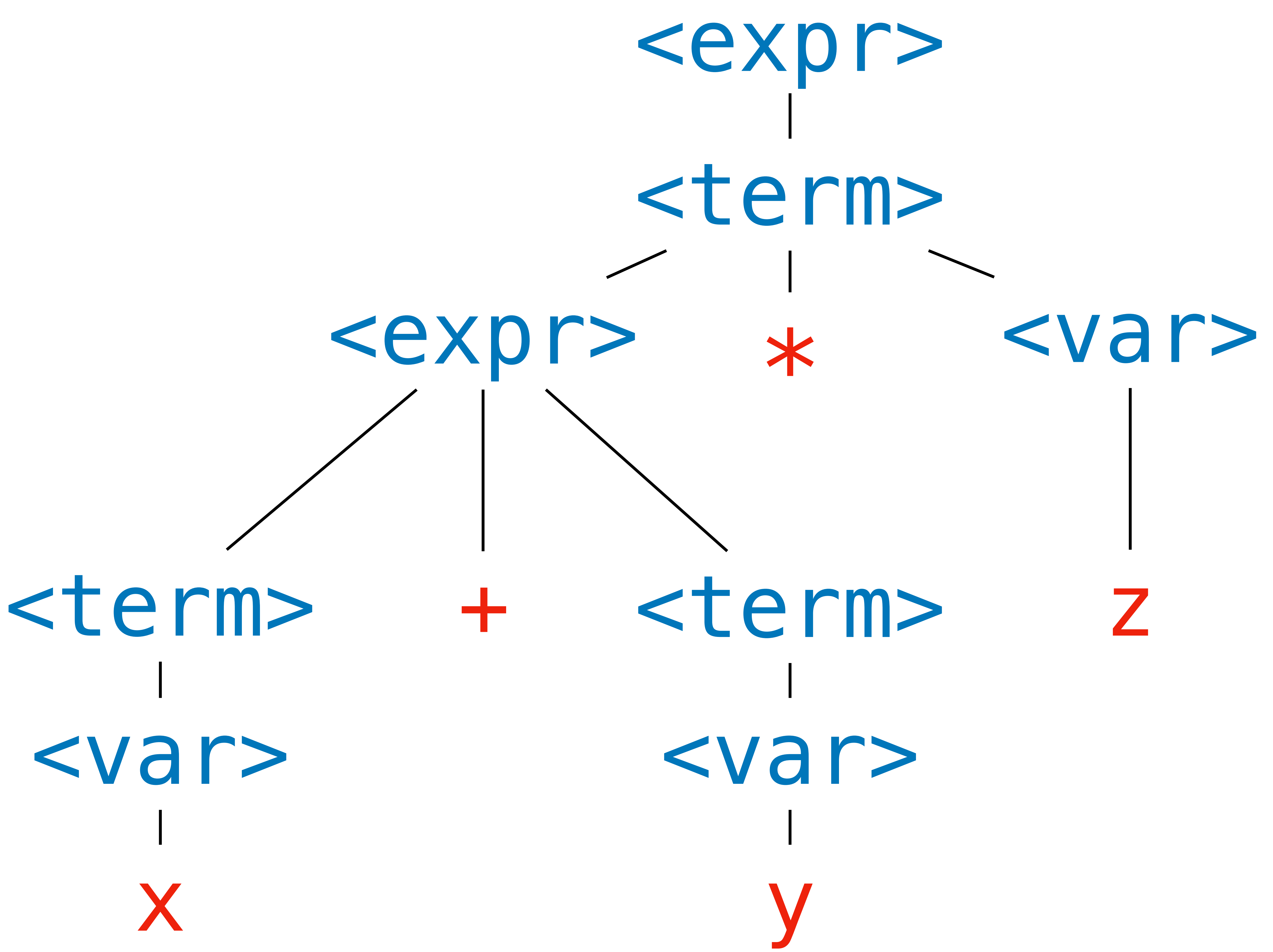
%left PLUS, MINUS
%left TIMES, DIVIDE
```

In most situations, we actually won't deal with associativity and precedence in this way

Using a parser generator we'll often be able to *specify* the precedence of an operator

The Issue of Parentheses Returns

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```



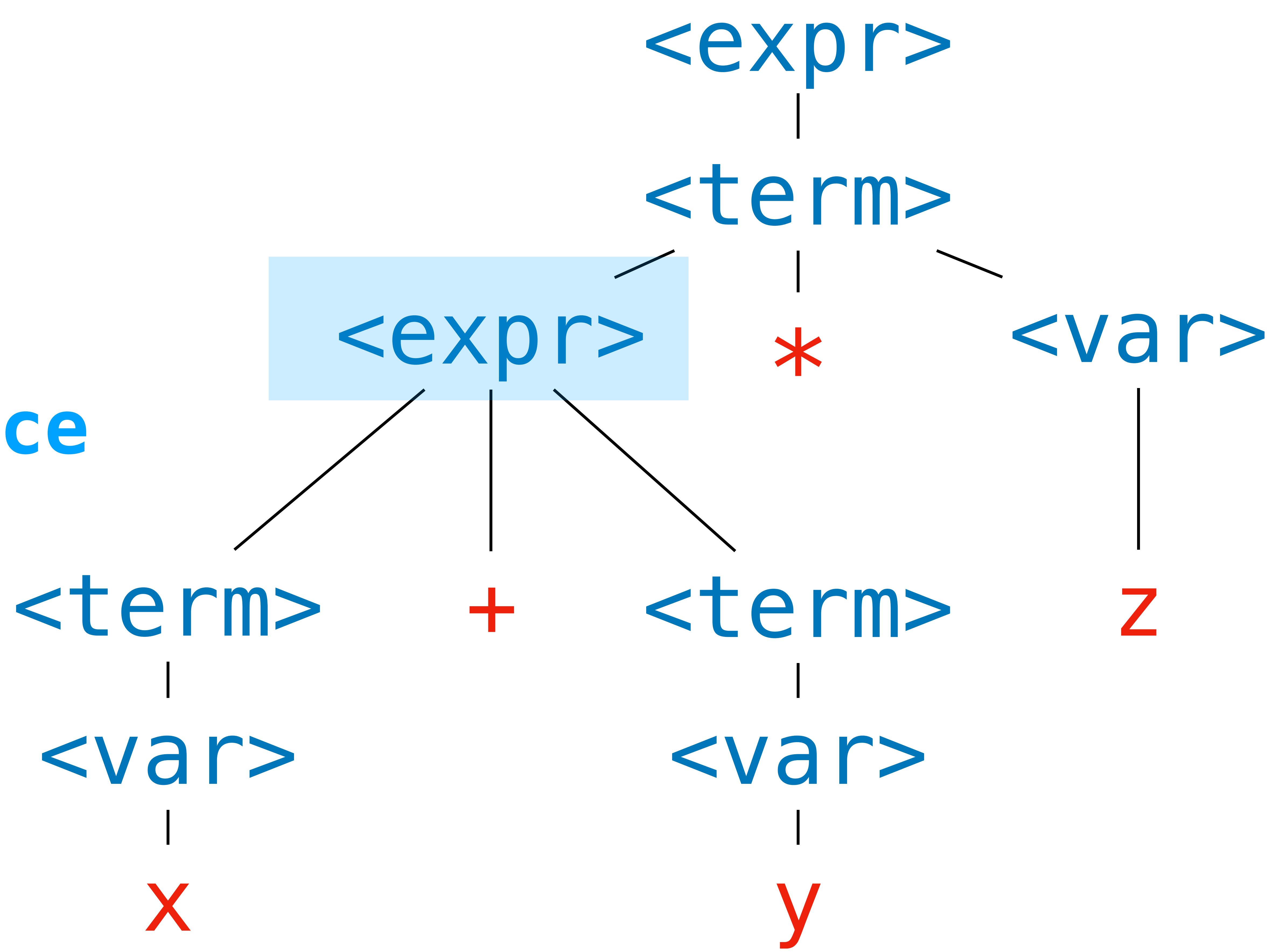
"multiply the sum of x and y with z"

Question. *Can we derive this parse tree?*

The Issue of Parentheses Returns

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```

No, we need to introduce parentheses again.



"multiply the sum of x and y with z"

Question. *Can we derive this parse tree?*

Dealing with Parentheses

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <pars>
<pars> ::= <var> | ( <expr> )
<var>  ::= x | y | z
```

We further factor out the part of the rule for parentheses. Note that any expression can appear in the parentheses

(This is a circular, or mutually recursive, production rule)

Example

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <pars>
<pars> ::= <var> | ( <expr> )
<var>  ::= x | y | z
```

(x + y) * z

Summary

When we specify a PL (e.g., in the projects) you will be given a **BNF grammar**

You will need to know how to translate this into a parser

So you will need *practice reading BNF specifications*

To avoid ambiguity, we make choices beforehand about the **fixity**, **associativity** and **precedence**