Assignment 3 Written Solutions

CAS CS 320: *Concepts of Programming Languages*

1 Typing Derivation (I)

$$\dfrac{\dfrac{}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{int}\}\vdash\texttt{y}:\texttt{int}}\,\text{(var)}\quad\dfrac{}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{int}\}\vdash\texttt{[]}:\texttt{int list}}\,\text{(nil)}\quad\dfrac{\dfrac{}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{int}\}\vdash\texttt{x}:\texttt{int}}\,\text{(var)}\quad\dfrac{}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{int}\}\vdash\texttt{[]}:\texttt{int list}}\,\text{(nil)}}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{bool}\}\vdash\texttt{x :: []}:\texttt{int list}}\,\text{(cons)}}{\{\texttt{x}:\texttt{int},\texttt{y}:\texttt{bool}\}\vdash\texttt{if y then [] else x :: []}:\texttt{int list}}\,\text{(if)}$$

2 Typing Derivation (II)

$\Gamma = \{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}$ where τ can be any type.

$$\dfrac{\dfrac{}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{g}:\texttt{int}\ \texttt{->}\ \tau}\,\text{(var)}\quad\dfrac{\dfrac{}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{f}:\tau\ \texttt{->}\ \texttt{int}}\,\text{(var)}\quad\dfrac{\dfrac{}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{g}:\texttt{int}\ \texttt{->}\ \tau}\,\text{(var)}\quad\dfrac{}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{0}:\texttt{int}}\,\text{(int)}}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{g 0}:\tau}\,\text{(app)}}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{f (g 0)}:\texttt{int}}\,\text{(app)}}{\{\texttt{f}:\tau\ \texttt{->}\ \texttt{int},\texttt{g}:\texttt{int}\ \texttt{->}\ \tau\}\vdash\texttt{g (f (g 0))}:\tau}\,\text{(app)}$$

3 URM Programs

$P = \texttt{J 0 1 2 I 0 I 1}$

$$\dfrac{\dfrac{}{\langle\, P\,,\,\texttt{[(0,5);(1,5)]}\,,\,0\,\rangle\longrightarrow\langle\, P\,,\,\texttt{[(0,5);(1,5)]}\,,\,2\,\rangle}\,\text{(jump-eq)}\quad\dfrac{\dfrac{}{\langle\, P\,,\,\texttt{[(0,5);(1,5)]}\,,\,2\,\rangle\longrightarrow\langle\, P\,,\,\texttt{[(0,5);(1,6)]}\,,\,3\,\rangle}\,\text{(incr)}\quad\dfrac{}{\langle\, P\,,\,\texttt{[(0,5);(1,6)]}\,,\,3\,\rangle\longrightarrow^{\star}\langle\, P\,,\,\texttt{[(0,5);(1,6)]}\,,\,3\,\rangle}\,\text{(refl)}}{\langle\, P\,,\,\texttt{[(0,5);(1,5)]}\,,\,2\,\rangle\longrightarrow^{\star}\langle\, P\,,\,\texttt{[(0,5);(1,6)]}\,,\,3\,\rangle}\,\text{(trans)}}{\langle\, P\,,\,\texttt{[(0,5);(1,5)]}\,,\,0\,\rangle\longrightarrow^{\star}\langle\, P\,,\,\texttt{[(0,5);(1,6)]}\,,\,3\,\rangle}\,\text{(trans)}$$